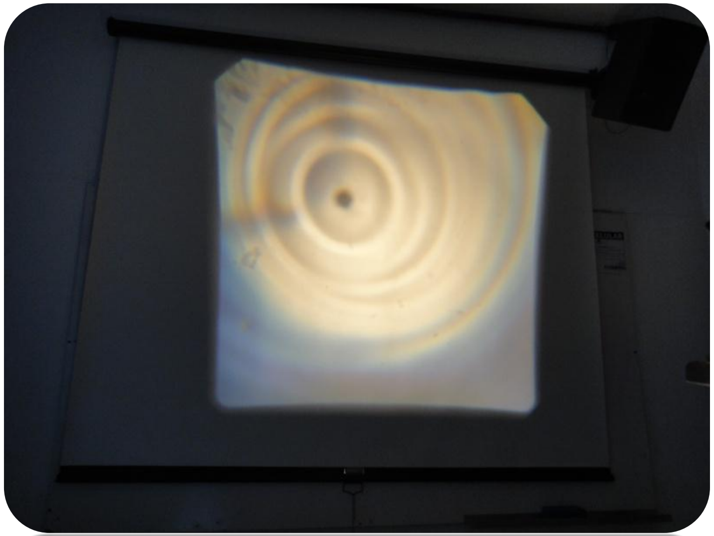
Team of Brazil

Problem 05
Bright waves



reporter:

Bárbara Cruvinel Santiago

Problem 05 Bright waves

Illuminate a water tank. When there are waves on the water surface, you can see bright and dark patterns on the bottom of the tank. Study the relation between the waves and the pattern.

Contents

Introduction
- Caustics
- Image point
- Catastrophe theory
- General features

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- Experimental data
- Theory validation

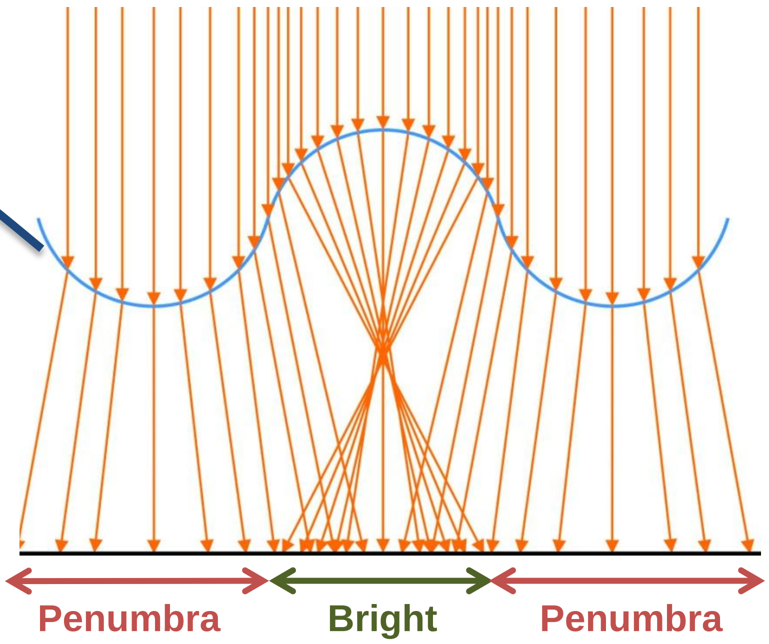
Conclusion
- Analysis
- Comparison

Caustics

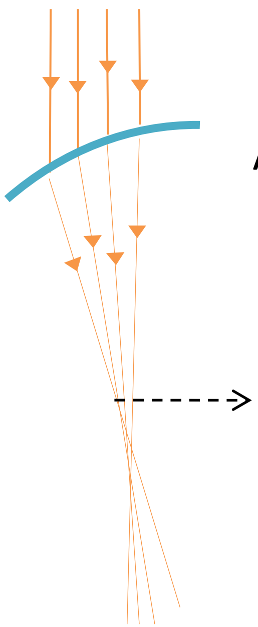
- Envelope of light rays reflected or refracted by a curved surface or object, or the projection of that envelope of rays on another surface.

Waves act like lenses

Caustics are formed

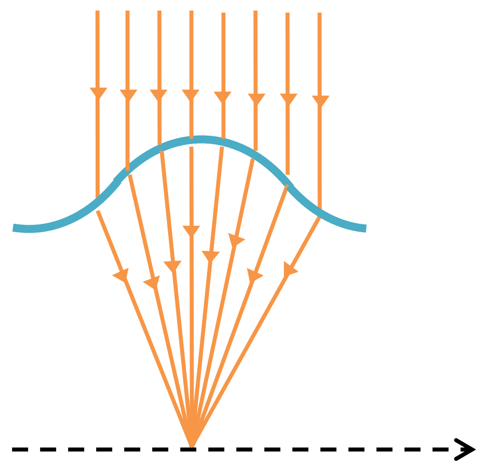


Depth



About 2 or 3 cm. *

Critical depth: 1st ray crosses the 2nd

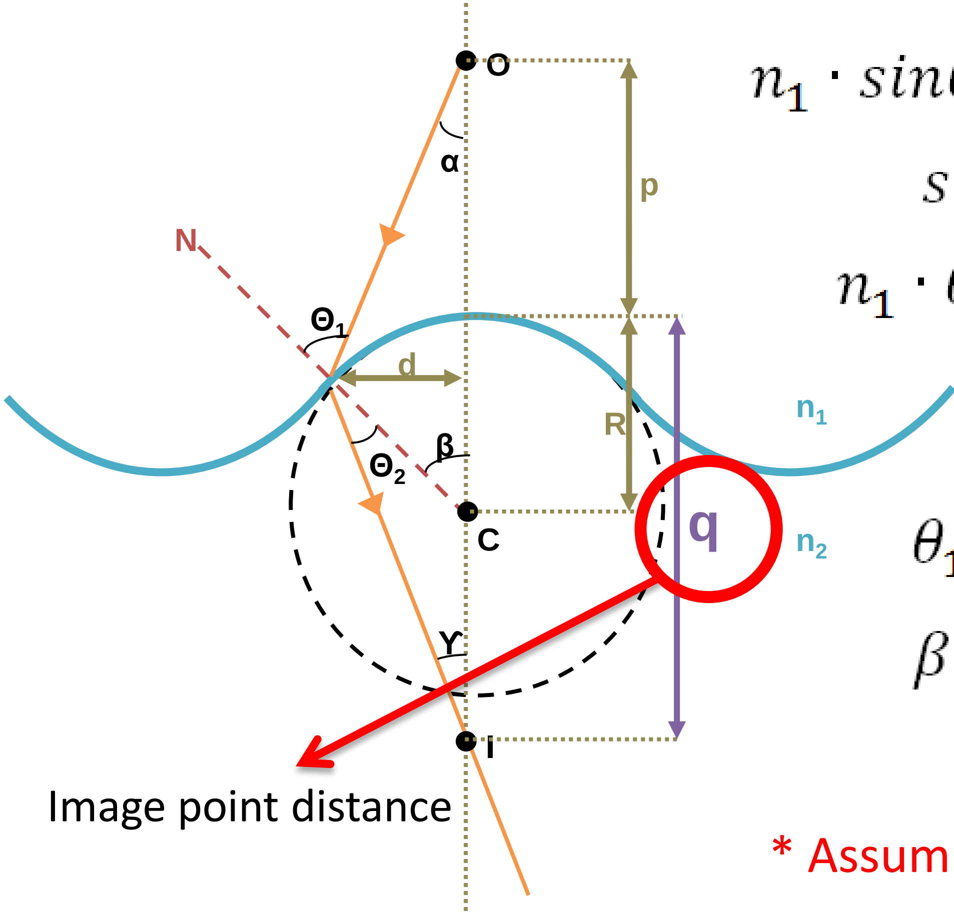


From this distance on, the brightness decreases.

Image point distance: maximum brightness

* WALKER, J. "Shadows Cast on the Bottom of a Pool Are Not Like Other Shadows. Why?". 1988

Depth



$$n_1 \cdot sin\theta_1 = n_2 \cdot sin\theta_2$$

$$sin\theta \cong \theta \text{ *}$$

$$n_1 \cdot \theta_1 = n_2 \cdot \theta_2$$

$$\theta_1 = \alpha + \beta$$

$$\beta = \theta_2 + \gamma$$

$$tan\theta \cong \theta \text{ *}$$

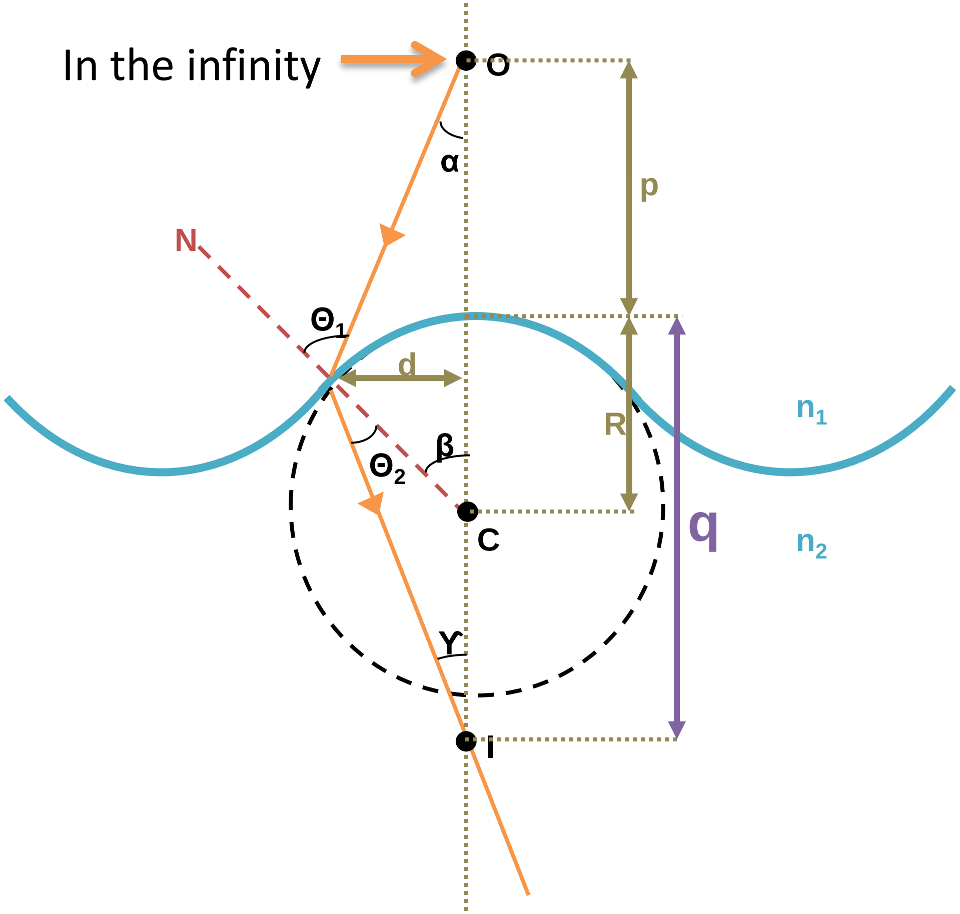
$$tan\alpha = \alpha = \frac{d}{p}$$

$$tan\beta = \beta = \frac{d}{R}$$

$$tan\gamma = \gamma = \frac{d}{q}$$

* Assuming small angles

Depth



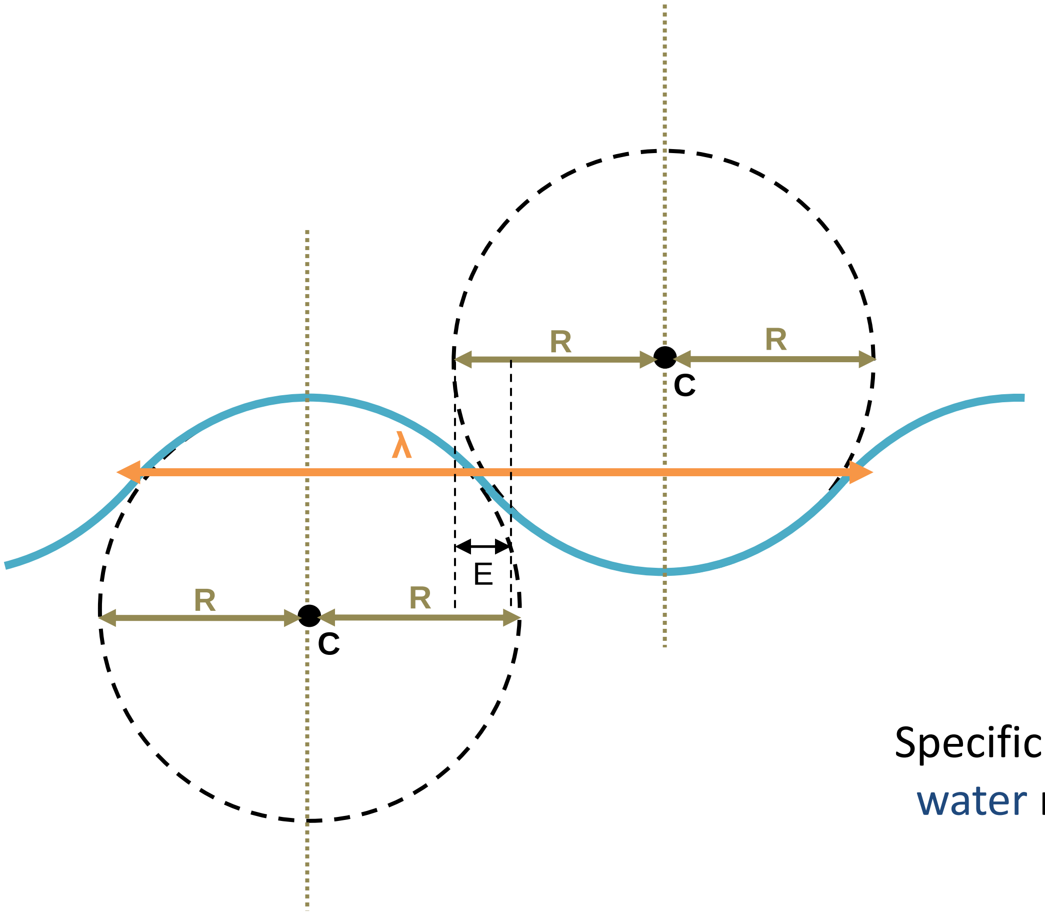
$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$$

$$\frac{n_1}{p} = 0$$

$$\frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$$

$$q = \frac{n_2}{(n_2 - n_1)} R$$

Depth



$$R = \frac{\lambda + E}{4}$$

$$R = \frac{\lambda}{4}$$

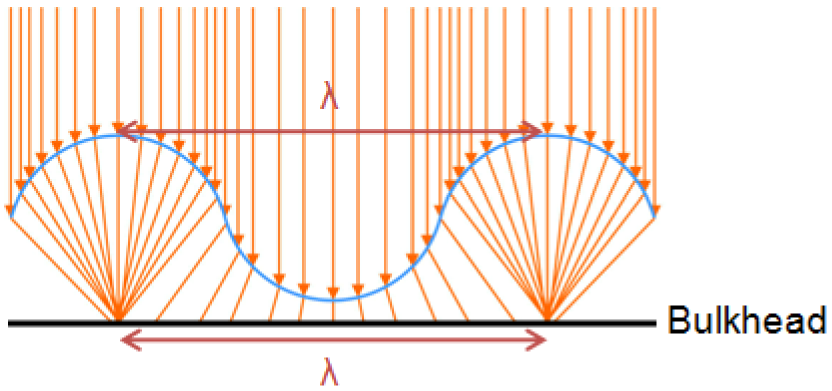
$$q = \frac{n_2 \cdot \lambda}{4(n_2 - n_1)}$$

$$q = \lambda$$

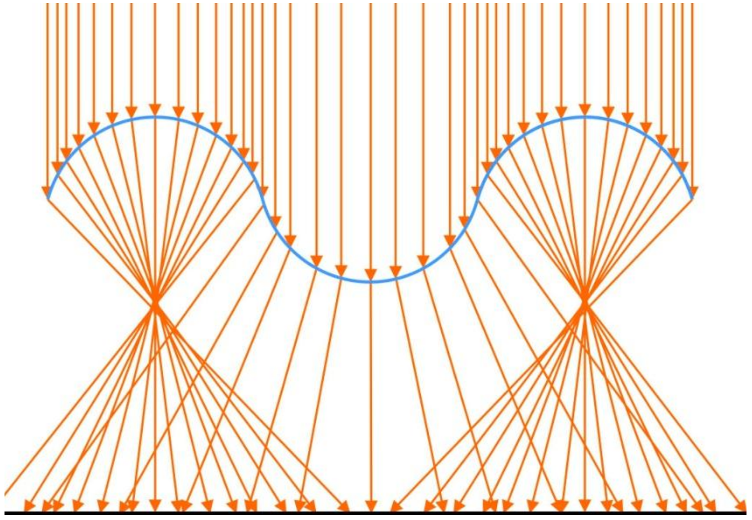
Specific case of water n = 4/3.

Width of the pattern

- Bulkhead in **q**.

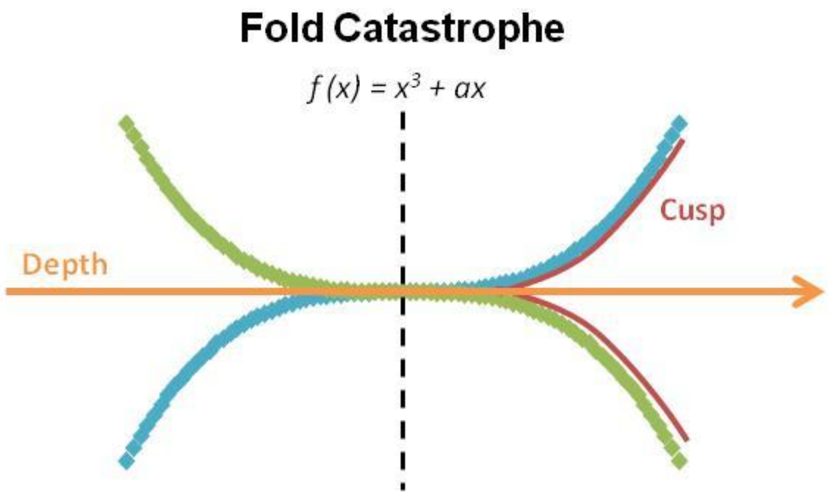


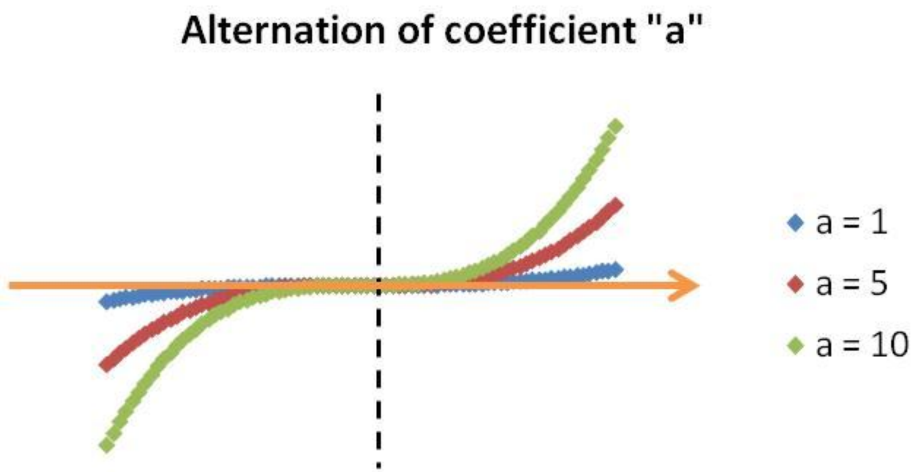
- Other depths: catastrophe theory



Width of the pattern

- Fold catastrophes united by cusps.





$$f(x) = x^3 + ax$$

$$H = x + q$$

$$L = 2 \cdot |f(x)|$$

Varies the catastrophe opening.

Width of the pattern

General expression for bright pattern's width behavior:

$$v = \lambda f$$

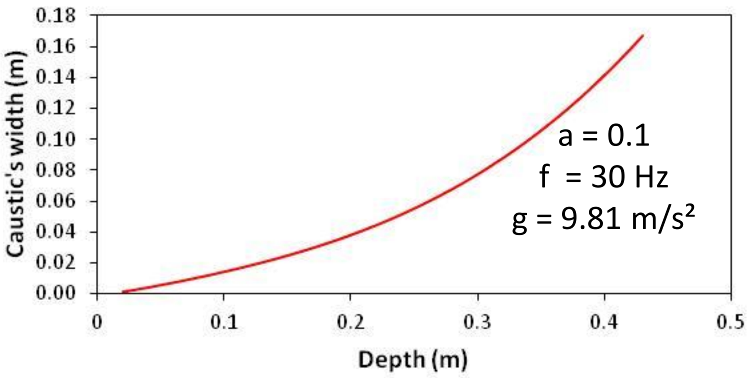
$$v = \sqrt{gH}$$

$$\lambda = \frac{\sqrt{gH}}{f}$$

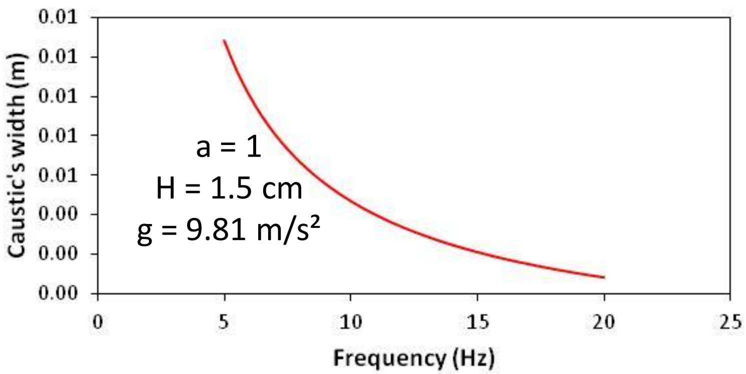
$$q = \frac{\sqrt{gH}}{f}$$

$$L = 2\left[\left(H - \frac{\sqrt{gH}}{f}\right)^3 + a\left(H - \frac{\sqrt{gH}}{f}\right)\right]$$

Depth variation behavior



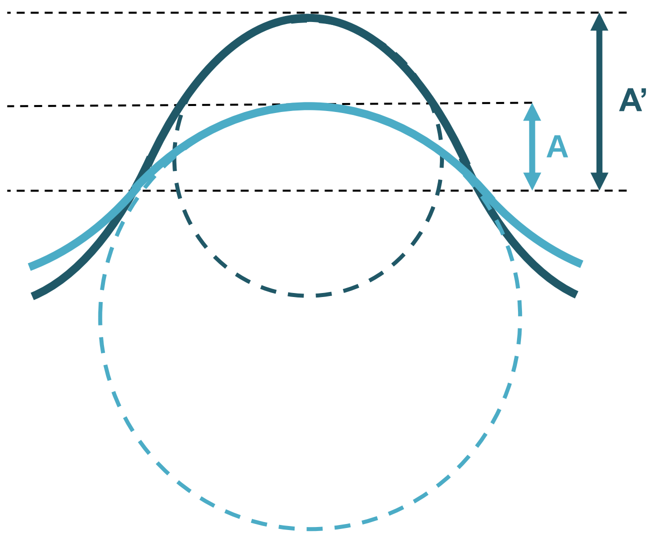
Frequency variation behavior



Other wave's characteristics

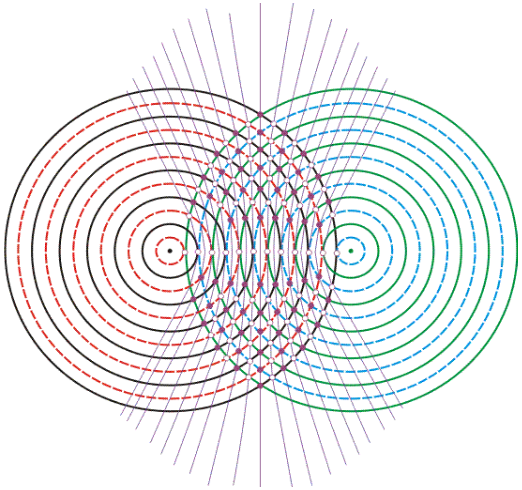
Influences:

- λ: size of the pattern
- A: radius
- T, v and f: periodic interchange between bright and dark patterns.
- Type of wave: circular or linear patterns.



Interference

Two punctual sources:

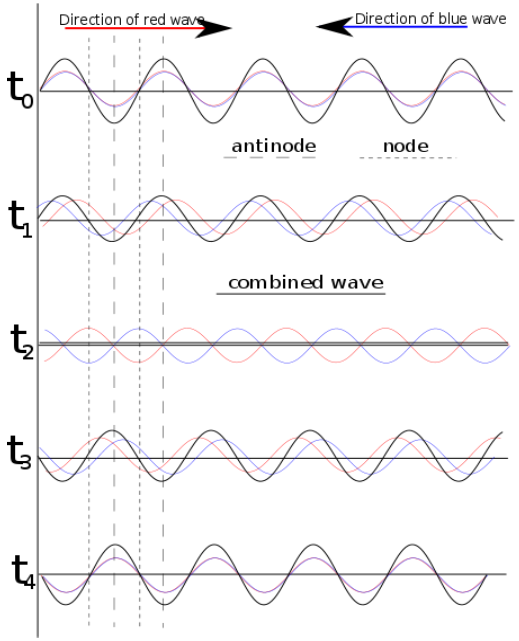


$$n = \frac{\Delta x}{\lambda}$$

- **Constructive:** n = 0, 1; 2; 3...
- **Destructive:** n = 0,5; 1,5; 2,5; 3,5...

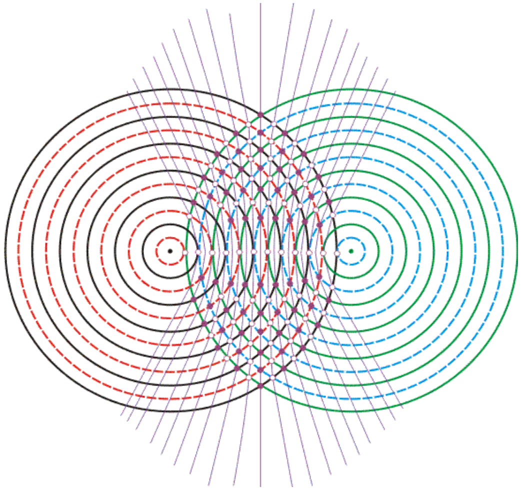
Reflected water waves:

Same frequency and opposite directions.



Interference

Two punctual sources:



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Reflected water waves:

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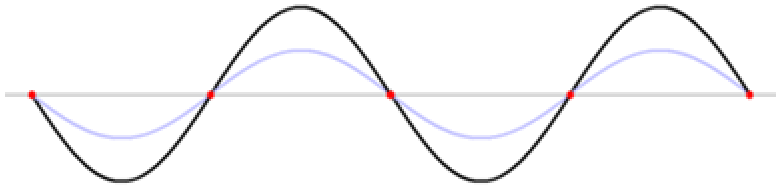


Standing waves are produced

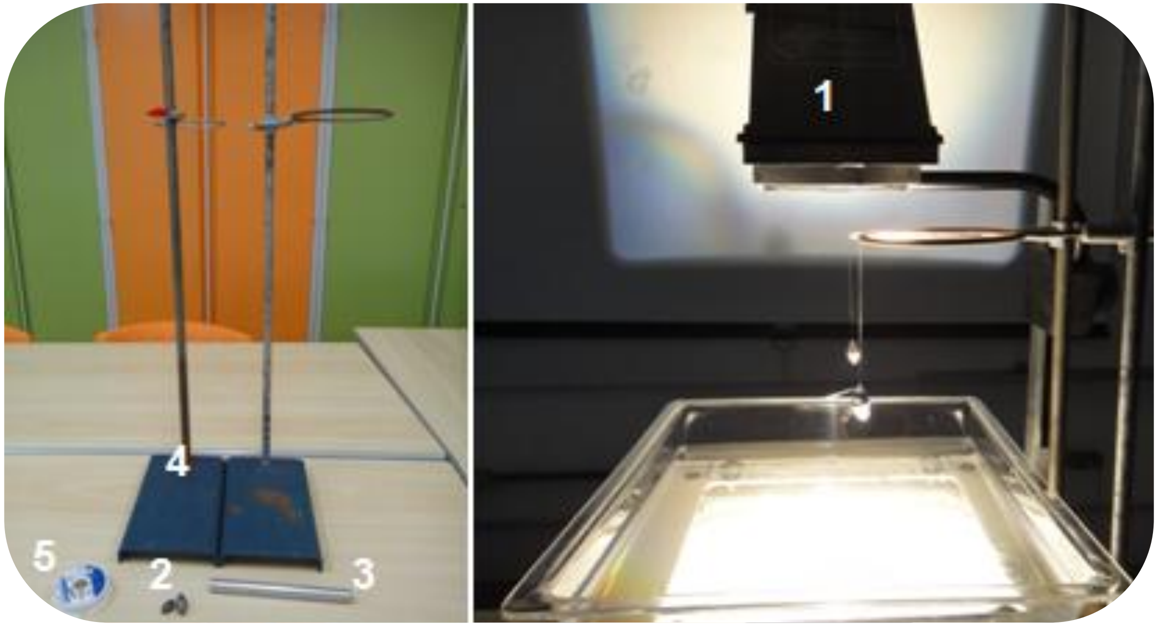
↓

Easier to analyze experimentally.

Material

1. Ripple tank
2. Lead weights
3. Cylinder
4. Metal support
5. Nylon

- Acoustic boxes
- Water
- Aquarium
- Computer
- Wave generator



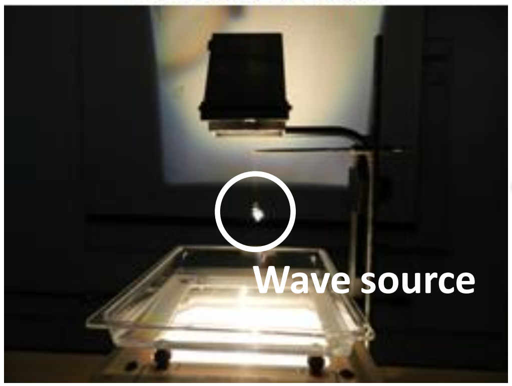
Experimental description

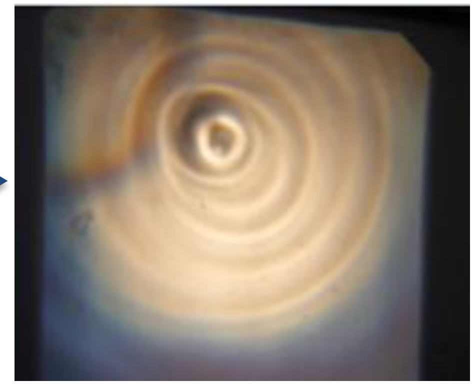
- **Experiment 1:** analysis of the used source.
- **Experiment 2:** bright pattern interference analysis.
- **Experiment 3:** obtaining the coefficient **a** of the catastrophe theory.
- **Experiment 4:** depth variation analysis.
- **Experiment 5:** frequency variation analysis.

Experiment 1: analysis of the used wave source

- **Source**
- Interference
- Coefficient a
- Depth
- Frequency

Punctual source





Extensive source

Patterns differ in geometry

Wave fronts near wave's source

↓

High energy/amplitude

↓

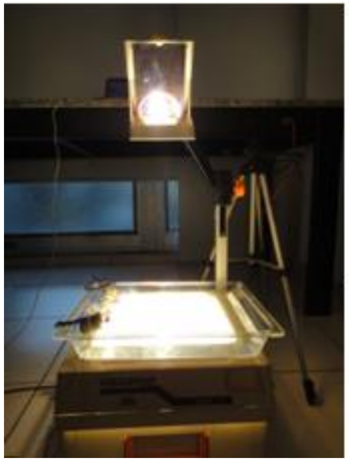
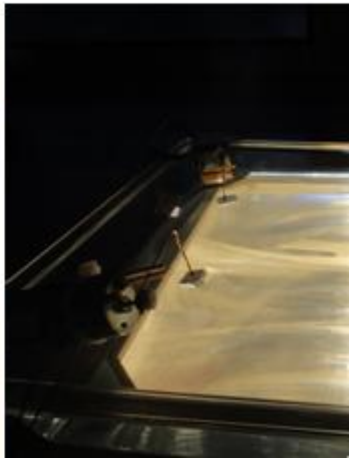
Different "q"

↓

Different brightness

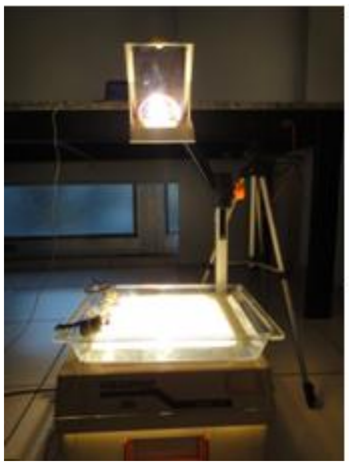
Experiment 2: interference analysis

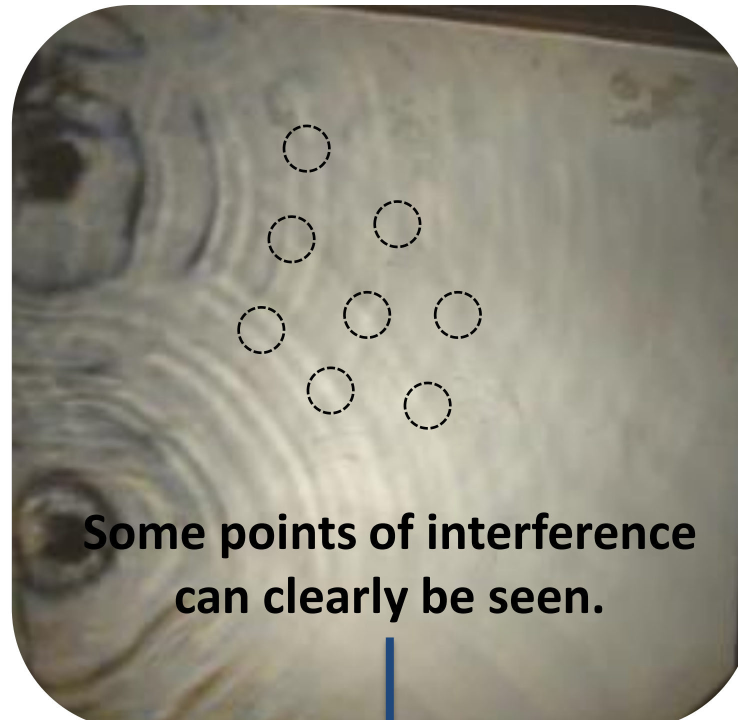
- Source
- Interference
- Coefficient **a**
- Depth
- Frequency


Experiment 2: interference analysis

- Source
- Interference
- Coefficient **a**
- Depth
- Frequency

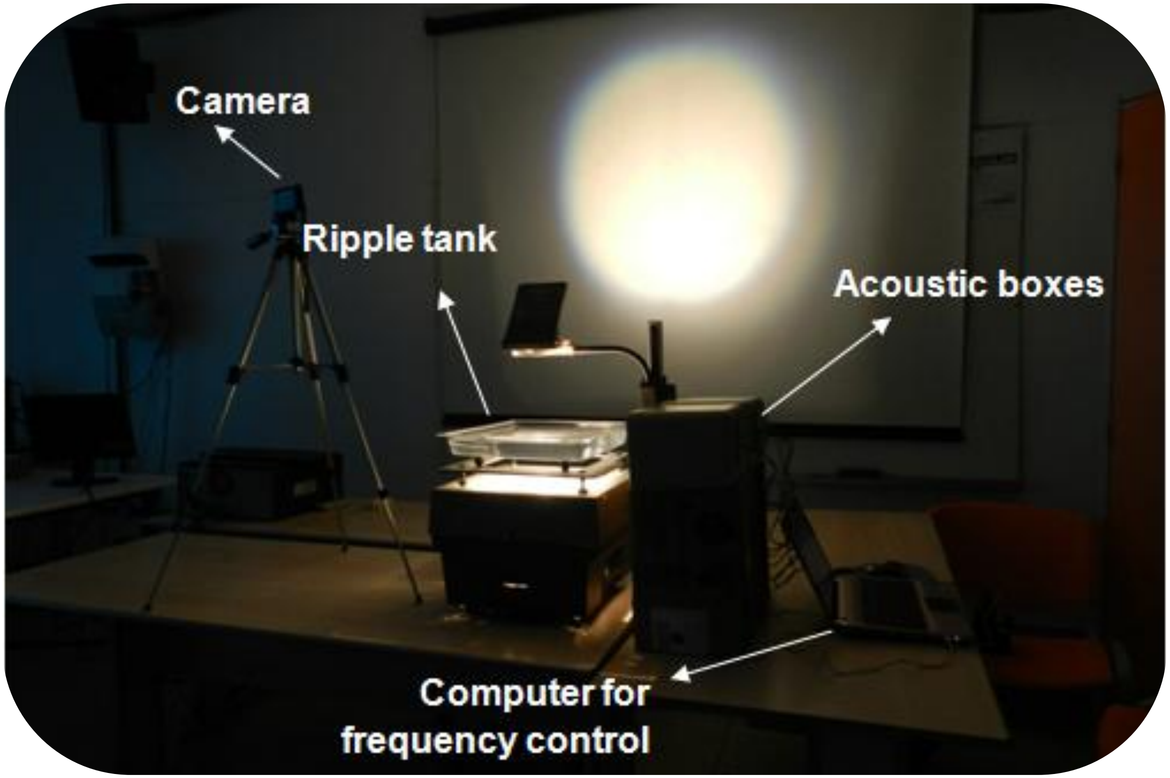




Change in the pattern's geometry

Experiment 3: coefficient "a"

- Source
- Interference
- Coefficient a
- Depth
- Frequency



Experiment 3: coefficient “a”

Source
Interference
Coefficient a
Depth
Frequency

	Caustic width (cm)	a
1st	0.10	1.41E-03
2nd	0.11	1.56E-03
3rd	0.12	1.70E-03
4th	0.10	1.41E-03
5th	0.11	1.56E-03
6th	0.10	1.41E-03
7th	0.10	1.41E-03
Average	**0.11**	**1.50E-03**
Standard deviation	0.008	1.11E-04

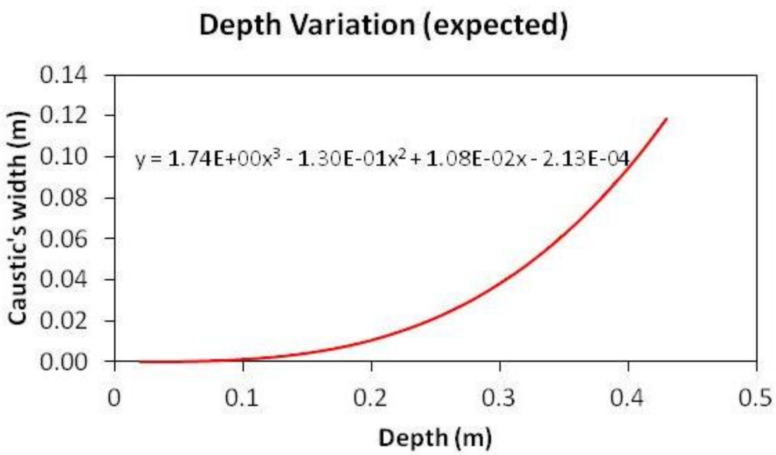
a = catastrophe theory coefficient (caustic opening)

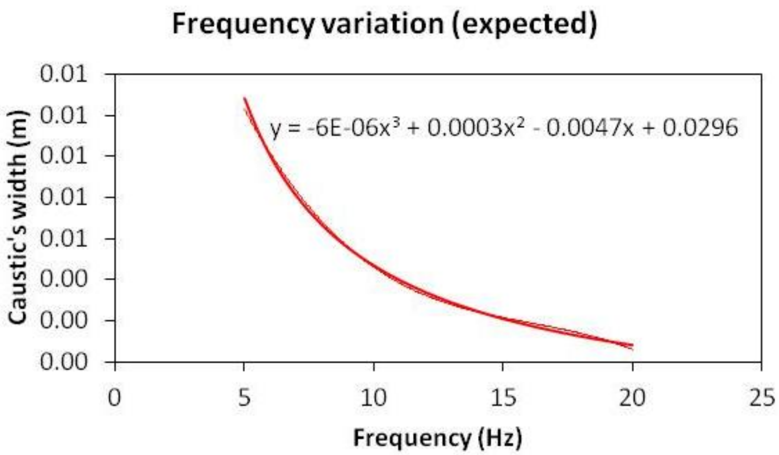
Experiment 3: coefficient "a"

- Source
- Interference
- Coefficient a
- Depth
- Frequency

Expected behavior applying "a" gotten previously:

General equation applied





Used in comparison (next experiments)

Experiment 4: depth variation

- Source
- Interference
- Coefficient **a**
- Depth
- Frequency

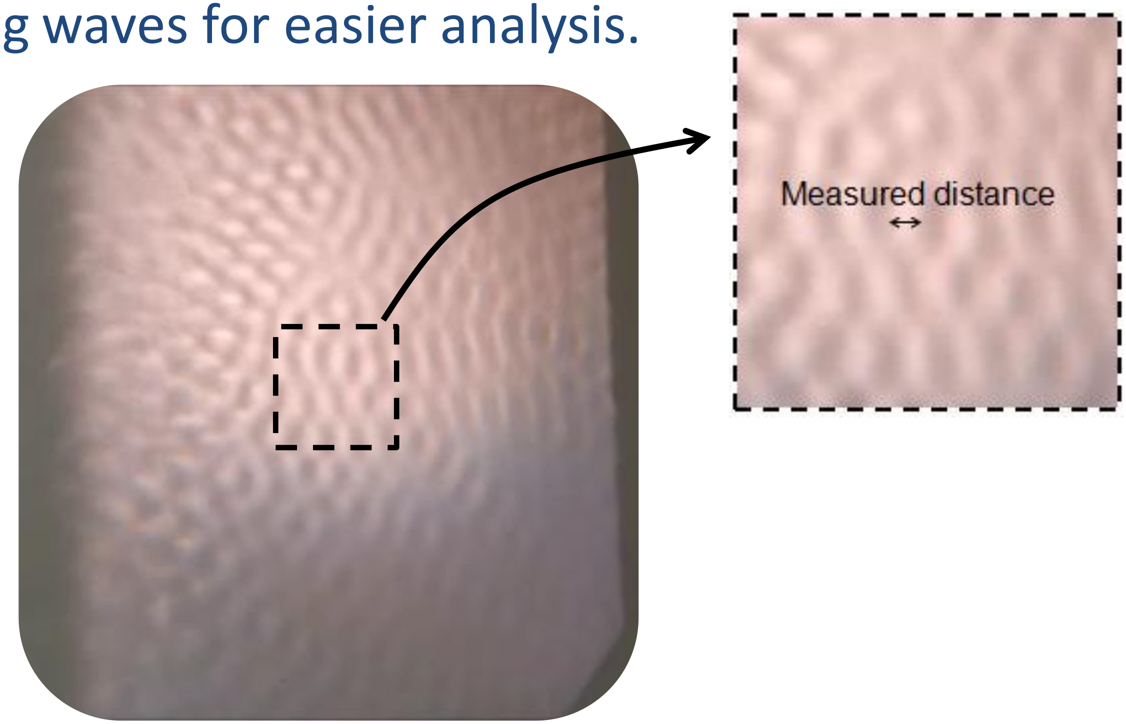
Experiment 4: depth variation

- Source
- Interference
- Coefficient **a**
- Depth
- Frequency

Experiment 4: depth variation

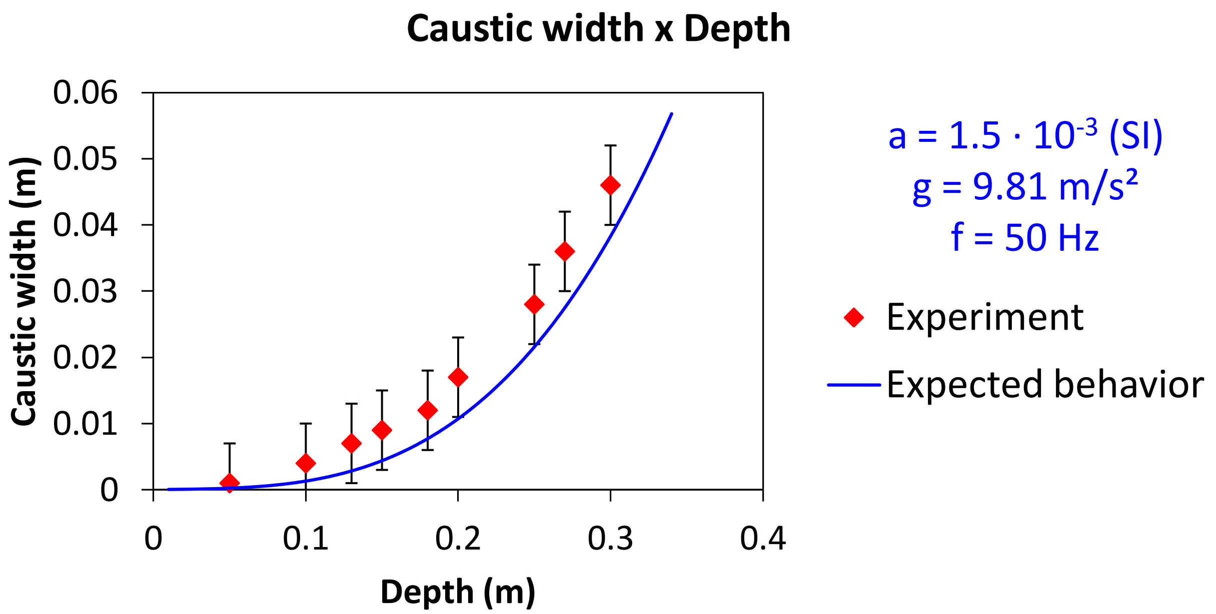
- Source
- Interference
- Coefficient **a**
- Depth
- Frequency

Interference by reflection produce standing waves for easier analysis.



Experiment 4: depth variation

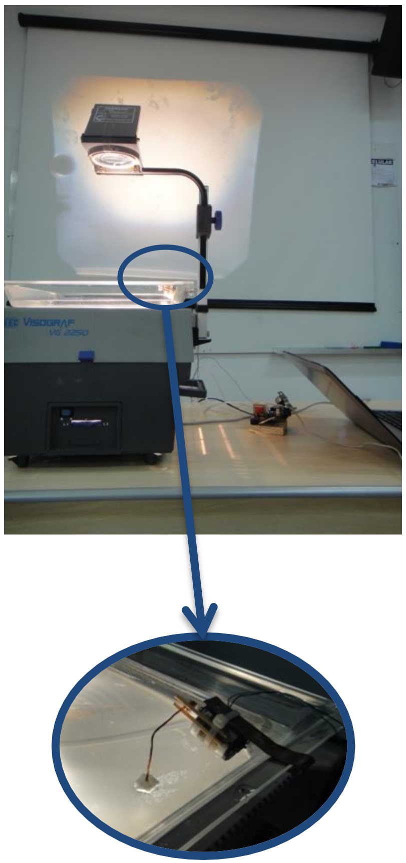
- Source
- Interference
- Coefficient **a**
- Depth
- Frequency

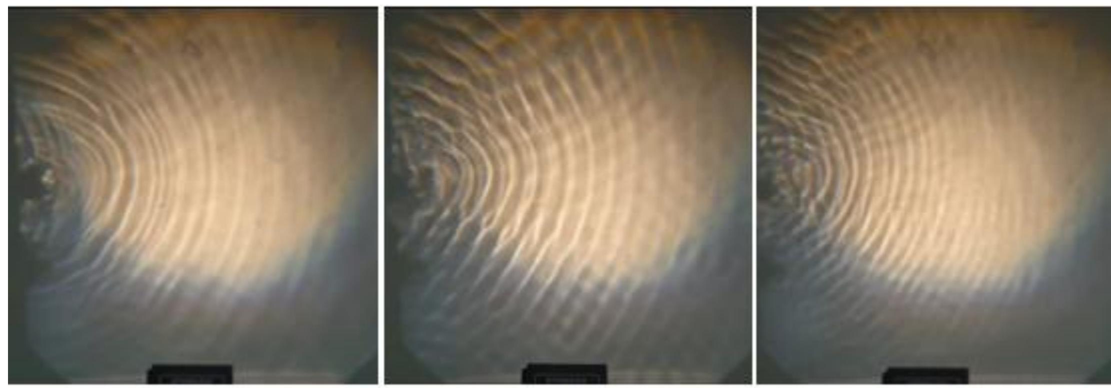


Standard deviation = 0.006 m

Experiment 5: frequency variation

- Source
- Interference
- Coefficient **a**
- Depth
- Frequency



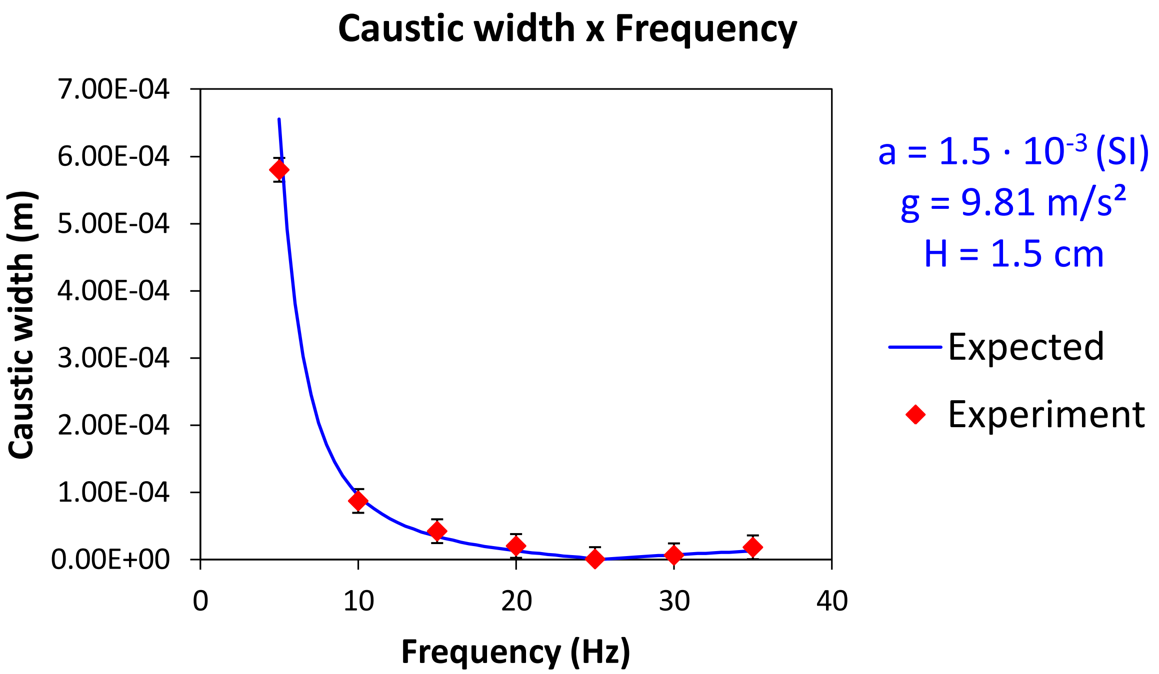


f = 5 Hz   f = 15 Hz   f = 25 Hz

Decreasing width with increasing frequency

Experiment 5: frequency variation

- Source
- Interference
- Coefficient **a**
- Depth
- Frequency



Standard deviation = $1.77 \cdot 10^{-5}$ m

Conclusion

- The patterns are formed by caustics.

depth/wavelength/amplitude/frequency	→	distance “q”/bright pattern’s length
interference	→	pattern‘s length and/or geometry
type of wave/wave source	→	pattern‘s geometry
period/frequency/velocity	→	bright/dark periodic interchange

- $a \approx 1.5 \cdot 10^{-3}$ (exp. 3) and characterizes the opening of the caustic. Predictions with this coefficient fitted experimental data.
- Behavior of the patterns in function of the depth and frequency were showed and complies with the expected behavior (exp. 4/5).

References

- 1. SERWAY, Raymond A.; JR., John W. Jewett – Princípios de Física – Volume 4 – Óptica e Física Moderna – Thomson Learning – 2007
- 2. http://optica.machorro.net/Optica/SciAm/PoolShadows/1988-07-fs.html
- 3. http://dic.busca.uol.com.br/result.html?t=10&ref=homeuol&ad=on&q=c%E1ustica&group=0
- 4. http://www.physicstutorials.org/home/waves/water-waves
- 5. http://www.atoptics.co.uk/fz535.htm
- 6. http://mathworld.wolfram.com/FoldCatastrophe.html
- 7. http://www.iag.usp.br/~marcelo/agg232/
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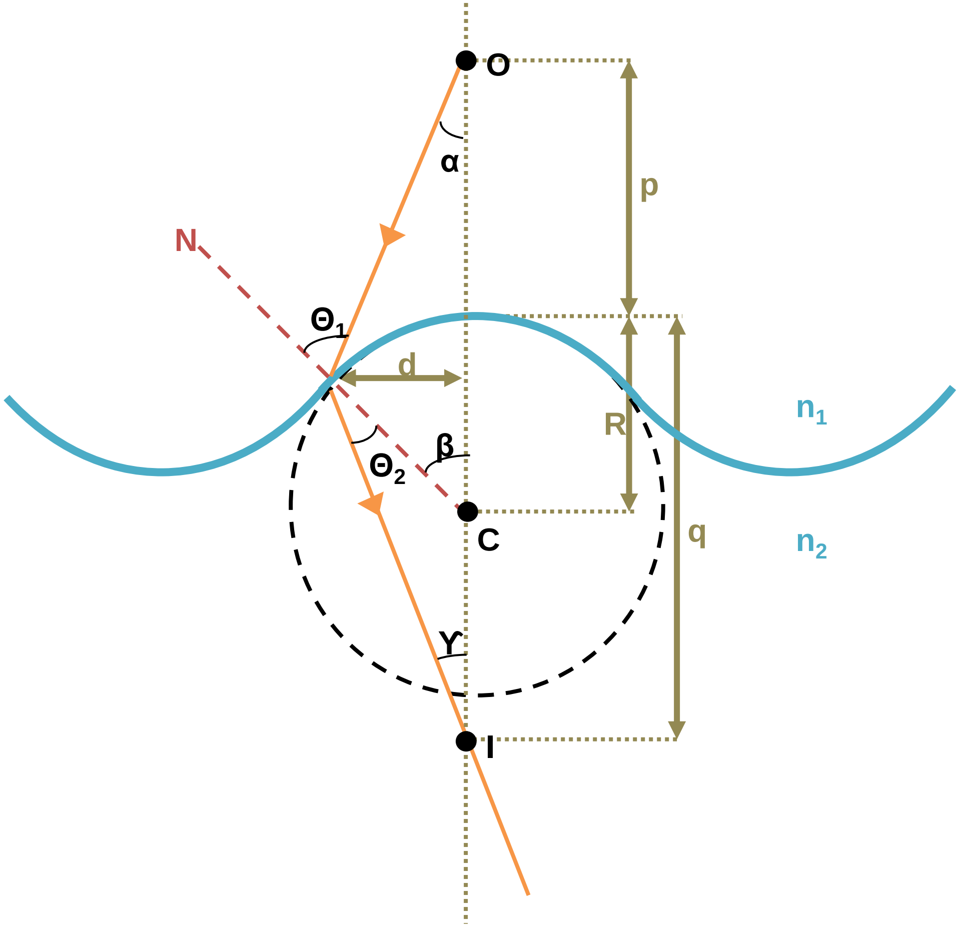
- 9. http://www.infoescola.com/fisica/onda-estacionaria/

Thank you!

Depth



$$n_1 \cdot sen\theta_1 = n_2 \cdot sen\theta_2$$

$$sen\theta = \theta$$

$$n_1 \cdot \theta_1 = n_2 \cdot \theta_2$$

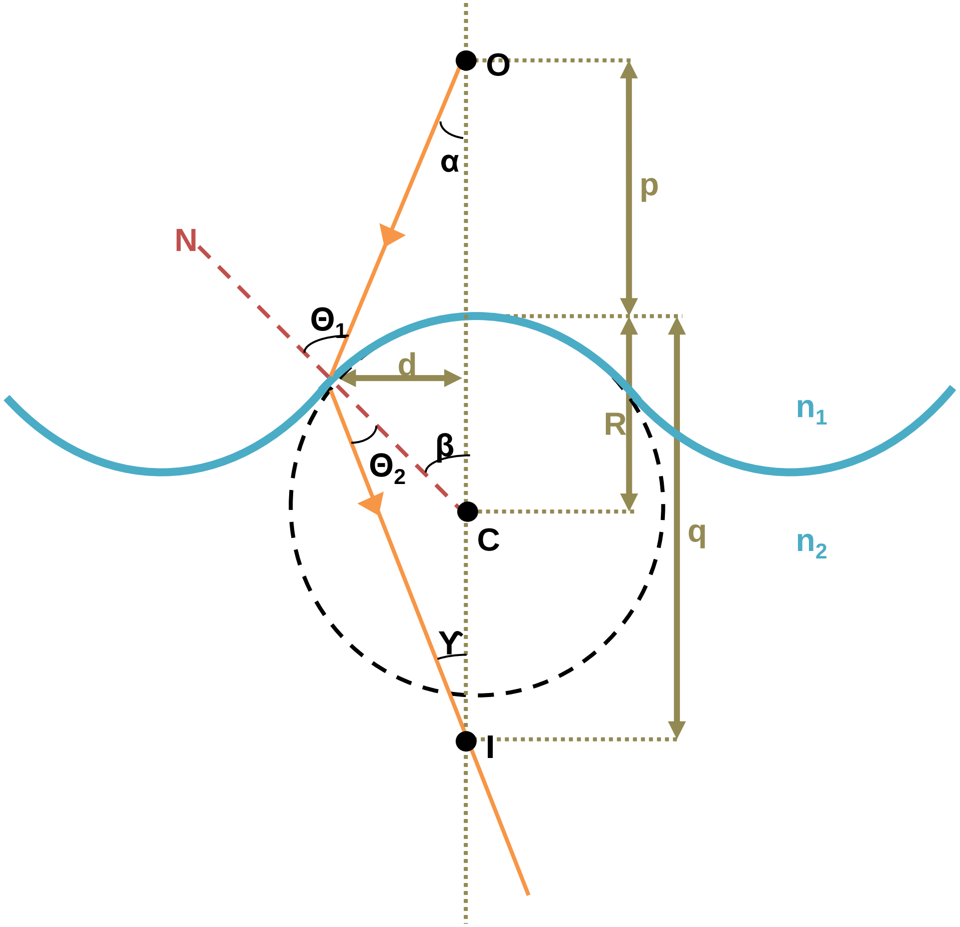
$$\theta_1 = \alpha + \beta$$

$$\beta = \theta_2 + \gamma$$

$$n_1 \cdot (\alpha + \beta) = n_2 \cdot (\beta - \gamma)$$

$$n_1 \cdot \alpha + n_2 \cdot \gamma = (n_2 - n_1) \cdot \beta$$

Depth



$$n_1 \cdot \alpha + n_2 \cdot \gamma = (n_2 - n_1) \cdot \beta$$

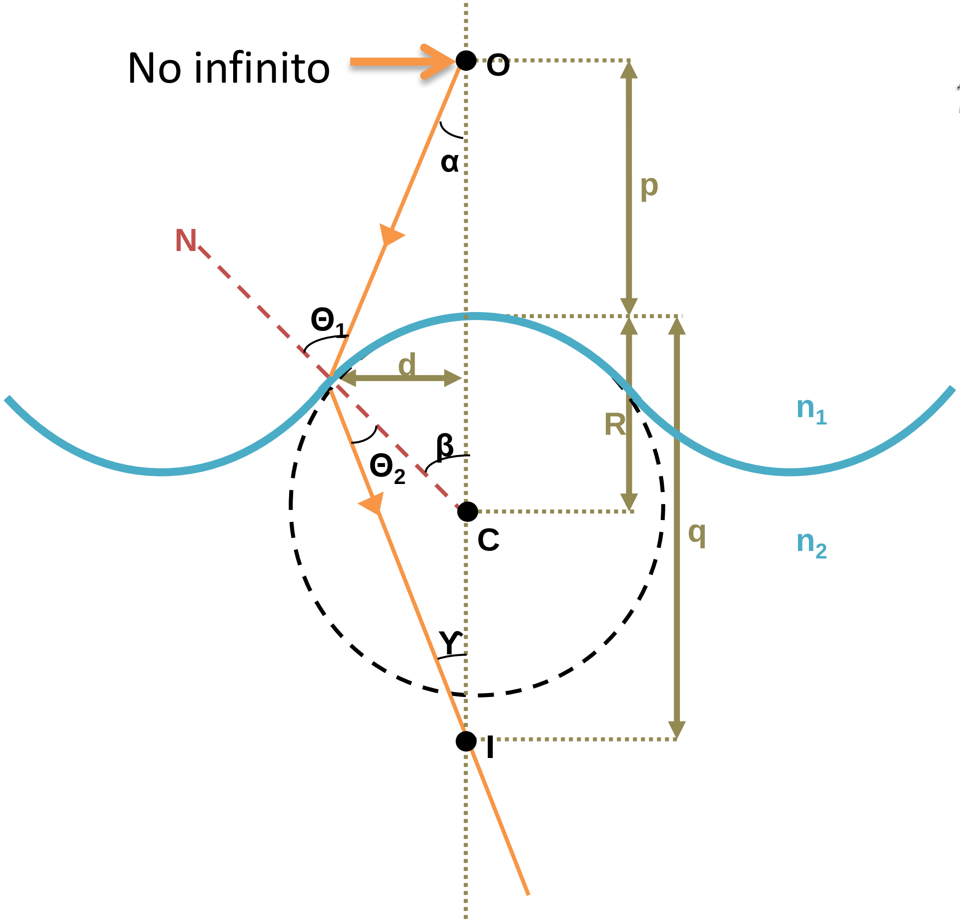
$$tg\theta = \theta$$

$$tg\alpha = \alpha = \frac{d}{p}$$

$$tg\beta = \beta = \frac{d}{R}$$

$$tg\gamma = \gamma = \frac{d}{q}$$

Depth



$$n_1 \cdot \frac{d}{p} + n_2 \cdot \frac{d}{q} = (n_2 - n_1) \cdot \frac{d}{R}$$

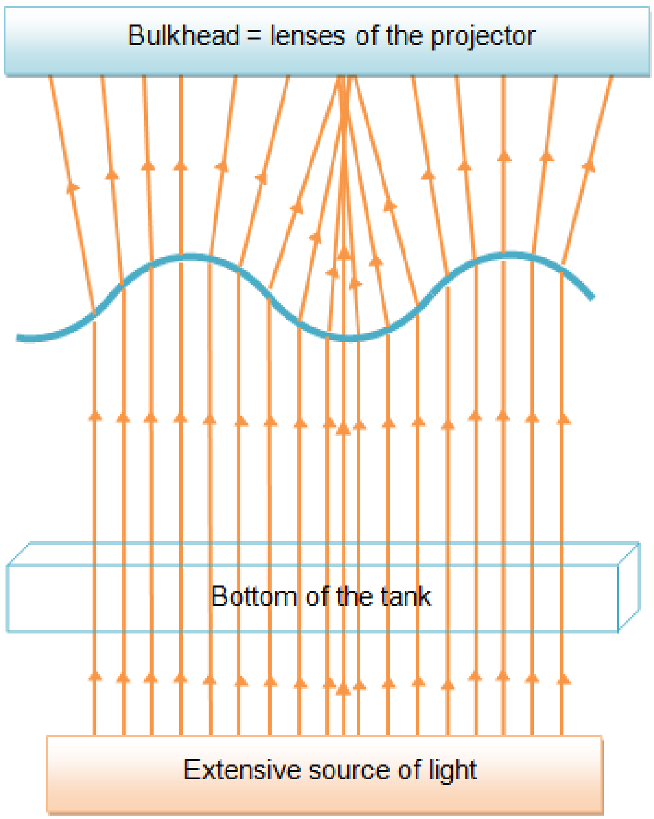
$$\frac{\boldsymbol{n_1}}{\boldsymbol{p}} + \frac{\boldsymbol{n_2}}{\boldsymbol{q}} = \frac{(\boldsymbol{n_2} - \boldsymbol{n_1})}{\boldsymbol{R}}$$

$$\frac{n_1}{p} = 0$$

$$\frac{n_2}{q} = \frac{(n_2 - n_1)}{R}$$

$$\boldsymbol{q} = \frac{\boldsymbol{n_2}}{(\boldsymbol{n_2} - \boldsymbol{n_1})}\boldsymbol{R}$$

Differences when the tank is lit by its bottom



- One more refraction.
- Role of water lenses is inverted.
- Convergence above the waves.
- The bulkhead is the lenses of the projector.
- The behavior in function of the depth continues the same (also shown by exp. 4).

Deduction of length in function of depth

$$v = \lambda f$$
$$v = \sqrt{gH}$$
$$\lambda = \frac{\sqrt{gH}}{f} \rightarrow q = \frac{\sqrt{gH}}{f}$$

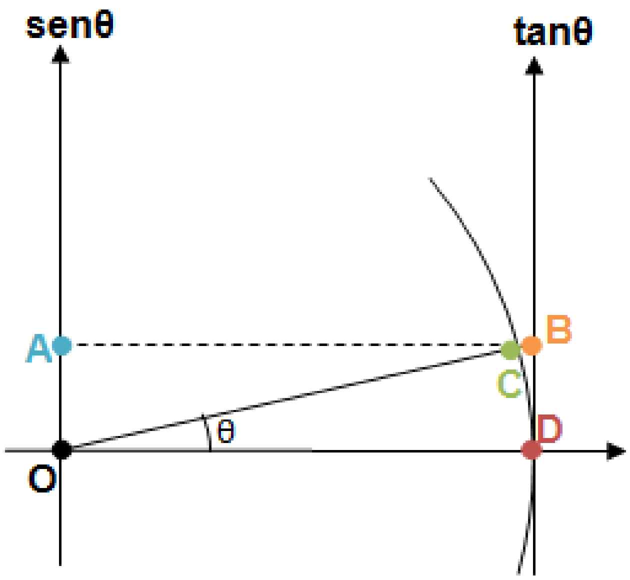
$$H = x + q$$

$$f(x) = x^3 + ax$$

$$L = 2 \cdot |f(x)|$$

$$L = 2\left[\left(H - \frac{\sqrt{gH}}{f}\right)^3 + a\left(H - \frac{\sqrt{gH}}{f}\right)\right]$$

Trigonometric approximations

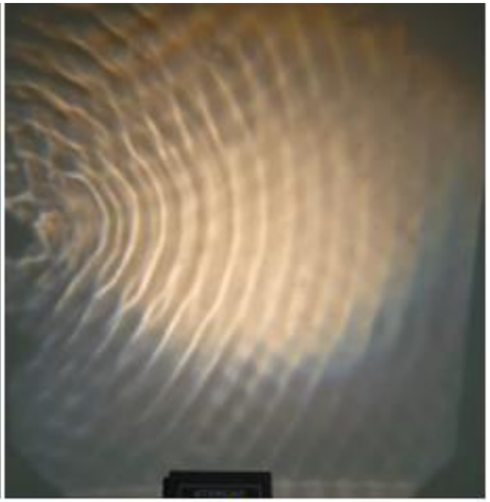


$$\sin\theta = \tan\theta = \theta$$

Different fluids

- **Refraction index:** convergence is altered and, thus, **q** is altered.
- **Absorbance:** bright pattern intensity changes.
- **Viscosity:** wave features, as wave length, velocity, frequency etc are altered.

Oil



Water

Capillary waves

- Are generated by surface tension effects mainly.
- With a high amplitude, we consider gravitaonal wave, for the high energy used.
- Software for our expriments: Audacity
  - The maximum amplitude (1 in arbitrary units) of the software was used.