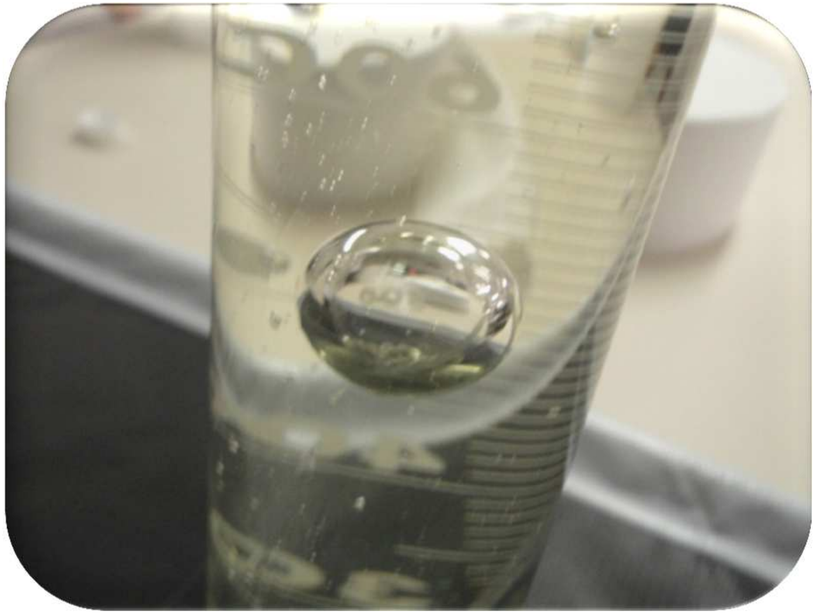
Problem 16
Rising bubble



reporter:

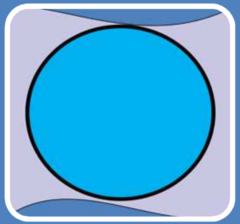
Ibraim Rebouças

Problem 16
Rising bubble

A vertical tube is filled with a viscous fluid. On the bottom of the tube, there is a large air bubble. Study the bubble rising from the bottom to the surface.

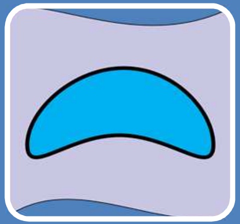
Contents

Kinds of bubbles



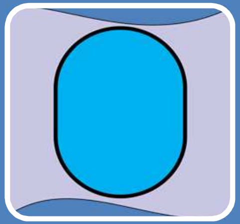
Spherical bubble

- Very viscous fluids.
- Governed by stokes low



Spherical cap bubble

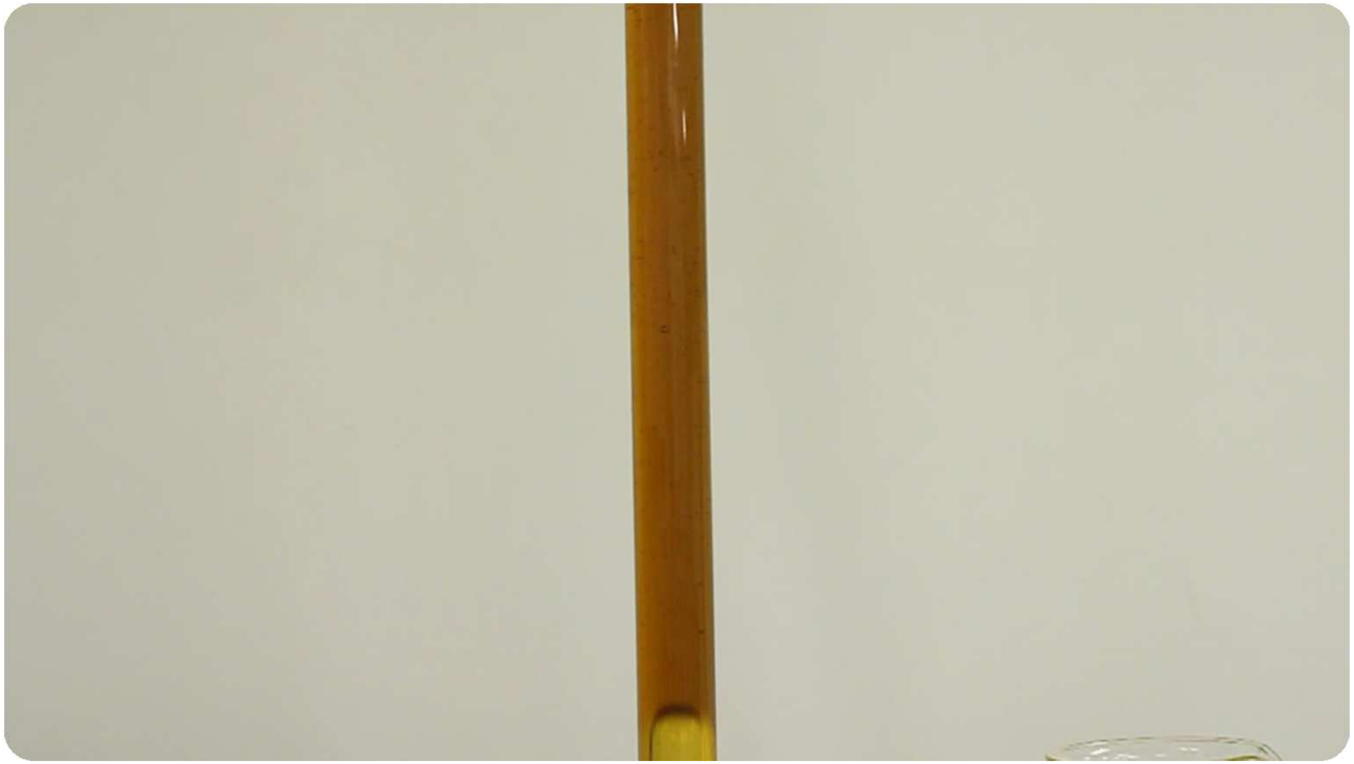
- Stable to moderated viscous fluids.



Taylor's bubble

- Height bigger than the diameter of the pipe.
- Constant velocity for all volumes.

The phenomenon



Viscosity

- It’s the measurement of the resistance to a shear stress.

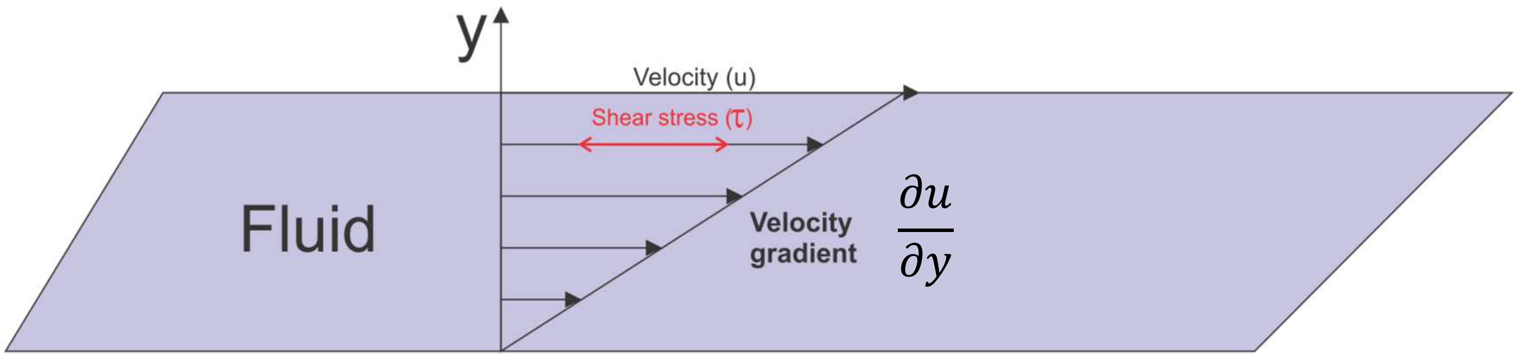
The viscosity is measured by a coefficient.

$$\tau \propto \frac{\partial u}{\partial y} \Rightarrow \tau = \mu \frac{\partial u}{\partial y}$$

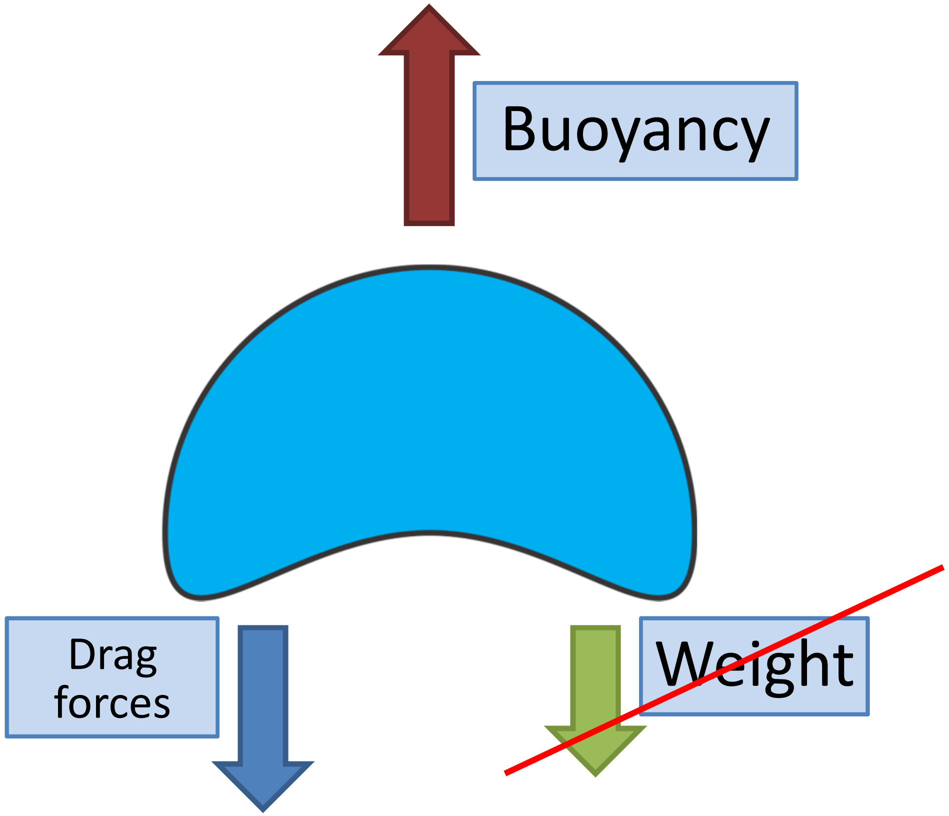
μ is called absolute viscosity coefficient.

Kinematic viscosity $\nu = \frac{\mu}{\rho}$

ρ is de density of the fluid



Forces acting on the bubble



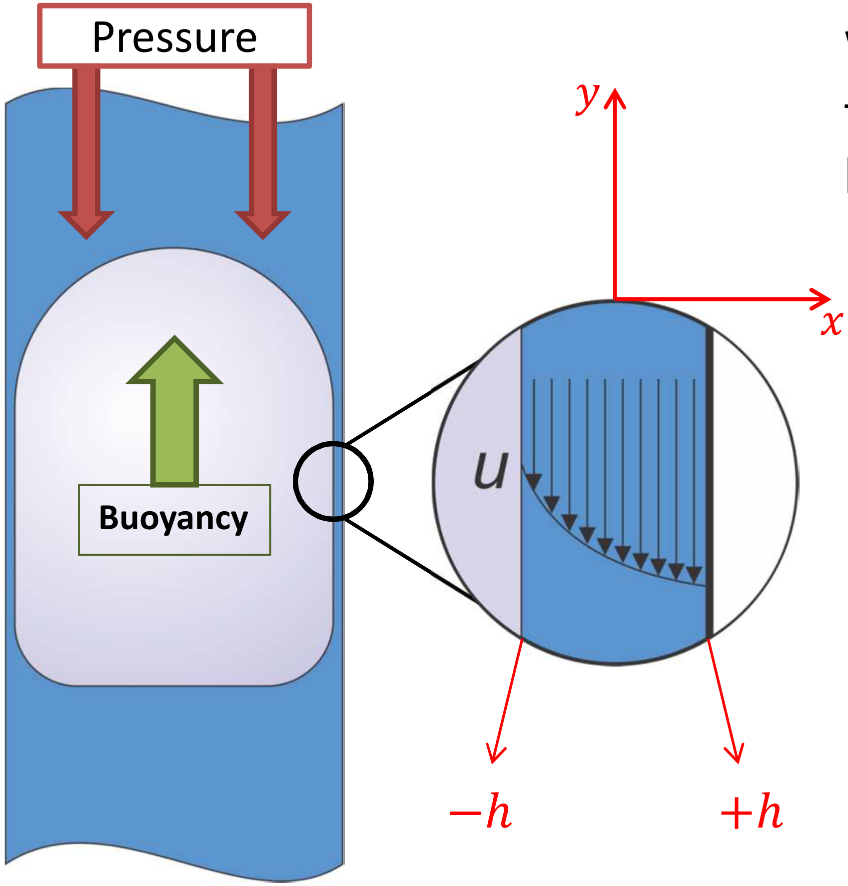
Forces acting on the bubble

$$\overbrace{F_d = \frac{1}{2} A U^2 C_d}^{Newton's\ equation}$$

$$\overbrace{F_d = -bv}^{Stoke's\ drag}$$

Both are wrong, because they do not consider the boundary effects.

Border of the bubble

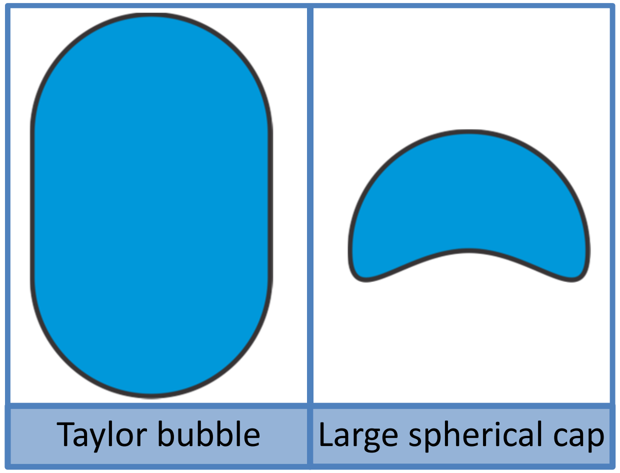


We must have conservation of mass for the liquid ahead and behind the bubble.

$$\mu \frac{d^2 u}{d x^2} = \frac{\partial p}{\partial y} = \text{constant} < 0$$

$$u = -\frac{dp}{dy}\frac{h^2}{2\mu}\left(1 - \left[\frac{x}{h}\right]^2\right)$$

Large spherical cap or Taylor bubble velocity



- The velocity is constant and does not depend on the volume of the bubble.

For Taylor bubble or large spherical cap bubbles, the velocity can be described by the following equation:

$$\frac{U}{\sqrt{gD}} = -\frac{8}{3}\frac{\nu}{\sqrt{gD^3}} + \frac{\sqrt{2}}{3}\left(1 + \frac{32\nu^2}{gD^3}\right)^{1/2}$$

Terminal velocity (m/s)

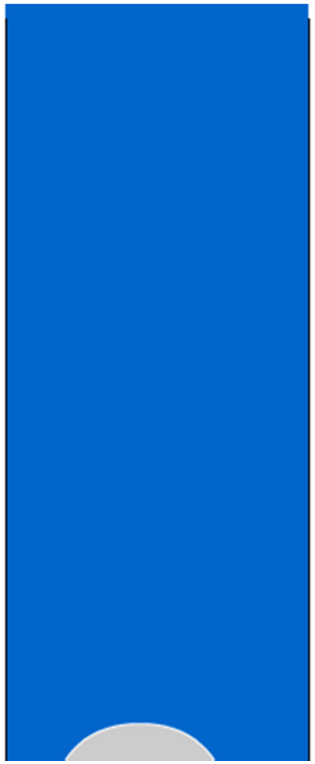
Diameter of the bubble

For low viscosity fluids(e.g. Water), the equation can be written as:

$$U = \frac{\sqrt{2}}{3}\sqrt{gD}$$

Spherical cap bubbles

- Diameter smaller than the tube



→ Lateral oscillation

Wave length

$$\lambda_c = 2\pi L_c$$

Capillary length

$$L_c = \sqrt{\frac{\gamma}{\Delta\rho g}}$$

$$L_c = \sqrt{\frac{\gamma}{\Delta\rho g}}$$

Surface tension coefficient (γ)

Difference of density (air-liquid) ($\Delta\rho$)

Drag coefficient

$$C_d = 6 + \frac{32}{R_e}$$

$$R_e = \frac{UD}{\nu}$$

Drag force

$$F_d = C_d \frac{\pi}{2} \rho U^2 R^2$$

Rise velocity of spherical cap

At the equilibrium: $F_d = \mathrm{B}$

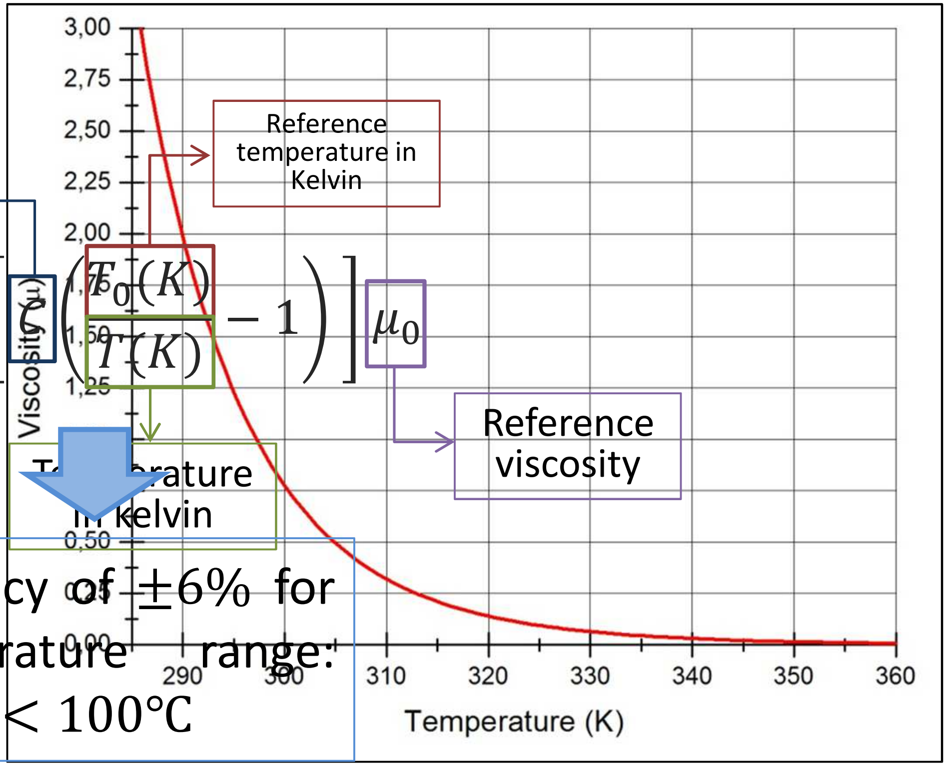
$$\begin{cases} F_d = C_d \dfrac{\pi}{2} \rho U^2 R^2 \\ B = \rho g V \end{cases}$$

$$\Rightarrow \quad C_d \frac{\pi}{2} \rho U^2 R^2 = \rho g V$$

Finally: $$\boxed{U\left(\frac{32\nu}{D} + 6U\right) = 2gV}$$

Influence of temperature

- The temperature acts changing the viscosity of the fluid and the density of the air inside the bubble.



$$\mu \approx exp\left[C\left(\frac{T_0(K)}{T(K)} - 1\right)\right]\mu_0$$

Viscosity parameter

Reference temperature in Kelvin

Temperature in kelvin

Reference viscosity

Accuracy of $\pm 6\%$ for temperature range: $0 < T < 100°\text{C}$

Experimental description

Experiment 1	• Vary the volume of the bubble
Experiment 2	• Vary the temperature
Experiment 3	• Vary the diameter of the tube
Experiment 4	• Influence of surfactant

Materials

- Burette 50 ml
- Pipette 10 ml
- Glass tube 250 ml
- Syringe 10 ml
- Rubber tube
- Camera
- Computer software's

Experiment 1: Variation of the volume

Volume

Temperature

Diameter

Tube diameter: 15 mm
Syringe: 20 ml
Fluid: Glycerin
Temperature: 20°C

Source of errors:

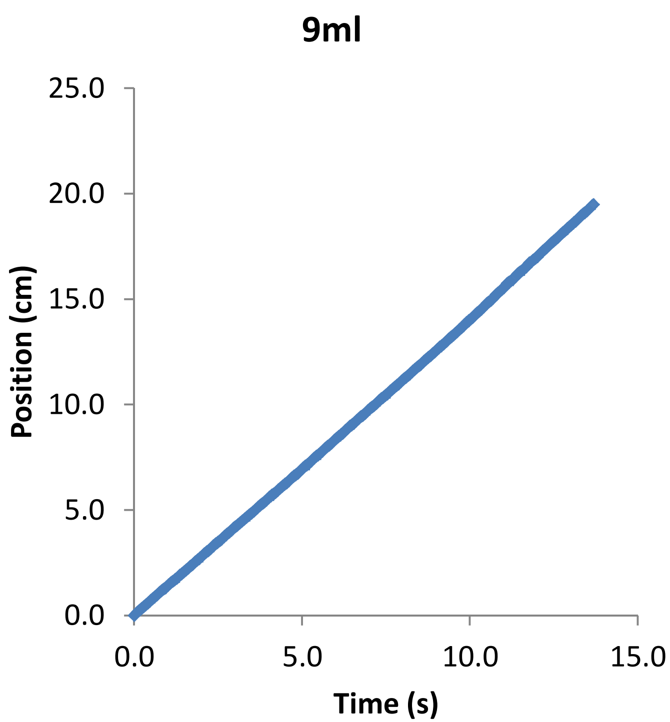
- Measuring ruler and conversion scale: $\pm 0.5mm$
- Volume of the syringe: $\pm 0.5\ ml$

Experiment 1: Variation of the volume

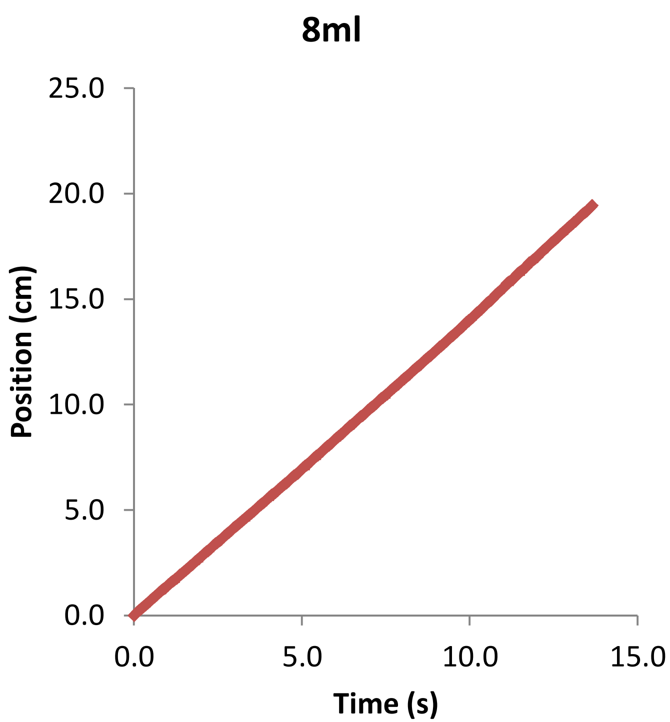
Volume

Temperature

Diameter



$$Vm = 0.01425\ \text{m/s}$$



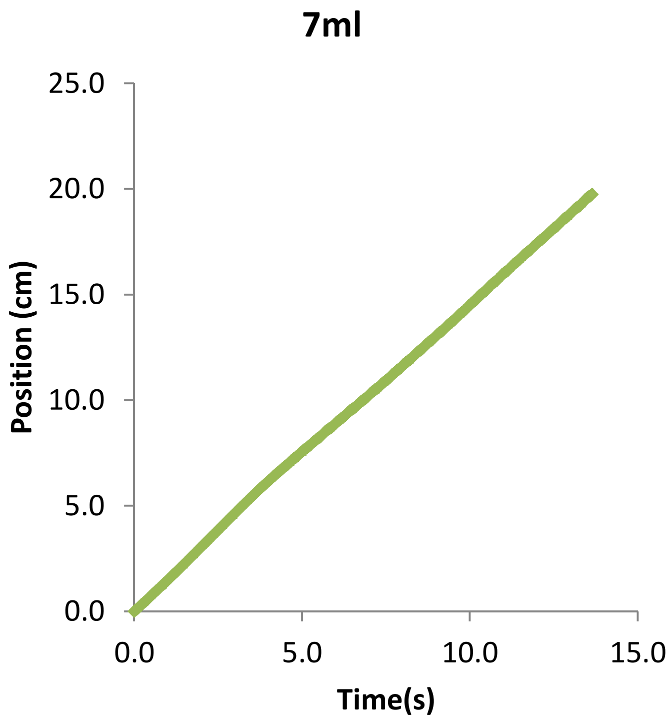
$$Vm = 0.01449\ \text{m/s}$$

Experiment 1: Variation of the volume

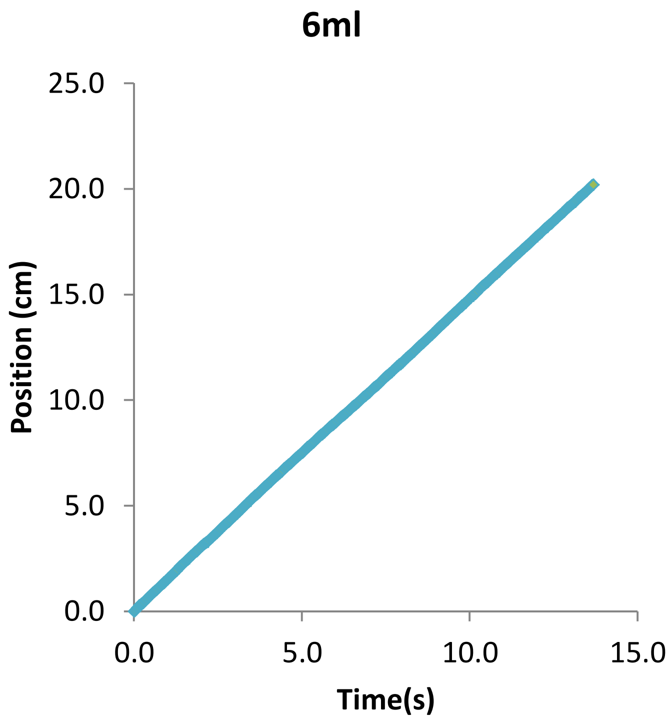
Volume

Temperature

Diameter



$Vm = 0.01445$ m/s



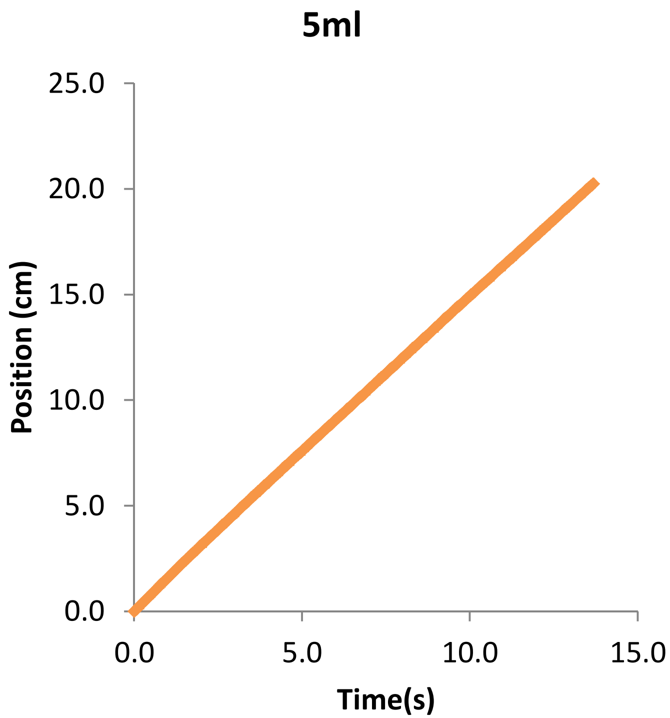
$Vm = 0.01477$ m/s

Experiment 1: Variation of the volume

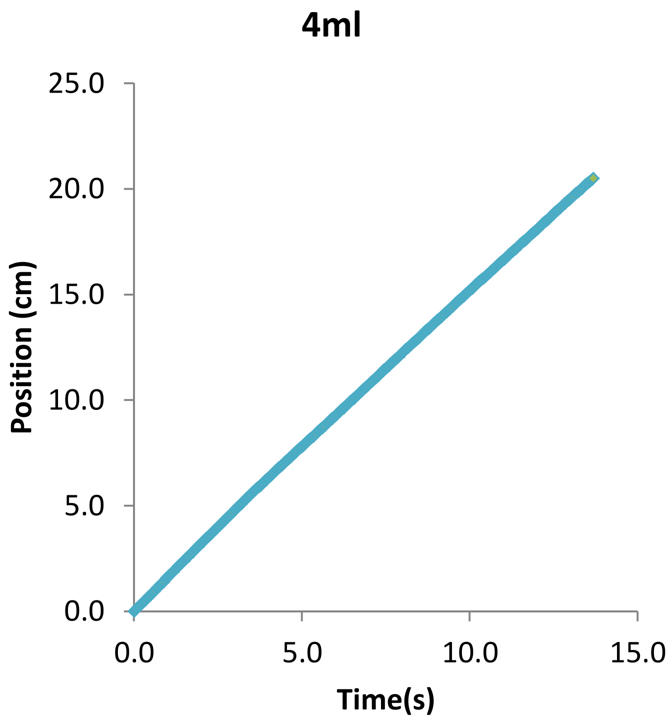
Volume

Temperature

Diameter



$$Vm = 0.01480 \text{ m/s}$$



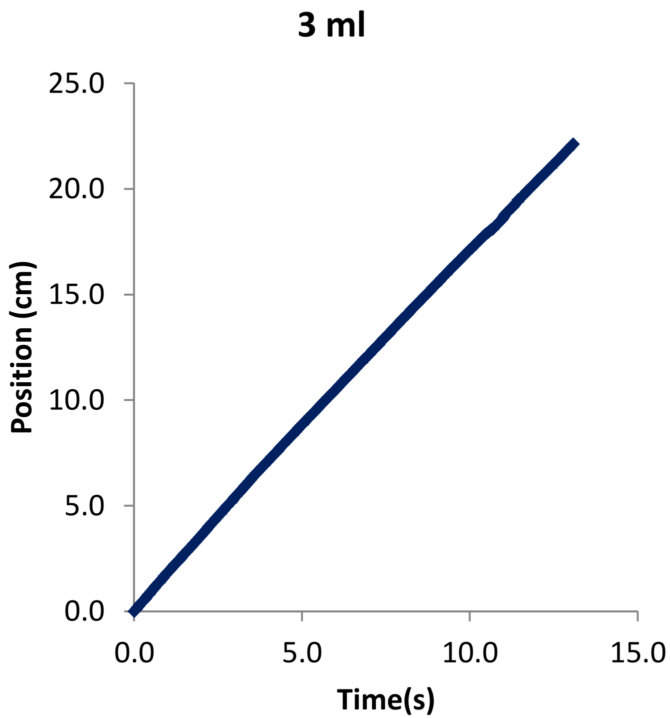
$$Vm = 0.01498 \text{ m/s}$$

Experiment 1: Variation of the volume

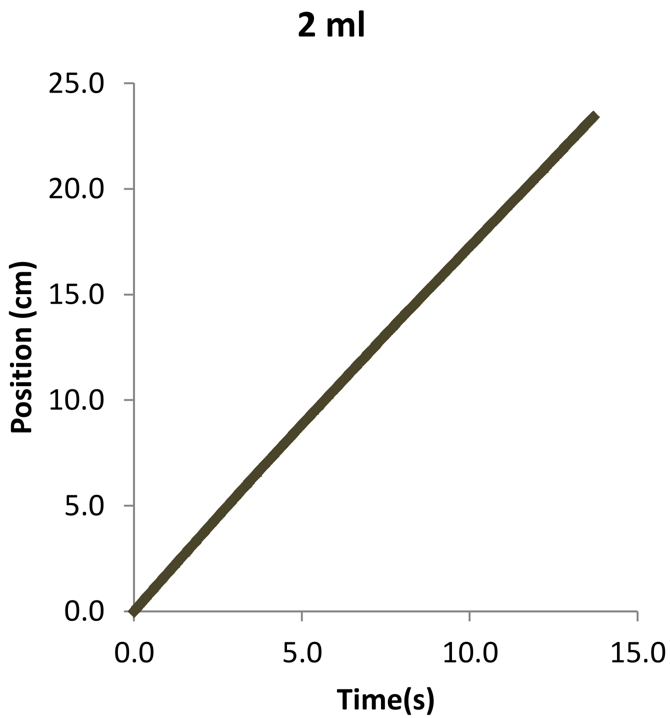
Volume

Temperature

Diameter



$Vm = 0.01640$ m/s



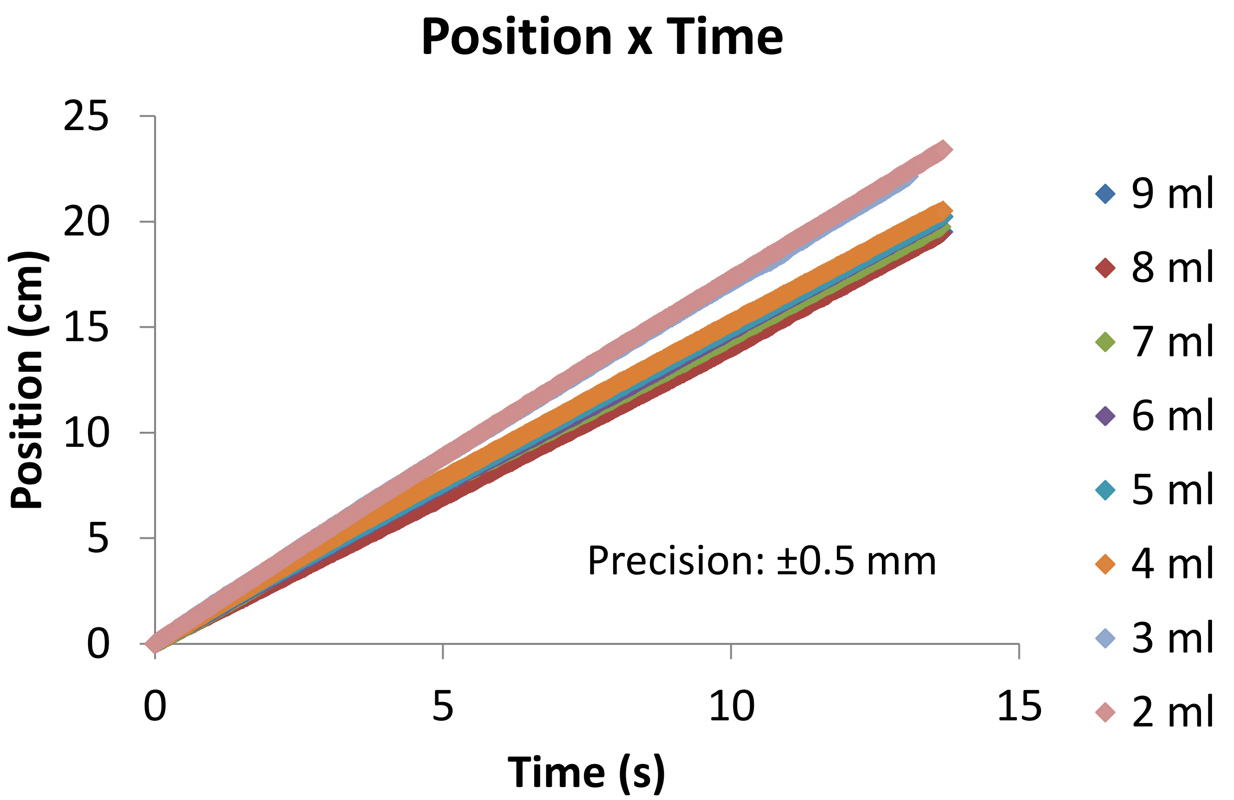
$Vm = 0.01710$ m/s

Experiment 1: Variation of the volume

Volume

Temperature

Diameter

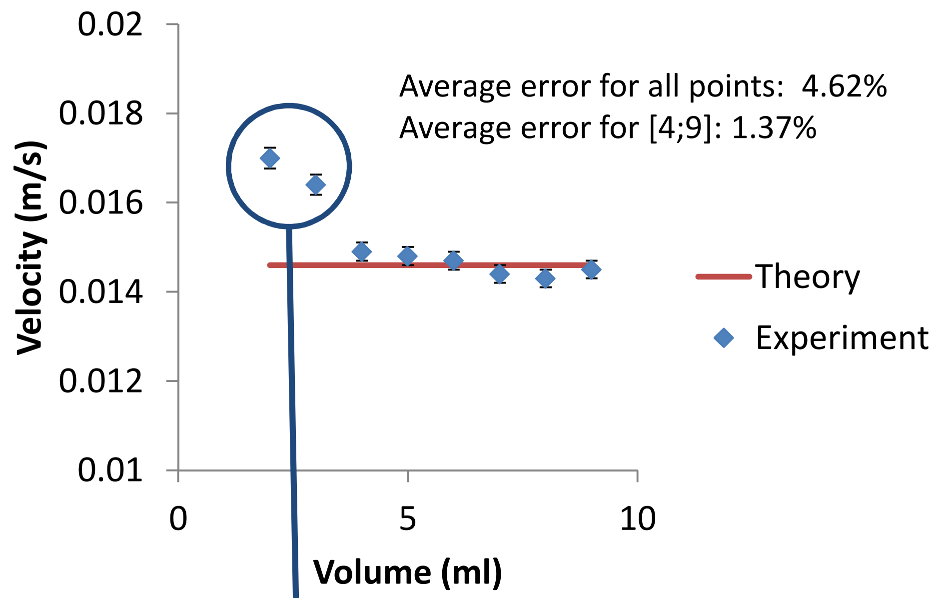


Experiment 1: Variation of the volume

- Volume
- Temperature
- Diameter

Volume (ml)	Experimental velocity (m/s)	Theoretical velocity (m/s)
9	0.0142	*0.0146*
8	0.0144	*0.0146*
7	0.0144	*0.0146*
6	0.0147	*0.0146*
5	0.0148	*0.0146*
4	0.0149	*0.0146*
3	0.0164	*0.0146*
2	0.0171	*0.0146*
Average	***0,0151***	***0.0146***

Experiment agrees with the theory



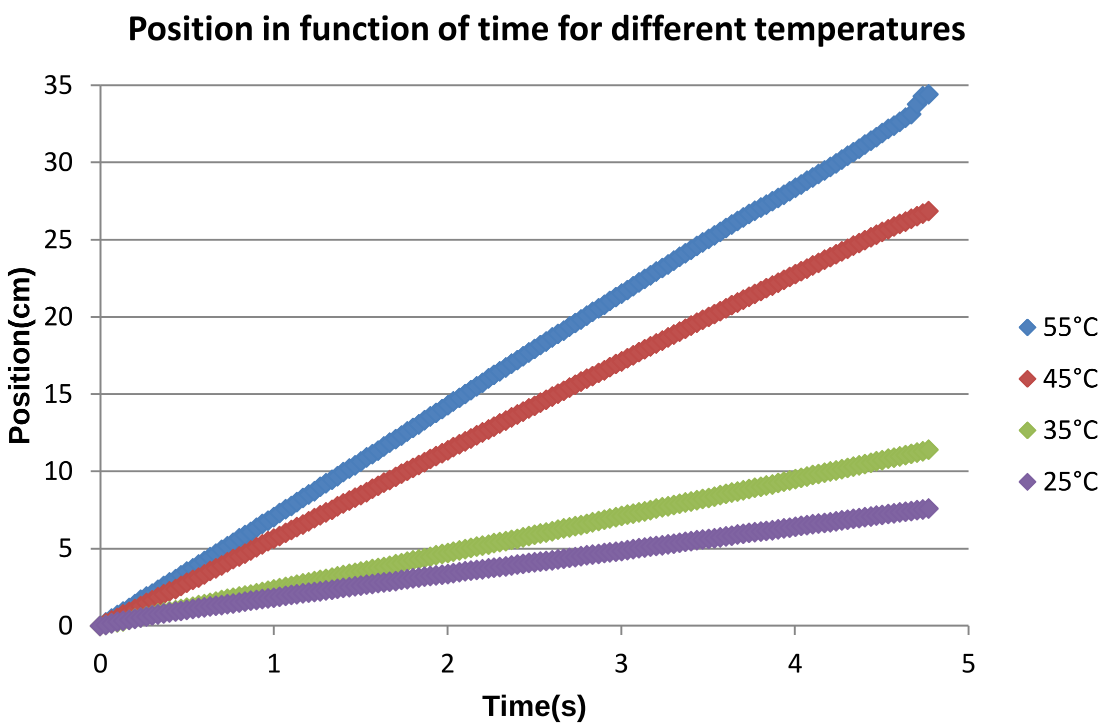
Out of the regime

Experiment 2: Variation of the temperature

Volume

Temperature

Diameter



Experiment 2: Variation of the temperature

- Volume
- Temperature
- Diameter

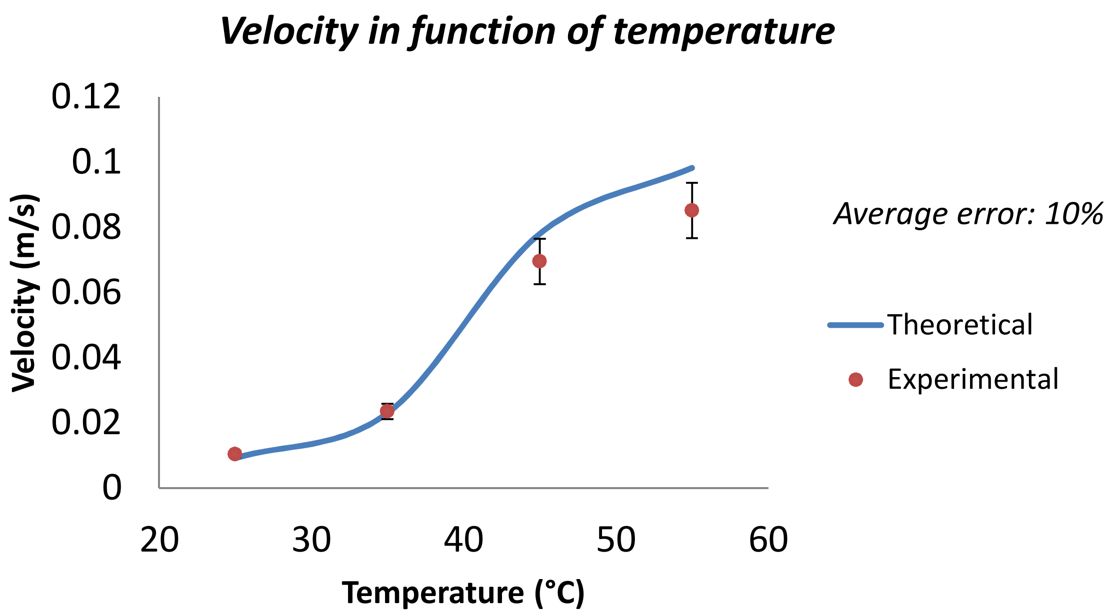
Temperature T (°C)	Calculated Viscosity (m²/s) $* 10^{-4}$	Predicted Velocity (m/s)	Experimental velocity
55.0	0.409	0.0981	*0.0850*
45.0	0.876	0.0779	*0.0694*
35.0	1.973	0.0229	*0.0234*
25.0	4.685	0.00910	*0.0103*

Experiment 2: Variation of temperature

Volume

Temperature

Diameter



Experiment 3: Variation of the diameter

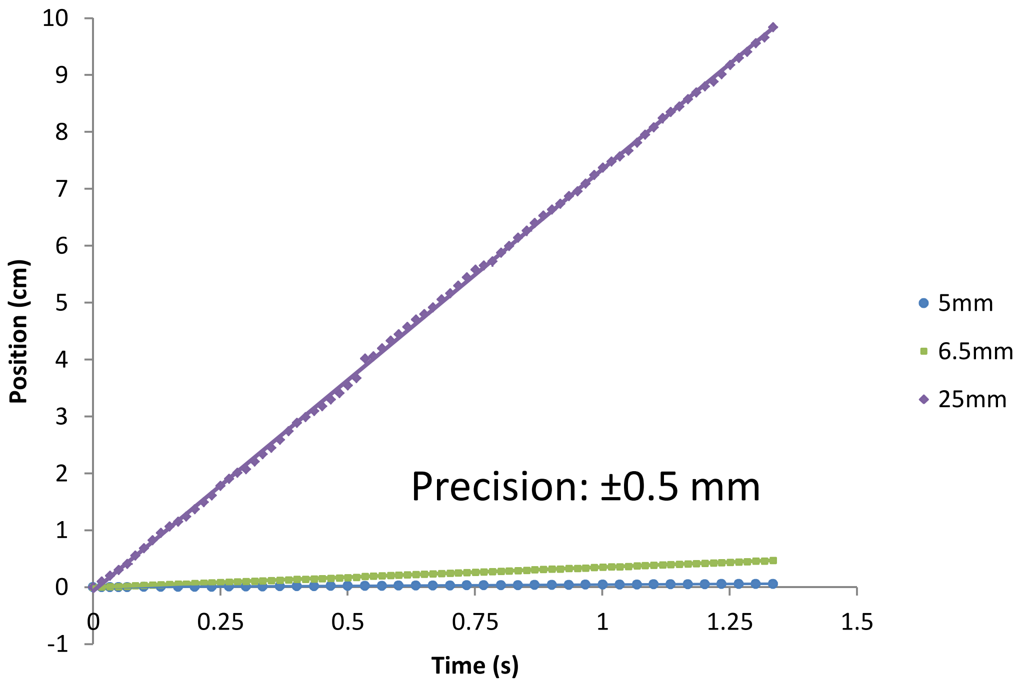
Fluid: Soy bean oil

Kinematic viscosity at 20°C (ν): $7.5 * 10^{-5}$

Volume

Temperature

Diameter



Experimental velocities

0,193 m/s

0,0703 m/s

0,0240 m/s

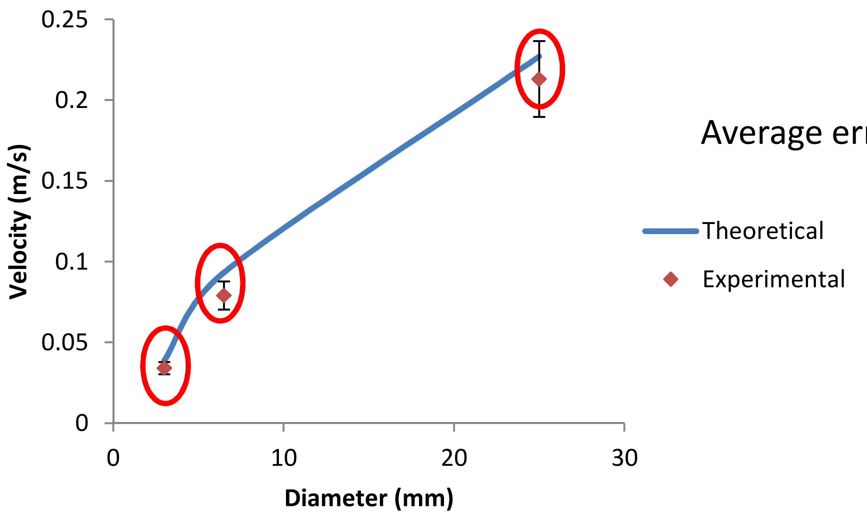
Experiment 3: Variation of the diameter

Volume

Temperature

Diameter

Diameter (mm)	Calculated Velocity (m/s)	Experimental Velocity (m/s)
25.0	0.2270	0.2130
6.5	0.0932	0.0790
5.0	0.0387	0.0340

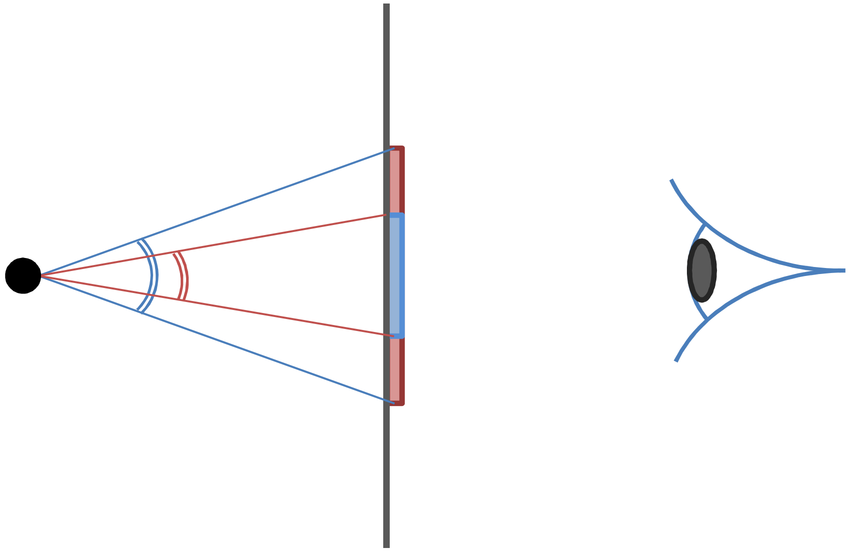


Average error: 11%

Conclusion

- Diameter of the tube
- Viscosity
- Temperature
- Diameter

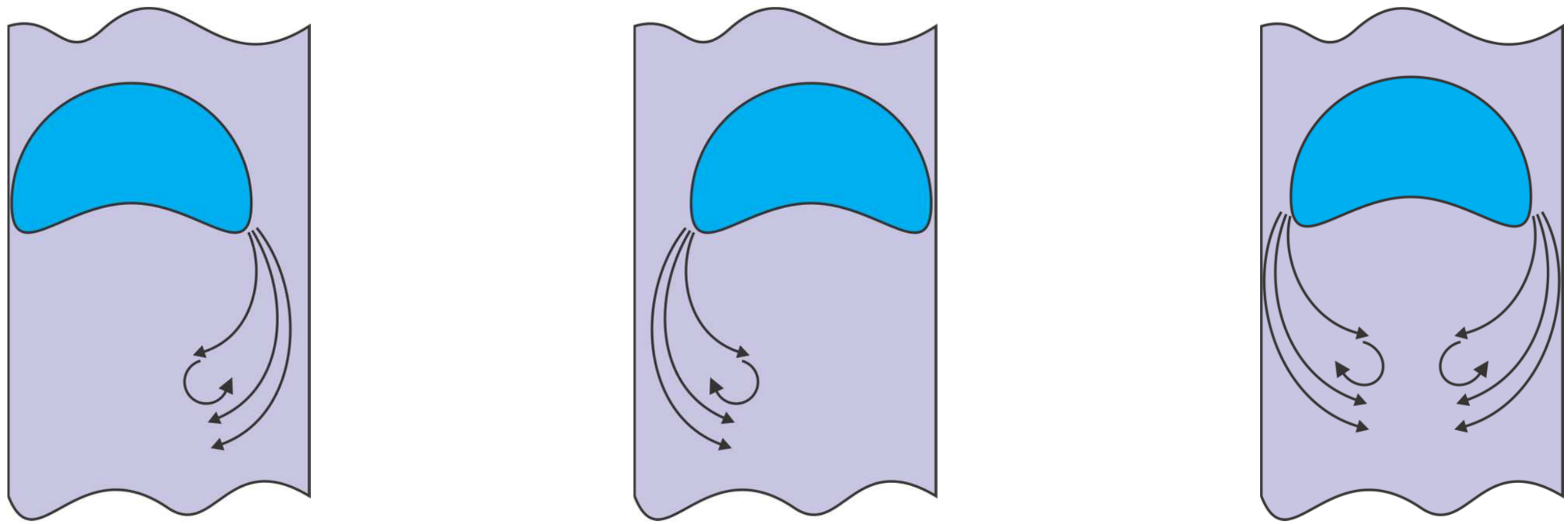
References



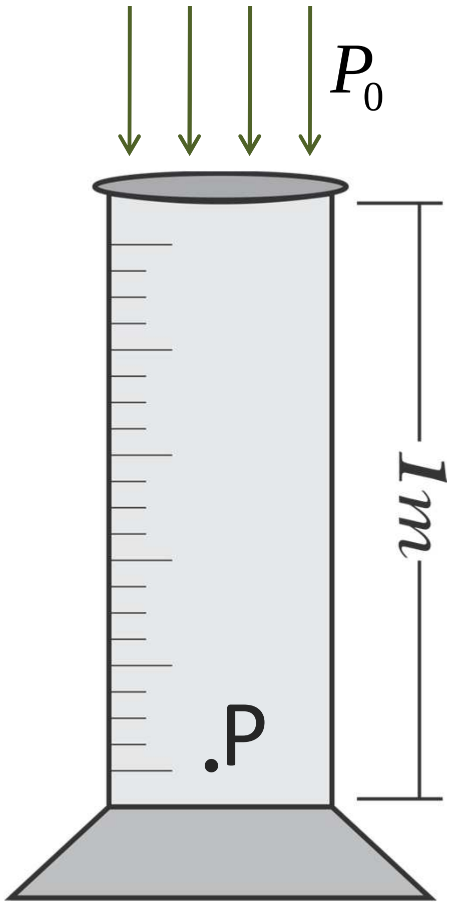
Thank you!

Lateral Flow

- For bubble measures comparable to the tube.



Pressure and volume variation



$$P_P = \rho.g.h + P_0$$

- For the following parameters:
  - Fluid: Water
  - Length of beaker: 1m

$\rho_{H_2O} = 10^3\ kg/m^3$
$P_0 = 1{,}10^5\ Pa$
$g = 9.81\ m/s^2$

$$h = 0 \Rightarrow P = P_0$$
$$\Delta P = \rho.g.h = 1.10^3 Pa$$

$$\frac{P_1.V_1}{T_1} = \frac{P_2.V_2}{T_2} \Rightarrow V_2 = 1.1\ V_1$$

The weight of the bubble

•Considering a bubble with a volume of $2\times10^{-5}m^3$ (20mL):

Buoyancy / Weight

$$B = \rho_l V g$$

$$B = 10^3 \times 2\times10^{-5} \times 10 = 2\times10^{-1}$$

$$\left| \begin{array}{l} W = mg \\ \rho_g = m/V \end{array} \right. \Rightarrow W = \rho_g V g = 2.4\times10^{-4}$$

Relation between them

$$\frac{W}{B} = \frac{2.4\times10^{-4}}{2\times10^{-1}} = 1.2\times10^{-3}$$

Conclusion

$$W = 0.6\%\text{ of } B\text{, so } W \approx 0$$

- http://www.pmmh.espci.fr/~jbico/TP/taylor50bulle.pdf

Fluid (l)	C	Fluid (l)	C
Ammonia	1.05	Freon 12	1.76
Ethanol	5.72	Benzene	4.34
Gasoline	3.68	Mercury	1.07
Glycerin	28.0	Methanol	4.63