Team of Brazil

Problem 03
String of beads

reporter:

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Problem 03
String of beads

A long string of beads is released from a beaker by pulling a sufficiently long part of the chain over the edge of the beaker. Due to gravity the speed of the string increases. At a certain moment the string no longer touches the edge of the beaker (see picture). Investigate and explain the phenomenon.

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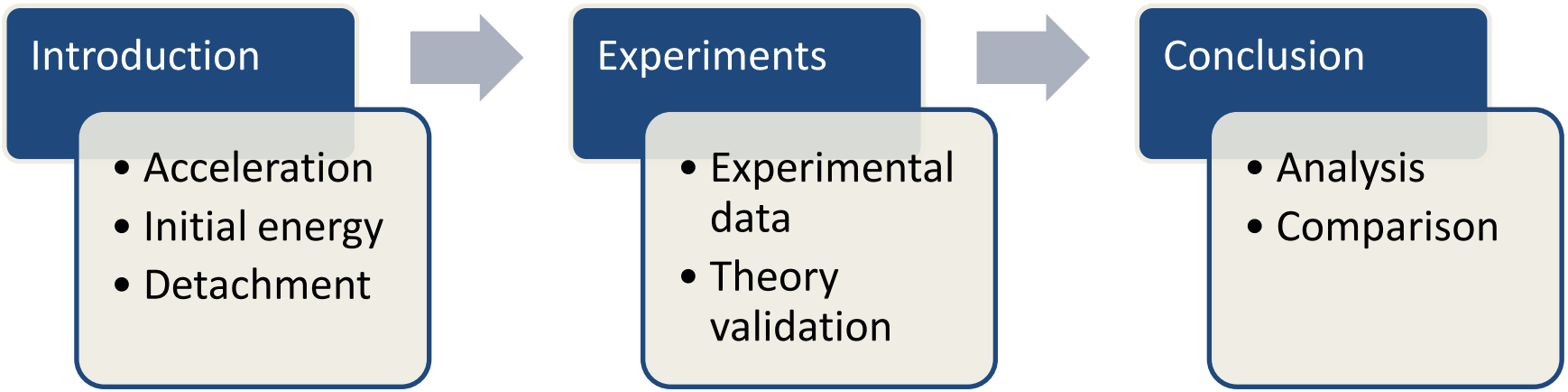
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• Acceleration
• Initial energy
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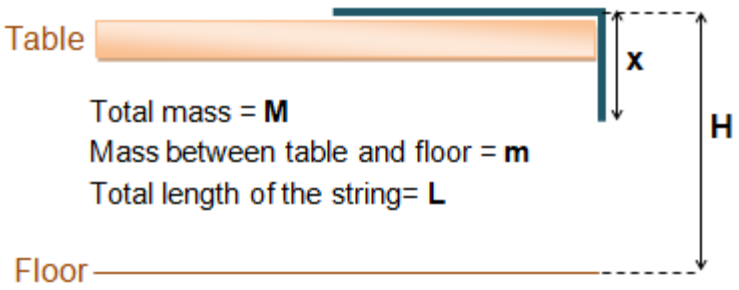
Introduction

- "sufficiently long part of the chain": there is a minimum energy to overcome inertia.
- "the speed of the string increases": two cases
  - Initial height > String's length: 2 phases for acceleration behavior
  - Initial height < String's length: 3 phases for acceleration behavior
- "the string no longer touches the edge of the beaker": there is a mechanism which makes the chain detach from the beaker.

Acceleration and motion features



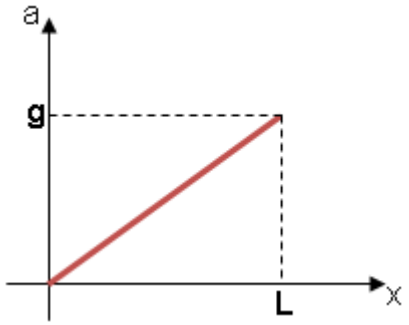
$$m = \frac{x}{L} M$$

$$F = W_x$$

$$M \cdot a = \frac{x}{L} M g$$

$$\boldsymbol{a = \frac{x}{L} g}$$

When H>L



$$v_1{}^2 = v_0{}^2 + 2A$$

$$v_1^2 = v_0^2 + 2\frac{gL}{2}$$

$$\boldsymbol{v_1 = \sqrt{v_0^2 + gL}}$$

$$v_1 = \sqrt{gL}$$

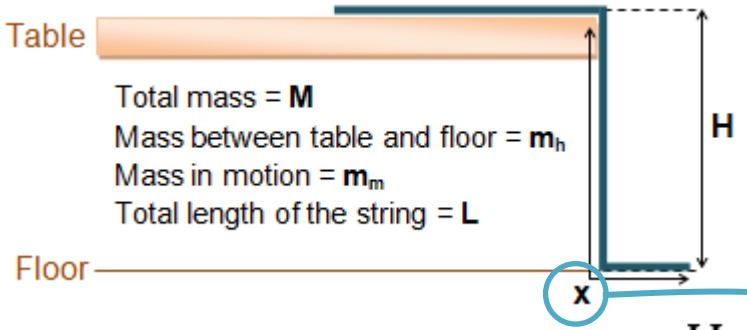
Acceleration and motion features

When H<L

- 1st part: till x = H

$$a = \frac{x}{L} g$$

- 2nd part:



$$m_m = \frac{L - x + H}{L} M \qquad m_h = \frac{H}{L} M$$

$$F = W_h$$

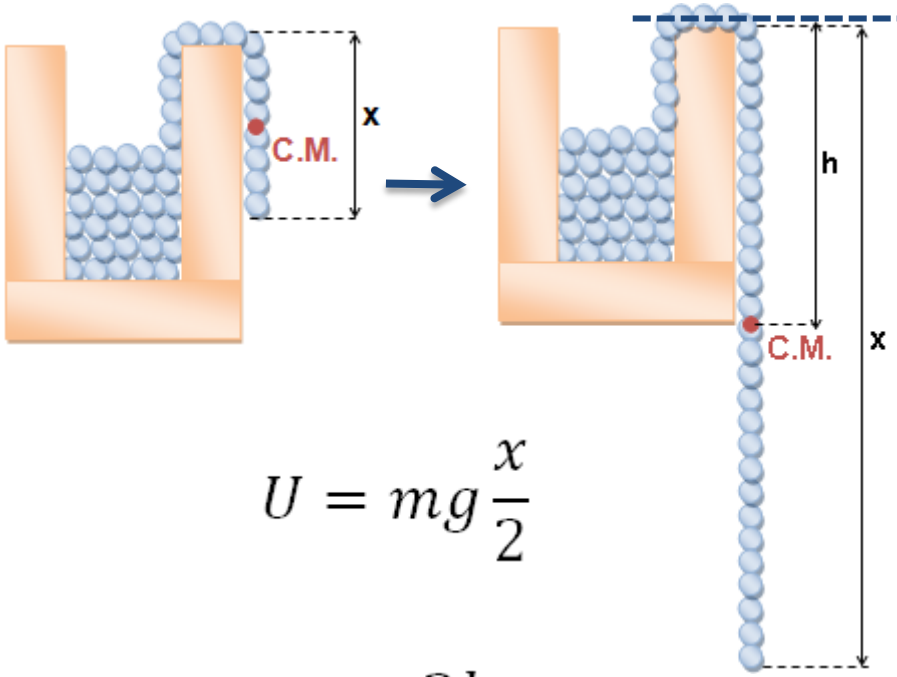
$$\frac{L - x + H}{L} M \cdot a = \frac{H}{L} M g$$

$$a = \frac{H}{L - x + H} g$$

- 3rd part: $a = g$

Total **length that already left the beaker**, including what already touched the floor.

Initial energy to overcome inertia



Referential for energy calculation.

$$U = mg\frac{x}{2}$$

$$x = 2h$$

$$U = \frac{x}{L}M \cdot g\frac{x}{2}$$

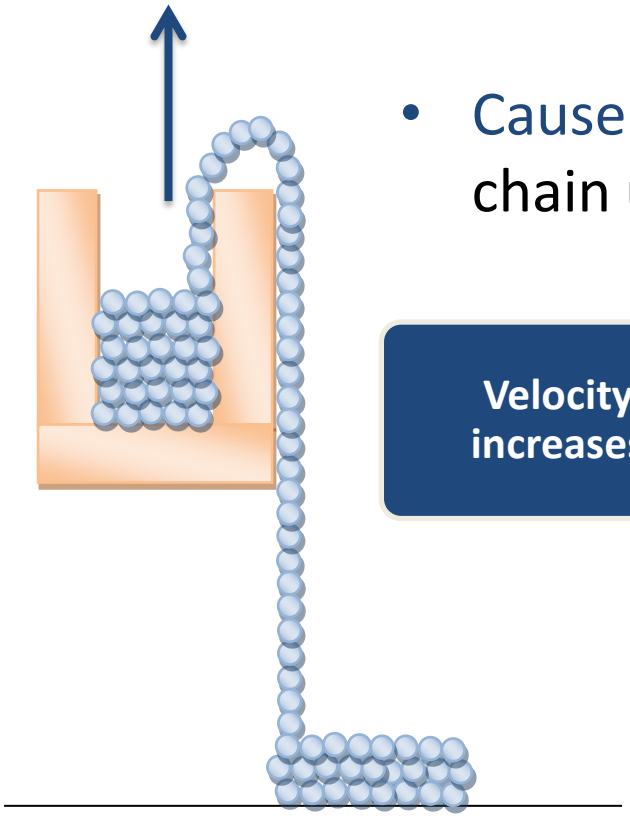
$$U = Mg\frac{x^2}{2L}$$

$$U = Mg\frac{(2h)^2}{2L}$$

Initial energy:

$$\boldsymbol{U = 2Mg\frac{h^2}{L}}$$

Detachment

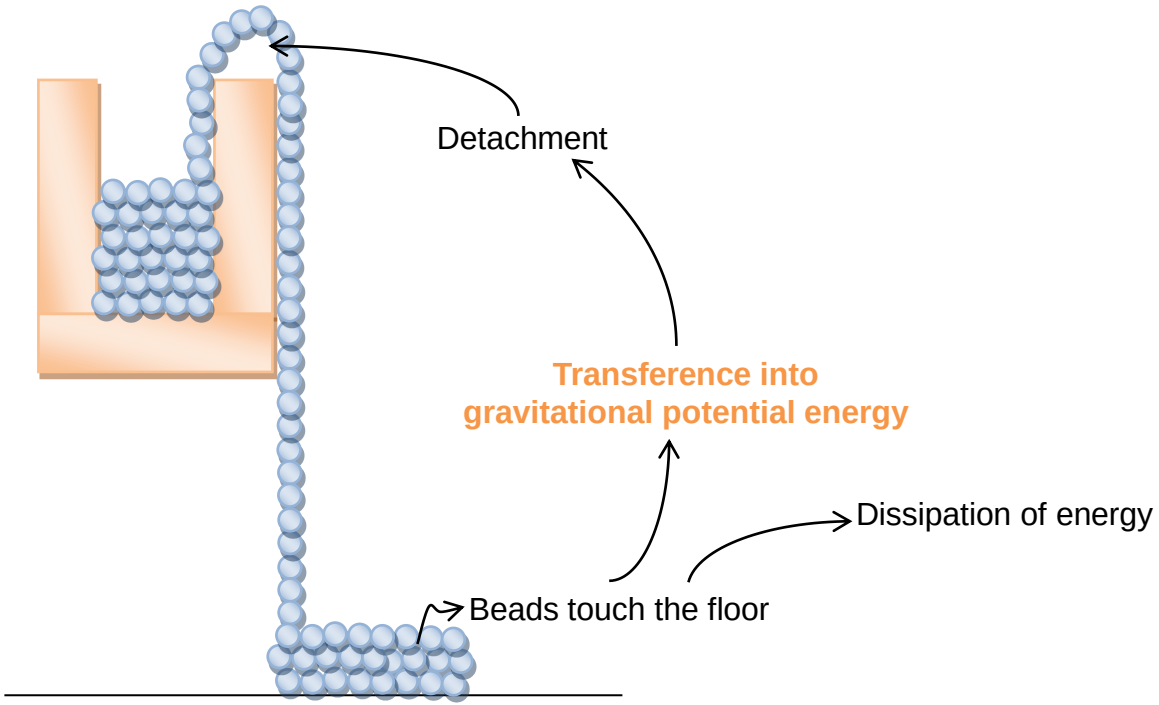


- Cause: tension causes an impulse that pull the chain upwards.

Velocity increases → **Tension increases** * → Detachment height increases with time

*WANG, C. W.; YASUI, K.. Falling chains. University of California. 20 Feb 2006

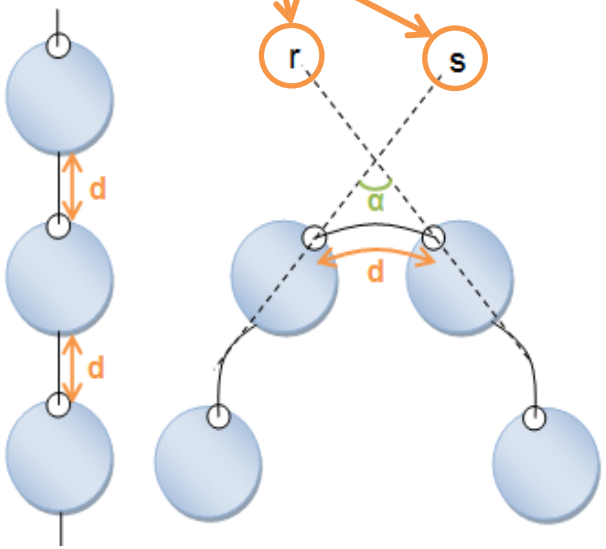
Detachment

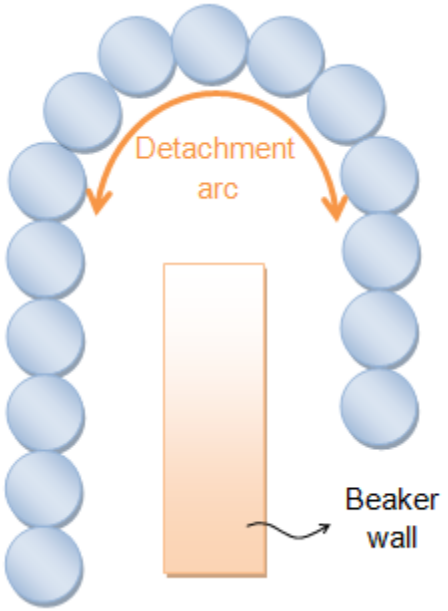


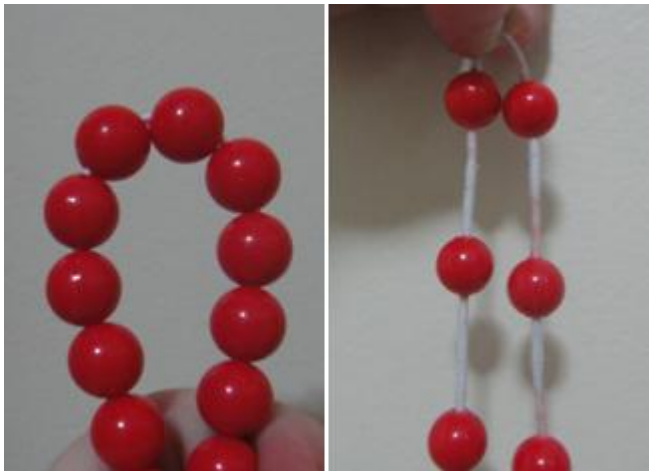
The height of the beaker influences the height of detachment above the beaker's border.

Detachment

Cut diametrically the beads

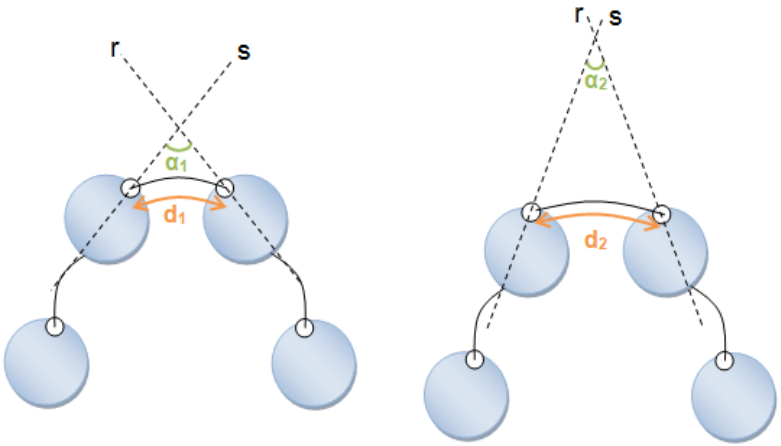






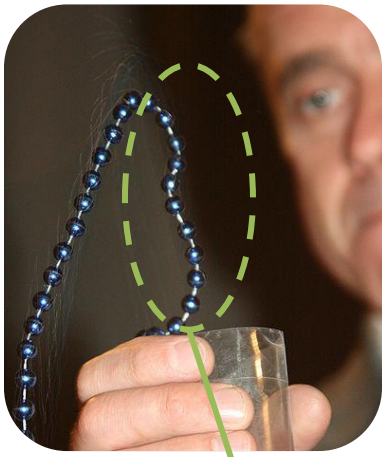
Larger angle → Larger detachment arc → There's a maximum arc of tension

Detachment



$d_1 < d_2$ → $\alpha_1 > \alpha_2$ → Higher tension

Larger bead's size → Larger angle → Higher tension



Takes different shapes due to the arrangement inside the beaker.

Material

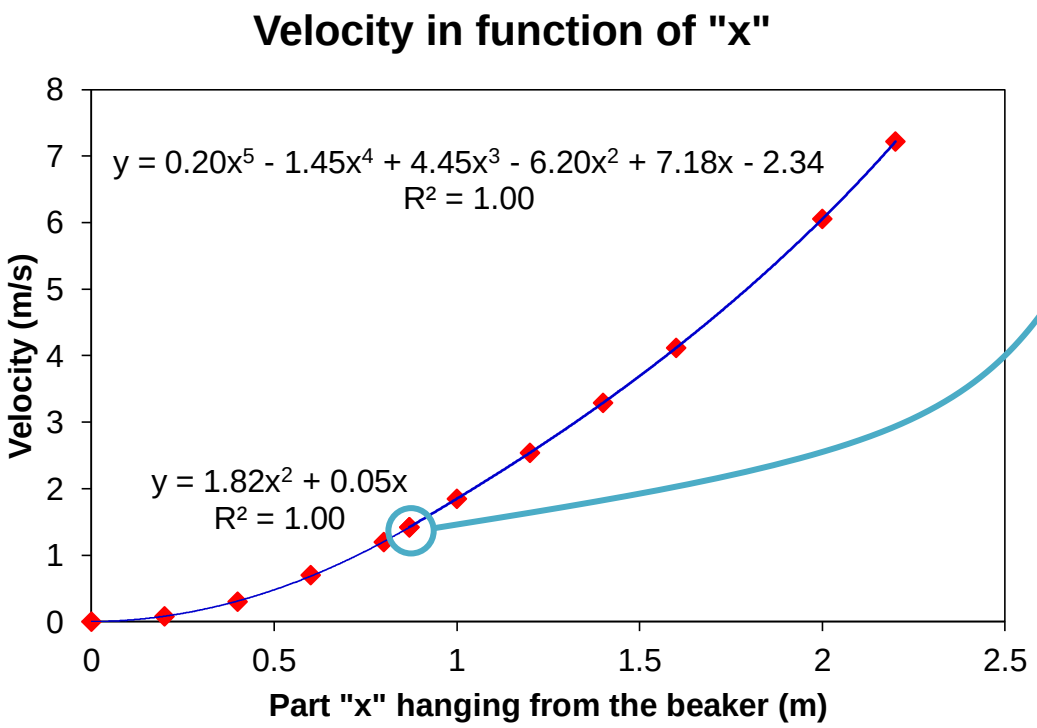
- Beakers
- Chains (beads and nylon)
- Table
- Camera
- Measuring tape (precision of ±0.05 cm)
- Precision scale (precision of ±0.1 g)

Experimental description

- **Experiment 1:** acceleration analysis.
- **Experiment 2:** initial energy analysis.
- **Experiment 3:** detachment analysis.
- **Experiment 4:** detachment evolution with time.
- **Experiment 5:** height of launch variation.
- **Experiment 6:** beaker dimensions variation.
- **Experiment 7:** variation of the distance between beads.

Experiment 1: acceleration analysis

- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads



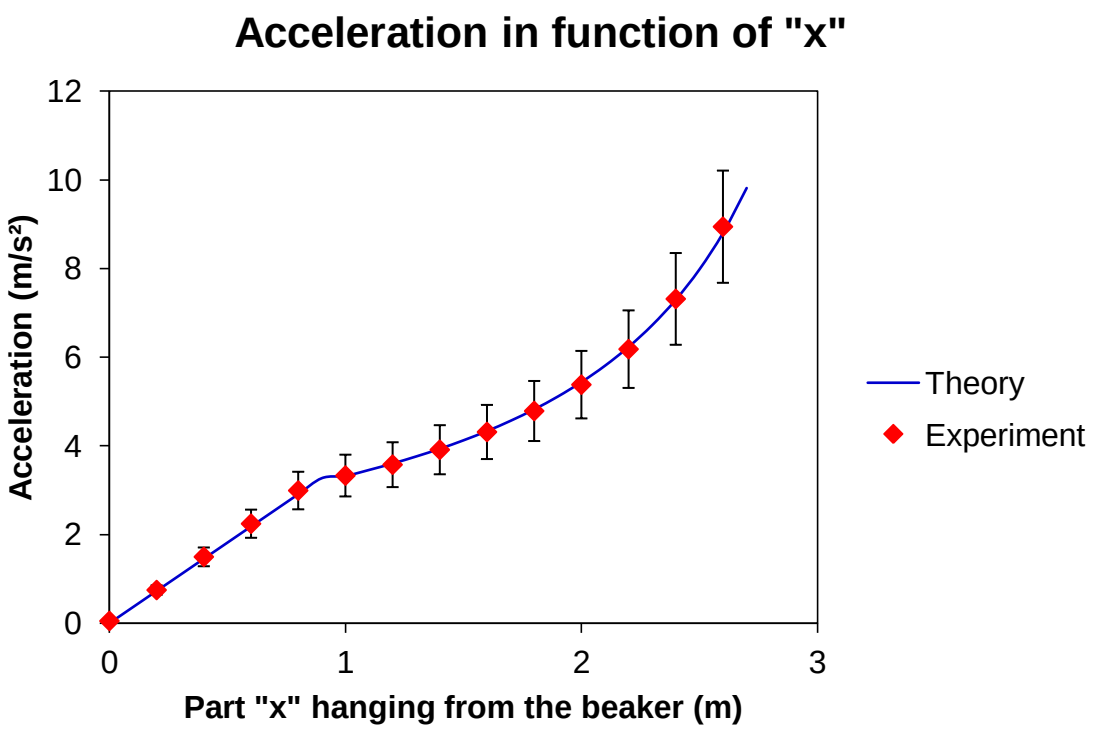
$$\lim_{x \to 0{,}87^-} x = 1.42$$

$$\lim_{x \to 0{,}87^+} x = 1.42$$

Function is differentiable

Experiment 1: acceleration analysis

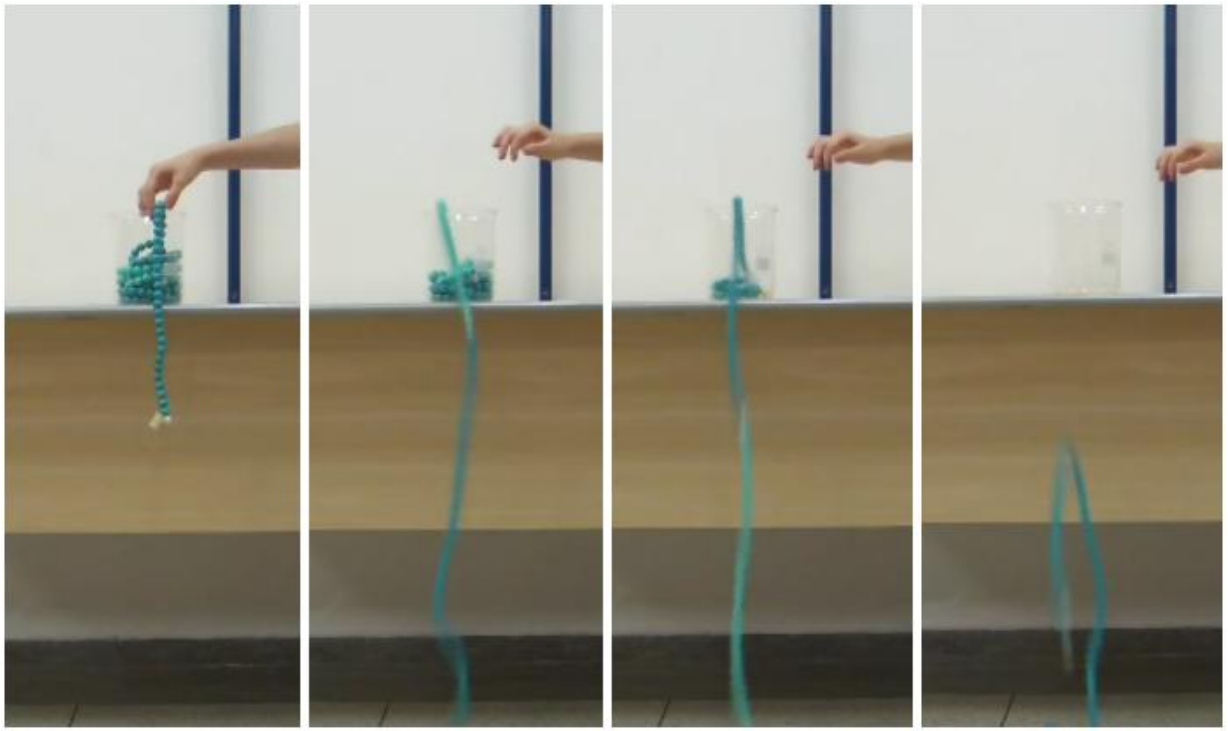
- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads



Source of errors: measuring tape (±0.5 mm) and conversion scale.

Experiment 1: acceleration analysis

- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads



Experiment 2: initial energy analysis

- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads

Minimum energy (J)
0.044
0.038
0.033
0.038
0.038
0.044
0.038

Standard deviation: 0.004 J

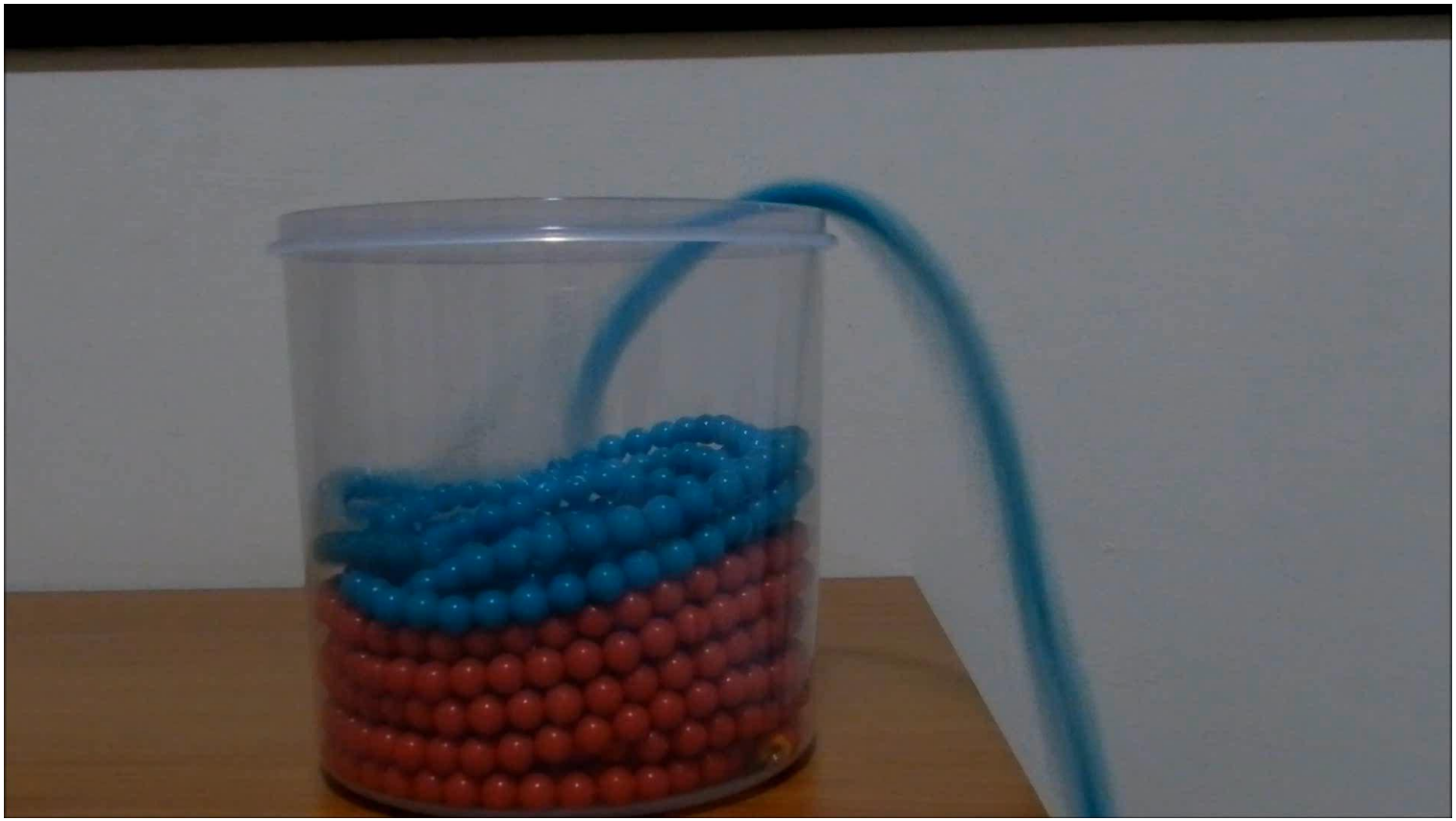
Average: 0.039 J ↔ Theoretical: 0.038 J

Experiment agrees with theory

Chain mass: 270 g
Chain length: 2.7 m
Beaker height: 14 cm
Source of error: measuring tape (±0.5 mm) e of the scale (±0.1 g).

Experiment 3: detachment analysis

- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads

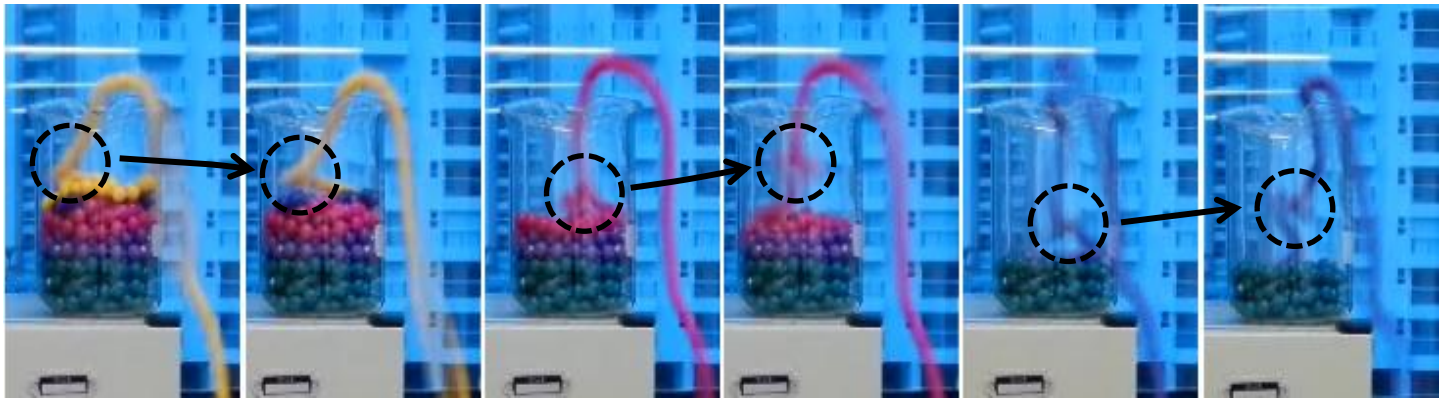


Experiment 3: detachment analysis

- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads

Experiment 3: detachment analysis

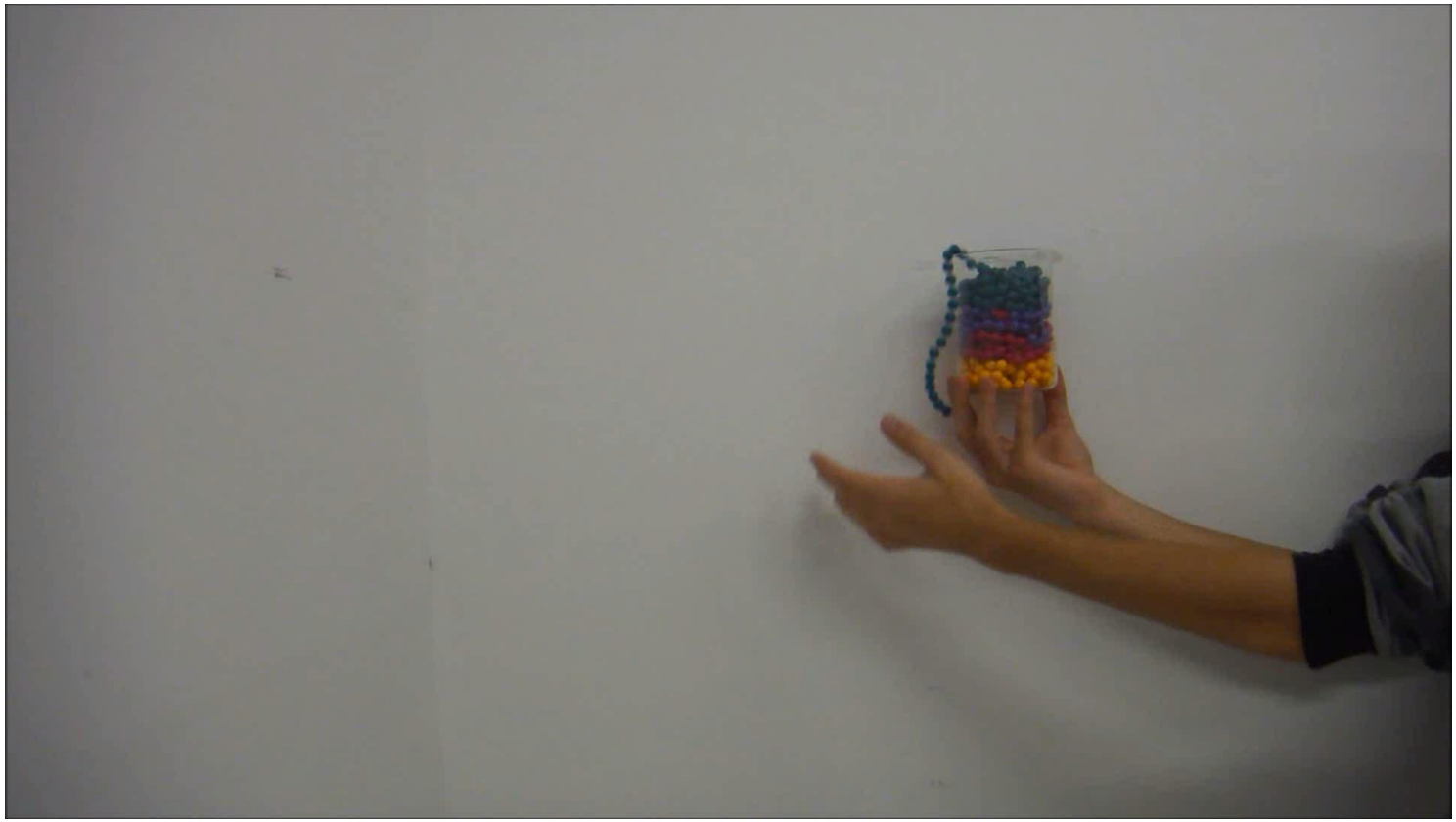
- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads



Possible to see the relation between patterns

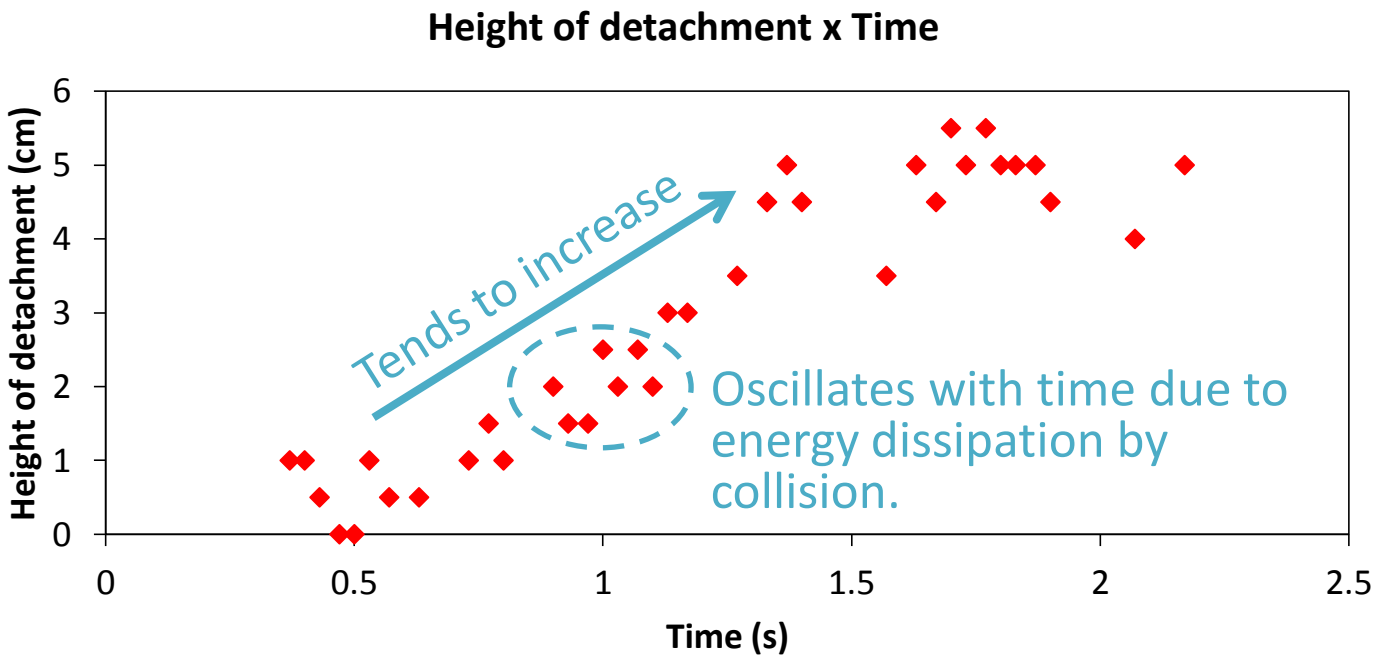
Experiment 4: evolution of the detachment with time

- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads



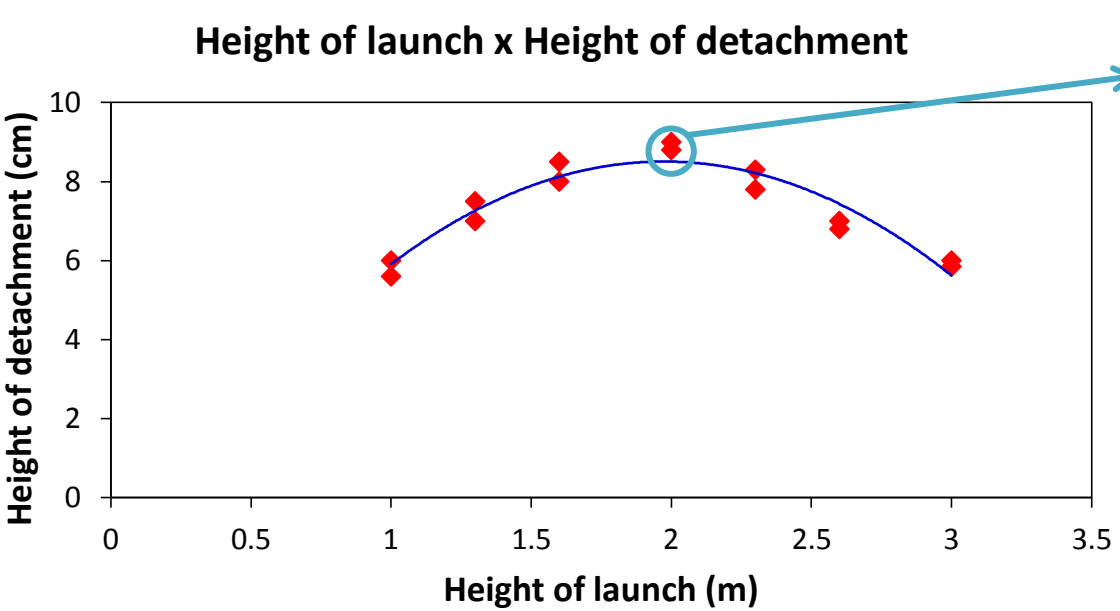
Experiment 4: evolution of the detachment with time

- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads



Experiment 5: height of launch variation

- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads



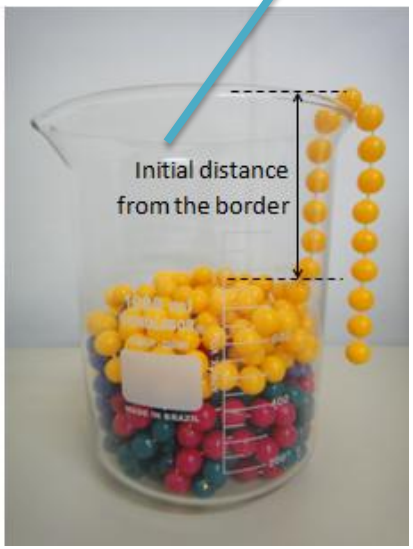
Optimum point

Dissipation starts to overcome the transference into potential energy.

Experiment 6: beaker dimensions variation

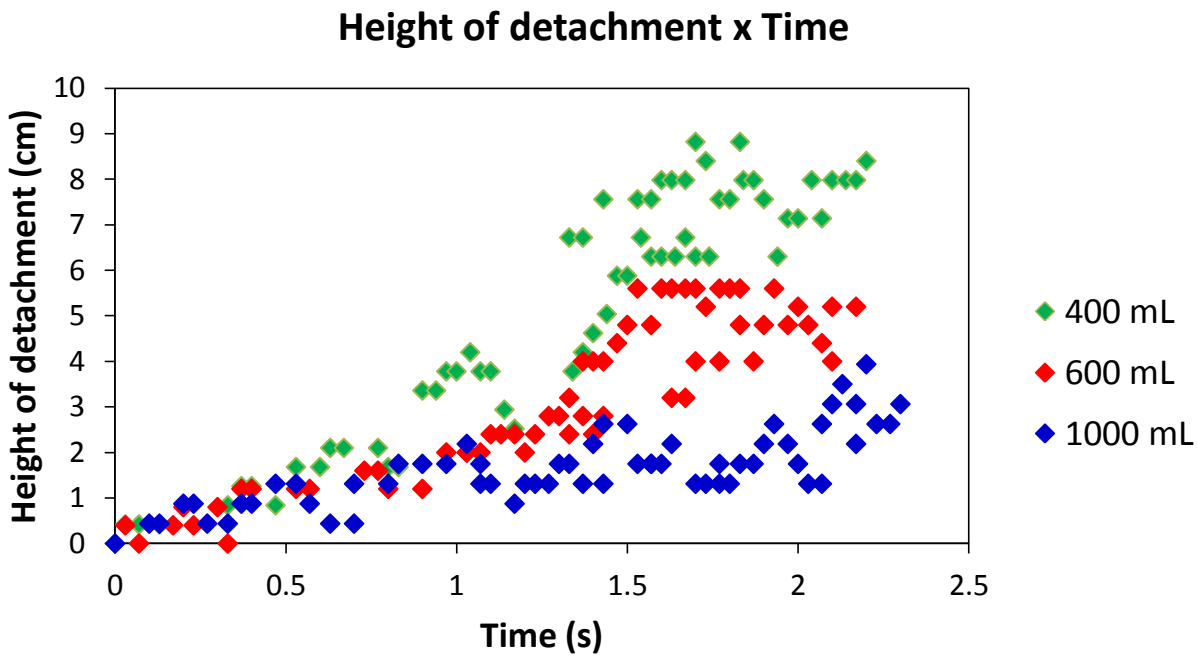
- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- **Beaker dimensions**
- Distance between beads

It is varied.



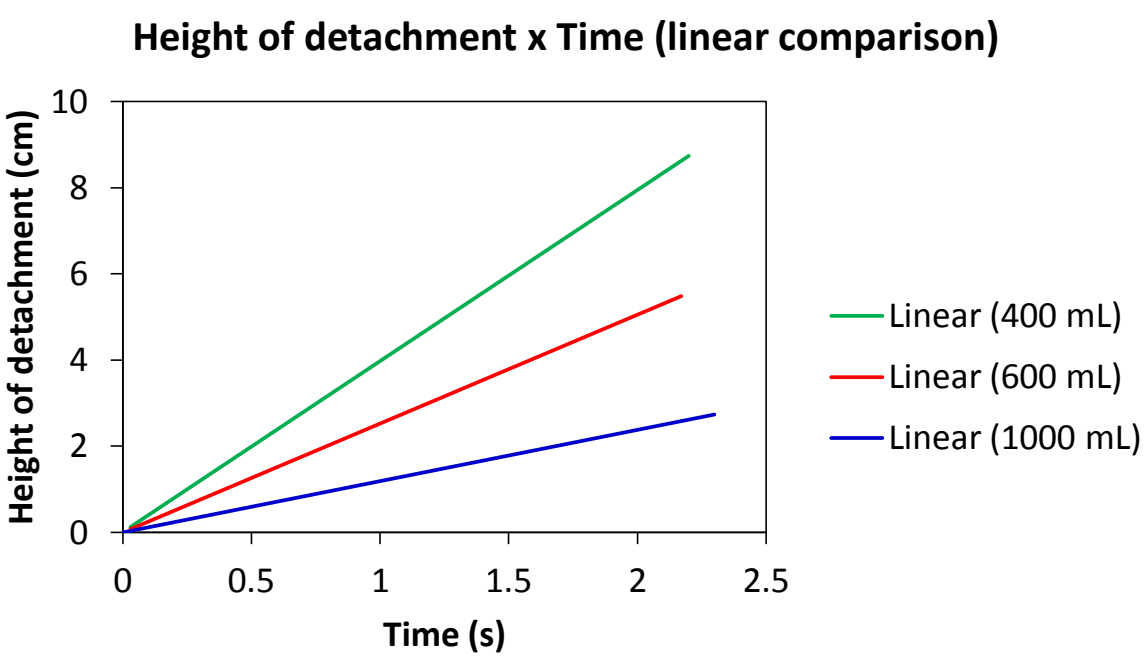
Experiment 6: beaker dimensions variation

- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads



Experiment 6: beaker dimensions variation

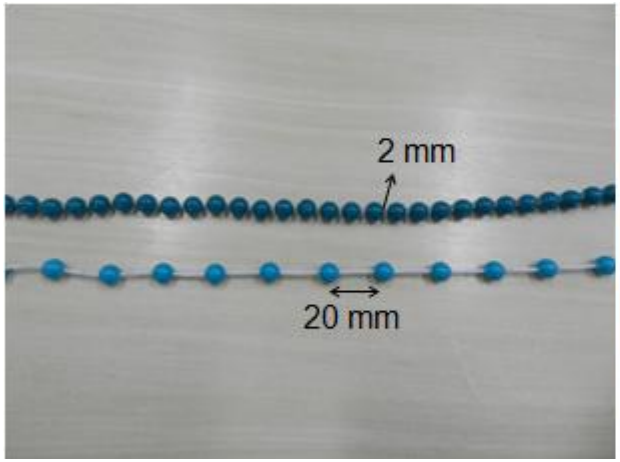
- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- **Beaker dimensions**
- Distance between beads



Larger initial distance from the border → Larger distance for the detachment to overcome → Lower detachment height

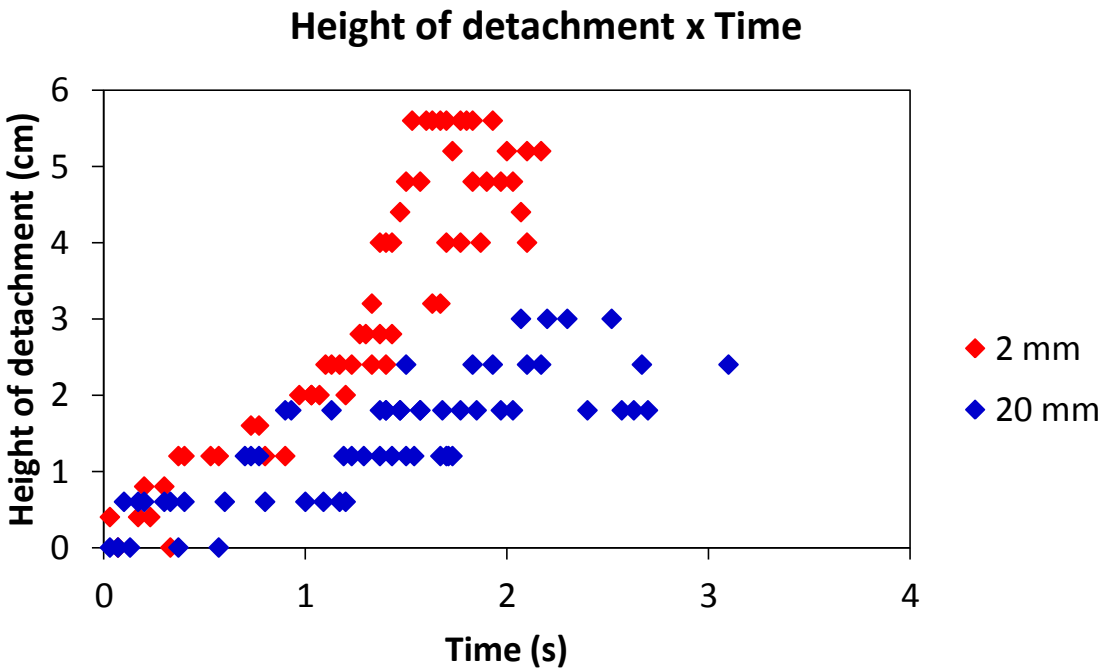
Experiment 7: distance between beads

- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads



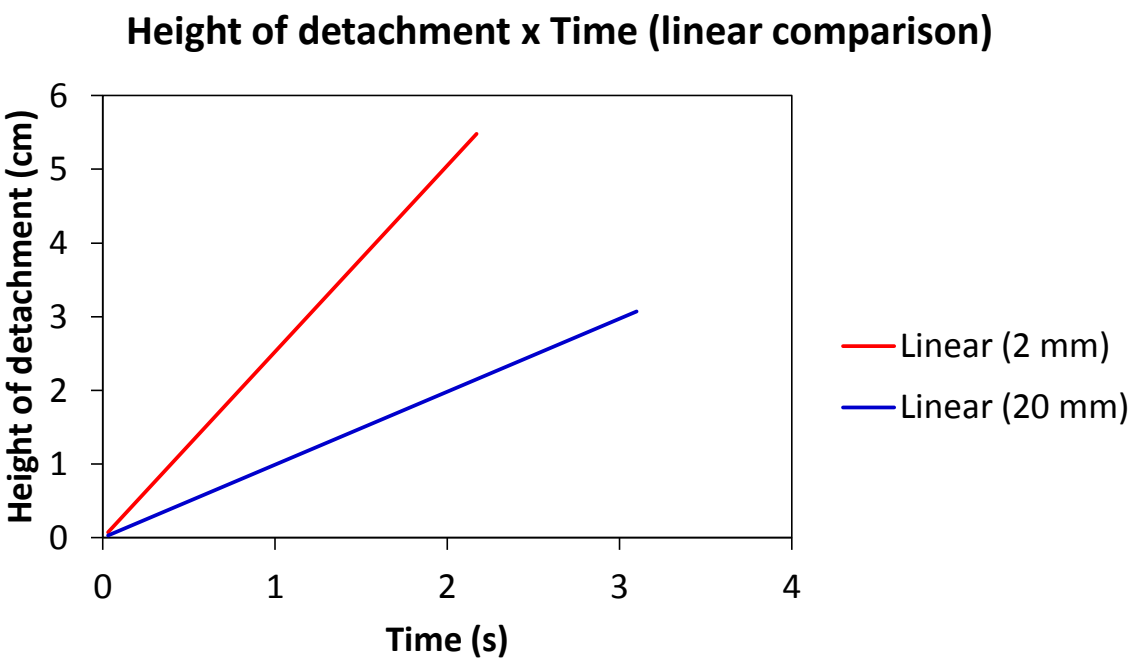
Experiment 7: distance between beads

- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads



Experiment 7: distance between beads

- Acceleration
- Initial energy
- Detachment shape
- Detachment in time
- Height of launch
- Beaker dimensions
- Distance between beads



Larger distance between beads → Lower tension → Lower detachment height

Conclusion

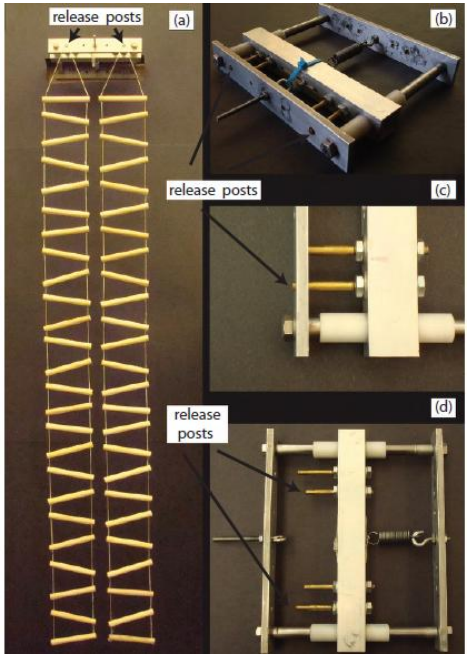
- Initial energy: related to the position of the center of mass (exp. 3).
- Acceleration increases until g is reached in three phases (exp. 2).
- Detachment:
  - Impulse of the tension pulls the string upwards;
  - Different shapes due to chain's arrangement (exp. 4);
  - Energy conversion (exp. 5);
  - Initial height (exp. 5);
  - Beaker's dimensions (exp. 6);
  - Distance between beads (exp. 7).

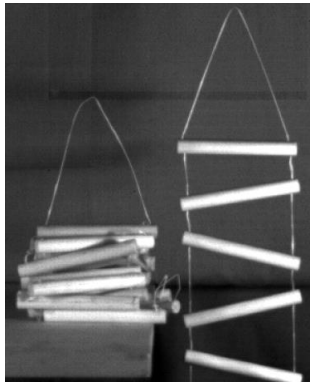
References

- 1. GREWAL, Anoop; JOHNSON, Phillip; RUINA, Andy. A chain that accelerates rather than slows due to collisions: how compression can cause tension. Cornell University. 13 mar. 2011.
- 2. http://mathworld.wolfram.com/Catenary.html
- 3. WANG, Chun Wa; YASUI, Kosuke. Falling chains. University of California. 20 Feb 2006

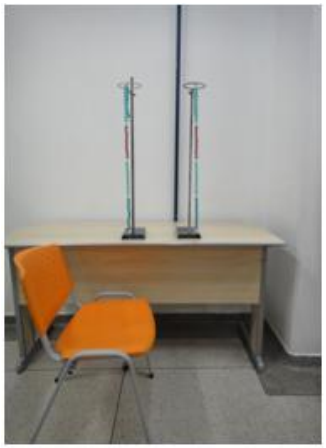
Thank you!

Summary of the experiment in reference 1

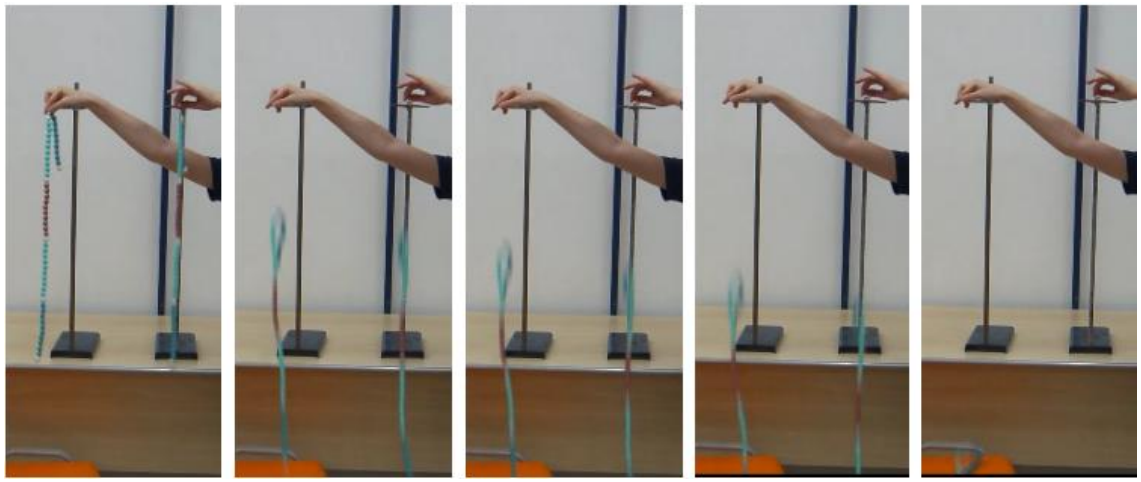




Experiment: collision analysis

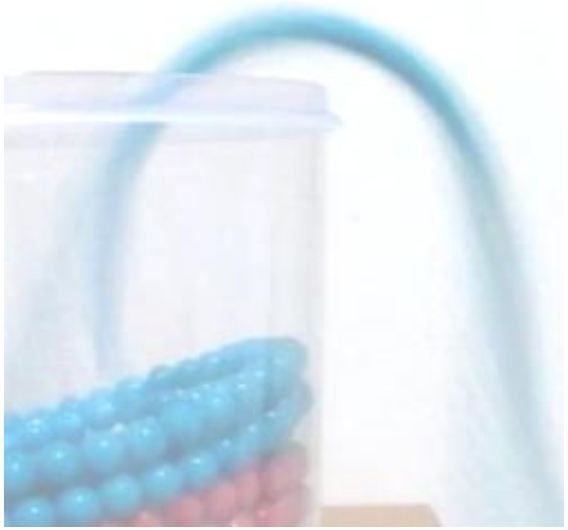


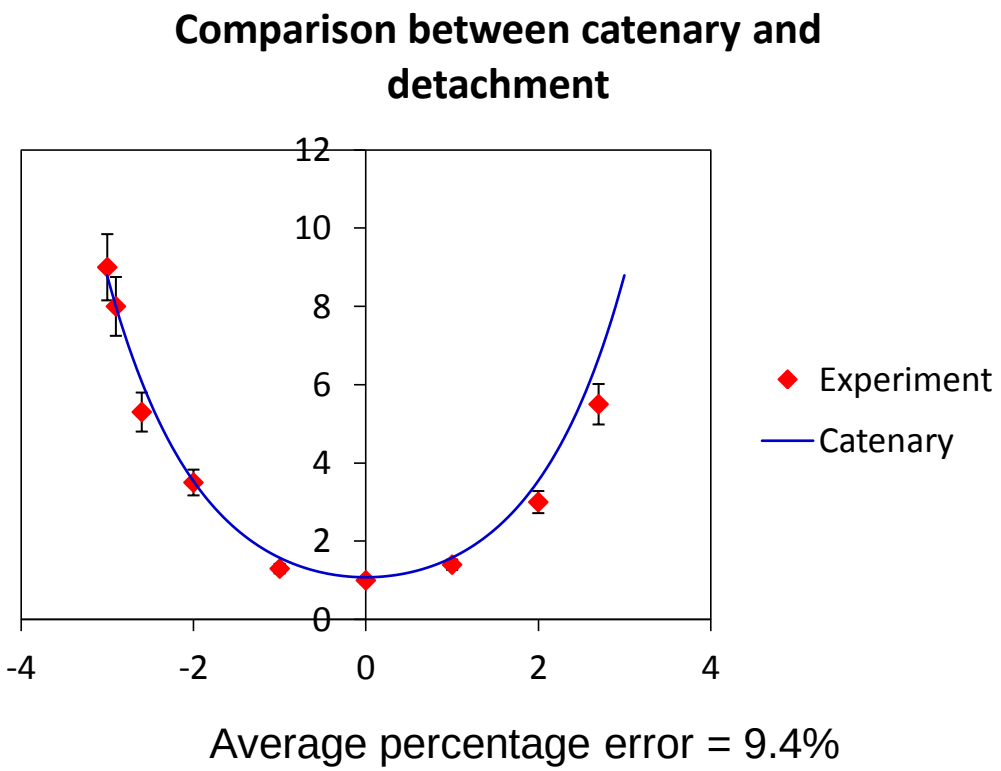
Experimental setup



Collision → Beads continue synchronized → Acceleration equal to g

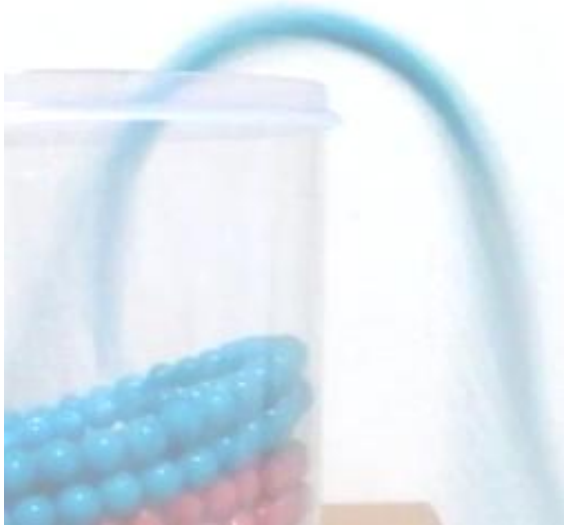
Experiment: detachment shape (upper part)

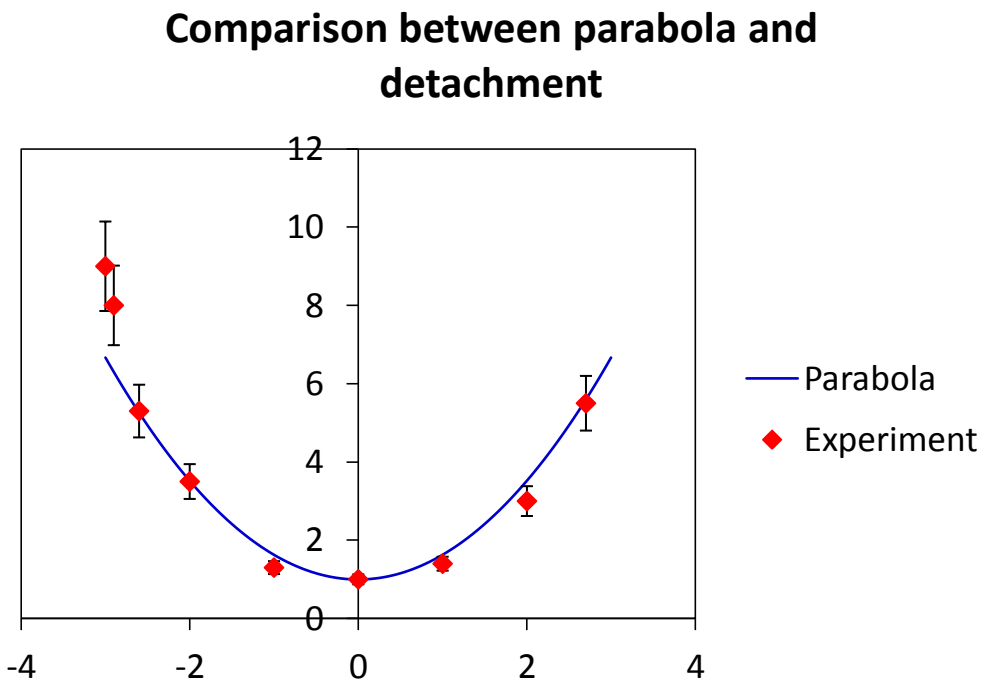




Average percentage error = 9.4%

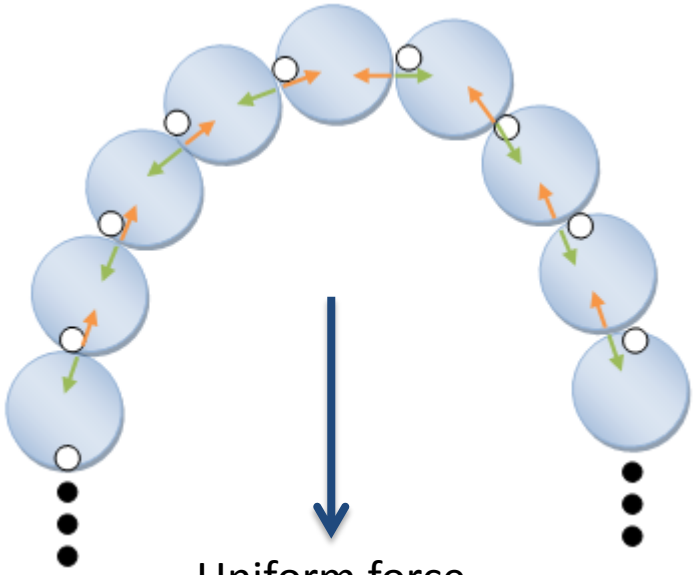
Experiment: detachment shape (upper part)





Average percentage error = 12.7%

Explanation: detachment shape (upper part)

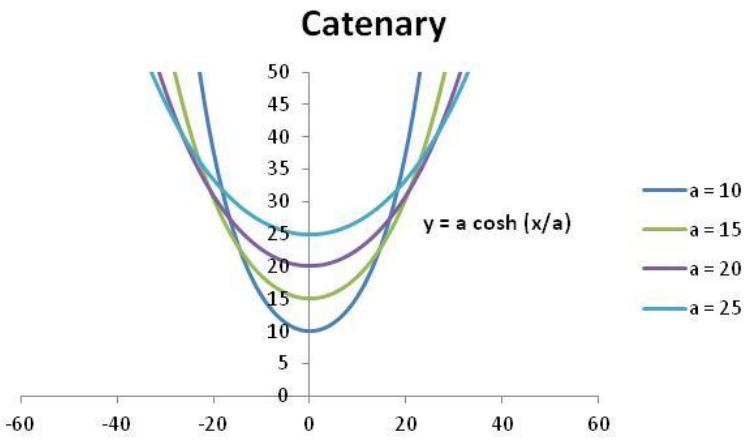


Uniform force distribution

Catenary

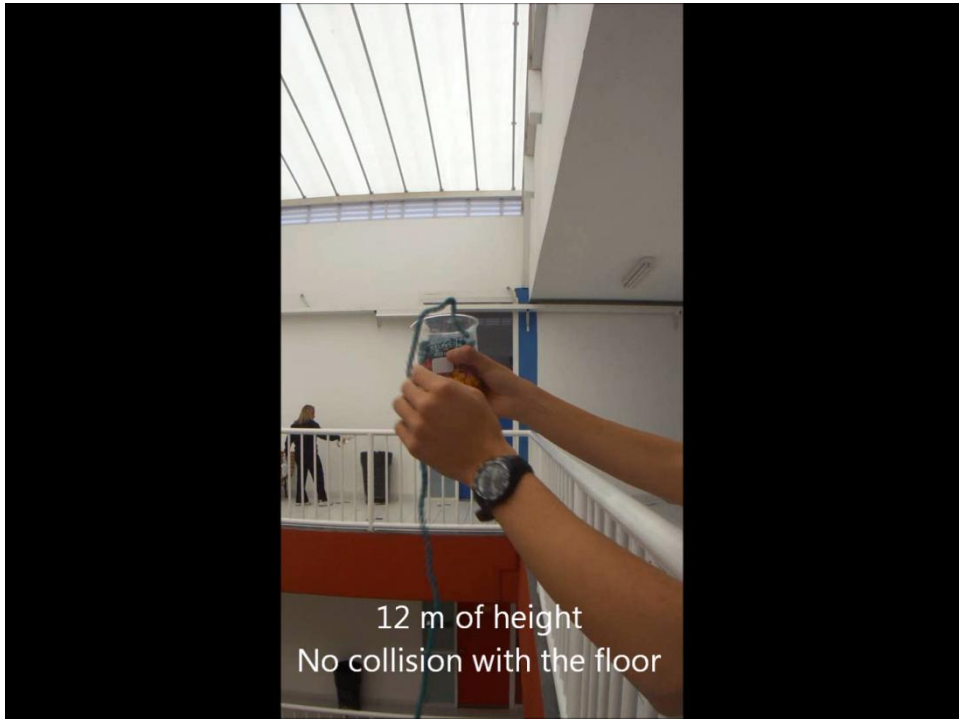
$$y = a \cdot \cosh\left(\frac{x}{a}\right)$$

$$y = a \cdot \frac{e^{\frac{x}{a}} - e^{-\frac{x}{a}}}{2}$$



Detachment without collision

- The chain is released from 12 m of height.
- There's no collision with the floor, but the detachment occurs.
- Slow motion video:



Experiment 2: initial energy calculation

Theoretical value

$$U = 2Mg\frac{h^2}{L}$$

Chain mass (M): 270 g
Chain length (L): 2.7 m
Beaker height (h): 14 cm

Experimental value

- The chain is pulled from rest bead by bead.
- h = half the minimum length pulled for motion beginning.
- Value gotten applying last formula.
- Acquired data: next slide

Experiment 2: initial energy calculation

	Half the minimum length
	0.15
	0.14
	0.13
	0.14
	0.14
	0.14
	0.15
Average	0.14
Theoretical	0.14
Standard Deviation	0.007

Half the minimum length measured