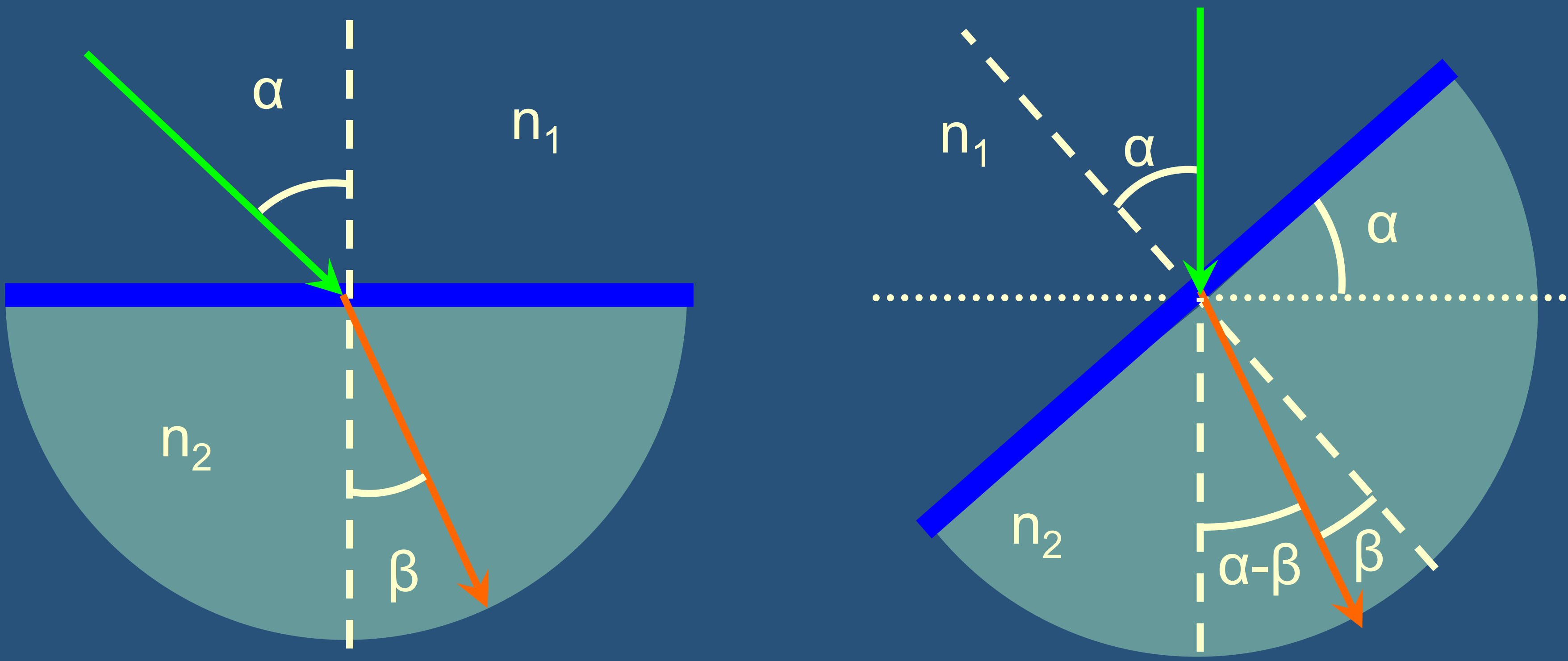
# Bright waves

Illuminate a water tank. When there are waves on the water surface, you can see bright and dark patterns on the bottom of the tank. Study the relation between the waves and the pattern.

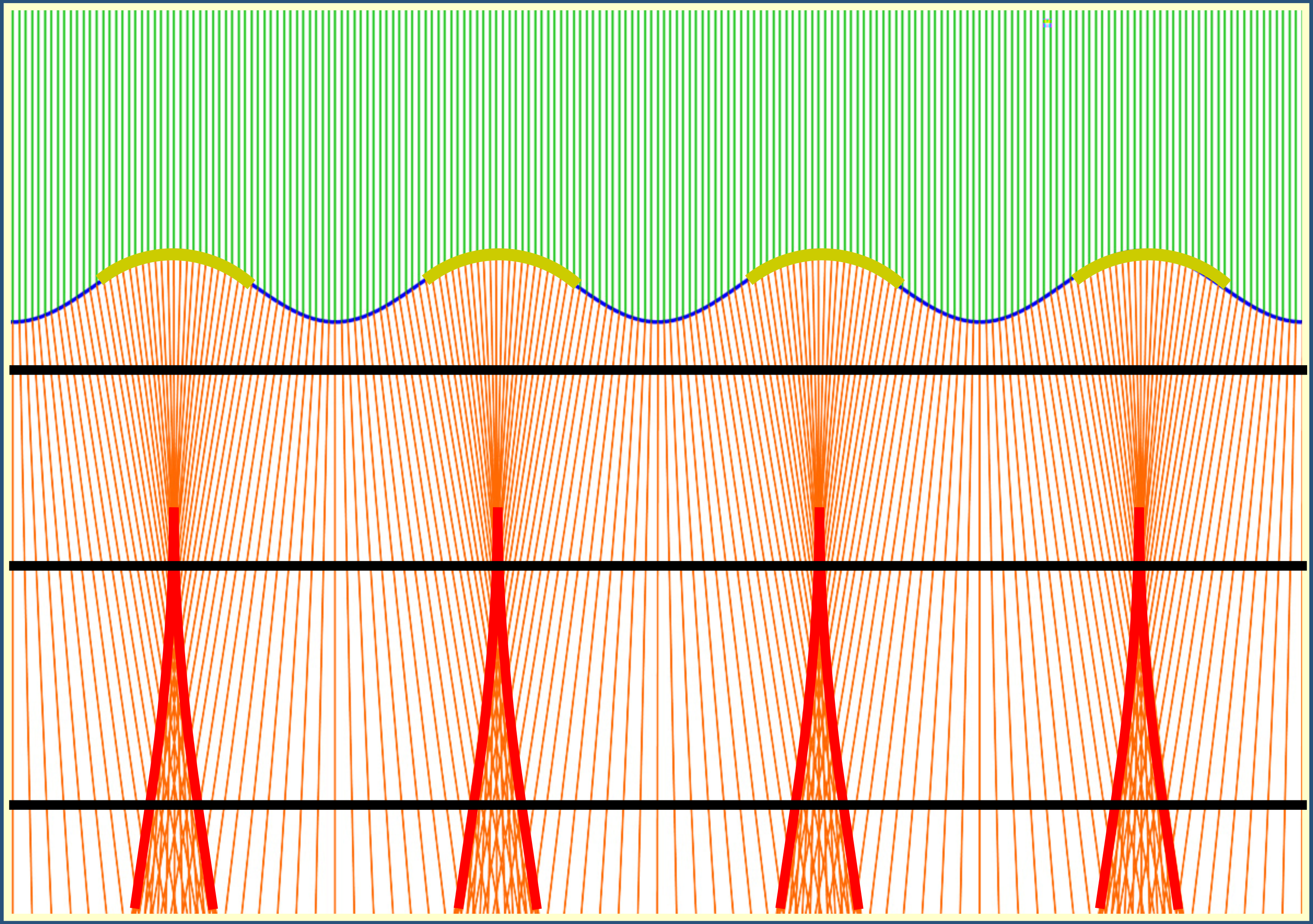
Vyacheslav Matyunin, Russia

# Snell's law of refraction

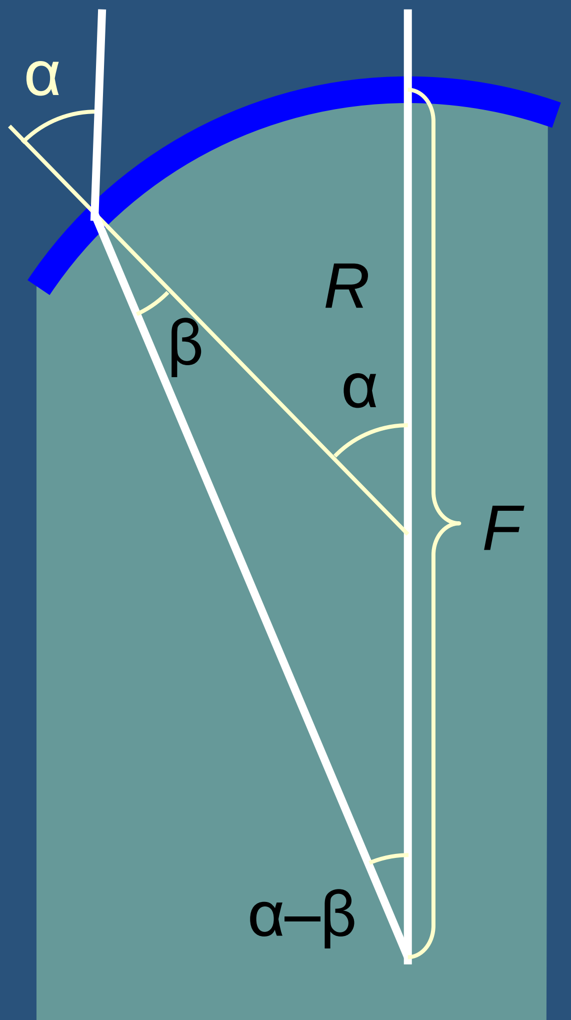



$$\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}$$

# Caustics as envelopes of light rays

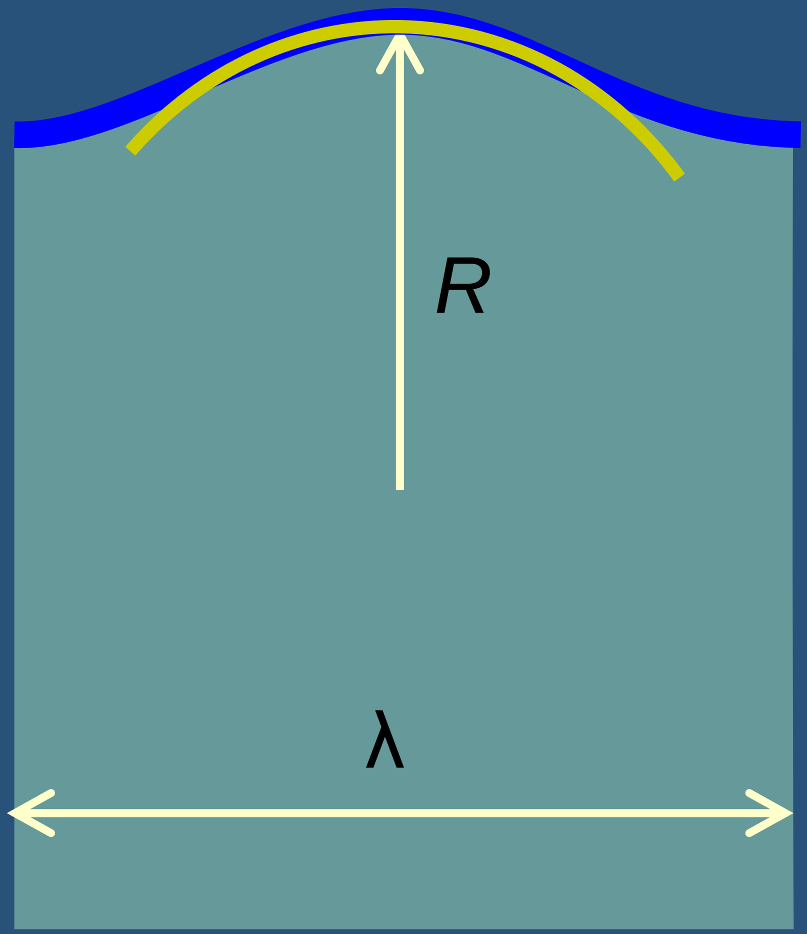


# Focal distance




- In the approximation of small angles:
- $\alpha = n\beta$
- $F(\alpha - \beta) = R\alpha$
- So $F(n - 1) = Rn$
- If $n = 4/3$, than $F = 4R$

# Radius of curvature of the sine wave

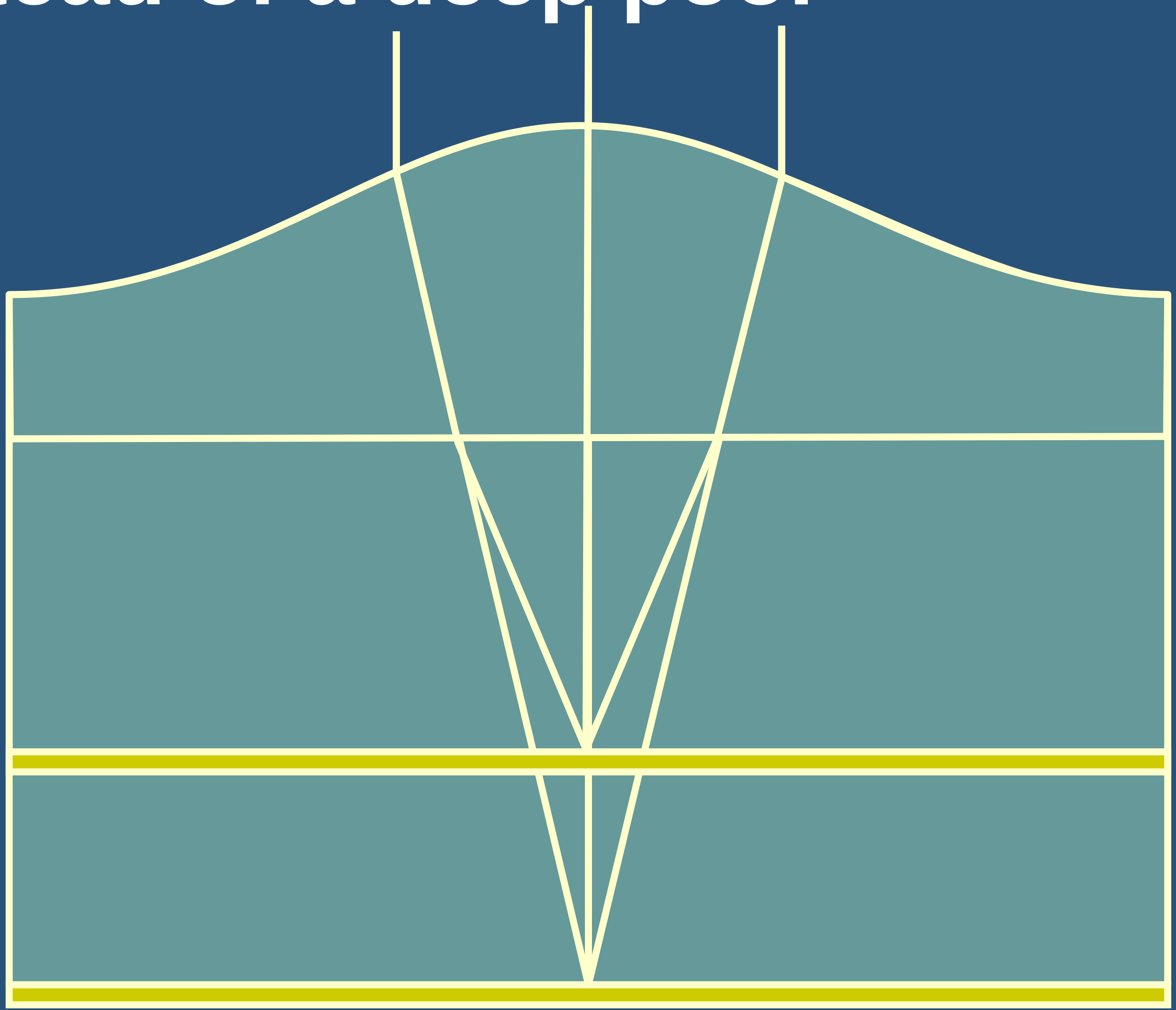



$$f(x) = A\cos\left(\frac{2\pi x}{\lambda}\right)$$

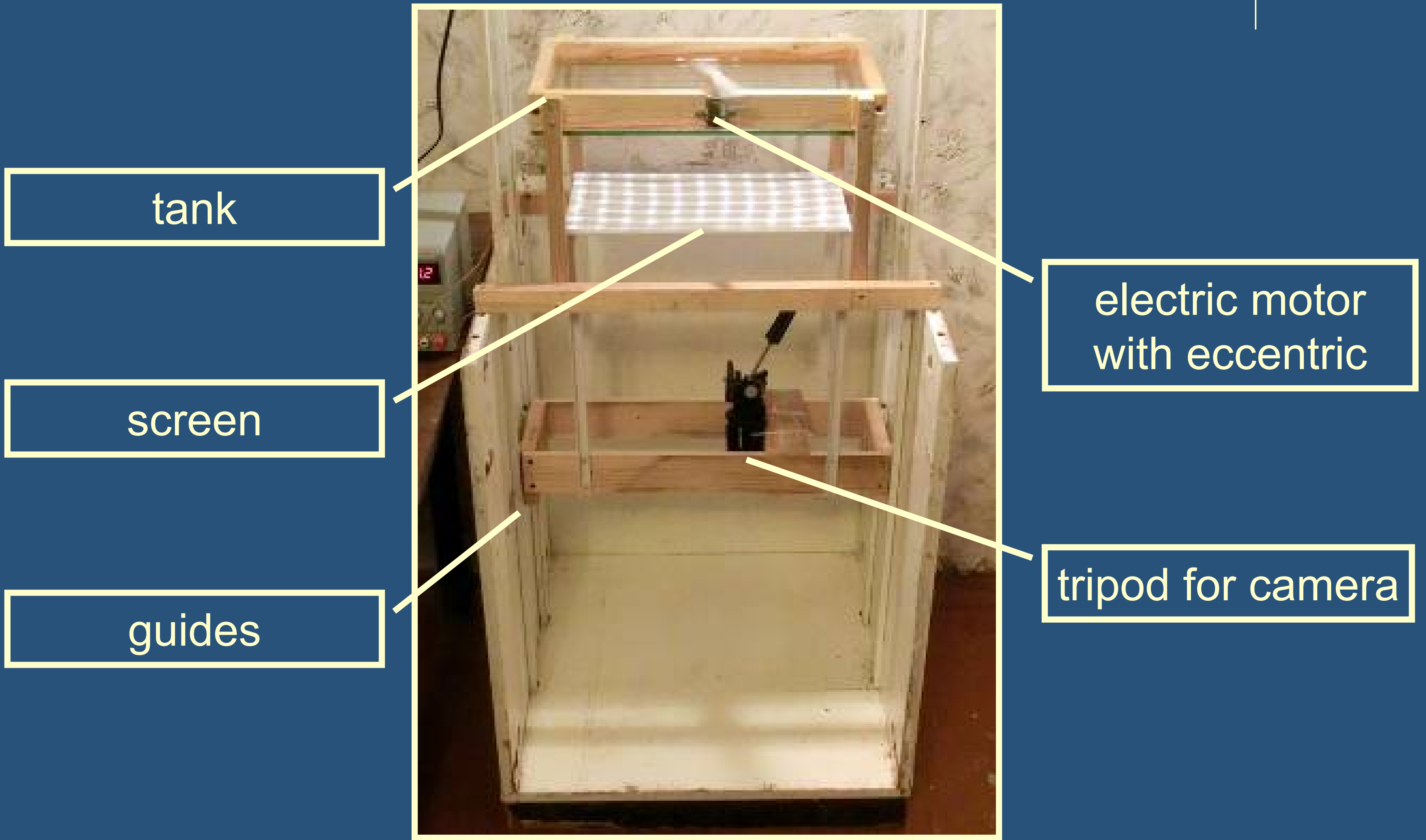
$$\frac{1}{R} = A\cdot\left(\frac{2\pi}{\lambda}\right)^2$$

$$R = \frac{\lambda^2}{4\pi^2 A} \approx \frac{\lambda^2}{40A}$$

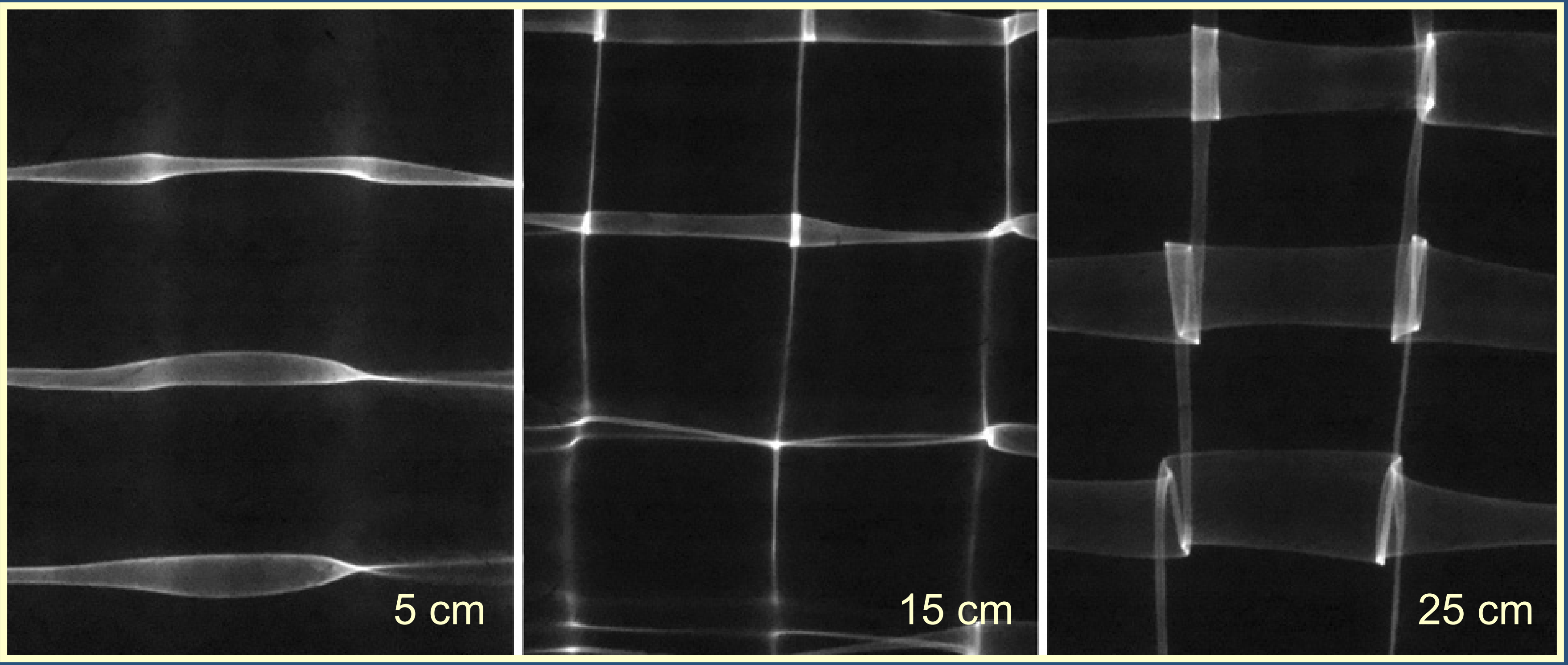
# Shallow tank instead of a deep pool



# Experimental setup

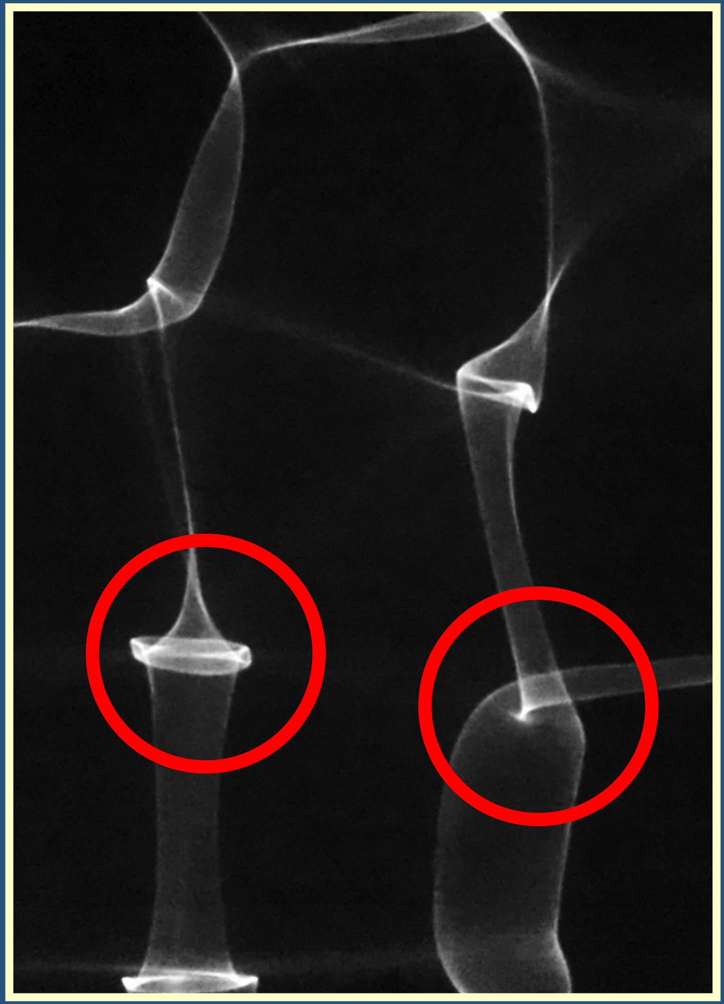


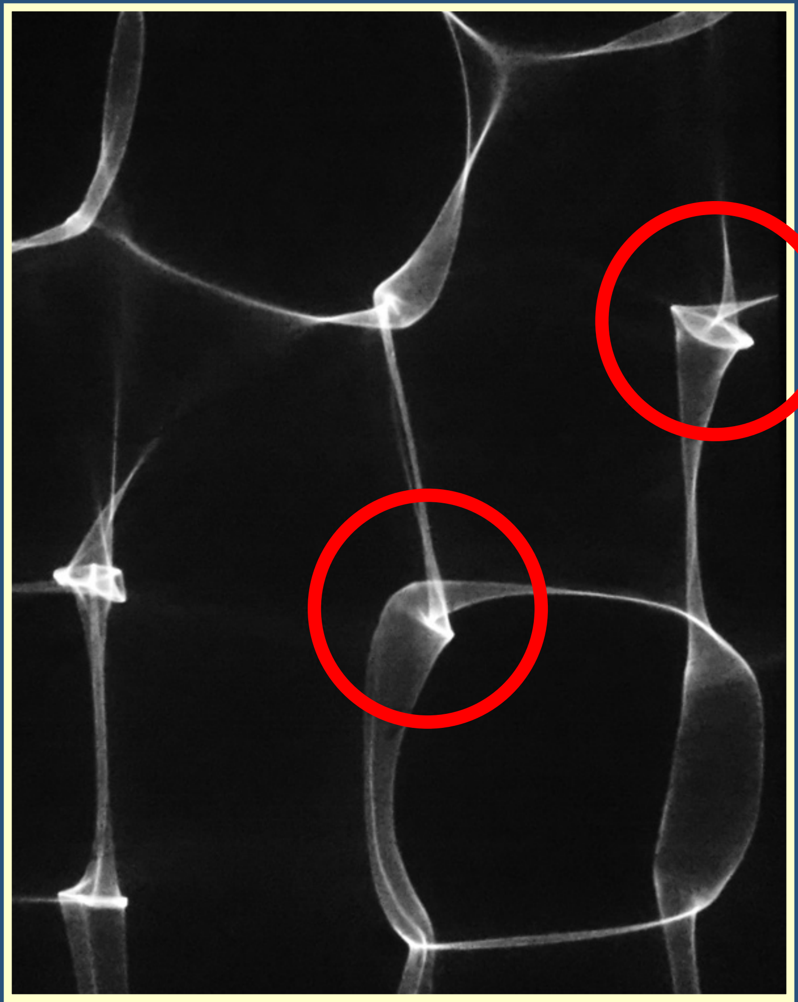
# Caustics of standing waves in a rectangular tank



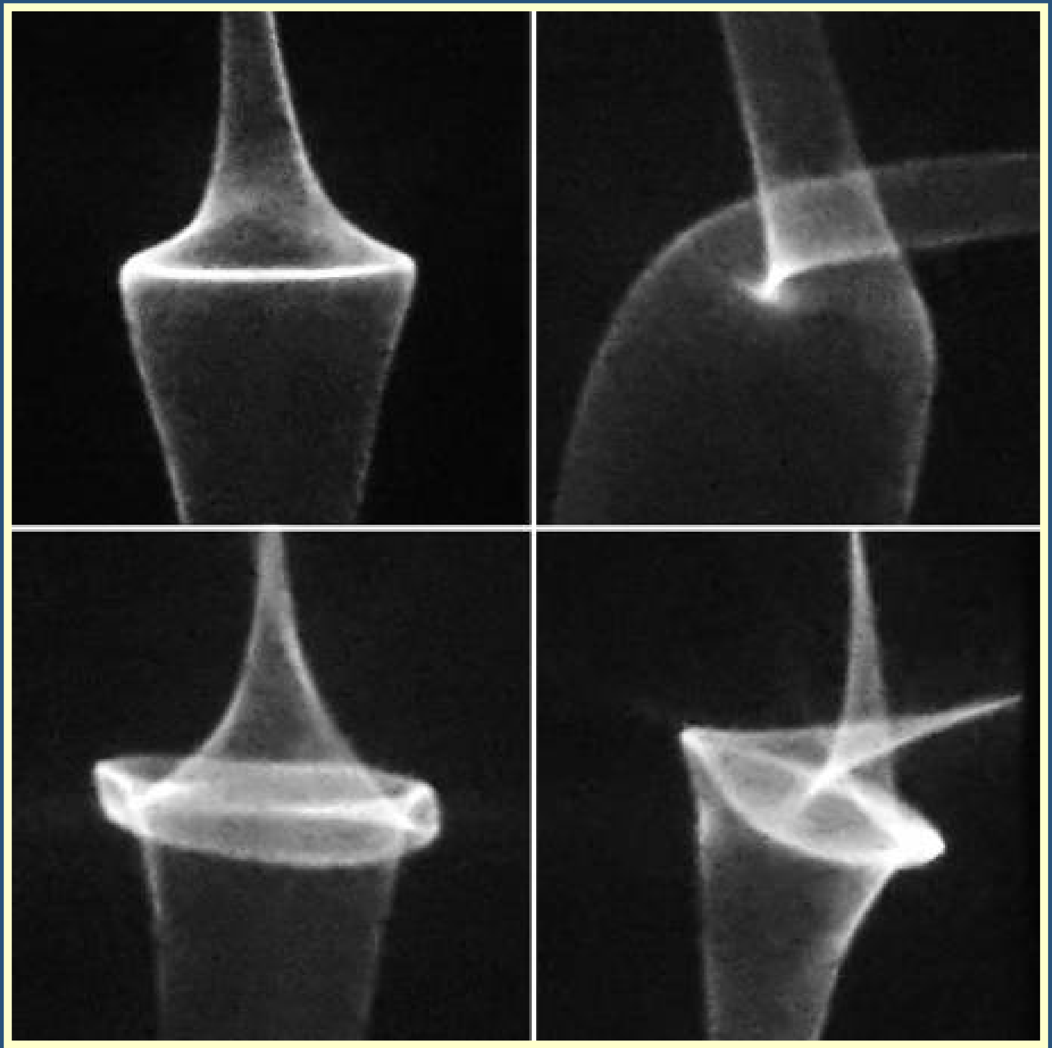

Depth = 2 cm, frequency = 10 Hz, wavelength = 3 cm.

# Disturbed rectangular network

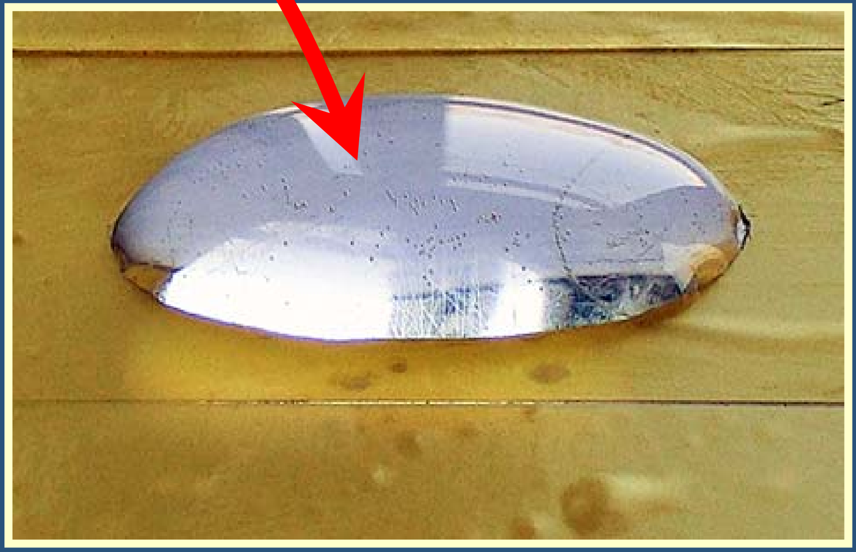




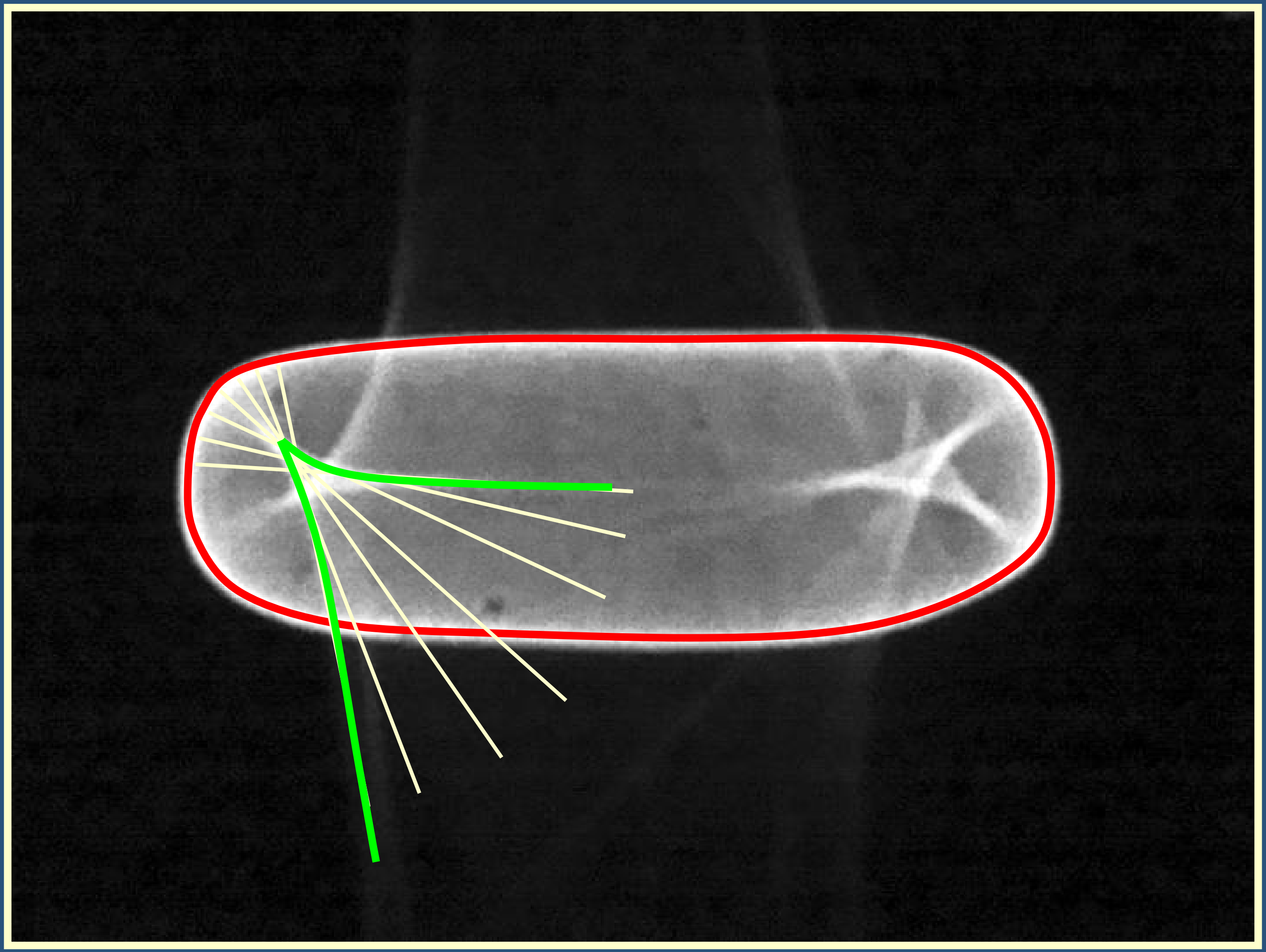
# Typical nod patterns



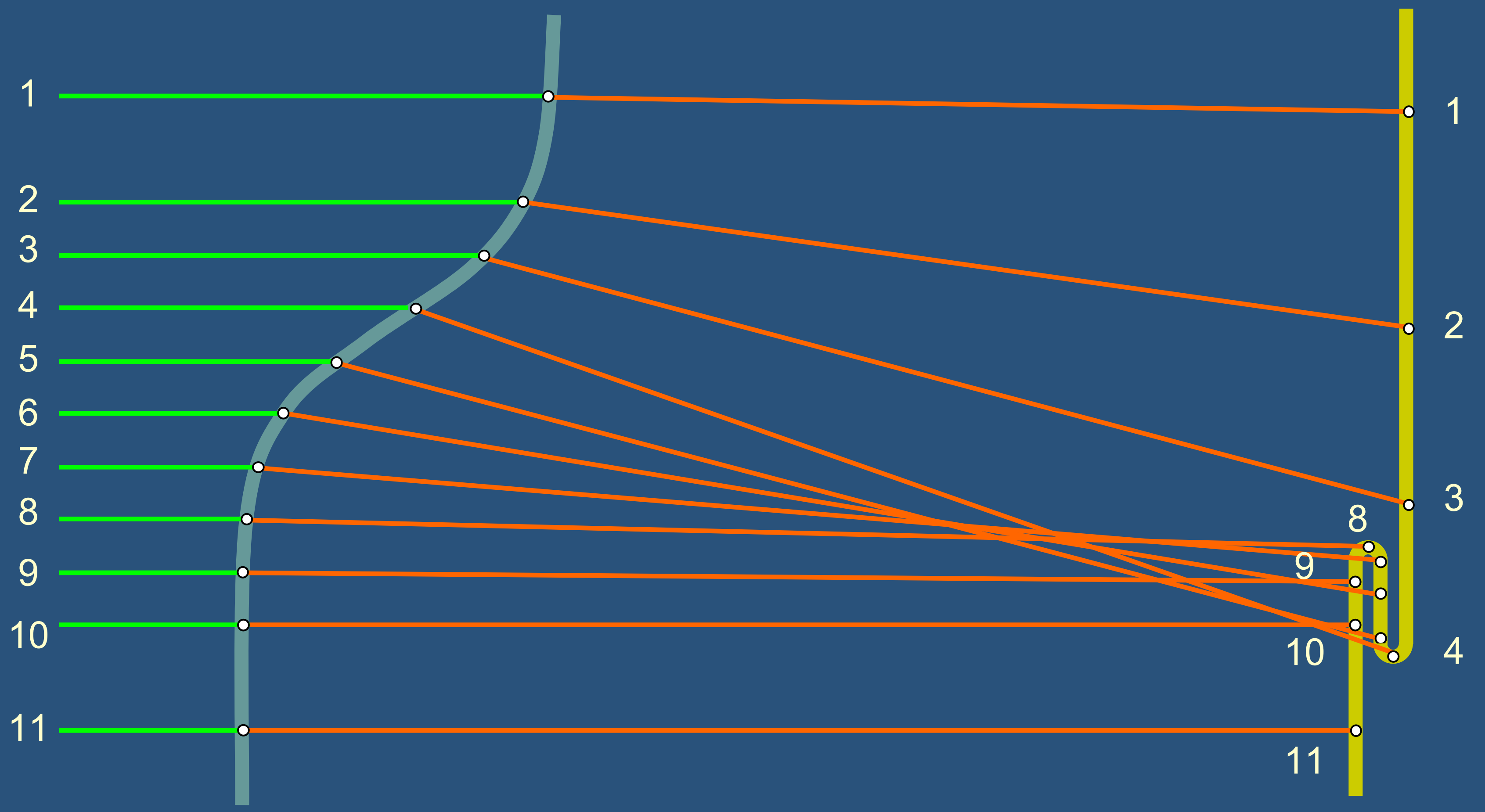
# Intersection of two wave crests => an oval drop



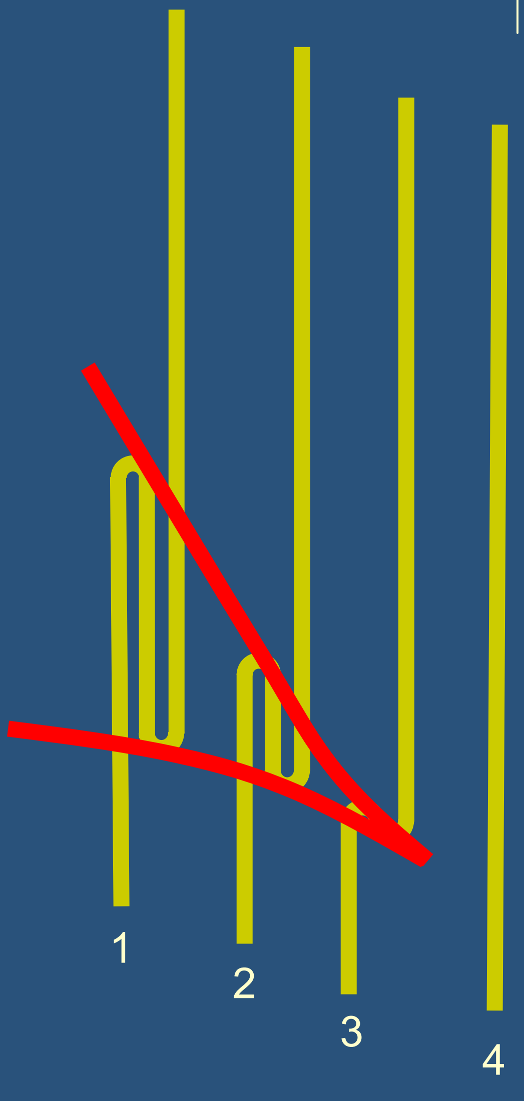
# Caustic pattern of an oval drop



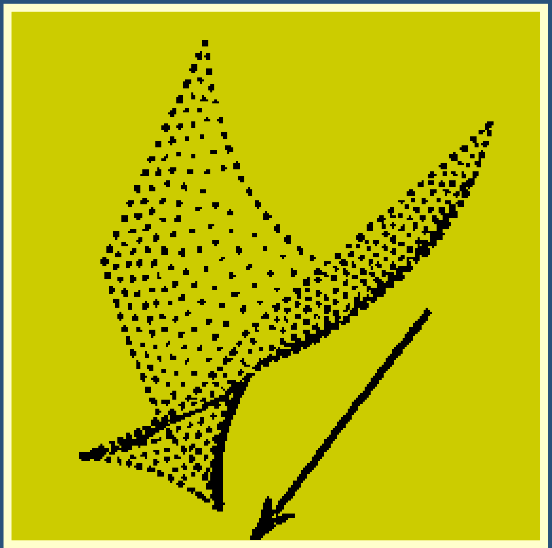
# Catastrophe theory: caustic as a fold catastrophe



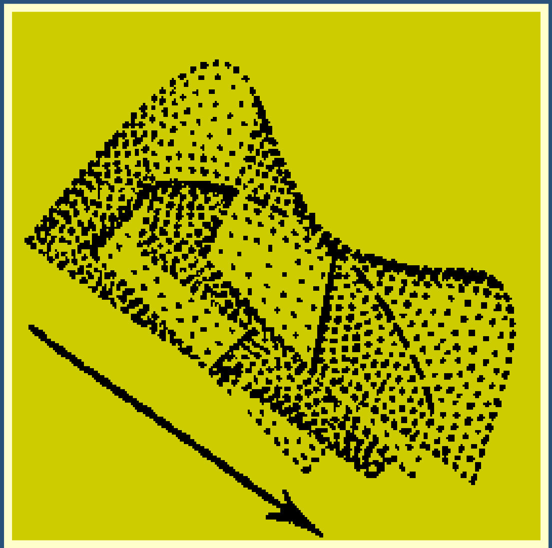
# Cusp catastrophe as junction of two folds

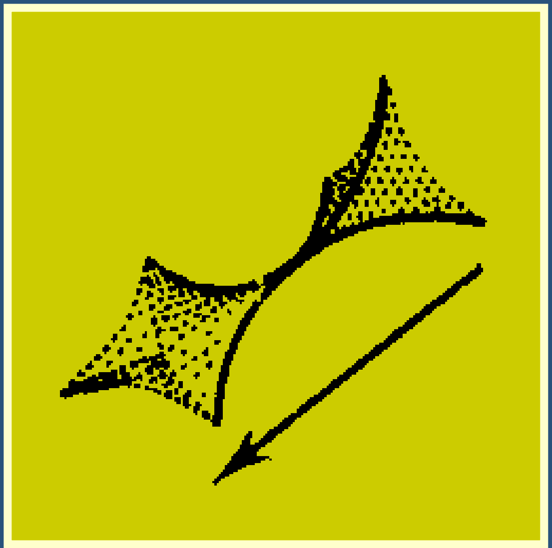
# Three typical catastrophes in 3-D space



Swallowtail

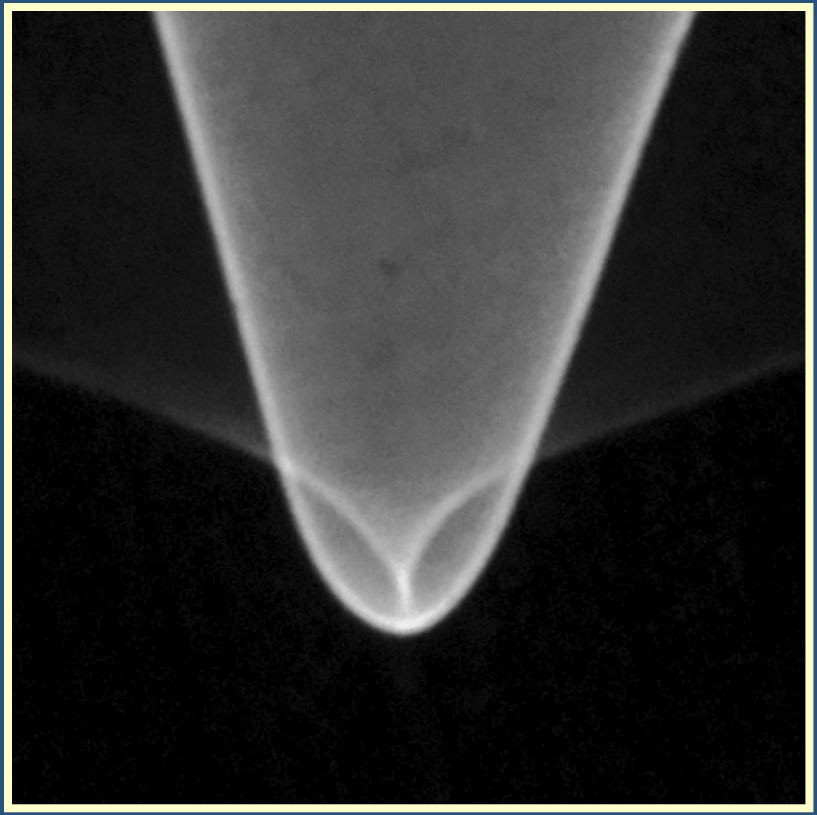


Hyperbolic umbilic

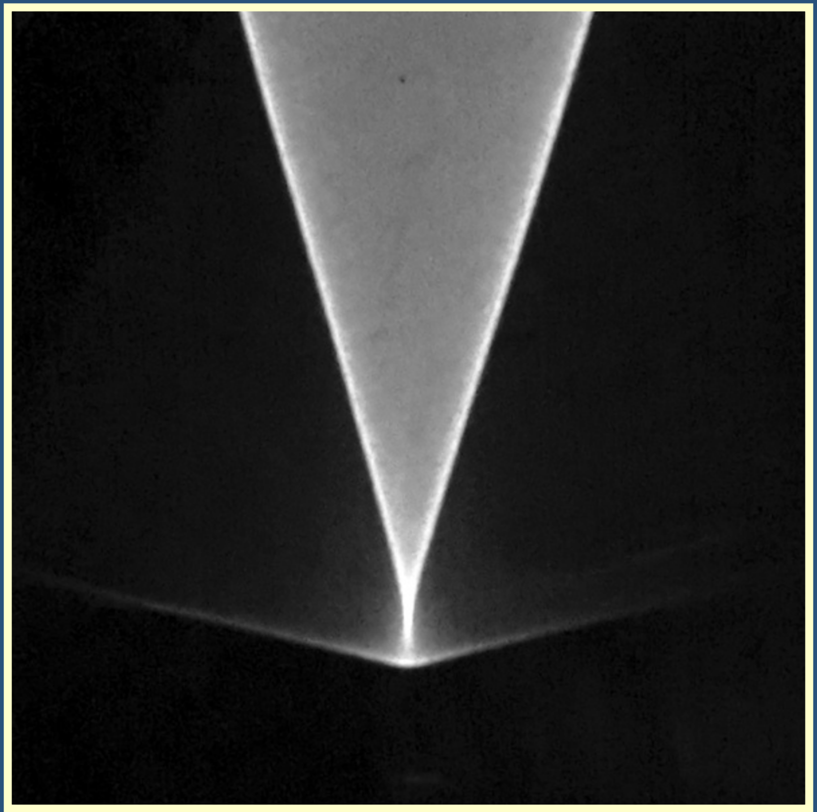


Elliptic umbilic

# Hyperbolic umbilic

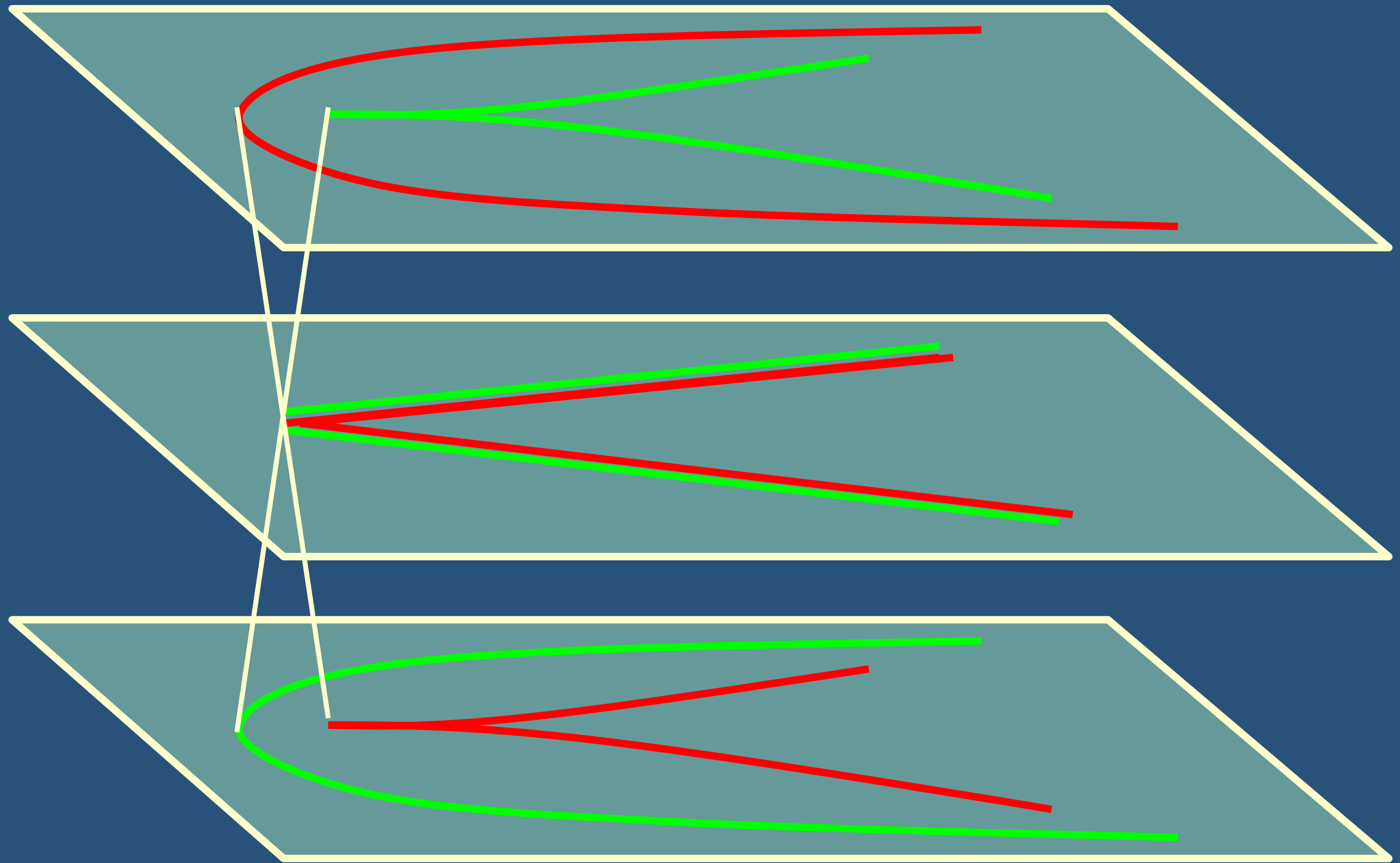


Small depth

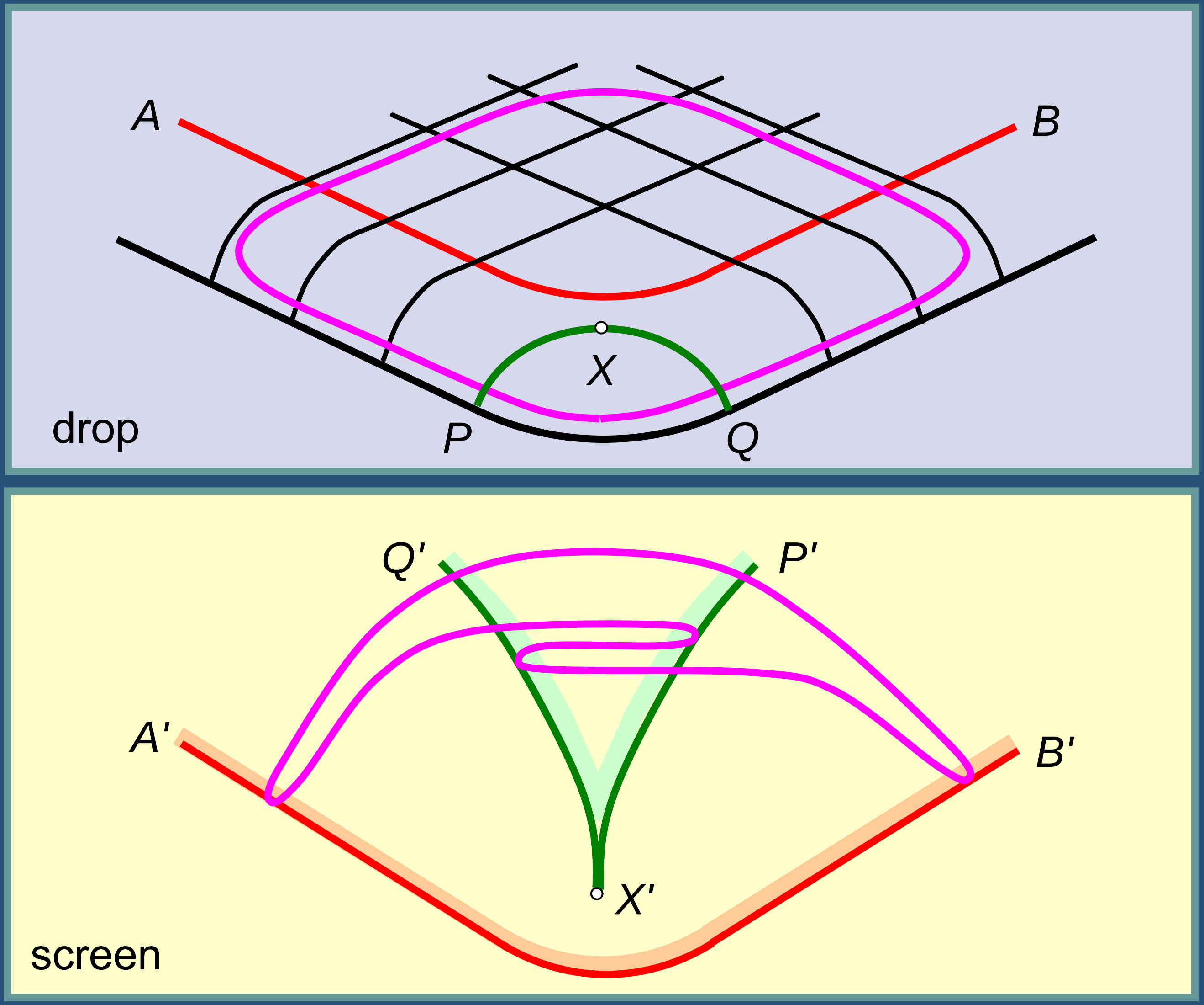


Large depth

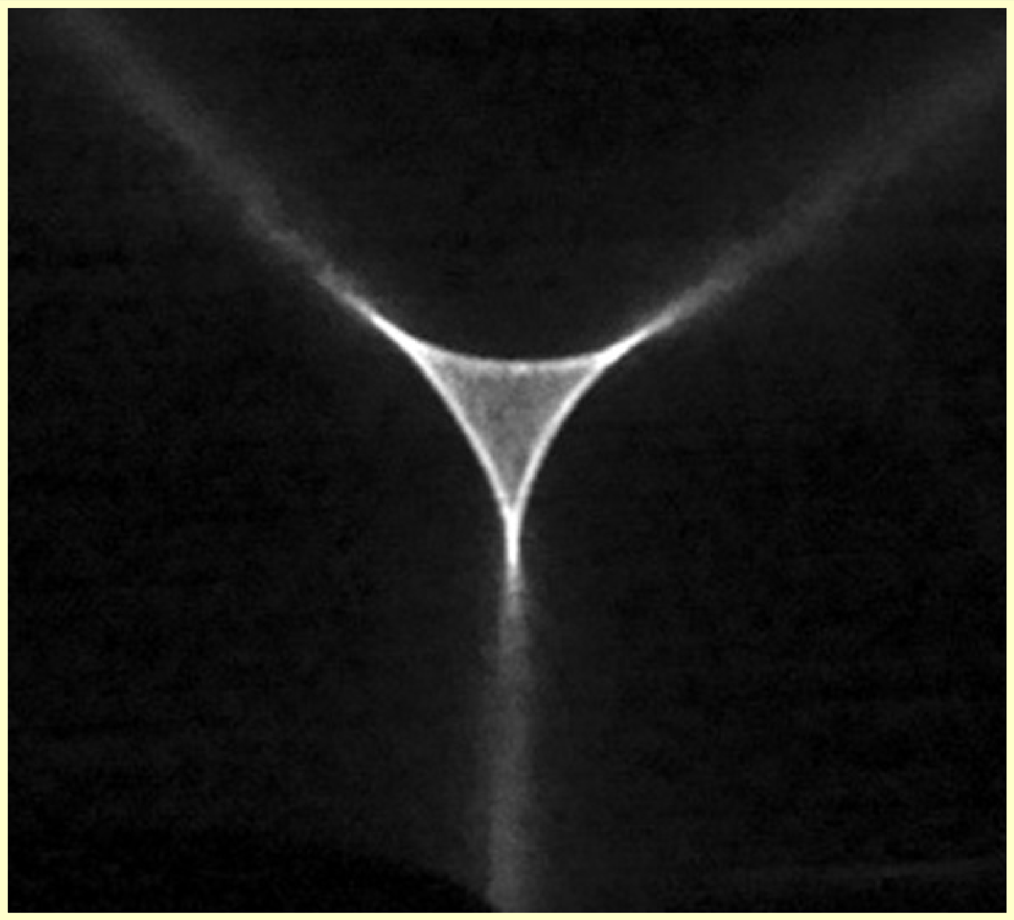
# Parallel sections of the hyperbolic umbilic



# Hyperbolic umbilic: what shape has a drop?

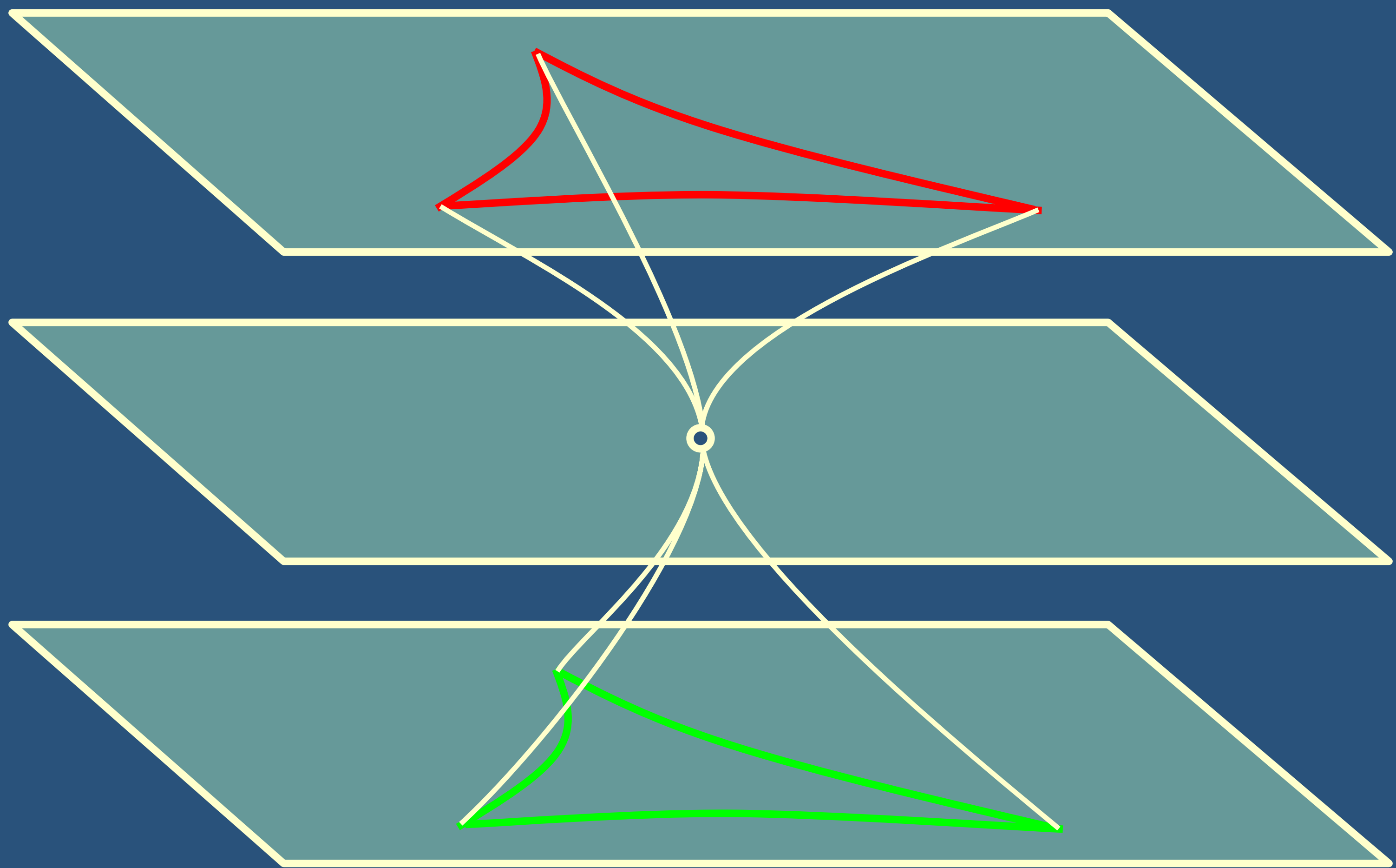


# Elliptic umbilic

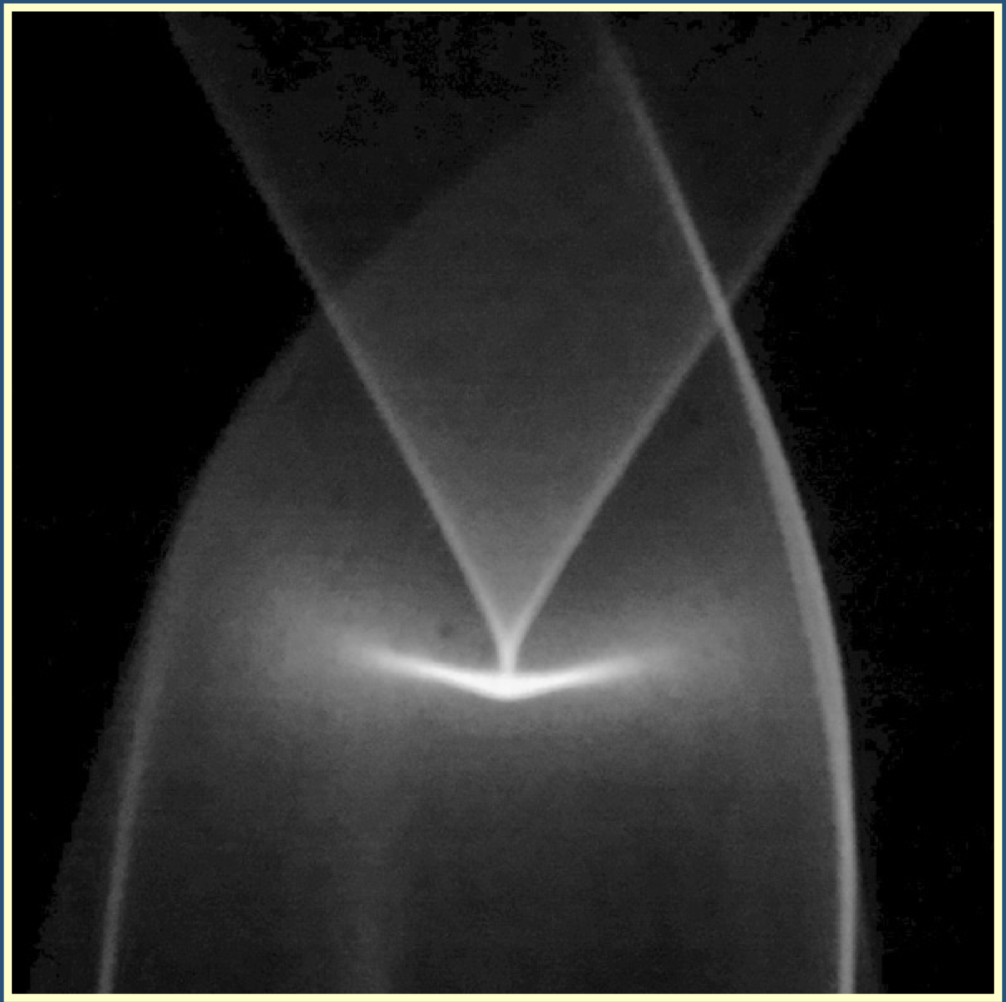


Small depth

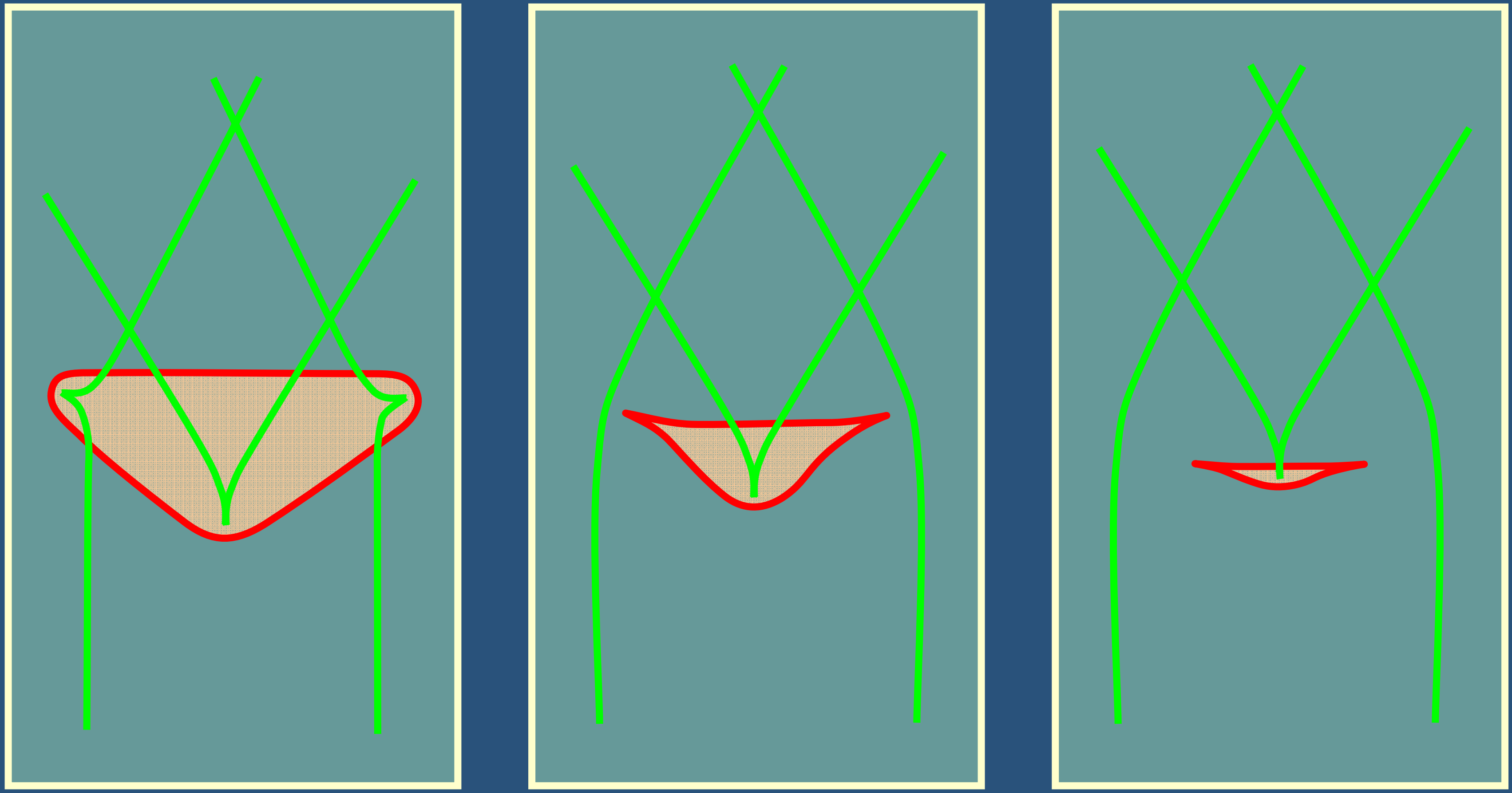
# Parallel sections of the elliptic umbilic



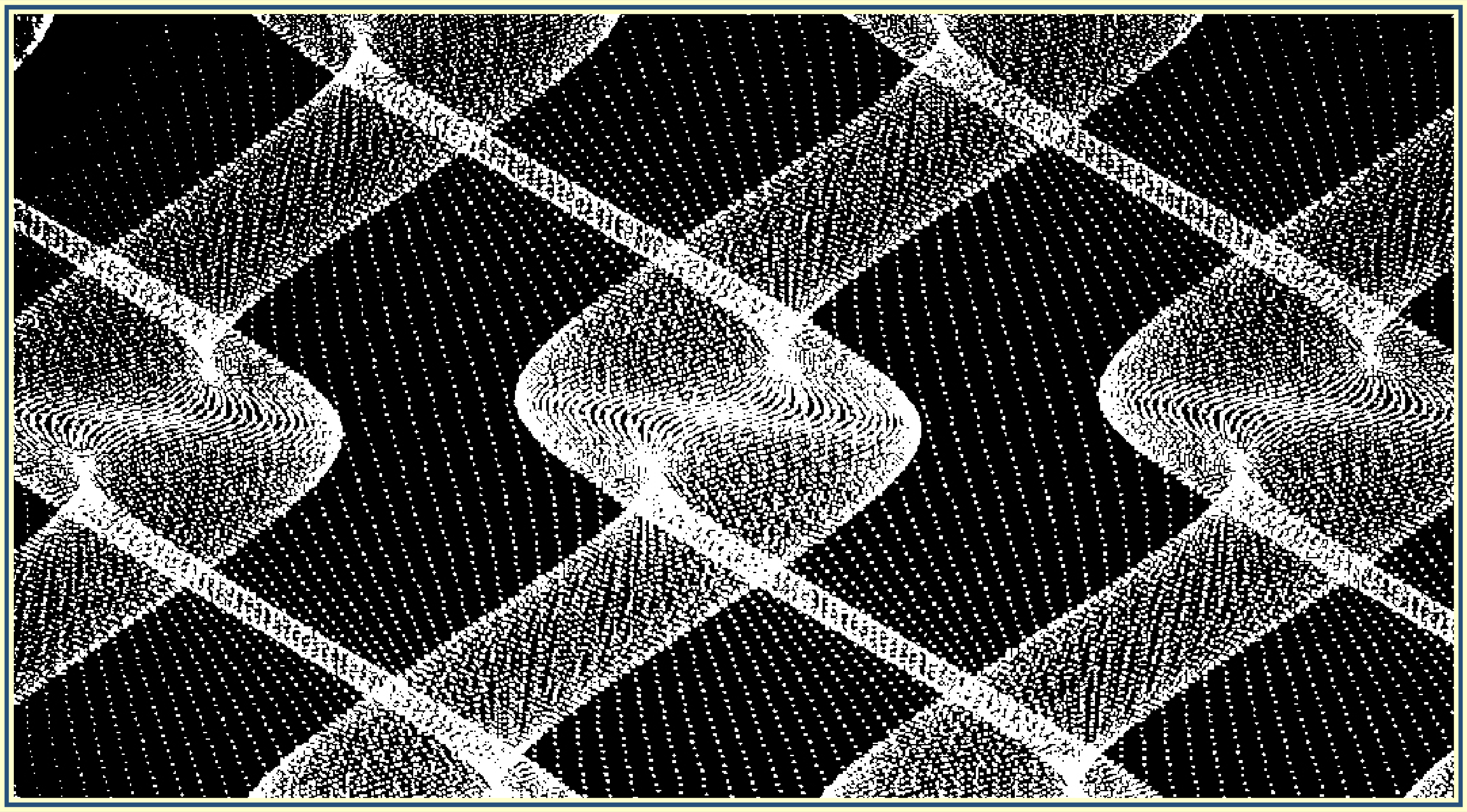
# What shape has a drop that produces this pattern?



# Solution:
# a triangular drop

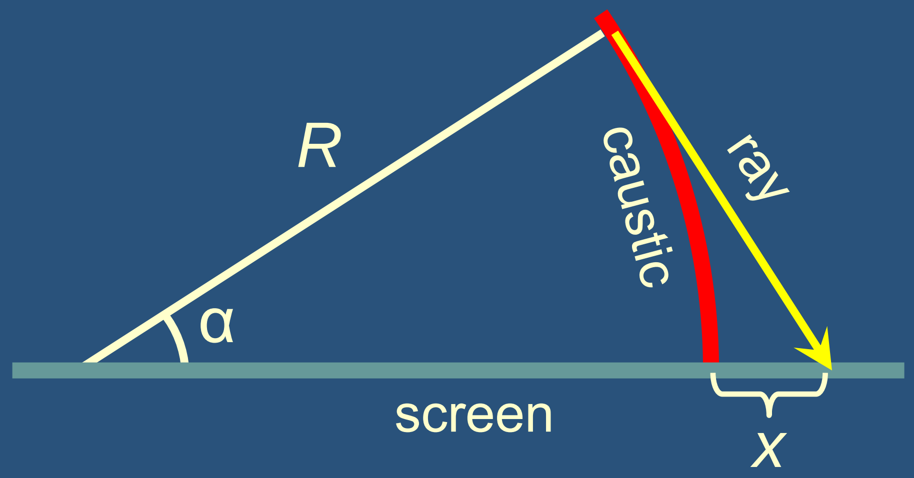


# Computer simulation: two sine waves



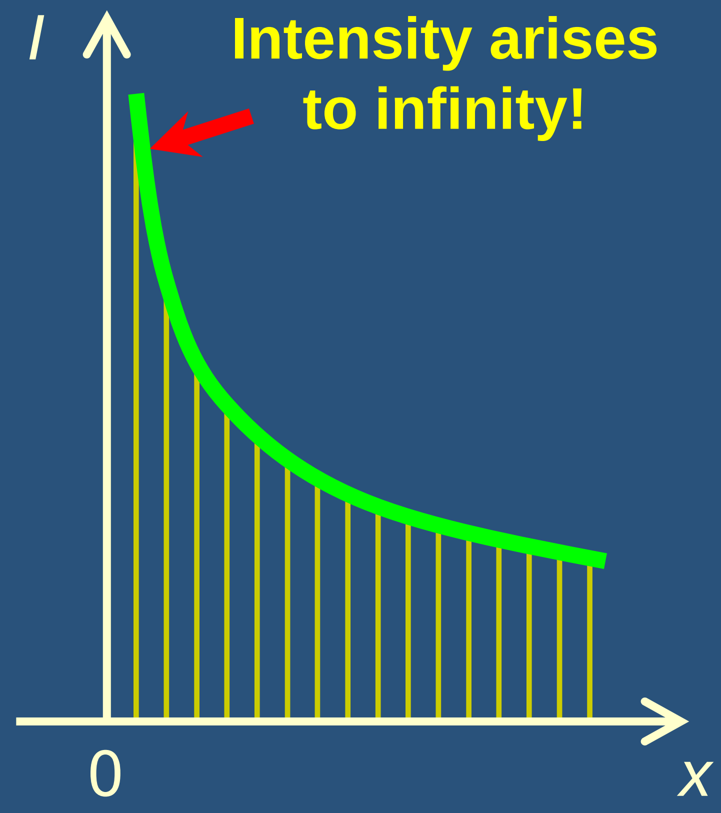
# Computer simulation: hyperbolic umbilic

# Intensity of light near the caustic

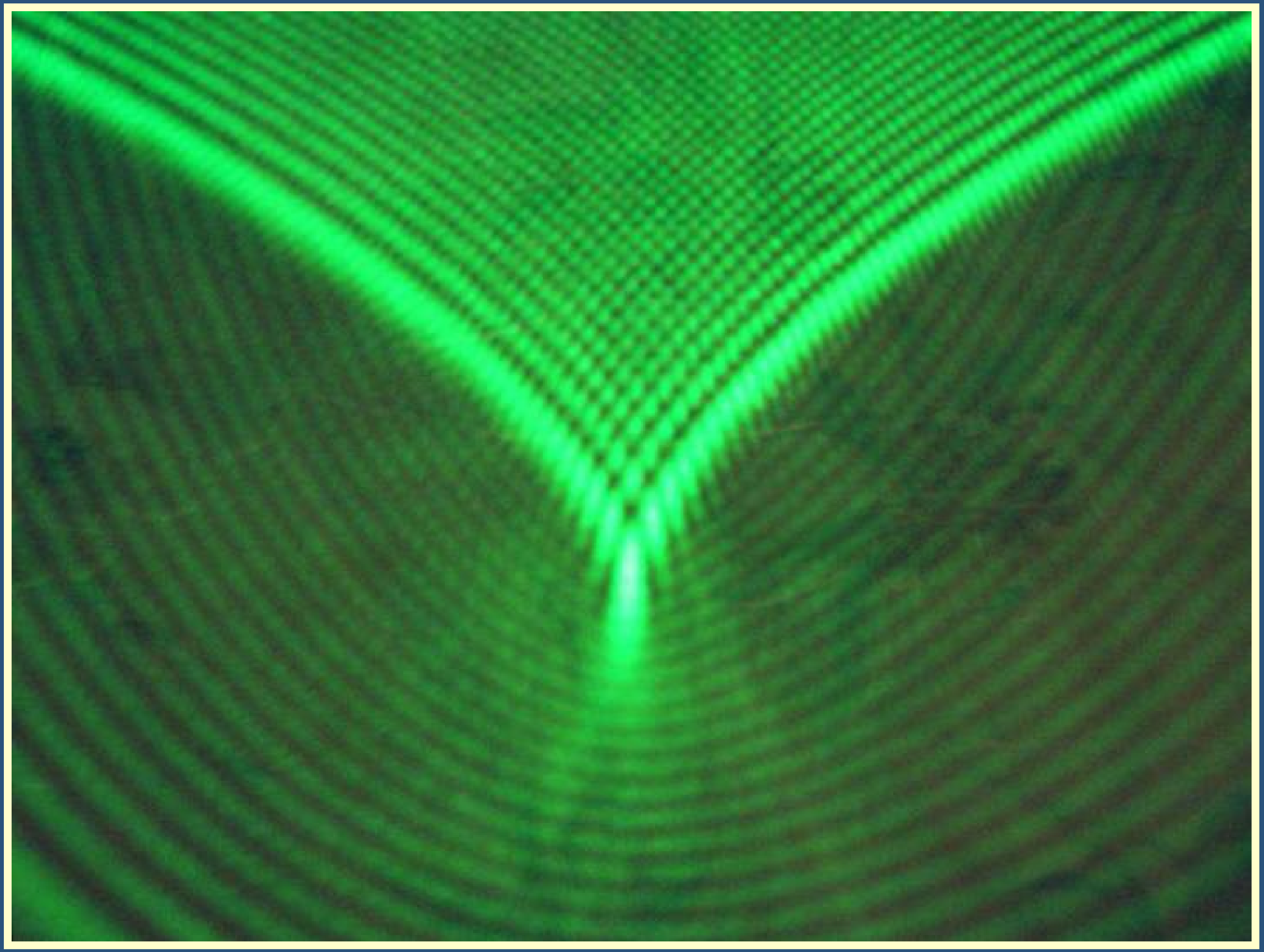



$$x \sim \alpha^2$$

$$I(x) \sim \frac{d\alpha}{dx} \sim \frac{1}{\sqrt{x}}$$



# Solving this paradox in wave optics



# Summary

- Elementary explanation using caustics
- Experimental setup and basic experiments
- Typical patterns
- Turning to a static model: an oval drop
- Connection with the catastrophe theory
- Obtaining umbilic
- Computer simulation
- Wave optics effects

# Bibliography


- Arnold V. I. (1992) *Catastrophe theory*.
- Nye J. F. (1999) *Natural focusing and fine structure of light: Caustics and wave dislocations.*
- Poston T., Stewart I. (1978) *Catastrophe theory and its applications*.
- Upstill C. (1979) “Light caustics from rippling water”. *Proceedings of the Royal Society A*, **365**, 95–104