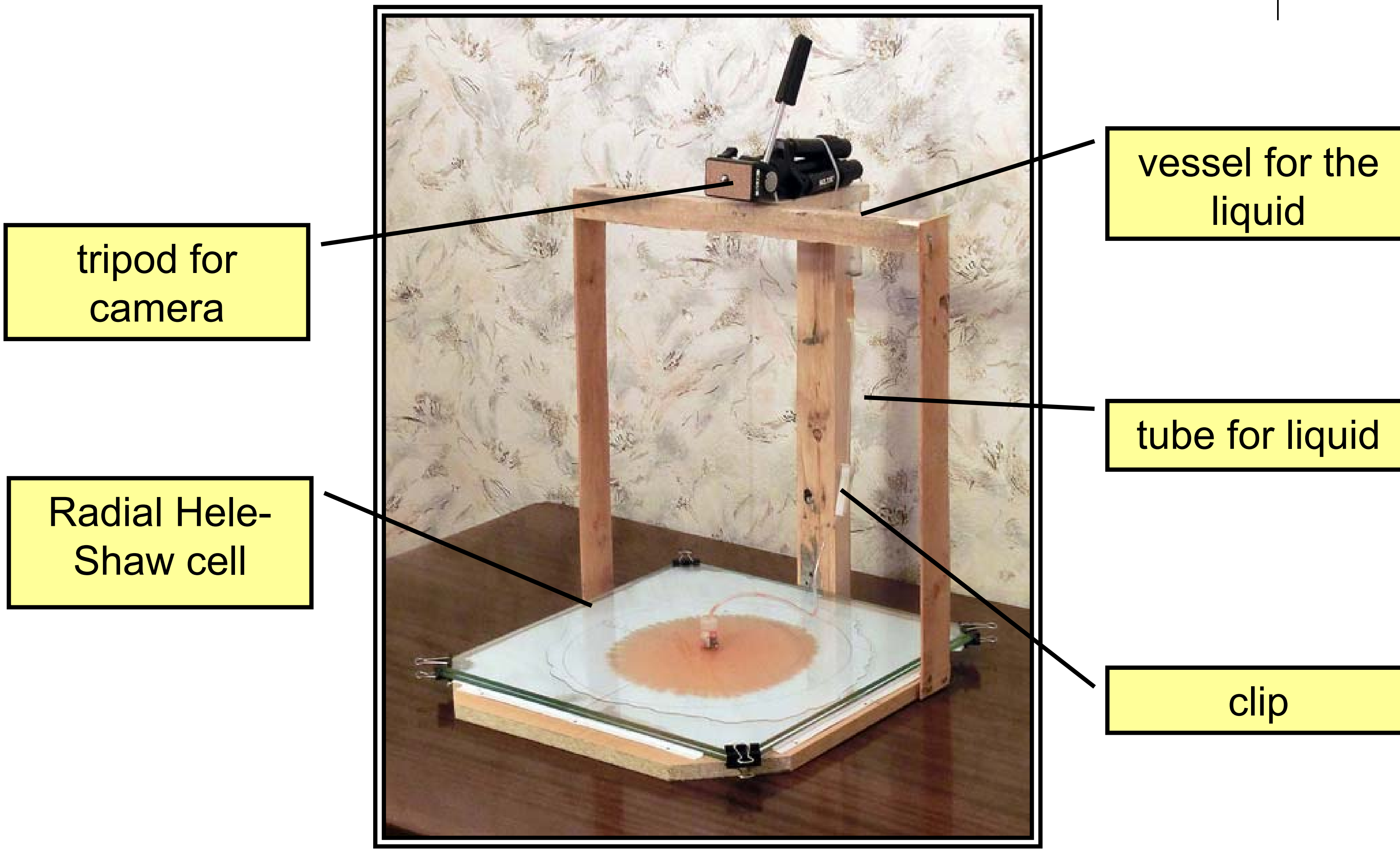
Flat flow

Fill a thin gap between two large transparent horizontal parallel plates with a liquid and make a little hole in the centre of one of the plates. Investigate the flow in such a cell, if a different liquid is injected through the hole.

Yuliya Sorochikhina, Russia

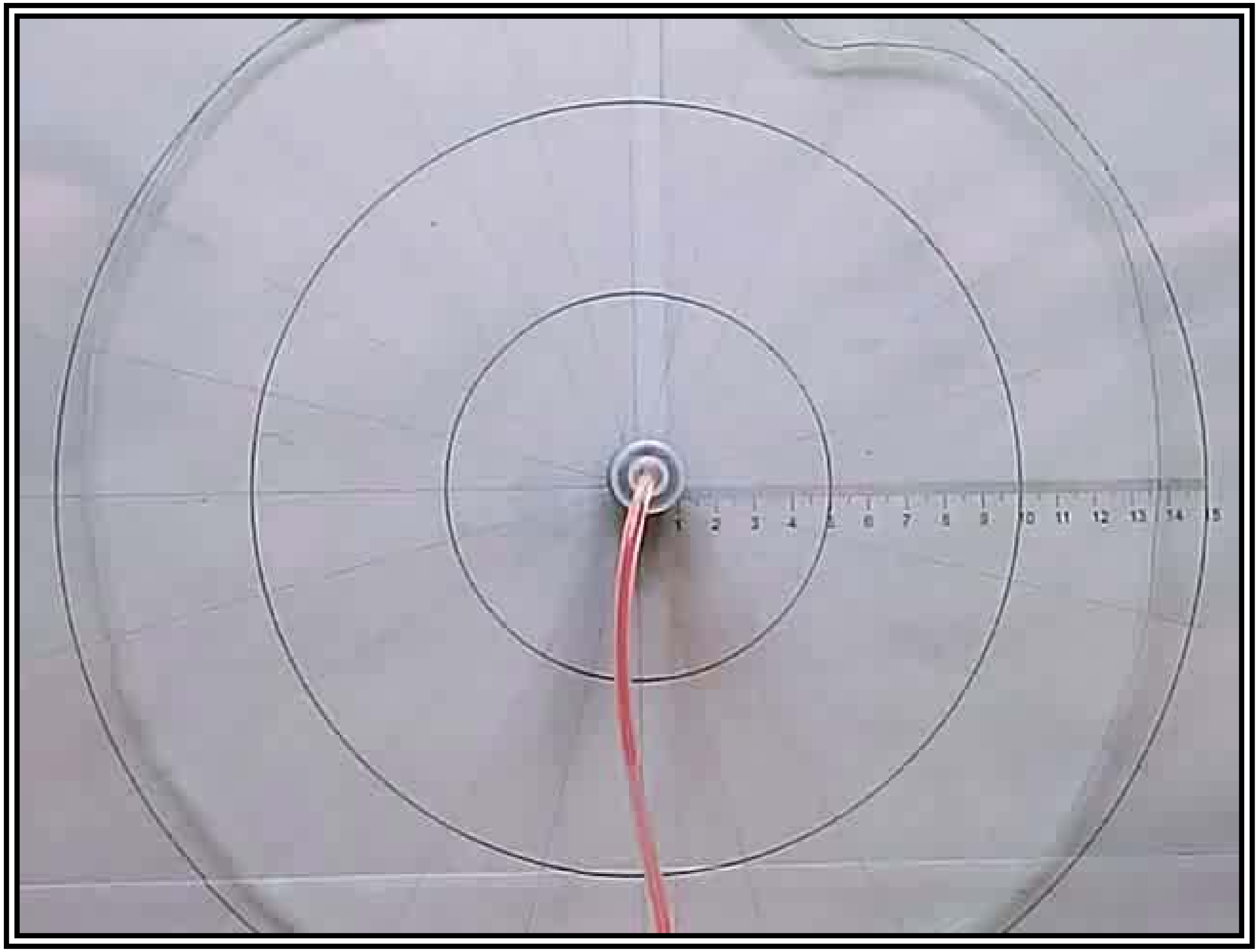
Experimental setup



Immiscible fluids: Air / glycerin

The gap = 1,2 mm

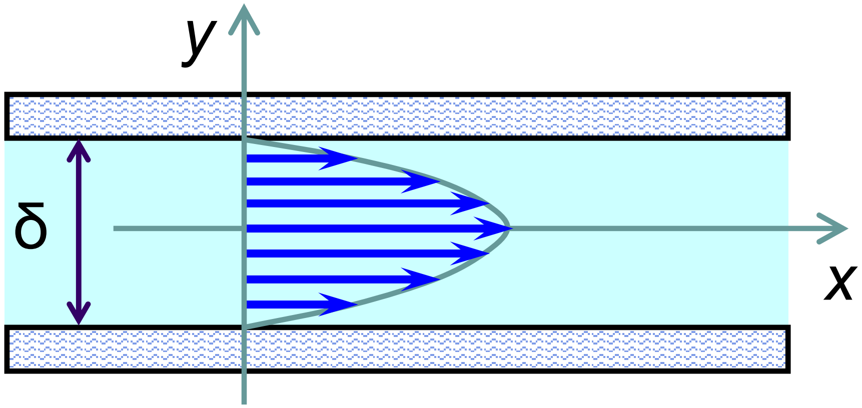
Miscible fluids: water / glycerin



The ratio of viscosities = 850 : 1. The gap = 1,2 mm. Pressure = 0,04 bar.

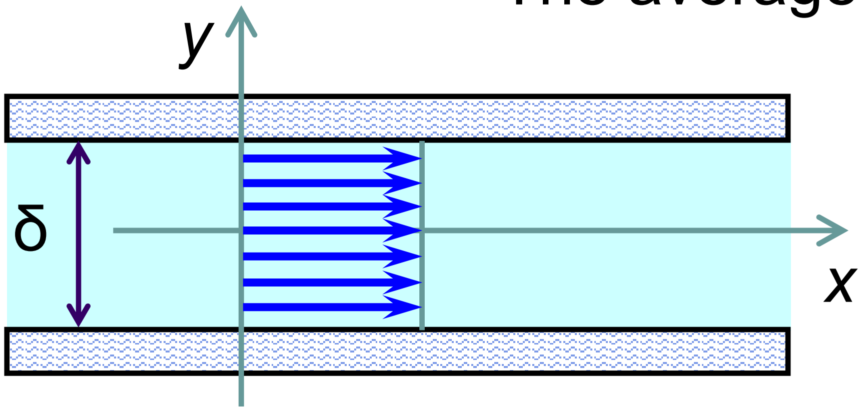
Darcy's law

Viscous flow in the gap between the plates:



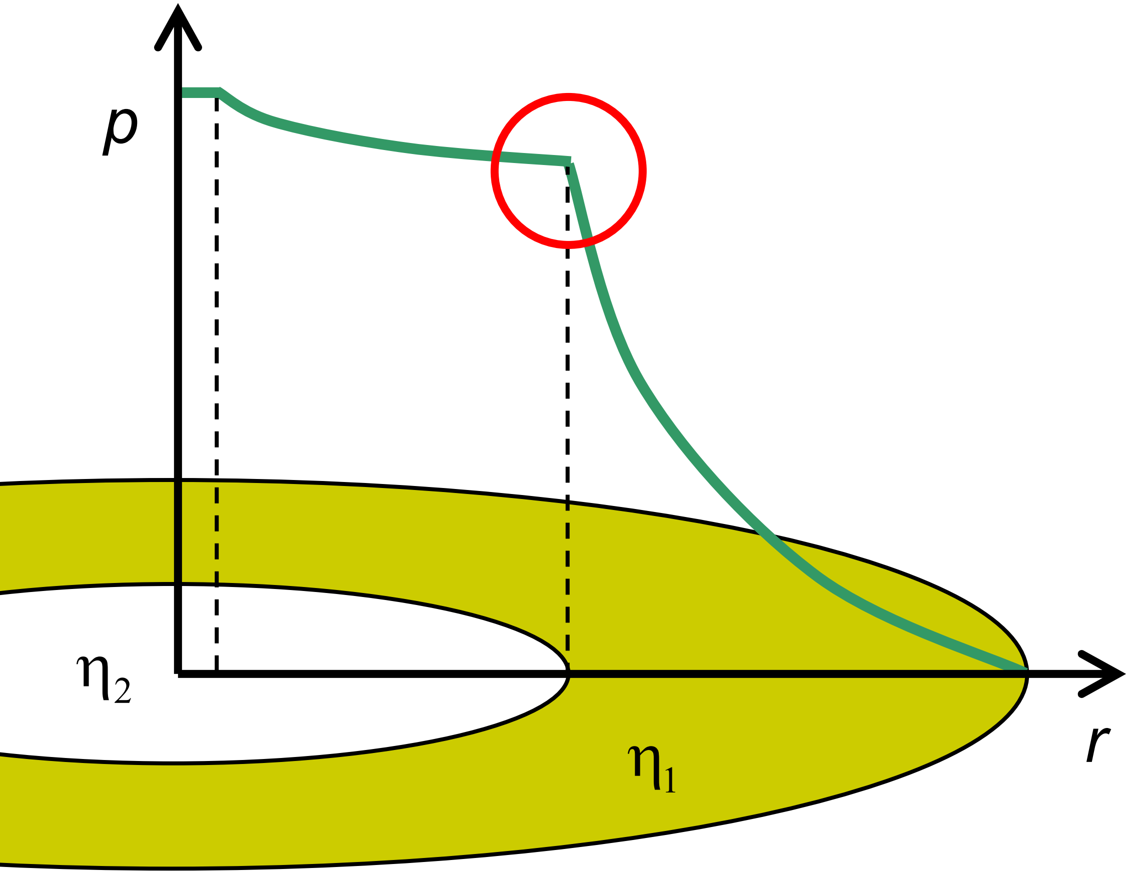
$$v(y) = -\frac{\delta^2}{8\eta}\cdot\frac{\Delta p}{\Delta x}\cdot\left(1-\frac{4y^2}{\delta^2}\right)$$

The average over the gap:



$$v = -\frac{\delta^2}{12\eta}\cdot\frac{dp}{dx}$$

Radial Hele-Shaw cell: pressure vs. radius



The cell is radial:

$$v \cdot 2\pi r = \text{const} = Q$$

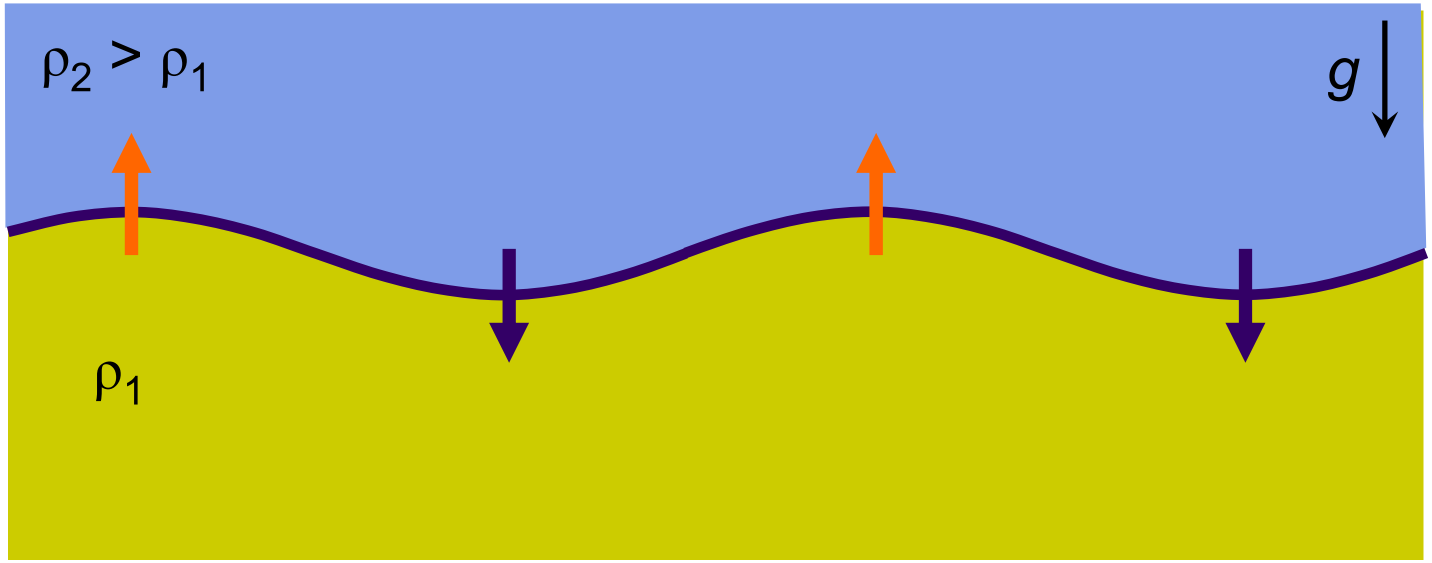
Darcy's law:

$$\frac{dp}{dr} = -\frac{12\eta v}{\delta^2} = -\frac{6\eta Q}{\pi\delta^2 r}$$

Integration:

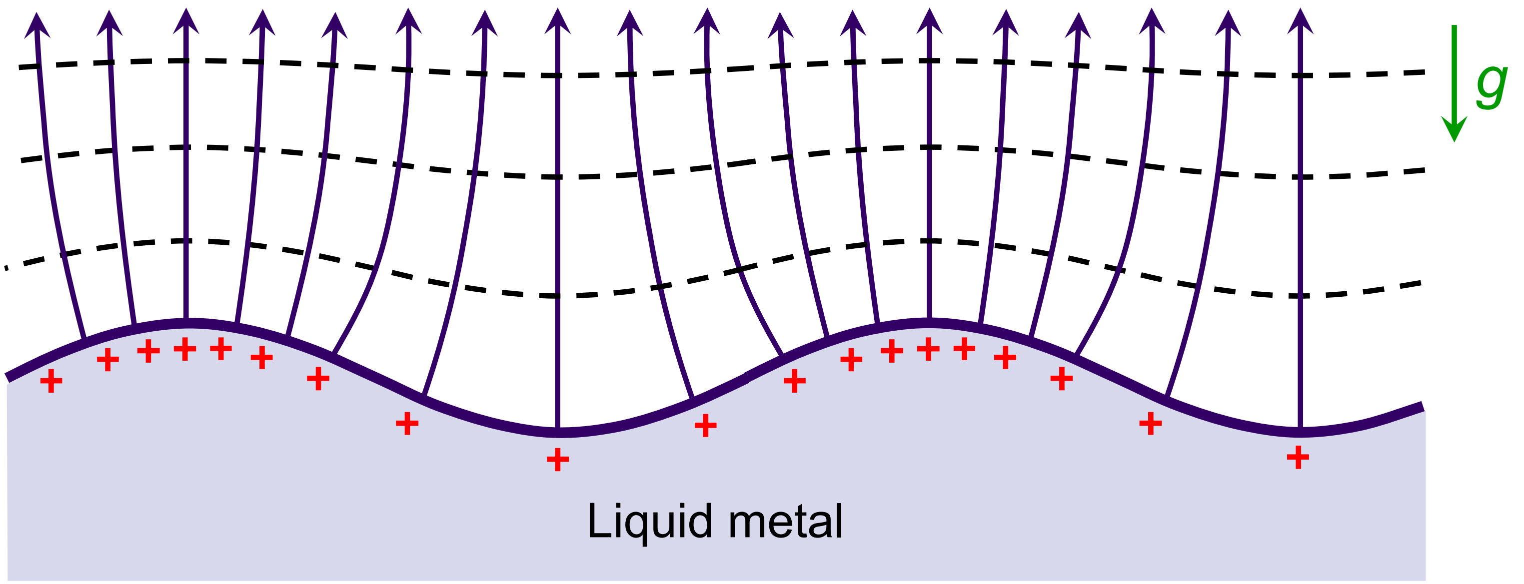
$$p(r) = \frac{6\eta Q}{\pi\delta^2} \cdot \ln\frac{r^*}{r}$$

Analogy 1: Rayleigh-Taylor instability



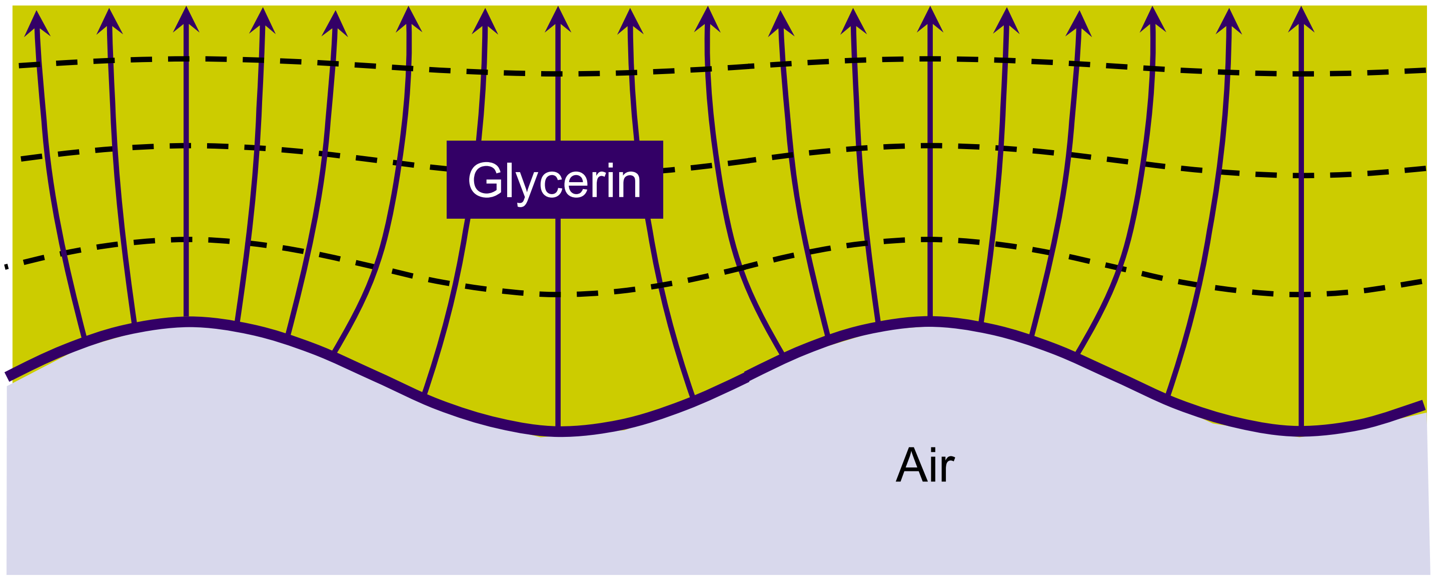
The shorter the wavelength, the faster disturbance increases.
Stabilizing factors for short waves:
A) immiscible fluids — surface tension;
B) miscible fluids — diffusion.

Analogy 2: Tonks-Frenkel instability



Above the fingers the electric field increases.
This leads to the further growth of the fingers.
Stabilizing factors:
A) for short waves — surface tension;
Б) for long waves — the force of gravity.

Saffman-Taylor instability



Before the protrusions the pressure gradient increases. This leads to the local velocity increasing and the further growth of protrusions.

Stabilizing factors for short waves:
A) Immiscible fluids — surface tension;
B) Miscible fluids — diffusion + 3D effects.

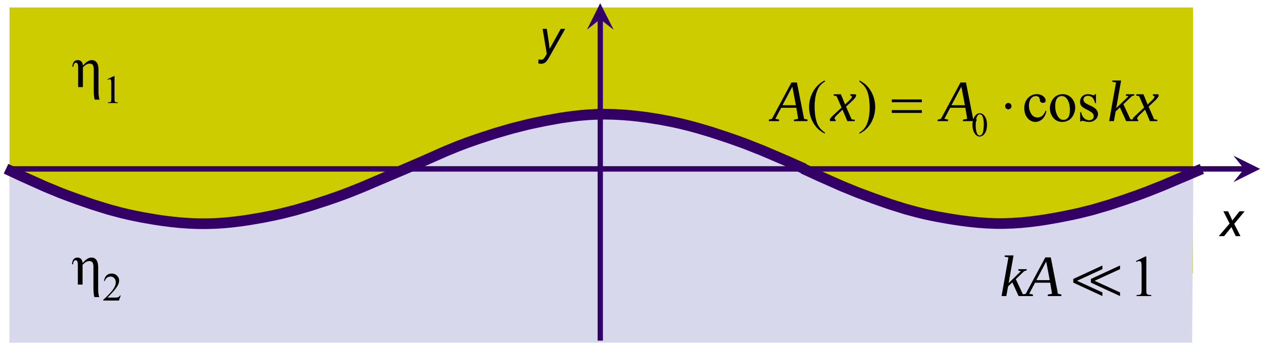
Growth of perturbations in the linear stage

Equations

$$\operatorname{div}\mathbf{v} = 0$$

$$\mathbf{v} = -\frac{\delta^2}{12\eta}\operatorname{grad} p$$

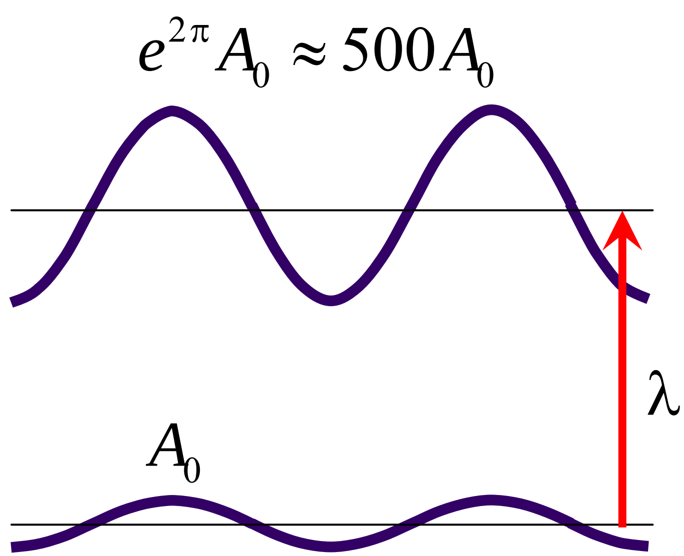
Initial conditions



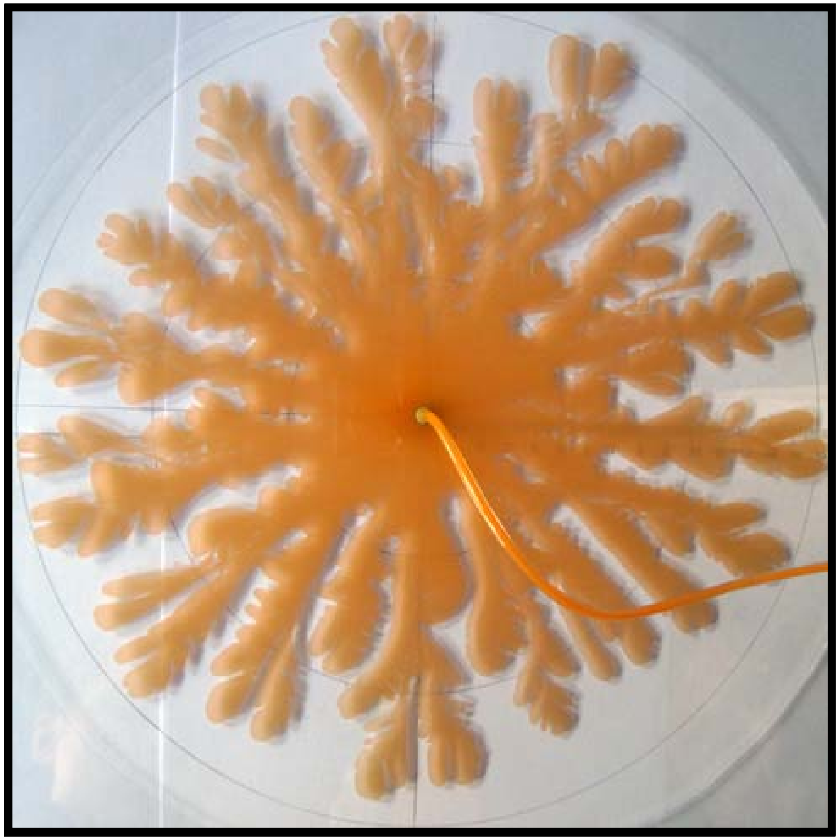
If stabilizing factors are absent, then

$$A(y) = A_0 \exp(\tilde{k}y)$$

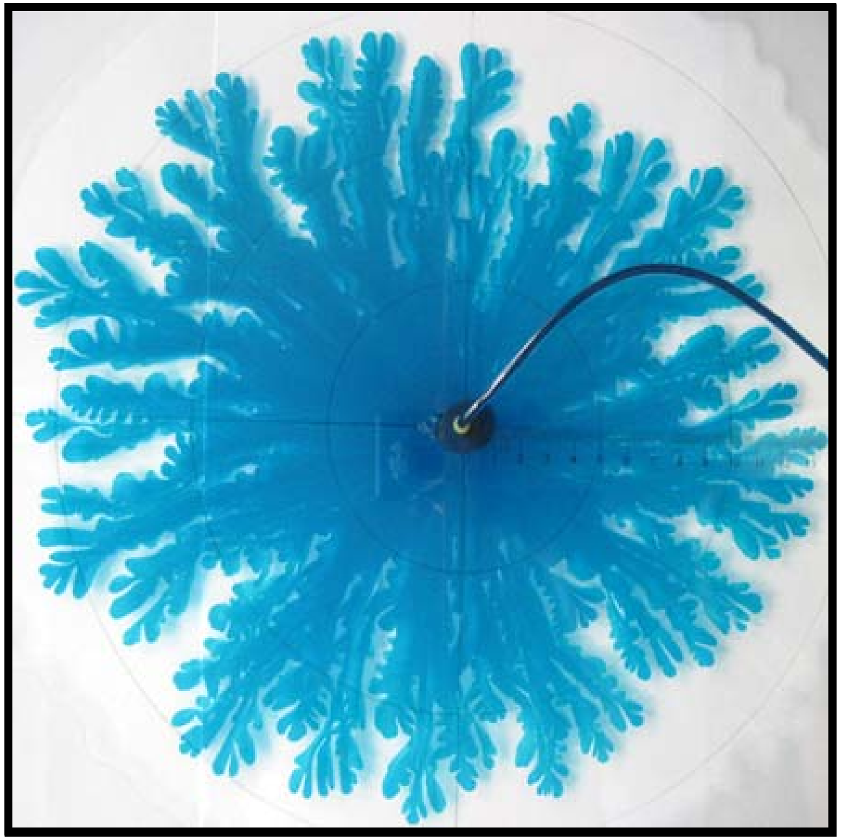
$$\tilde{k} = k\frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$



Dependence on gap: water / glycerin



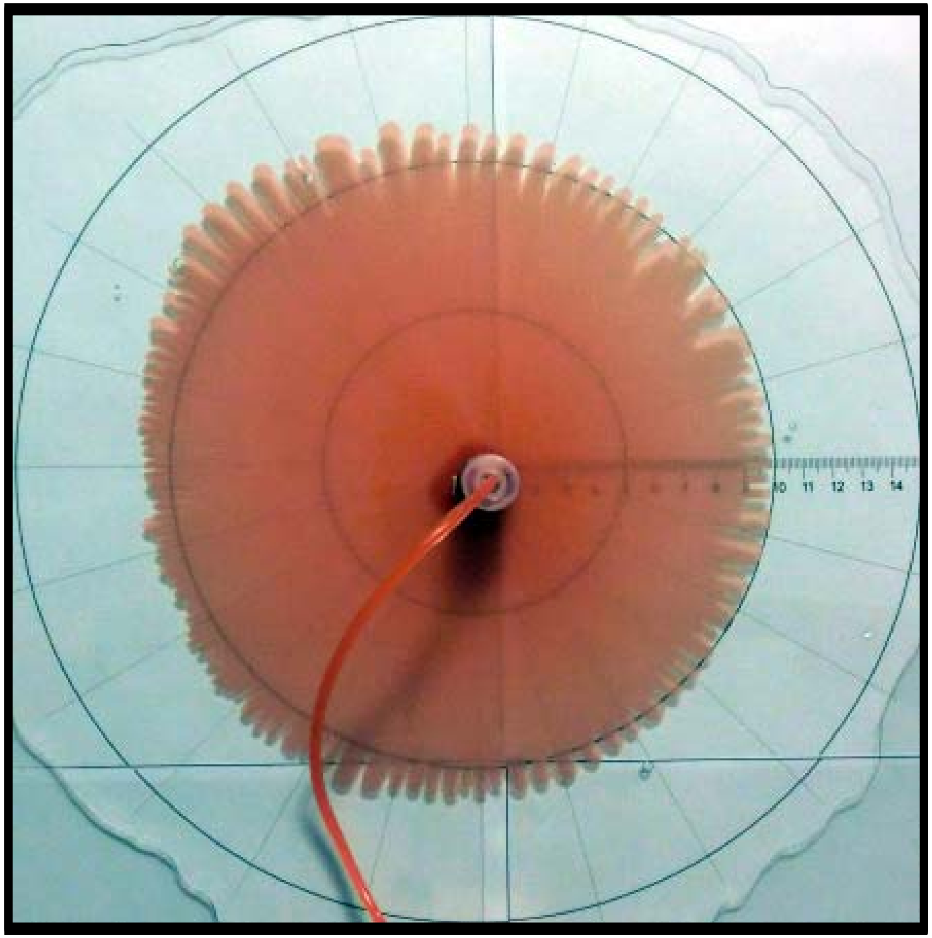
The gap 0,45 mm



The gap 0,22 mm

The ratio of viscosities = 850 : 1. Pressure = 0,1 bar.

Miscible fluids:
Glycerin / glycerin

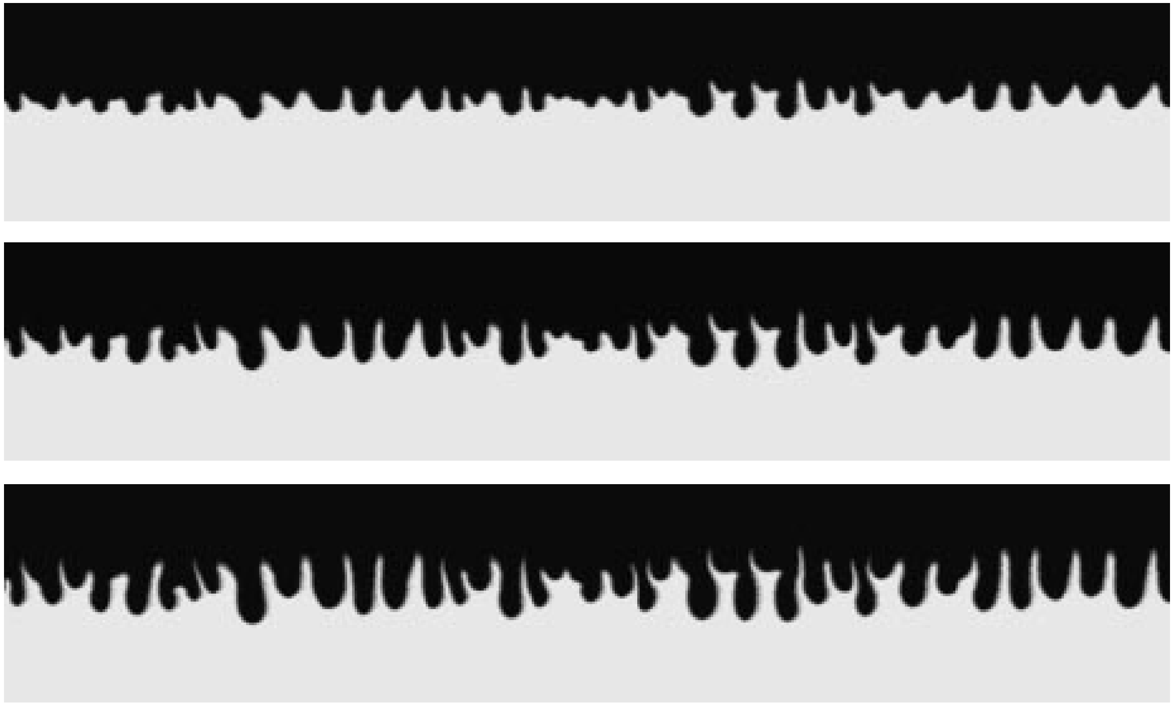


The ratio of viscosities = 9 : 1. The gap = 0,9 mm.
Pressure = 0,04 bar. Interval = 30 s.

Critical wavelength

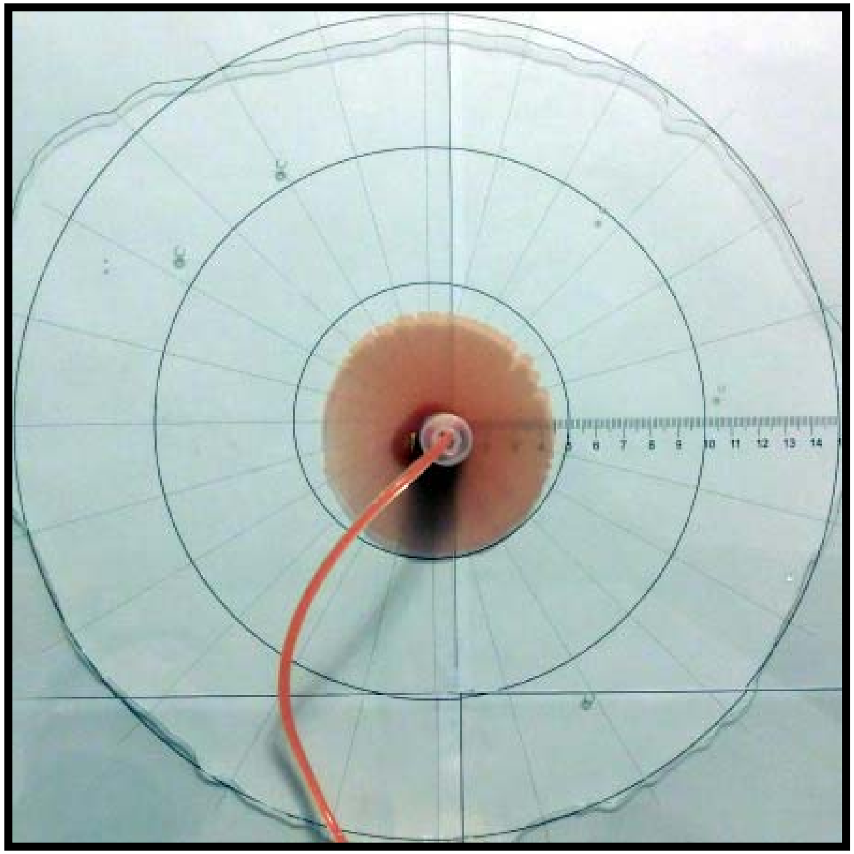
Paterson 1985, Fernandez a.o. 2002

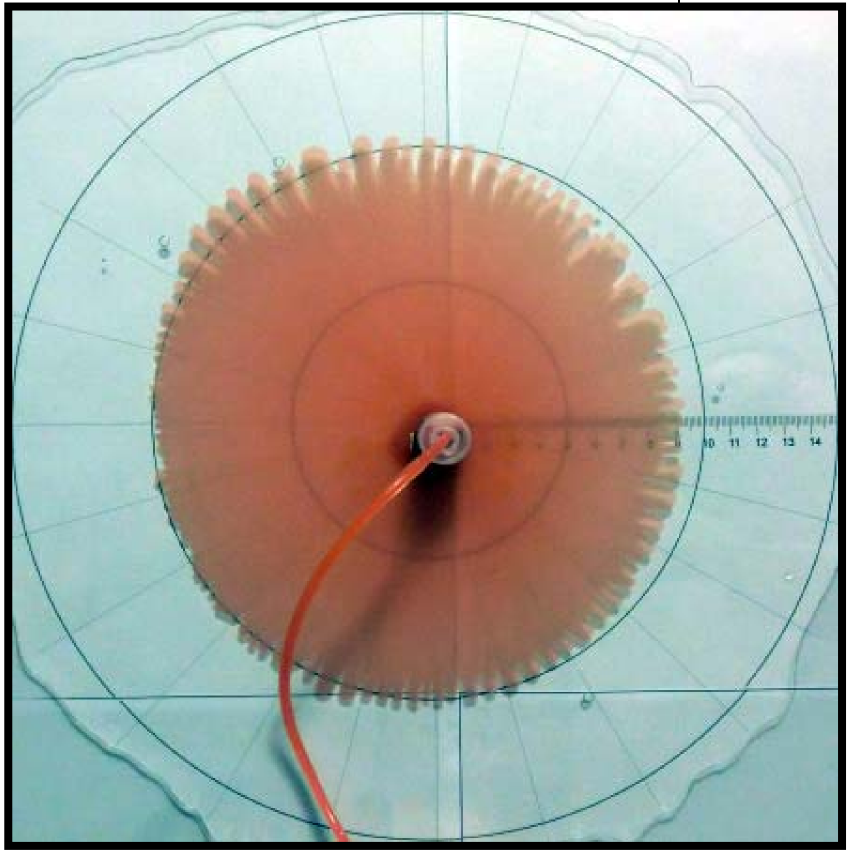
$$\lambda \approx 5\delta$$



Fernandez a.o. 2002

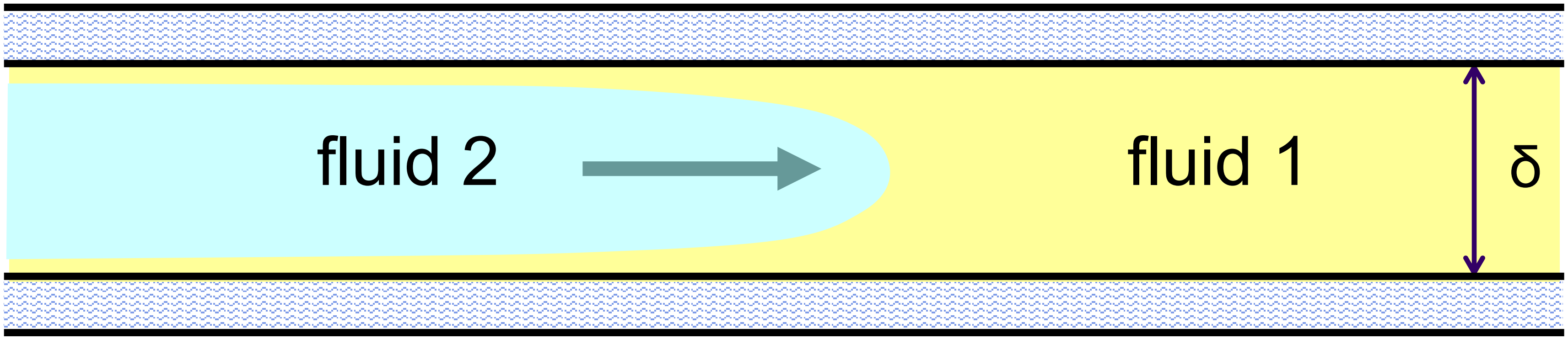
Appearance and the number of fingers





Critical radius = 3–4 cm. Number of fingers = 60–70.
Critical wavelength = 4–5 mm. The gap = 0,9 mm.

Penetration of miscible fluids

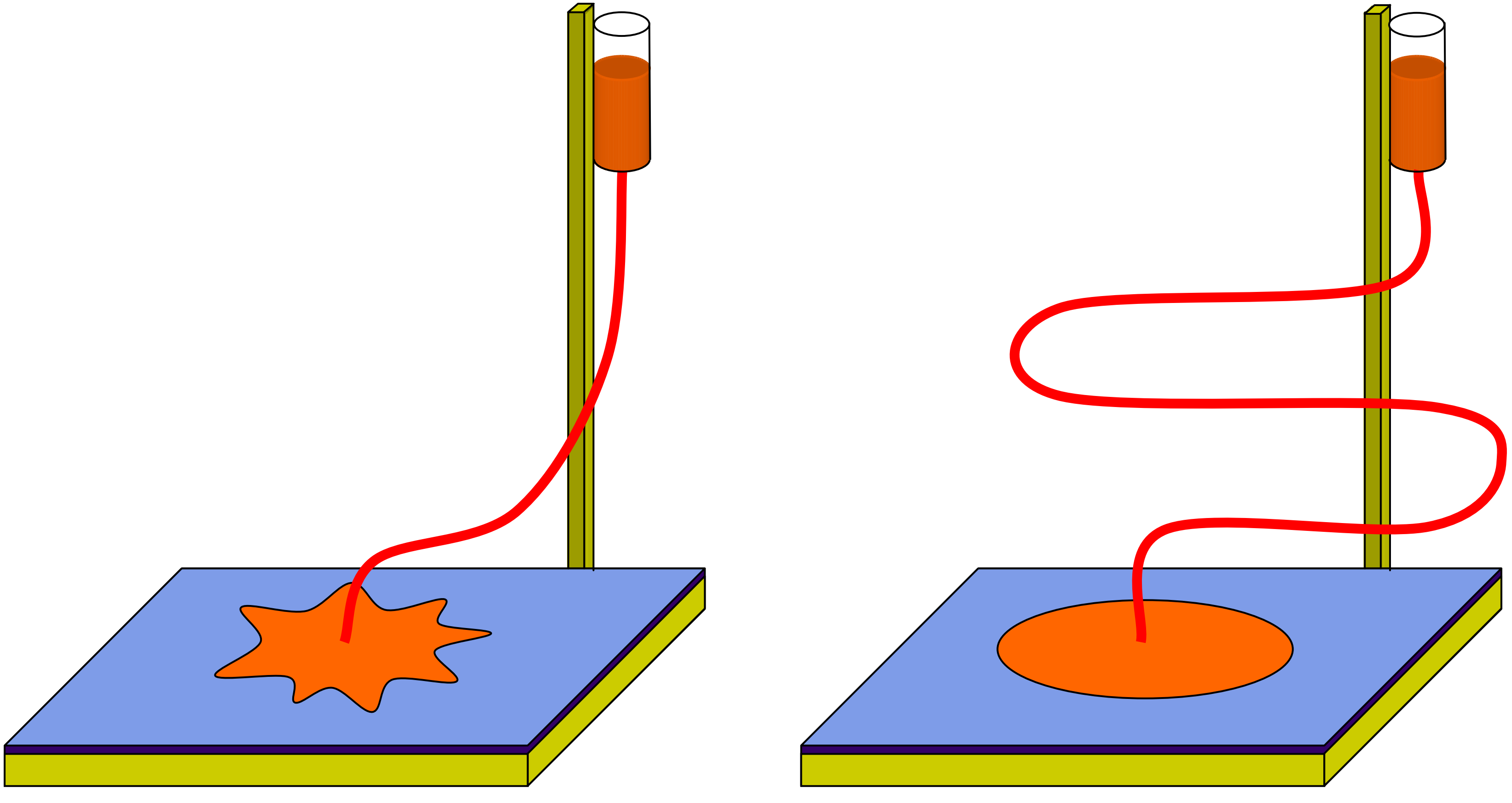


Stabilizing factors:

- 3-D profile of the boundary;
- diffusion.

Near 2-D boundary Darcy's law does not work.

Stabilizing factor for miscible fluids



Summary

- Hele-Shaw cell
- Phenomenon of radial fingering
- Darcy’s law
- Analogies with another instabilities
- Development of Saffman-Taylor instability: linear stage
- Immiscible and miscible fluids
- Critical wavelength and stabilizing factors

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