SLOVAKIA
IYPT '12

15.

Frustrating golf ball

Matej Badin

Problem

- It often happens that a **golf ball escapes** from the hole an instant after it has been ***putted*** into it. Explain this phenomenon and investigate the conditions under which it can be observed.

Content

- Definitions
- Analysis of the motion
  - Rolling, flight, collisions
- Simulation
- Experiment
- Conclusion

Our definitions

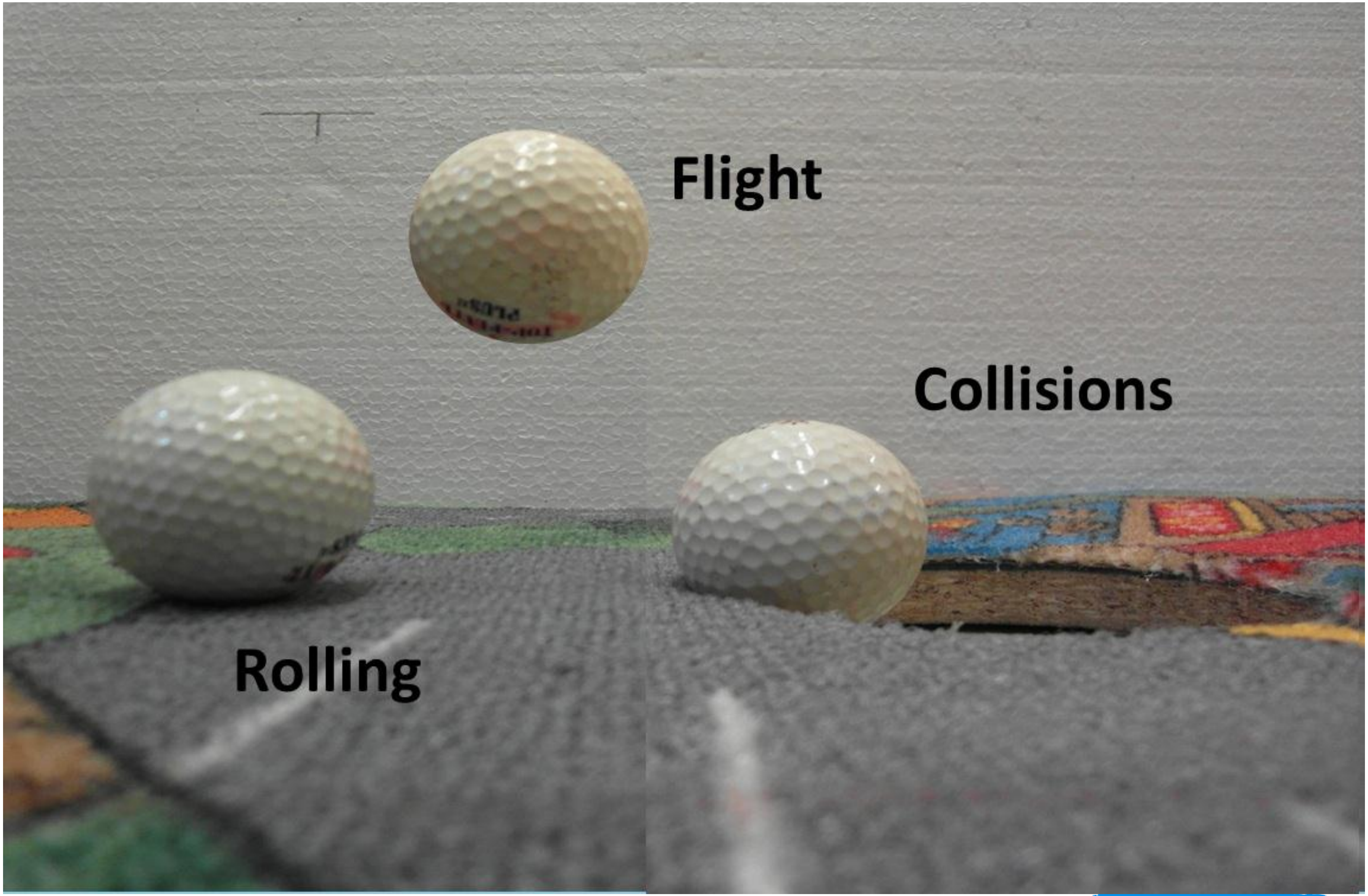
- Golf ball
- **USGA** norm – diameter not less than 4.267 cm
  - Our ball – d=**4.27** cm
- Hole – cylinder
  - Diameter = **10.8** cm
  - Depth = **10.2**cm

Putting green

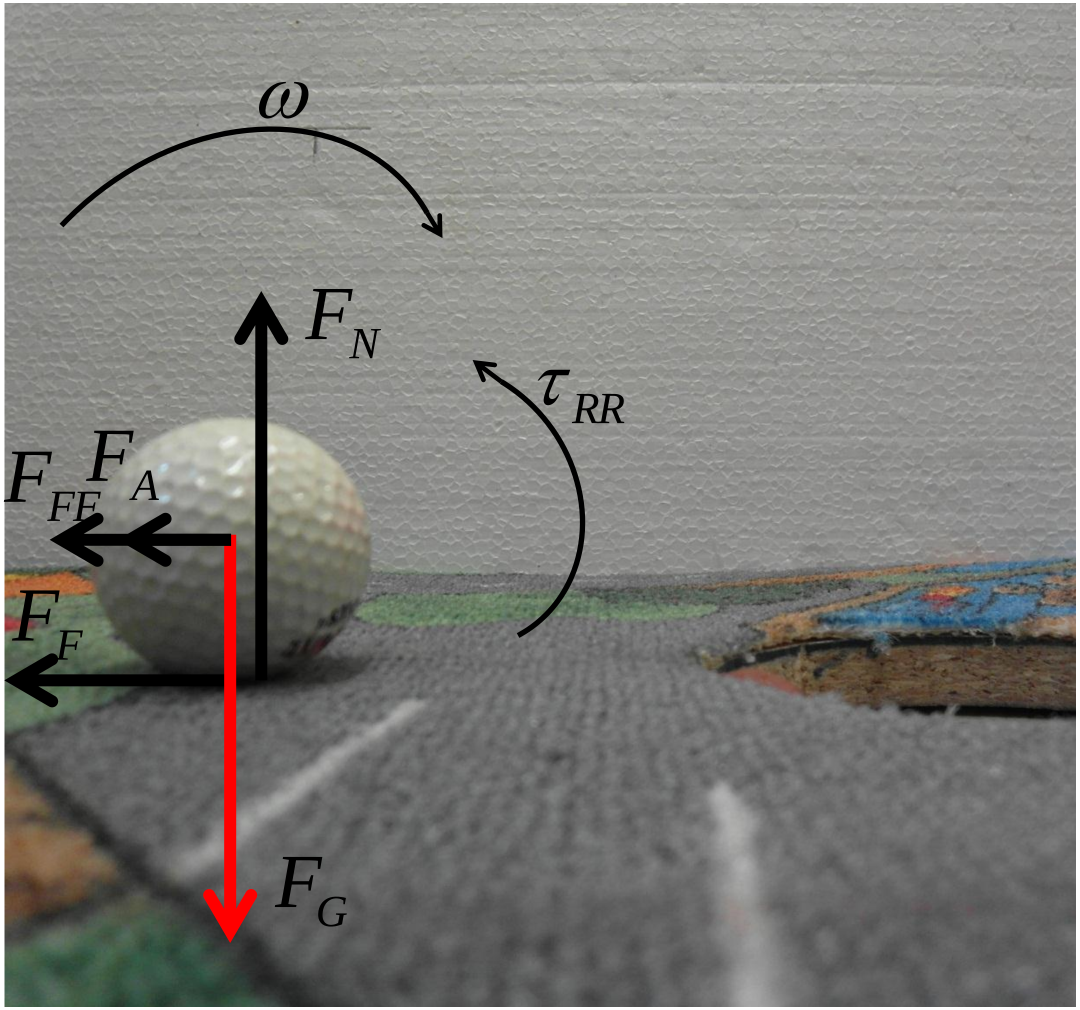
Structure of carpet

Our "green"

Analysis of the motion



Rolling



F_G - Gravity force

F_N - Normal force

F_F - Friction force

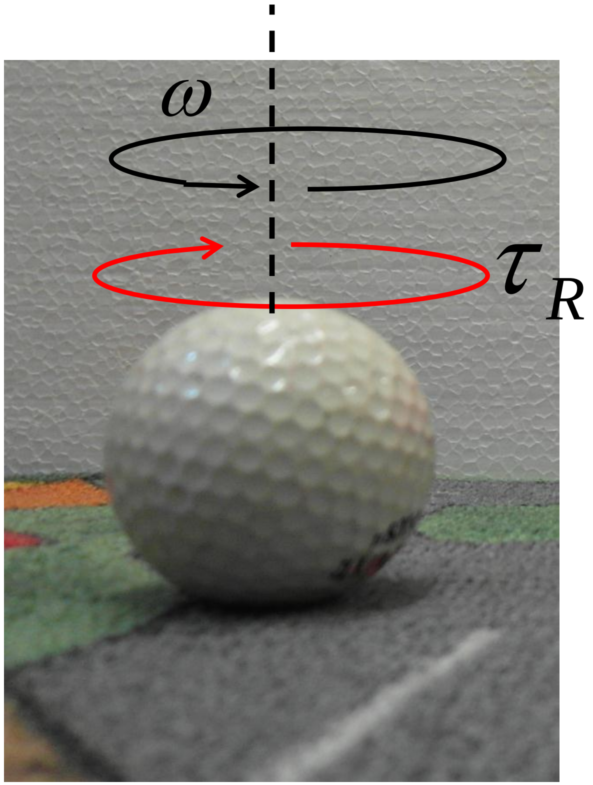
F_A - Air drag force

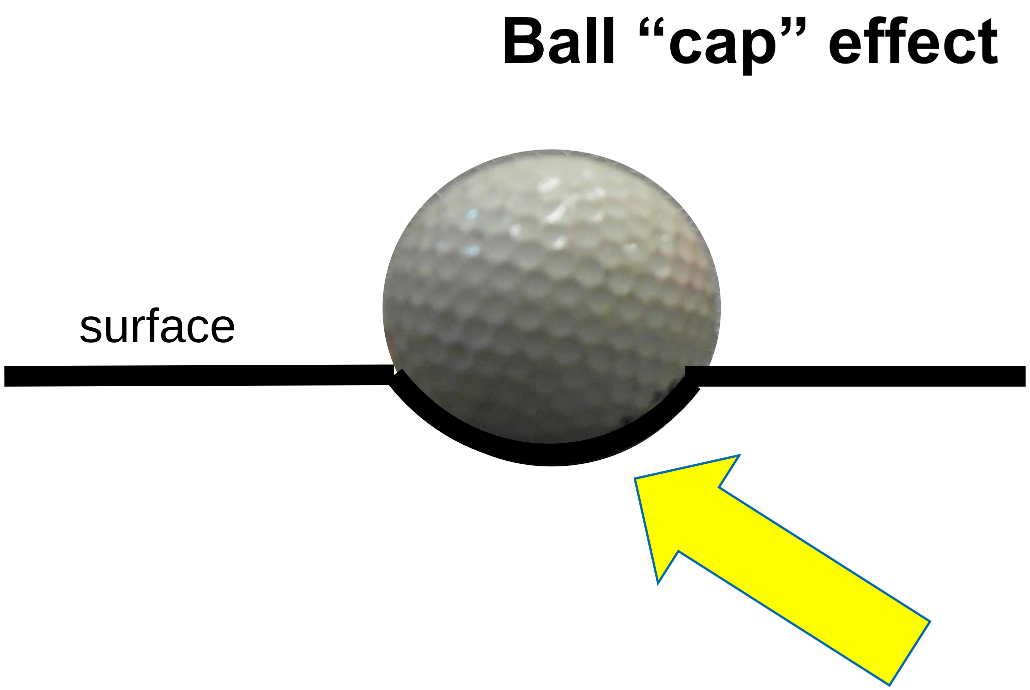
τ_{RR} - Rolling resistance torque

F_{FF} - Rolling resistance force

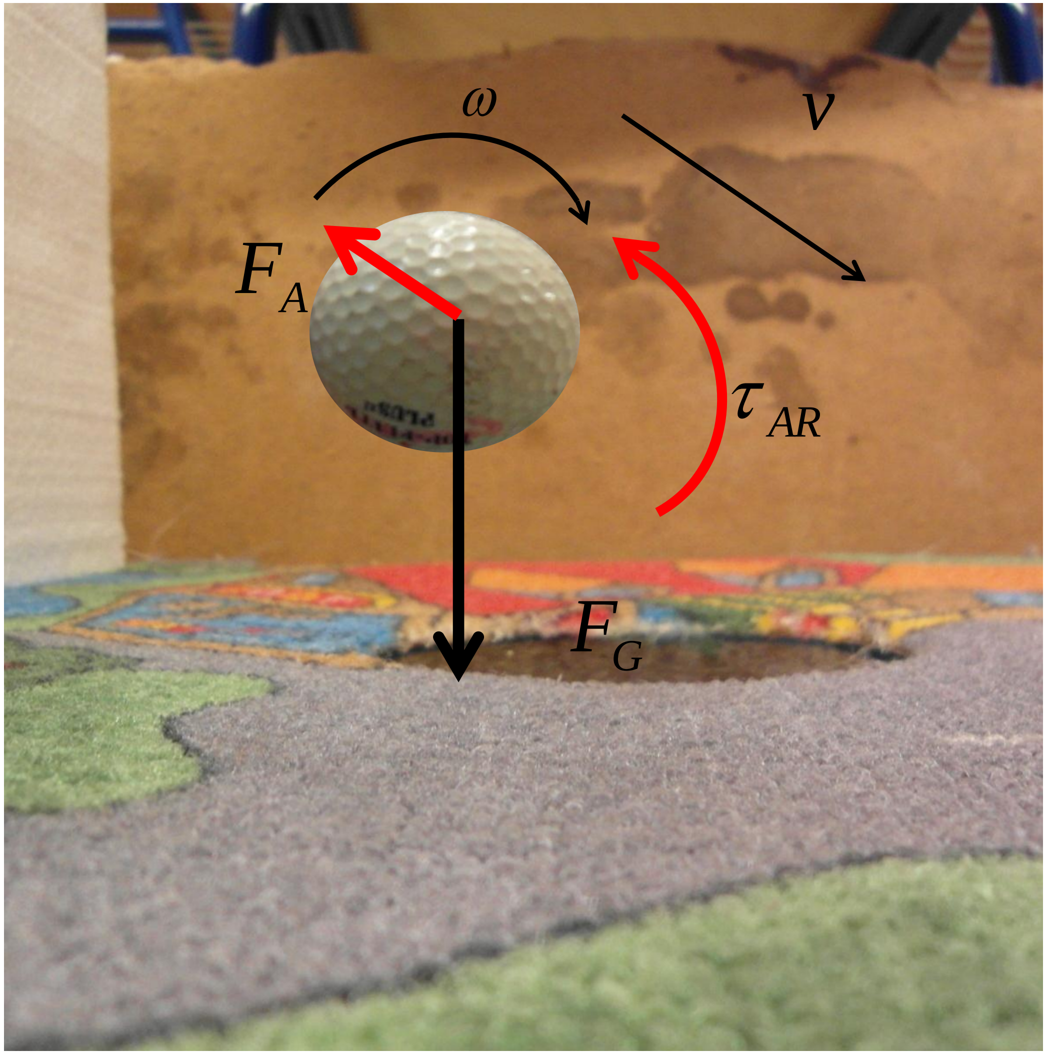
Another resistance in rolling

- Resistance **torque** to rotation around perpendicular axis to horizontal caused by **deformation of the surface or the ball**





Flight



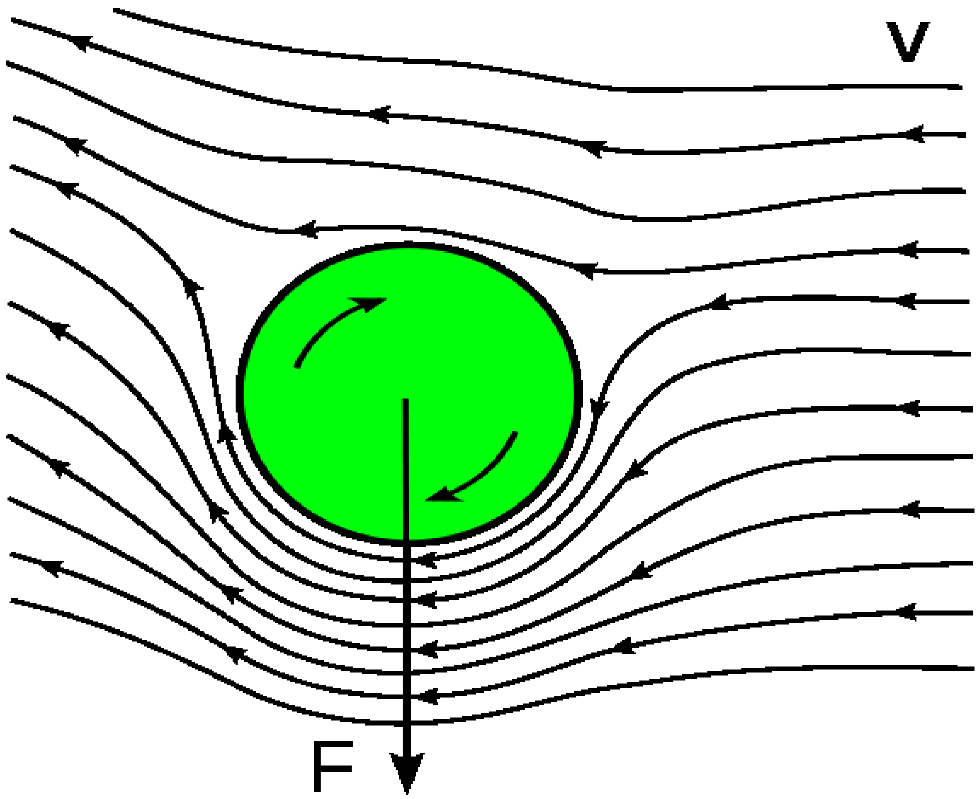
F_G - Gravity force

F_A - Drag force

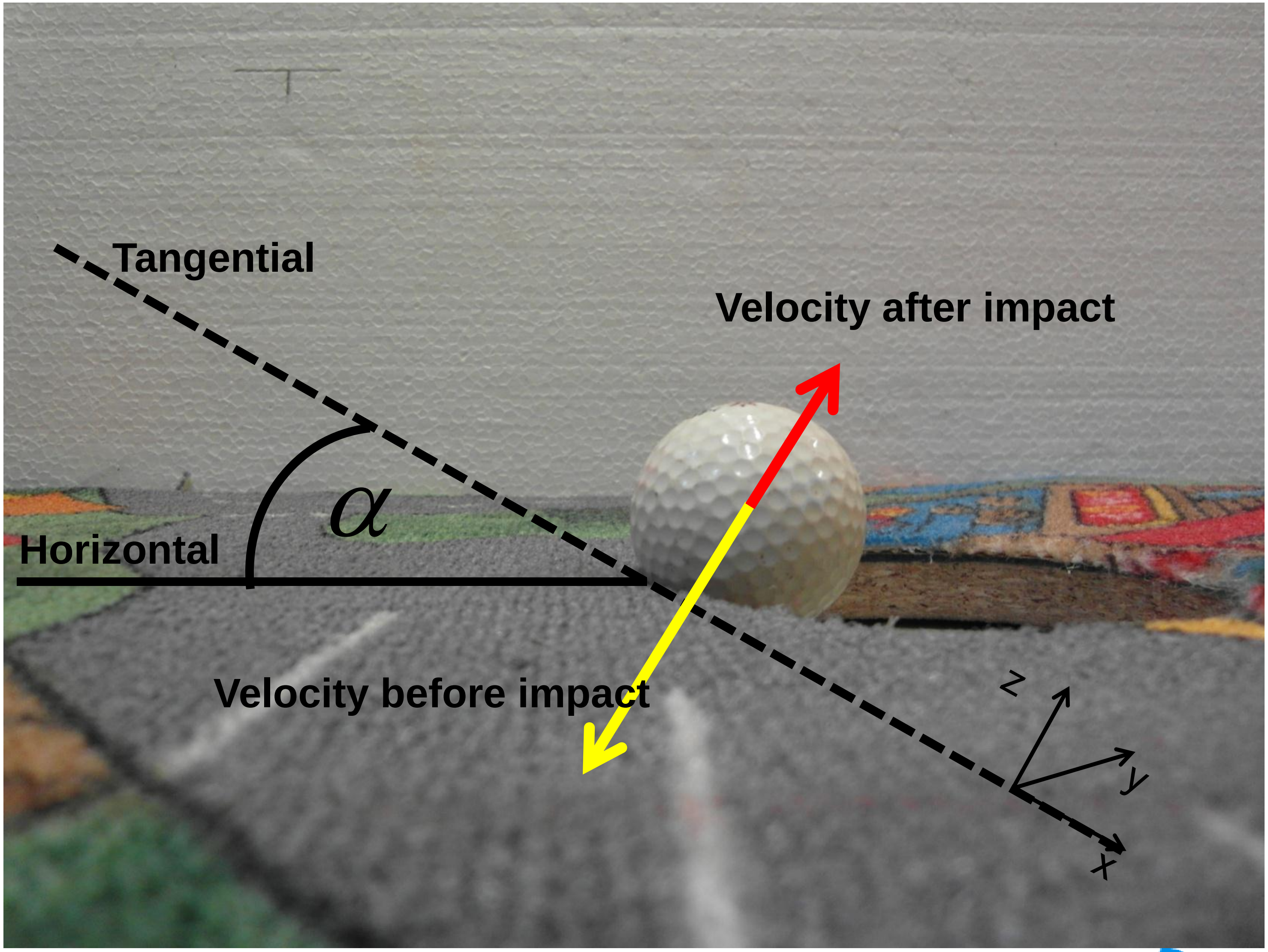
τ_{AR} - Drag torque

Magnus-Robins effect

- Force caused by **pressure difference** on forward and backward moving side of spinning object.



Collisions


Tangential
Velocity after impact
α
Horizontal
Velocity before impact
z
y
x

Collision

Translation motion and rotation

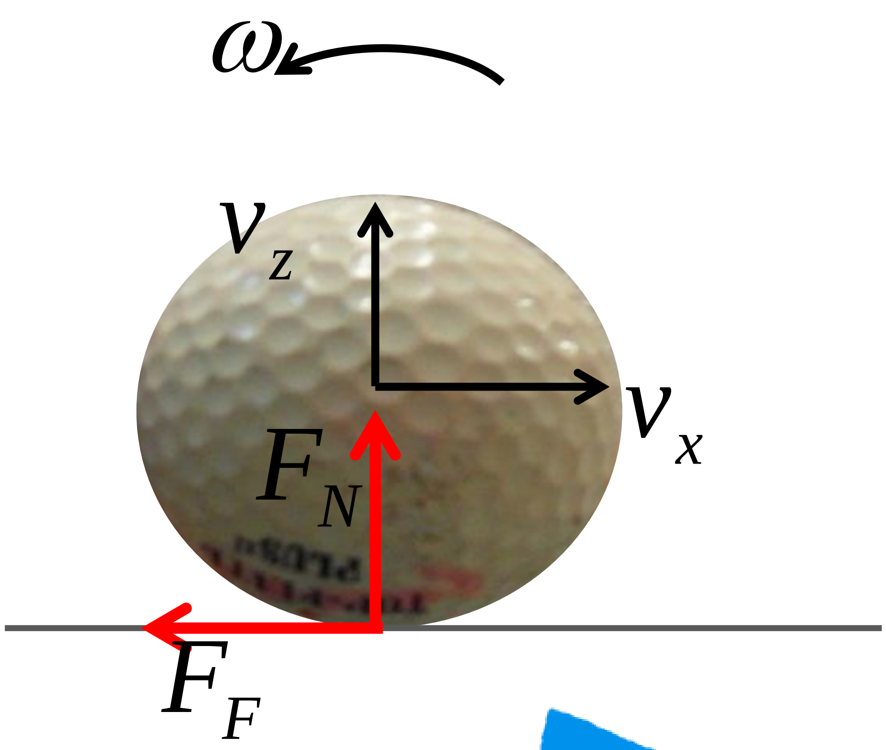
Normal force and friction

While slipping

$$F_F = fF_N$$

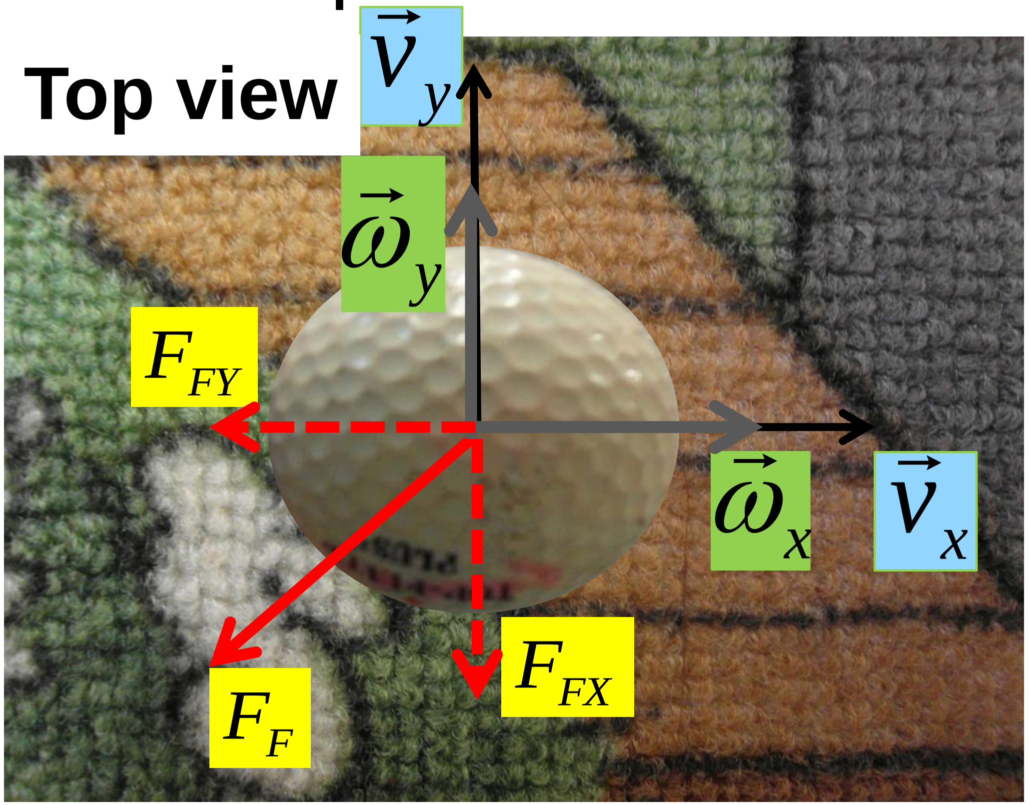
If not

$$F_F = 0$$



Slipping during collision

Direction of friction force is opposite to the direction of a contact point



During small time Δt :

$$f^2 F_N^2 = F_{FX}^2 + F_{FY}^2$$

$$\frac{F_{FX}}{F_{FY}} = \frac{v_x - \omega_y R}{v_y + \omega_x R}$$

We don't know how the normal force changes in time.

Used trick - simulation

- Use of Newton's laws
- Simulation over small increments of the momentum Δp
- Calculation of Δp_x , Δp_y from :

$$f^2 F_N^2 = F_{FX}^2 + F_{FY}^2 \Rightarrow f^2 \Delta p^2 = \Delta p_x^2 + \Delta p_y^2$$

$$\frac{F_{FX}}{F_{FY}} = \frac{v_x - \omega_y R}{v_y + \omega_x R} \Rightarrow \frac{\Delta p_x}{\Delta p_y} = \frac{v_x - \omega_y R}{v_y + \omega_x R}$$

Simulation

$$v_x \Rightarrow v_x - \frac{F_{FX}}{m}\Delta t = v_x - \frac{1}{m}\Delta p_x$$

$$v_y \Rightarrow v_y - \frac{F_{FY}}{m}\Delta t = v_y - \frac{1}{m}\Delta p_y$$

$$v_z \Rightarrow v_z + \frac{\Delta p}{m}$$

$$\omega_x \Rightarrow \omega_x - \frac{F_{FY}}{I}R\Delta t = \omega_x - \frac{R}{I}\Delta p_y$$

$$\omega_y \Rightarrow \omega_y - \frac{F_{FX}}{I}R\Delta t = \omega_y - \frac{R}{I}\Delta p_x$$

$$\omega_z \Rightarrow \omega_z - \frac{M_{(F_N)}}{I}\Delta t$$ **(Ball cap effect)**

Summary of coefficients

Rolling

- Rolling resistance *arm*
- Shape coefficient
- Contact area radius
- Frictions coefficients
- Moment of inertia
- Mass
- Radius

Fly

- Shape coefficients

Collisions

- Friction coefficient
- Coefficient of restitution on the edge (in dependence on angle)

Summary of effects

Rolling

- Rolling resistance
- ~~Air resistance~~
- ~~Air resistance torque~~
- Ball cap effect (deformation of ball or deformation of surface)

Fly

- ~~Magnus effect~~
- ~~Air resistance~~
- ~~Air resistance torque~~

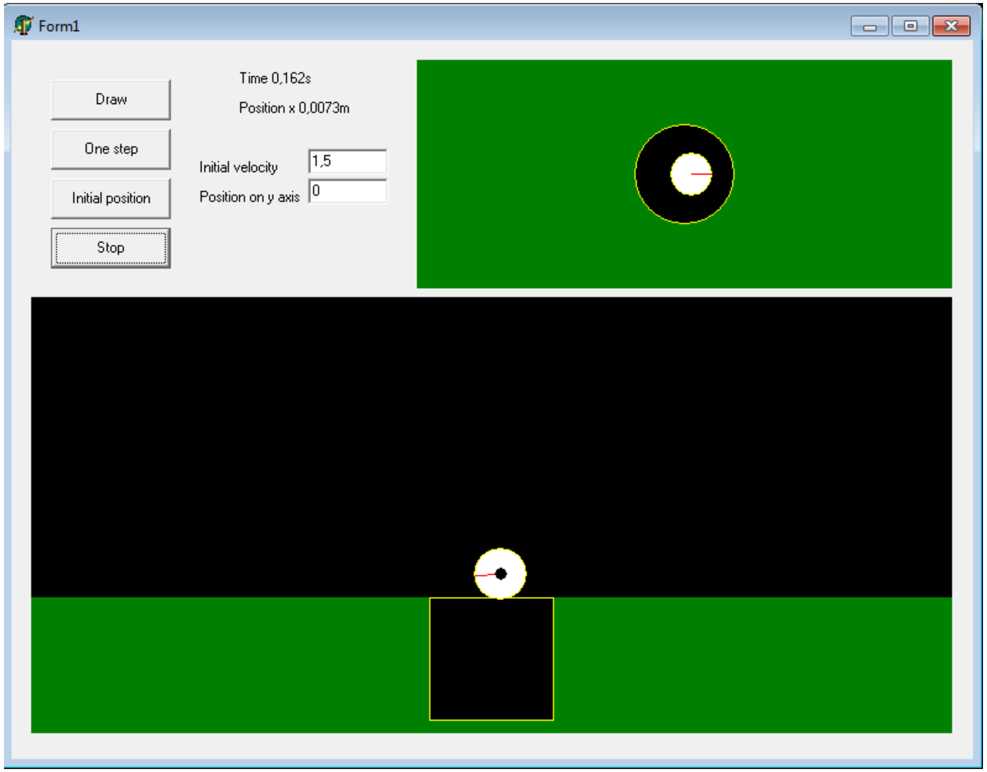
Collisions

- Sliding
- Ball cap effect

Negligible

Simulation

- Used described theory
- In collisions we rotated the frame of the reference

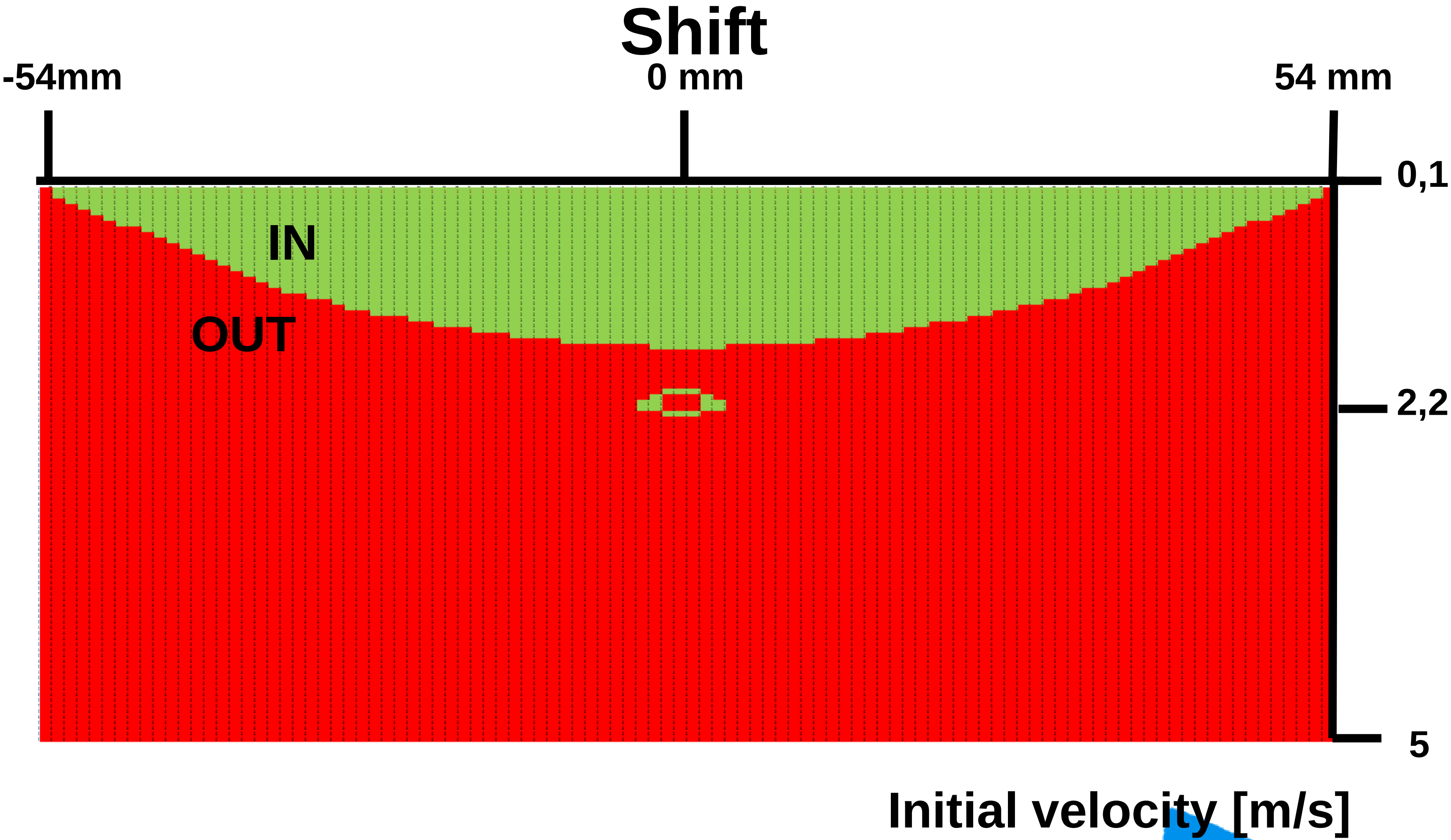


Experiments

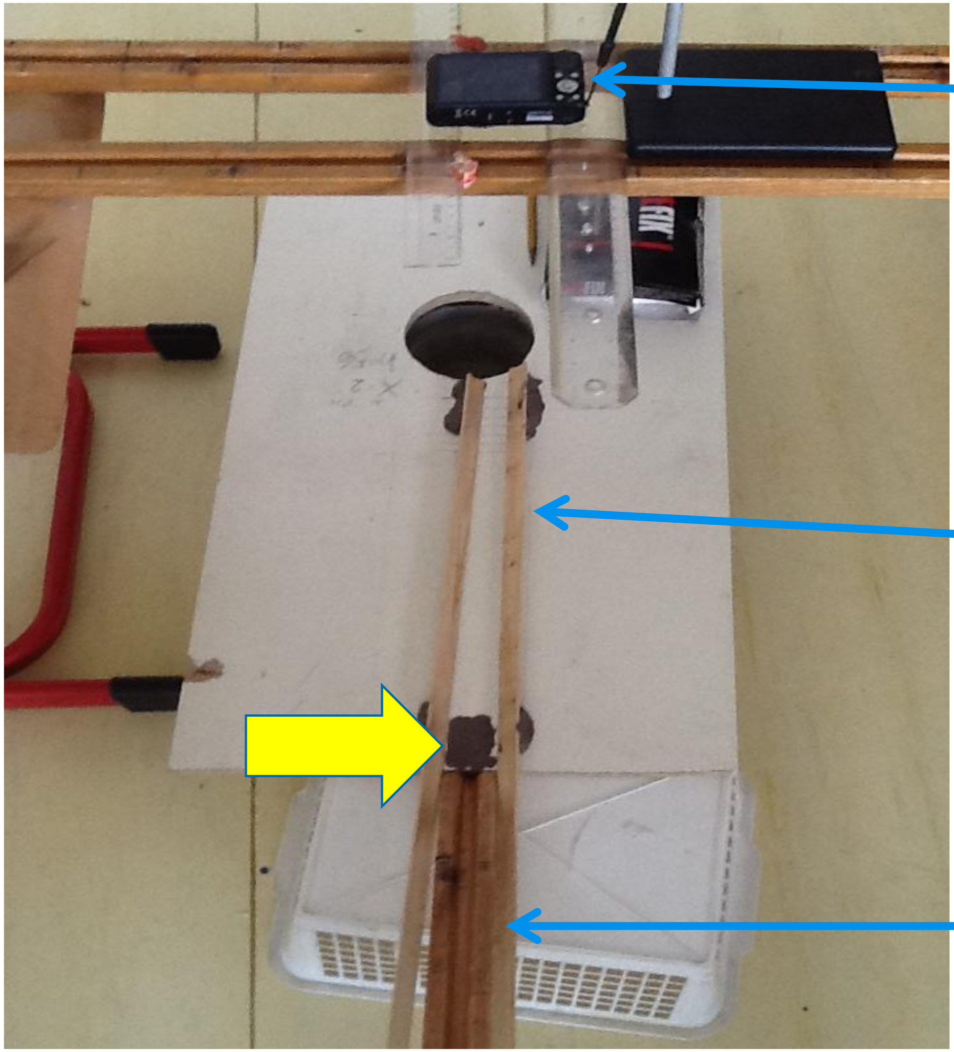
- Two surfaces

Smooth and smooth edges **Similar to grass and rough edges**

Wood simulation results



First experimental setup



High-speed camera

Rails – stabilisation of the ball

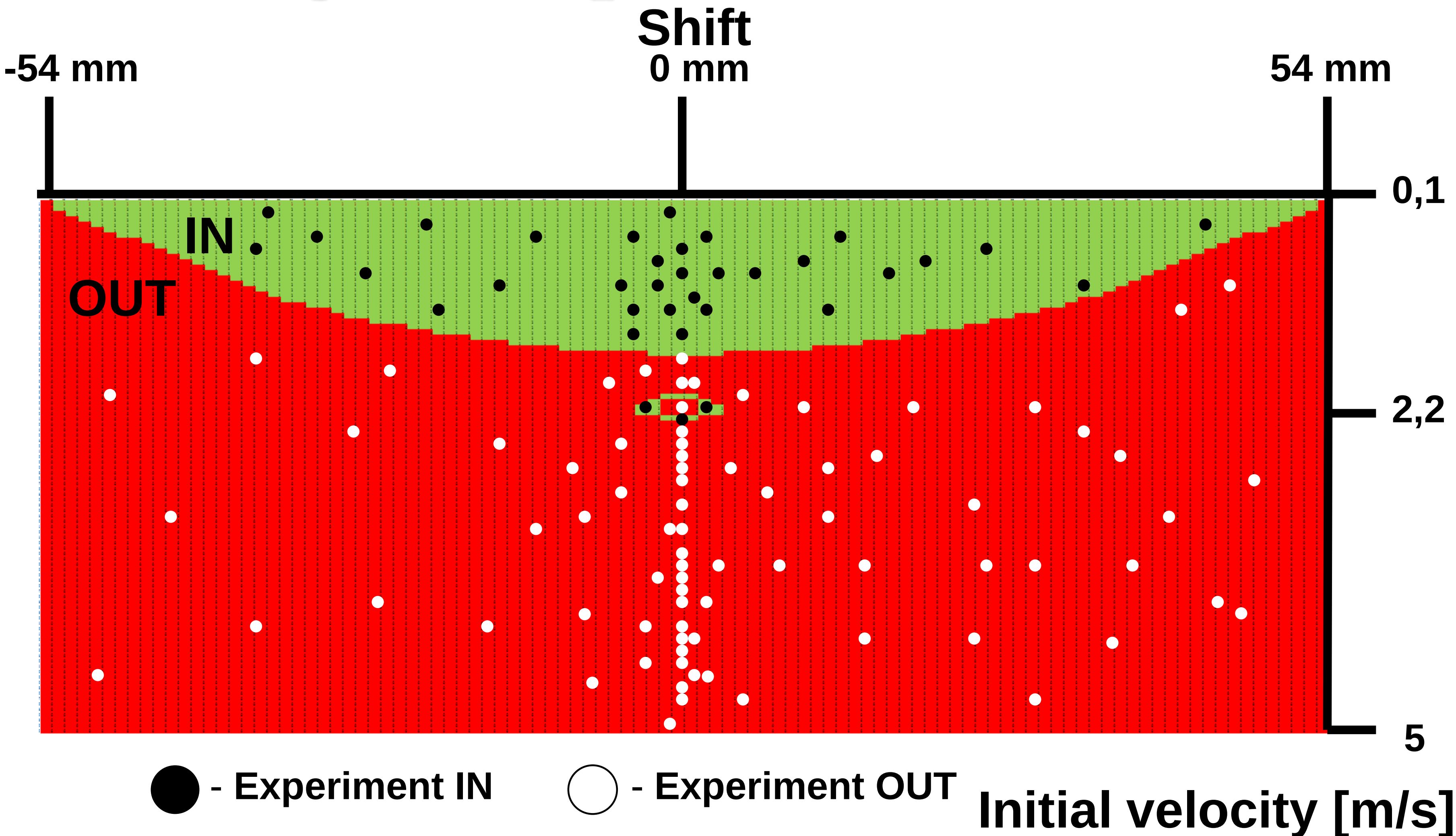
Inclined plane – regulation of the speed

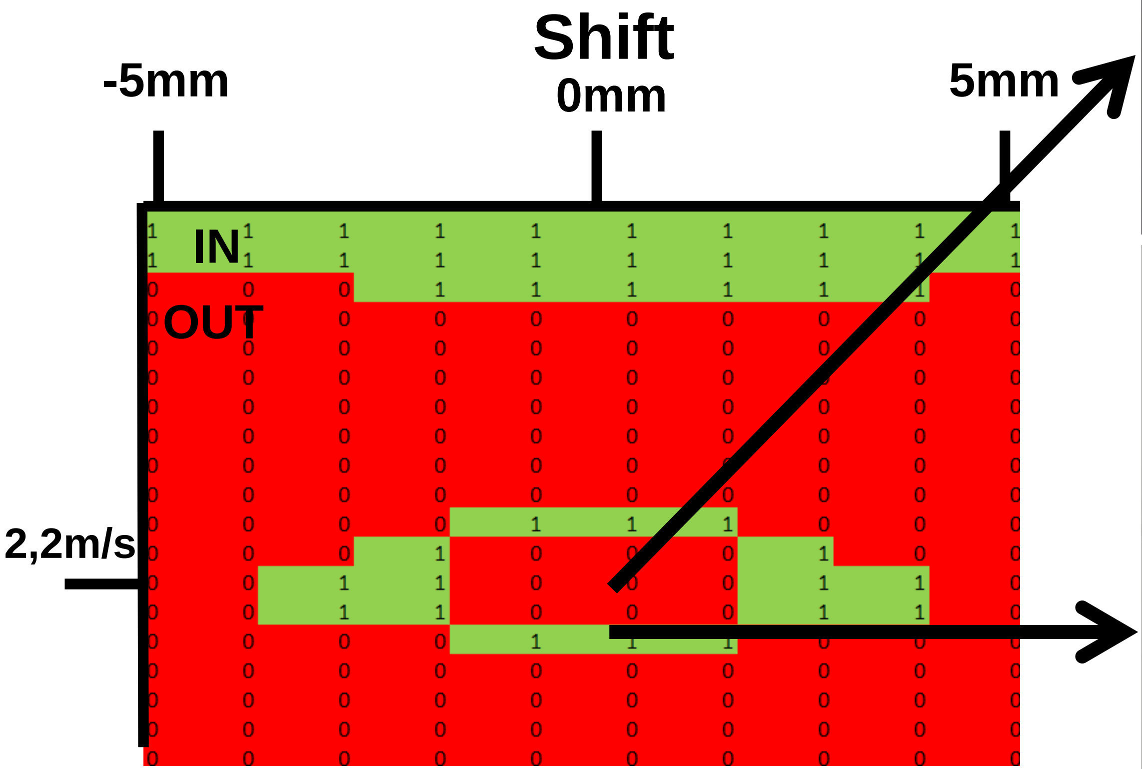
Second experimental setup

Inclined plane – regulation of the speed

Place of high-speed camera

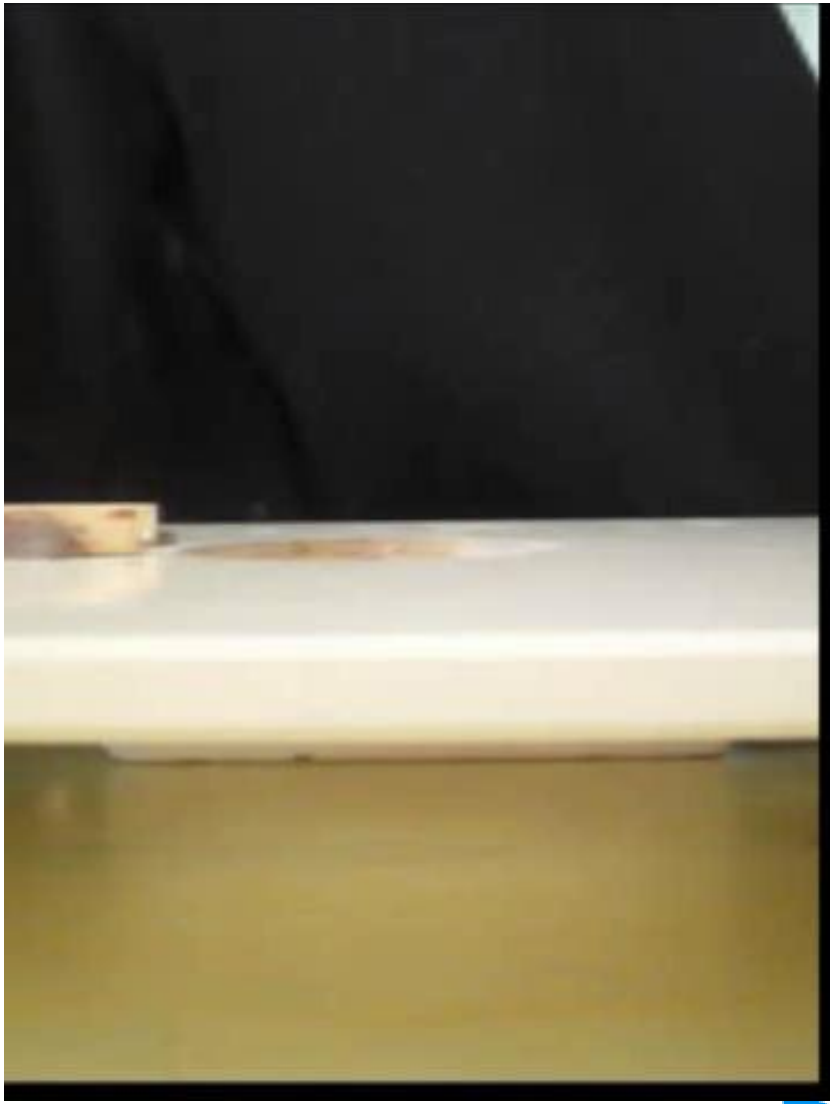
Theory vs. experiment




Shift
-5mm
0mm
5mm
IN
OUT
2,2m/s

Comparision - Escape

No shift
Velocity – 2,2 m/s



wood_escape.exe

Comparision - Jump

No shift
Velocity – 2,0 m/s

wood_jump.exe

Comparision – Too much

No shift
Velocity – 4,0 m/s

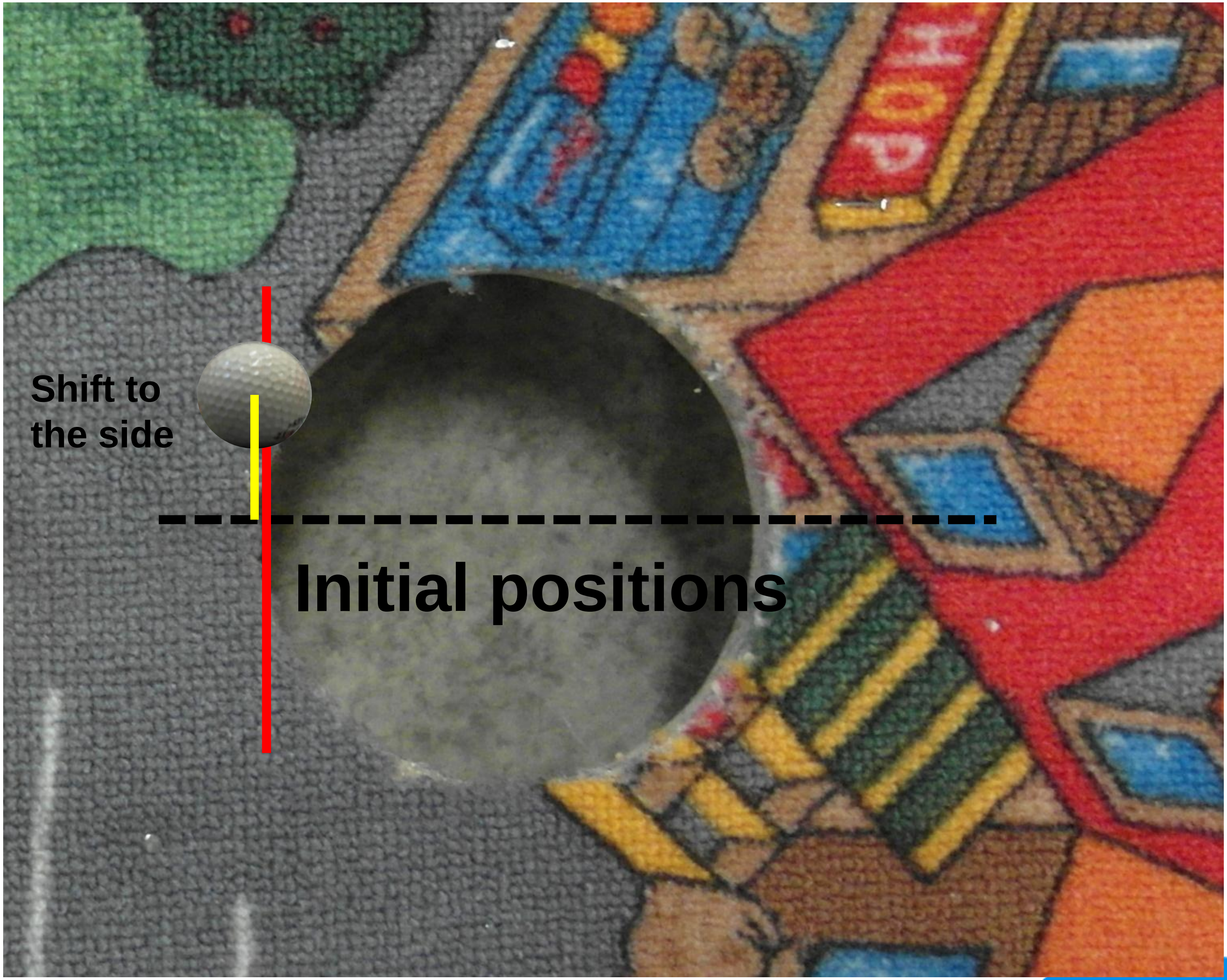
wood_from.exe

Comparision – Turn around

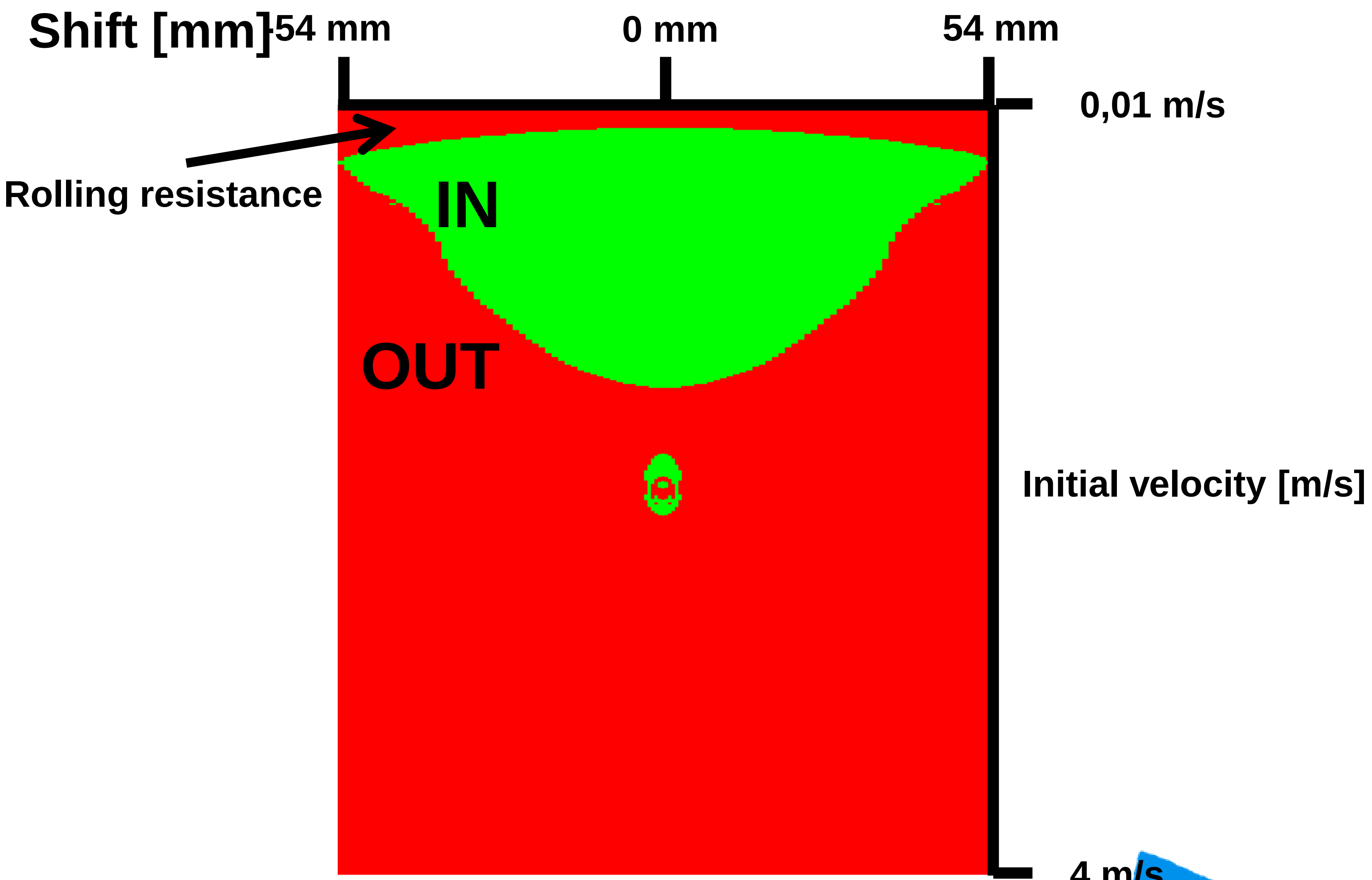
Shift – 35 mm
Velocity – 1,0 m/s

wood_around.exe

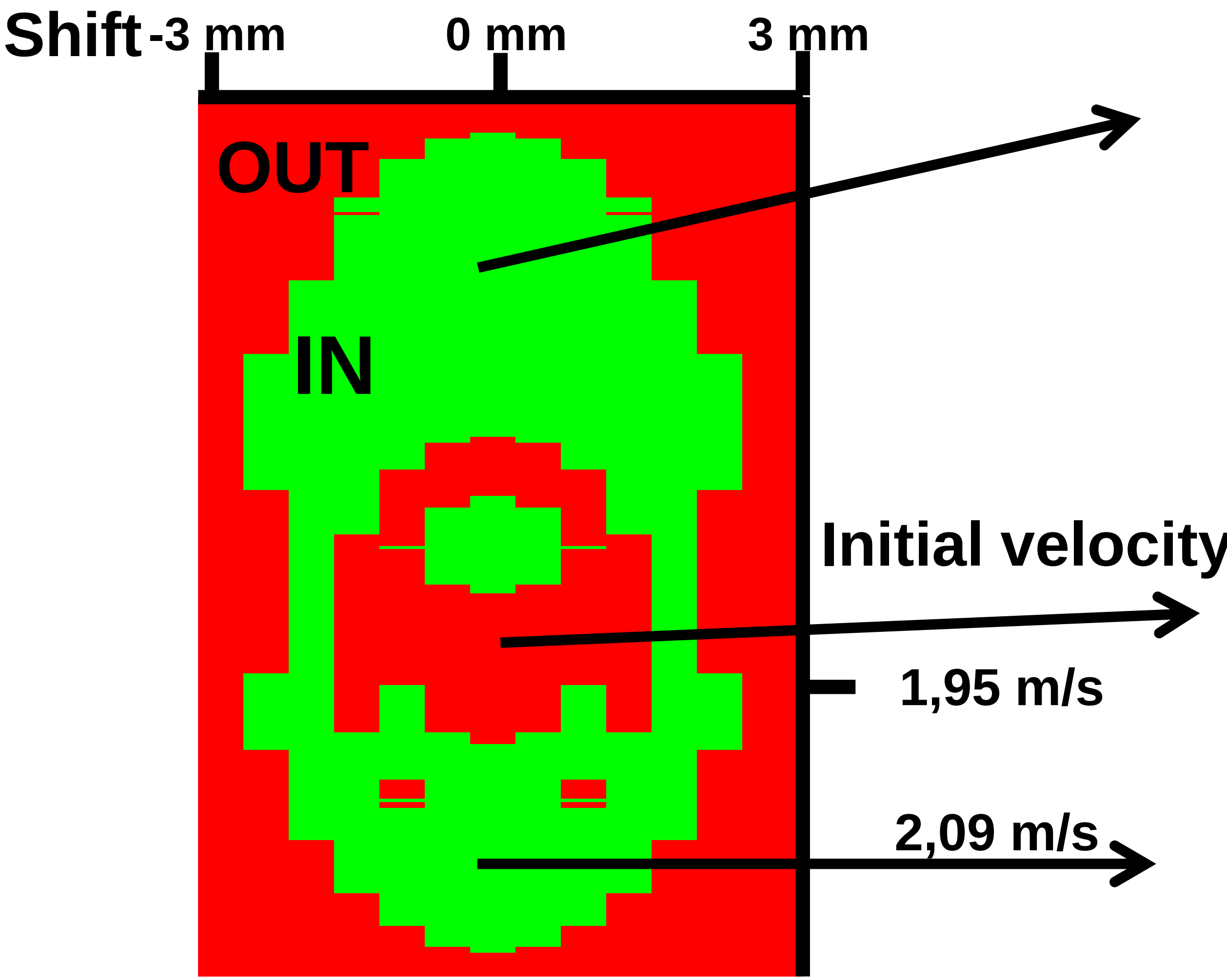
Carpet simulation

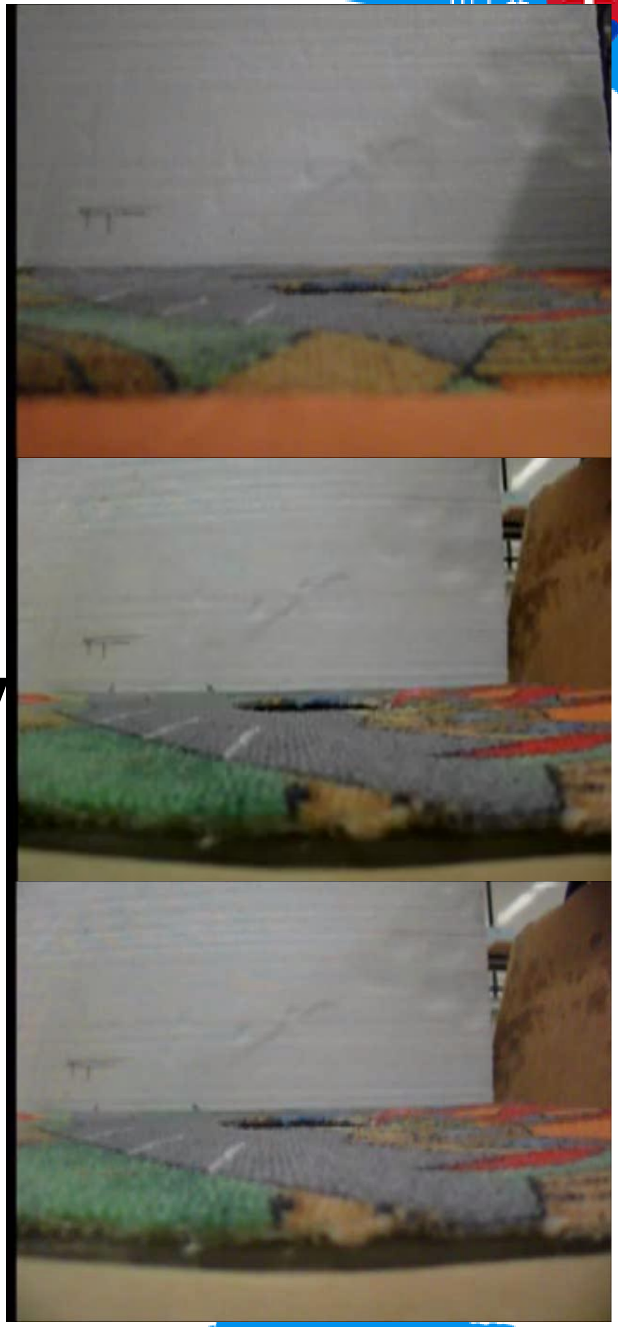


Carpet simulation result



Island or face?



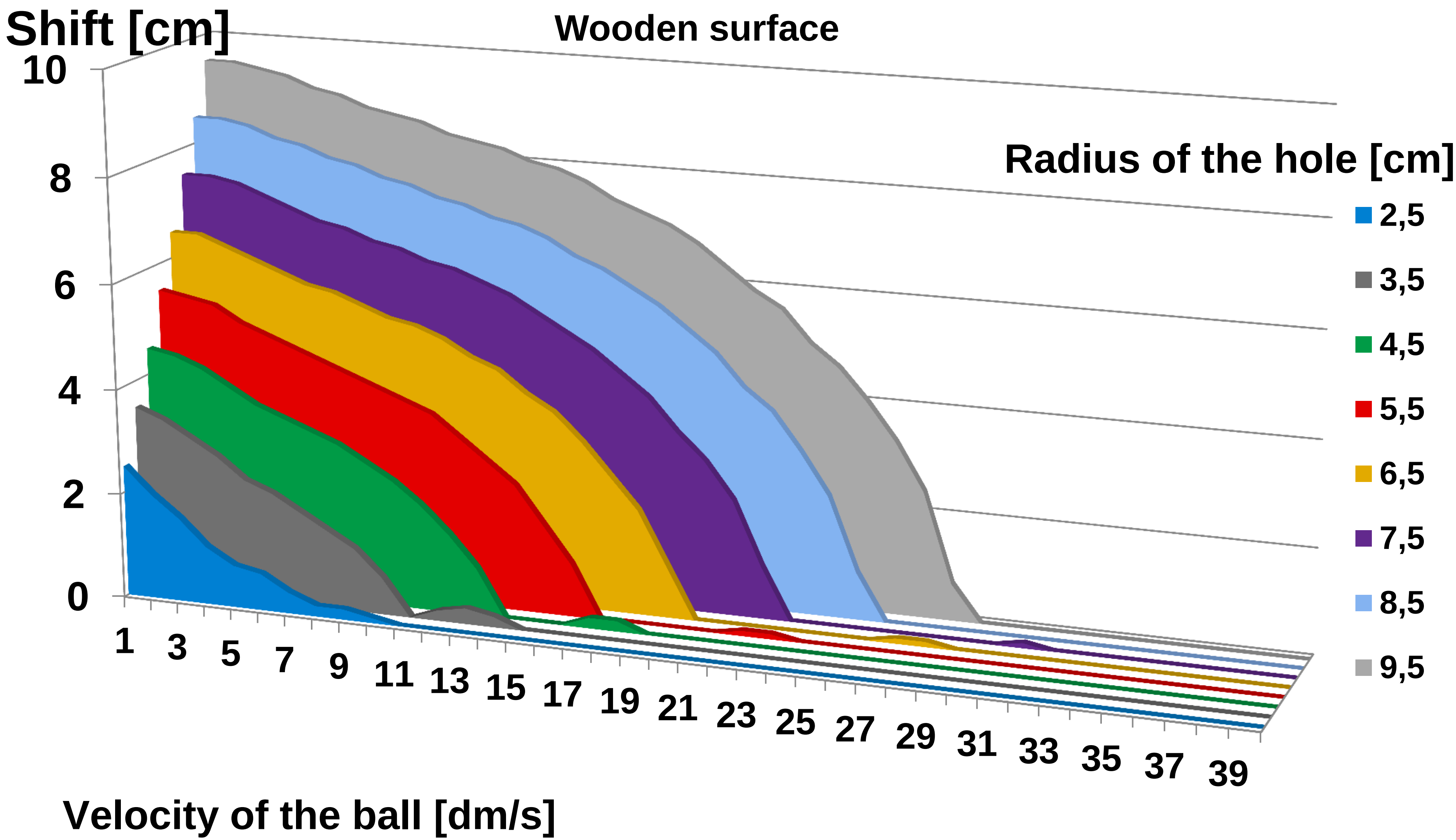


Carpet - Escape

No shift
Velocity – 1,94 m/s

carpet_escape.exe

Influence of diameter of the hole



Conclusion

- The most important parameters and effects are:
  - Rotation
  - Slipping during collision
  - Coefficient of restitution on the edge
  - Coefficient of restitution in the hole

Conclusion

- We developed the model of the motion and collisions
- We theoretically **predicted** the path of the ball
- Prediction **correlates** with experiment
- We explained the most important parameters under which can be phenomenon observable

THANK YOU FOR YOUR ATTENTION

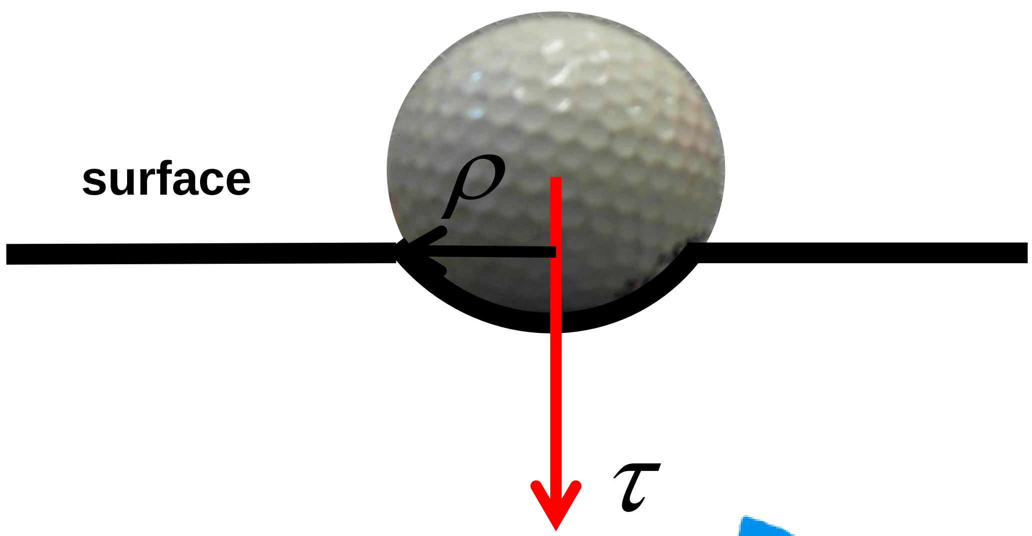
Ball cap effect

We assume that normal force is evenly distributed to a contact area.

Torque acts on ball in z-axis:

$$\tau = \frac{fF_N}{2\pi R(R-\sqrt{R^2-\rho^2})}\int_0^{R-\sqrt{R^2-\rho^2}} 2\pi\sqrt{(R^2-(R-x)^2)}^{\,2}\,dx$$

$$\tau = fF_N\,\frac{\frac{2}{3}R^3-\rho^2\sqrt{R^2-\rho^2}}{R^2-R\sqrt{R^2-\rho^2}}$$



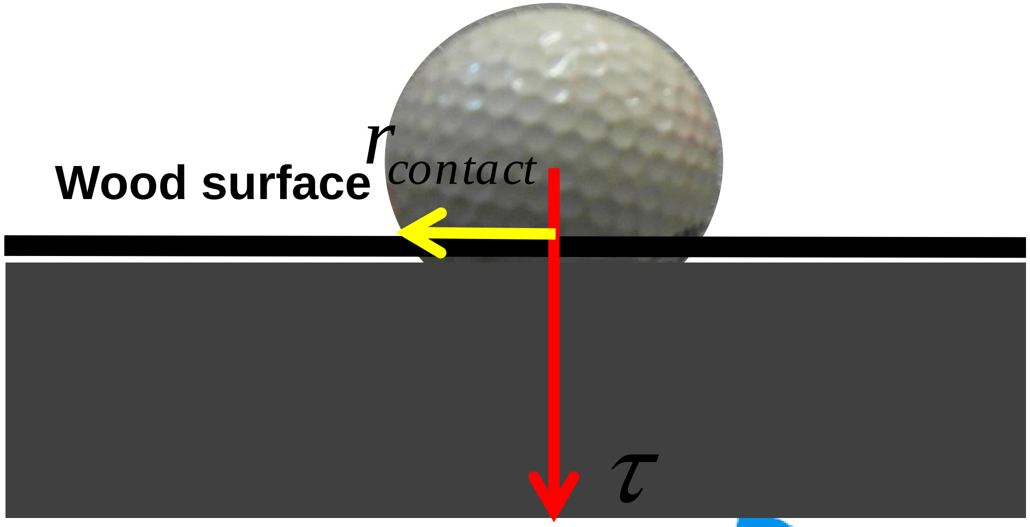
Deformation of the ball

We assume that normal force is evenly distributed to a contact area.

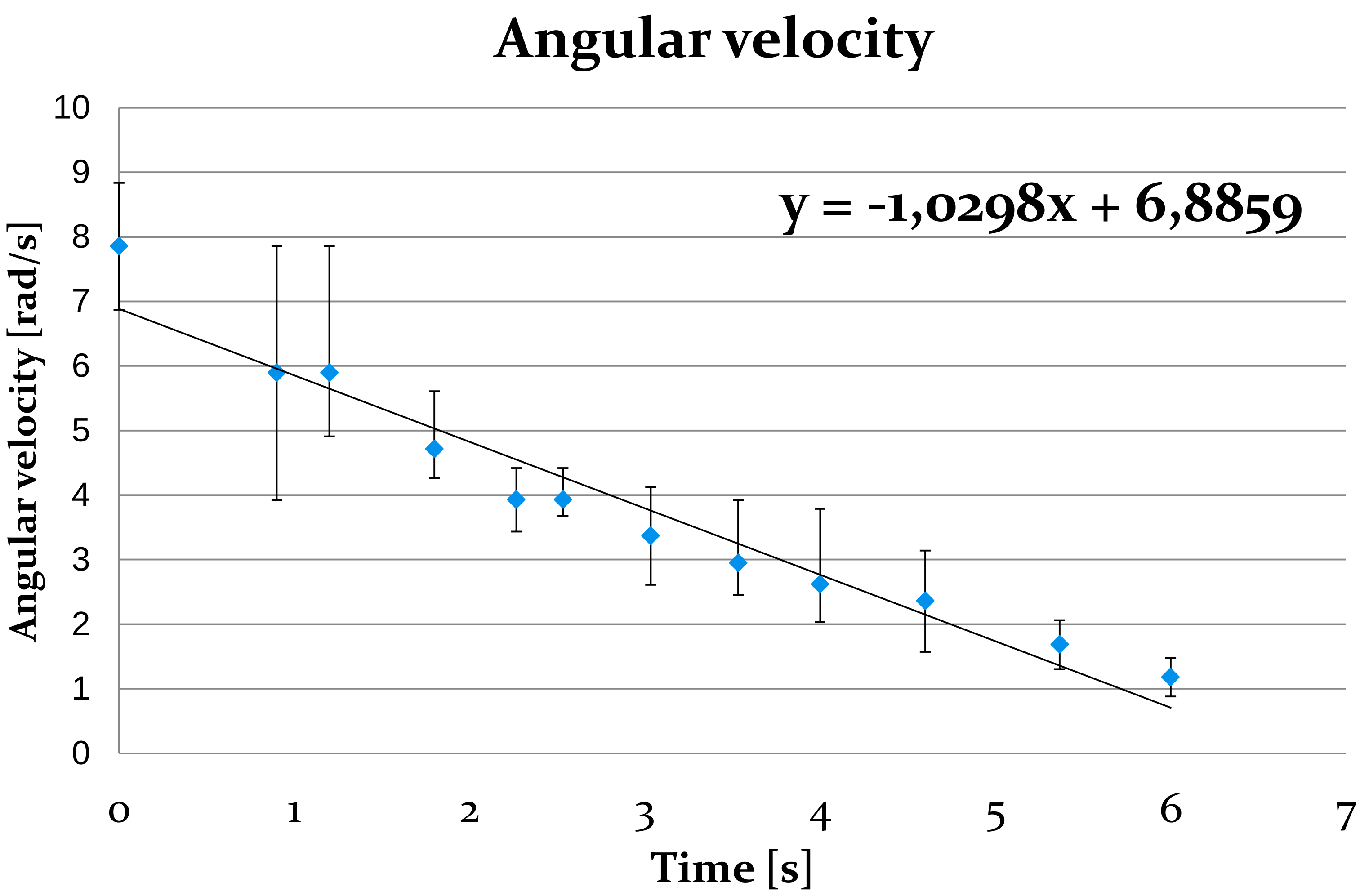
Torque acts on ball in z-axis:

$$\tau = \frac{fF_N}{\pi r_{contact}^2} \int_0^{r_{contact}} 2\pi x^2 \, dx$$

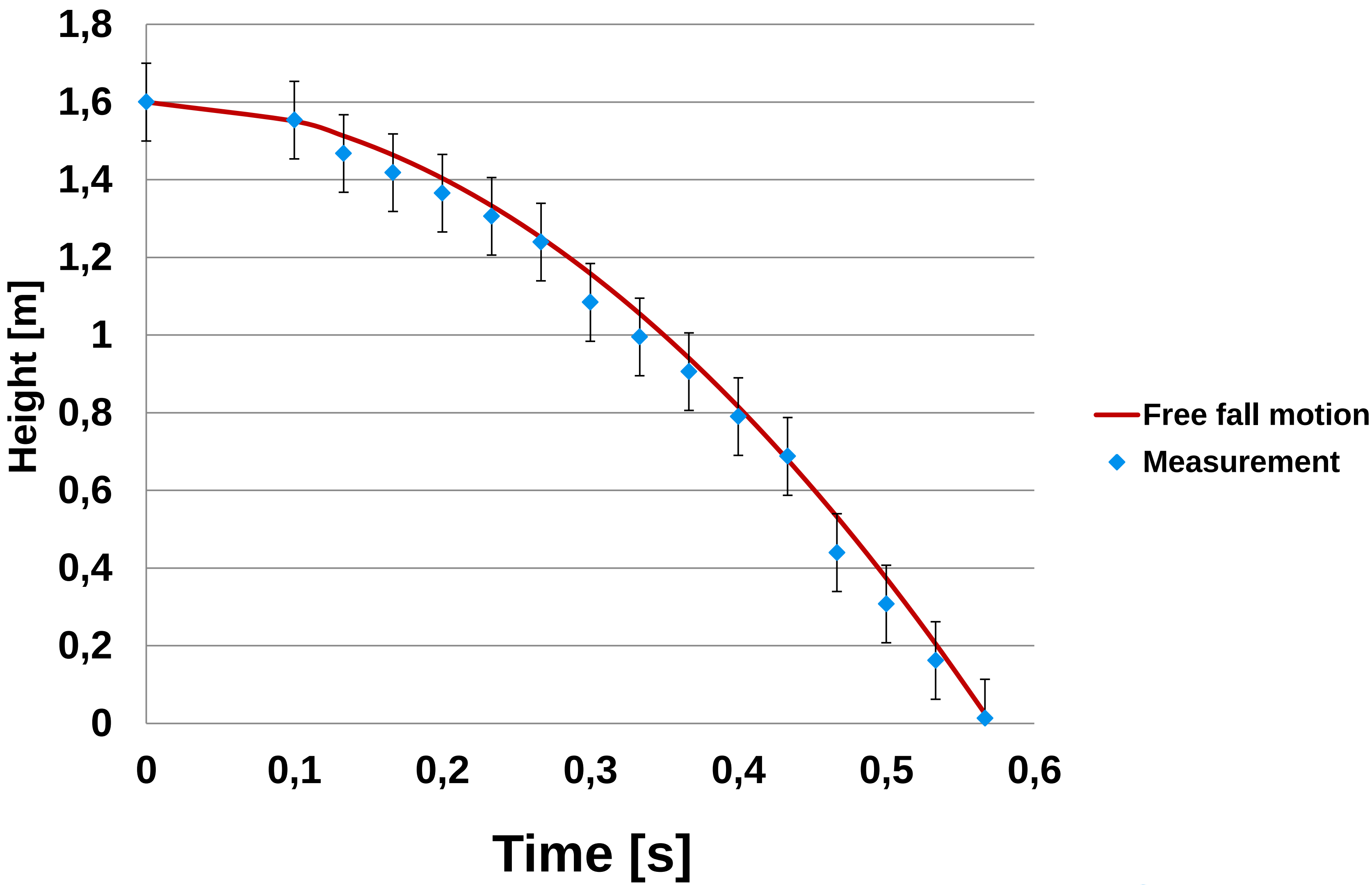
$$\tau = \frac{2}{3} fF_N r_{contact}$$



Measurement of coefficients



Drag


1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
Height [m]
0
0,1
0,2
0,3
0,4
0,5
0,6
Time [s]
Free fall motion
Measurement

Magnus effect

Detaily explained and investigated in (also with other forces) :

Borg, Karl et al. " *Forces on a spinning sphere moving in a rarefied gas."* Physics of Fluids Volume 15, Number 3 (March 2003): 736-41

$$\vec{F} = f(\vec{\omega} \times \vec{v})$$

Effect is **unobservable** at this small range of velocities (0 – 5 m/s)

Just try to spin the ball and let it fall. There is no shift to the side.

Also calculated in : **Barber** *"Golf Ball Flight Dynamics"* Final project, Cornell University

Result = **The effect is relevant only in the motion with long trajectories.**

Summary of coefficients

Carpet

- Rolling resistance *arm - 0.0015 m*
- Shape coefficient (Air drag) – 0.1
- Coefficient of restitution Ball with
  - Carpet 0.4
  - Hole bottom 0.8
  - Hole walls, edges 0.55
- Friction coefficients - ball with
  - Carpet 0.23
  - Hole bottom 0.12
  - Hole walls 0.1
  - Hole edges 0.4

- Ball cap effect – contact radius ball on :

  Carpet - 0.0004 m

  Bottom –0.0001 m