3.

String of beads

Michal Hledík

3. String of beads

A long string of beads is released from a beaker by pulling a sufficiently long part of the chain over the edge of the beaker.

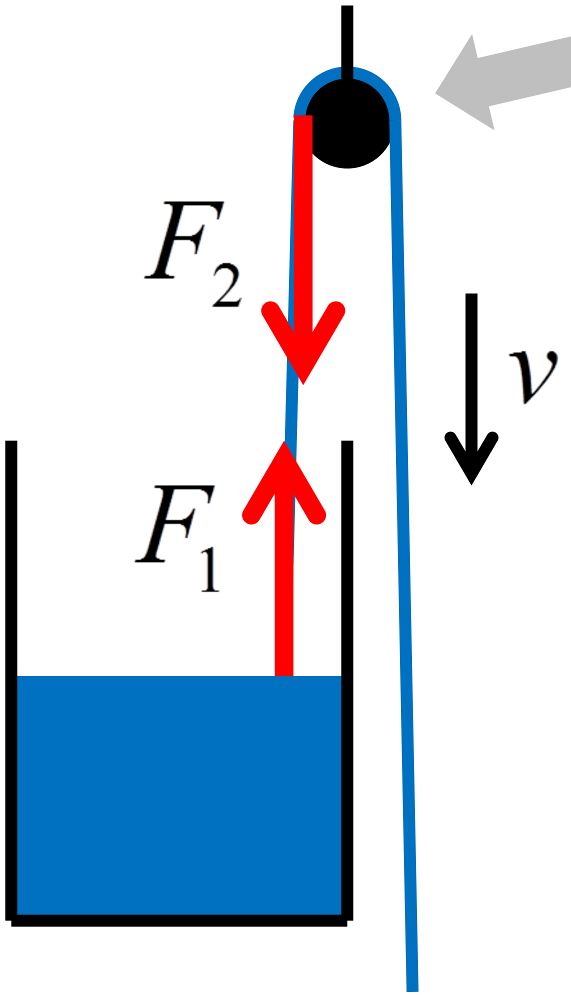
Due to gravity the speed of the string increases. At a certain moment the string no longer touches the edge of the beaker.

Investigate and explain the phenomenon.

Content

1. Mechanism of levitation
2. Simulation
3. Investigating interesting aspects of the phenomenon

Simple 1D model



Imaginary pulley at the top, constant velocity

Analyzing tension in the string:

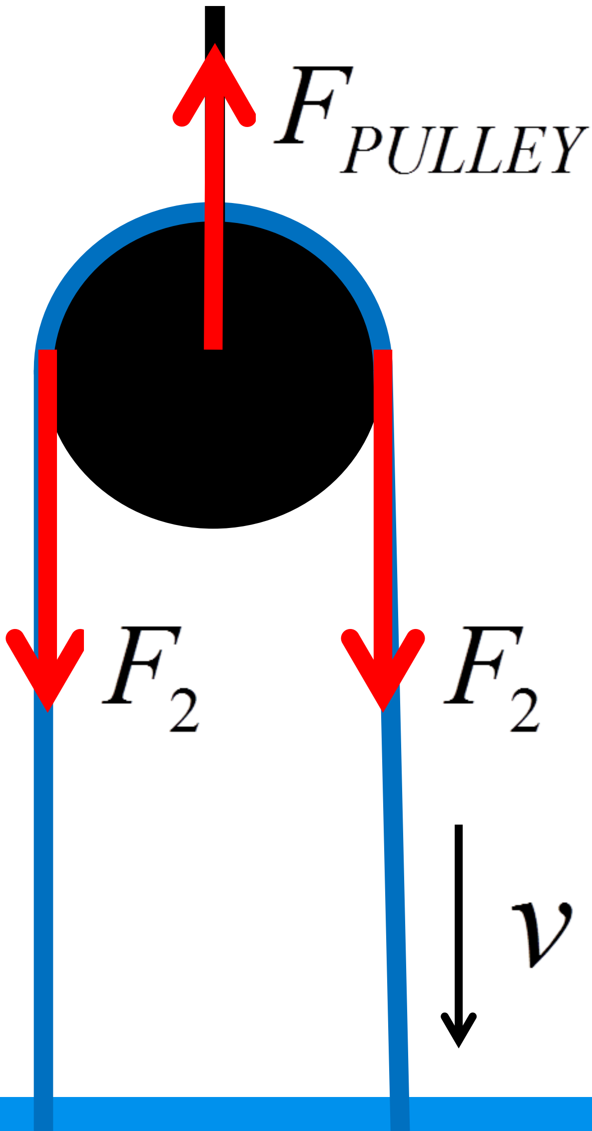
$$F_1 = v^2\sigma$$

(momentum needed to pull new beads)

$$F_2 = v^2\sigma + h\sigma g$$

(to balance the gravity)

Simple 1D model



Rate of change of momentum:

$$2v^2\sigma = 2F_2 - F_{PULLEY}$$

Resulting F_{PULLEY} :

$$F_{PULLEY} = 2h\sigma g$$

What if there is no pulley?

What is the source of energy?

Gravity potential?

- It would levitate in 1D model

Air drag effects? (Magnus effect…)

- Works with heavier and smaller beads

Bending stiffness of thread

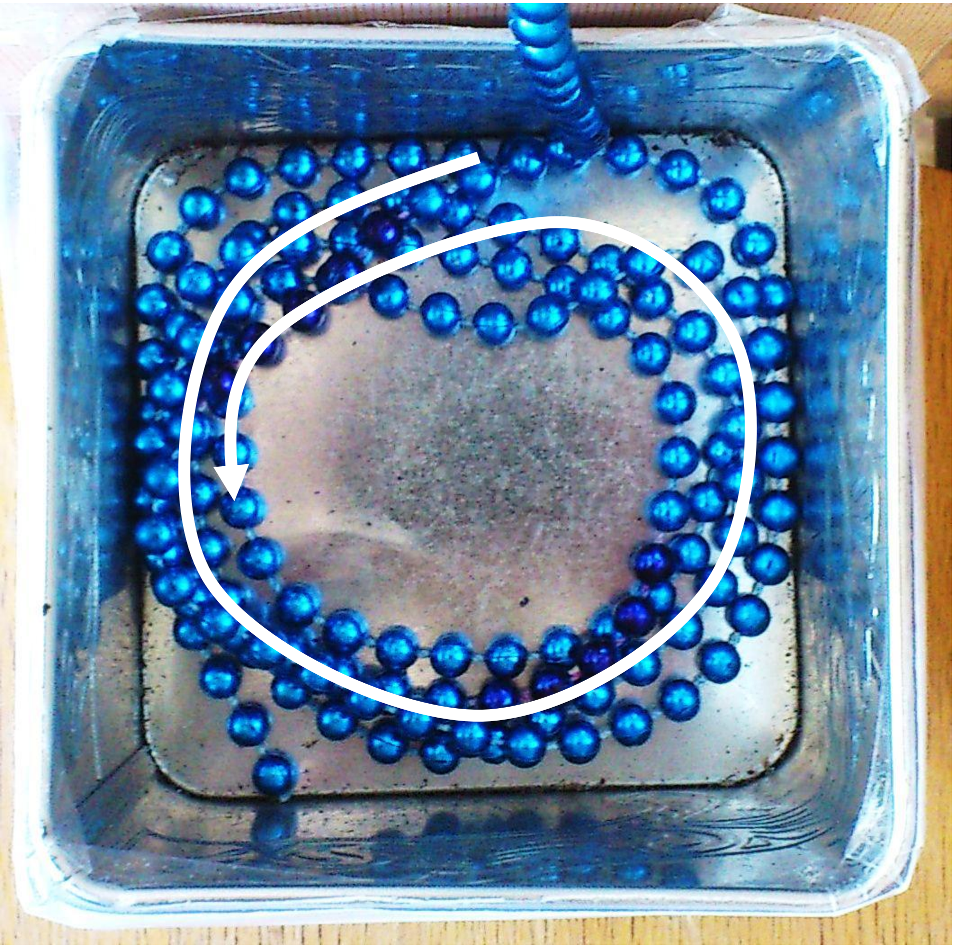
Thread – non-zero **bending stiffness**

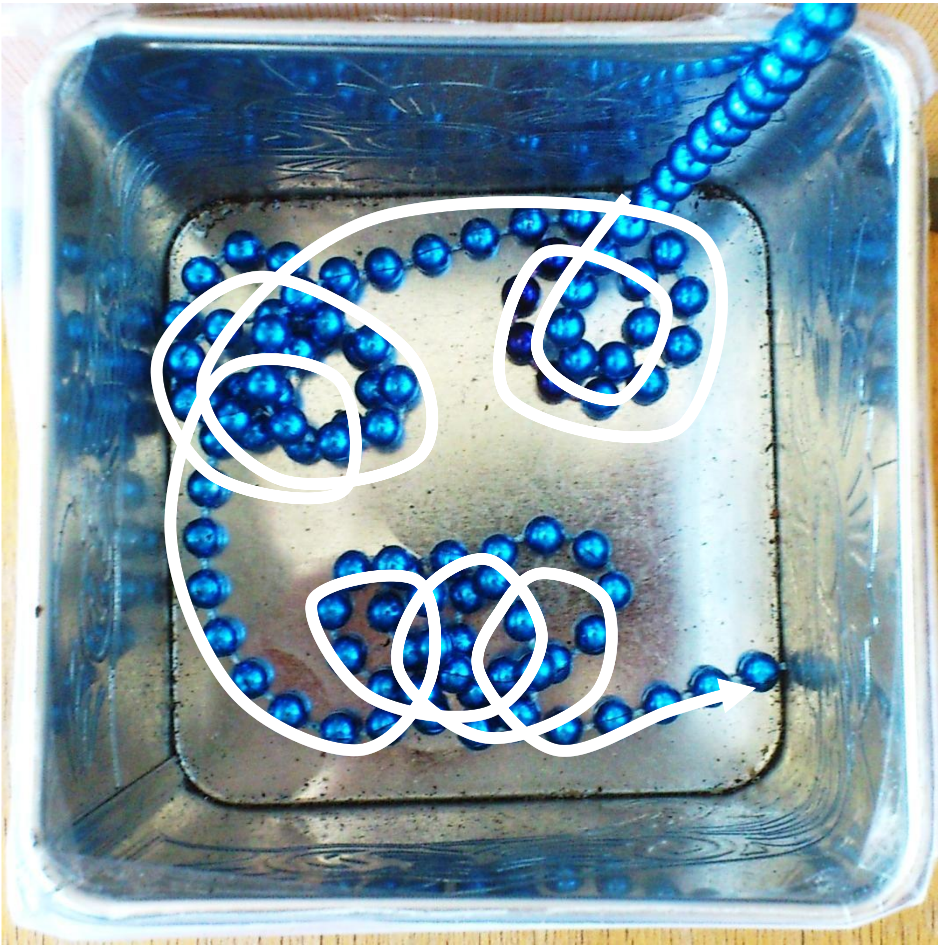
Small…

Significant compared to mass of beads

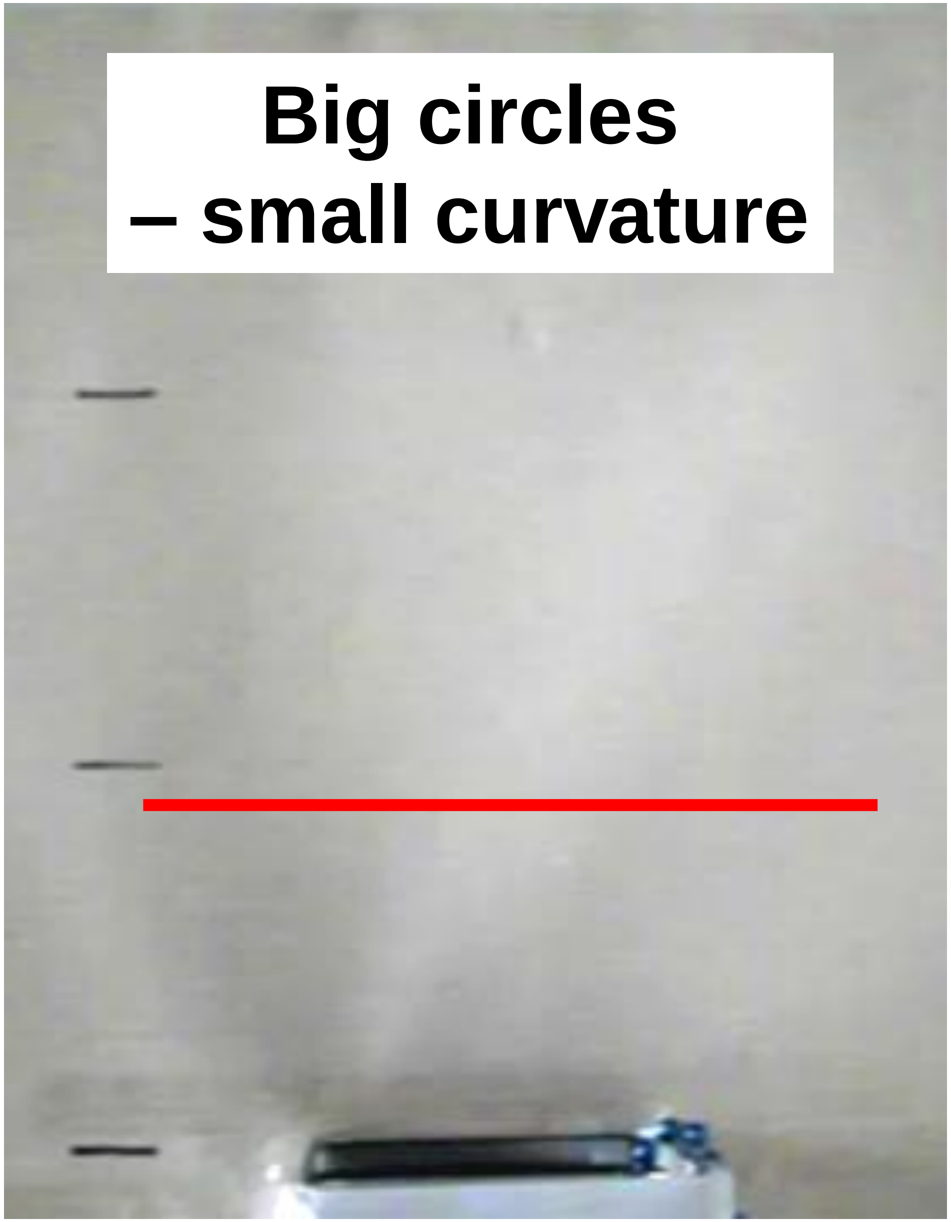
Let's test it

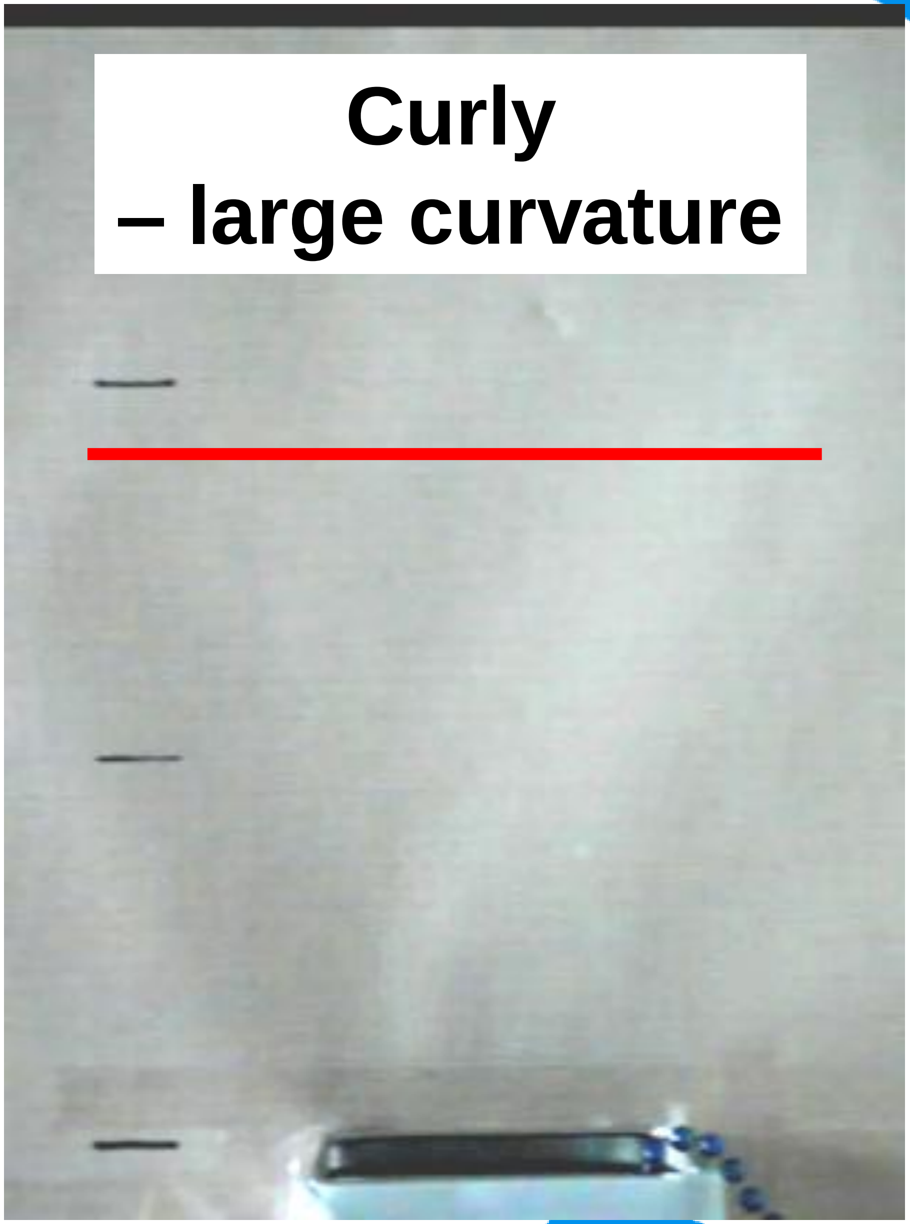
Different initial setting



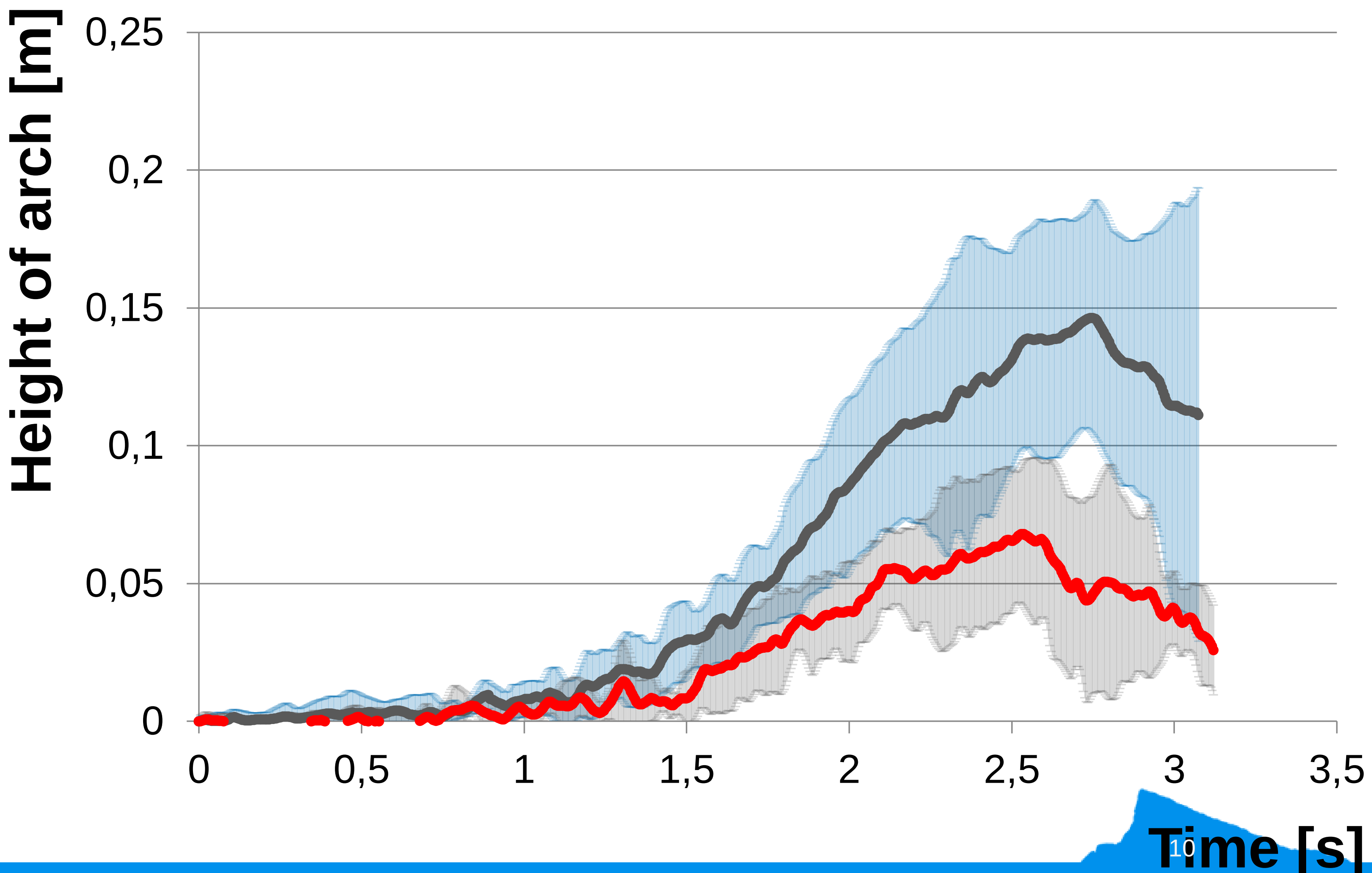


Big circles – small curvature **vs.** **Curly – large curvature**

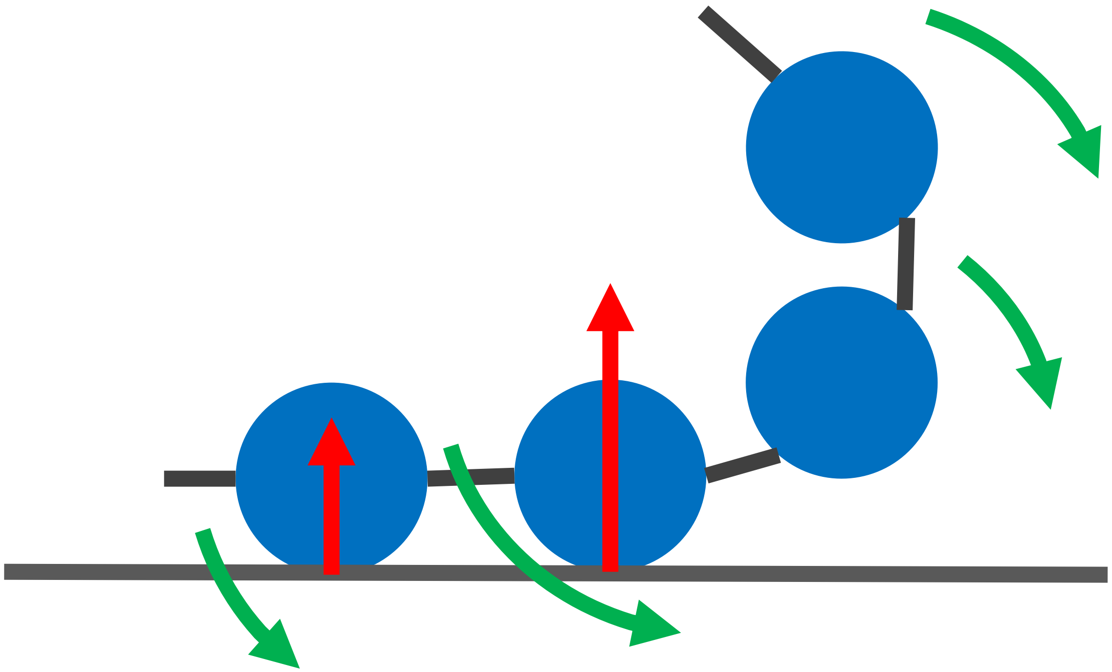

Big circles
– small curvature


Curly
– large curvature

More measurements, average height

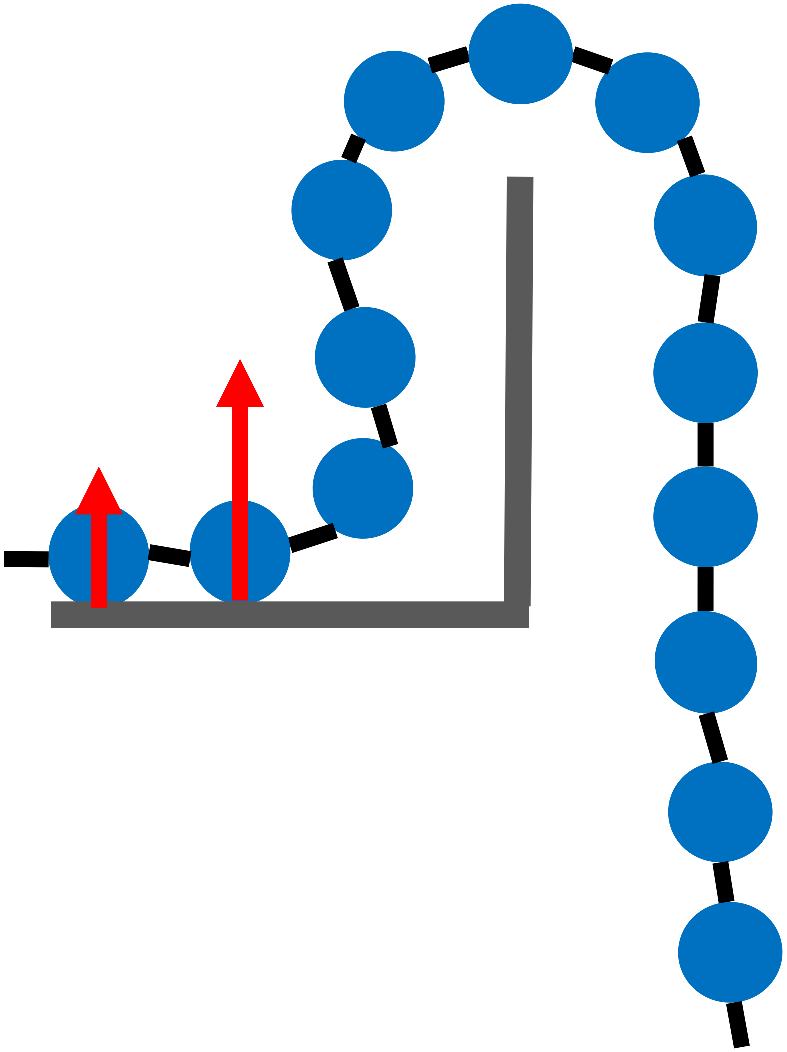


How does it work?

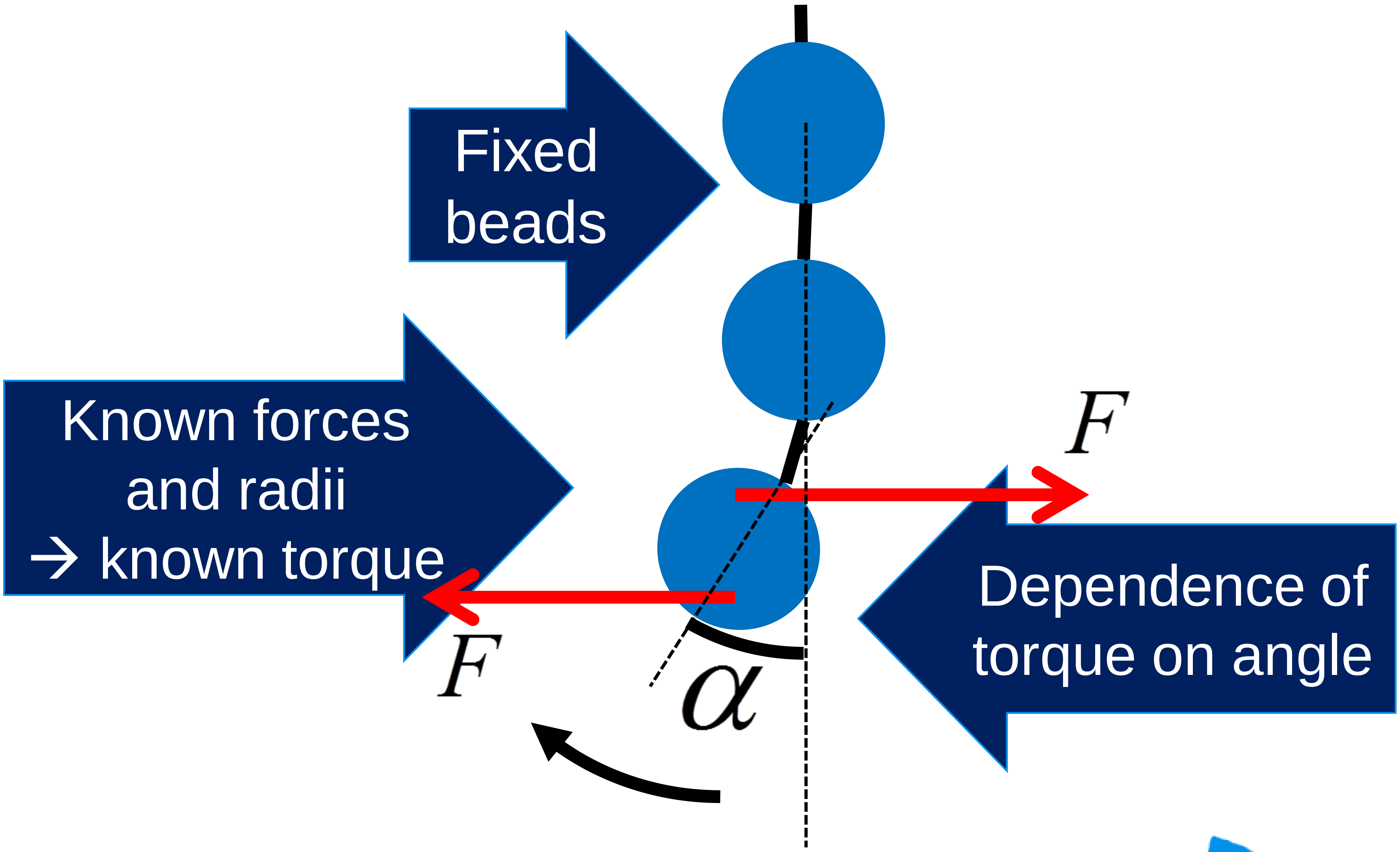


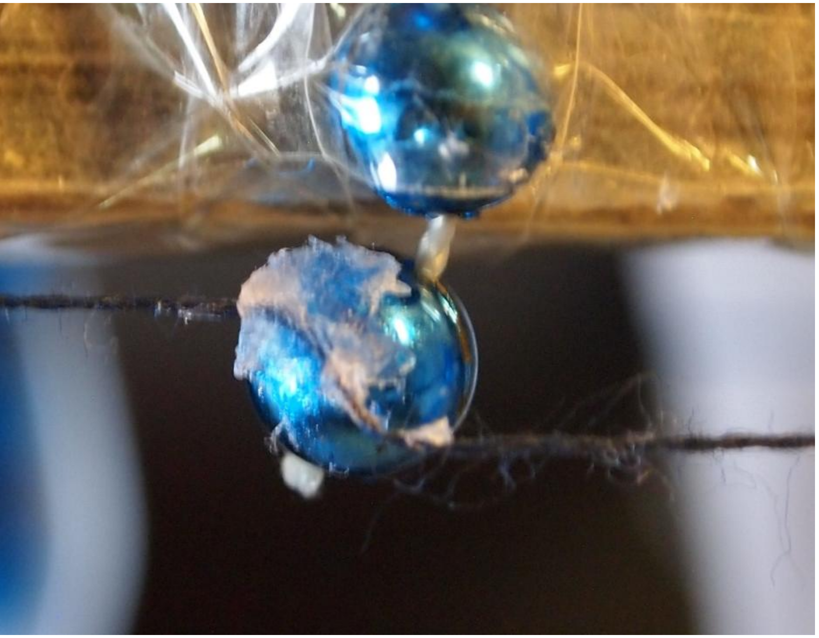
- String is trying to straighten
- Reaction – **normal force**

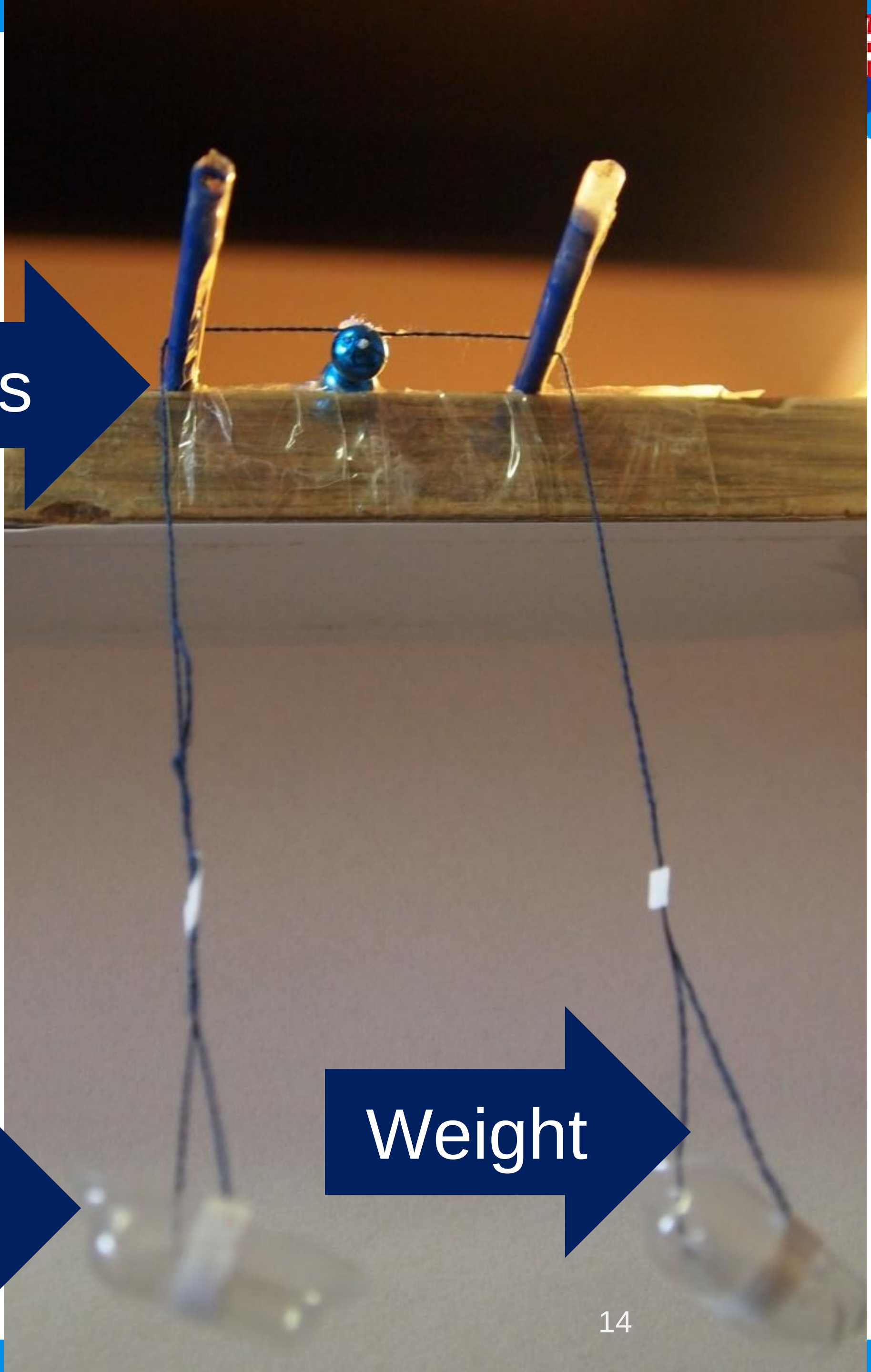
- Extra momentum upwards
- **Higher speed** upwards
- **Height of arch increases**



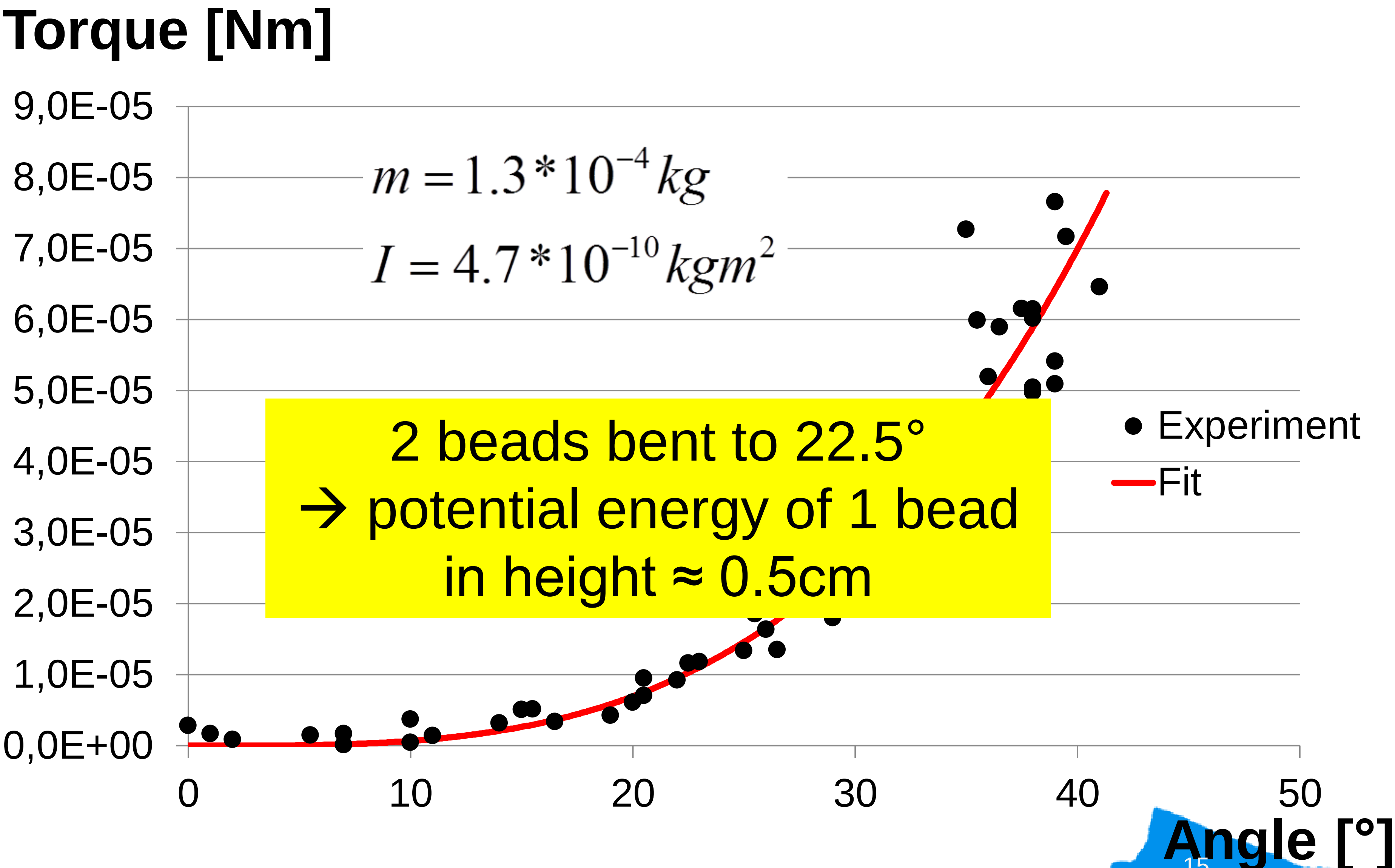
Bending stiffness measurement






Beads
Weight
Weight

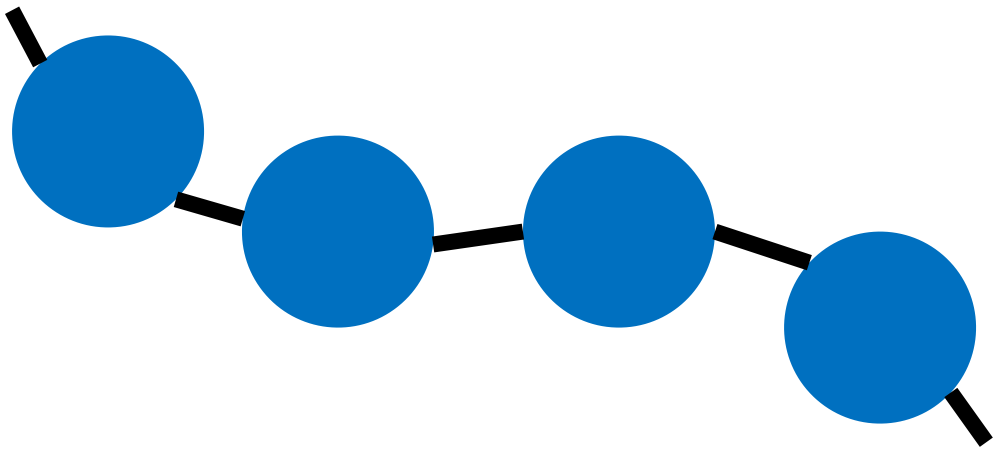
Result


Torque [Nm]
9,0E-05
8,0E-05
7,0E-05
6,0E-05
5,0E-05
4,0E-05
3,0E-05
2,0E-05
1,0E-05
0,0E+00
0
10
20
30
40
50
Angle [°]
Experiment
Fit
$m = 1.3 * 10^{-4} kg$
$I = 4.7 * 10^{-10} kgm^2$
2 beads bent to 22.5°
→ potential energy of 1 bead
in height ≈ 0.5cm

Developing a theory

- We know the important effect
- Analytic theory?
- Too complicated process
  → **simulation**

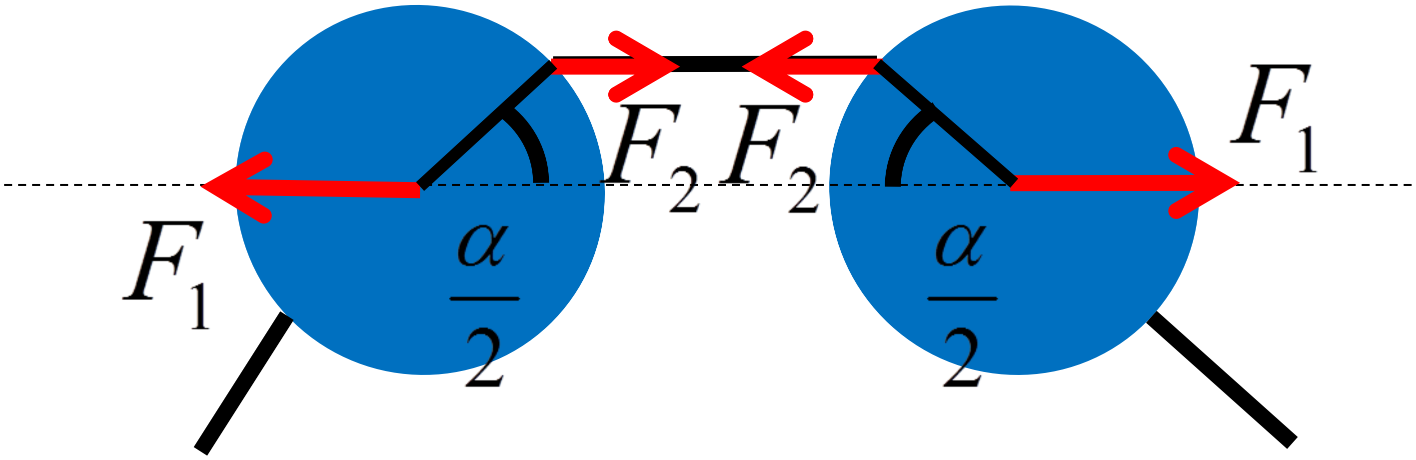
3D Simulation



Forces:

- Gravity
- Thread
- **Straightening**
- **Damping**

Simulation – straightening



- Keeping the length of thread constant
  → relation of F_1, F_2
- Data from bending stiffness measurement
  → torque (depending on angle)

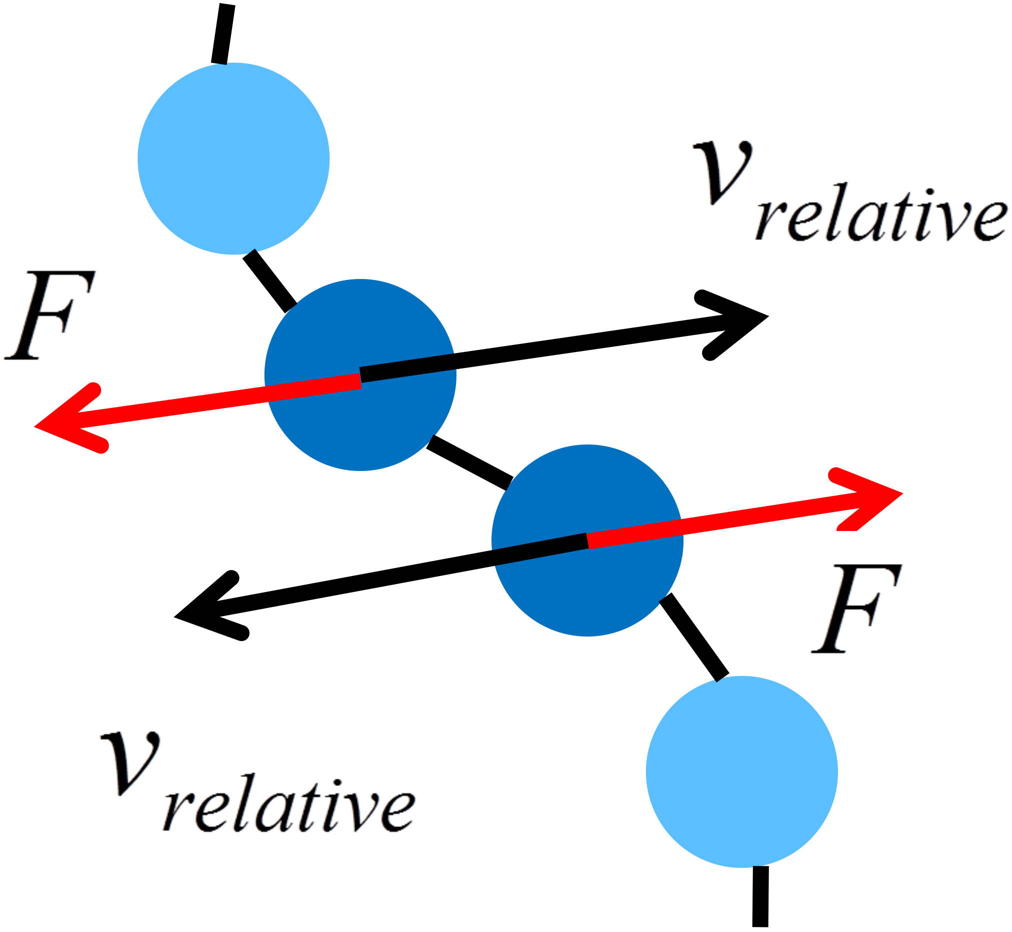
$$F_2 = \frac{\tau}{\sin\alpha} \qquad F_1 = F_2\left(\frac{mr^2}{I}\sin^2\alpha + 1\right)$$

Simulation – damping

Relative velocity according to adjacent bead

→ Force in opposite direction

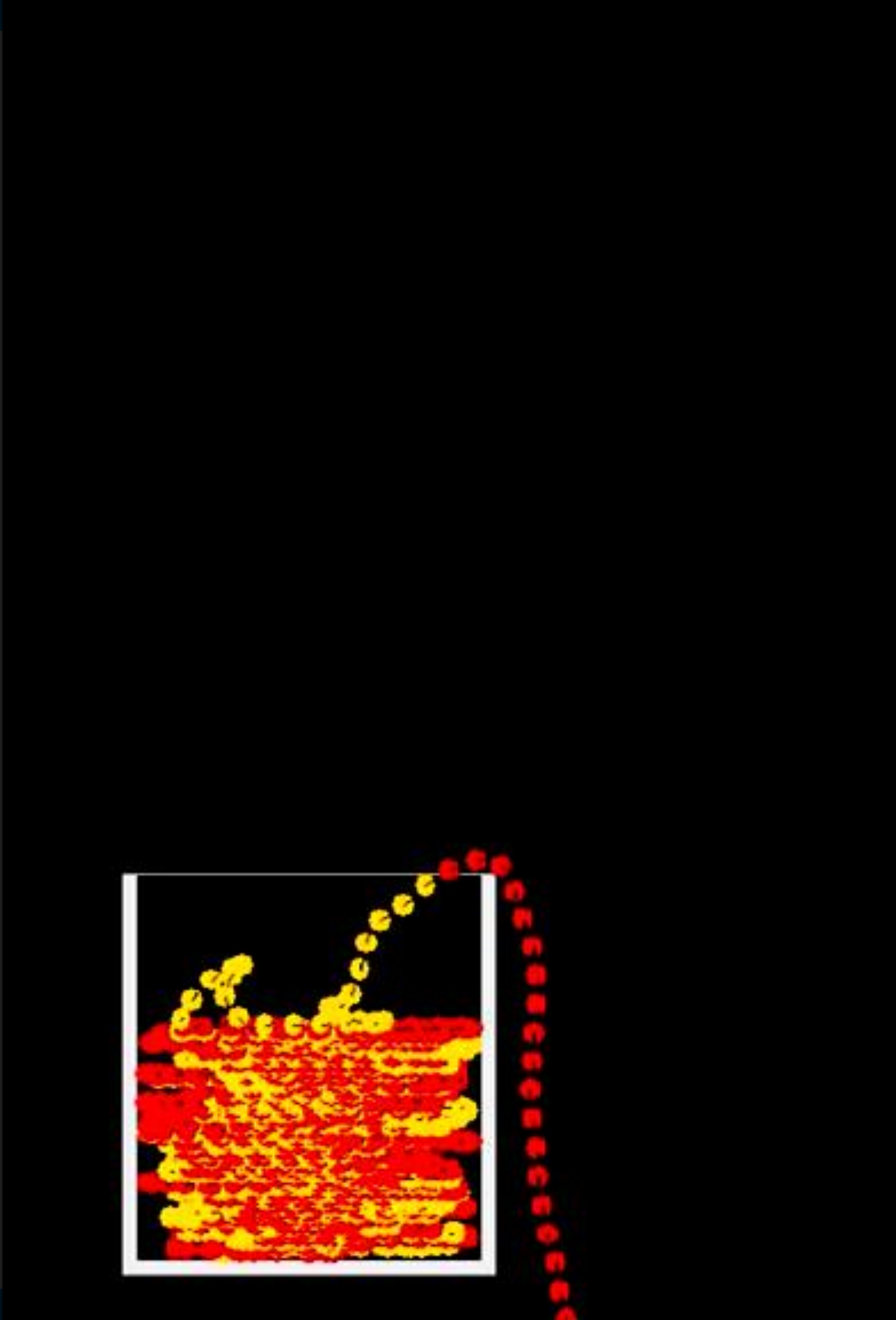
$$F = -Cv_{relative}$$



Simulation input

- **Geometrical properties** of string and beaker
- Experimental data – **bending stiffness**
- Fitted coefficient of damping
- Initial distribution of beads – random
  - Letting the string fall to the beaker
  - Small random side velocity for instability

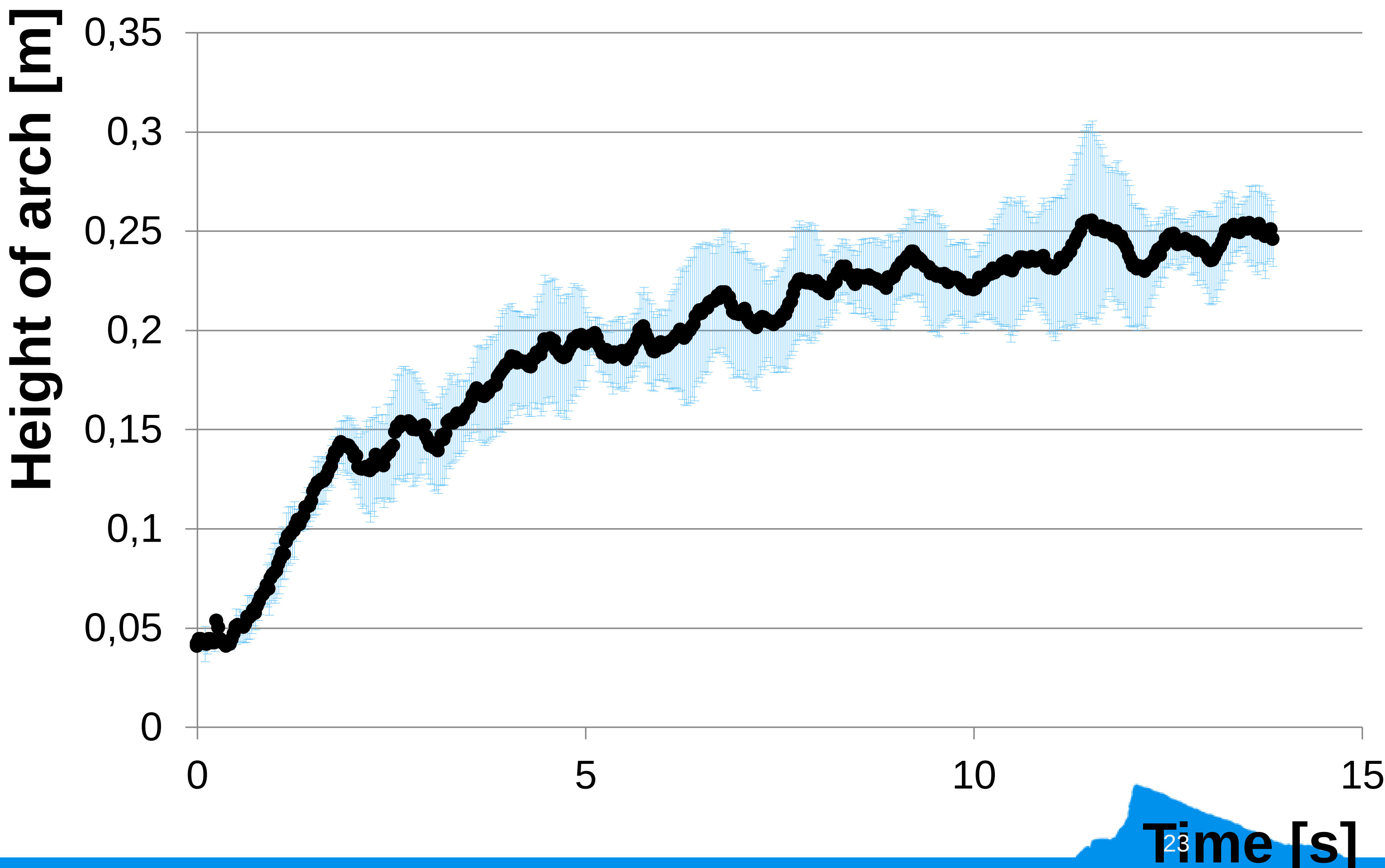
SLOVAKIA



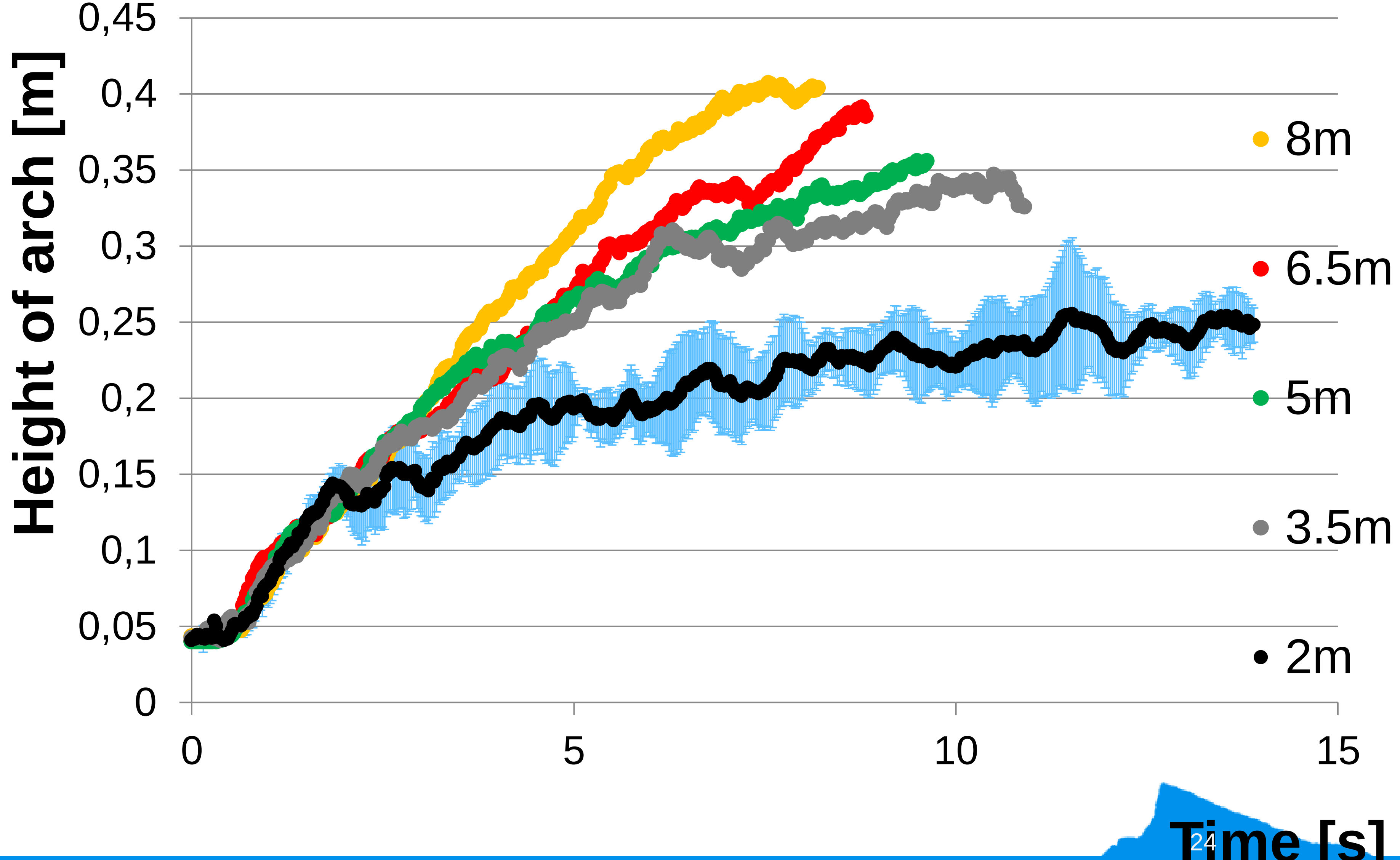
What if…?

- Very long string
- Influence of height over the floor
- Different strings/ropes
- Different beakers

Very long string



Height of the beaker over the floor



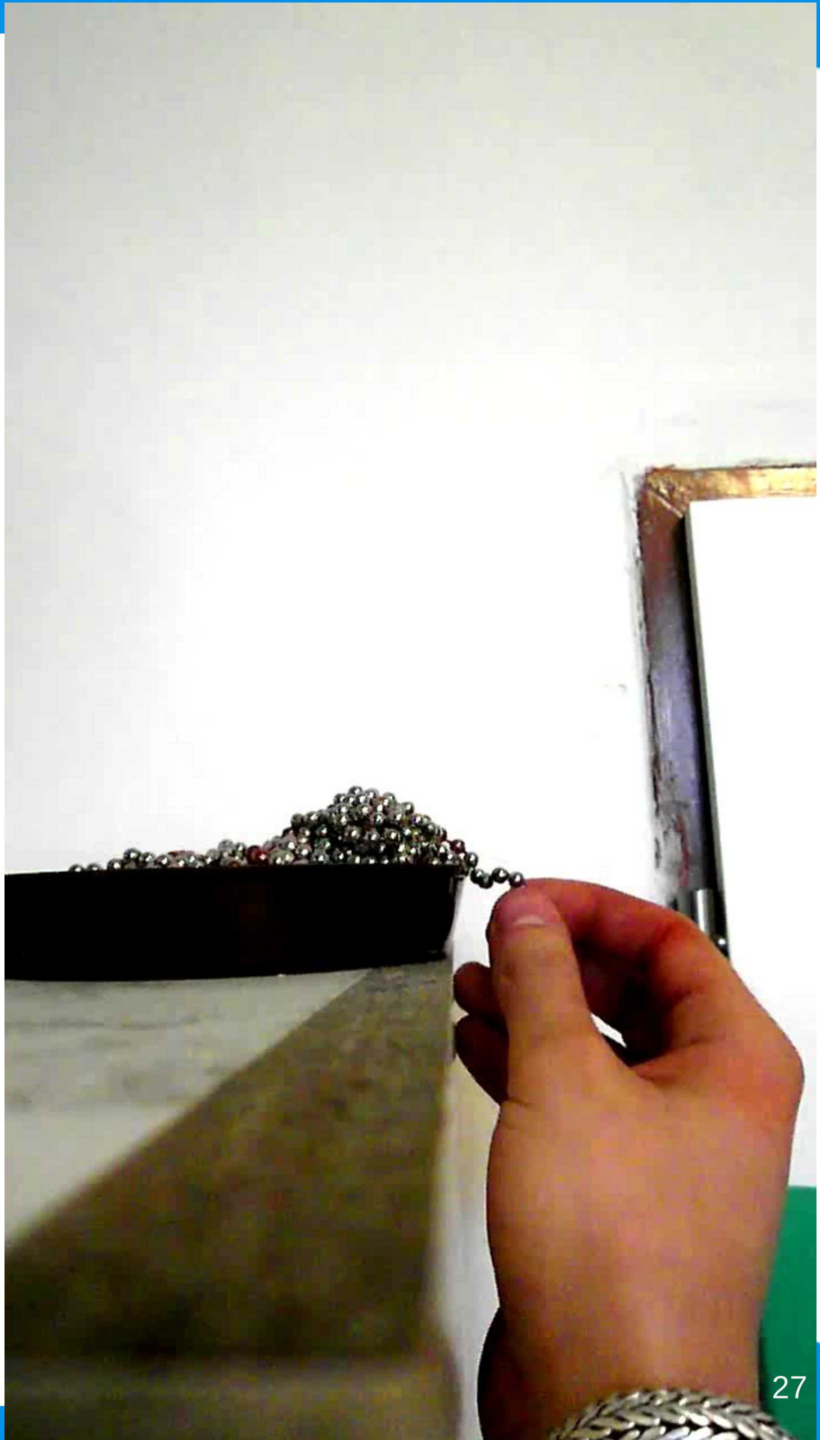
Different strings

- Metal beads – work

- Thread, climbing rope – do not work

Beaker – high walls, small hole

Beaker
– low walls



Stable shape of the arch

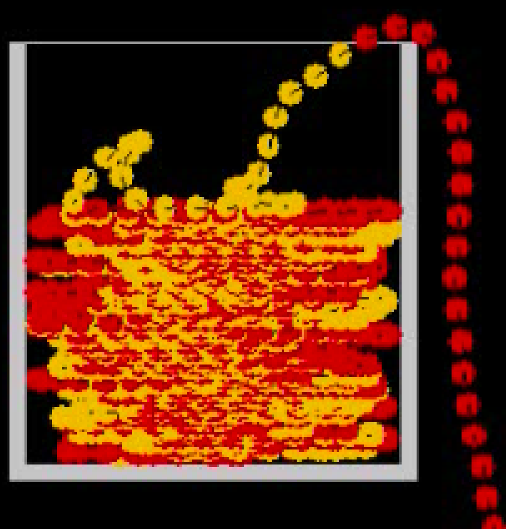
- Well known effect – **Lariat chain**
- Speed of transverse waves = speed of the string
- Waves appear to be stable

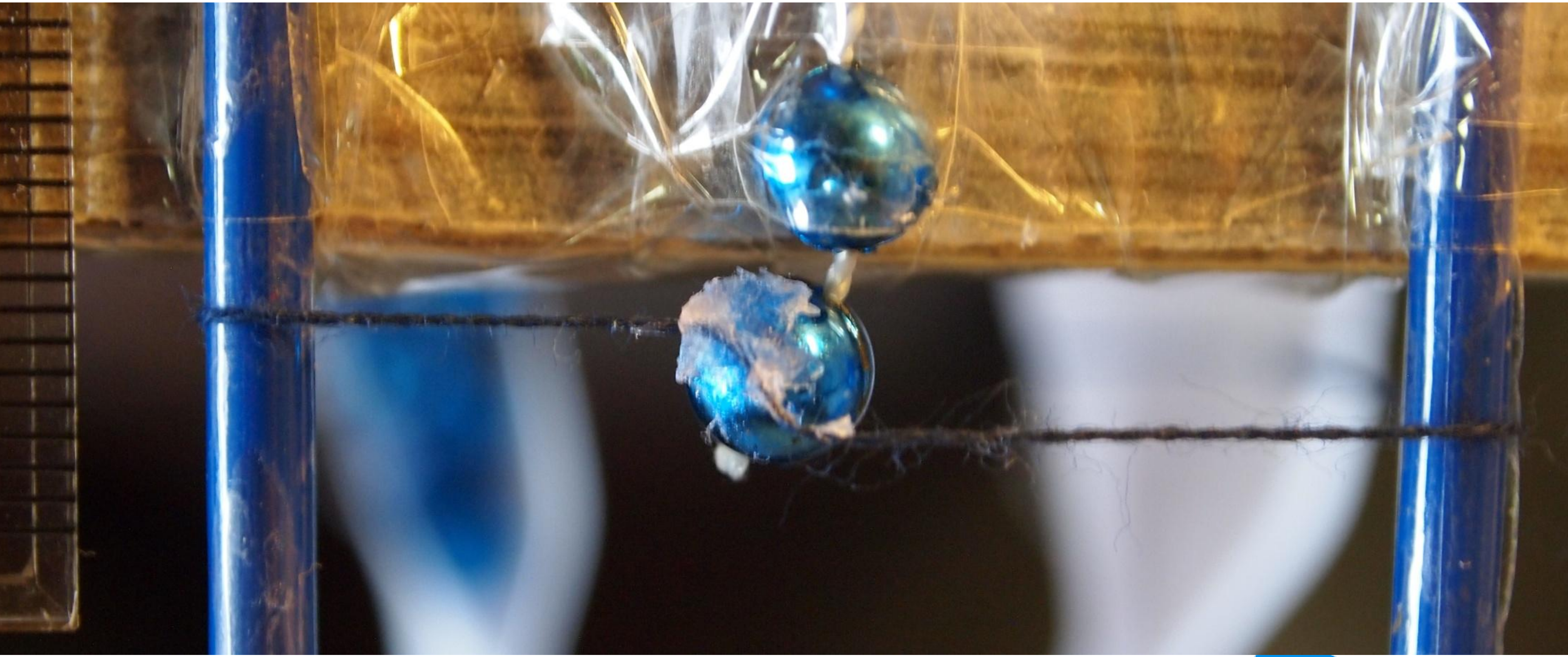
Conclusion

- Found important effect of bending stiffness
- Explained mechanism behind arch formation
- Developed a 3D numerical model of phenomenon
- Investigated relevant parameters

APPENDICES

3. String of beads





SLOVAKIA
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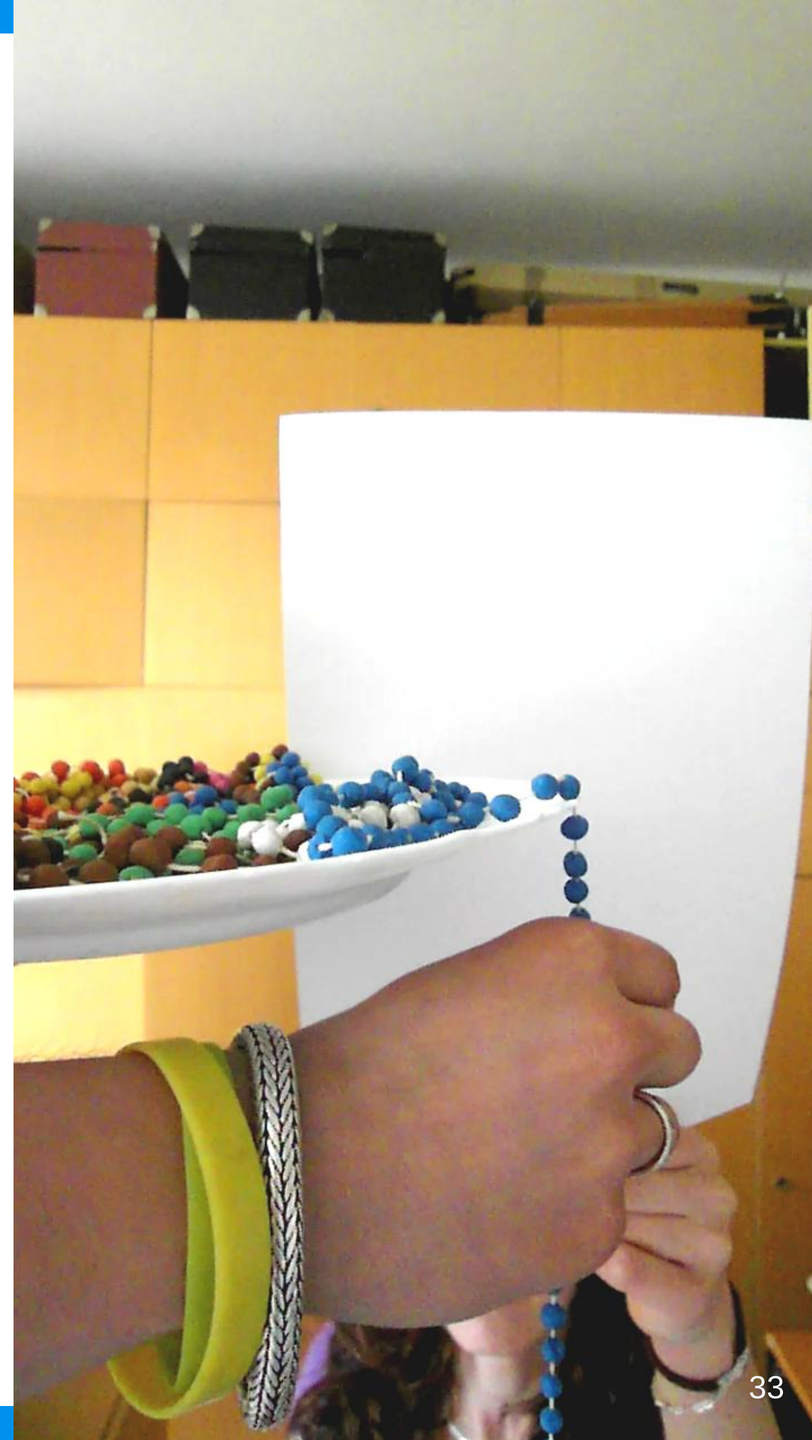
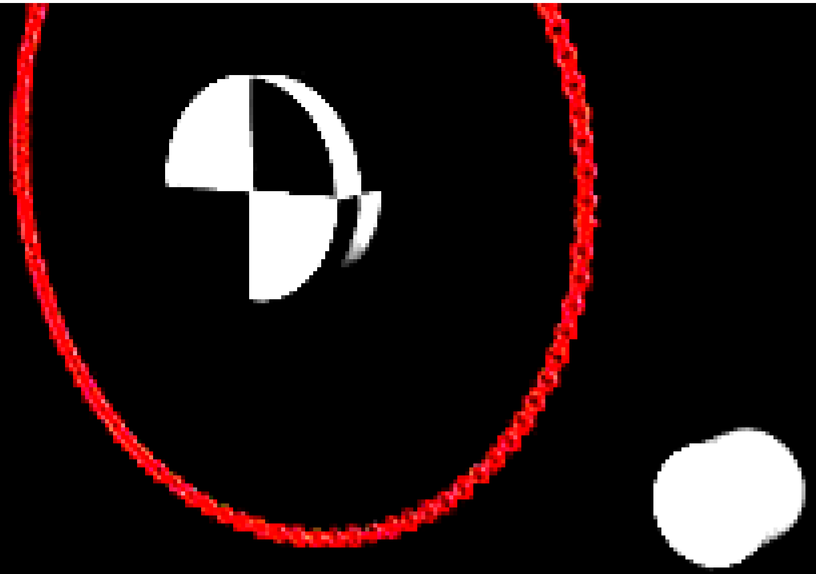
Plasticine beads – do not work



Climbing rope

Thread



http://www.youtube.com/watch?v=sRkI4qOWB7A