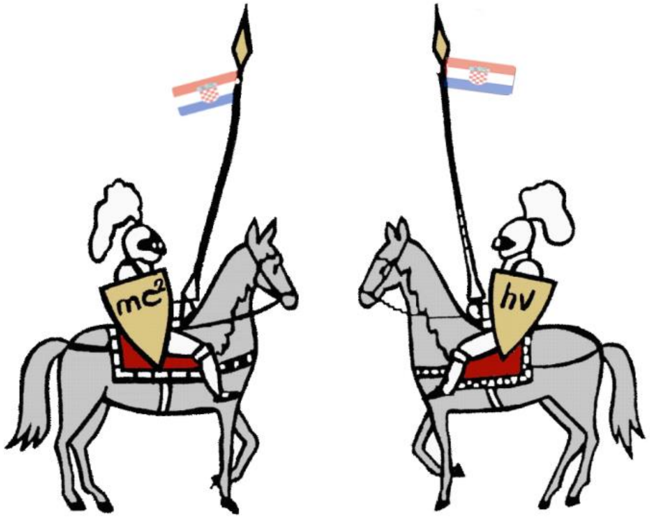
IYPT 2013

TEAM OF CROATIA

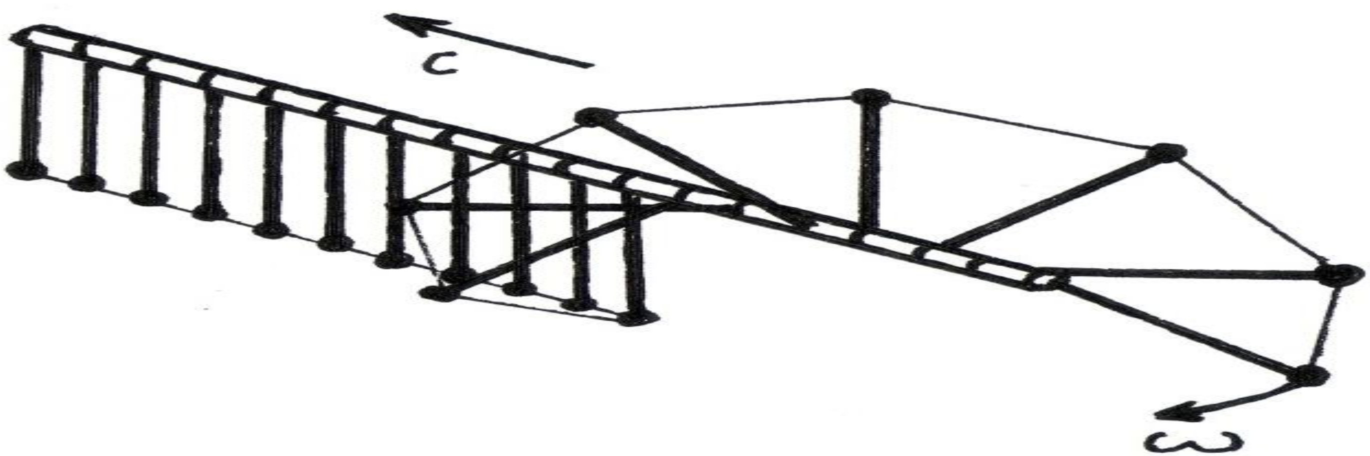
4. SOLITON

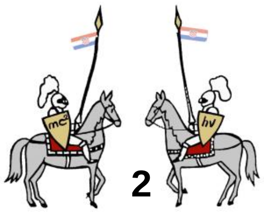
Reporter: Domagoj Pluščec



Problem

A chain of similar pendula is mounted equidistantly along a horizontal axis, with adjacent pendula being connected with light strings. Each pendulum can rotate about the axis but can not move sideways (see figure). Investigate the propagation of a deflection along such a chain. What is the speed for a solitary wave, when each pendulum undergoes an entire 360º revolution?

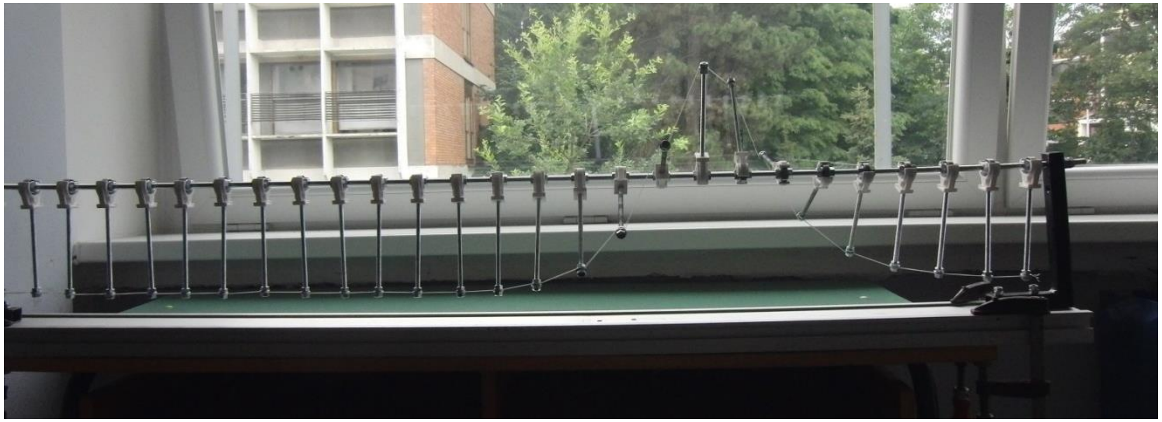




Pendulum system

Summary

- Theory
  - What is a soliton?
  - Model of the system
- Experimental
  - Apparatus
  - Methods
- Results
- Conclusion

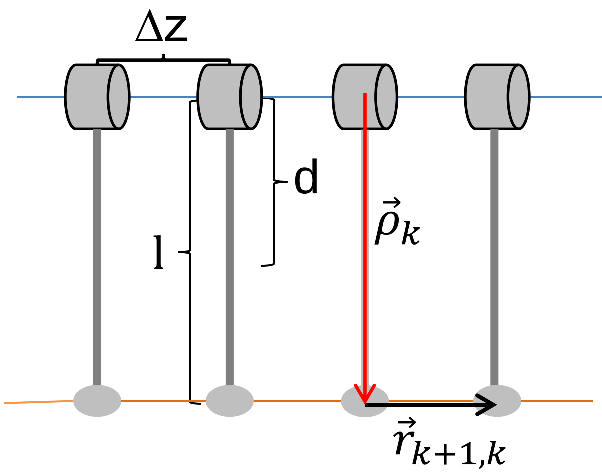


Equilibrium position

Soliton

- Wave packet that travels trought medium with unchanged shape
- Form of solution of some nonlinear partial diferential equations (Sine-gordon equation, KdV equation)
- They can interact with other solitons, and emerge from the collision unchanged except for a phase shift

Theory – basic concepts



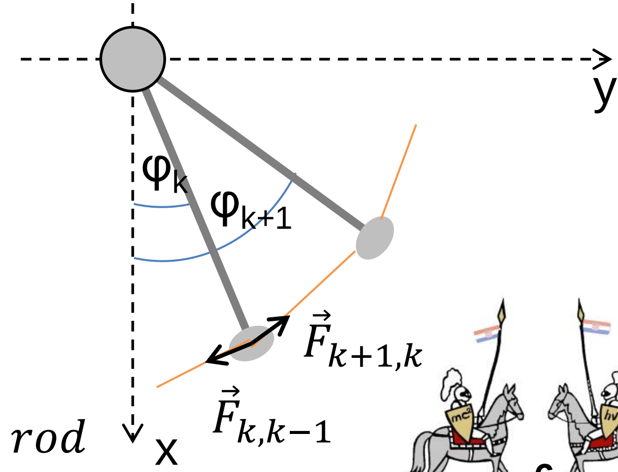
$$I\alpha_k = \sum \tau$$

$$I\alpha_k = \tau_g + \tau_{k+1,k} + \tau_{k,k-1}$$

$$\tau_g = -mgdsin(\varphi_k)$$

$$\vec{\tau}_{k+1,k} = \vec{l} \times \vec{F}_{k+1,k}$$

Analogly derived



$$\vec{\rho}_k = \mathrm{l} * (\cos(\varphi_k)\vec{\imath} + \sin(\varphi_k)\vec{\jmath})$$

$$\vec{r}_{k+1,k} = \vec{\rho}_{k+1,k} + \Delta z\vec{k}$$

Hooke's law:

$$F_{k+1,k} = k(r_{k+1,k} - \Delta z)$$

τ_g – *gravitational torque*

$\tau_{k+1,k}$ – *torque from next pendulum*

k – *coefficient of elasticity of the string*

d – *distance between center of gravity and horizontal rod*

I – *moment of inertia of the pendulum*

Theory – our model

- Torque from elastic string:

$$\tau_{k+1,k} = kl^2 A_{k+1,k}\sin(\varphi_{k+1,k})$$

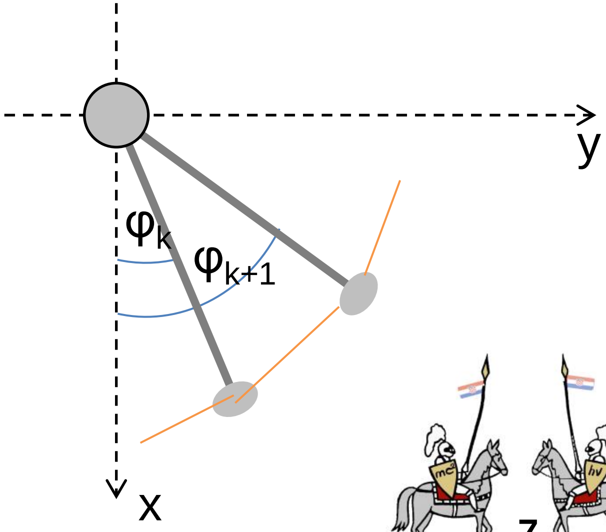
- Equation of the motion of the pendulum:

$$\boldsymbol{I\alpha_k = -mgdsin\varphi_k + kl^2(A_{k+1,k}\,sin\,\varphi_{k+1,k} - A_{k,k-1}\,sin\,\varphi_{k,k-1})}$$

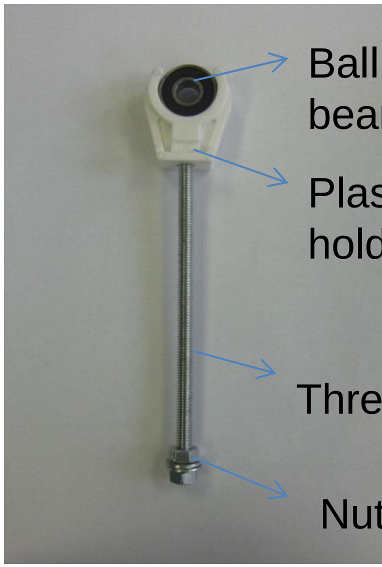
- We solved it numerically with computer (programing languages – python and quick basic) for every pendulum in some time interval
    - initial conditions from experimental setup (initial position and velocities for system of pendulums)

Substitution:

$$A_{n+1,n} = \left(1 - \frac{1}{\sqrt{1 + B^2 sin^2 \frac{\varphi_{n+1,n}}{2}}}\right); B = \frac{2l}{\Delta z}$$

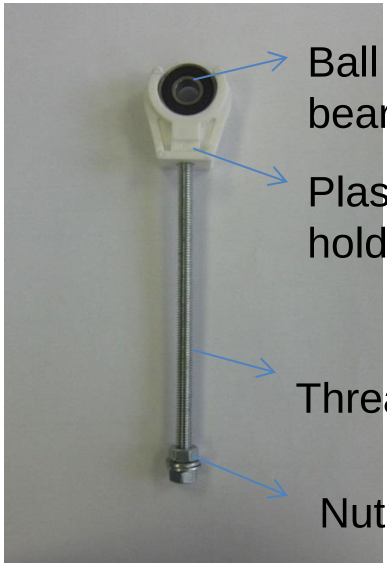


Apparatus



- HS camera
- Custom stand
  - Reduce vibrations of horizontal rod
- Ruler
- Software (ImageJ, self-made programs for numerical analysis)
- Balance

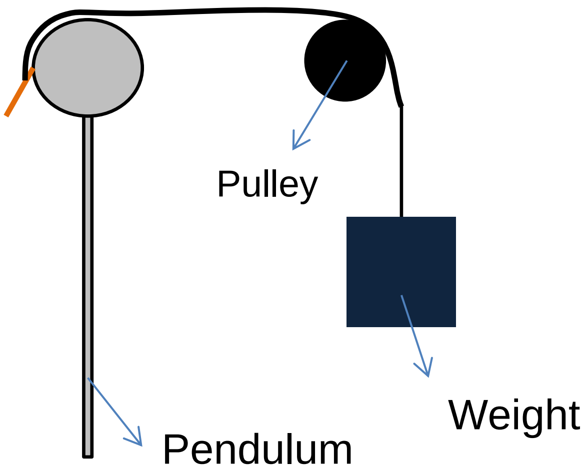
Characteristics of our system

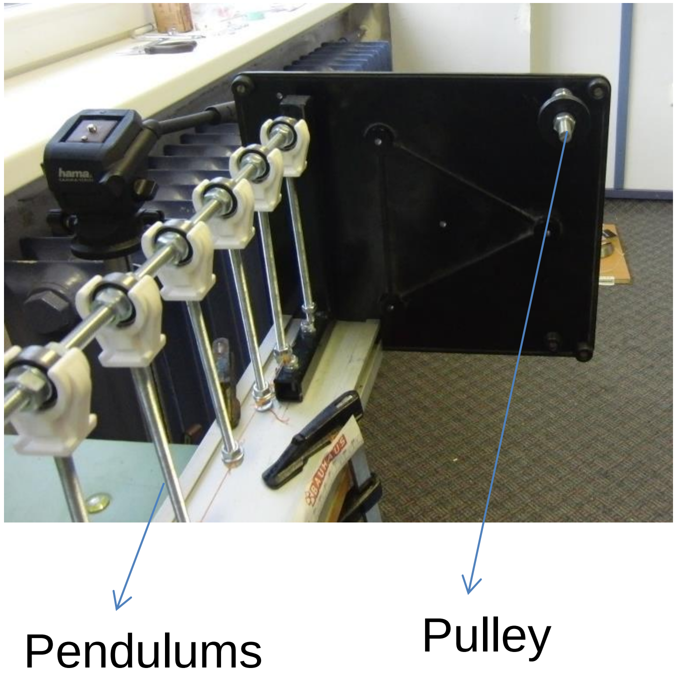


- Pendulum
  - Length = 14.5 cm
  - Mass = 45 g
  - Distance from centre of the gravity to the rotational center = 6.1 cm
  - Moment of inertia = $3.69 * 10^{-4}$ kgm^2
  - Internal radius of a ball bearing = 8 mm
  - External radius of a ball bearing = 22 mm
- System
  - Distance between pendulums 5.3 cm
  - Number of pendulums = 27

Method

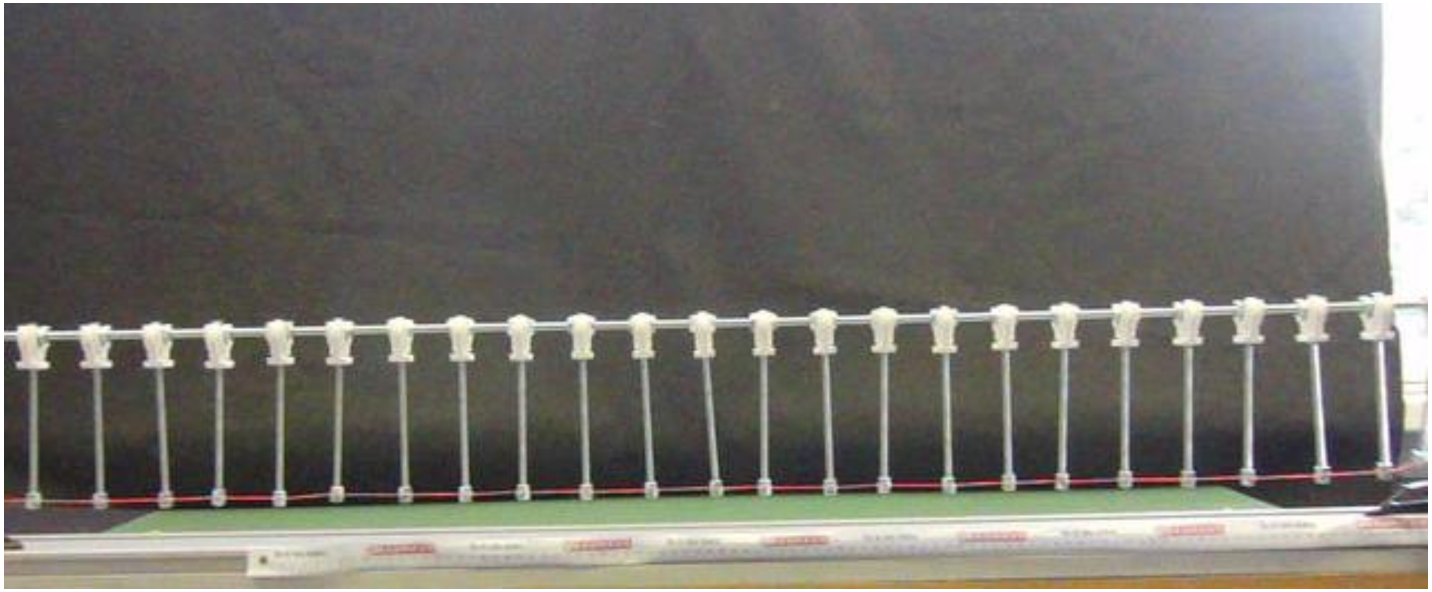
- Weight was connected to the first pendulum
- We released the weight to give initial speed
- Weight would disconnect from pendulum





- The wave was filmed by high speed camera (120 -420 fps)

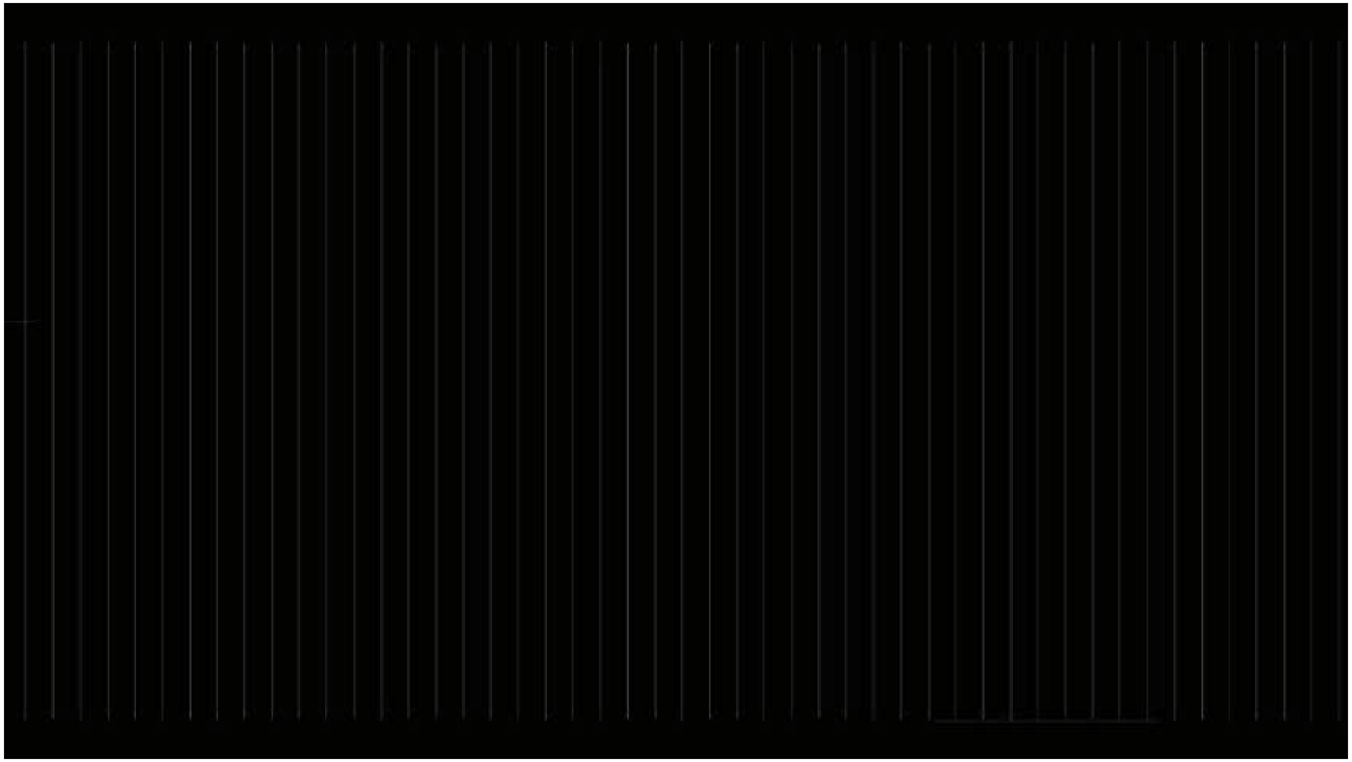
Traveling wave



120 fps

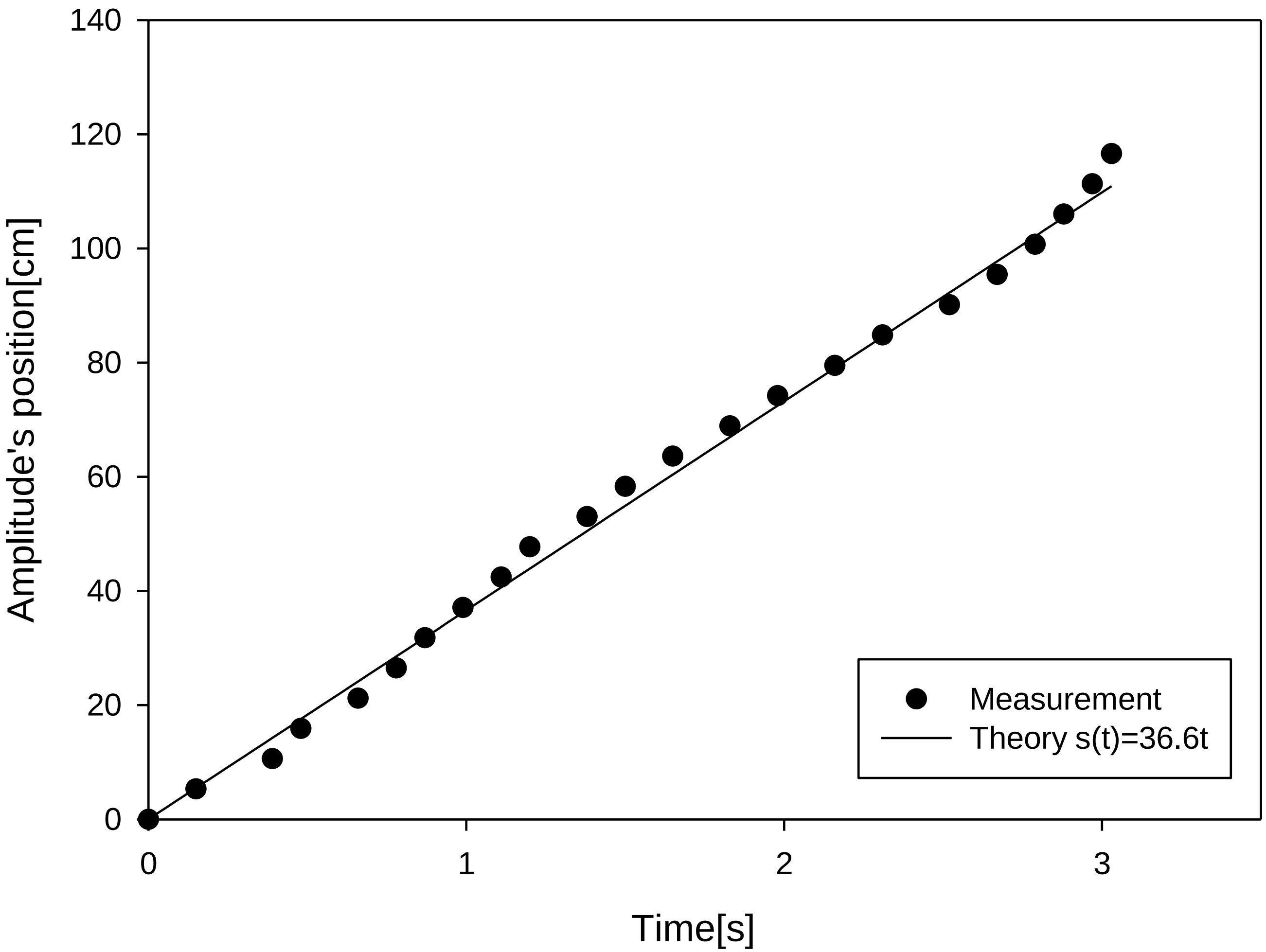
Speed of the wave – displacement of the amplitude of the wave divided by the time interval

Theoretical wave

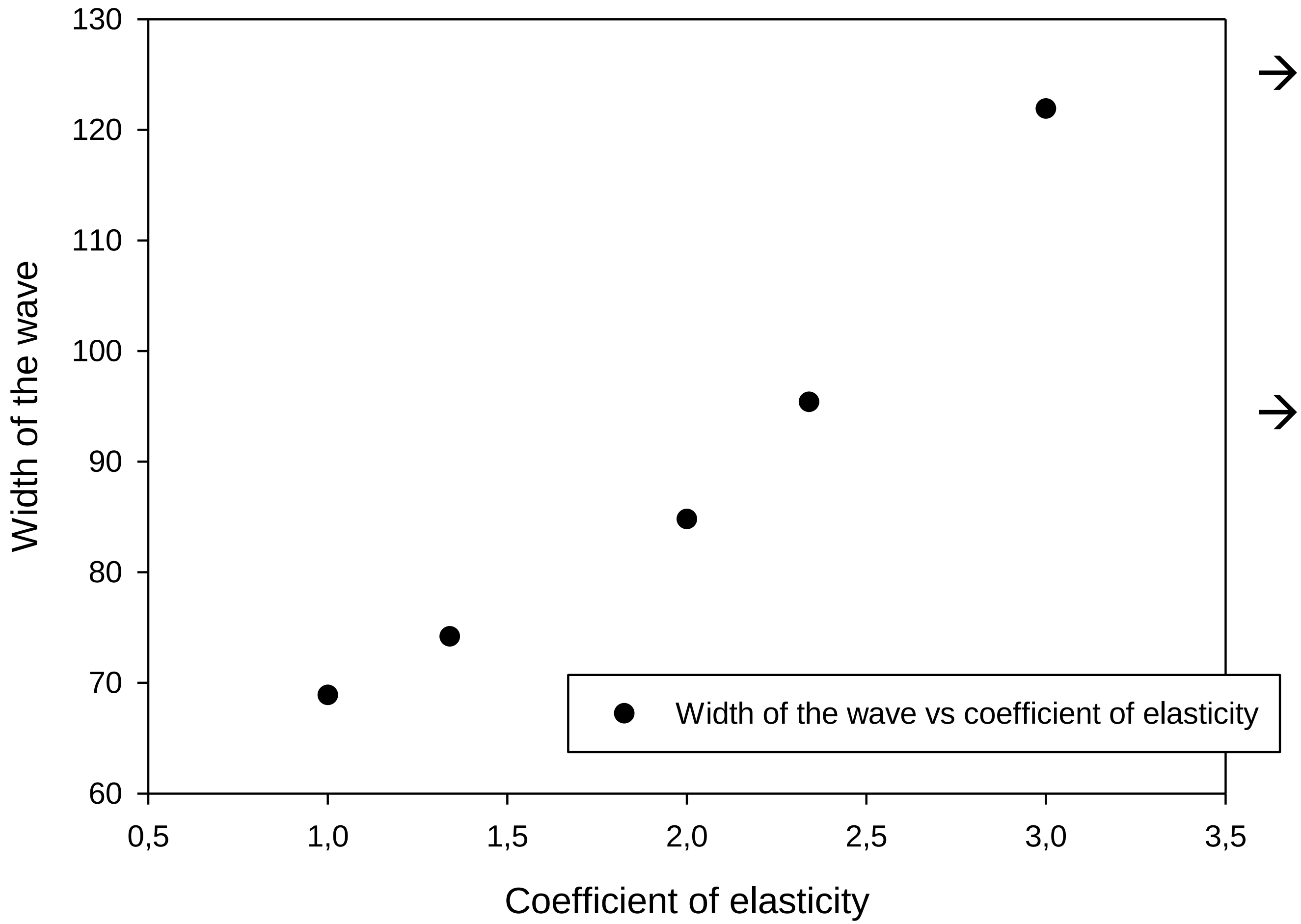


- We created system with the same initial conditions
- Every white line represents one pendulum

Comparison

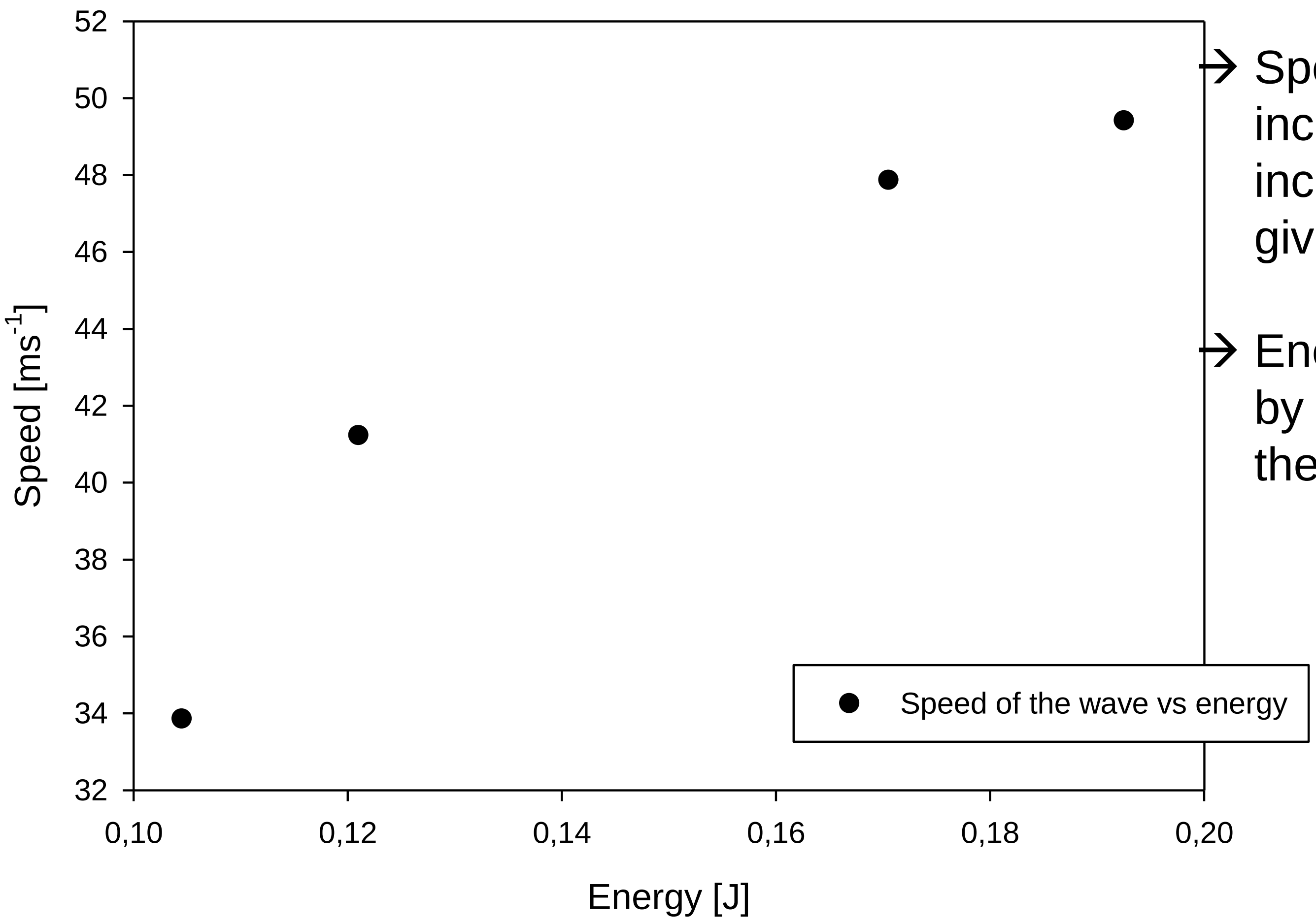

Amplitude's position[cm]
140
120
100
80
60
40
20
0
0
1
2
3
Time[s]
Measurement
Theory s(t)=36.6t

Width of the wave vs coefficient of elasticity



- → Width of the wave increases with coefficient of elasticity
- → Energy given to the system was same for every measurement

Speed of the wave vs energy given to the system



- → Speed of the wave increases with increasing energy given to the system
- → Energy was changed by changing mass of the weight

Collision – clockwise-counterclockwise

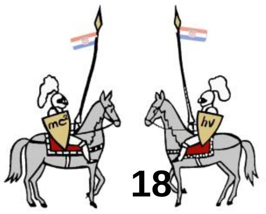
Solitons repel each other – they reflect with phase shift = π
In collision they behave like a fixed end

Collision – clockwise-clockwise

Solitons pass trought each other – they don’t add them selves by superposition principle

Conclusion

- Pendulum system was constructed
- We have modeled the system theoreticaly and shown the correlation between experiment and theory
- Speed of the wave increases with initial energy given to the system
- Speed of the wave increases with increasement of coefficient of elasticity of the string
- Two types of collision were observed
  - Same direction of rotation → pass through each other
  - Opposite direction of rotation → reflection on fixed end



Literature

[1] Andrey Miroshnichenko, Solitons and Soliton collisions, http://www.mpipksdresden.mpg.ge/~andreym/solitons

[2] Peter S. Lomdahl, What is soliton, http://www.fas.org/sgp/othergov/doe/lanl/pubs/00326980.pdf

[3] System of Pendulums: A Realization of Sine-Gordon Model, http://demonstrations.wolfram.com/SystemOfPendulumsARealizationOfTheSineGordonModel/

[4] Clay James Grewcoe, Analysis of soliton solutions of Sine - Gordon equation.

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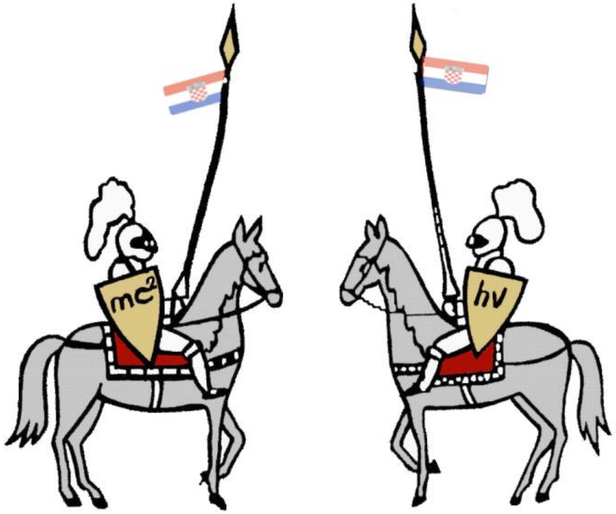
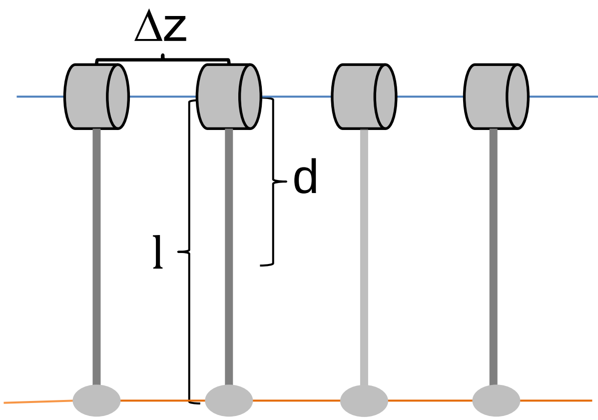
THANK YOU



Derivation of theory



Hooke's law:

$$F_{k+1,k} = k\left(r_{k+1,k} - \Delta z\right)$$

$$F_{k+1,k} = k\Delta z\left(\sqrt{1 + b^2 sin^2\left(\frac{\varphi_{k+1,k}}{2}\right)} - 1\right)$$

$$\Delta\vec{r} = \overrightarrow{r_{k+1}} - \overrightarrow{r_k}$$

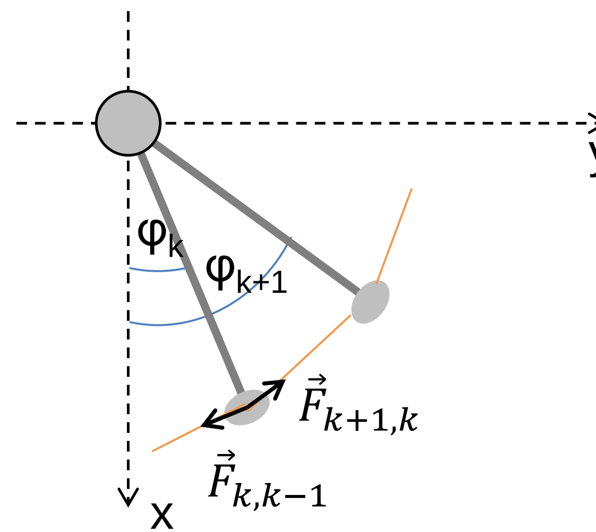
$$\overrightarrow{r_n} = \overrightarrow{\rho_n} + z_n\vec{k}$$

$$\overrightarrow{\rho_n} = l(\cos(\varphi_n)\vec{\imath} + \sin(\varphi_n)\vec{\jmath})$$

$$\left|r_{k+1,k}\right| = \sqrt{\overrightarrow{\rho_{k+1,k}} + \Delta z}$$

$$\left|r_{k+1,k}\right| = \Delta z\sqrt{1 + b^2 sin^2\left(\frac{\varphi_{k+1,k}}{2}\right)}$$

$$b = \frac{2l}{\Delta z}$$



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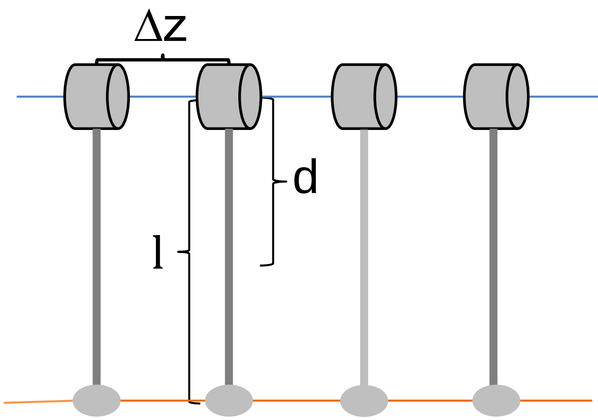


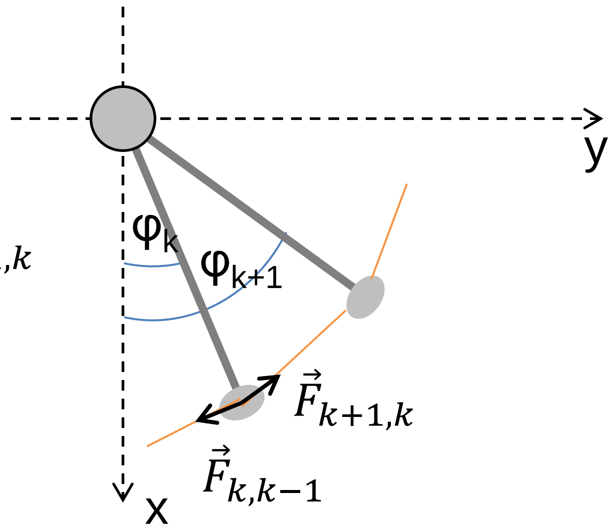
Hooke's law:

$$F_{k+1,k} = k\Delta z\left(\sqrt{1 + b^2 sin^2\left(\frac{\varphi_{k+1,k}}{2}\right)} - 1\right)$$

$$\vec{\tau}_{k+1,k} = \vec{l} \times \vec{F}_{k+1,k}$$

$$\vec{\tau}_{k+1,k} = F_{k+1,k} * \frac{l^2}{r_{21}} \sin(\varphi_{21})\, \vec{k}$$

$$\tau_{k+1,k} = kl^2\left(1 - \frac{1}{\sqrt{1 + \left(\frac{2l}{\Delta z}\right)^2 sin^2\frac{\varphi_{n+1,n}}{2}}}\right) sin\,\varphi_{k+1,k}$$



One pendulum rotation - damping