Fire hose

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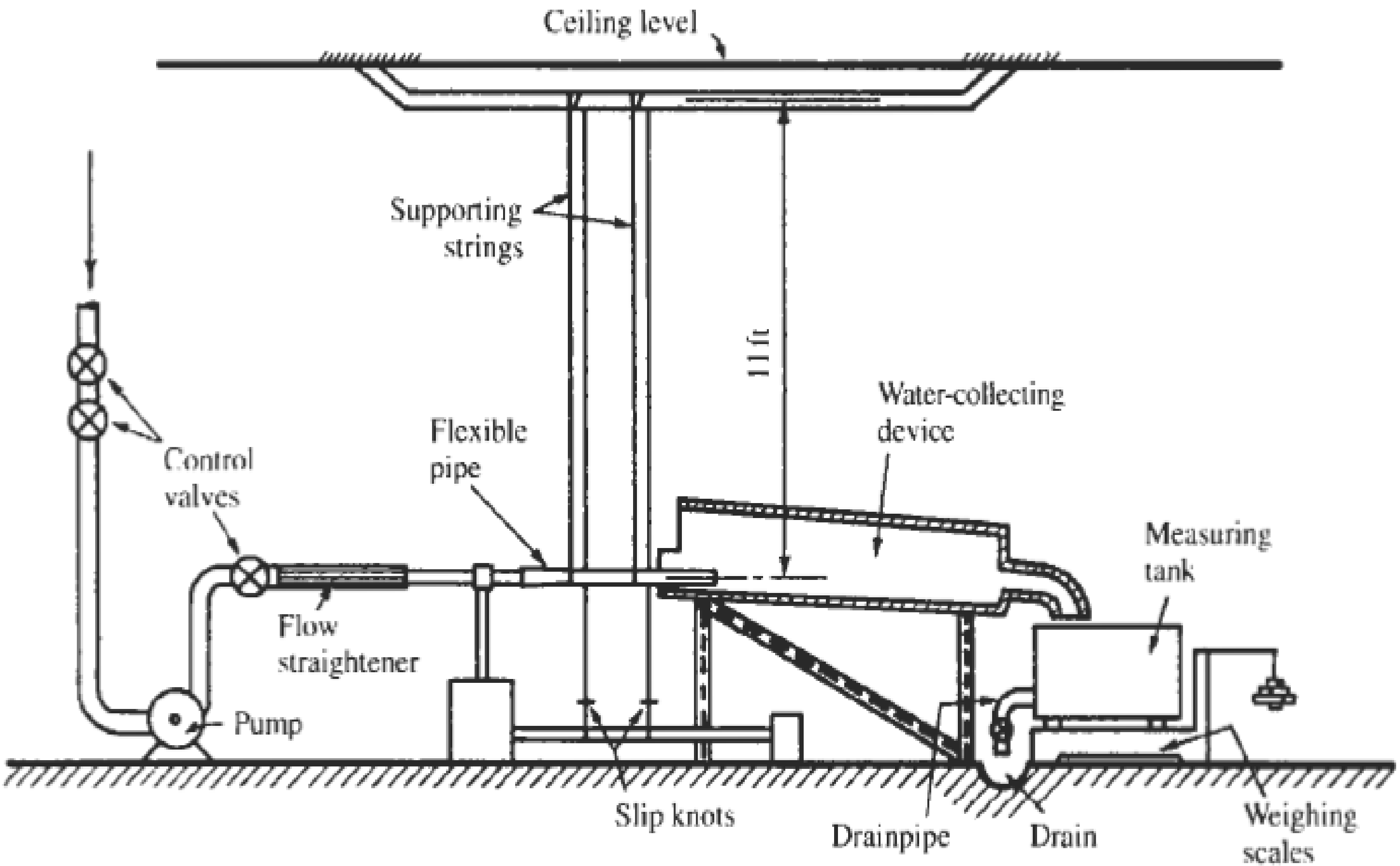
The problem

Consider a hose with a water jet coming from its nozzle. Release the hose and observe its subsequent motion. Determine the parameters that affect this motion.

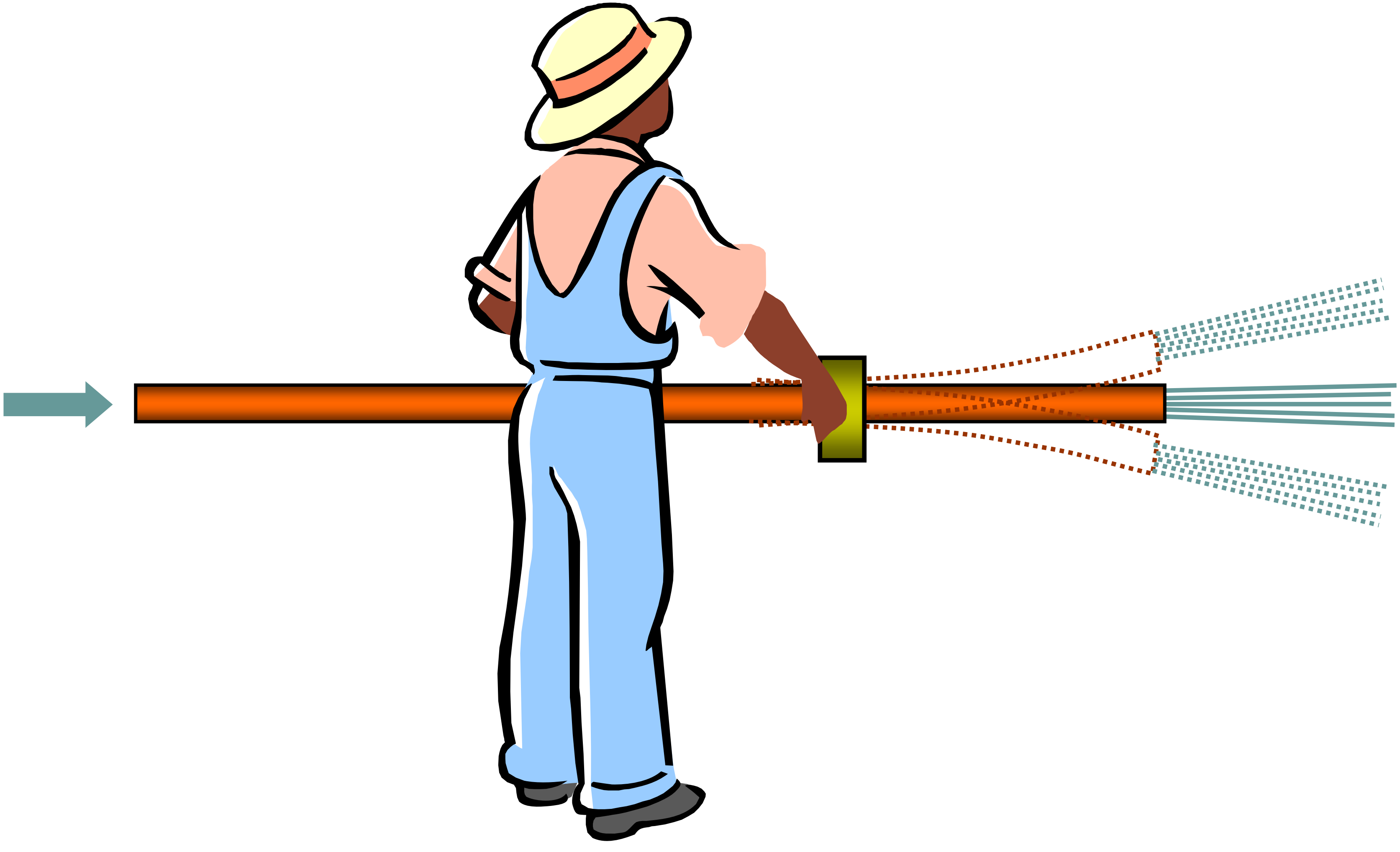
Previous investigations



Gregory R. W., Païdoussis M. P. (1966) "Unstable oscillation of tubular cantilever conveying fluid". *Proc. Roy. Soc. A.* **261**, 512–542.

First observations

Garden-hose instability



Oscillations (240 fps)

Theory

Acting forces

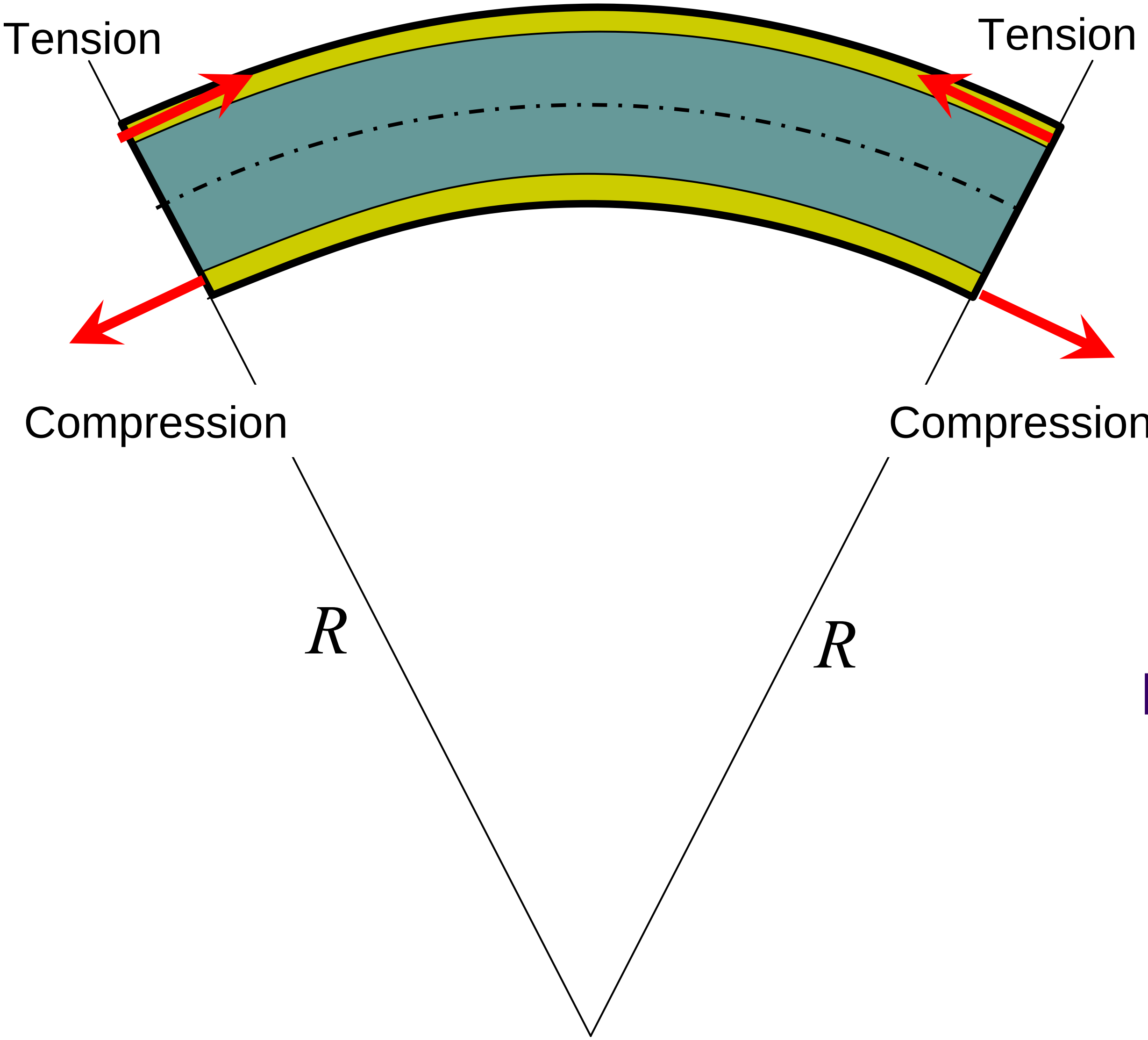
Take into account:

- Elastic force
- Centrifugal force
- Coriolis force

Neglect:

- ~~Gravity~~
- ~~Viscosity~~

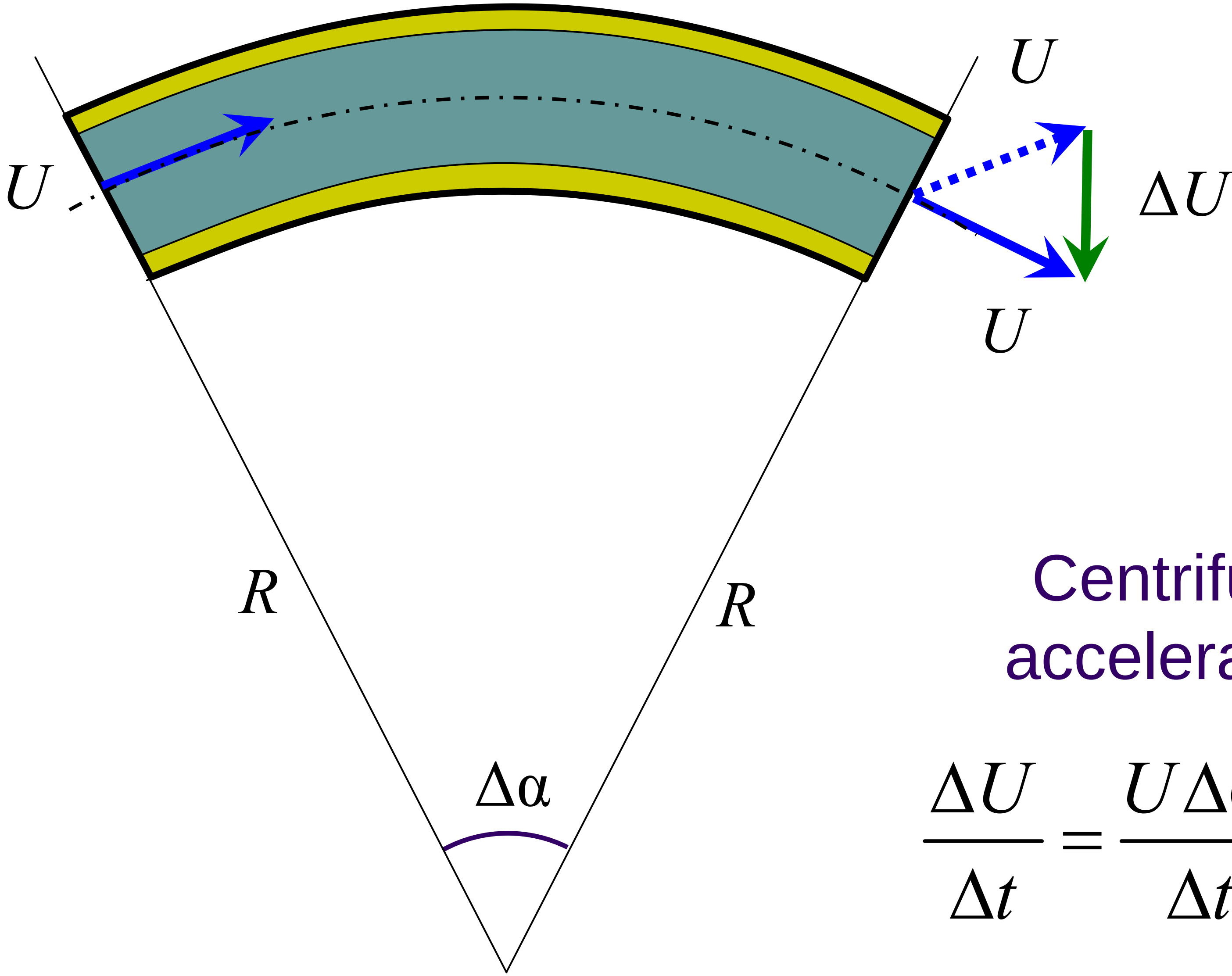
Elastic force



Bending moment:

$$M = \frac{EJ}{R}$$

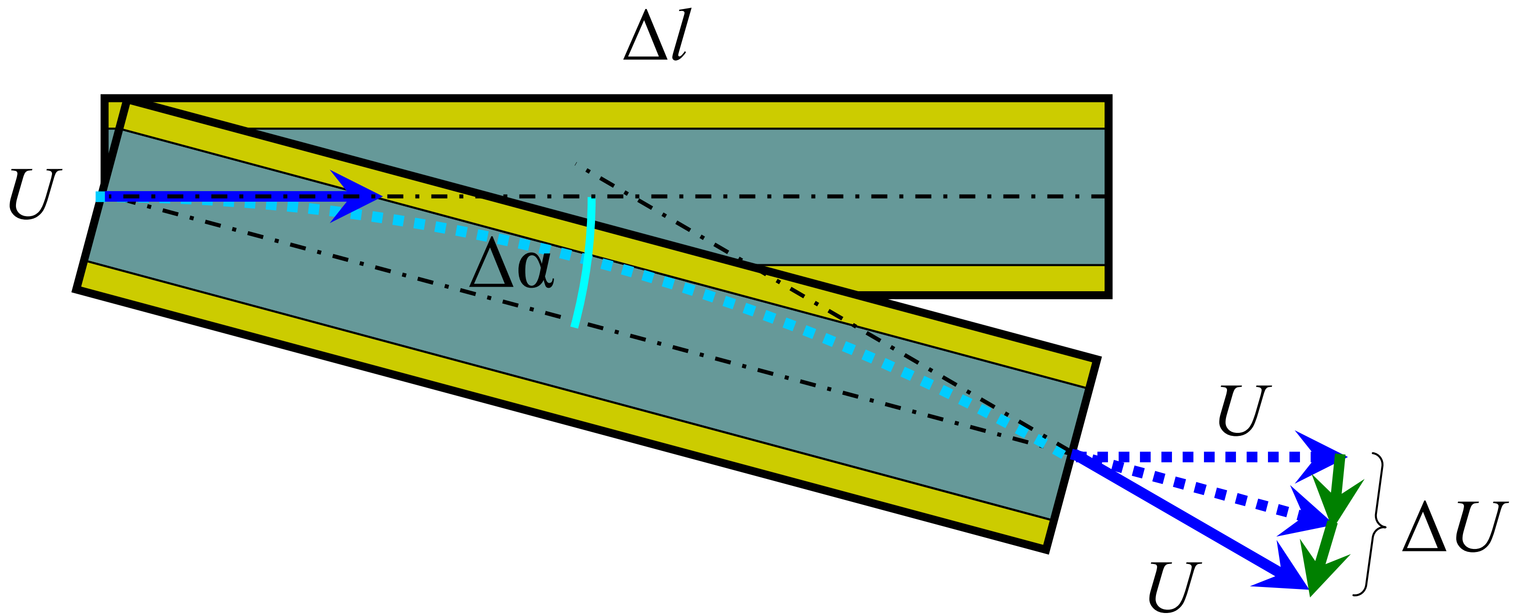
Centrifugal acceleration



Centrifugal acceleration:

$$\frac{\Delta U}{\Delta t} = \frac{U\,\Delta\alpha}{\Delta t} = \frac{U^2}{R}$$

Coriolis acceleration



$$\Delta U = U \cdot 2\Delta\alpha = 2U\omega \cdot \Delta t$$

Coriolis acceleration: $\dfrac{\Delta U}{\Delta t} = 2\omega U$

Equation of the hose motion

Mass × acceleration

Centrifugal force

$$(M+m)\frac{\partial^2 y}{\partial t^2} = -EJ\frac{\partial^4 y}{\partial x^4} - MU^2\frac{\partial^2 y}{\partial x^2} - 2MU\frac{\partial^2 y}{\partial x \partial t}$$

Flexural restoring force

Coriolis force

Gregory R. W., Païdoussis M. P. (1966)

Equation of the hose motion

These terms contain ***U***

$$(M+m)\frac{\partial^2 y}{\partial t^2} = -EJ\frac{\partial^4 y}{\partial x^4} - MU^2\frac{\partial^2 y}{\partial x^2} - 2MU\frac{\partial^2 y}{\partial x \partial t}$$

Dominates for large ***U***

$$\frac{MU}{L}\cdot\frac{U}{L}$$

Dominates for small ***U***

$$\frac{MU}{L}\cdot\omega$$

Dimensionless parameters

Coordinates $x = \xi \cdot L, \quad y = \eta \cdot L$

Velocity $U = u \cdot \sqrt{\dfrac{EJ}{ML^2}}$ Time $t = \tau \cdot \sqrt{\dfrac{(M+m)L^4}{EJ}}$

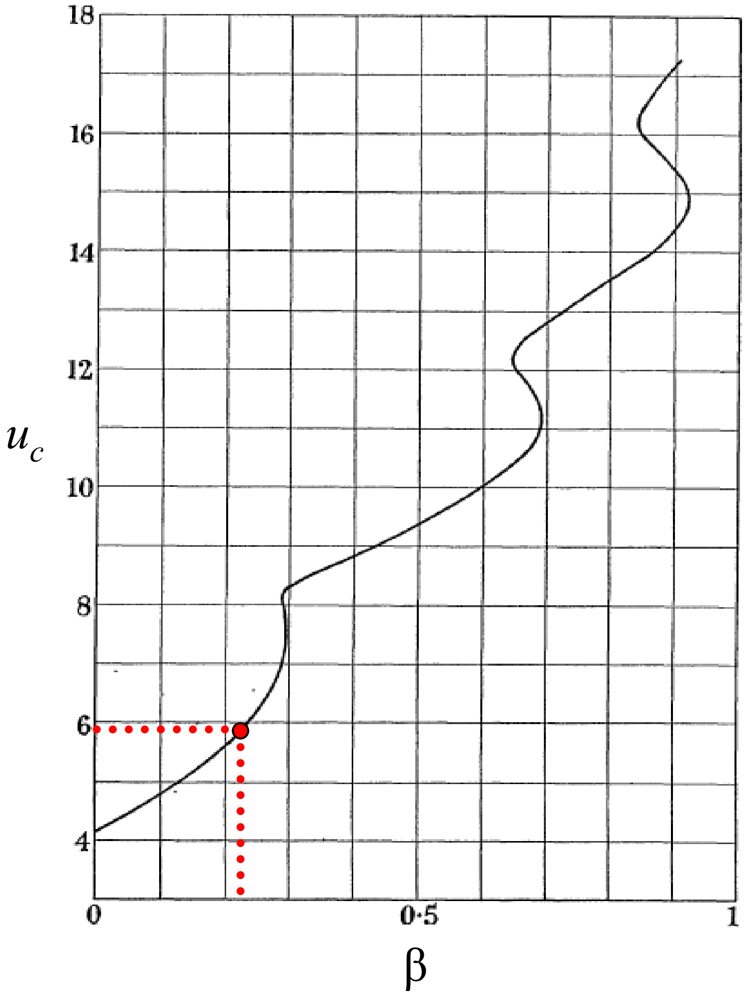
$$\frac{\partial^4 \eta}{\partial \xi^4} + u^2 \frac{\partial^2 \eta}{\partial \xi^2} + 2\beta^{1/2} u \frac{\partial^2 \eta}{\partial \xi \partial \tau} + \frac{\partial^2 \eta}{\partial \tau^2} = 0$$

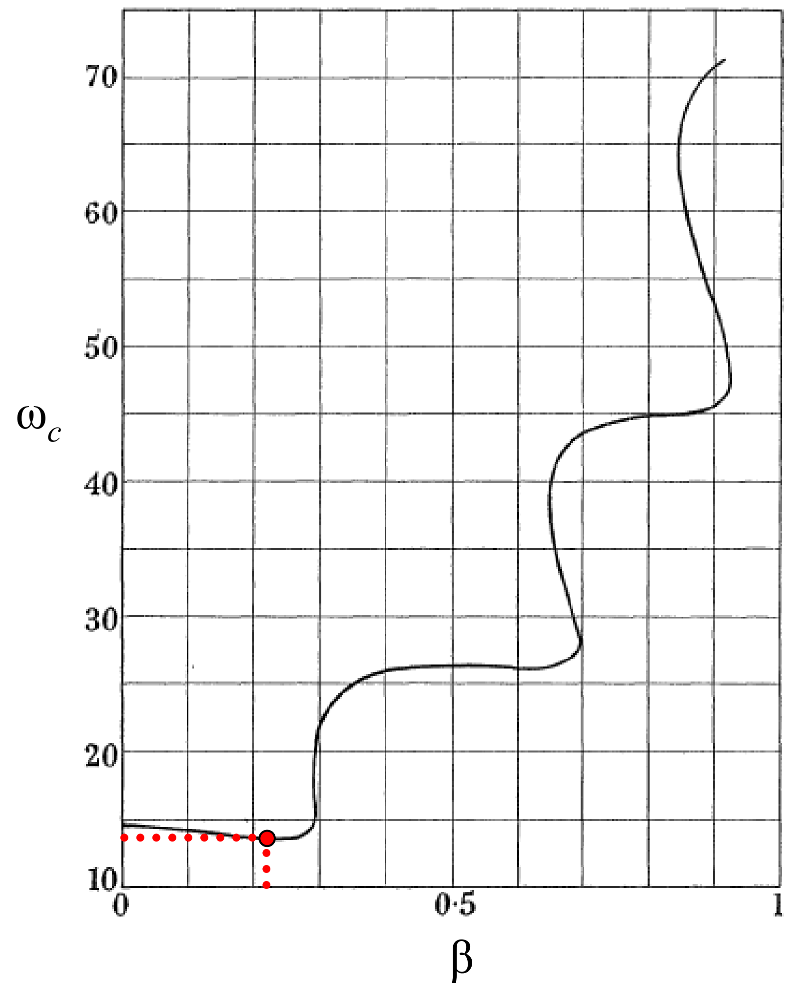
$$\beta = \frac{M}{M+m}$$

M: Mass of water per unit length

m: Mass of the tube per unit length

Critical velocity and frequency





Gregory R. W., Païdoussis M. P. (1966)

Theoretical prediction

Critical velocity

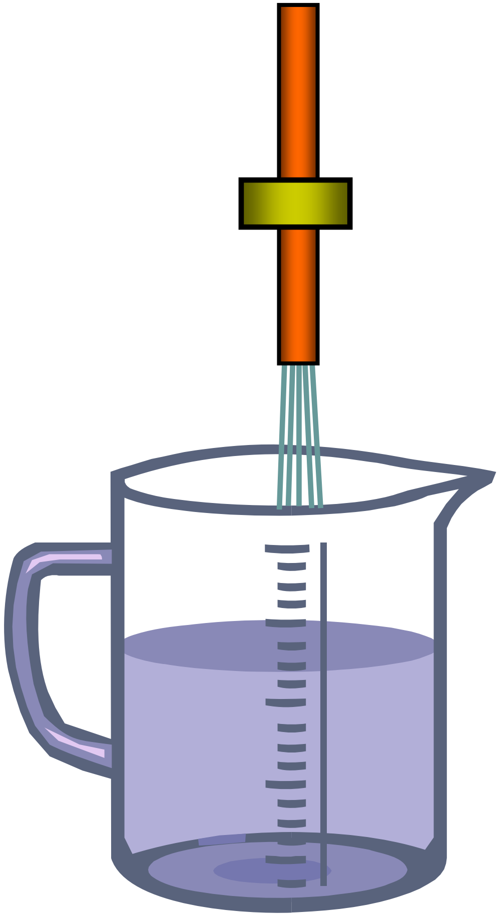
$$U_c = \sqrt{\frac{EJ}{ML^2}} \cdot u_c(\beta) \sim \boxed{\frac{1}{L}}$$

Frequency of oscillations

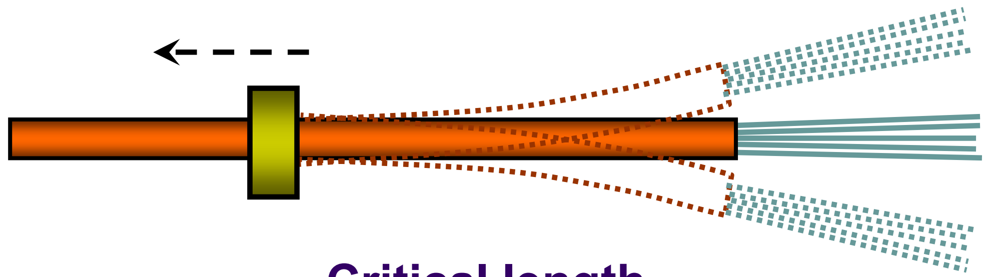
$$\Omega_c = \sqrt{\frac{EJ}{(M+m)L^4}} \cdot \omega_c(\beta) \sim \boxed{\frac{1}{L^2}}$$

Experiment

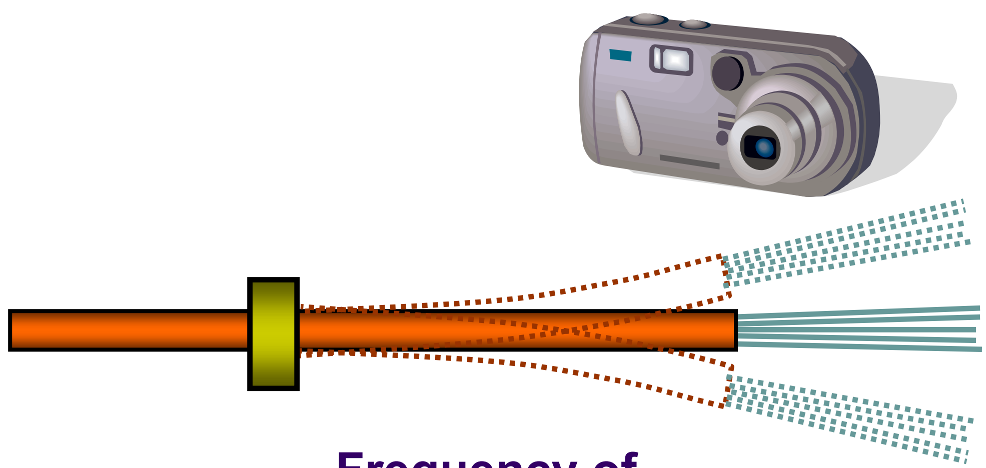
Experimental procedure



Water flow
and its velocity



Critical length
of a hose

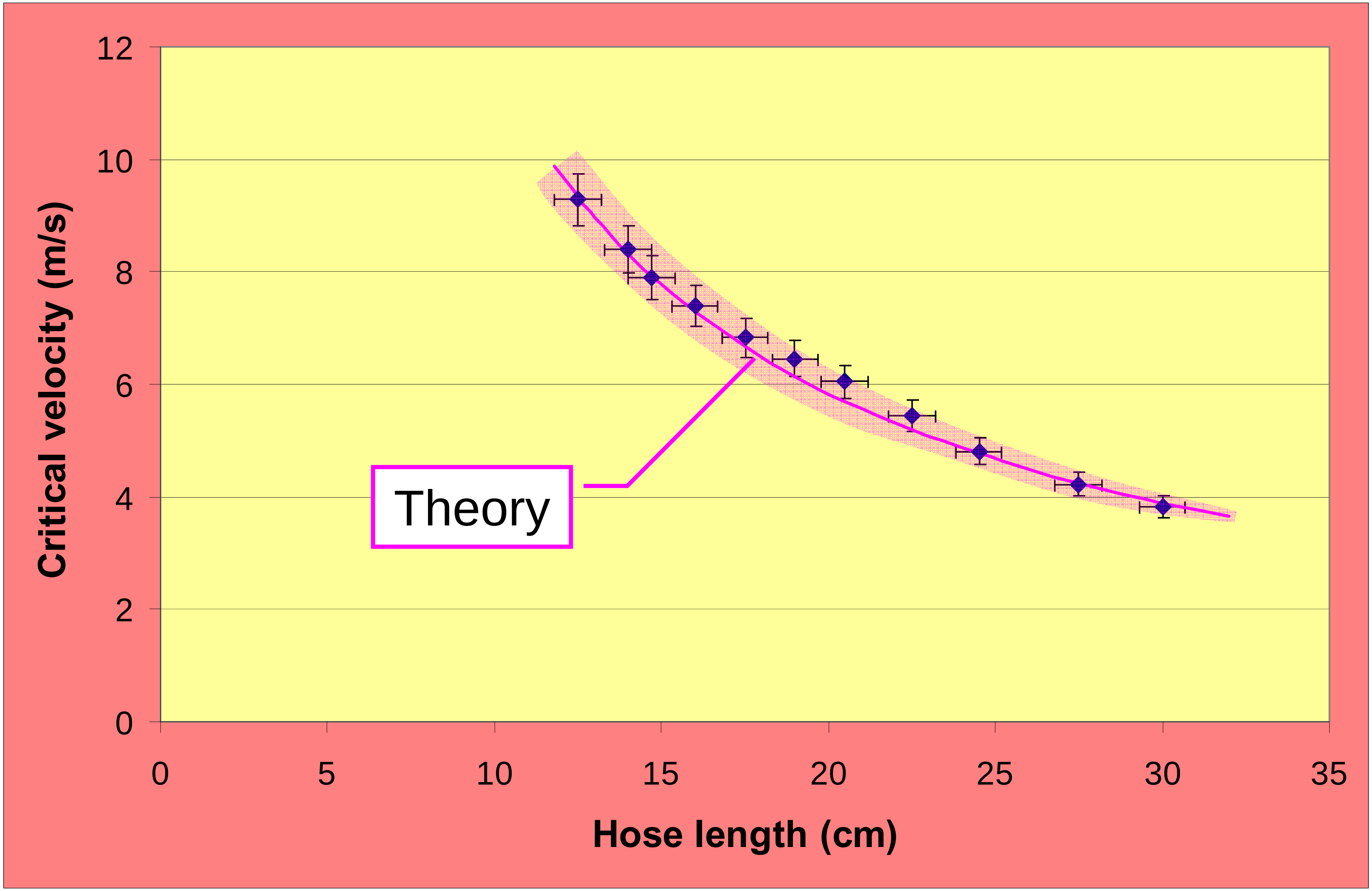


Frequency of
oscillations

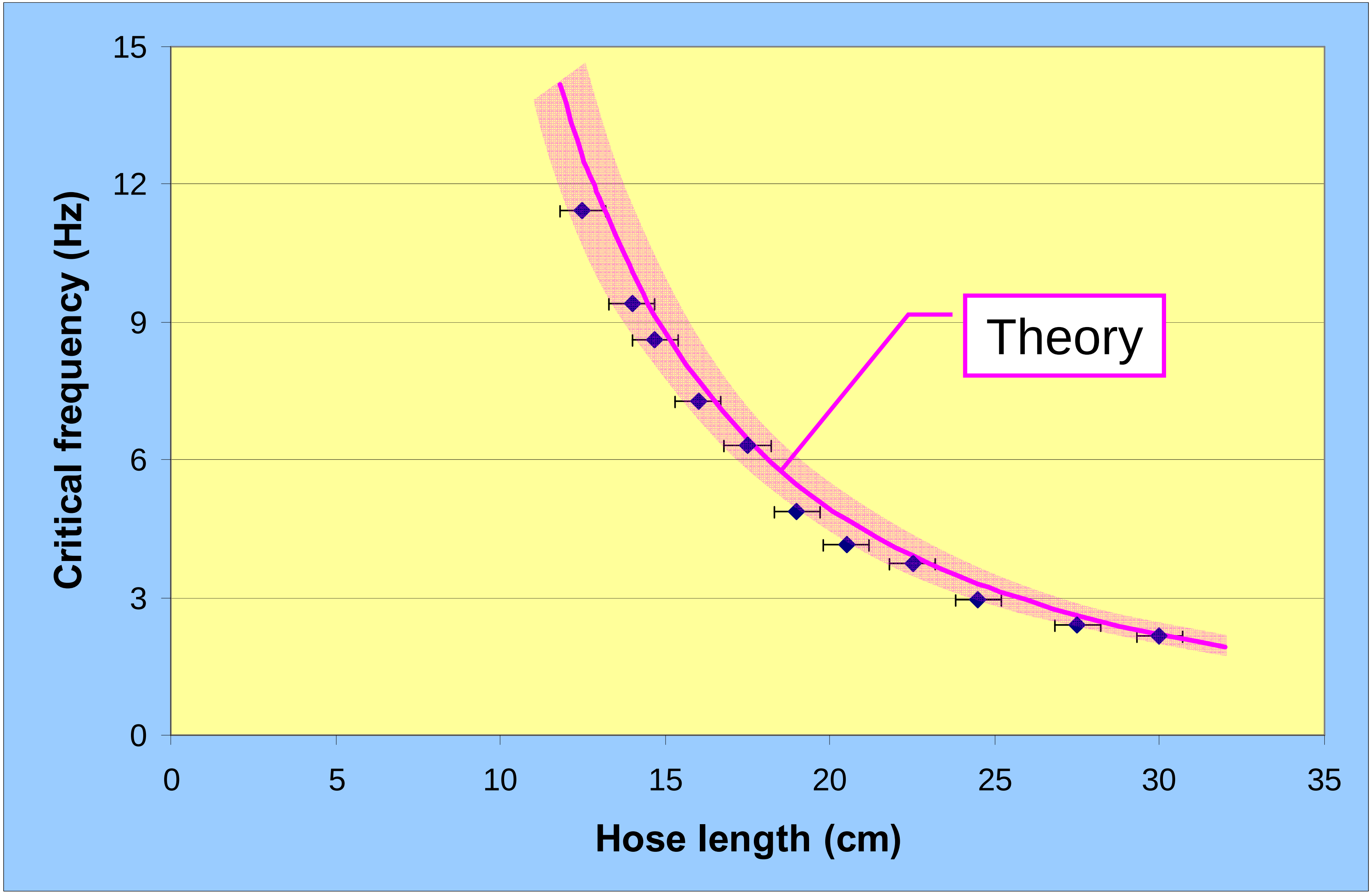
Parameters of the hose

- Mass of tube per unit length m = **22.7 g/m**
- Mass of water per unit length M = **6.4 g/m**
- Mass ratio **$\beta = M : (m + M) = 0.22$**
- Flexural rigidity EJ = **$2.5 \cdot 10^{-4}$ N·m²**

Critical velocity vs. length



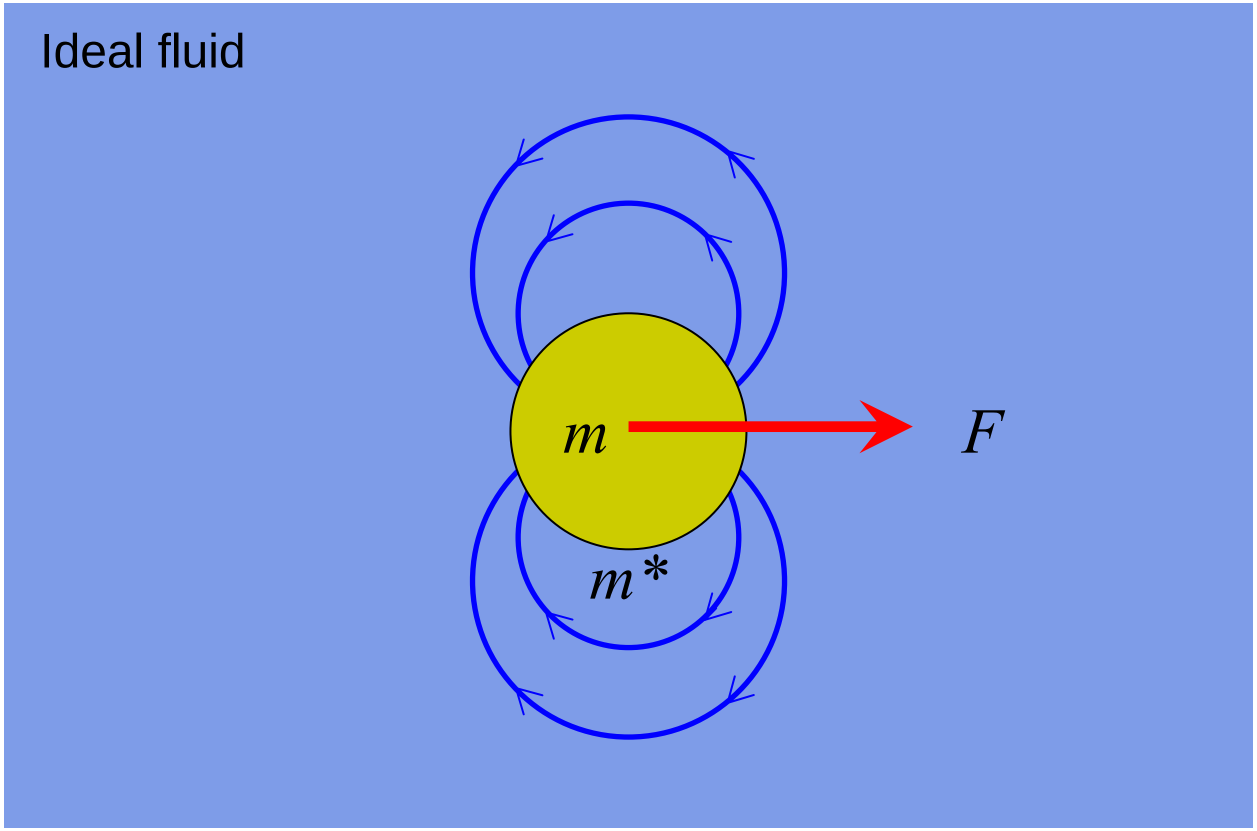
Critical frequency vs. length



Experiment with a submerged hose

Course of the experiment (video)

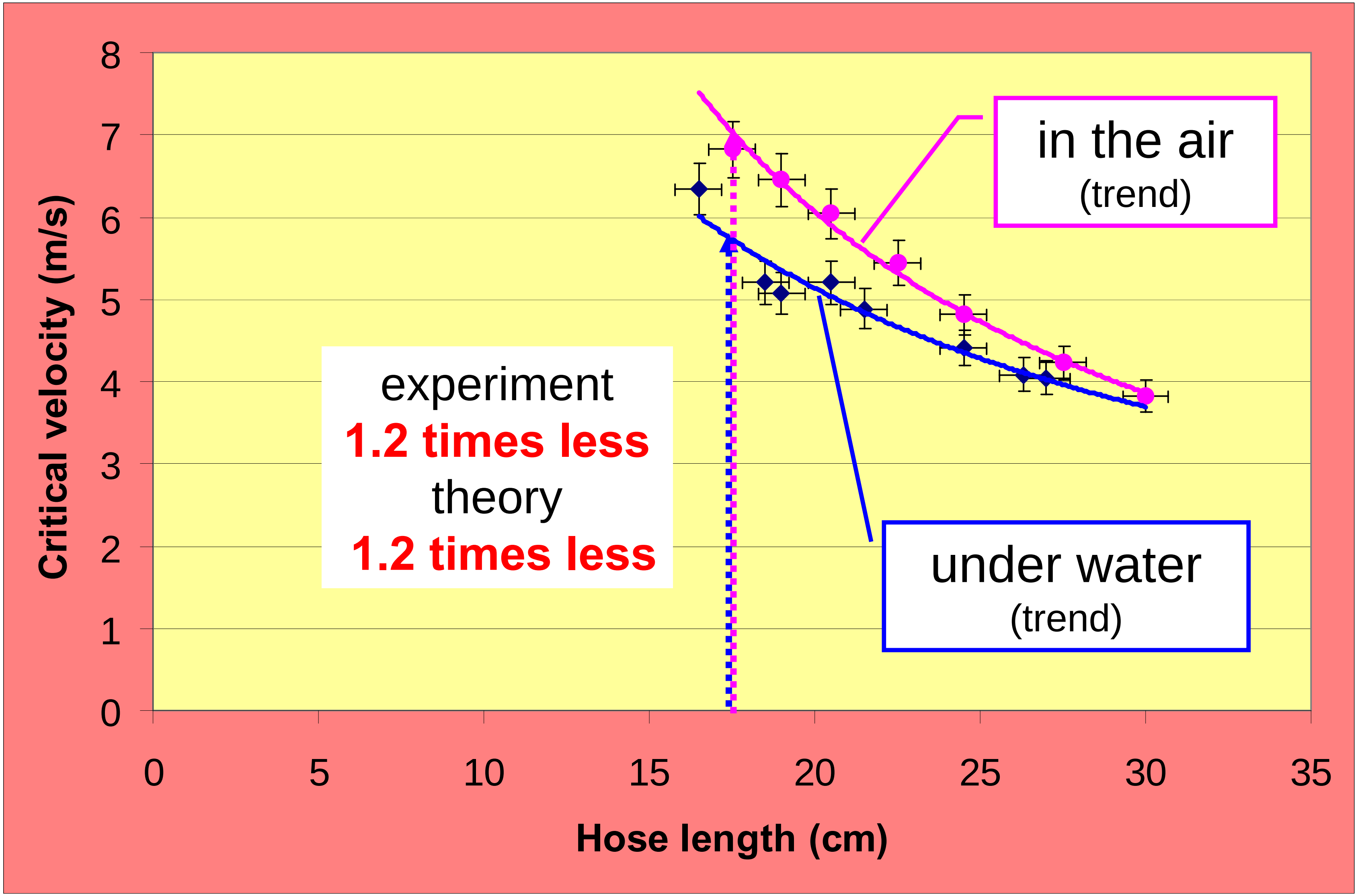
The added mass



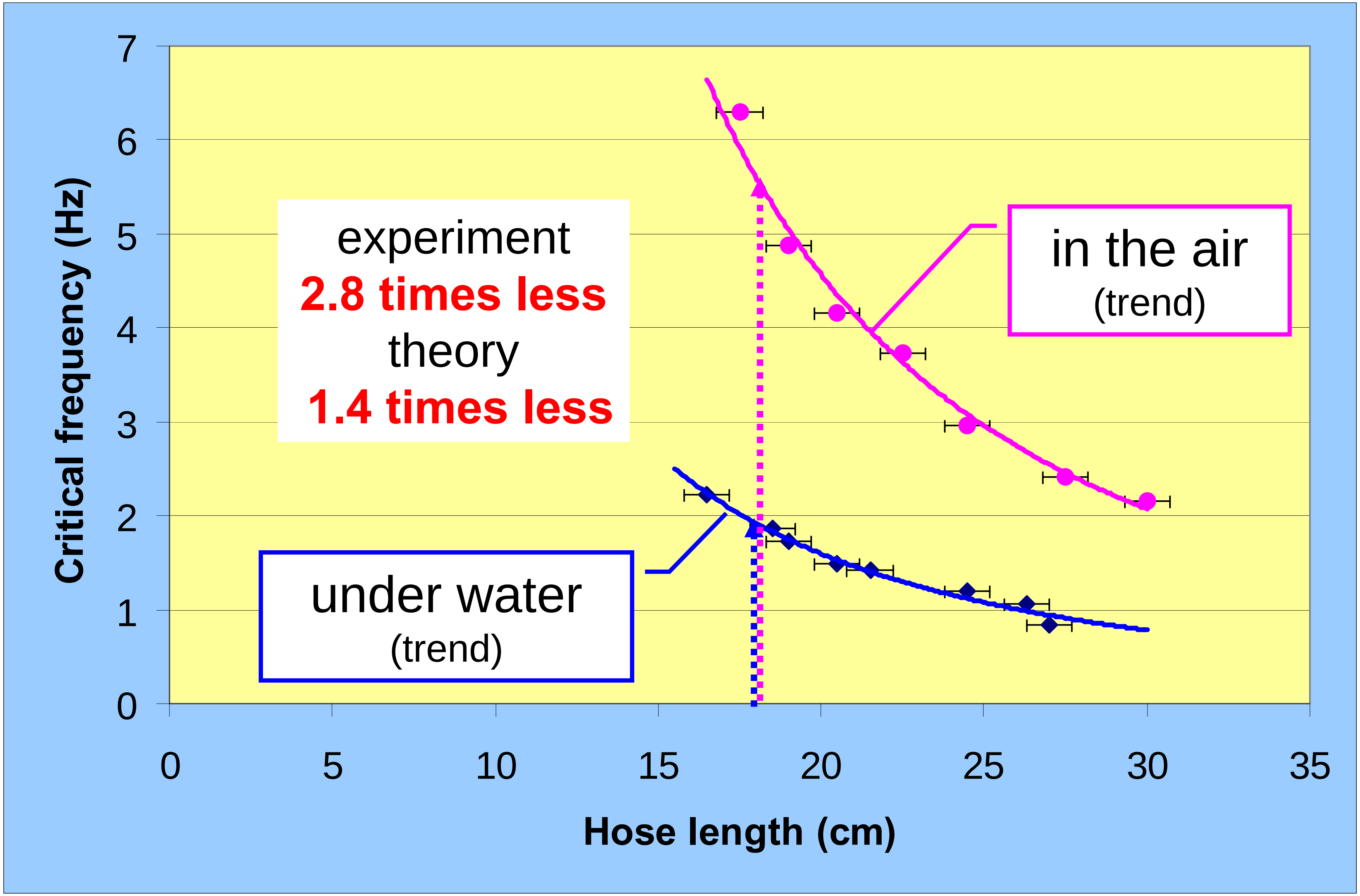
$$a = \frac{F}{m + m^*}$$

For a long cylinder $m^* = \rho V$

Critical velocity vs. length



Critical frequency vs. length



Reasons for this disagreement

- The theory of the added mass is built for the ideal fluid. In fact, the submerged hose moves at high Reynolds numbers, so the added mass can be increased through the nucleation of fluid vortices.
- All calculations are performed for the regime of growing linear oscillations. In contrast, all experimental measurements were made in the regime of steady nonlinear oscillations. And the oscillatory frequency in these two regimes can differ significantly.

Summary

Conclusions

- Garden-hose instability is caused by the **interplay of centrifugal and Coriolis forces** generated by the flow of water through the hose.
- Self-sustaining hose oscillations appear with a certain critical flow velocity.
- When the hose is submerged, its effective mass increases by the added mass of external water. As a result, in this case the critical oscillatory frecuency decreases significantly, unlike the critical flow velocity.

References

- Gregory R. W., Païdoussis M. P. (1966) “Unstable oscillations of tubular cantilevers conveying fluid”. *Proc. Roy. Soc. A*, **293**, 512–542.
- Païdoussis M. P. (1998) *Fluid-structure interactions. Slender structures and axial flow.*
- Doaré O., de Langre E. (2002) “The flow-induced instability of long hinging pipes”. *Eur. J. Mech. A*, **21**, 857–867.

Thank you for your attention!