# Hoops

Vitaliy Matiynin
Aleksandr Severinov
Vladislav Tumanov

Russia
IYPT

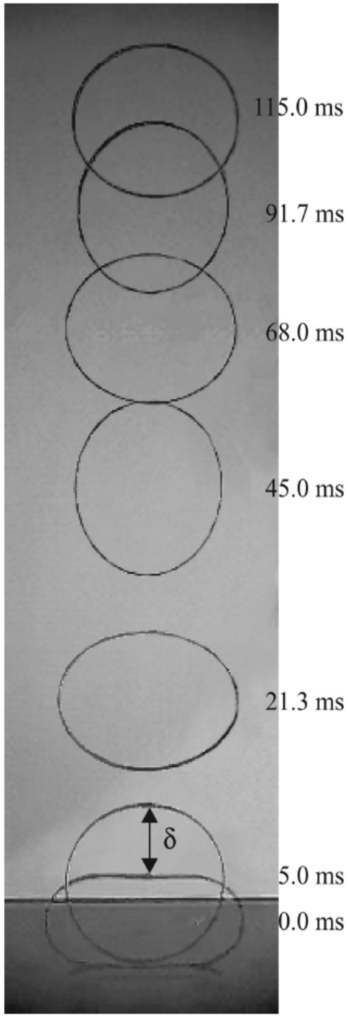
# The problem

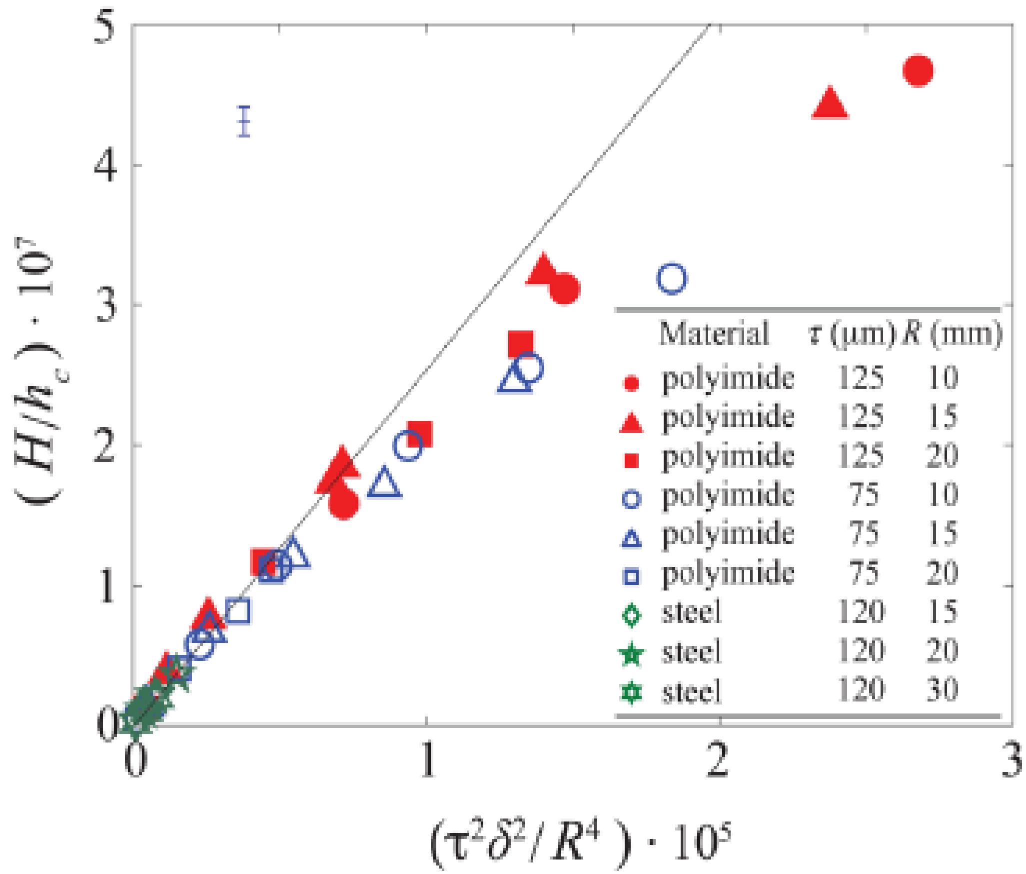


An elastic hoop is pressed against a hard surface and then suddenly released. The hoop can jump high in the air. Investigate how the height of the jump depends on the relevant parameters.

# Previous investigations






E. Yang, H.-Y. Kim (2012) “Jumping hoops”. *Am. J. Phys.* **80**, 19–23.

# Scaling law for the jump height

# Energy conservation law



Jump height

Initial elastic strain energy

$$H \simeq \frac{U}{mg}$$

Weight

Elastic strain energy: $U \simeq E\varepsilon^2 V$

Hoop weight: $mg = \rho g V$

$$H \simeq \frac{E\varepsilon^2}{\rho g}$$

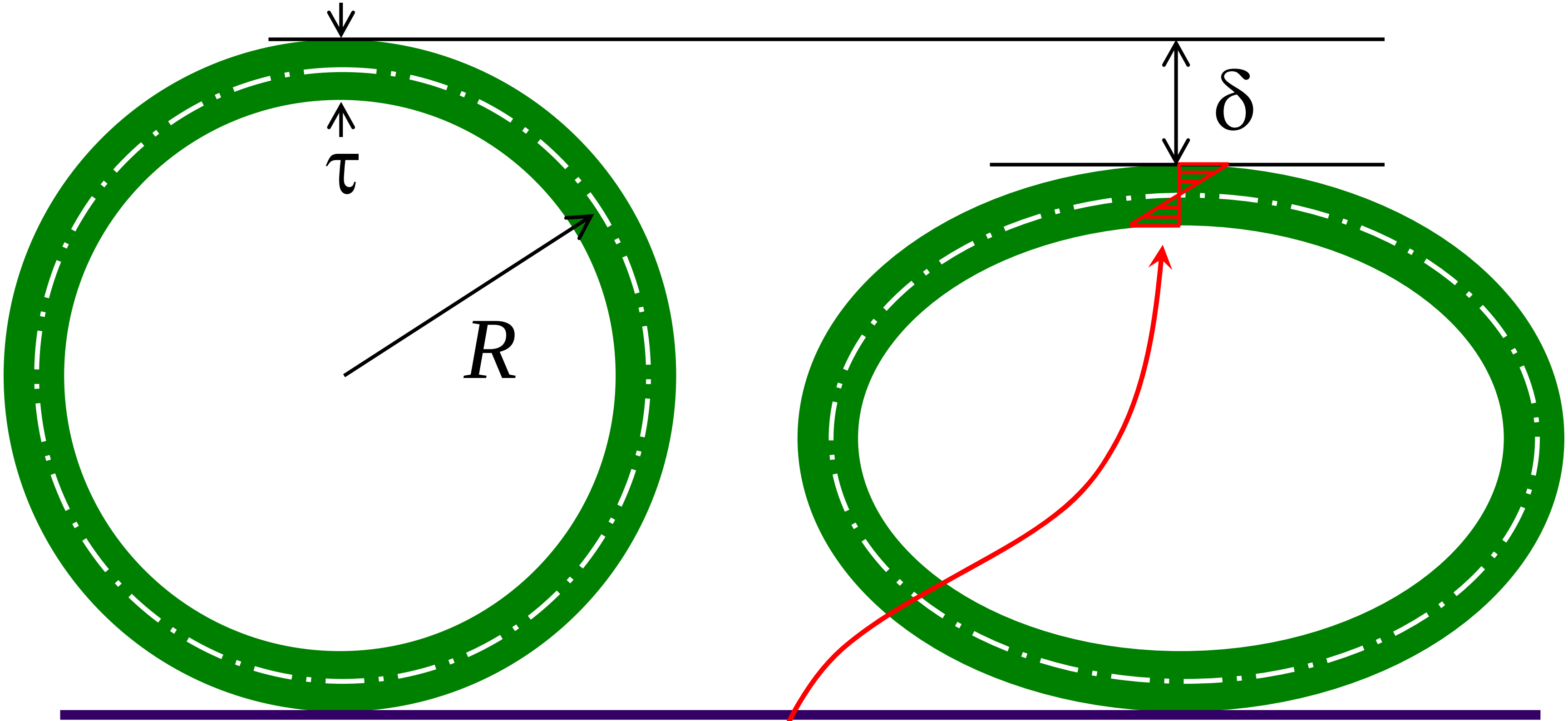
$E$ — Young's modulus

$\varepsilon$ — hoop strain

$\rho$ — material density

# Strain estimation






$\varepsilon \sim \delta$ Hooke's law

$\varepsilon \sim \tau$ the further from the neutral plane, the more strain

$$\varepsilon \simeq \frac{\tau \cdot \delta}{R^2}$$

# Scaling law for the jump height



Relative thickness

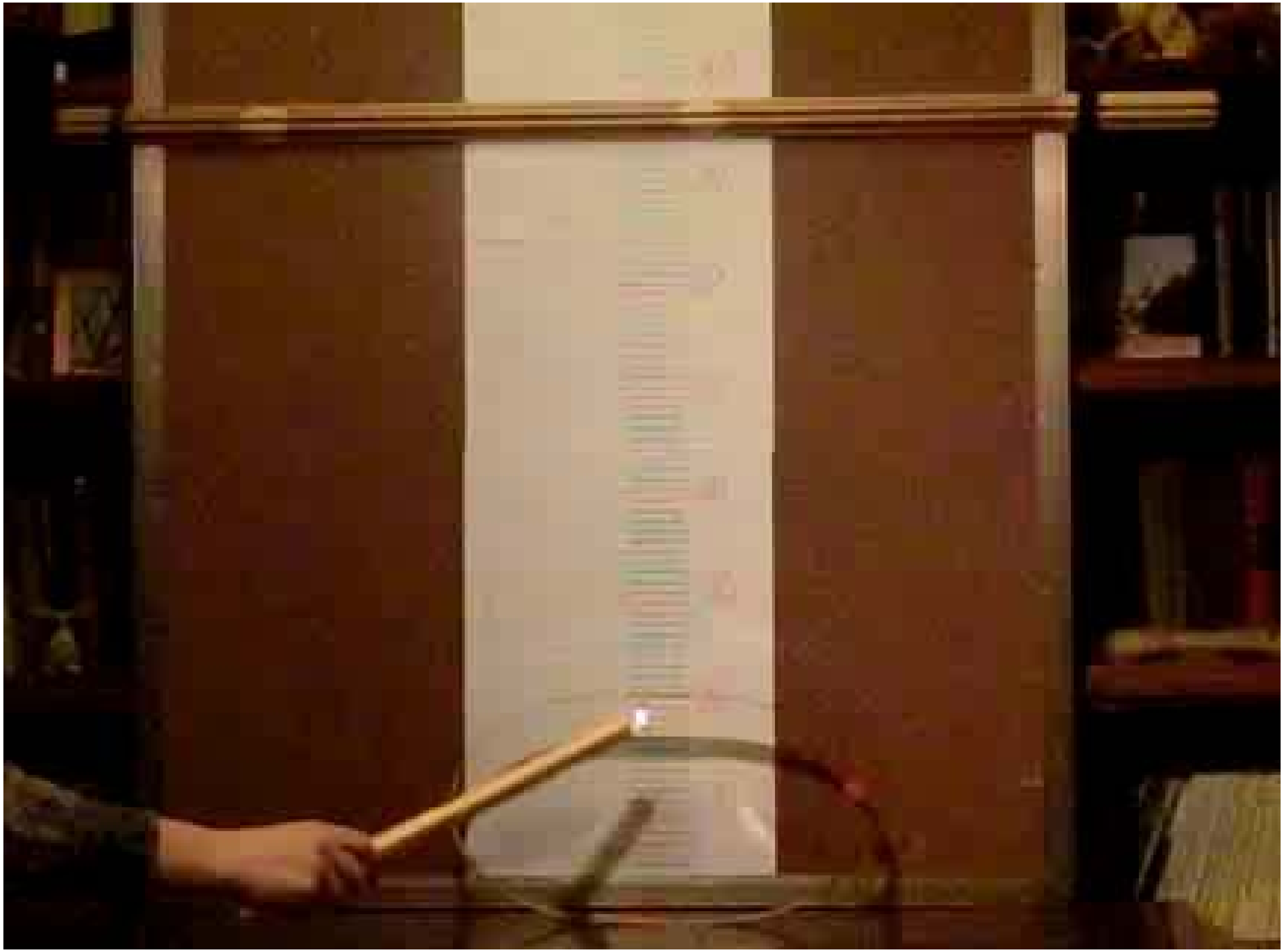
$$H \simeq \frac{E}{\rho g} \cdot \left(\frac{\tau}{R}\right)^2 \cdot \left(\frac{\delta}{R}\right)^2$$

Relative deformation

# Experimental changing the hoop’s thickness

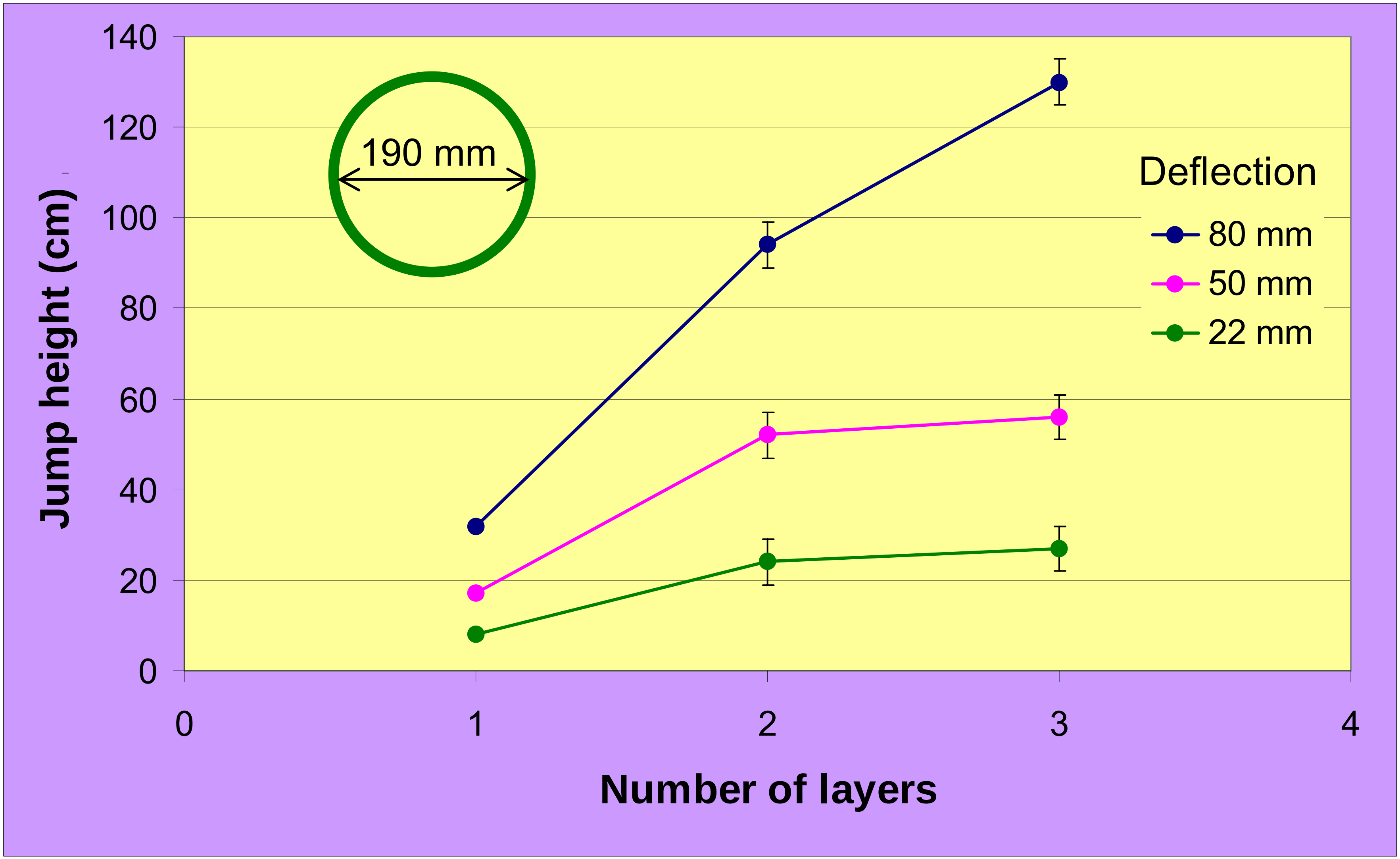
# Experimental procedure (240 fps)





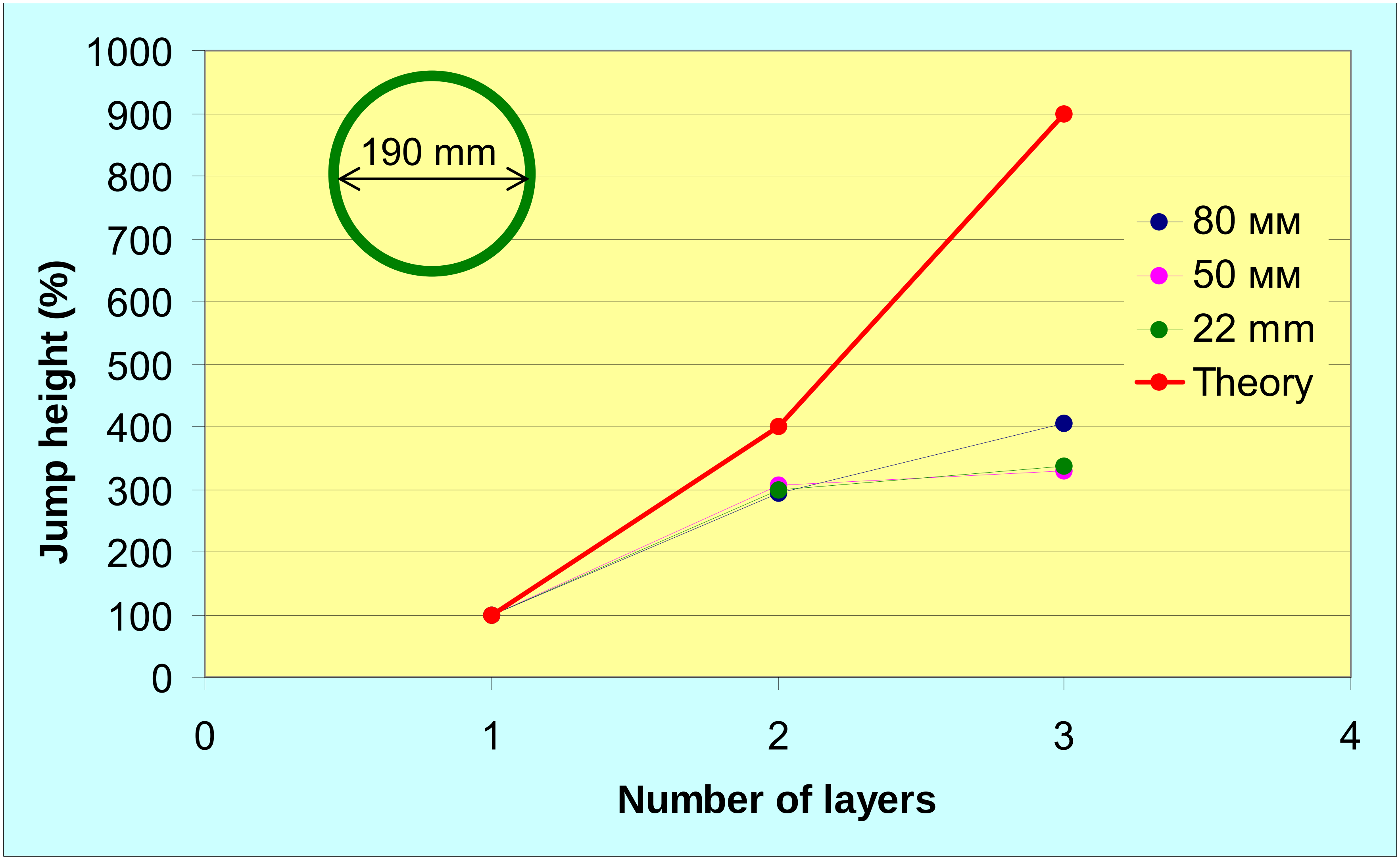
# Welded steel multilayer hoops





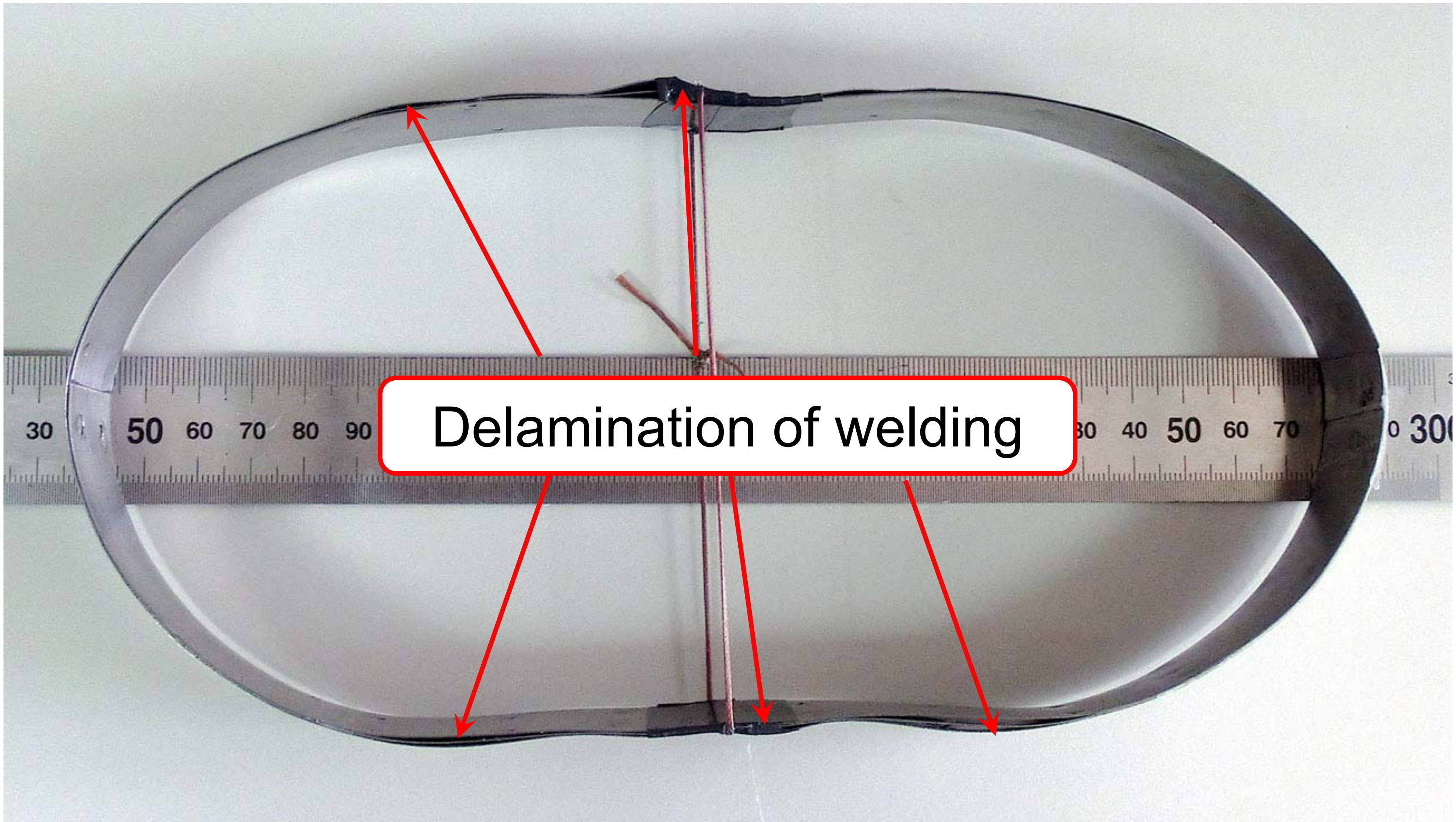
# Welded steel multilayer hoops



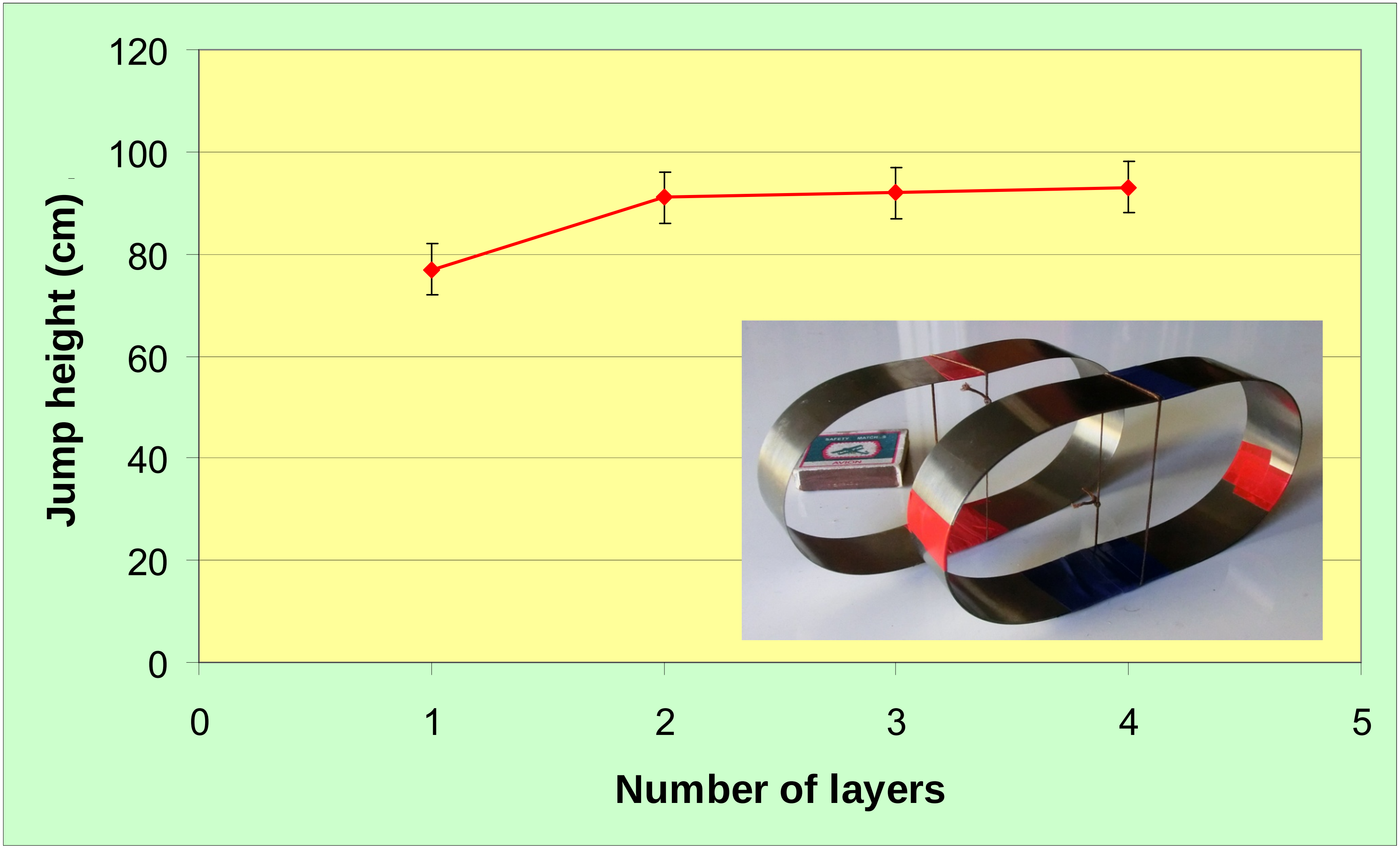


# Divergence with a theory





# Non-welded steel multilayer hoops



# Experimental changing the hoop’s deformation

# Hoops




**Steel ruler**

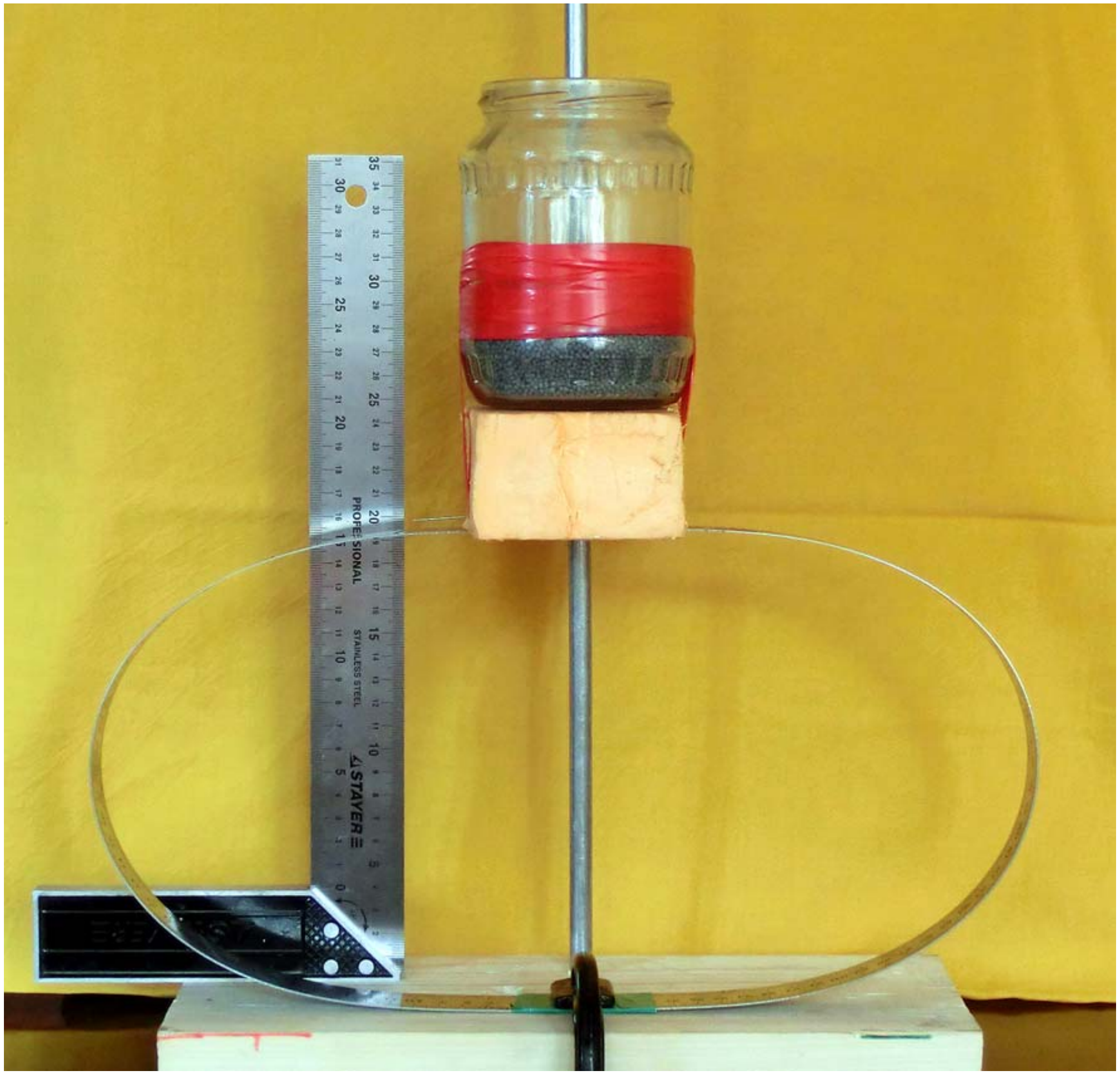
- Hoop diameter 30.5 cm
- Band width 2.8 cm
- Band thickness 0.7 mm
- Mass 154 g

**Plastic tube**

- Hoop diameter 54 cm
- Tube diameter 1.6 cm
- Mass 93 g

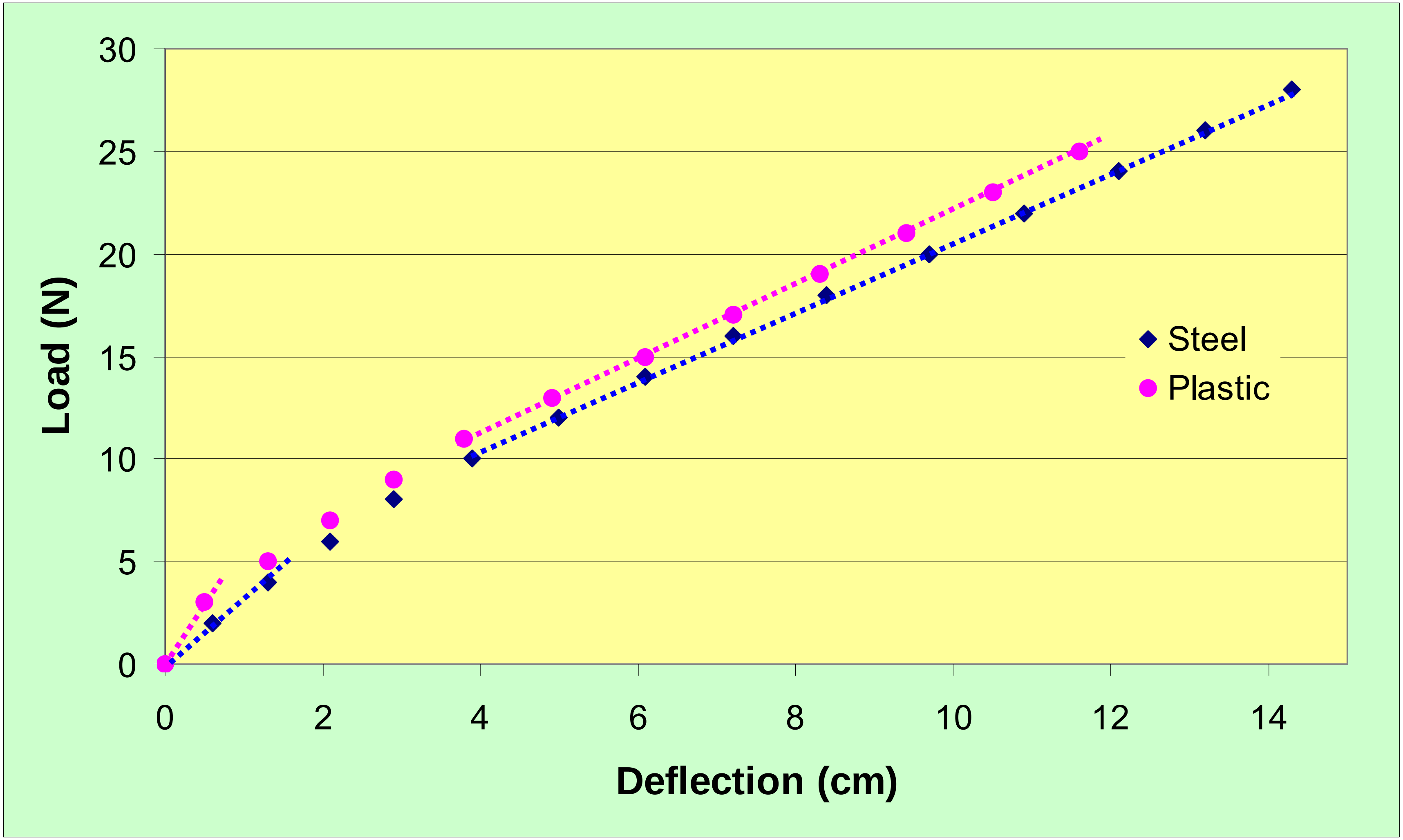
# Verification of Hooke’s law





# Load vs. deflection

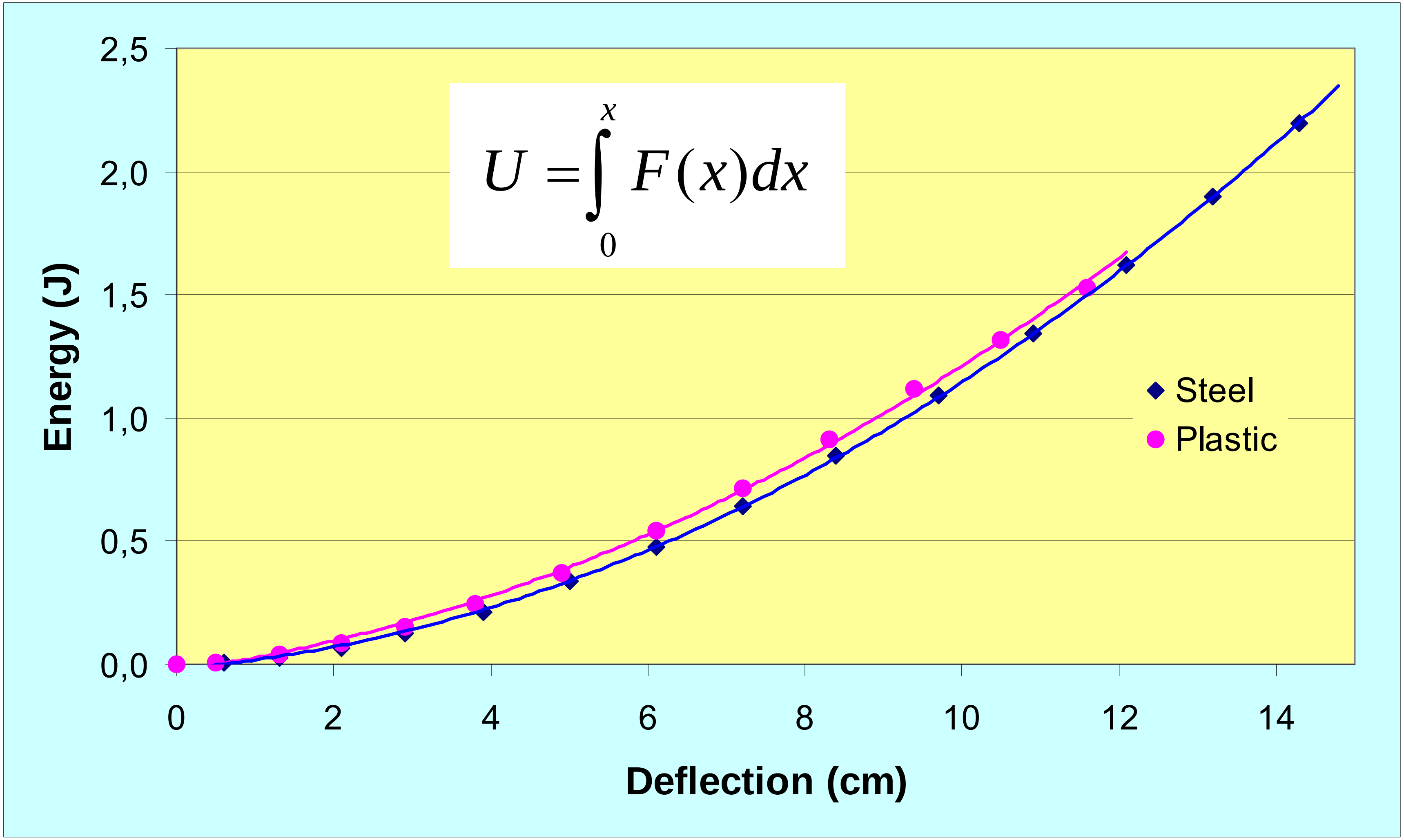





Hooke's law holds only approximately. For small deformations the hoop is tougher than for average ones.

# Elastic strain energy



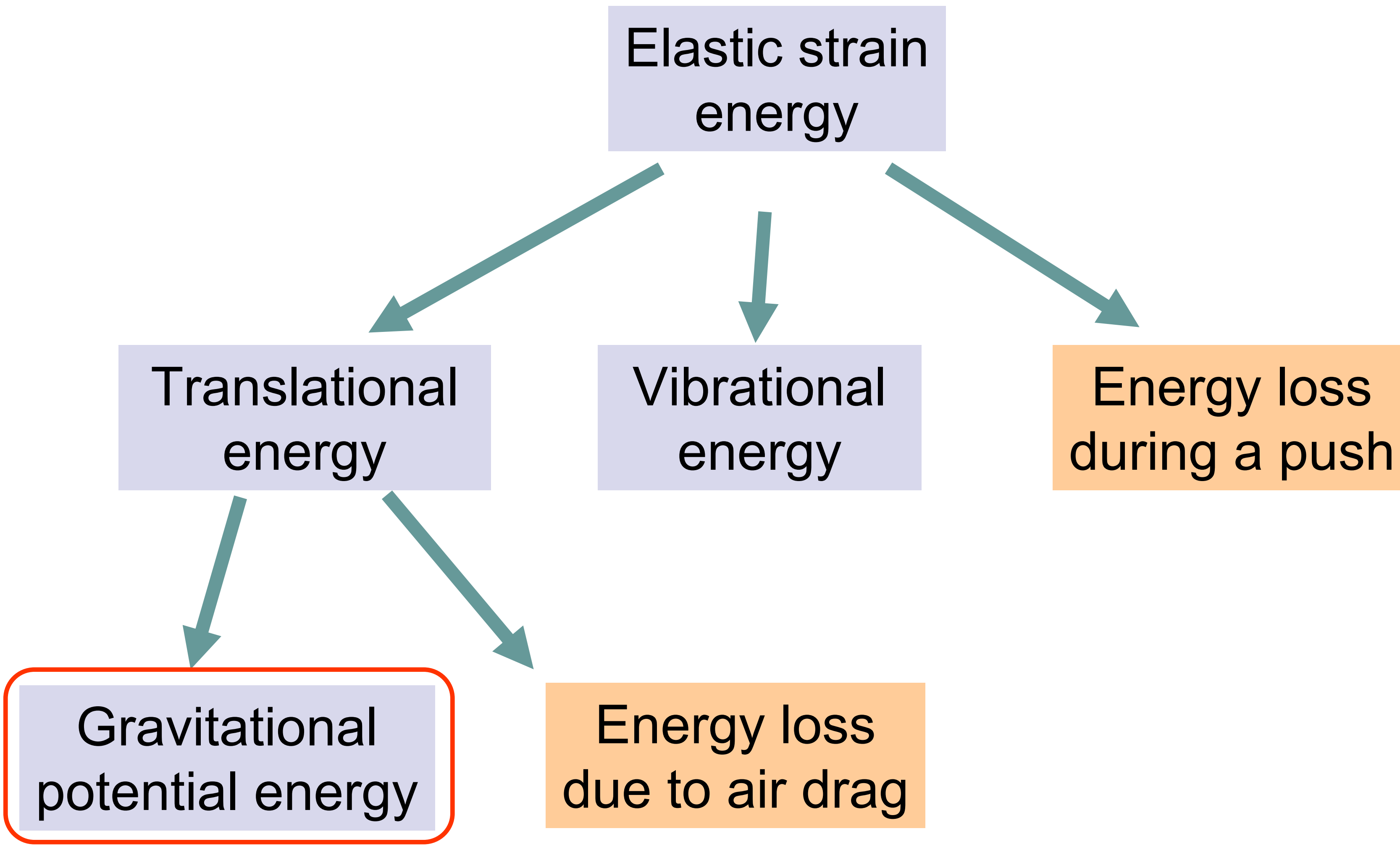



$$U = \int_0^x F(x)dx$$

For simplicity, we can assume that the energy stored in the hoop is proportional to the square of the deflection.

# Energy distribution (experiment)

# Energy distribution



# Effect of air drag

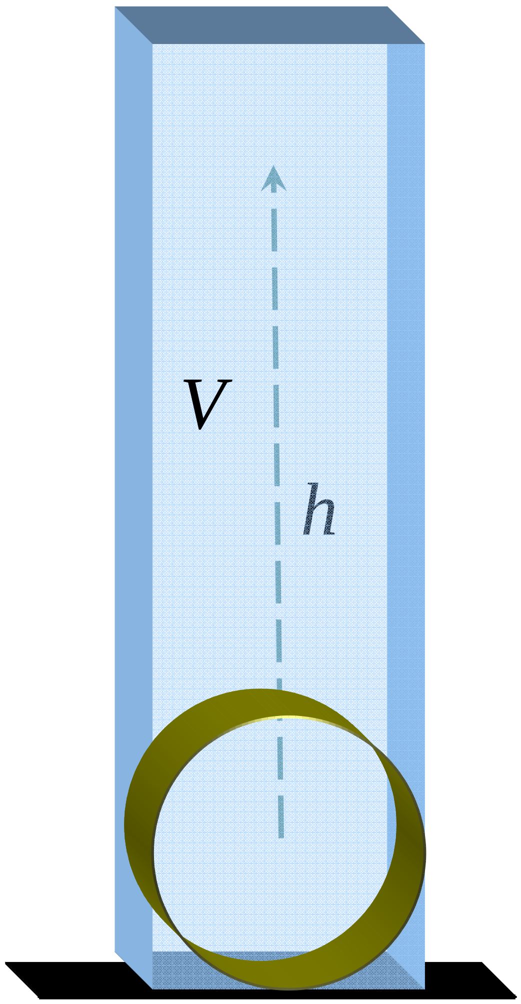
Condition to neglect air drag

$$m_{\text{air}} \ll m_{\text{hoop}}$$

Steel hoop

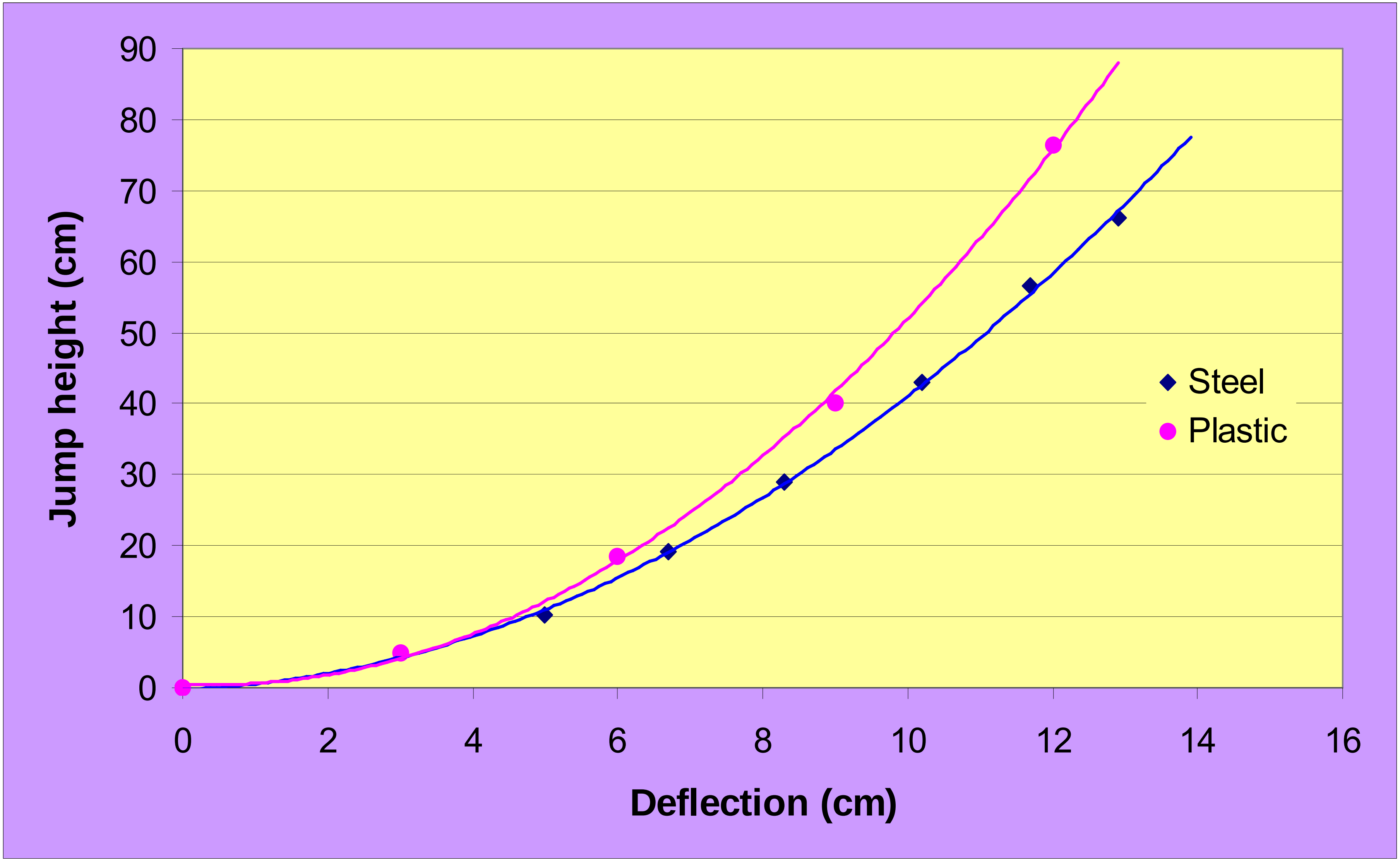
$$m_{\text{air}} = \rho V \approx 5\,\text{g}$$

$$m_{\text{hoop}} = 154\,\text{g}$$



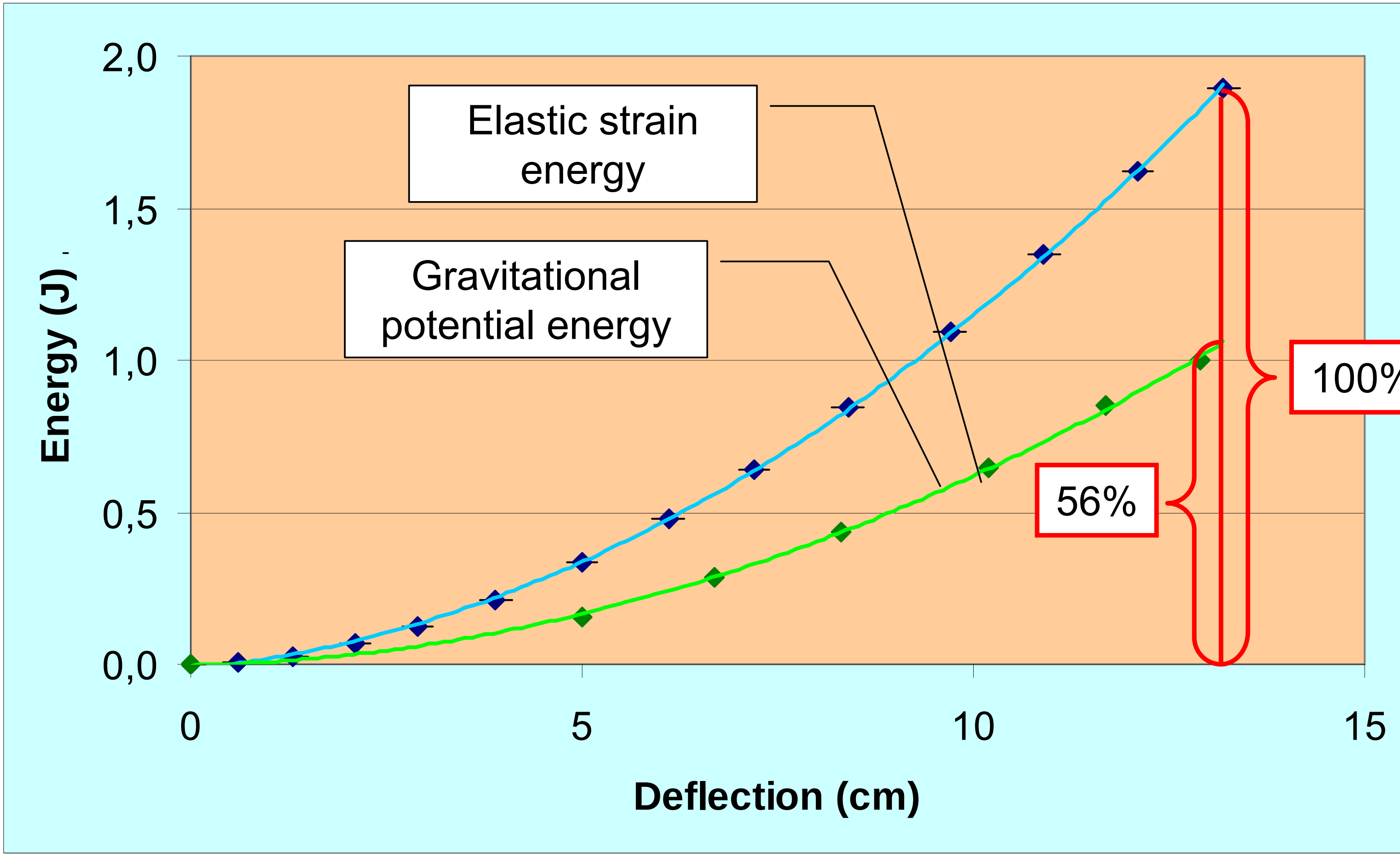
# Jump height vs. initial deformation





# Ratio of energies (steel hoop)

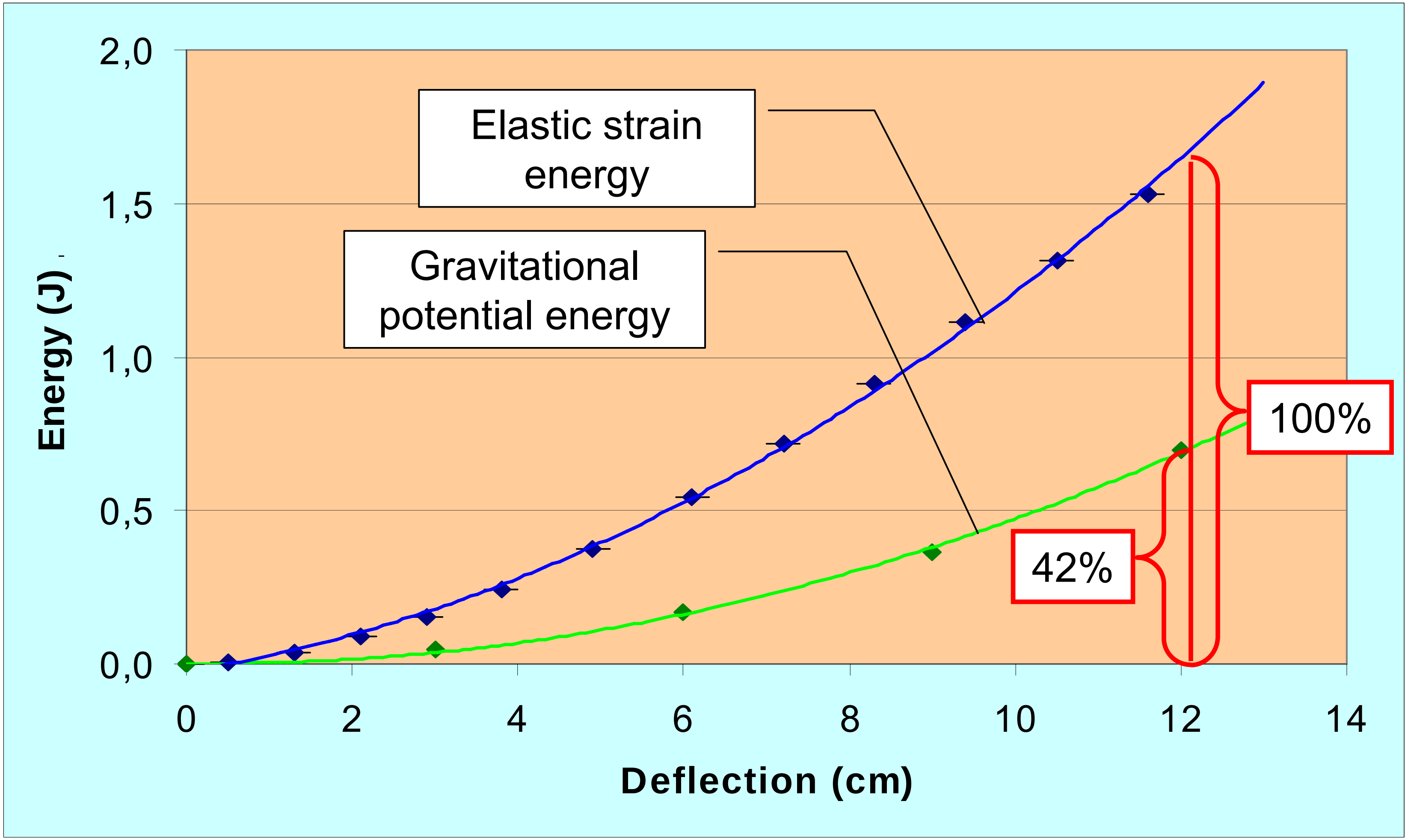





Both plots were approaching by a quadratic dependence, then coefficients were compared.

# Ratio of energies (plastic hoop)





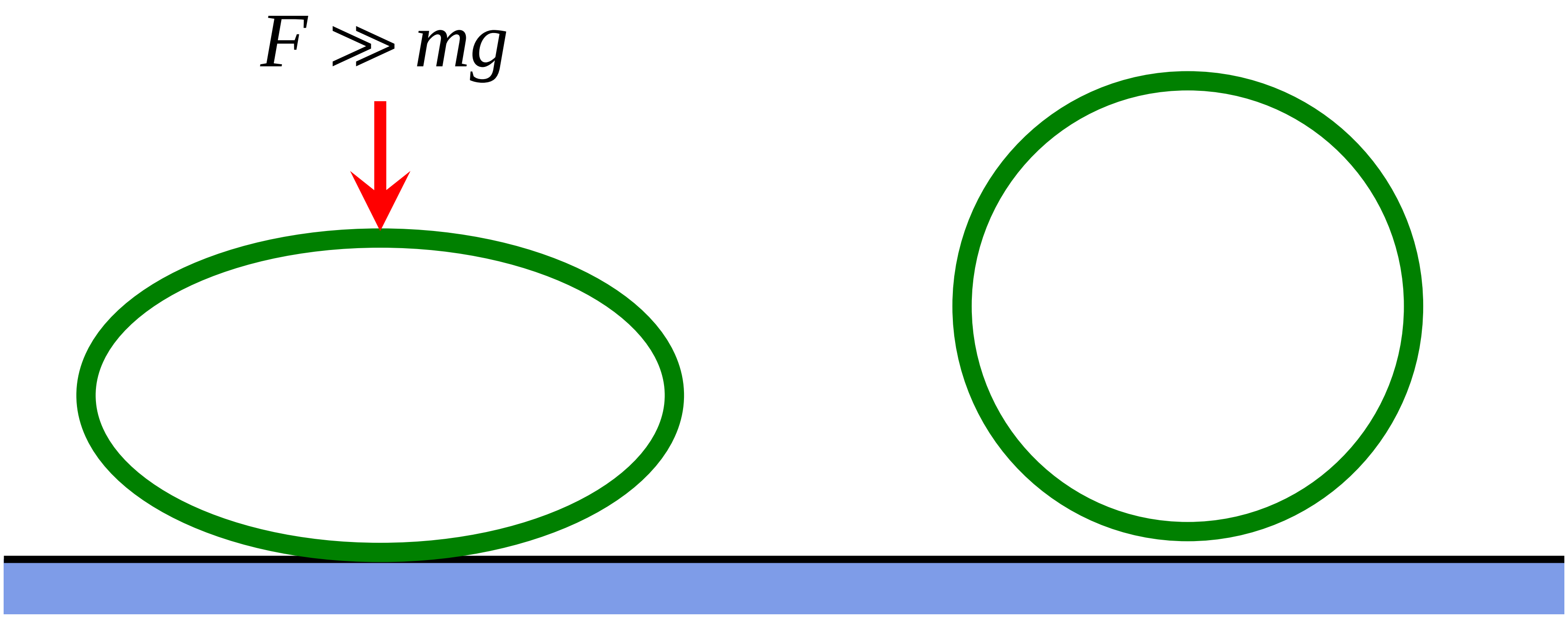

Both plots were approaching by a quadratic dependence, then coefficients were compared.

# Energy distribution (theory)

# First assumption



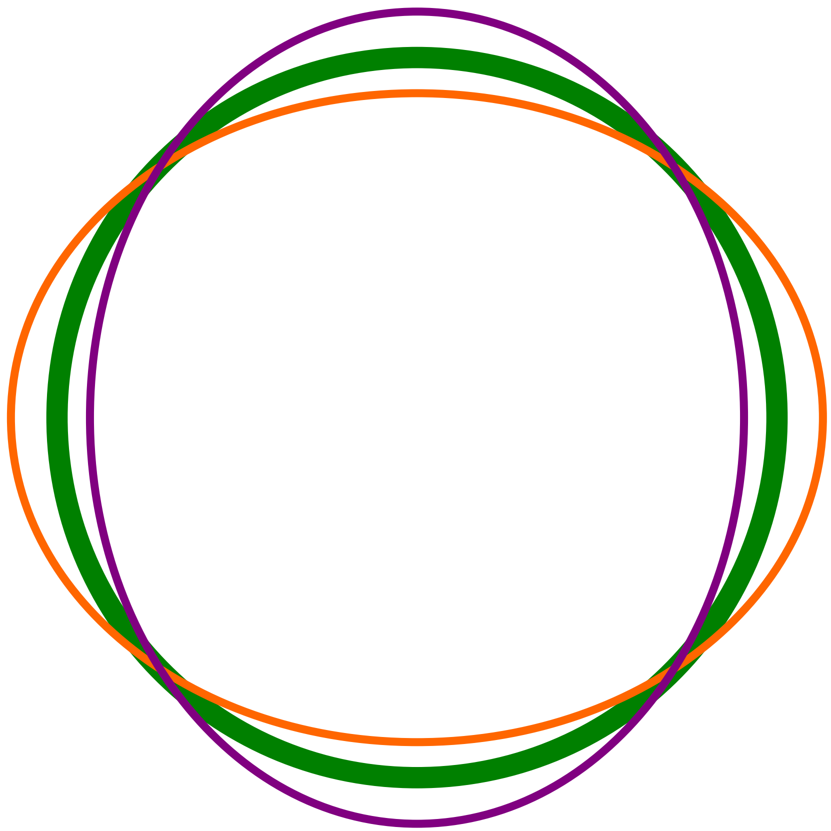
If $F >> mg$, then at the moment of rebound the hoop recovers its circular shape. Therefore, the initial elastic strain energy of the hoop is totally converted to the kinetic energy of its parts.



# Second assumption



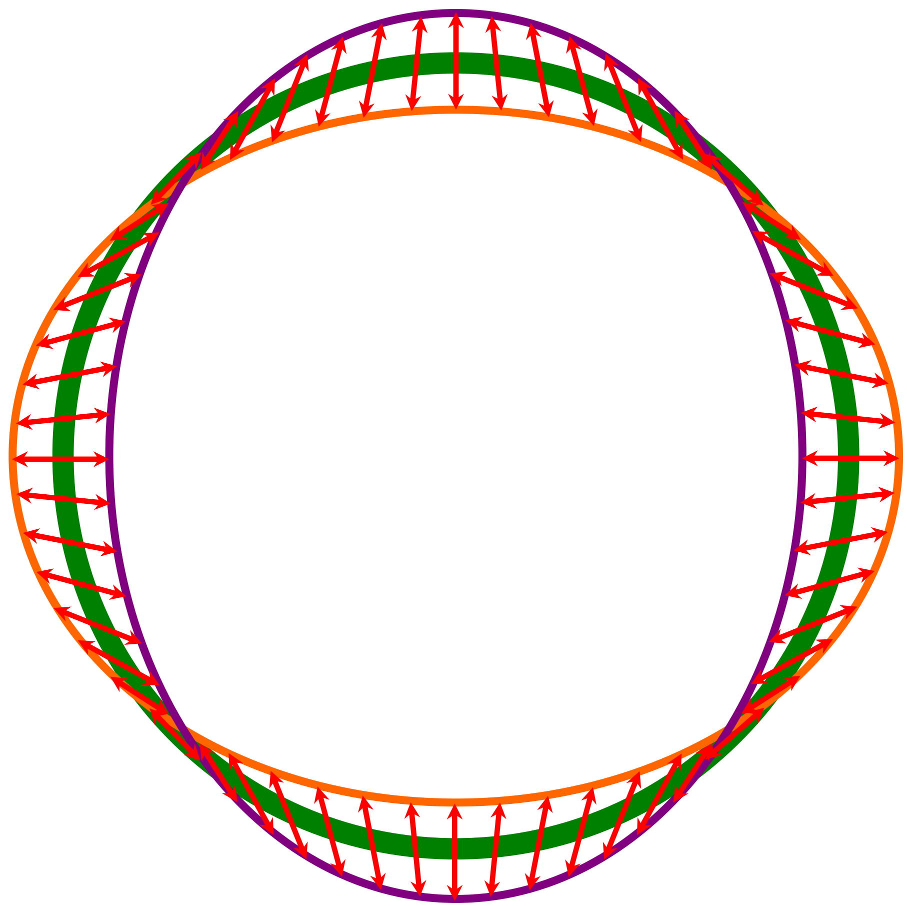
On rebound, only the first vibrational mode excited, in which the hoop in which the hoop vibrates between oblate and prolate elliptical shapes.



# Third assumption



With oscillations only the form of the hoop changes, and the total length of its arc remains constant.



# Theoretical prediction



Translational energy

$$E_{t,0} = \frac{8}{13}U$$

Vibrational energy

$$E_{v,0} = \frac{5}{13}U$$

Elastic strain energy

E. Yang, H.-Y. Kim (2012) "Jumping hoops". *Am. J. Phys.* **80**, 19–23.

# Comparison of results



Elastic strain energy → Translational energy

**Theory: 62%**

**Experiment:**

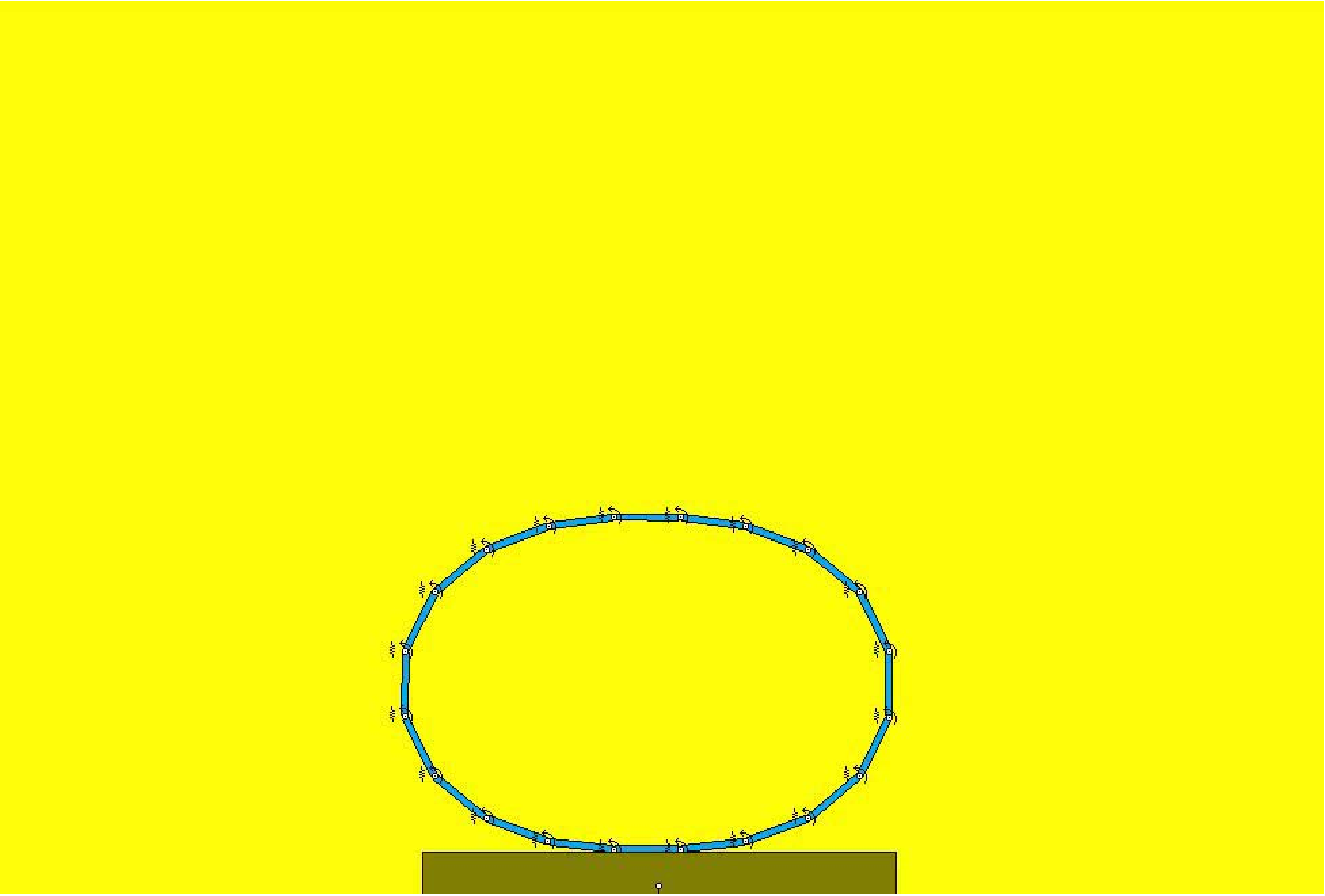
**56%** for the steel hoop

**42%** for the plastic hoop

# Computer simulation

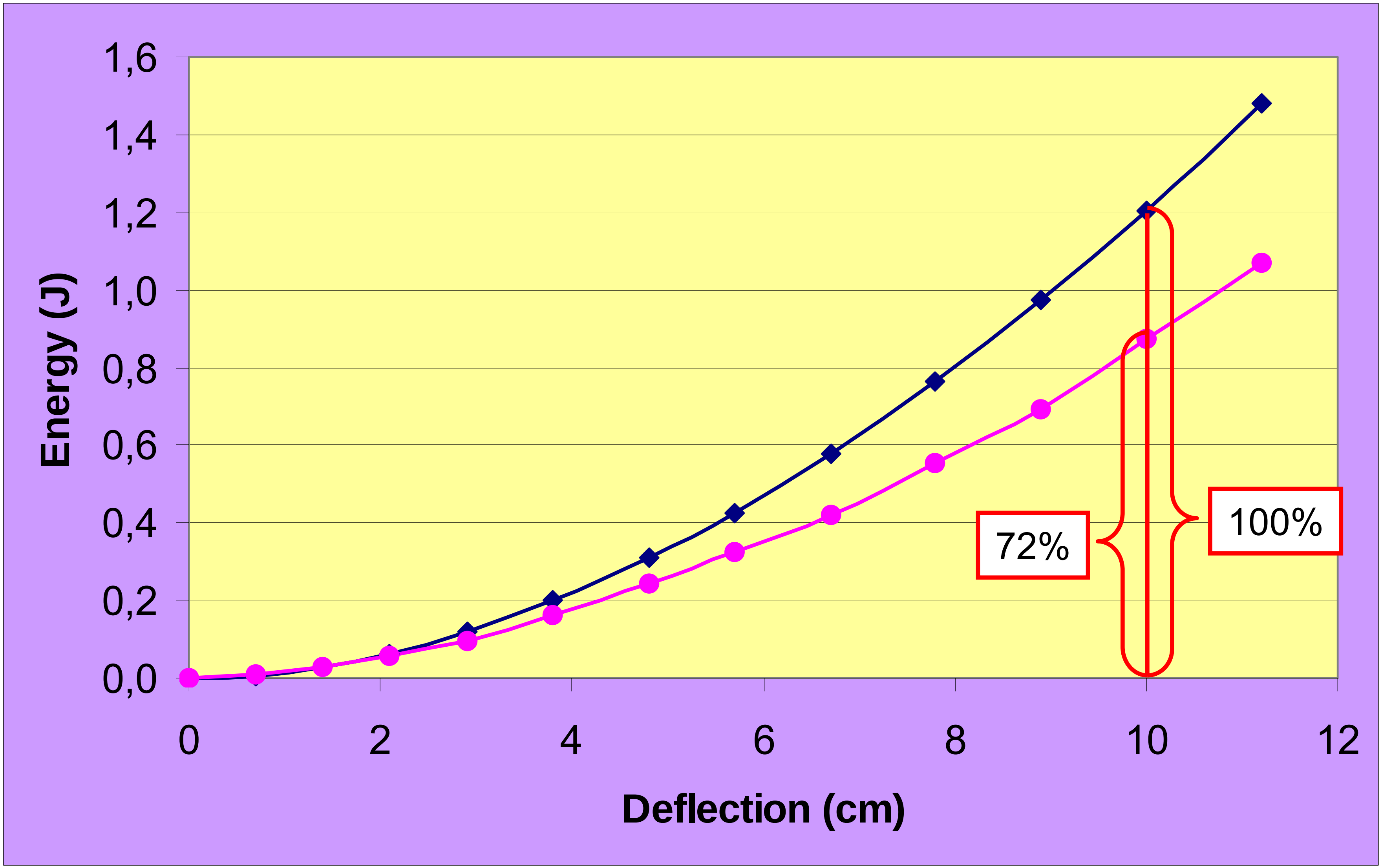
# “Hoop” of 8 members





# Ratio of energies





# Comparison of results



Elastic strain energy → Translational energy

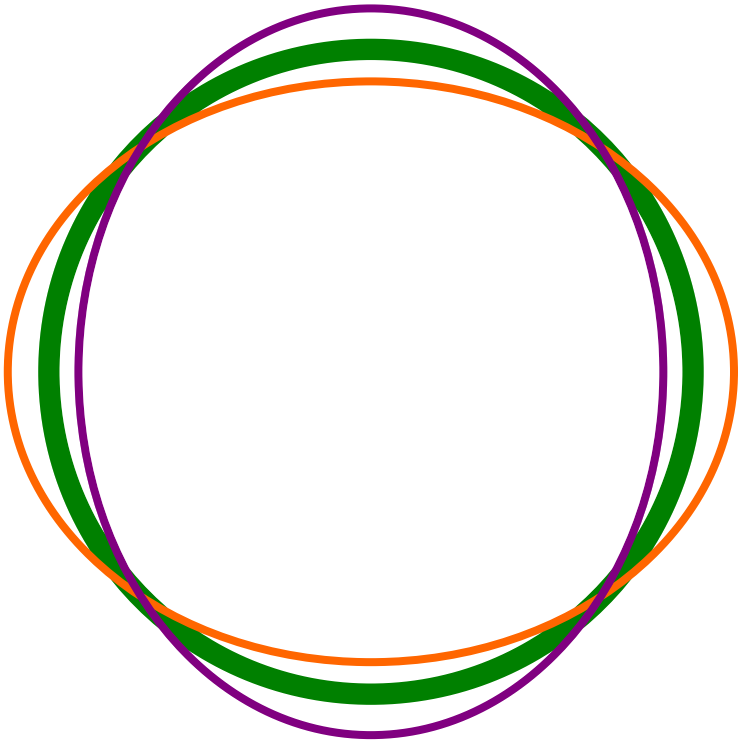
**Computer simulation: 72%**

**Theory: 62%**

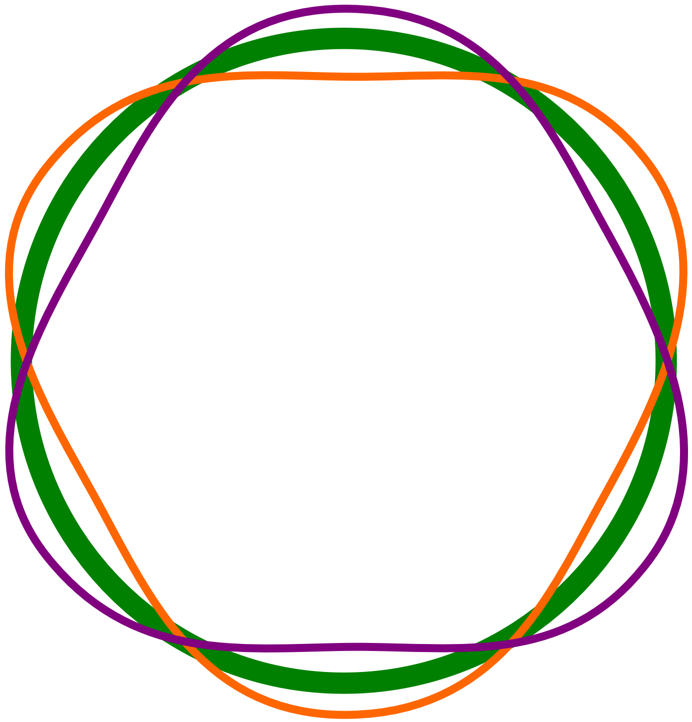
**Experiment:**
**56%** for the steel hoop
**42%** for the plastic hoop.

# Higher modes of vibration





2 wavelengths



3 wavelengths

# Summary

# Conclusions



- The height of a jump is proportional to the square of relative initial deflection and to the square of relative thickness of the hoop.
- When the hoop starts, its initial elastic strain energy is distributed between the translational energy and the vibrational energy. In a simple theoretical model, considering only the first vibration mode, the share of the translational energy is 62%.
- In a more accurate computer simulation this share is about 72%.

# References




- E. Yang, H.-Y. Kim (2012) “Jumping hoops”. *Am. J. Phys.* **80**, 19–23

# Thank you for your attention!

# Calculation principle



- During the rebound the elastic strain energy $U$ is converted to the kinetic energy of the hoop parts:

$$U = \int_0^{2\pi} \frac{\rho v^2}{2} \cdot R\,d\alpha$$

Kinetic energy per arc unit length ($\frac{\rho v^2}{2}$)

Element of the arc ($R\,d\alpha$)

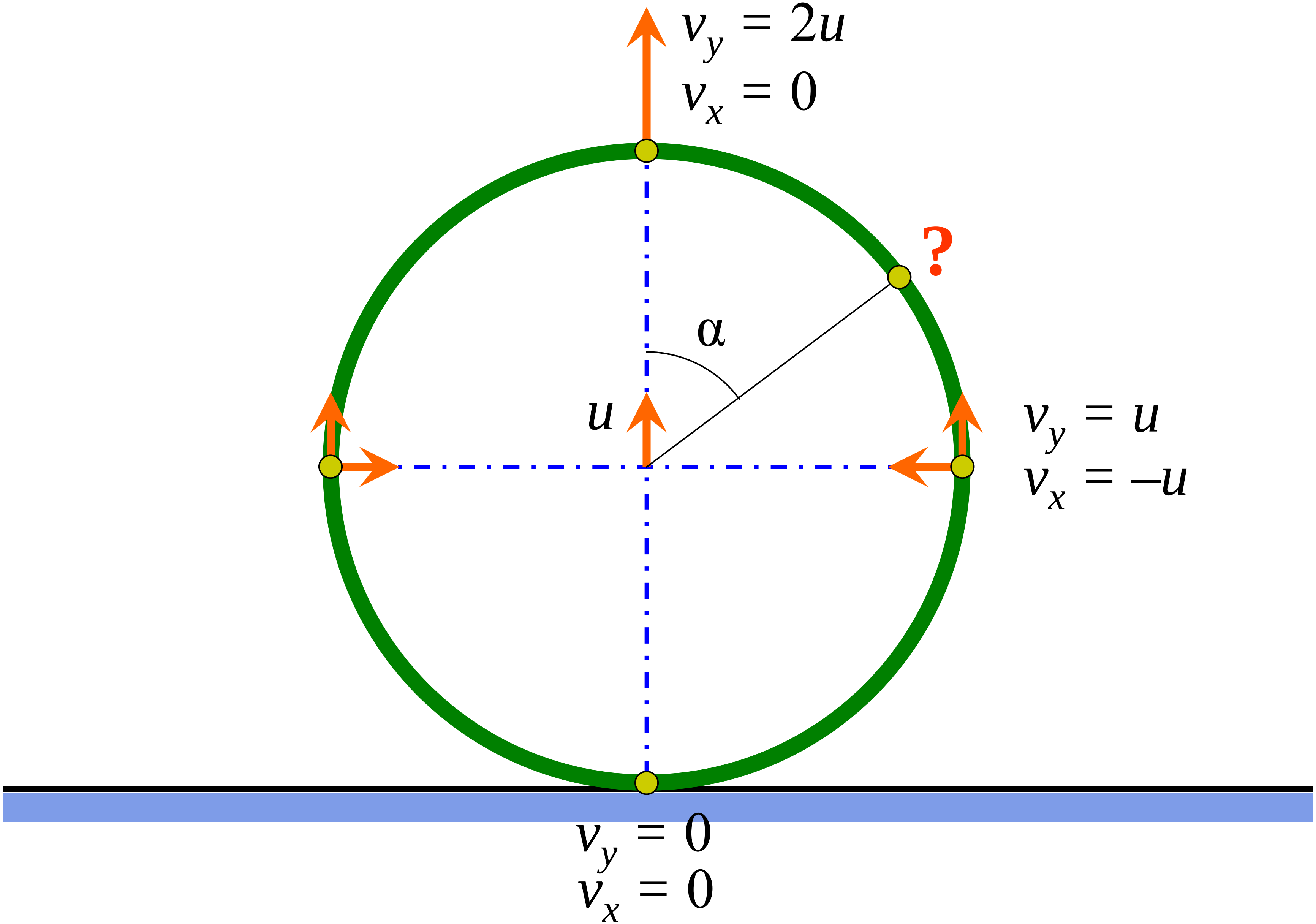
- Kinetic energy of the hoop parts distributed between the translational energy and the vibrational energy:

$$U = \frac{Mu^2}{2} + E_{osc}$$

Velocity of the mass center ($u$)

# Calculation 1

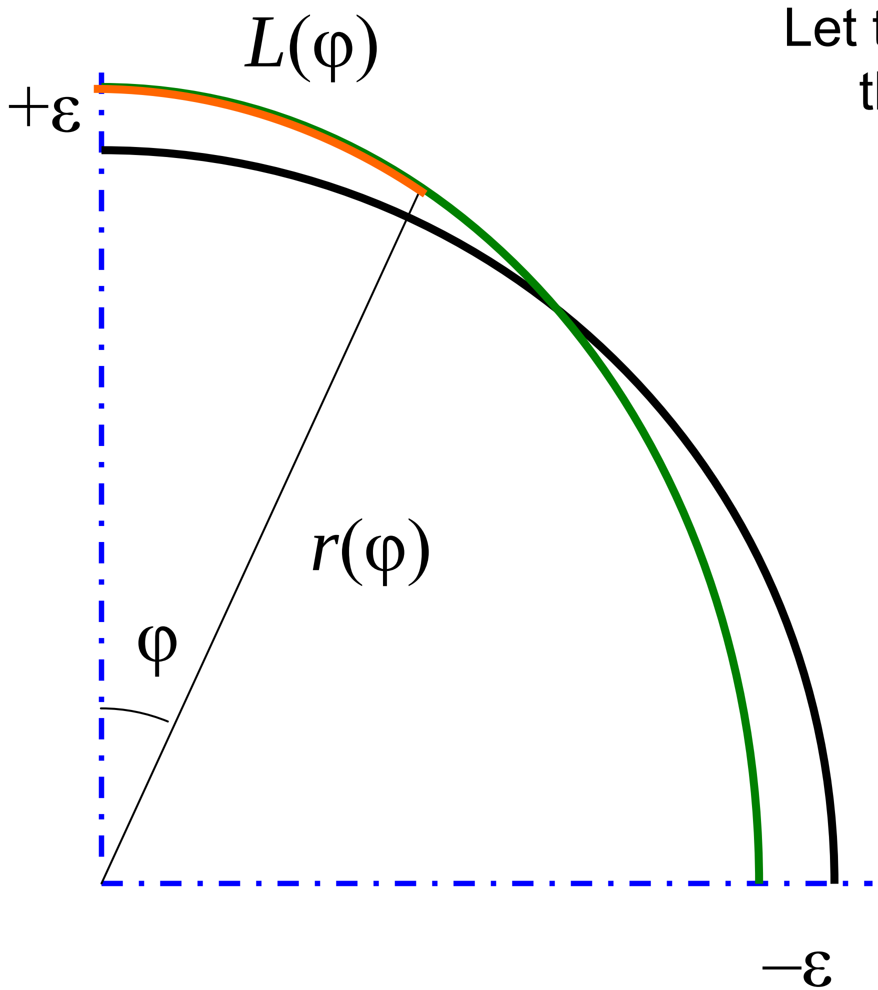




# Calculation 2



Pass in the center of mass frame and consider ¼ hoop arc.




Let the hoop deformation along the principal axes $\varepsilon << R$.

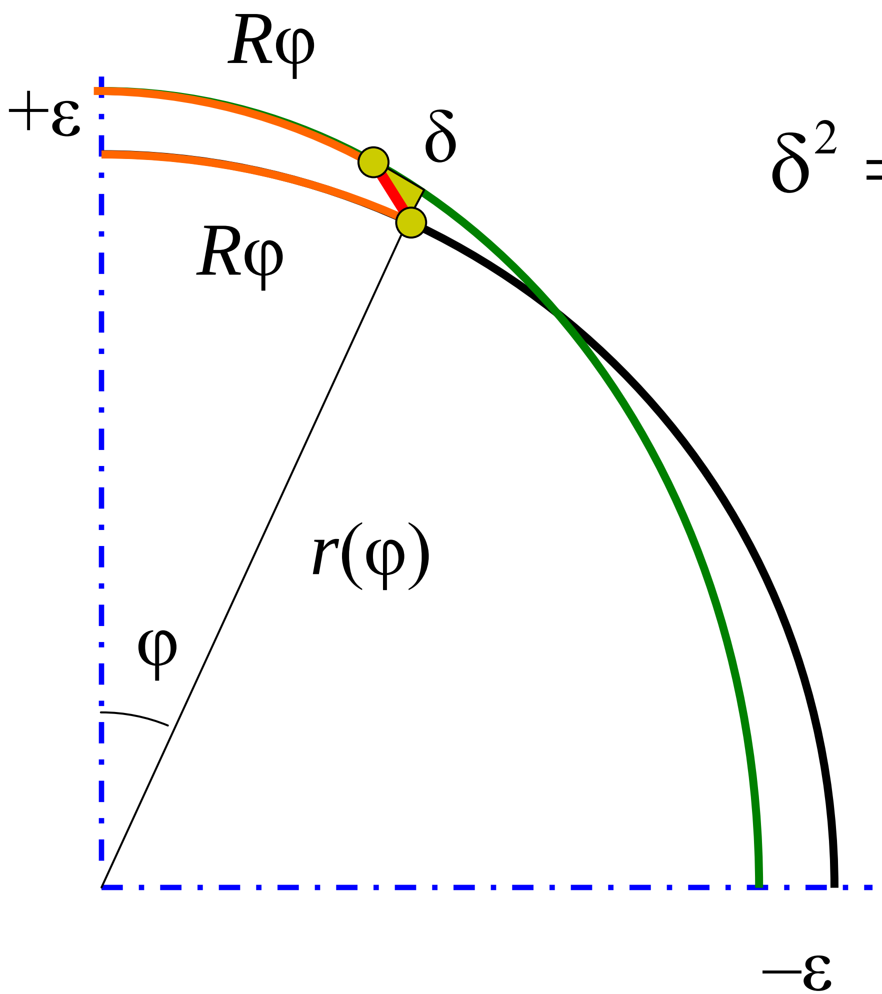
$$r(\varphi) = R + \varepsilon \cos 2\varphi$$

$$L(\varphi) = \int_0^{\varphi} (R + \varepsilon \cos 2\varphi) d\varphi =$$

$$= R\varphi + \frac{\varepsilon}{2} \sin 2\varphi$$

# Calculation 3

The hoop in its motion is not deformed along the ribbon.

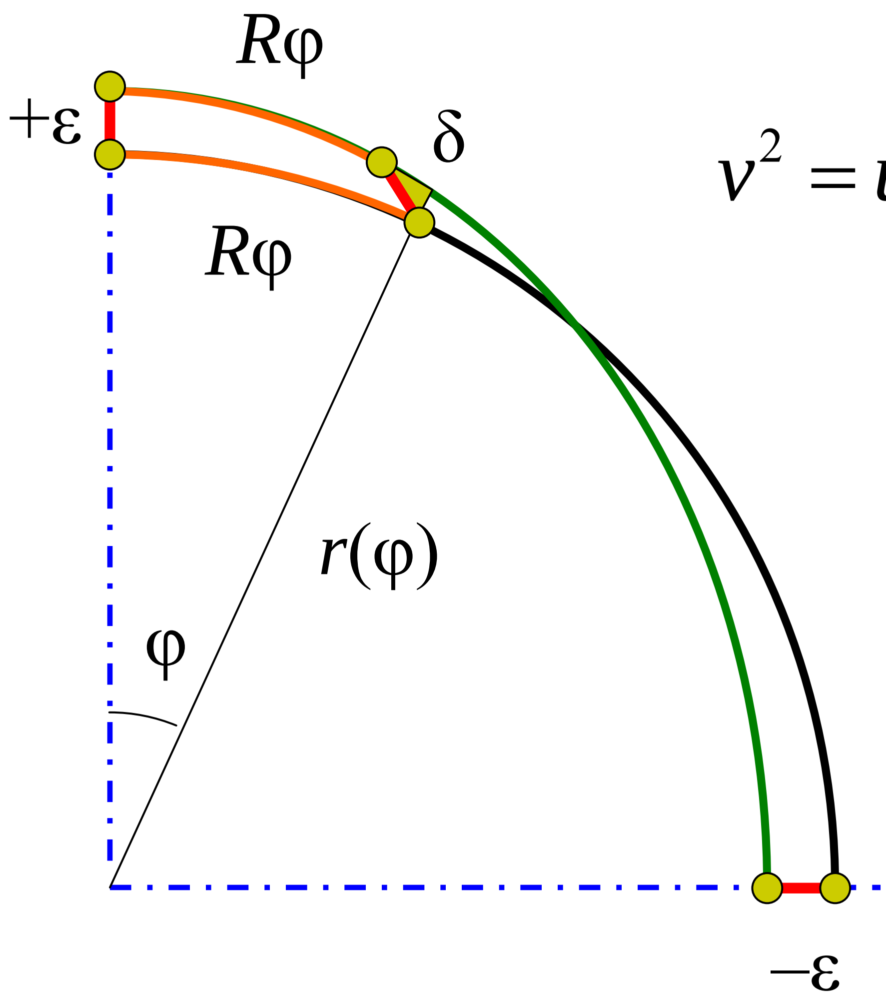



$$\delta^2 = \varepsilon^2 \cos^2 2\varphi + \frac{\varepsilon^2}{4}\sin^2 2\varphi =$$

$$= \varepsilon^2 \left( \frac{5}{8} + \frac{3}{8}\cos 4\varphi \right)$$

# Calculation 4

Amplitudes of velocities are proportional to the aptitudes of displacements.




$$v^2 = u^2 \frac{\delta^2}{\varepsilon^2} = u^2 \left( \frac{5}{8} + \frac{3}{8} \cos 4\varphi \right)$$

$$E_{osc} = \int_0^{2\pi} \frac{\rho v^2}{2} R d\varphi = \frac{5}{8} \cdot \frac{Mu^2}{2} = \frac{5}{8} E_{trans,0}$$