1

Invent Yourself

Kamila Součková

1 Invent Yourself

It is more difficult to bend a paper sheet, if it is folded “accordion style” or rolled into a tube.

Using a single A4 sheet and a small amount of glue, if required, construct a bridge spanning a gap of 280 mm.

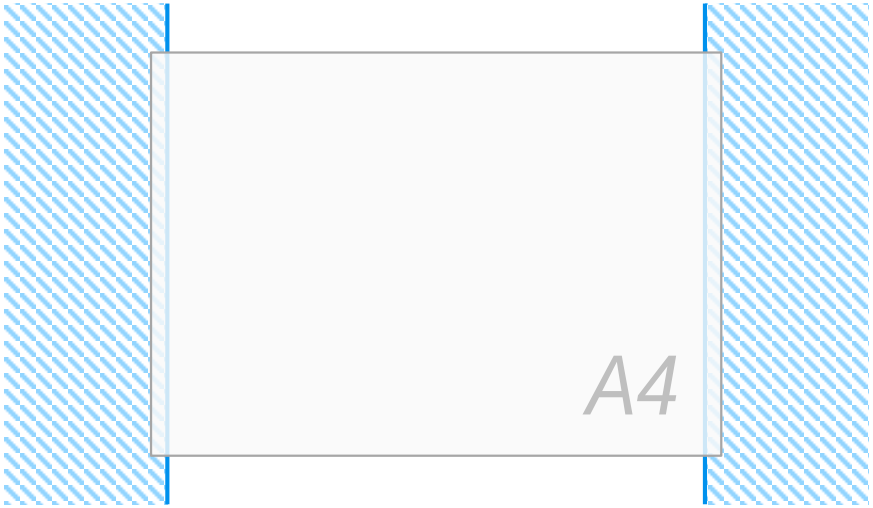
Introduce parameters to describe the strength of your bridge, and optimize some or all of them.

Paper

standard office paper

- areal density $\approx 80\ \mathrm{g/m^2}$
- thickness $\approx 0.1\ \mathrm{mm}$
- A4: $210\ \mathrm{mm} \times 297\ \mathrm{mm}$

our bridge:
280 mm



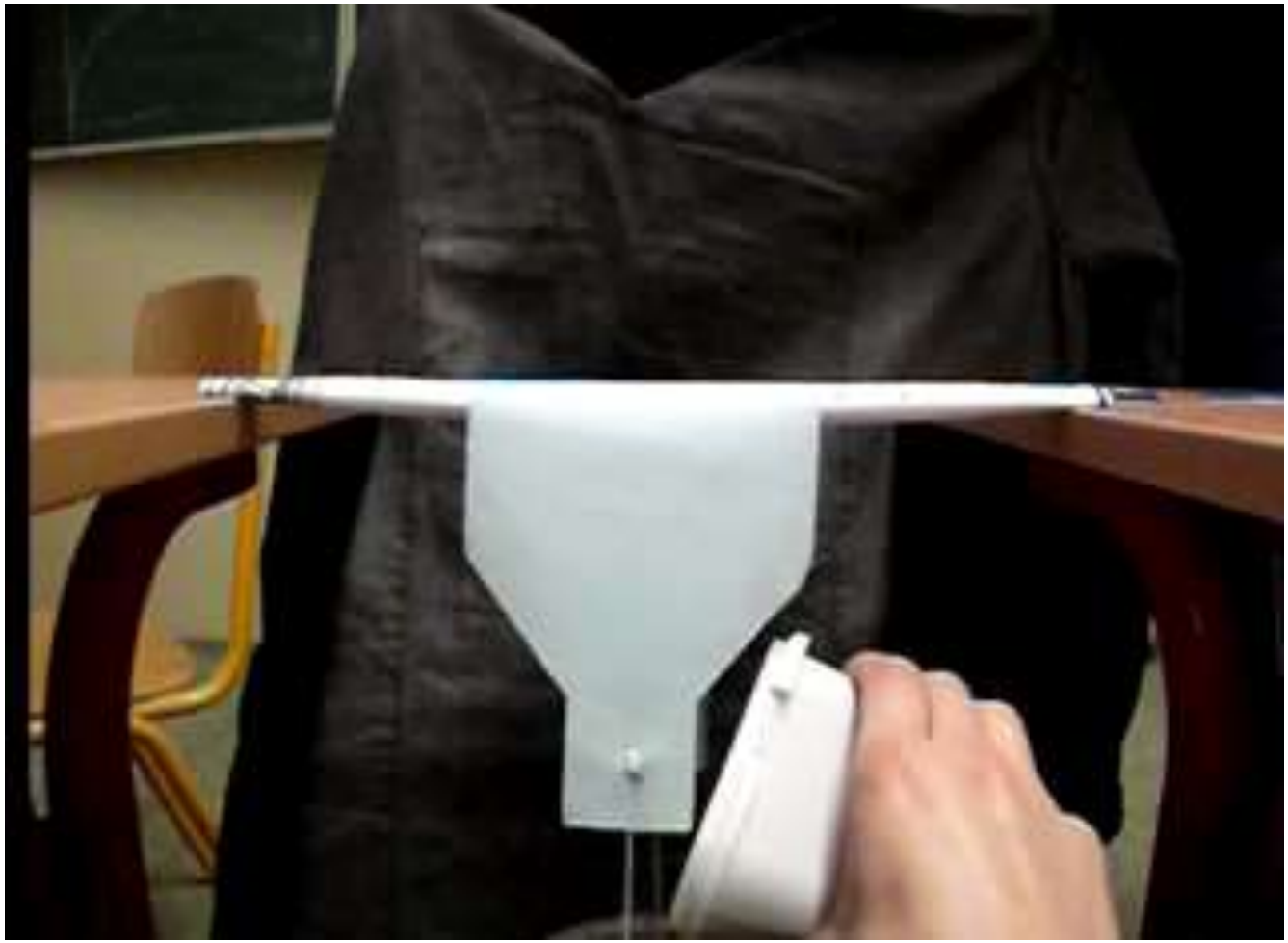
Definition of Strength

strength ≡ maximal load the bridge can hold in its center

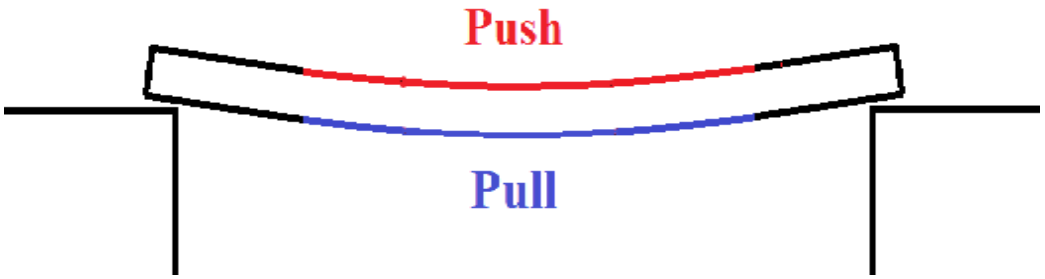
- load placed over constant width of the bridge (12 cm)

BRIDGE OPTIMIZATION

How a Simple Bridge Breaks

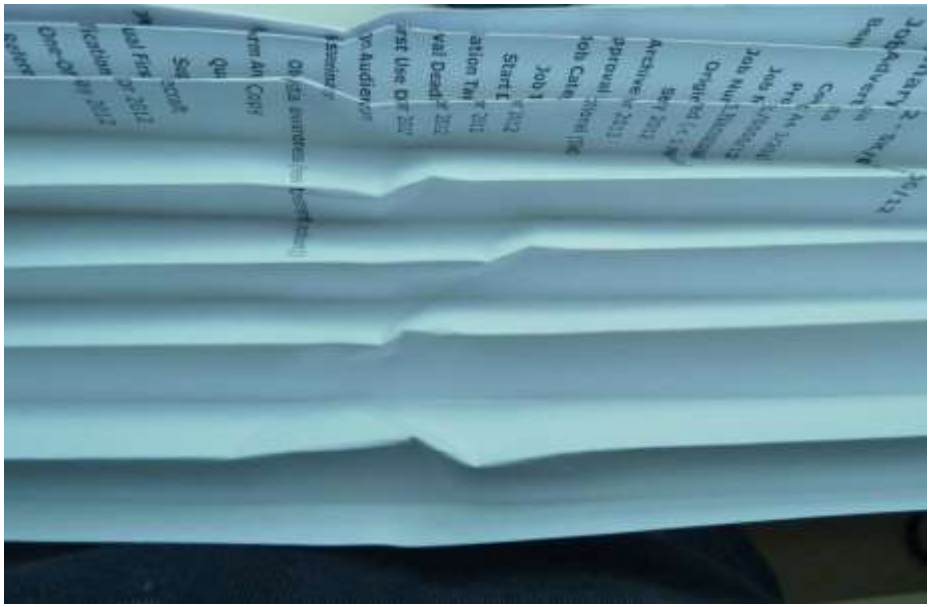


What Happened



paper:

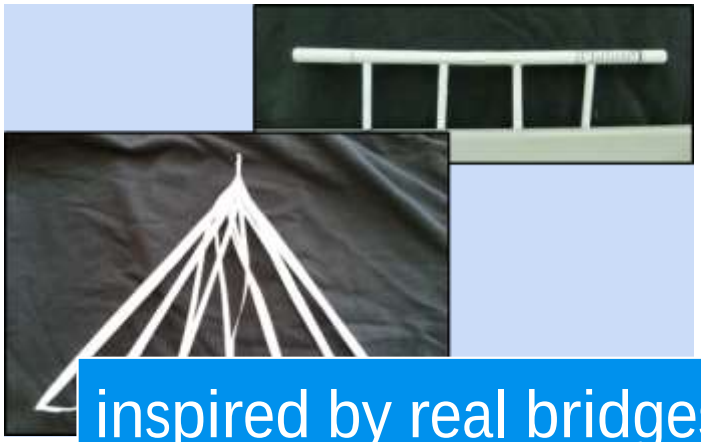
strong in pull,
weaker in push



Bridge Designs

accordion

tube



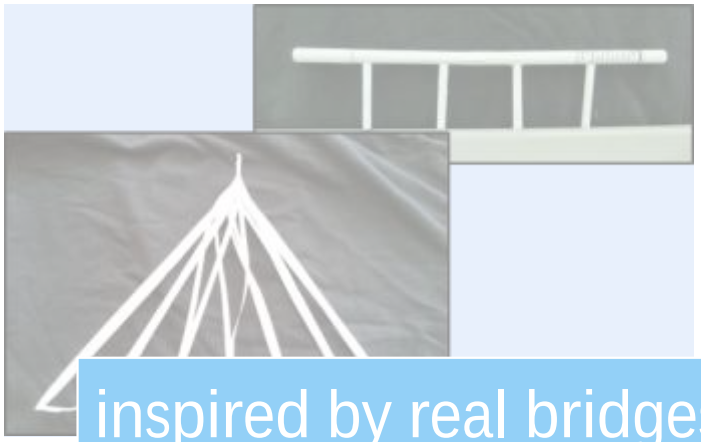
inspired by real bridges

?

paper-unique

accordion

tube

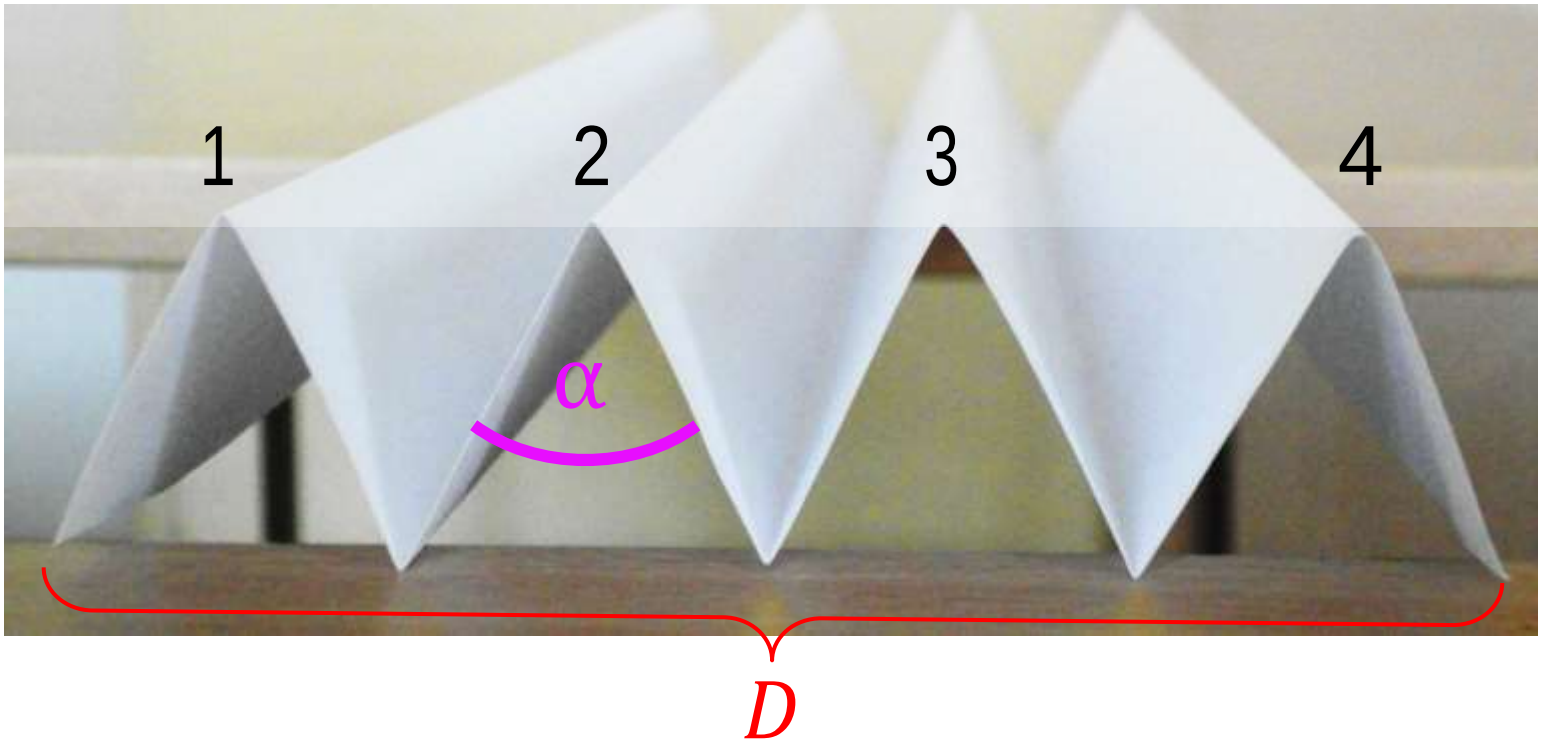


inspired by real bridges

?

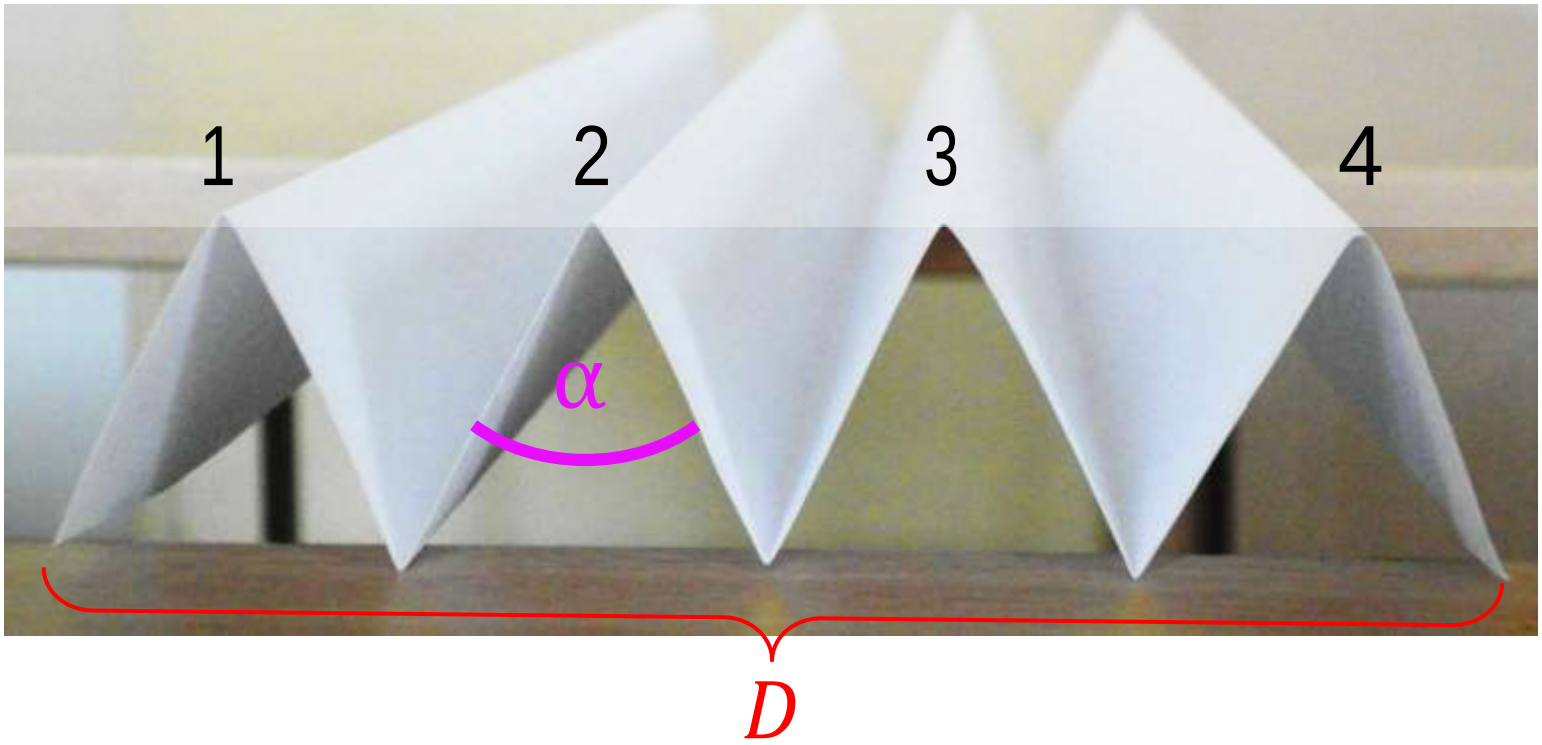
paper-unique

Accordion: Parameters



- number of dents N
- width of accordion D

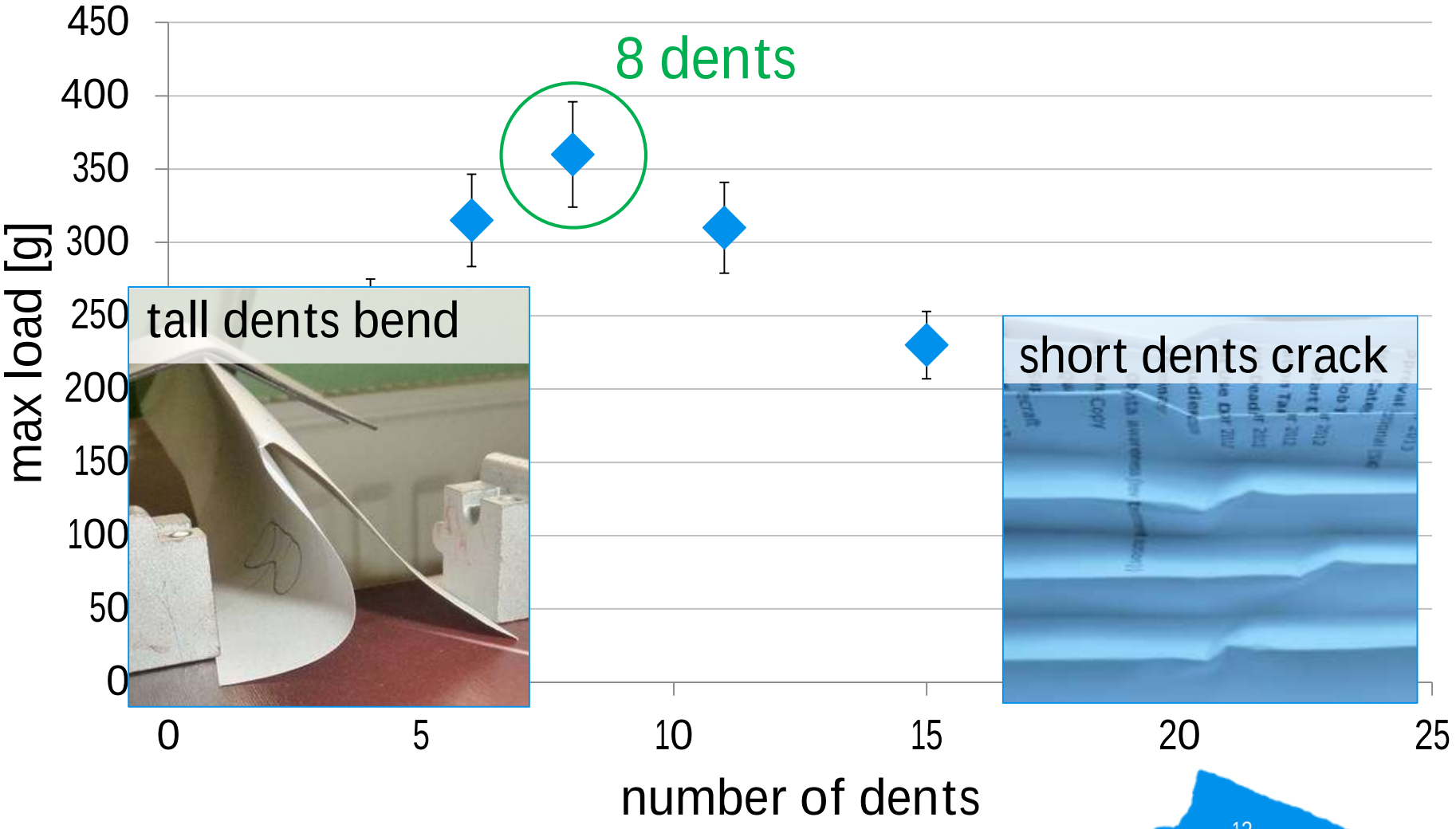
Accordion: Parameters



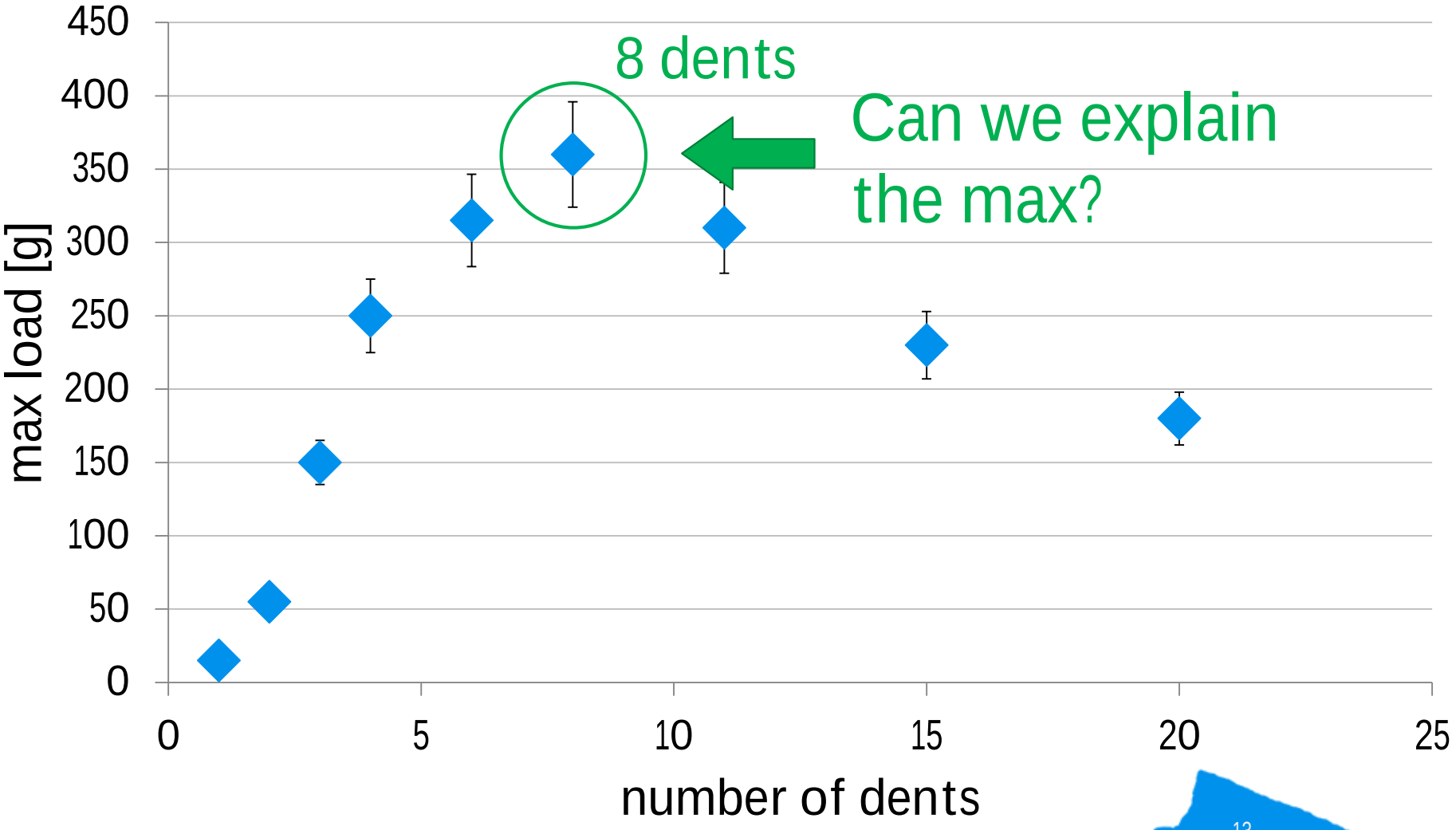
→ number of dents N

- width of accordion D

Max Load vs. # of Dents for $D = 8$ cm

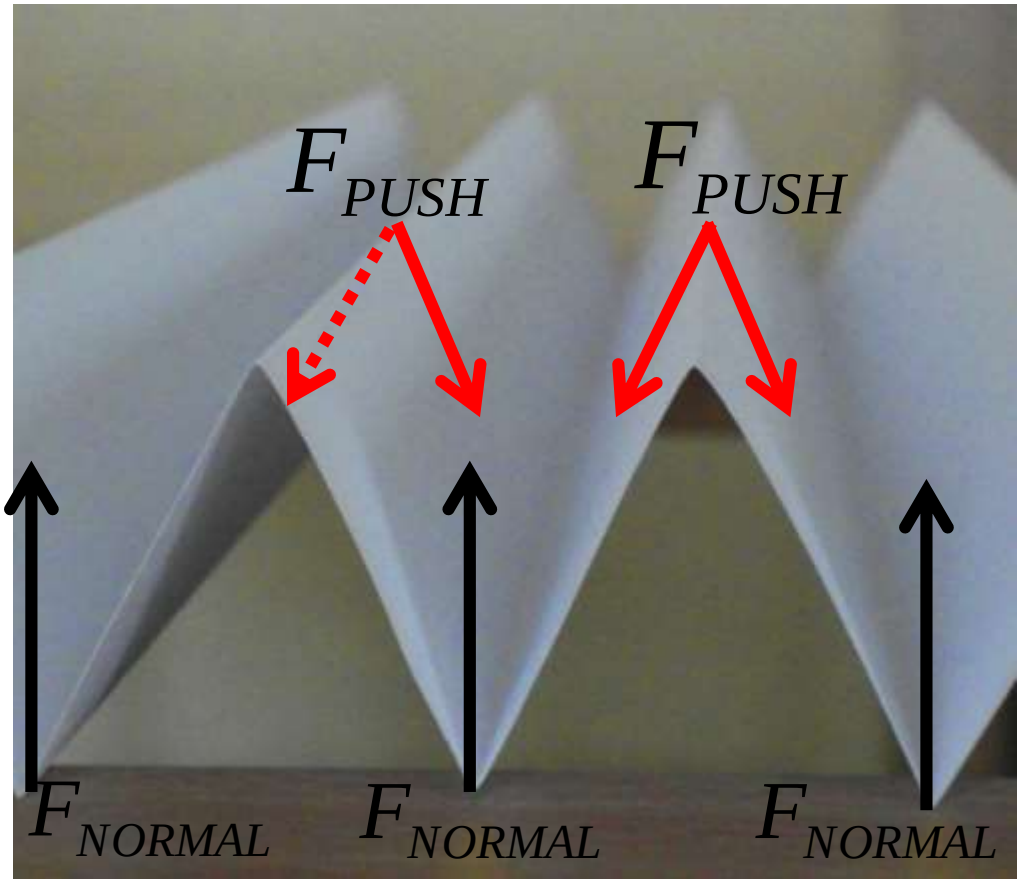


Max Load vs. # of Dents for $D = 8$ cm



Force analysis

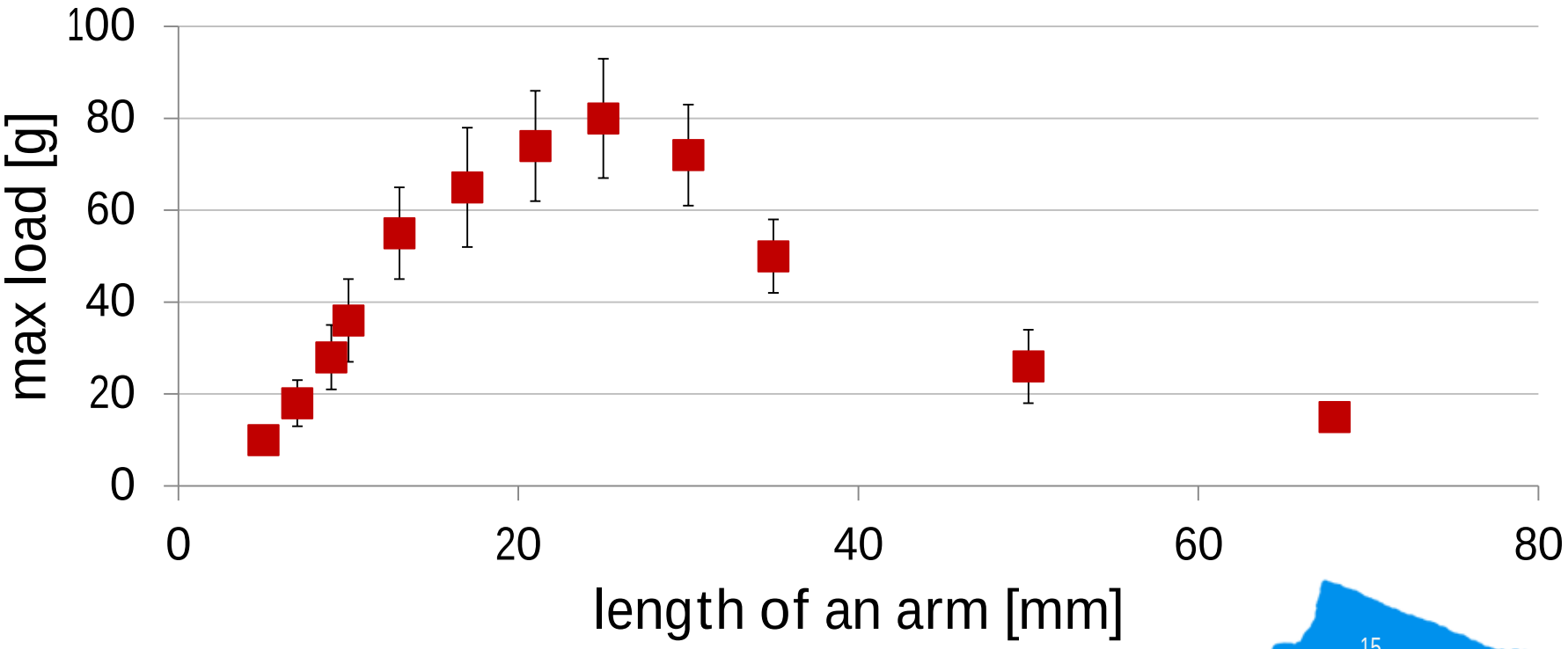
strength of bridge = strength of single dent × N



Accordion: Explaining the Max

strength of bridge = strength of single dent × *N*

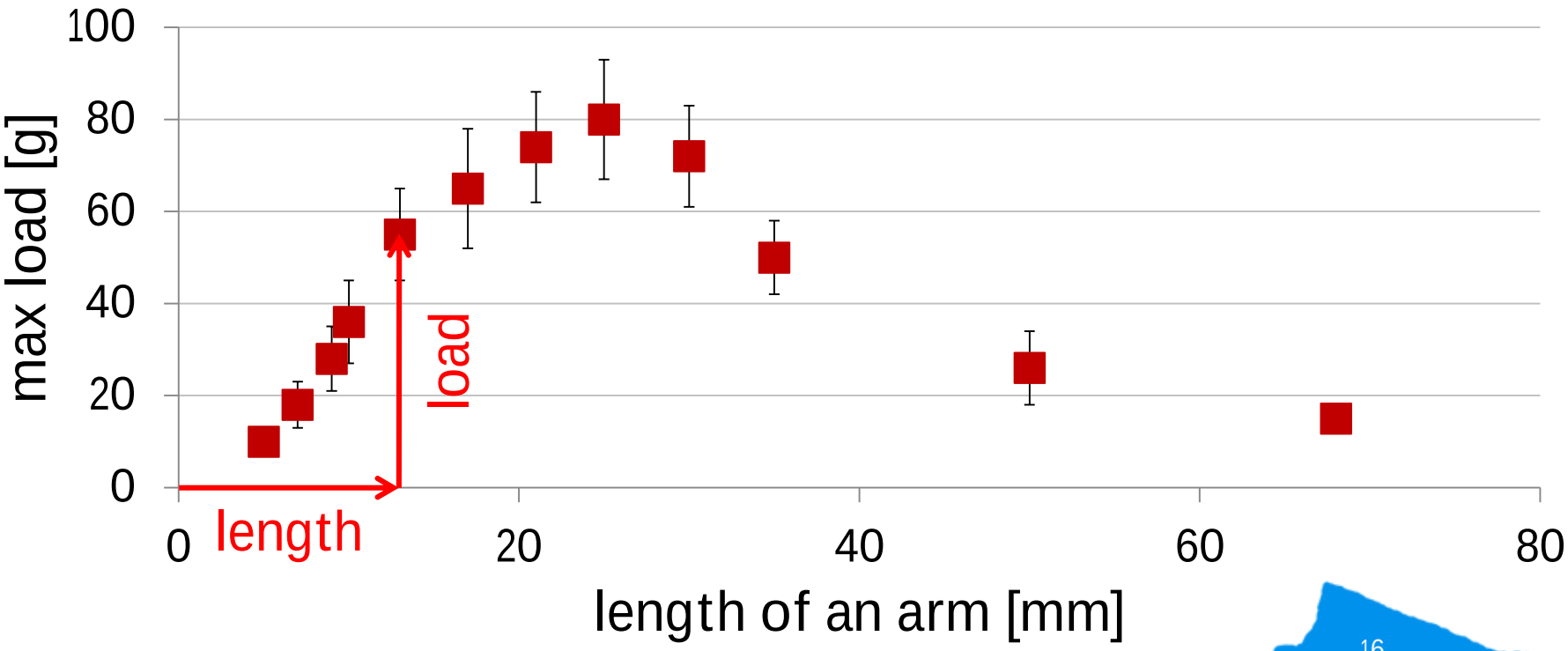
→ experiment: strength of a single dent



Accordion: Explaining the Max

strength of bridge = strength of single dent × N

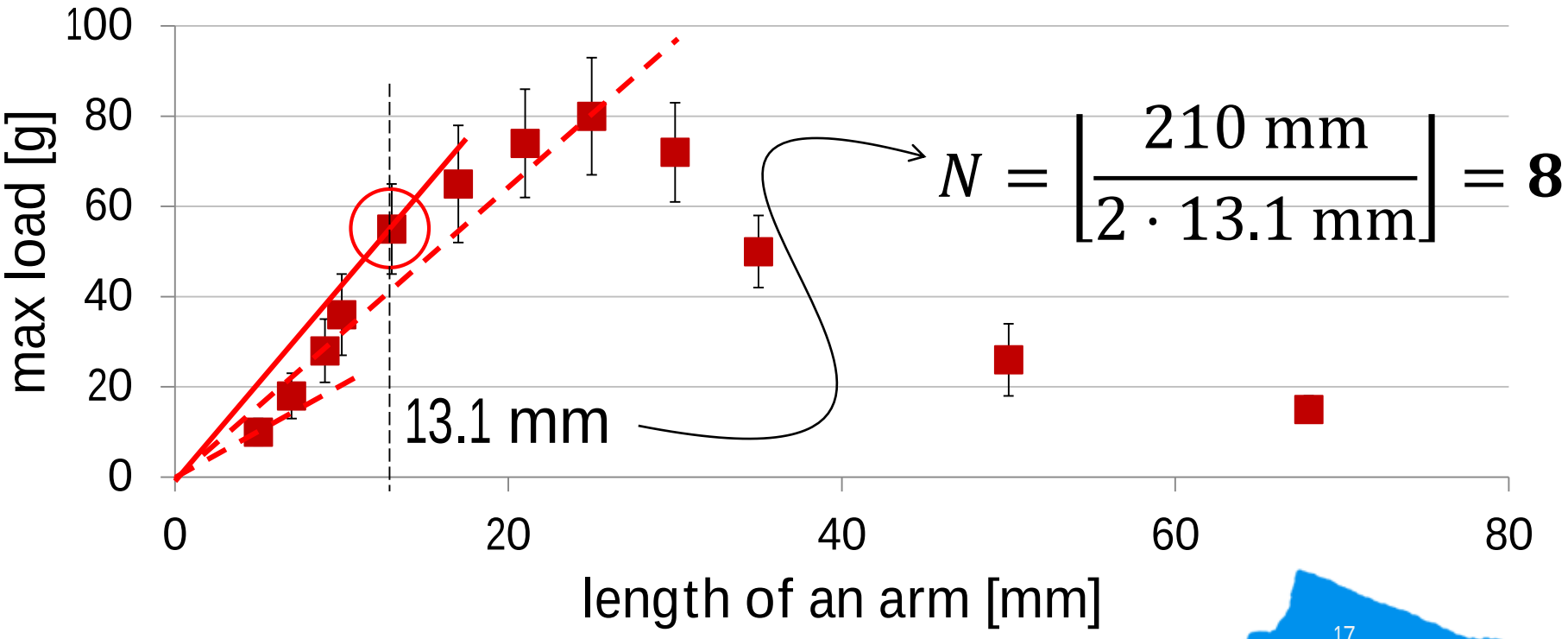
max: highest load/length ratio $\left(N \propto \frac{1}{\text{length}} \right)$



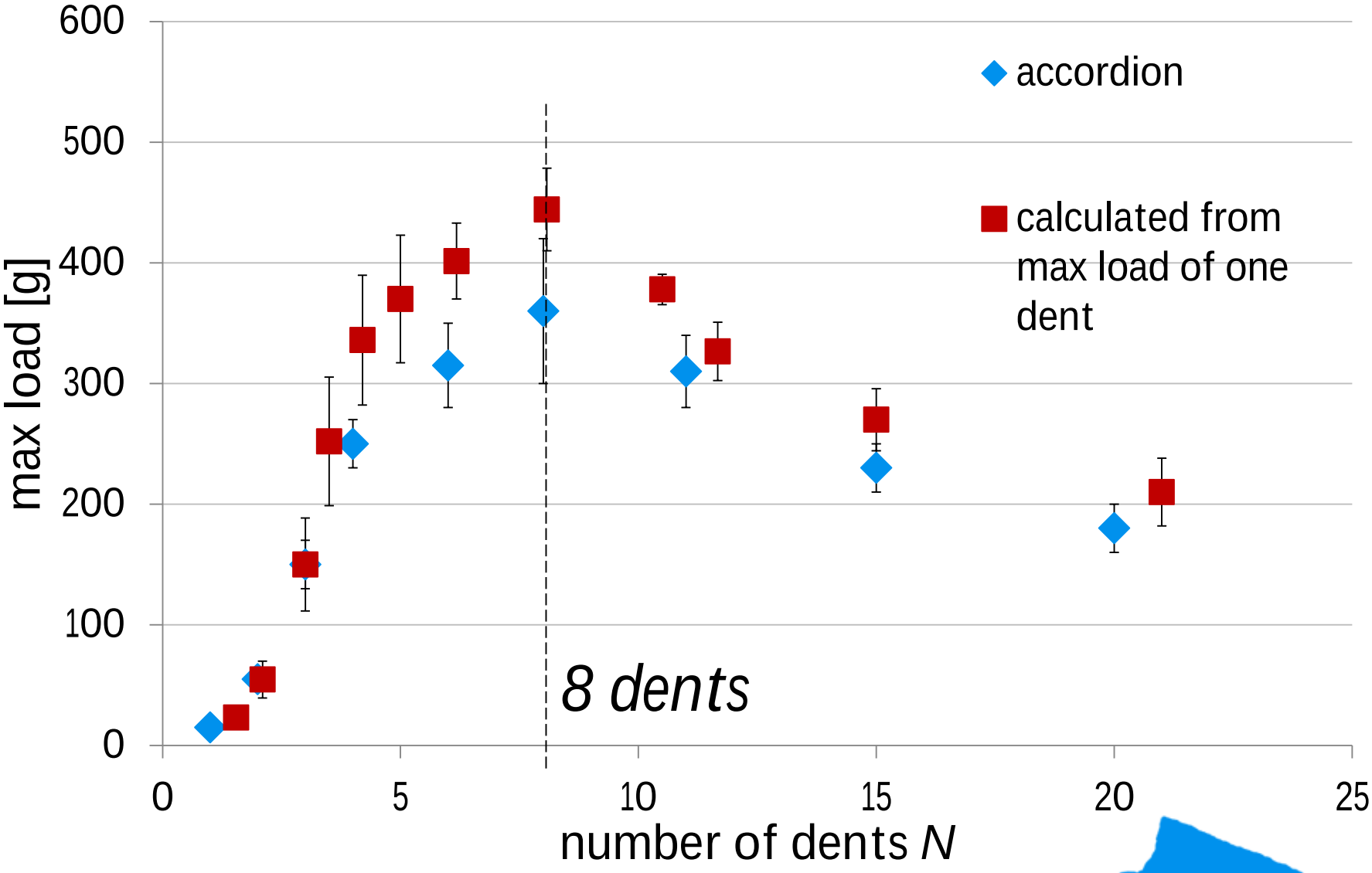
Accordion: Explaining the Max

strength of bridge = strength of single arm × *N*

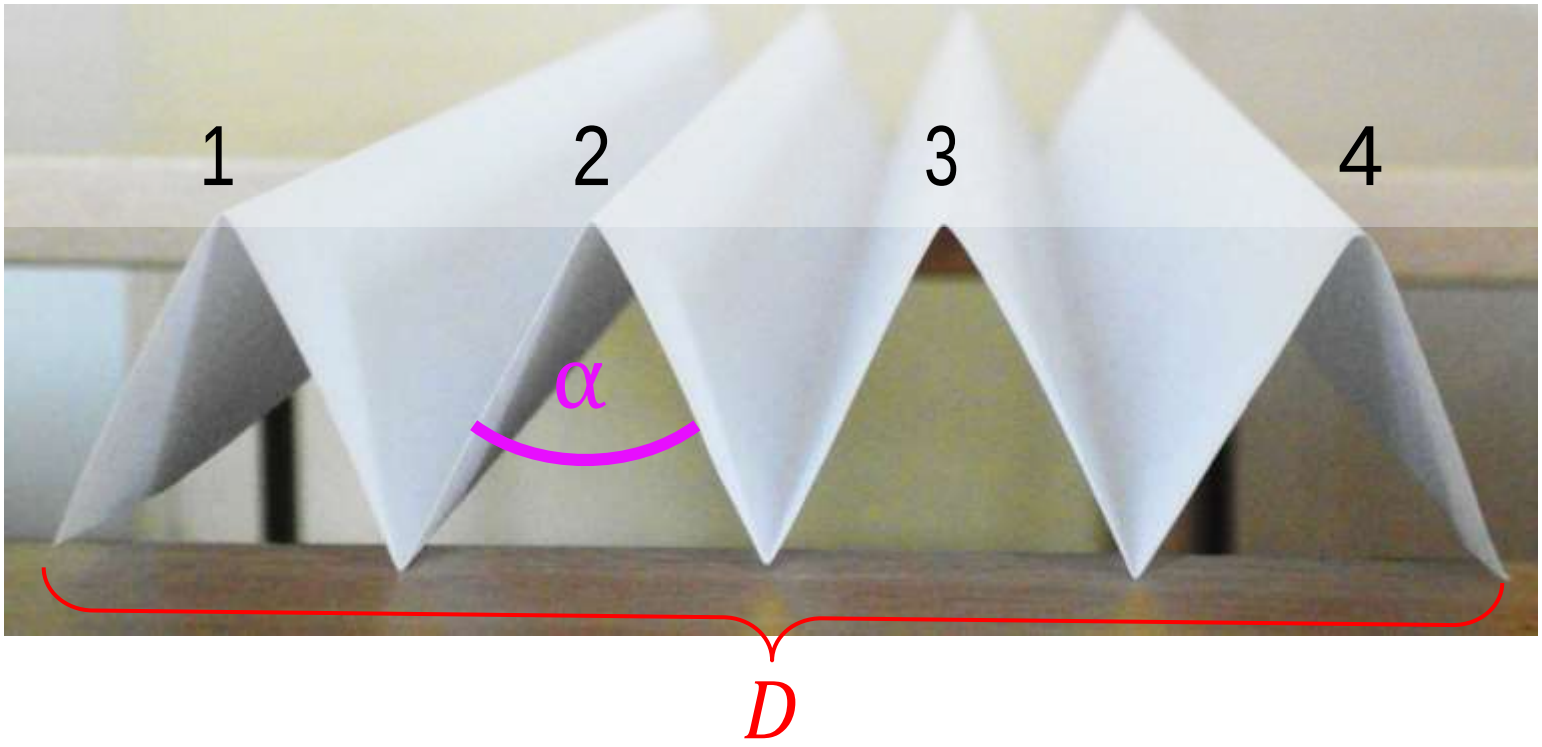
max: highest load/length ratio $\left(N \propto \frac{1}{\text{length}} \right)$



Measured Max Load vs. *N* × Single Dent

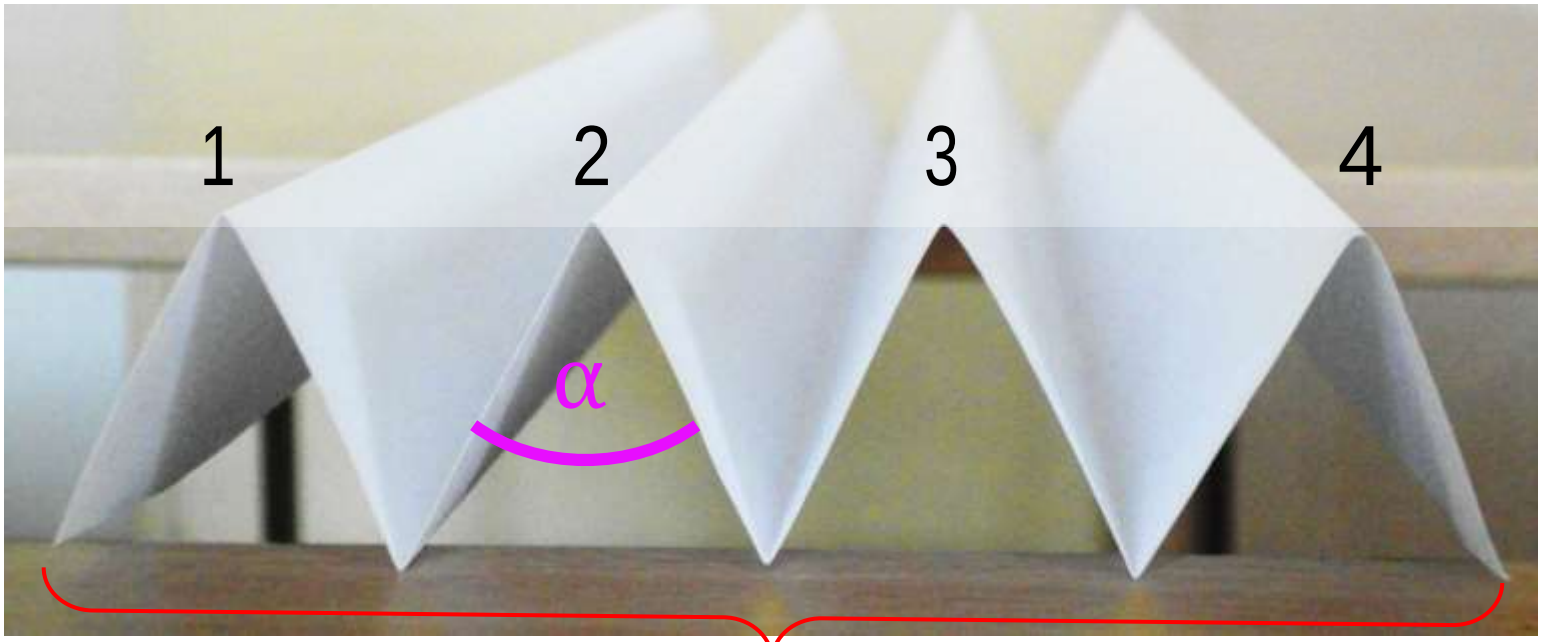


Accordion: Parameters



- number of dents N ✓
- width of accordion D

Accordion: Parameters

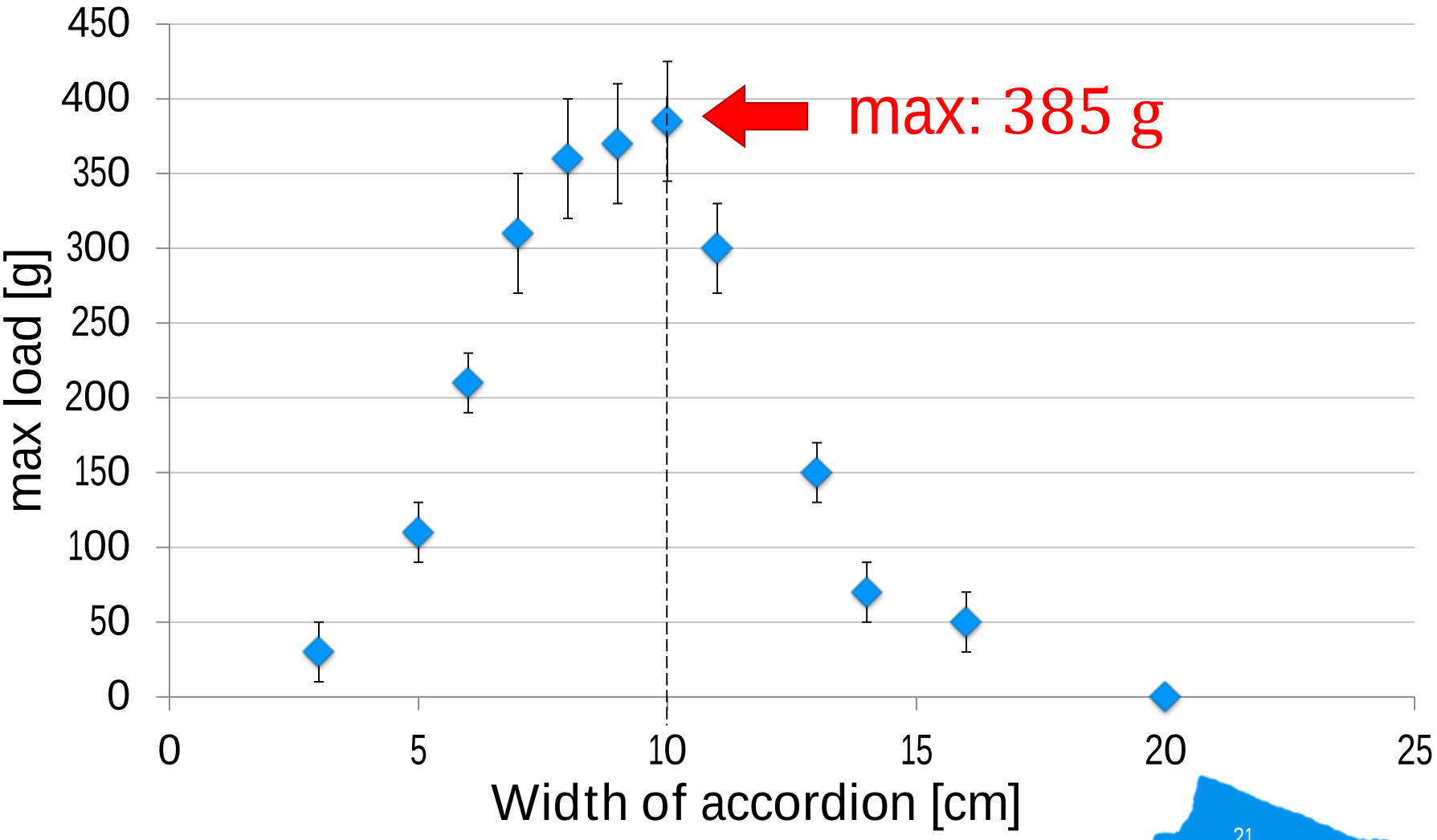


- number of dents N ✓

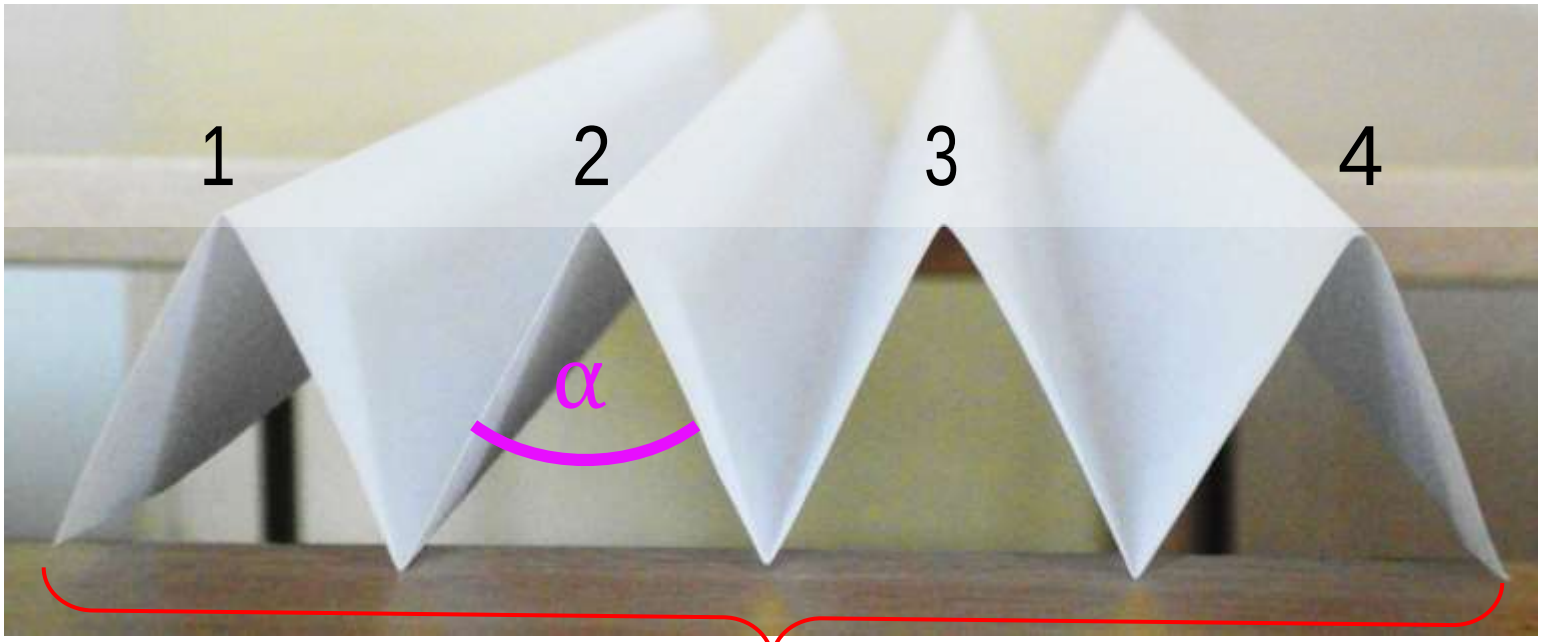
→ width of accordion $\boldsymbol{D}$

Max Load vs. Accordion Width

for 8 dents



Accordion: Parameters



- number of dents N ✓
- width of accordion D ✓

Accordion: Optimal Parameters

max:

- 8 dents
- $D = 10$ cm

385 g

Modifying the Accordion

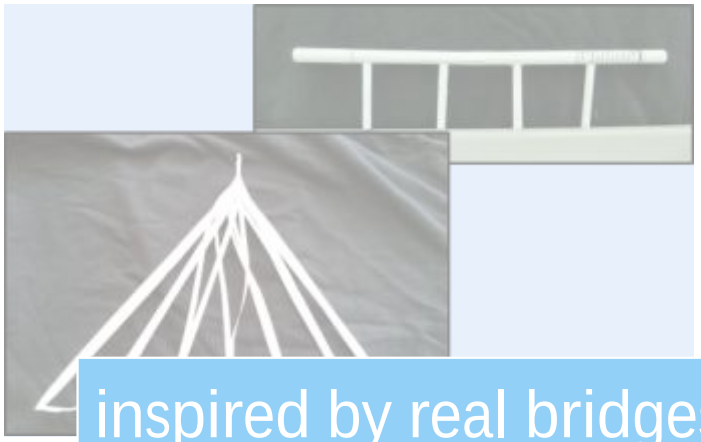
max load:
180 g

low stability – crashes near the center

→ cute, but useless

accordion

tube



inspired by real bridges

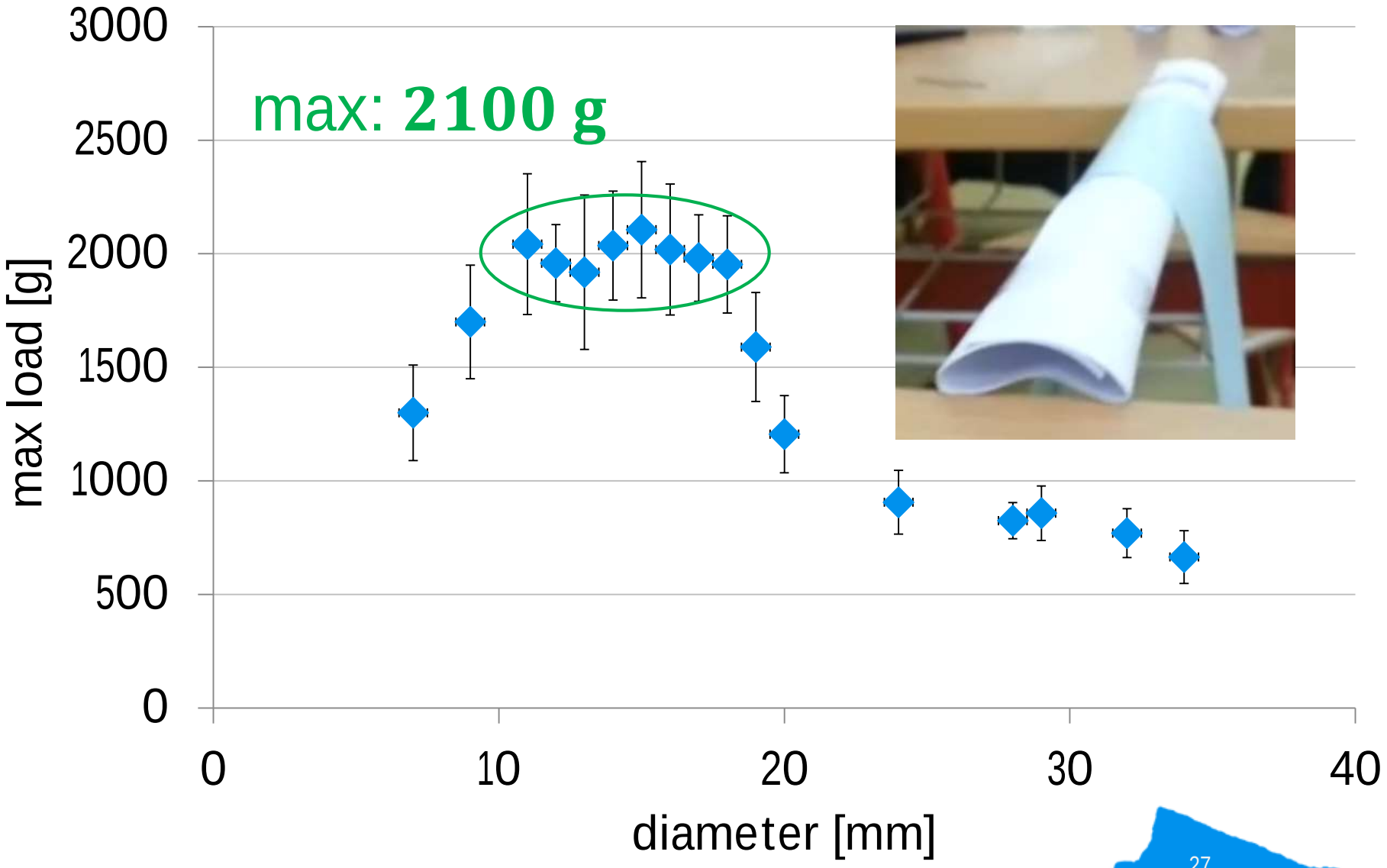
?

paper-unique

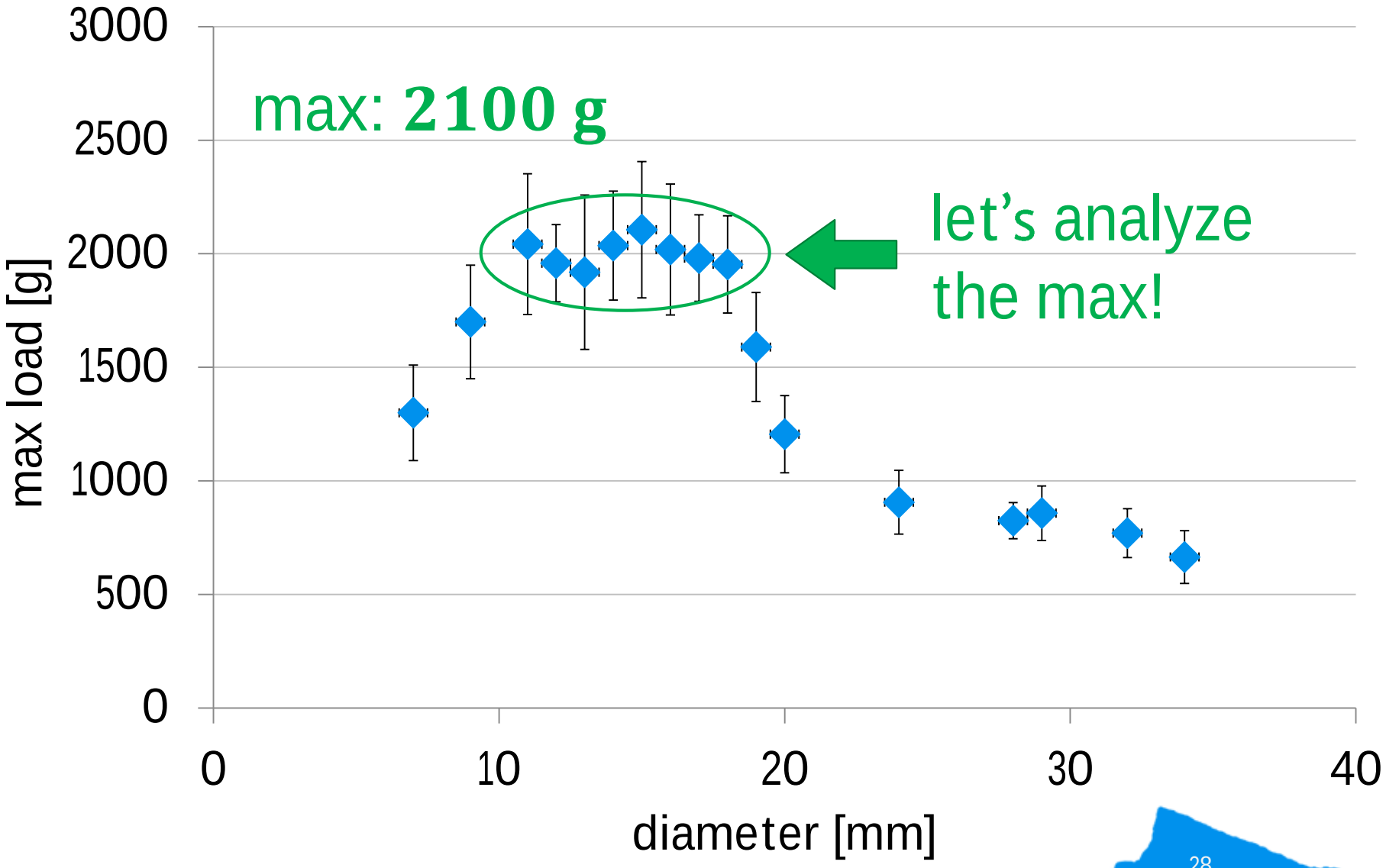
Tube: Parameters

- radius $r \Rightarrow$ number of layers $L = \frac{210\ \mathrm{mm}}{2\pi r}$

Max Load vs. Radius



Max Load vs. Radius



Analysis of Variables

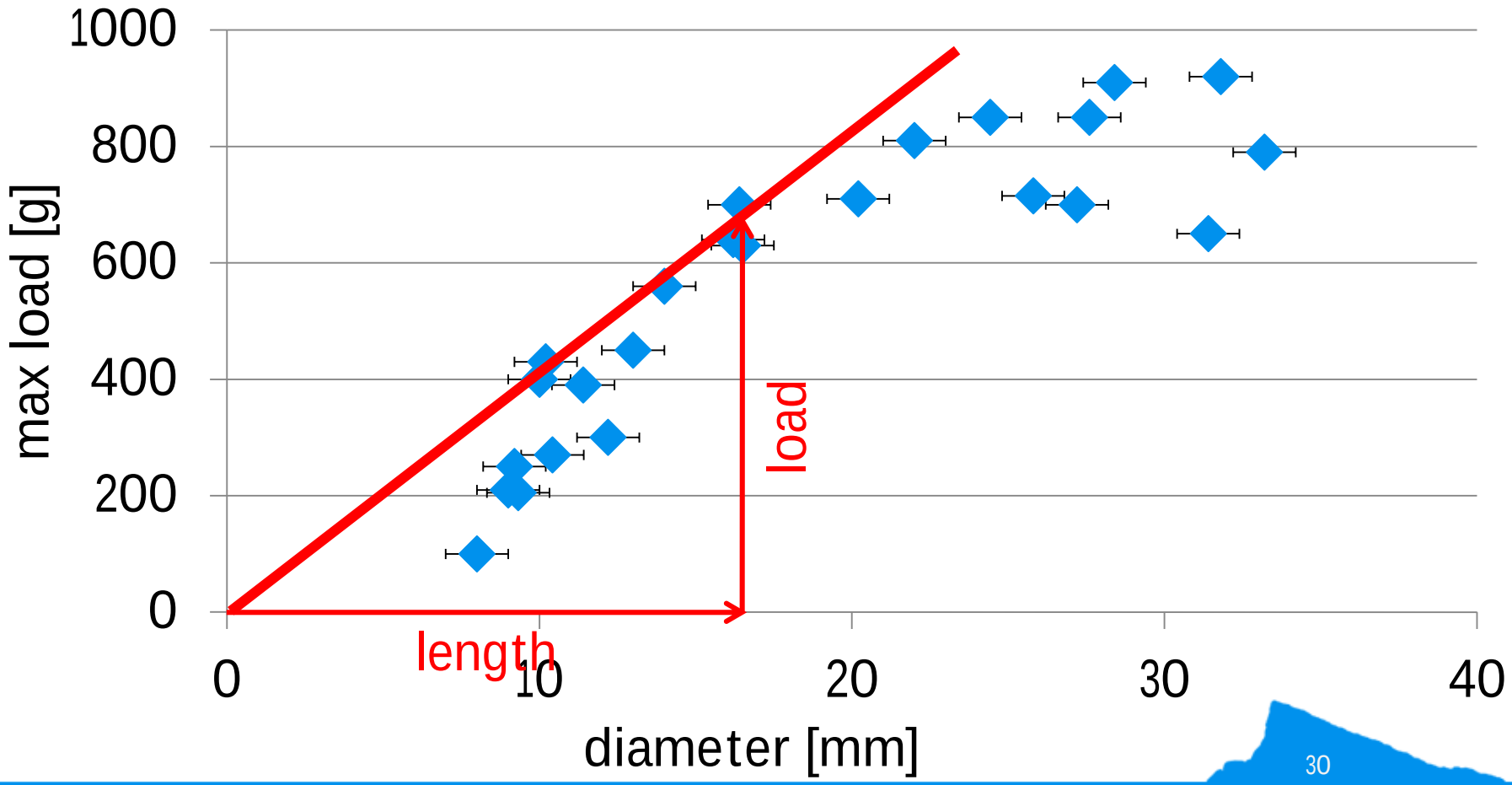
need to separate the effect of

- radius
- number of layers

→ change radius but not number of layers: experiment with double-layer tubes

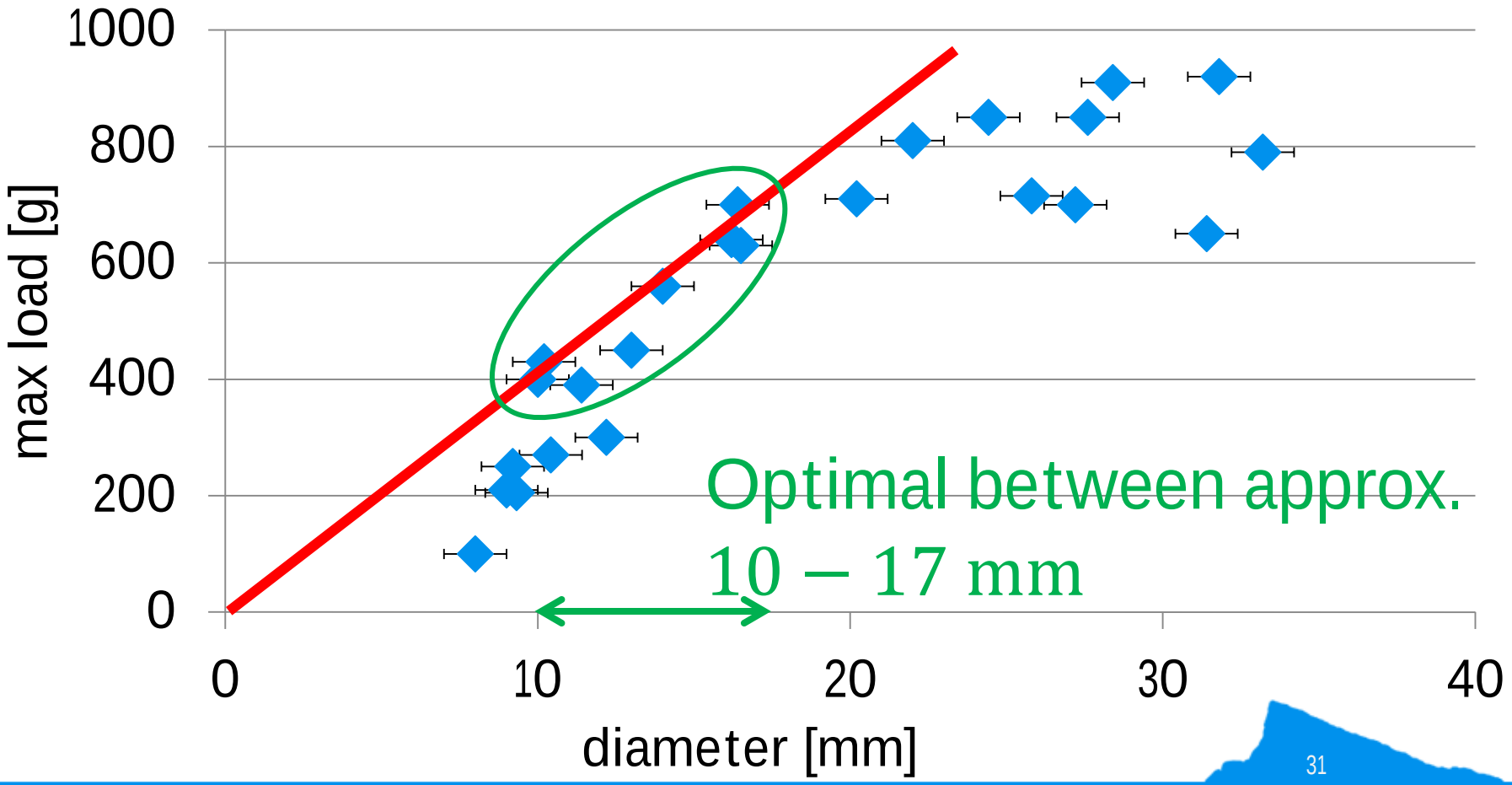
Max Load of Double-Layer Tubes

max: highest load/diameter ratio $\left(N \propto \frac{1}{\text{length}} \right)$

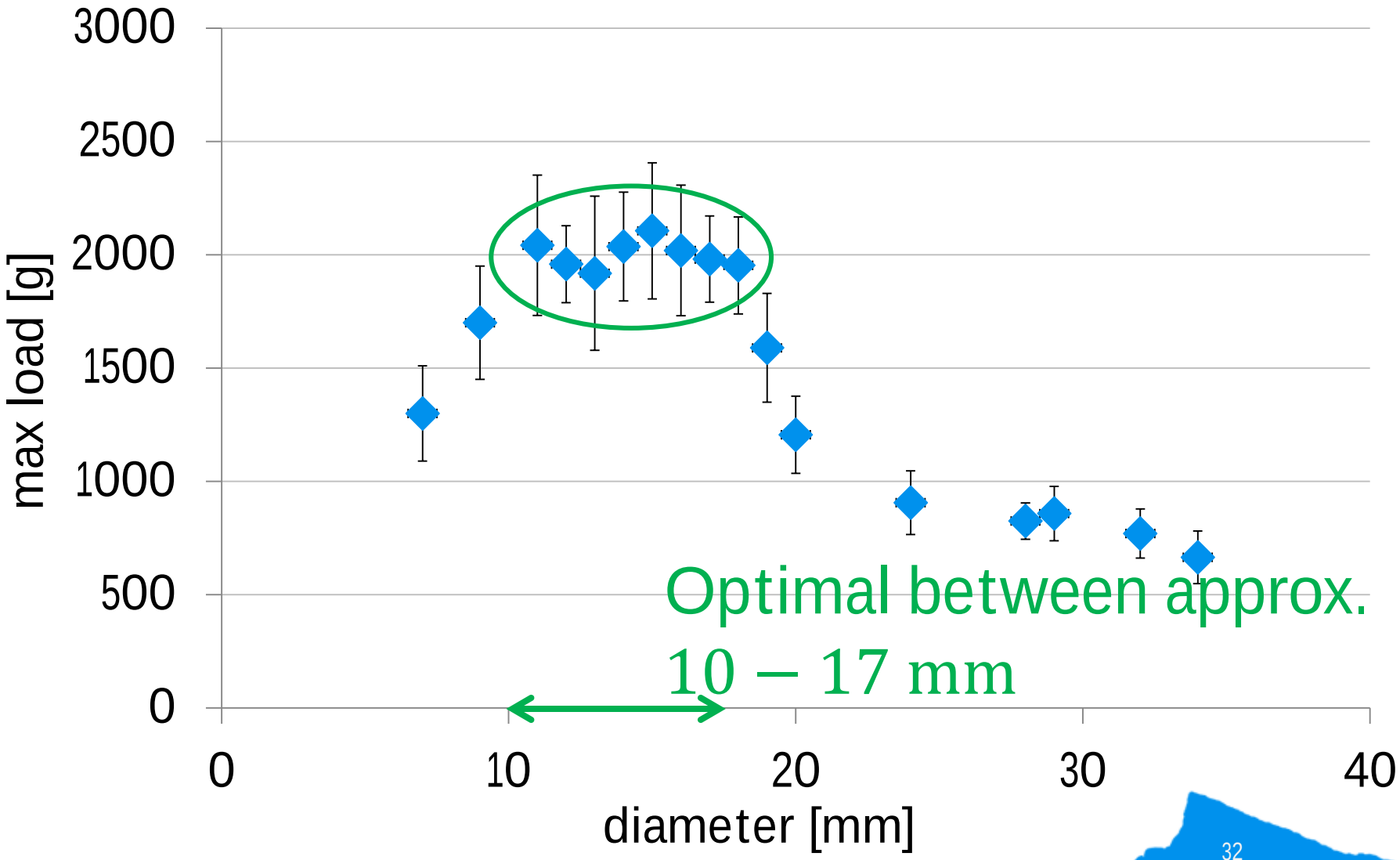


Max Load of Double-Layer Tubes

max: highest load/diameter ratio $\left(N \propto \frac{1}{\text{length}} \right)$



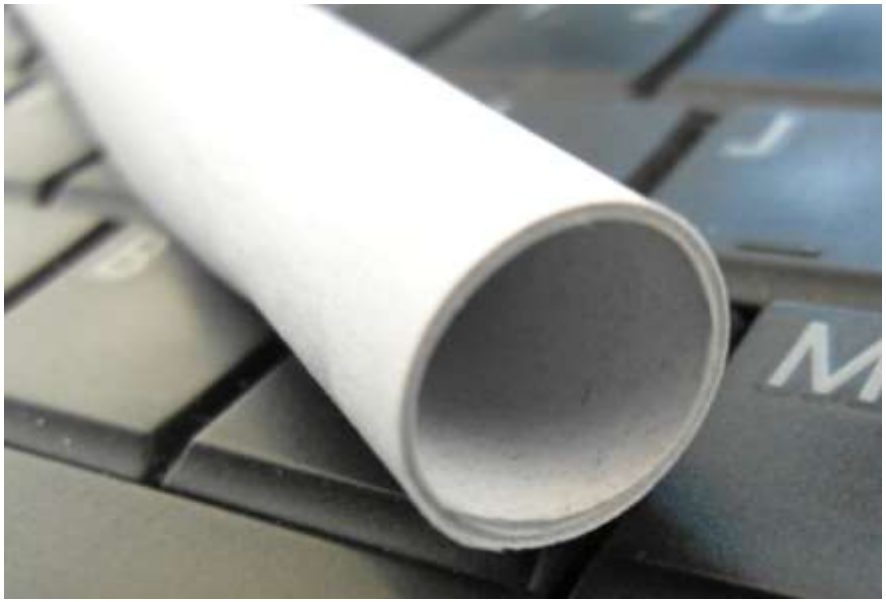
Max Load vs. Radius



Tube: Optimal Parameters

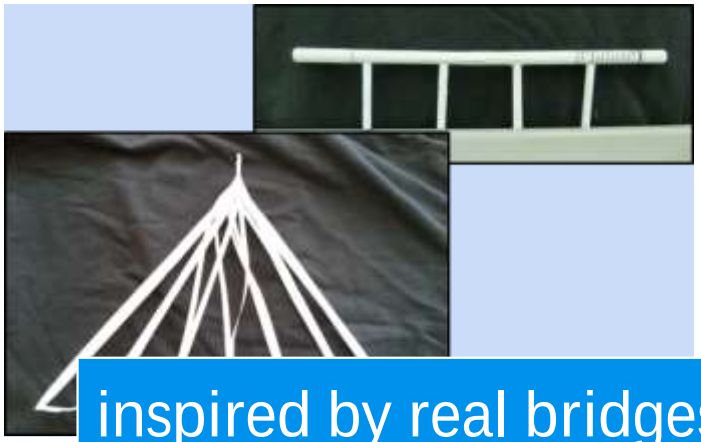
Maximal strength: with 15 mm diameter

2100 g



accordion

tube



inspired by real bridges

?

paper-unique

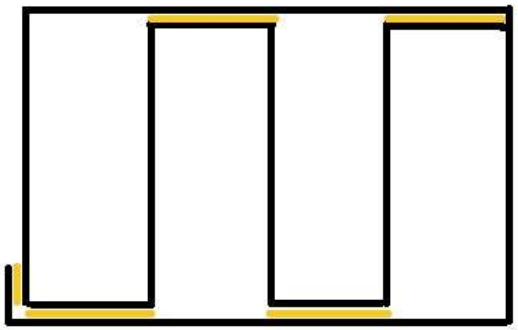
Bridges Inspired by Reality

- Different girders

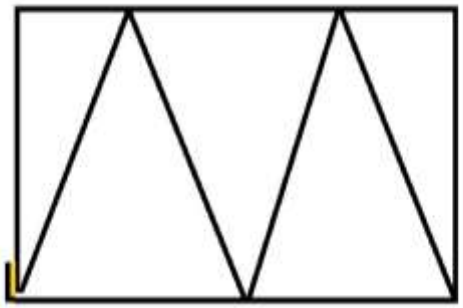
- Suspension bridge

- Pier bridge

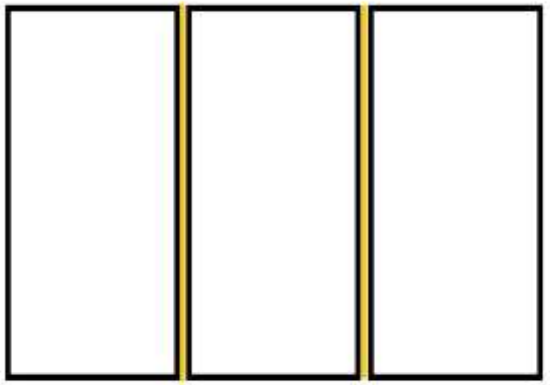
Experiments: Different Girders/Profiles



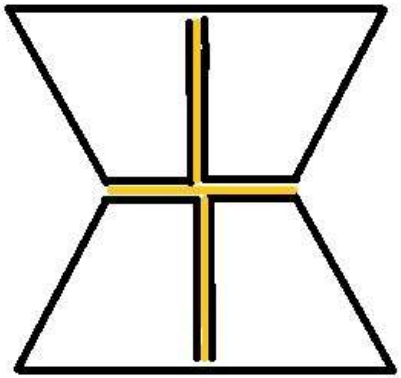
GLUE
PAPER
1025g



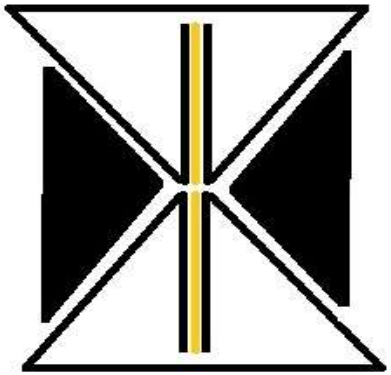
GLUE
PAPER
510g



GLUE
PAPER
640g



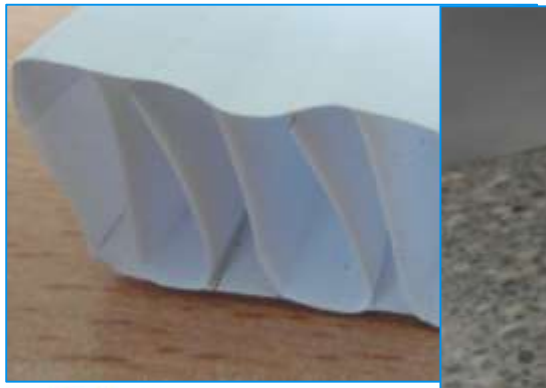
GLUE
PAPER
875g



GLUE
PAPER
490g

GLUE
PAPER
1040g

Experiments: Different Girders/Profiles

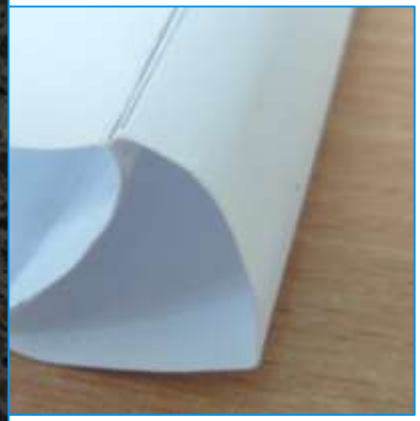


GLUE
PAPER
1025g

40g

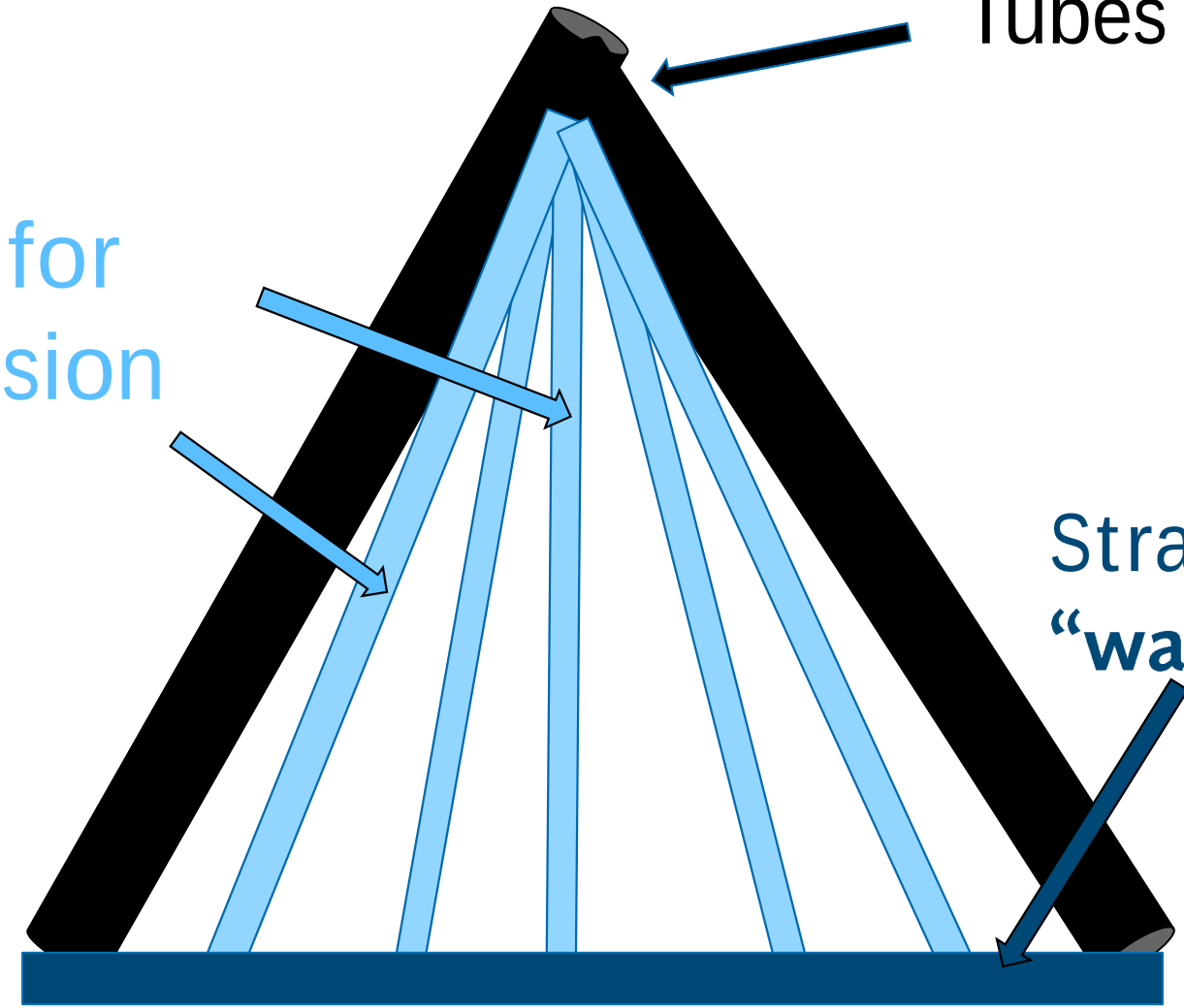
GLUE
PAPER
875g

PAPER
490g



1040g

Suspension Bridge



Suspension Bridge

limiting factor:
strength of

- supporting tubes
- suspension straps

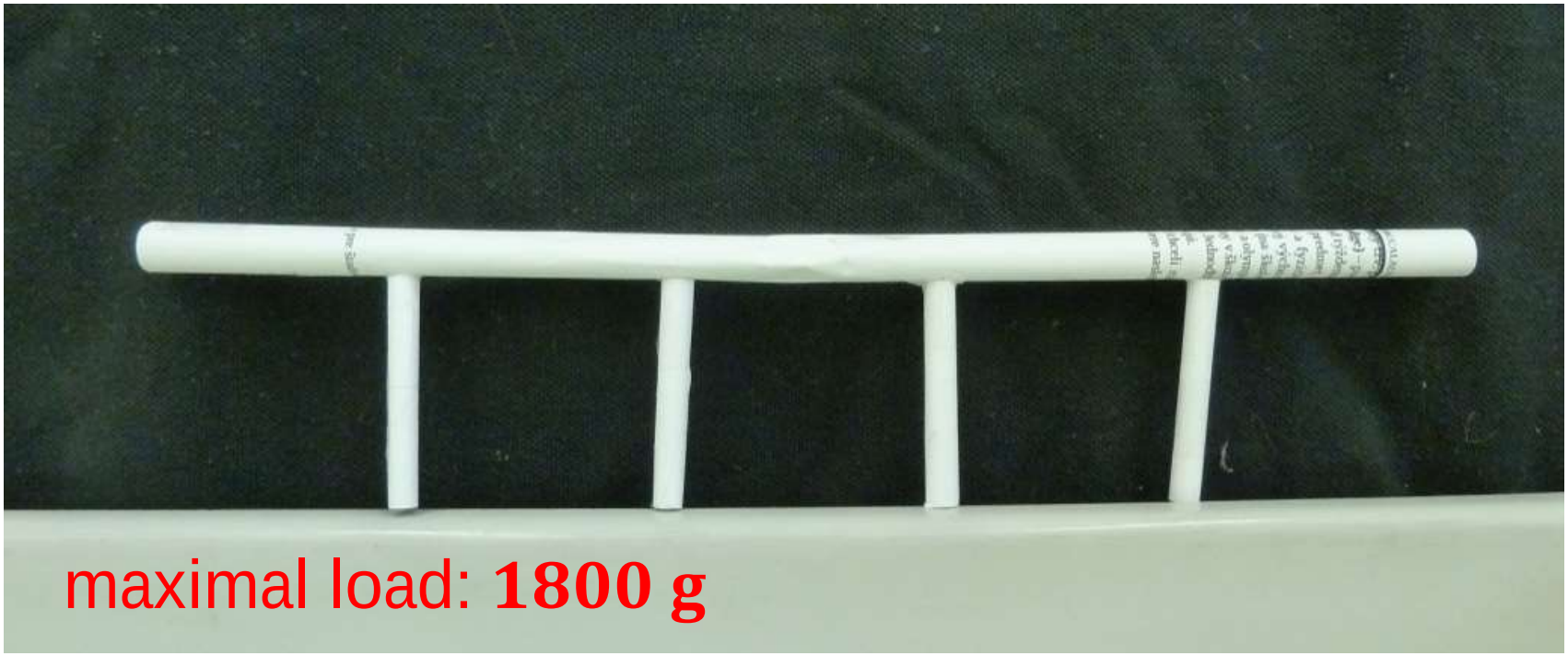
results:

	max load	
narrower straps	1000 g	*straps tore*
thinner tubes	900 g	*tubes bent*

Pier bridge

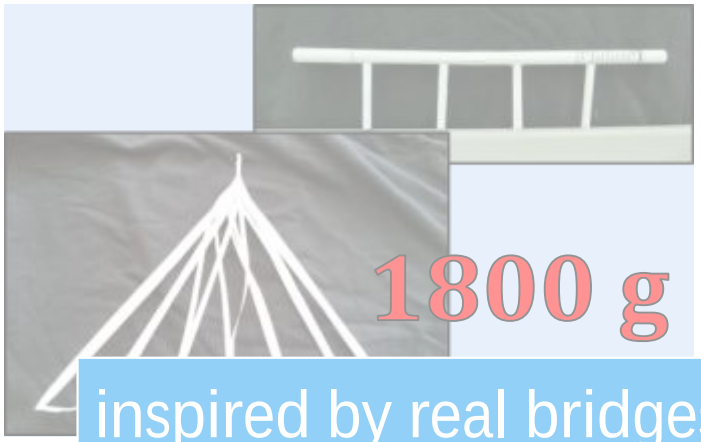
relevant parameter:

strength of the tube under bending and pressure



accordion

tube

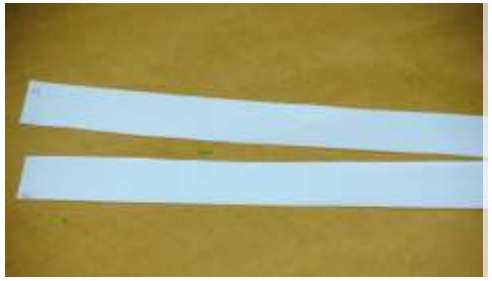
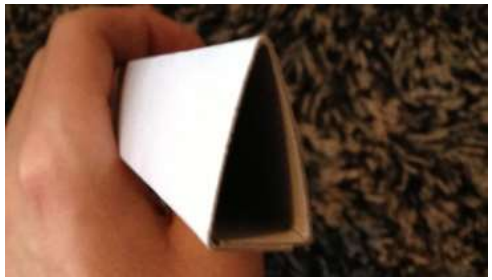


inspired by real bridges

?

paper-unique

Applicability of “Components”

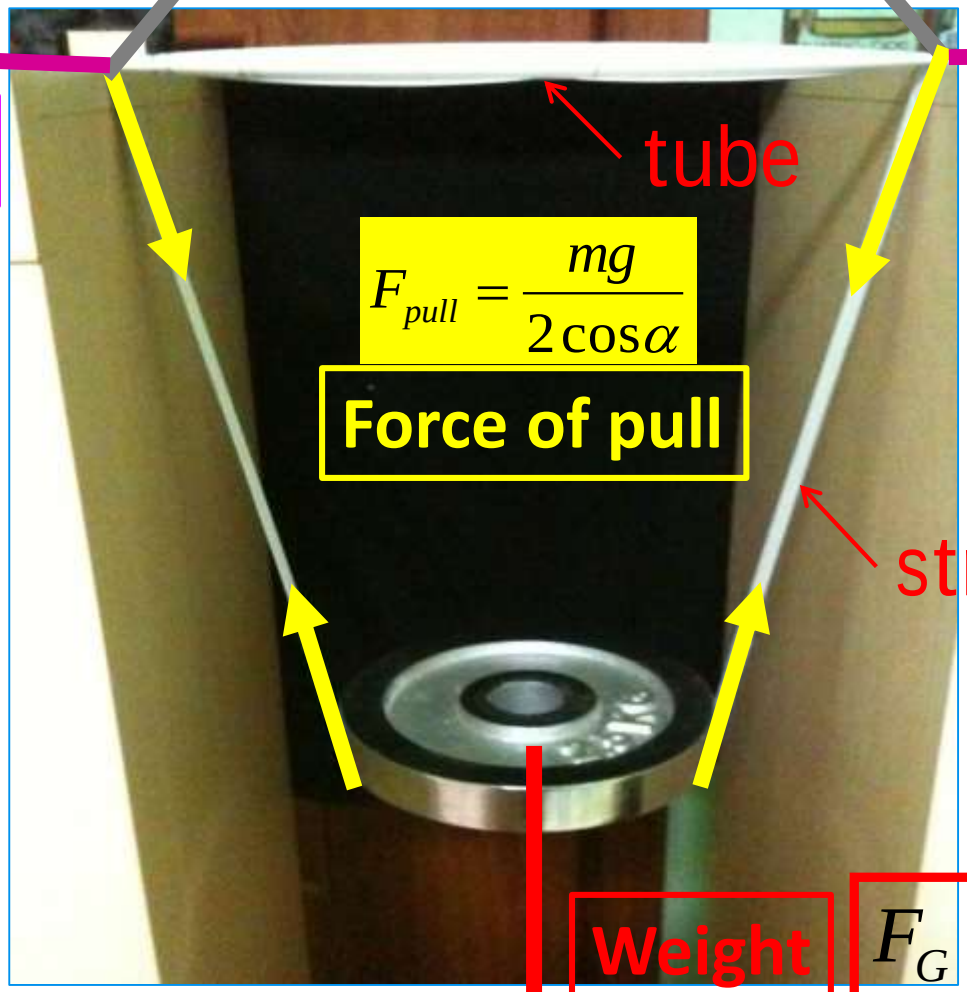
	bending	push	pull
	0 N	0 N	570 N
	21 N	100 N	570 N
	11 N	60 N	570 N

weak

weak

complex

"Hercules" Bridge Design



Normal force

Force of push

$$F_{pull} = \frac{mg}{2\cos\alpha}$$

Force of push

$$F_{pull} = \frac{mg}{2\cos\alpha}$$

tube

$$F_{pull} = \frac{mg}{2\cos\alpha}$$

Force of pull

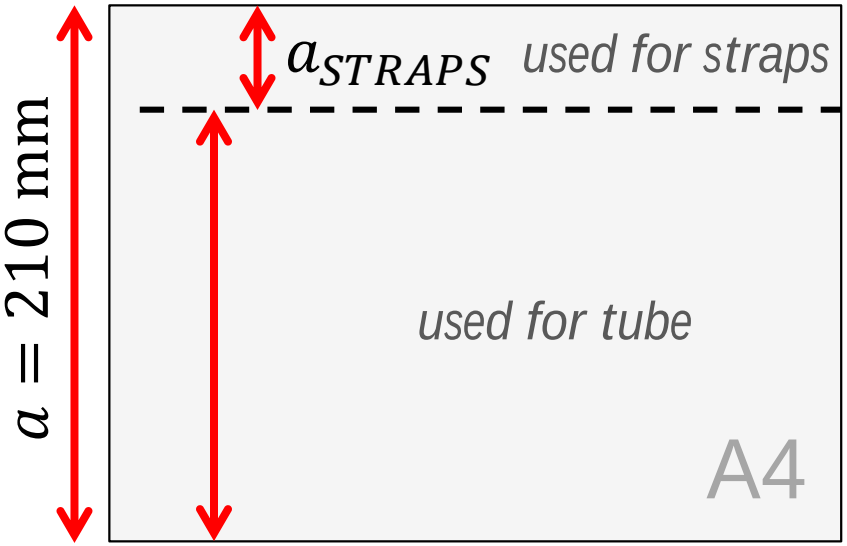
strap

Weight

$$F_G = mg$$

Hercules: Free Parameters

- % of paper used for straps/tube



- angle of straps α

Hercules: Optimizing the Design

straps:

$$\frac{mg}{2\cos\alpha} \leq wtG$$

tube:

$$\frac{mg}{2\cos\alpha} \leq N_{LAYERS}K$$

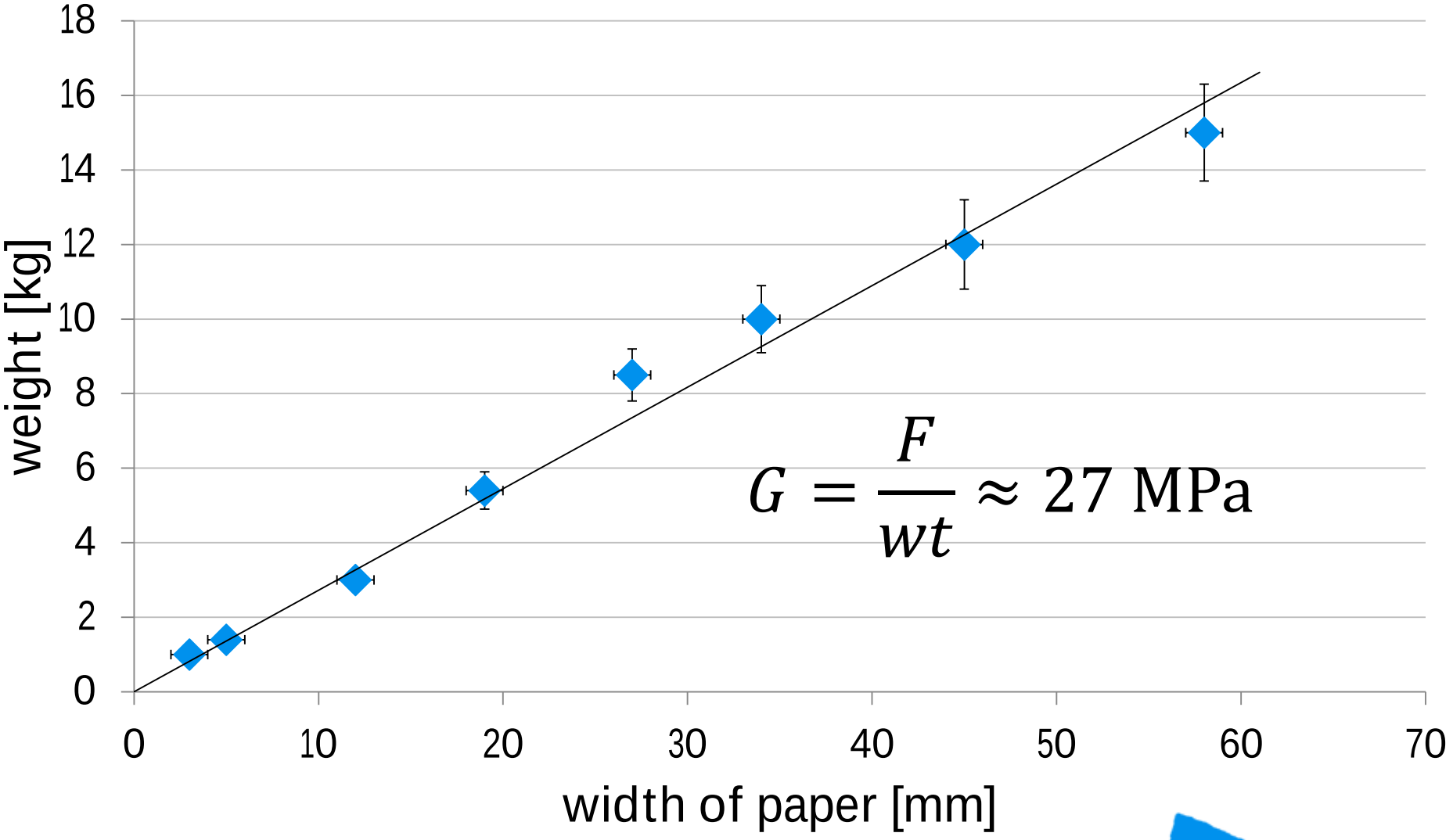
t: paper thickness
w: width of the strap
G: paper tensile strength
K: strength of a single tube layer

Tensile Strength of Paper Straps

no precise material constants known

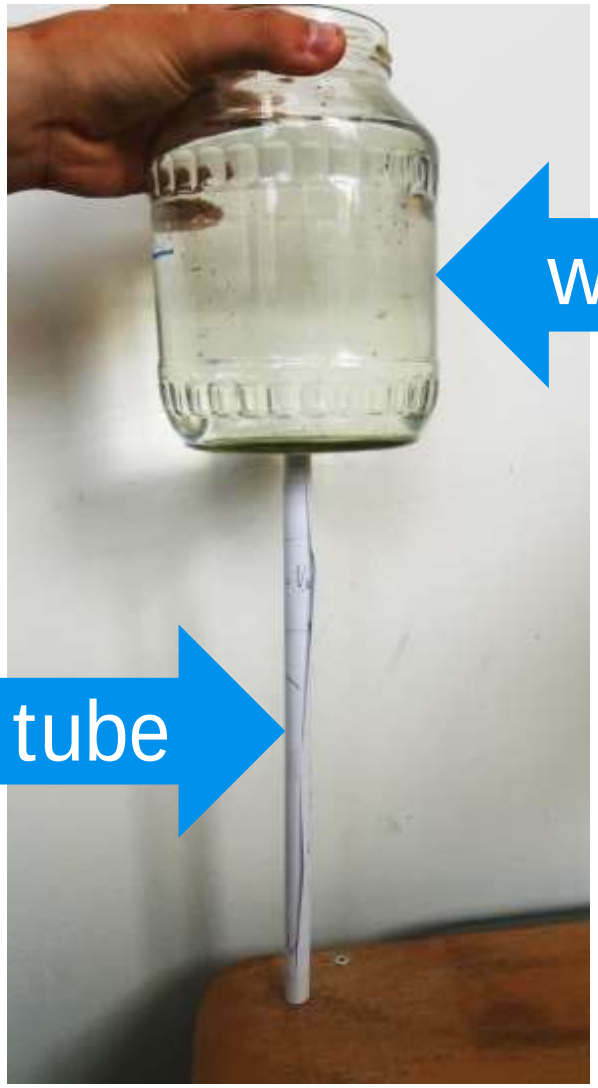
- *experiment*: changing strap width, measuring max load

Tensile Strength of Paper Straps

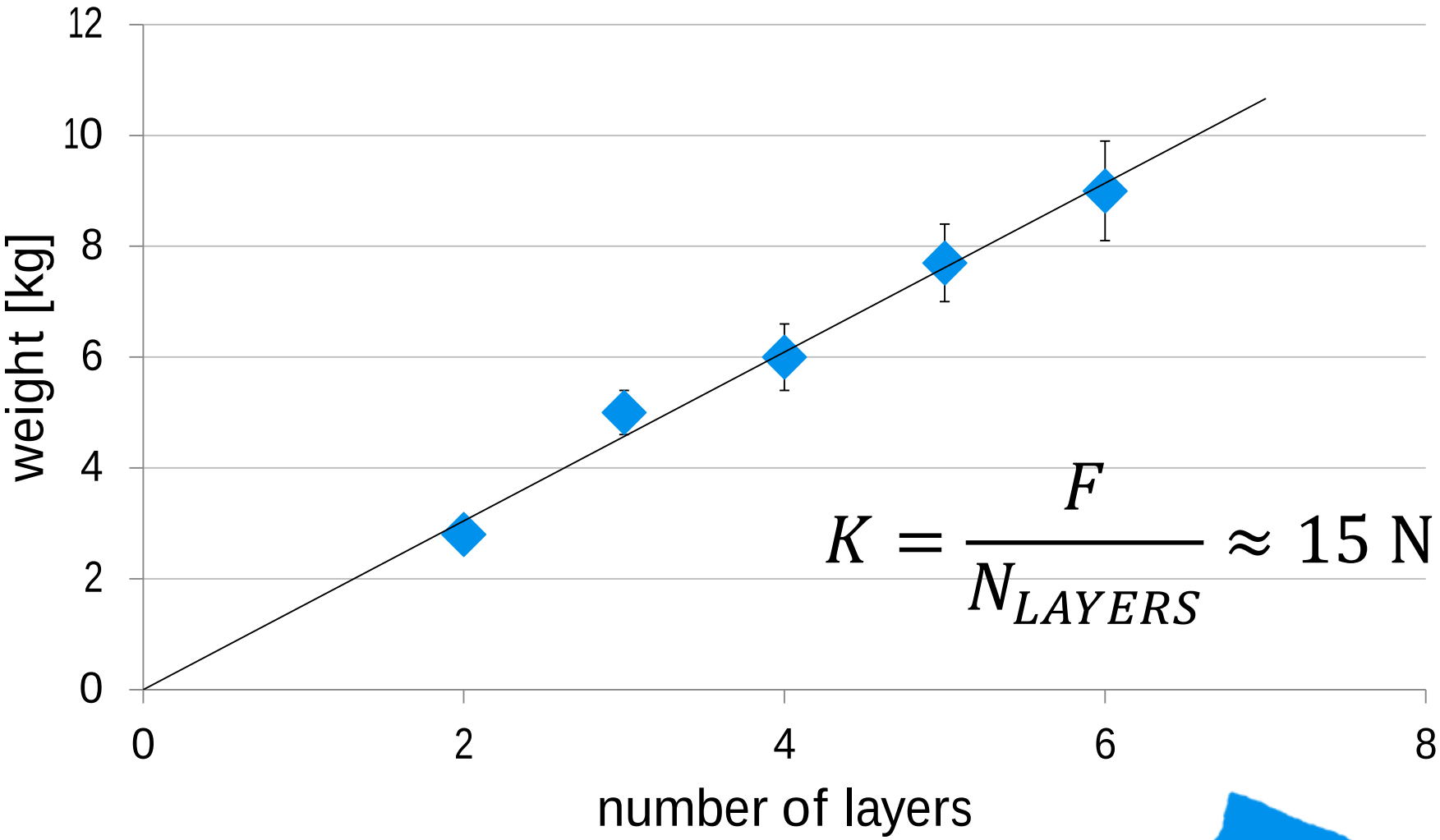


$$G = \frac{F}{wt} \approx 27\ \text{MPa}$$

Strength of a Single-Layer Tube

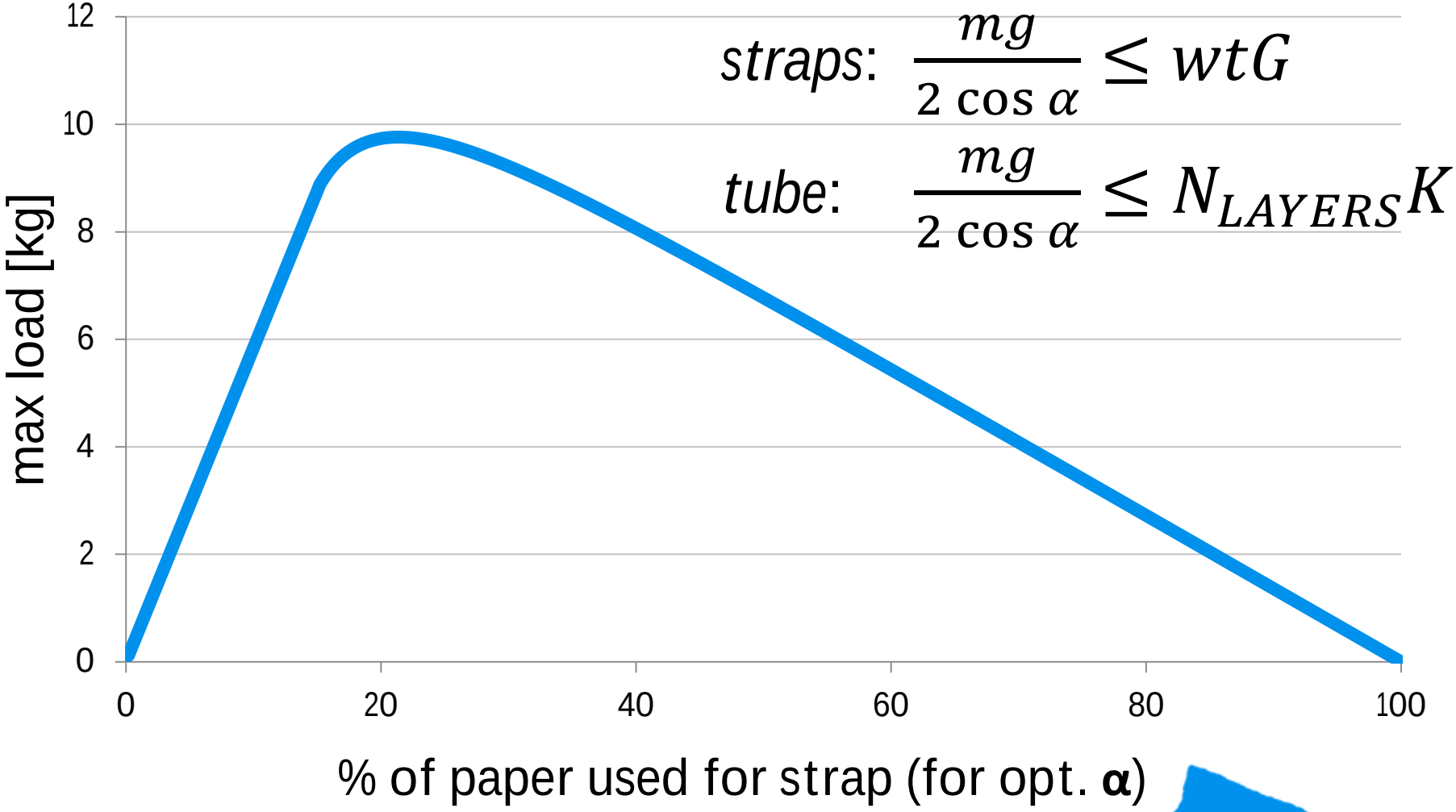


Strength of a Single-Layer Tube

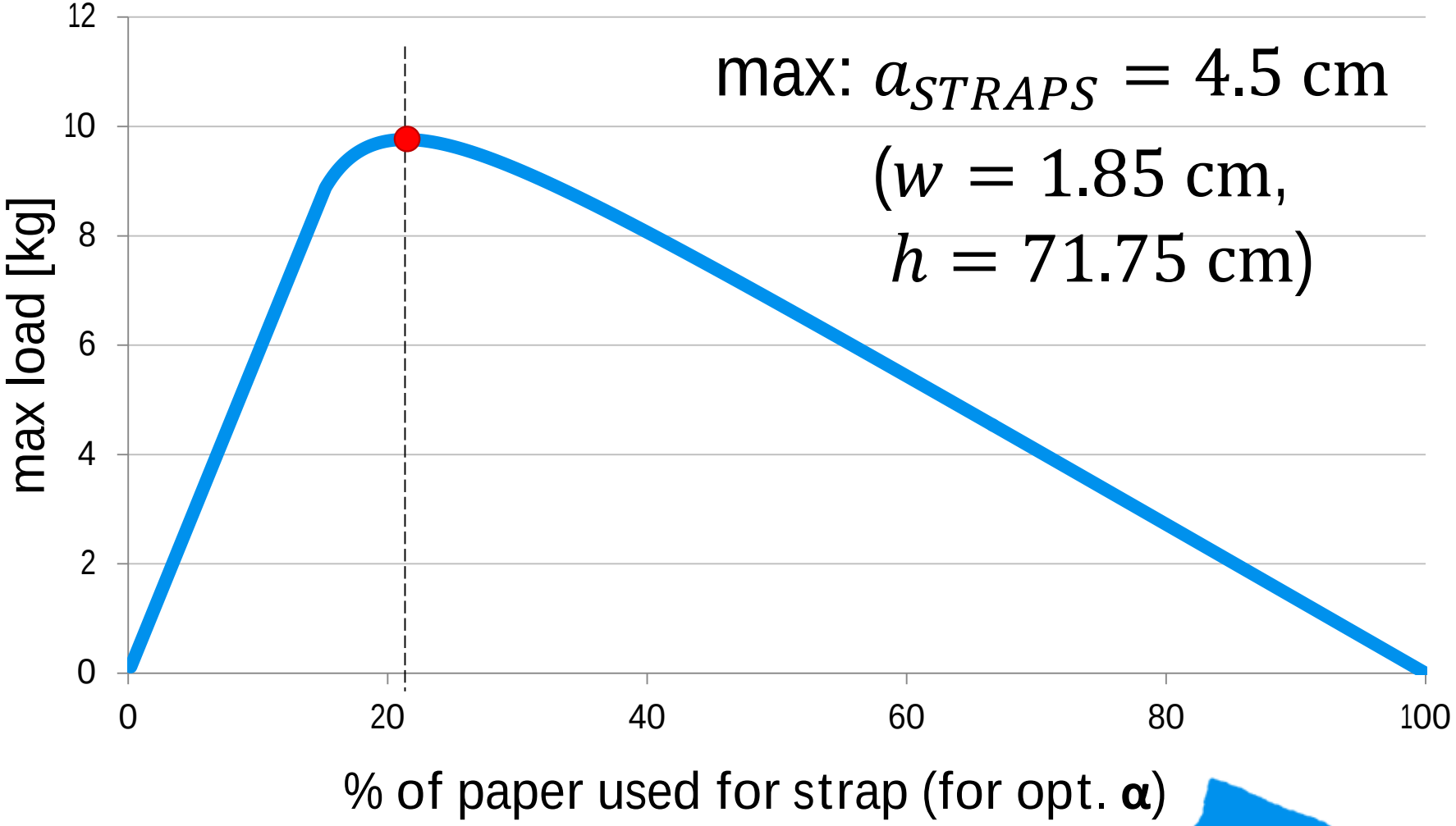


$$K = \frac{F}{N_{LAYERS}} \approx 15\ \mathrm{N}$$

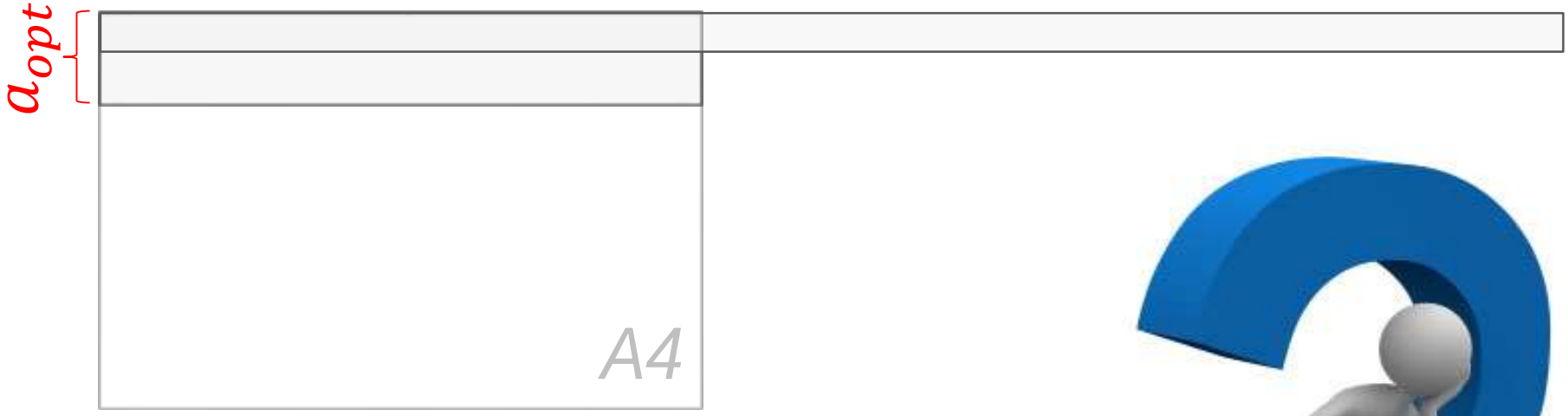
Hercules: Strength Prediction



Hercules: Strength Prediction

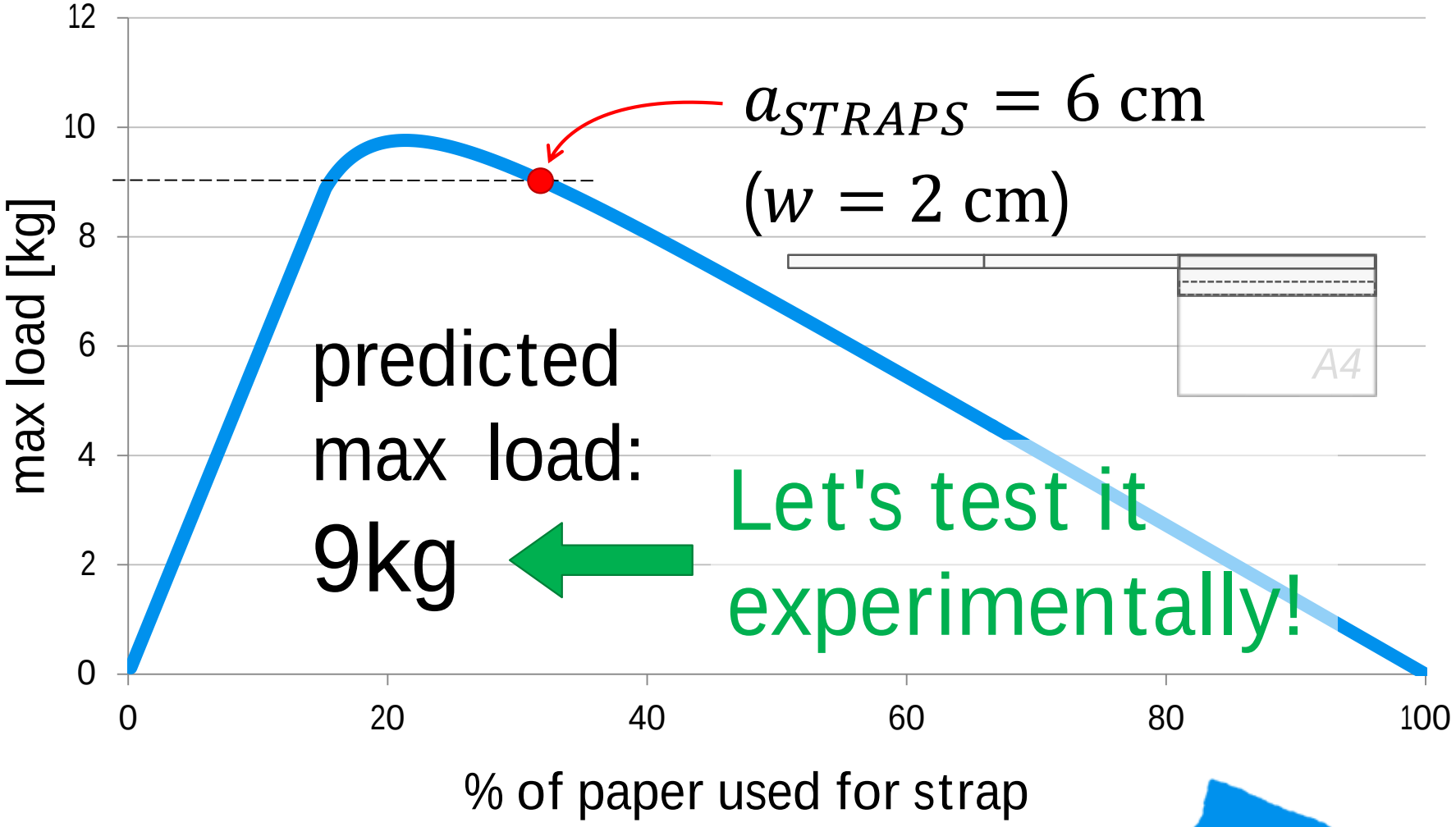


How to make a 1.85 × 71.75 cm strap out of a 4.5 × 29.5 cm paper ?



→ we chose best realizable parameters instead

Hercules: Strength Prediction



5 kg

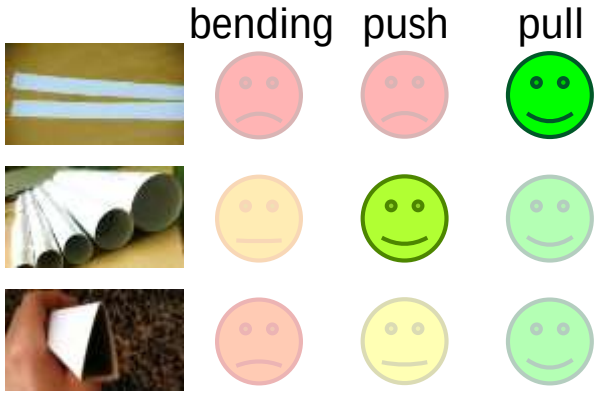
7.5 kg

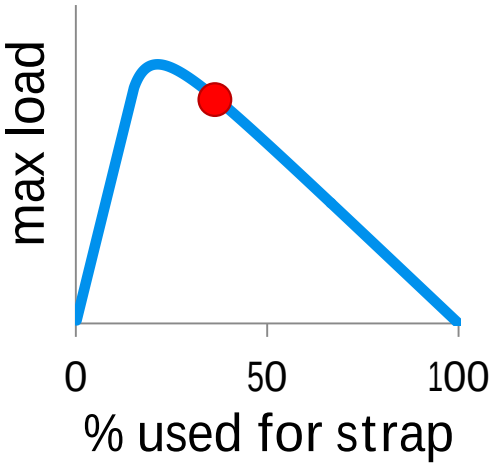
8.75 kg!

9 kg

Hercules: Summary

Applicability of "Components"

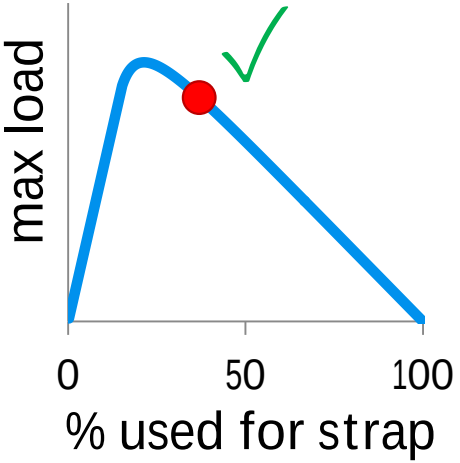




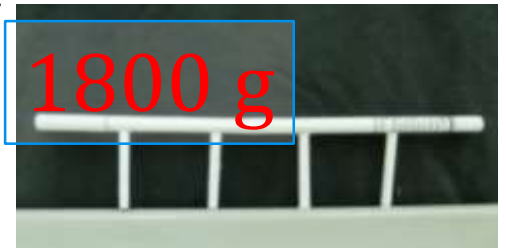
Conclusion

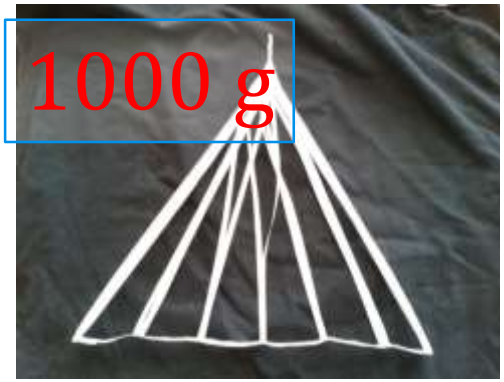
simple: theoretically explored, max explained

Hercules



inspired by reality

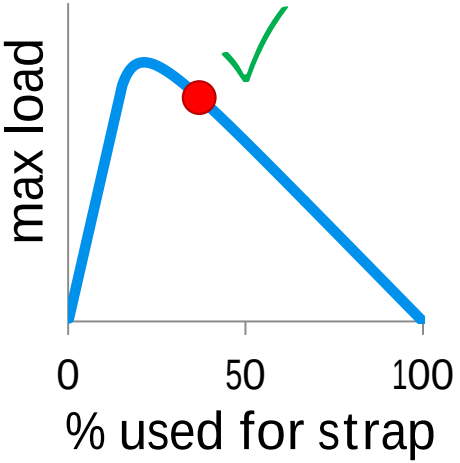




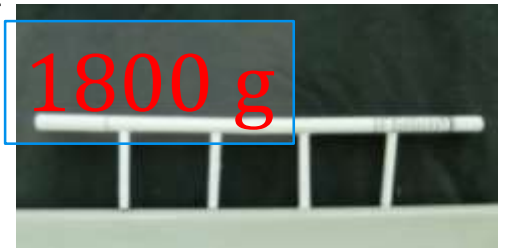
Thank you for your attention!

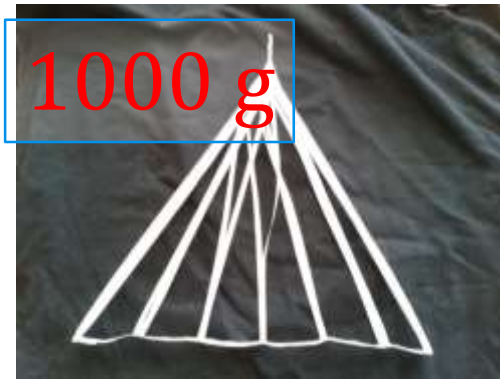
simple: theoretically explored, max explained

Hercules



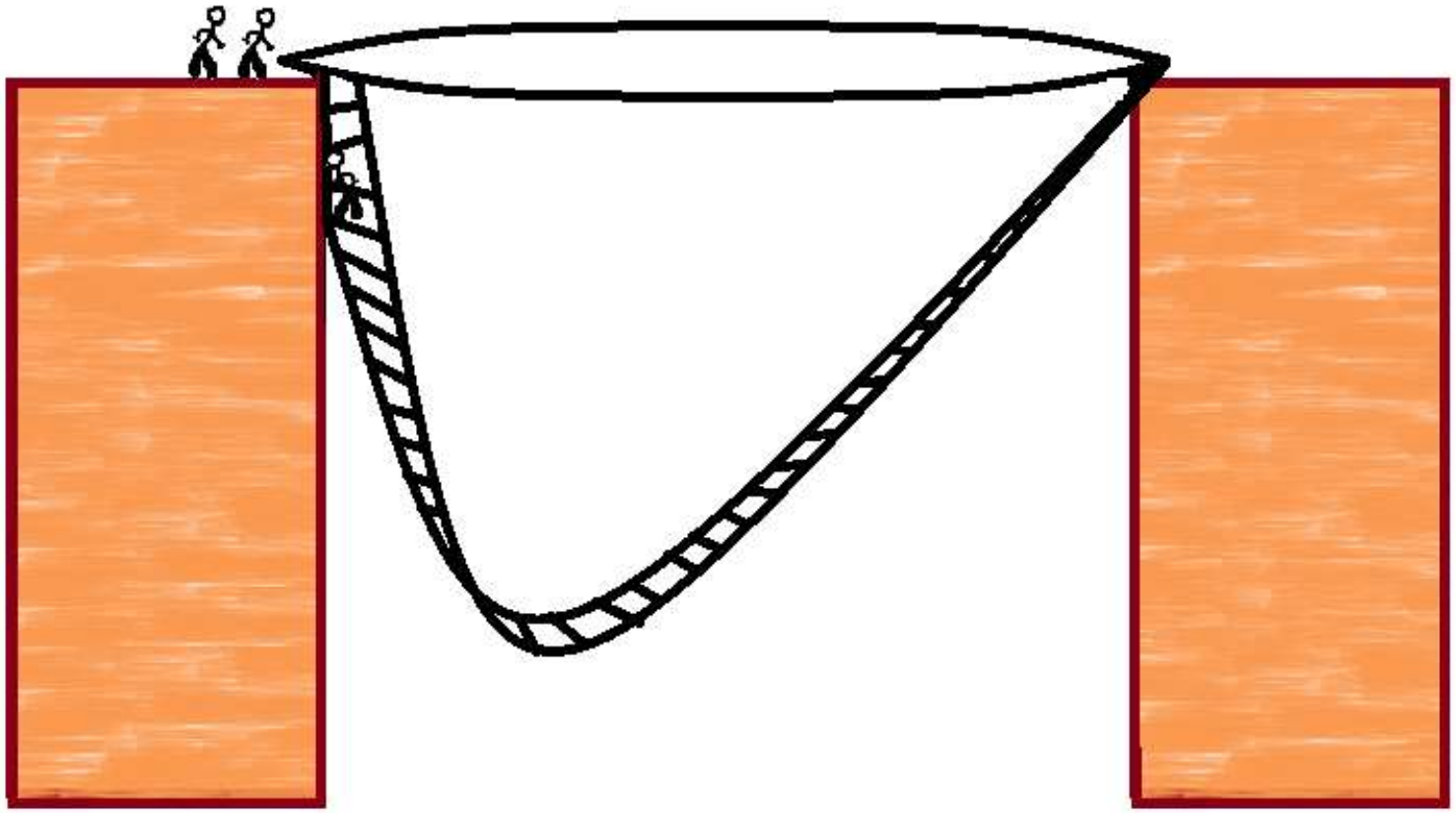
inspired by reality



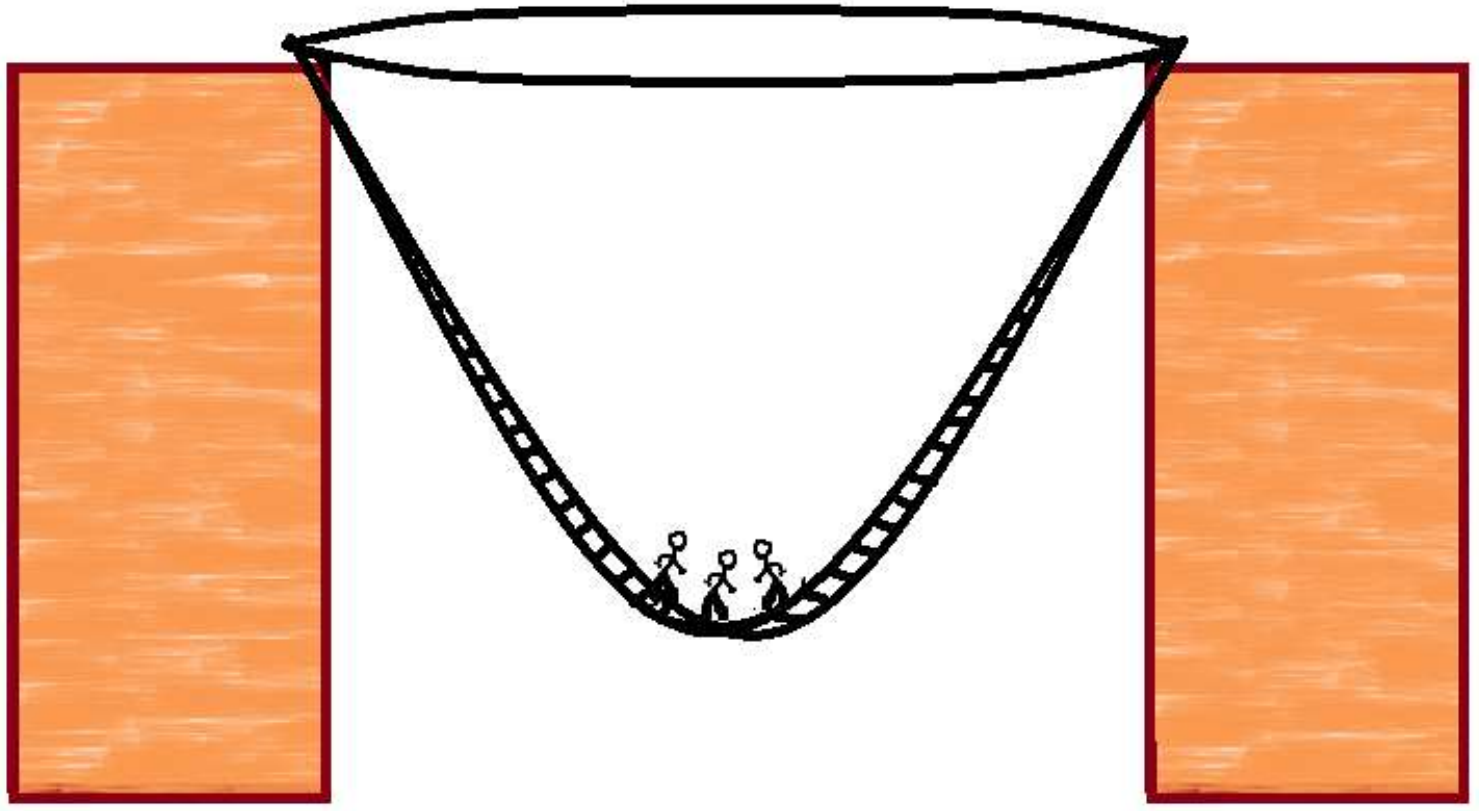


APPENDICES

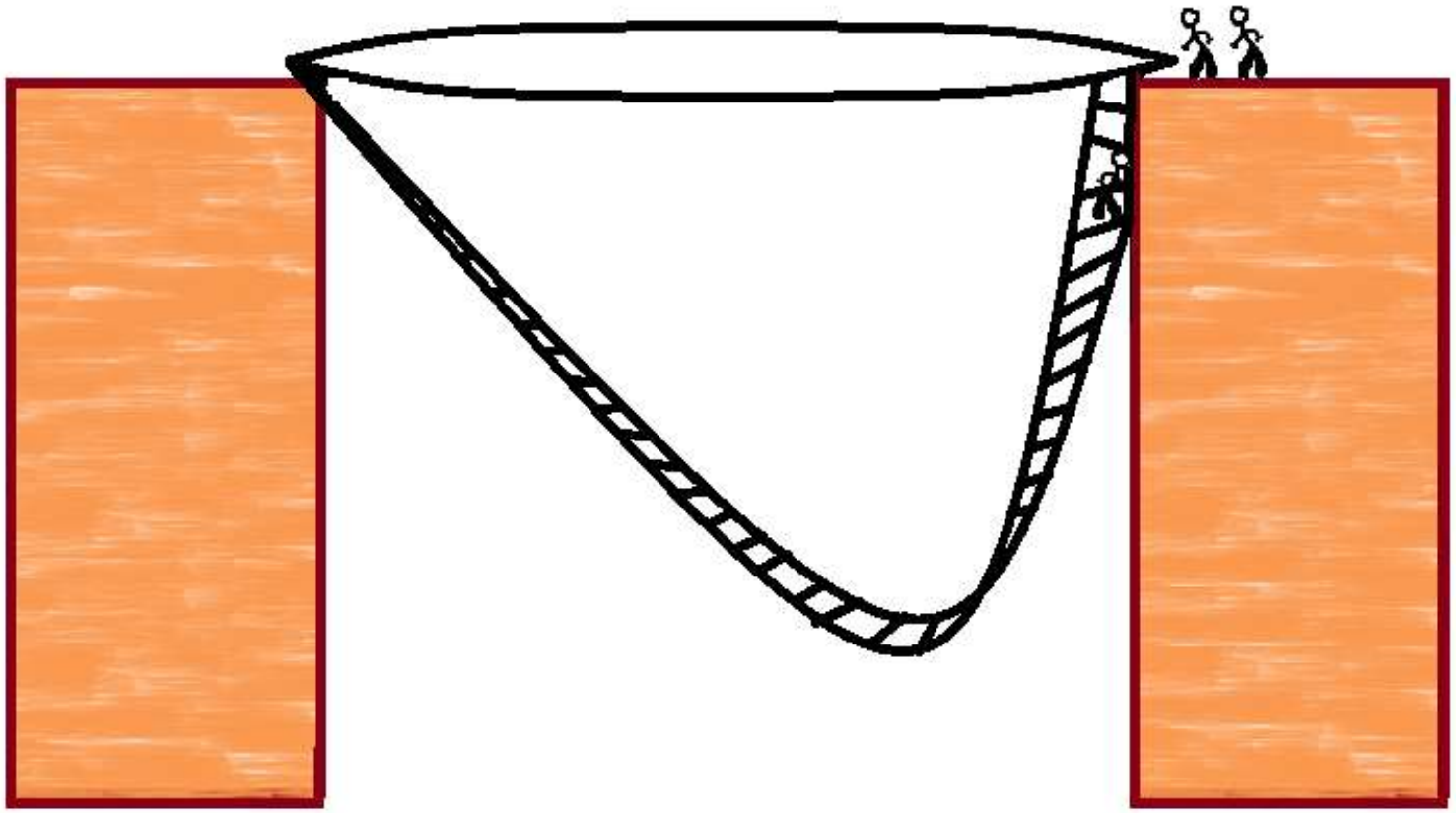
Hercules: User's Guide #1



Hercules: User's Guide #1

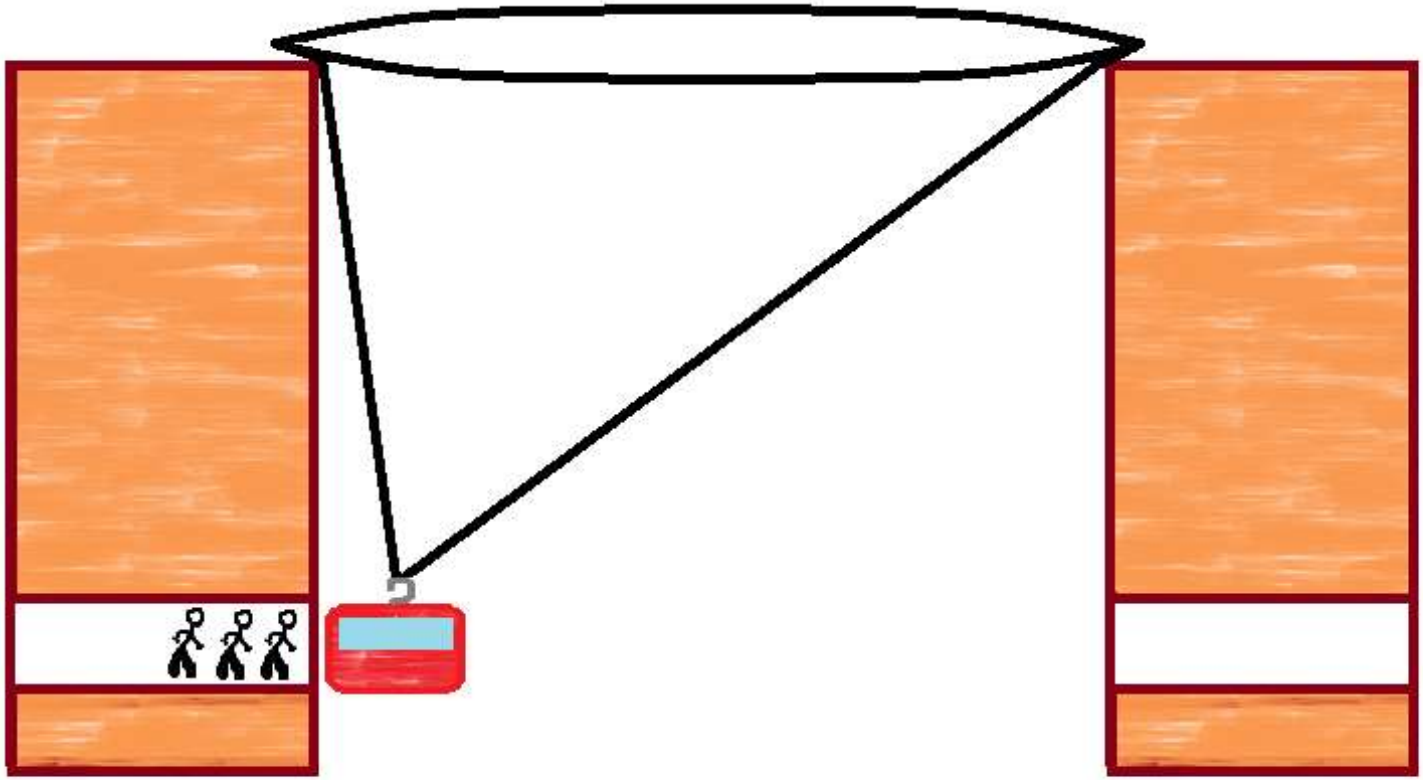


Hercules: User's Guide #1



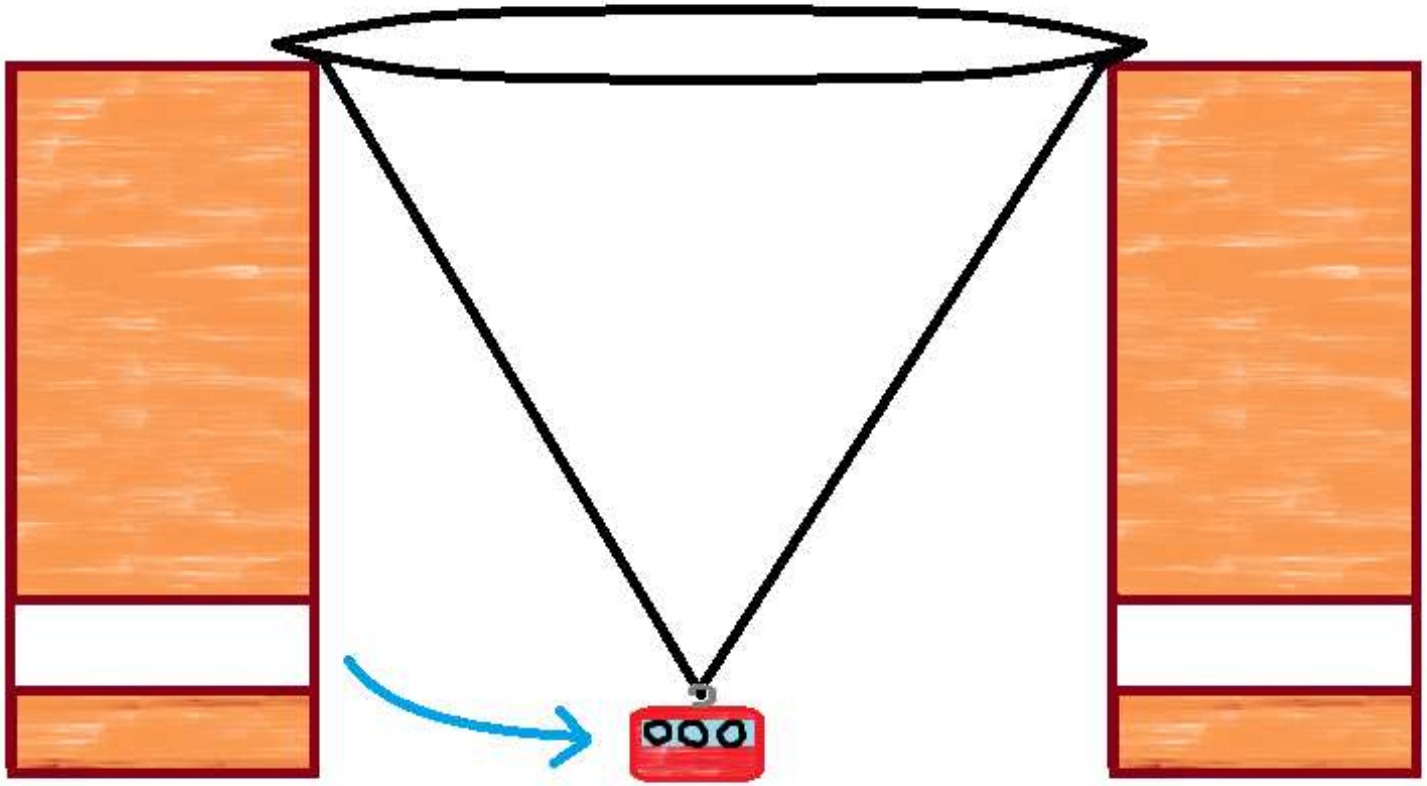
Hercules: User's Guide #2

Passenger ropeway!



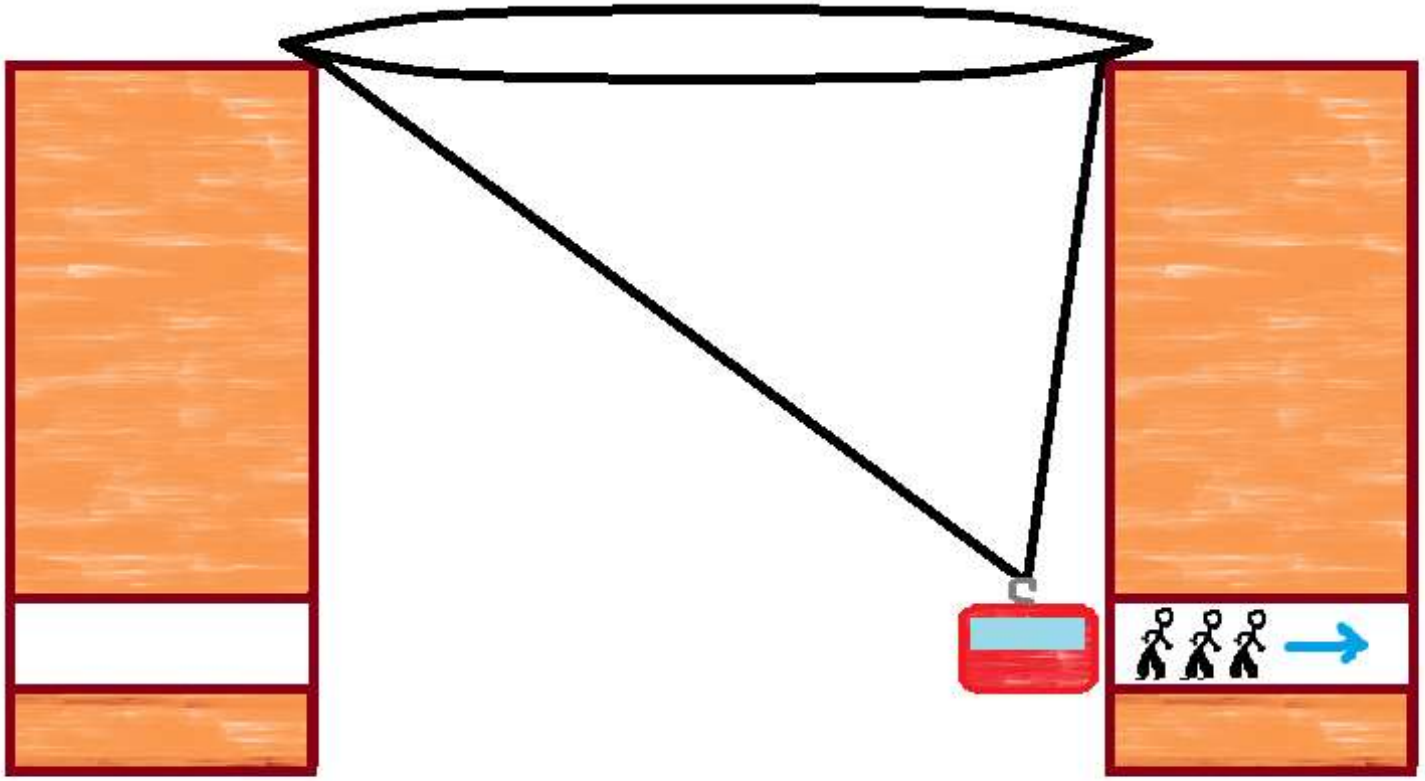
Hercules: User's Guide #2

Passenger ropeway!

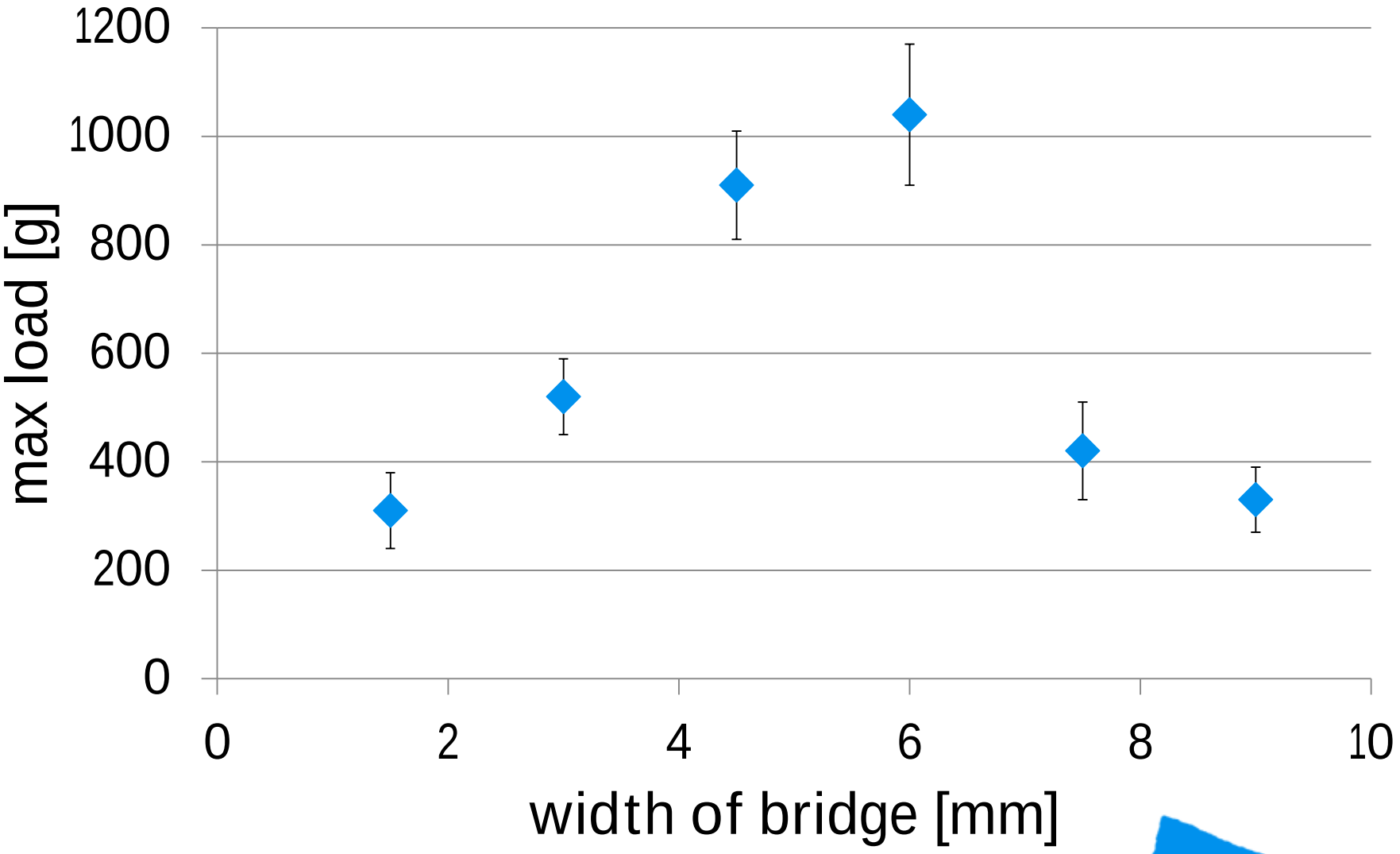


Hercules: User's Guide #2

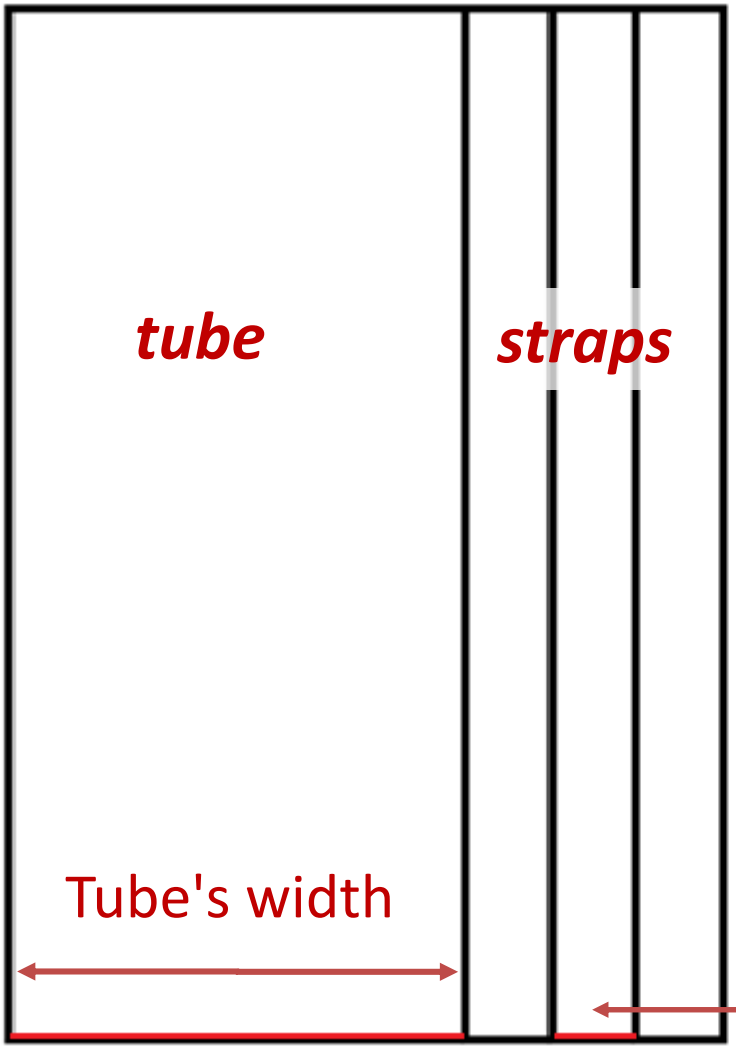
Passenger ropeway!



Strength of the 3-prism bridge



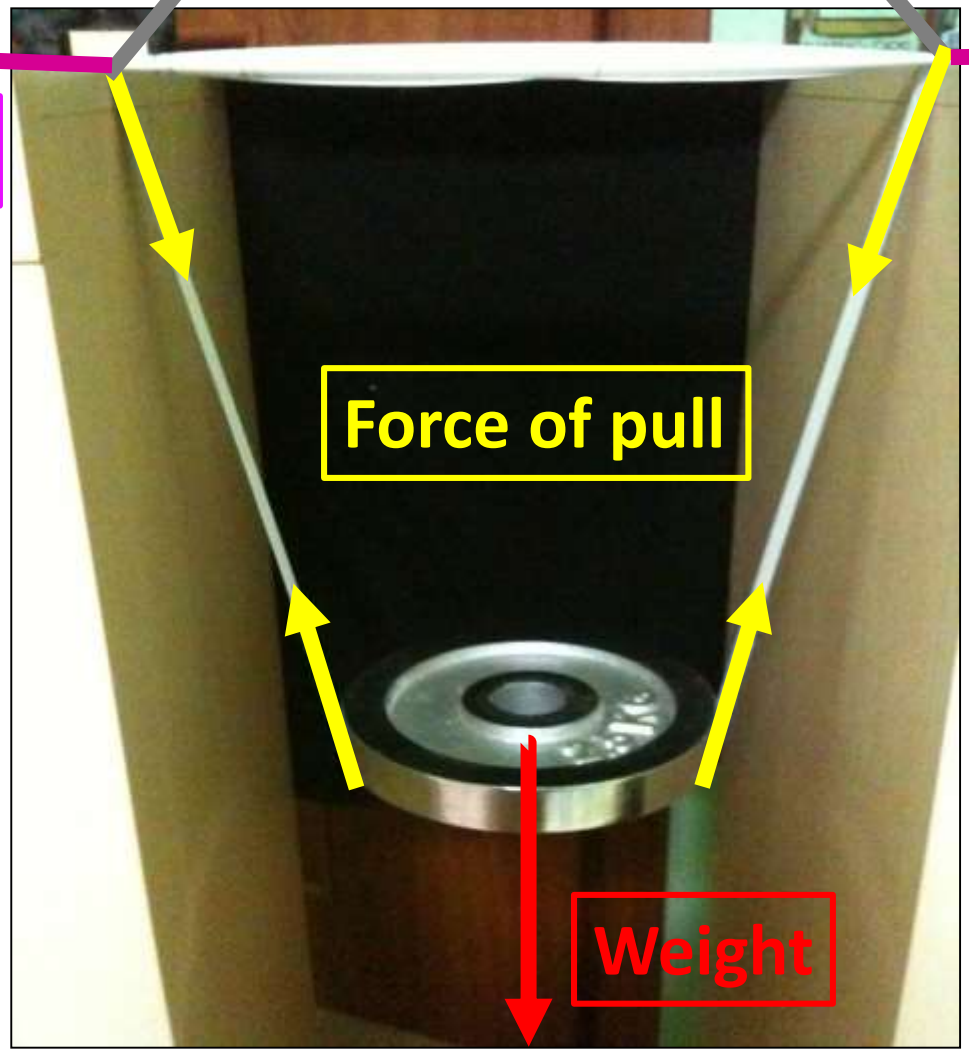
Construction of Hercules



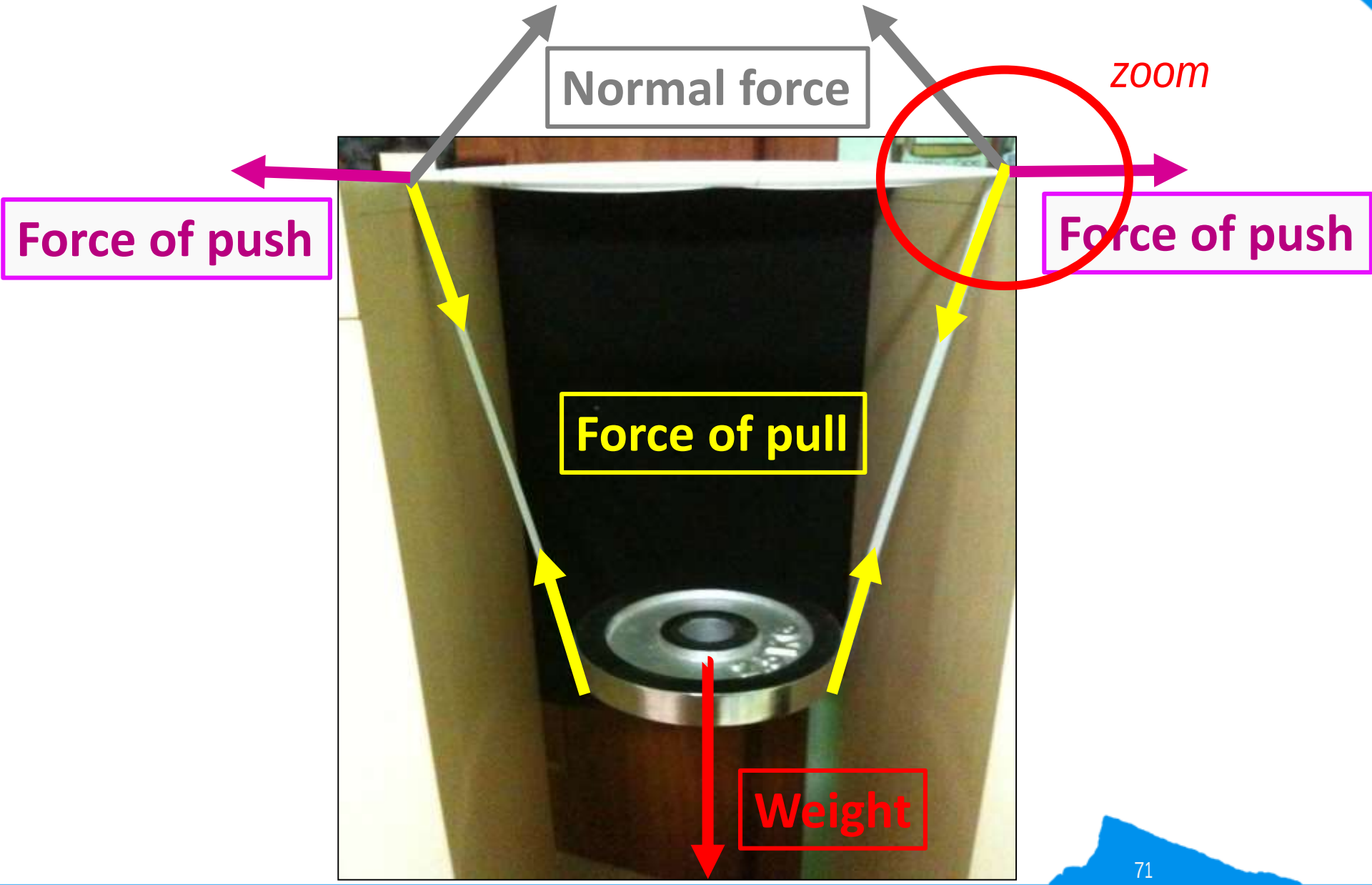
- sticking more straps into a longer one

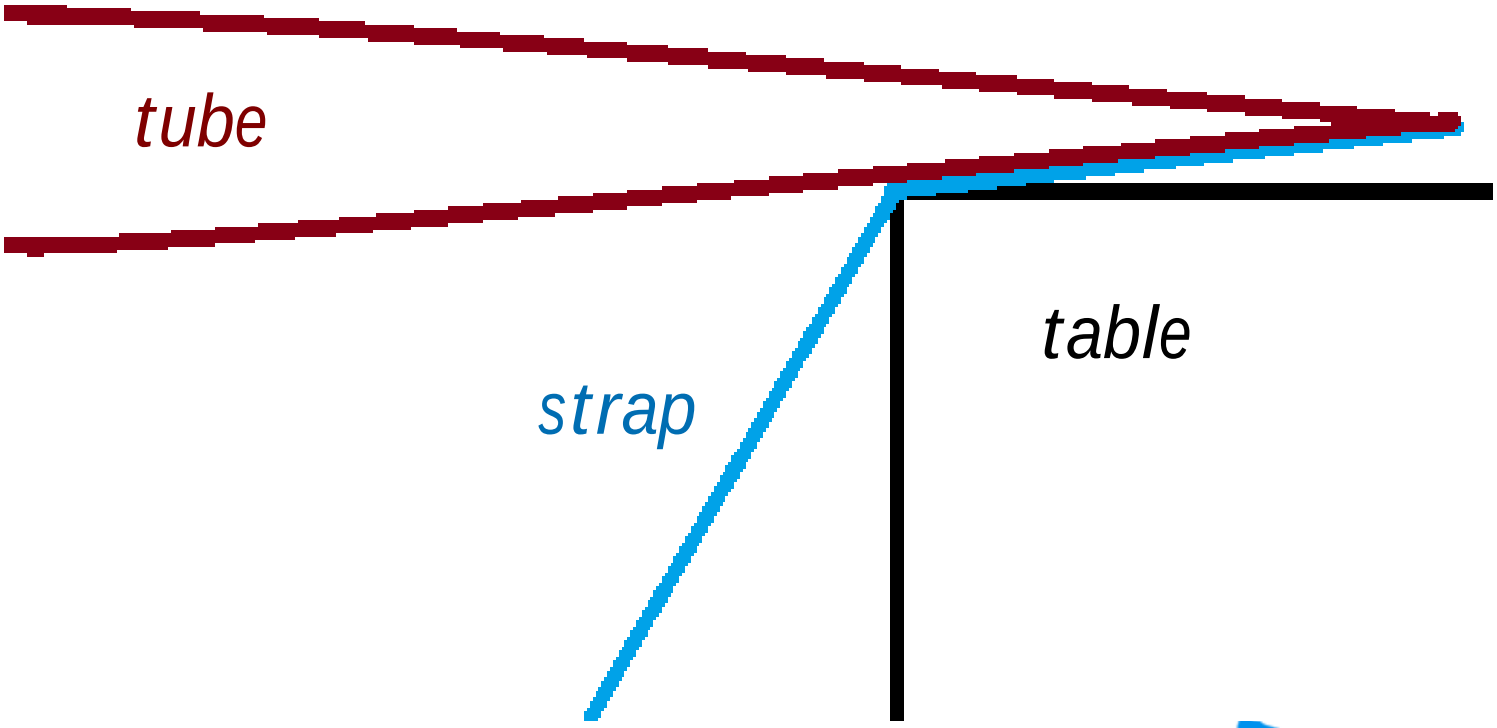
Width of the final strap

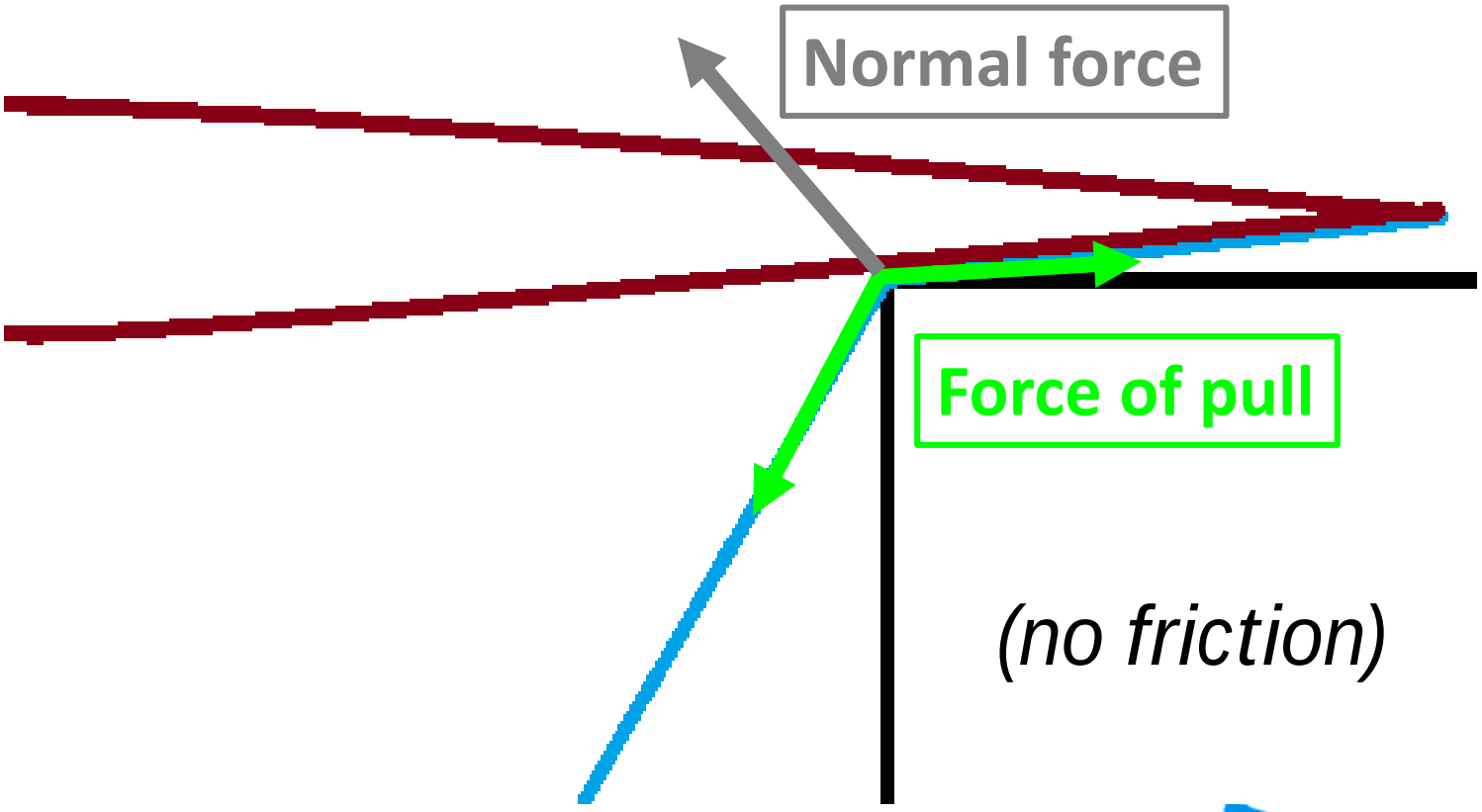
Forces in Hercules

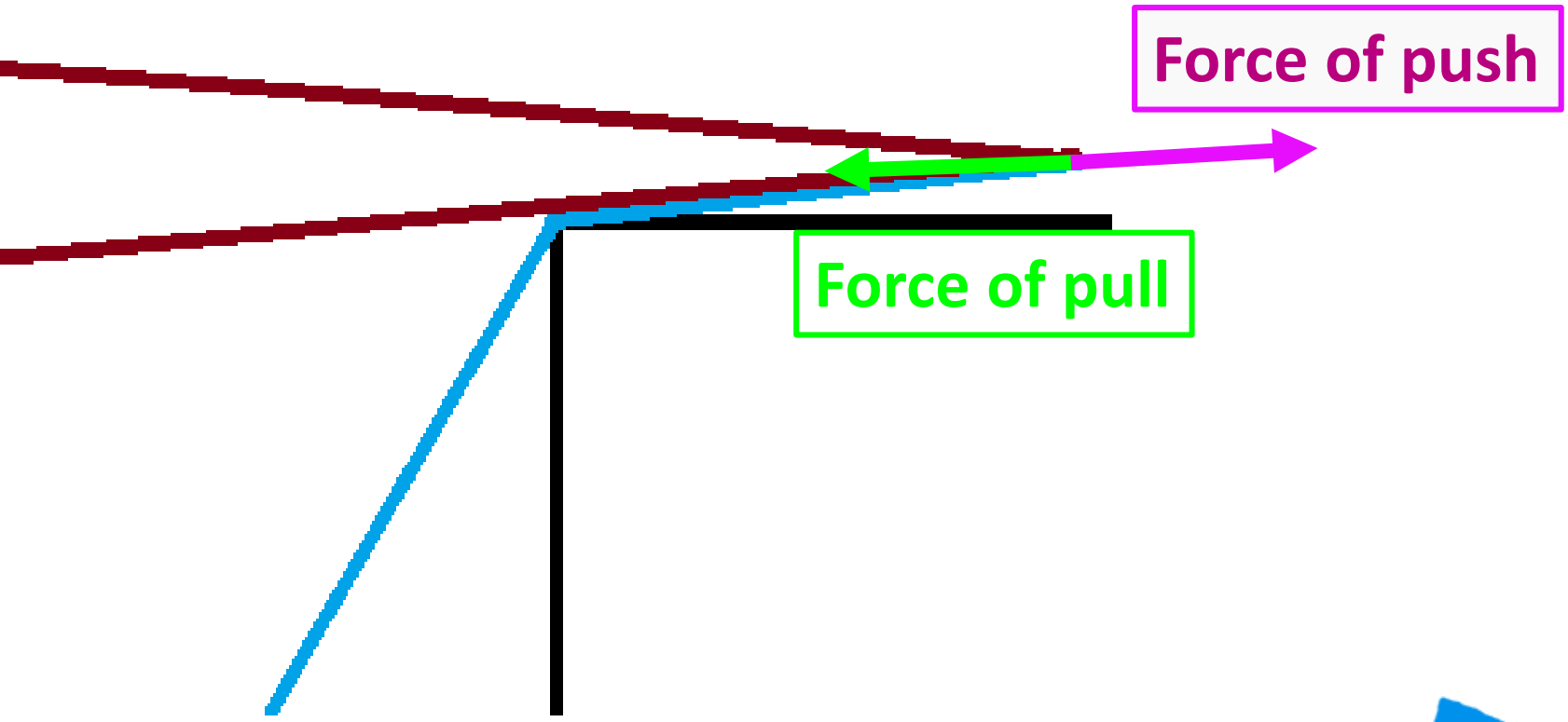


Forces in Hercules

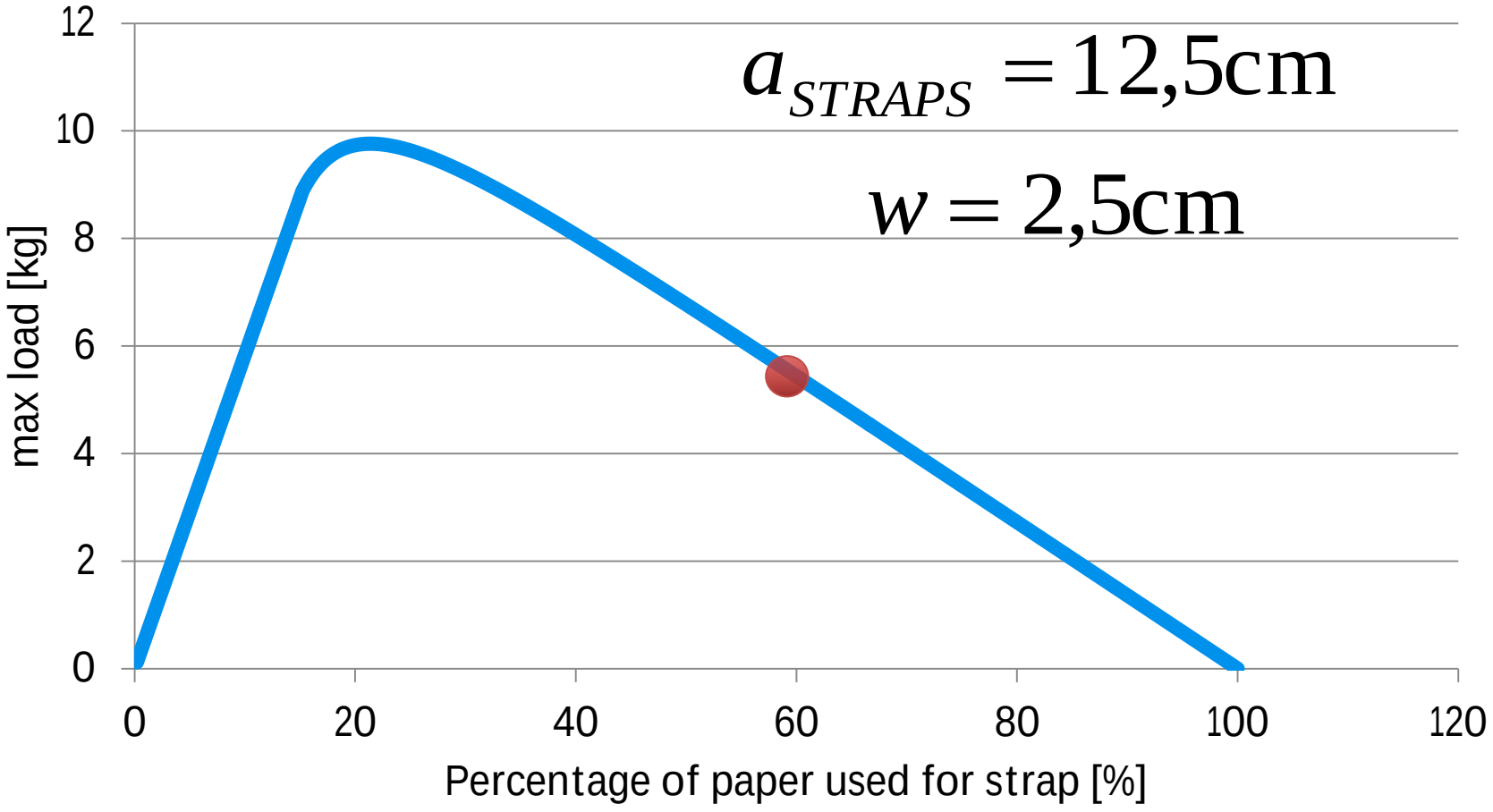



tube
strap
table


Normal force
Force of pull
(no friction)

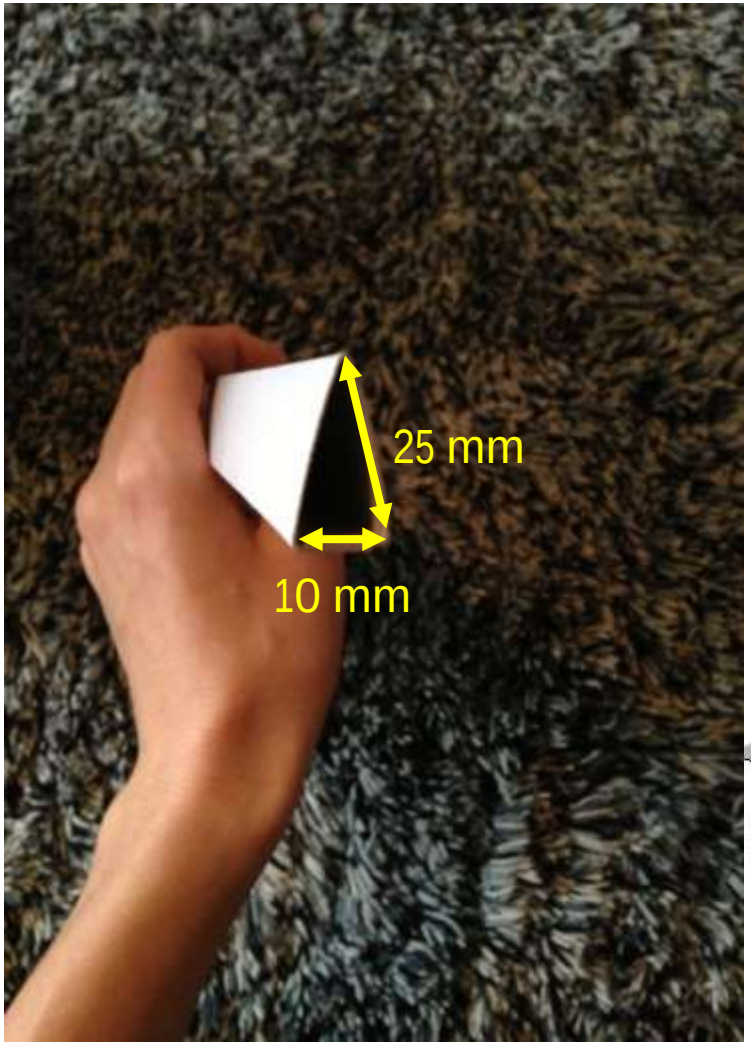

Force of push
Force of pull

Hercules: Other Experiment



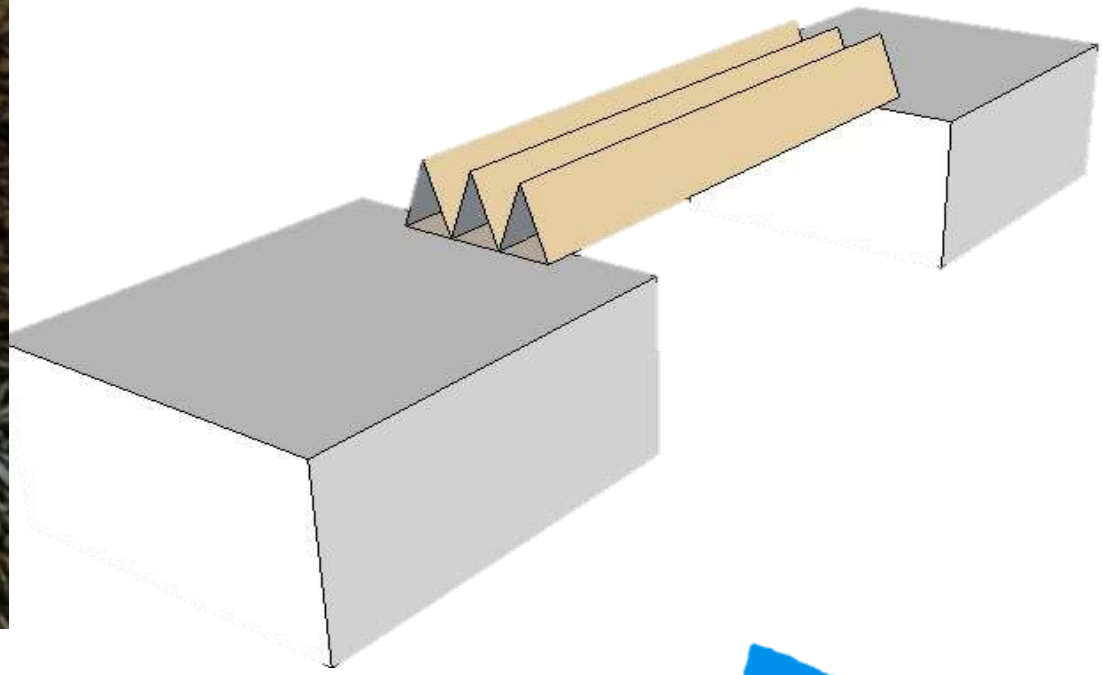
Hercules: Other Experiment

Different paper parameters



$$Areal\ density - 390\ g/m^2$$

$$m_{max} \leq 8{,}5\ kg$$



Different paper parameters

$$Areal\ density \leq 149\ g/m^2$$

Accordion with 10 dents (10 supporting points)

Maximal load: 750g