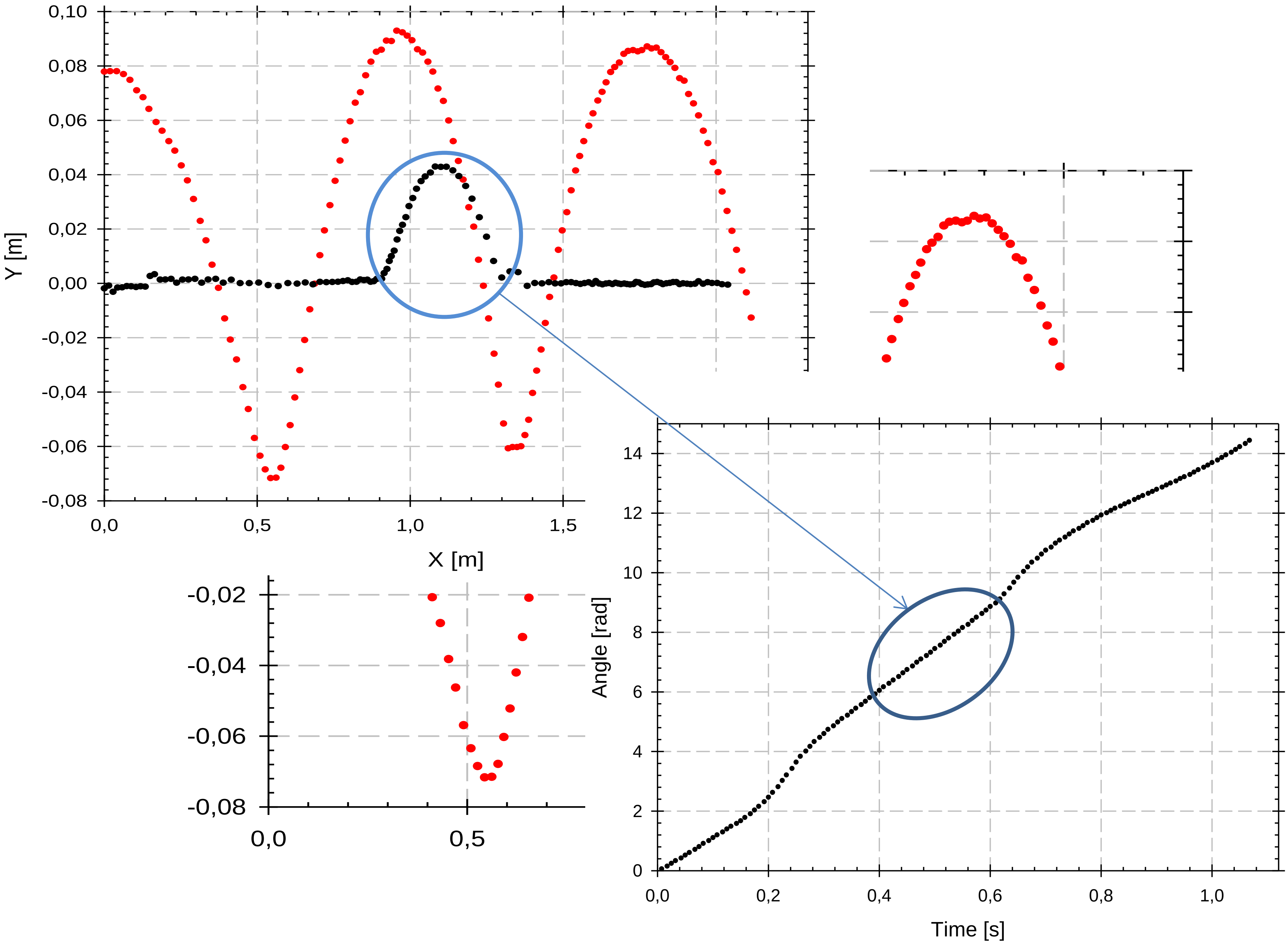
5. LOADED HOOP

"Fasten a small weight to the inside of a hoop and set the hoop in motion by giving it an initial push.

Investigate the hoop's motion."

Example of motion - video


0,10
0,08
0,06
0,04
0,02
0,00
-0,02
-0,04
-0,06
-0,08
Y [m]
0,0
0,5
1,0
1,5
X [m]
-0,02
-0,04
-0,06
-0,08
0,0
0,5
14
12
10
8
6
4
2
0
Angle [rad]
0,0
0,2
0,4
0,6
0,8
1,0
Time [s]

Outline

Experimental approach

- Hoop properties
- Giving initial push
- Parameters

↓

Theoretical modeling

- Kinematics and dynamics
- Modes of motion

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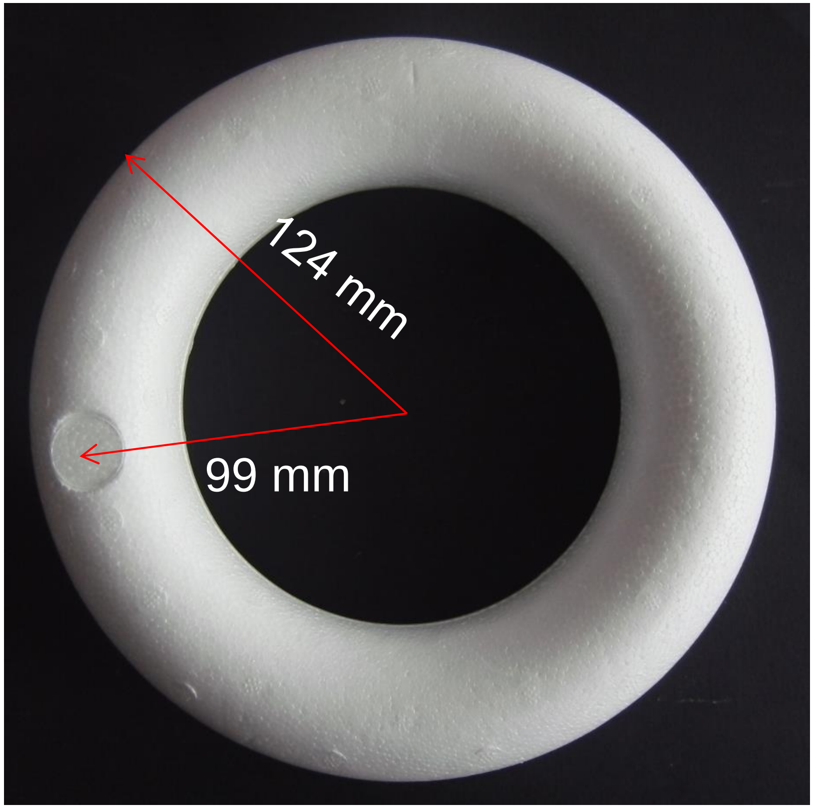
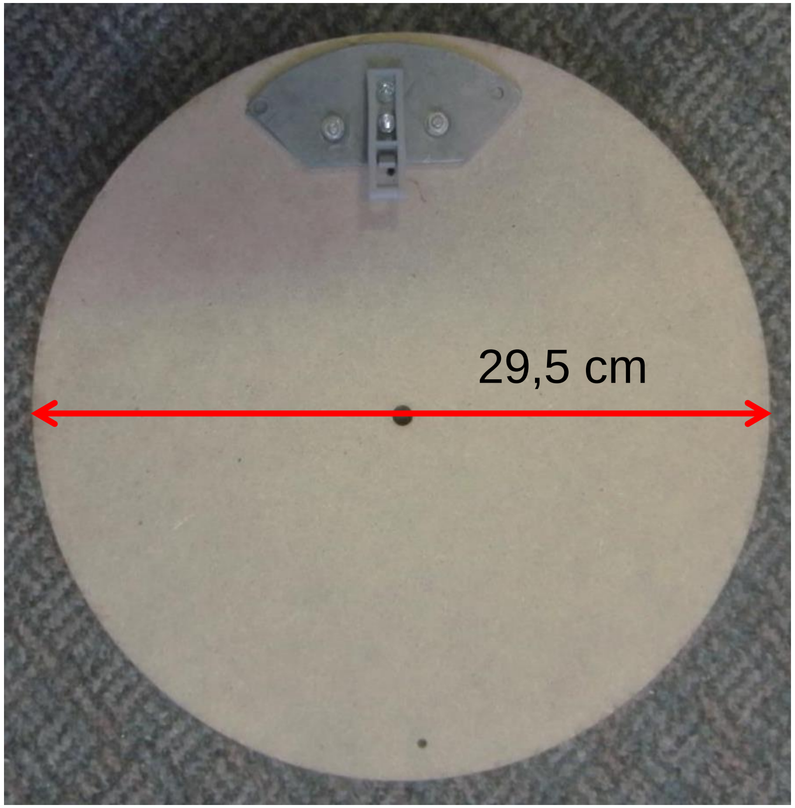
Comparison of experimental data and theory

- Modes of motion
- Dependence of angular frequency on angle for different weight mass
- Hop analysis
- Energy analysis

Hoop properties



$m_h = 24.4g$



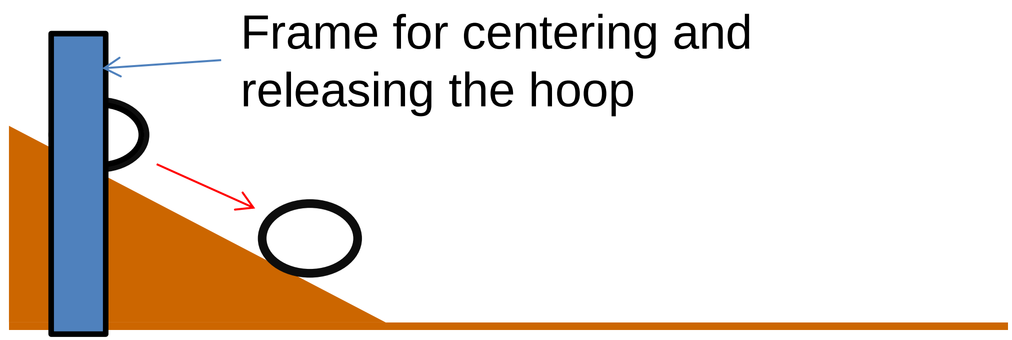
$m_h = 186\ g$

Hoop properties

	Hoop 1	Hoop 2
Material	Stryofoam	Wood
Shape	Toroidal	Full ring
Determing moment of inertia	Mathematicaly $I_{cm} = \left[\left(R - \frac{\Delta R}{2}\right)^2 + \frac{3}{4}\left(\frac{\Delta R}{2}\right)^2\right] m_o + m_u r^2$	Eksperimentaly $T = 2\pi\sqrt{\frac{I_0}{mgd}}$ $I_{cm} = I_0 - mgd$
Determing distance from center of the mass	Mathematicaly $r_{cm} = \frac{m_u}{M} r$	Eksperimentaly -hoop was stabilised on screwdriver

Experimental aproach

- Giving initial push
  - Incline

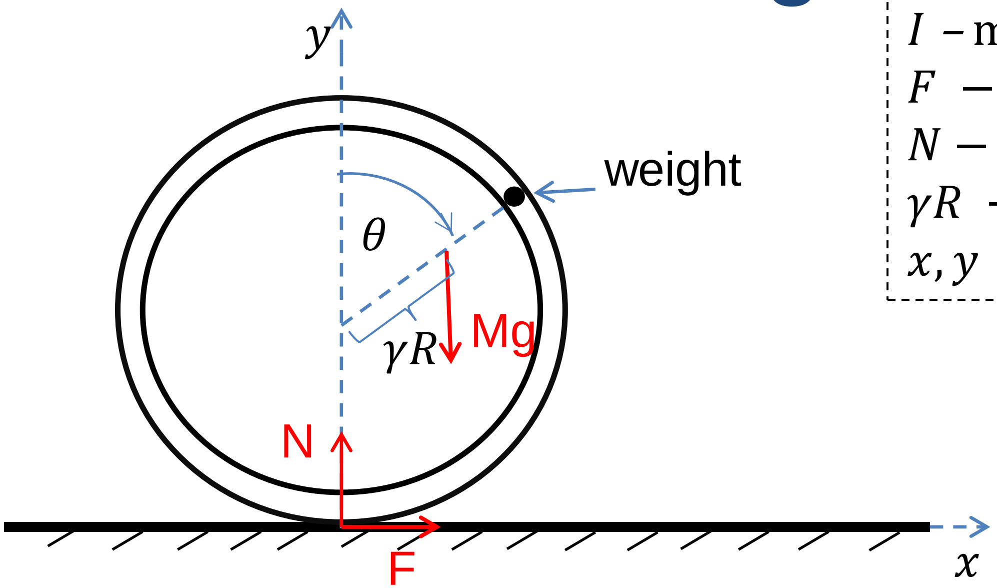


  - Colision

- Method of motion recording
  - 3 small areas marked with reflective stickers
  - Motion recorded in 120 fps
  - Stickers tracked in program for video analysis ImageJ

Theoretical modeling

m_h – *hoops mass*
m_w – *mass of the weight*
I – moment of inertia around center
F – friction force
N – normal force
γR – center of the mass distance
x, y – coordinates of the hoops center



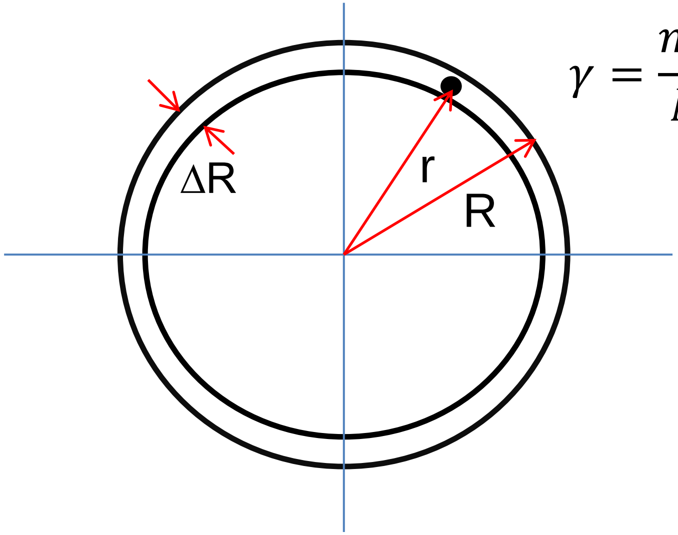
$$M = m_h + m_u$$

$$I = kMR^2$$

$$\gamma = \frac{m_w r}{MR} = \frac{r_{cm}}{R}$$

Kinematics of the hoop:

$$x_c = x + \gamma R sin(\theta)$$
$$y_c = y + \gamma R cos(\theta)$$



Dynamics of the hoop:

$$M\ddot{x}_c = F$$
$$M\ddot{y}_c = N - Mg$$
$$I\ddot{\theta} = m_w grsin(\theta) - FR$$

Modes of motion

- Rolling
- Rolling with slipping
- Hop

Rolling

- Properties of rolling
  - $x = R\theta,\ \dot{x} = R\omega, \ddot{x} = R\alpha$
  - $N > 0, y = 0, \dot{y} = 0, \ddot{y} = 0$
  - $|F| < \mu N$

- Solving equations for dynamics with assumption of rolling we obtain:
  - $\omega = \sqrt{\left(\frac{\eta+cos\theta_0}{\eta+cos\theta}\right)^2\left(\omega_0{}^2+\frac{g}{R}\right)-\frac{g}{R}}$
  - $\left(\frac{N}{M}\right) = g\left(1-\gamma\frac{\sin^2\theta}{\eta+cos\theta}\right)-\gamma R\omega^2\left(\frac{1+\eta cos\theta}{\eta+cos\theta}\right)$
  - $\left(\frac{F}{M}\right) = g\gamma sin\theta - k(R\omega^2+g)\frac{sin\theta}{\eta+cos\theta}$

$$\eta = \frac{1+k}{\gamma}$$

Rolling with slipping

- Properties of rolling with slipping
  - $\vec{F} = -\mu N \frac{\vec{v}_r}{|\vec{v}_r|}$ whereby $\vec{v}_r$ is relative velocity between hoop on the contact of the surface and the surface
  - $N = 0,\ y = 0, \dot{y} = 0, \ddot{y} = 0$

- Two cases:
  - $R\omega > \dot{x}$
    - The hoop is rotating more than it's translating
  - $R\omega < \dot{x}$
    - The hoop is translating more than it's rotating

Hop

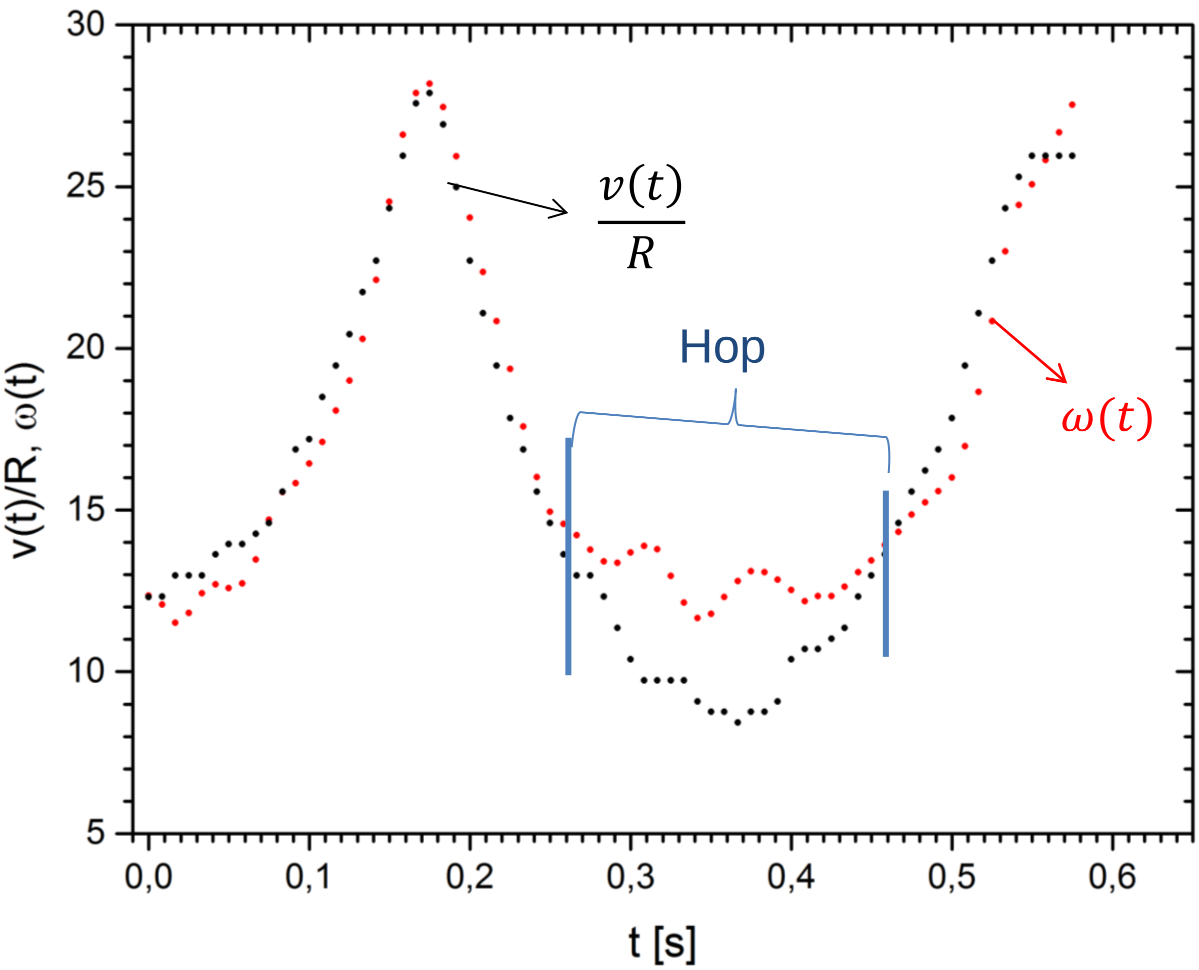
- The center of mass moving along a parabola with constant speed in the x direction

- Conservation of angular momentum

- Most probable hop when weight is on highest position

- $N = 0$

Determing hoop motion - 1



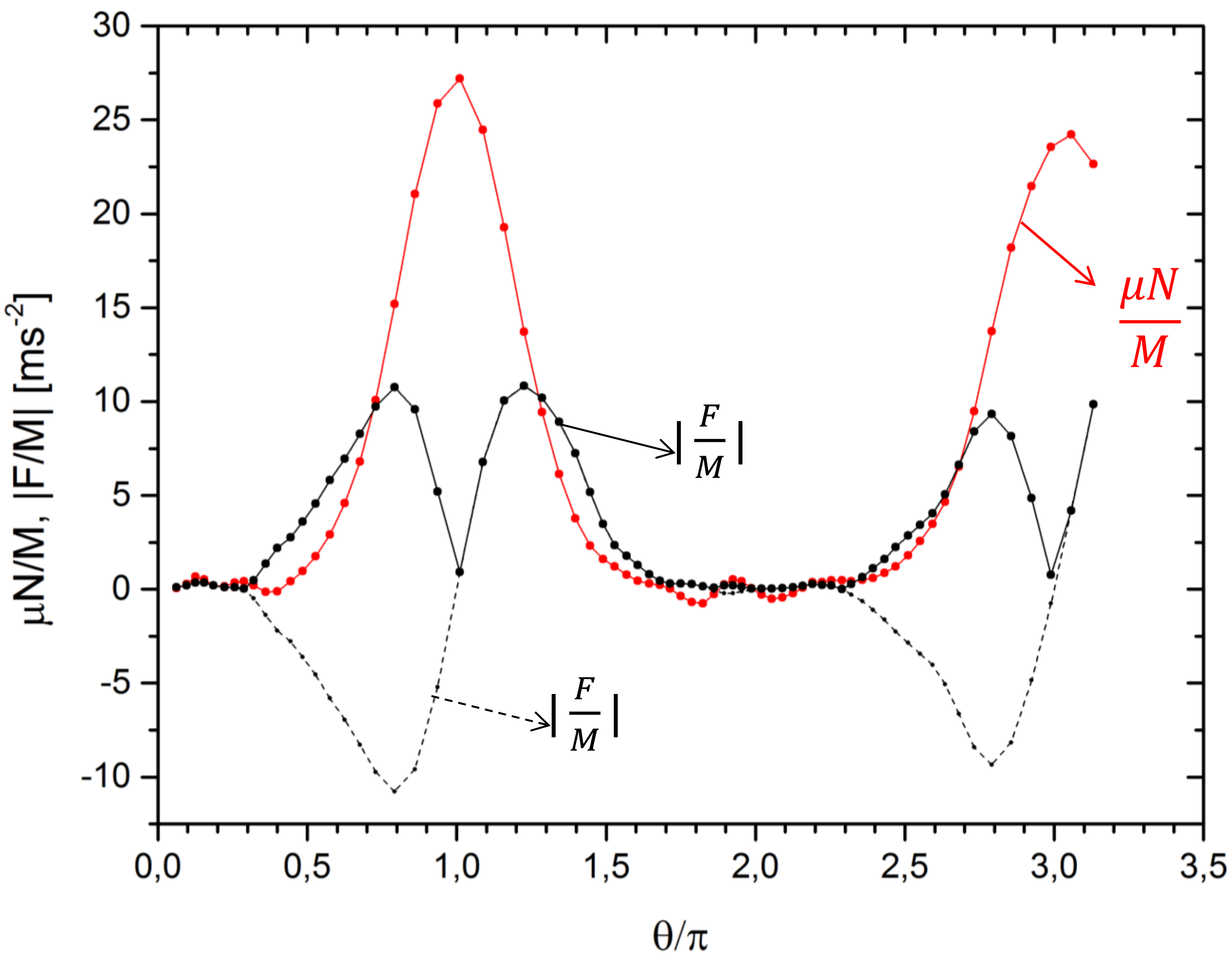
Rolling condition

- $\dot{x} = R\omega$

Hop

- $\omega - const$

Determing hoop motion - 1



Rolling condition:

$$|F| < \mu N$$

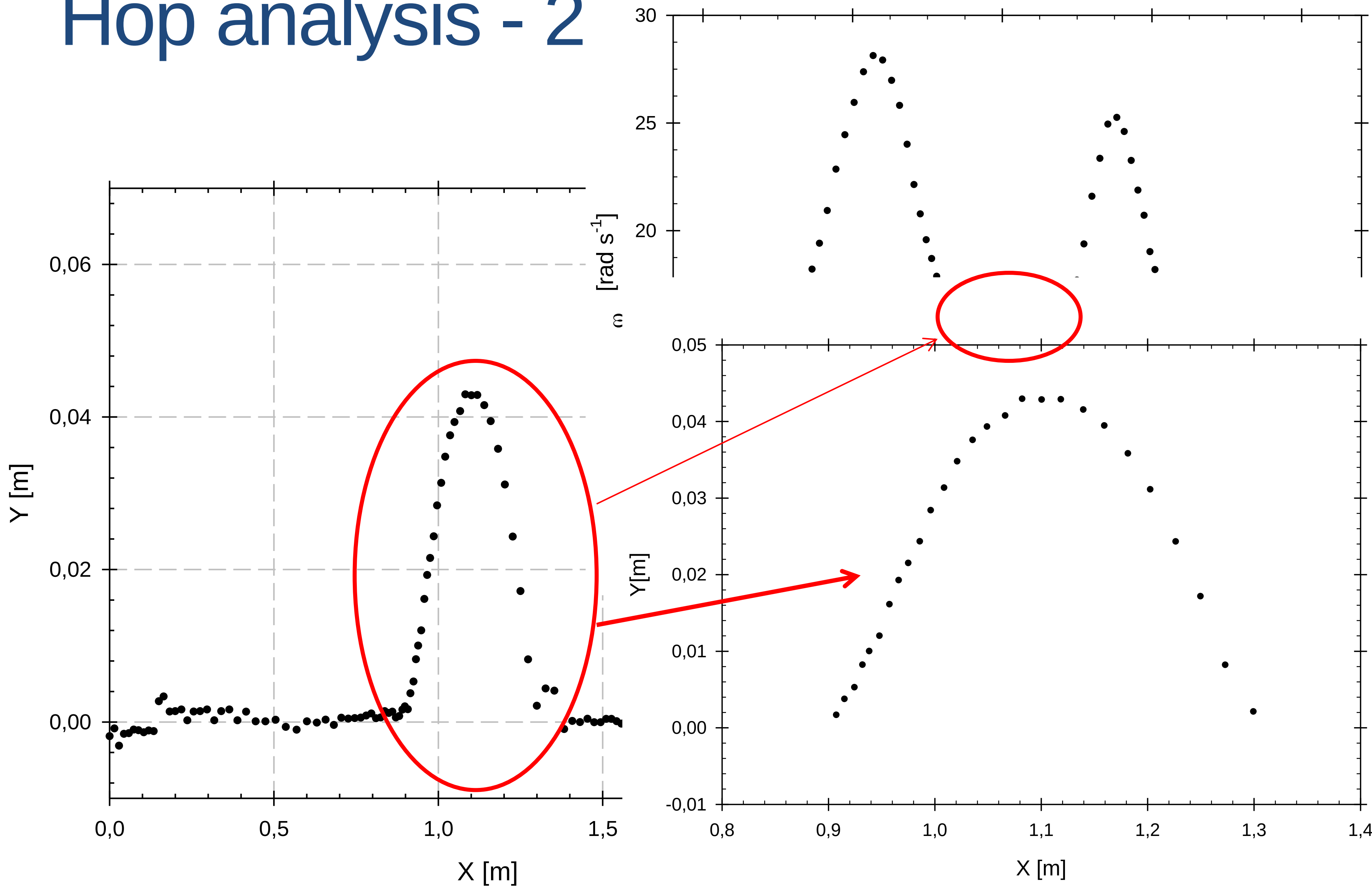
Rolling with sliping condition:

$$|F| > \mu N$$

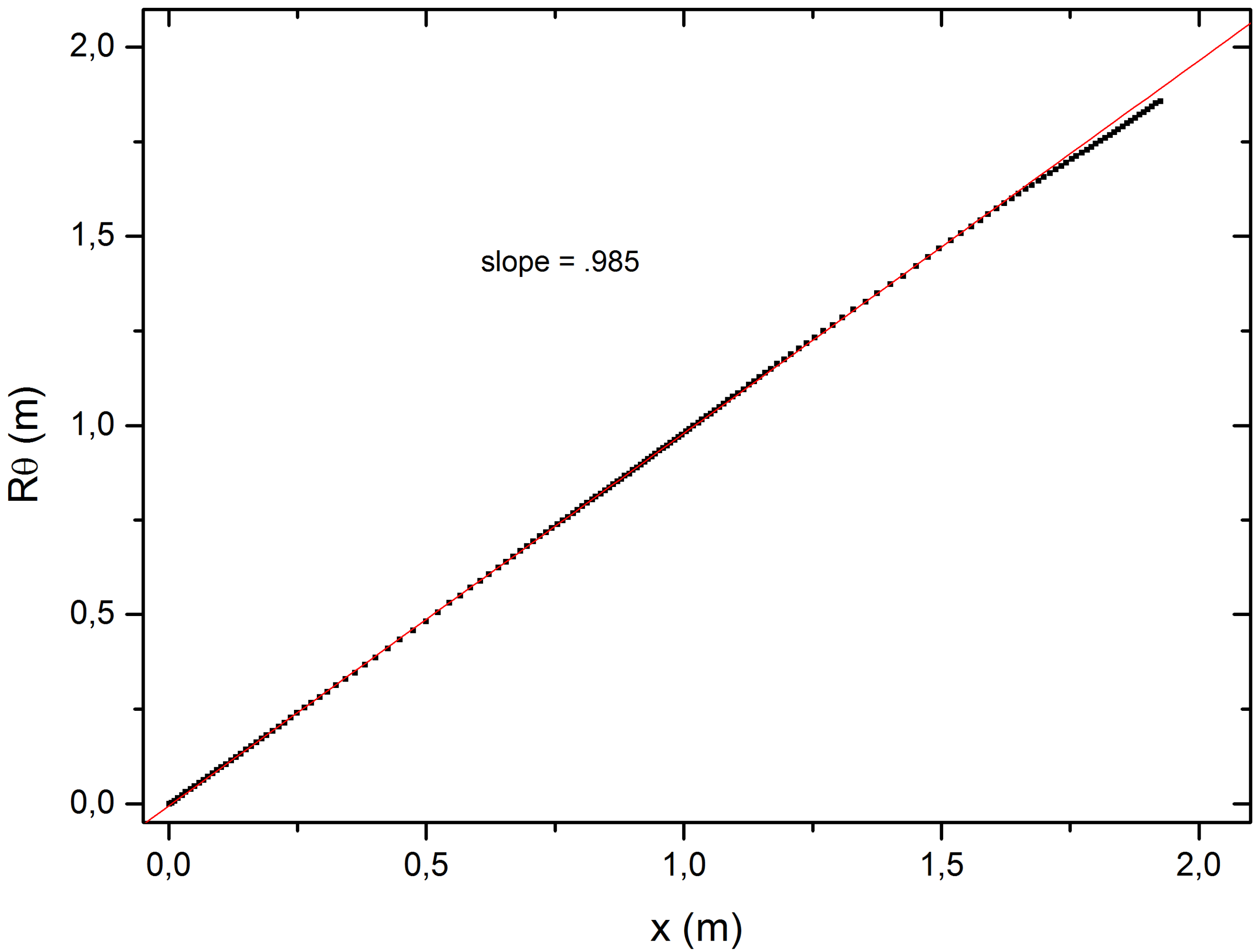
Hop:

$$N = 0$$

Hop analysis - 2



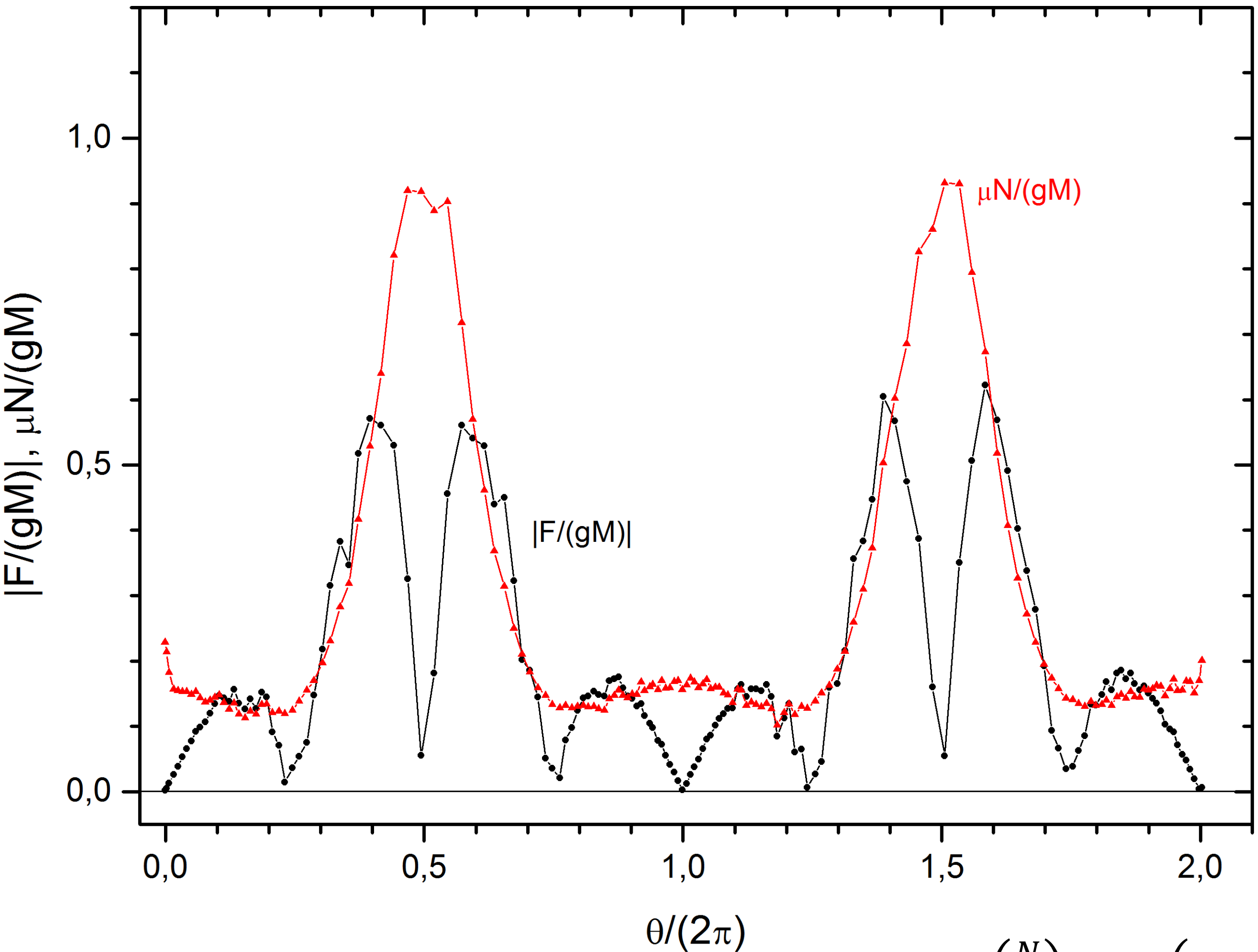
Analysis of motion without hop - 3



Rolling condition

- $x = R\theta$

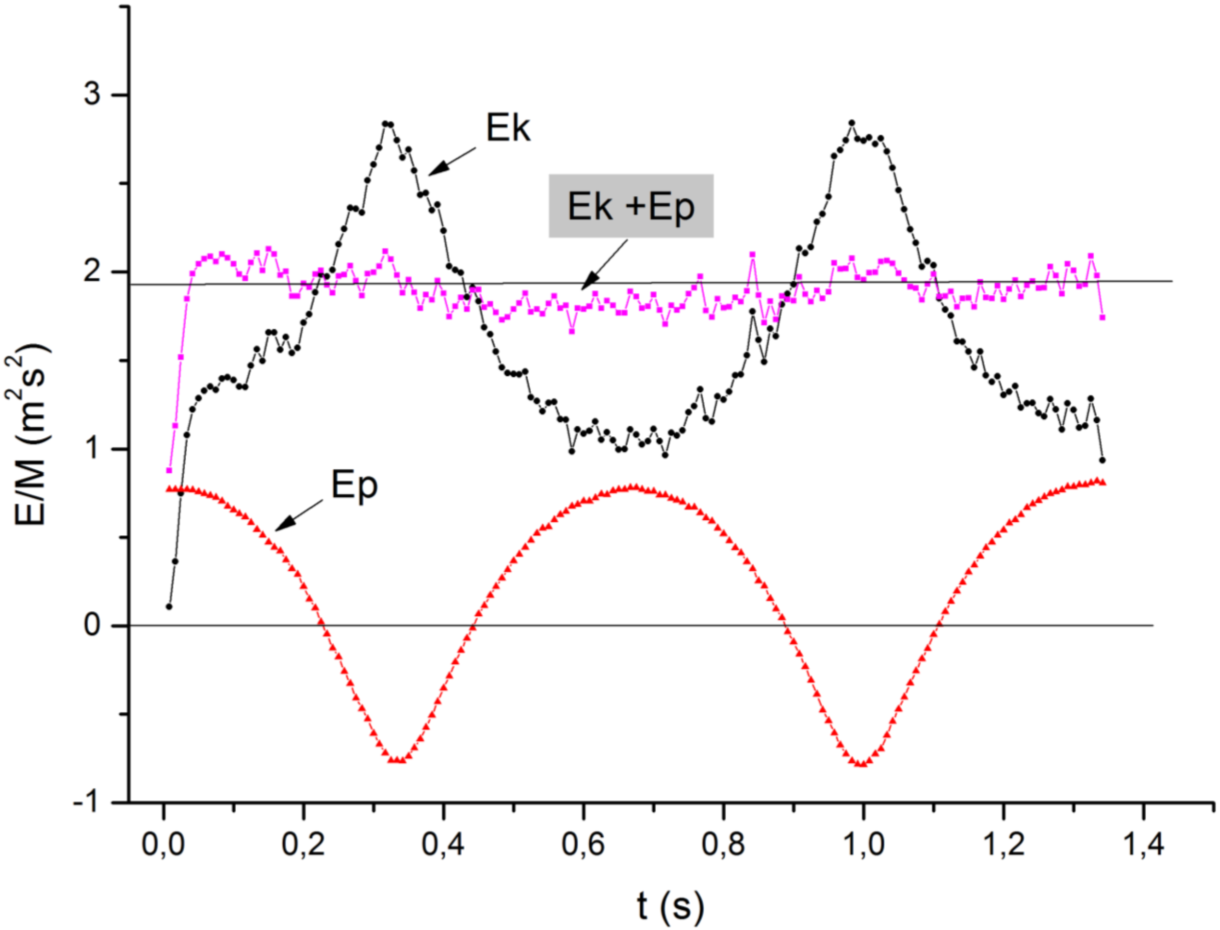
Analysis of motion without hop - 3



- The motion of the hoop can be considerated as rolling $|F| < \mu N$

- $\left(\frac{N}{M}\right) = g\left(1 - \gamma\frac{\sin^2\theta}{\eta + cos\theta}\right) - \gamma R\omega^2\left(\frac{1 + \eta cos\theta}{\eta + cos\theta}\right)$
- $\left(\frac{F}{M}\right) = g\gamma sin\theta - k(R\omega^2 + g)\frac{sin\theta}{\eta + cos\theta}$

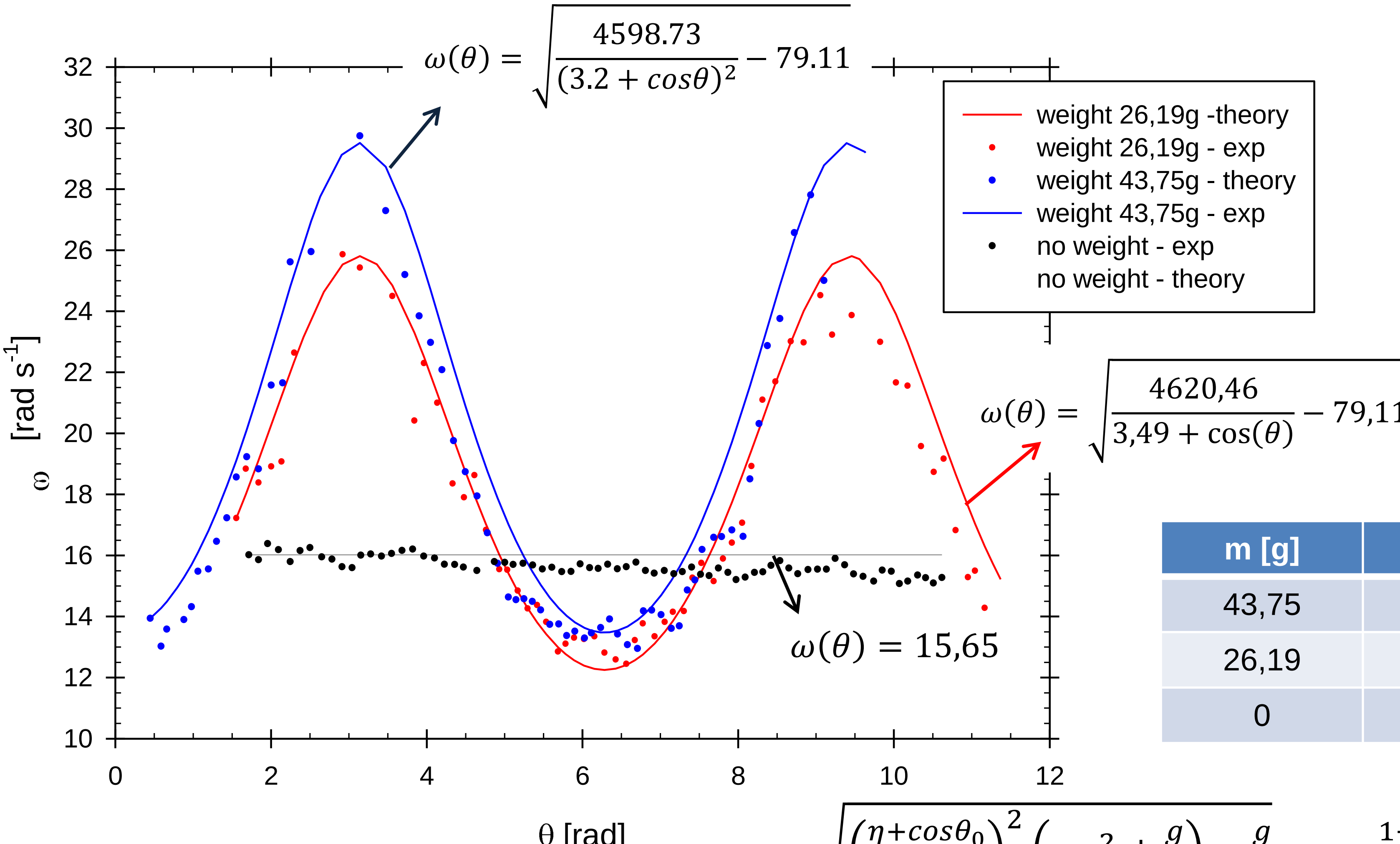
Conservation of energy



Energy of a hoop:

$$\frac{E_t}{M} = \frac{1}{2}(v_c^2 + k_c(\omega R)^2) + gy_c$$

Dependence of angular velocity on angle for different weight mass



m [g]	η
43,75	3.2
26,19	3.49
0	$\rightarrow \infty$

$$\omega = \sqrt{\left(\frac{\eta + cos\theta_0}{\eta + cos\theta}\right)^2 \left(\omega_0{}^2 + \frac{g}{R}\right) - \frac{g}{R}}, \eta = \frac{1+k}{\gamma}$$

Conclusion

- Three modes of motion were observed
  - Rolling
  - Rolling with sliping
  - Hop

- We analyticaly solved equation of motion for rolling case
  - $\omega = \sqrt{\left(\frac{\eta+cos\theta_0}{\eta+cos\theta}\right)^2\left({\omega_0}^2+\frac{g}{R}\right)-\frac{g}{R}}$, $\eta = \frac{1+k}{\gamma}$ and the solution fits the experimental data

- When the hoop hop its center of mass will move on parabolic trajectory and it will have constant angular momentum

Reference

- M. F. Maritz, W.F.D. Theron; Experimental verification of the motion of a loaded hoop; 2012 American journal of physics
- M. F. Maritz, W.F.D. Theron; The amazing variety of motions of a loaded hoop; Mathematical and Computer Modelling 47 (2008) 1077-1088
- Lisandro A. Raviola, O. Zarate, E. E. Rodriguez; Modeling and experimentation with asymmetric rigid bodies: a variation on disks and inclines; March 31, 2014
- W. F. D. Theron, Analysis of the Rolling Motion of Loaded Hoops, dissertation, March 2008

IYPT 2014
CROATIAN TEAM

THANK YOU

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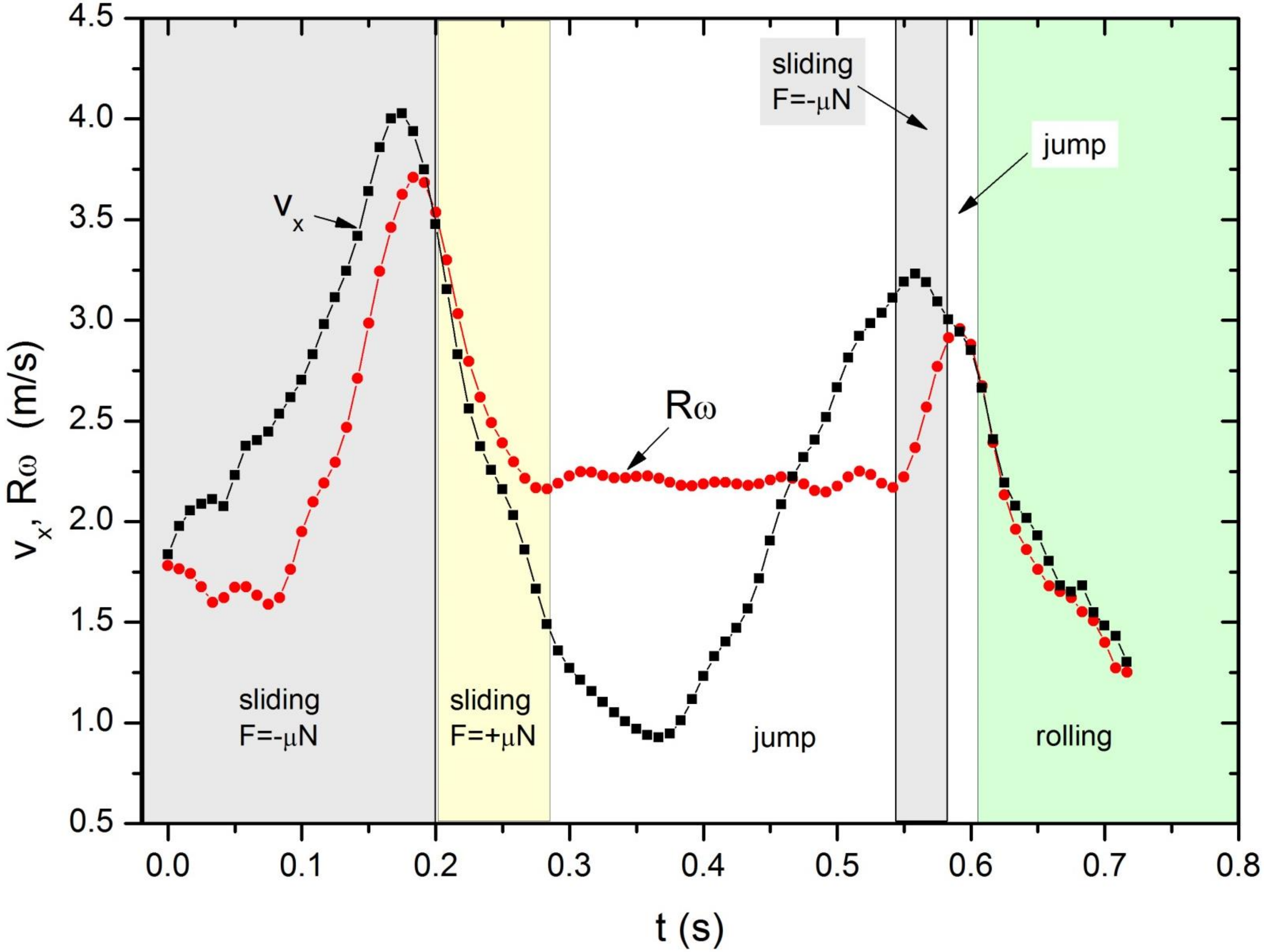
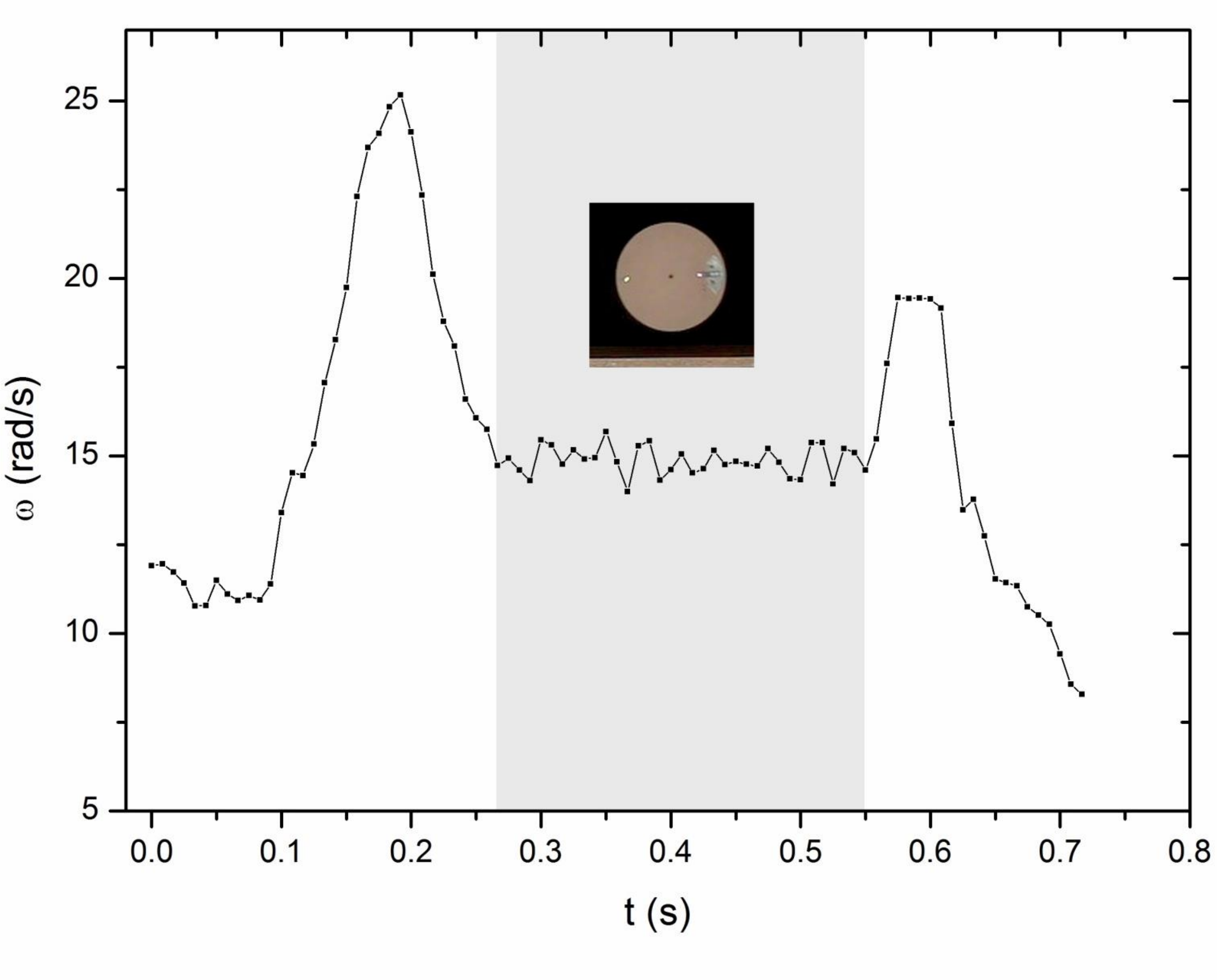
Hoop1

n	m_u[g]	M[g]	I[kgm^2]$\cdot 10^{-4}$	γ	k	η
1	17,8	42,2	4,33	0,337	0,657	4,92
2	26,2	50,6	5,17	0,414	0,654	4,01
3	35,5	59,9	6,1	0,474	0,652	3,49
4	44	68,4	6,95	0,515	0,65	3,2

Determing hoops motion – another example



Rotational velocity in time – another example



Energy – another example