16. MAGNETIC BRAKES

”When a strong magnet falls down a non ferromagnetic metal tube, it will experience a ***retarding force***.

Investigate the phenomenon.”

Outline

Qualitative explanation

↓

Quantitaive explanation

- Force modeling
- Dipole case
- Equation of motion

↓

Apparatus and experimental methods

- Measurement of magnetic flux change
- Measurement of magnet velocity

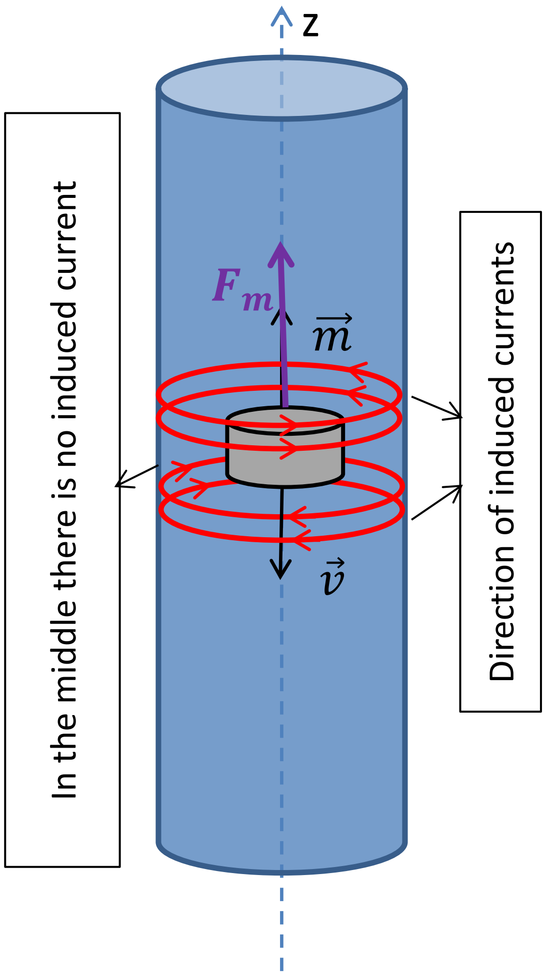
↓

Comparison of experimental data and theory

- Dependence of terminal velocity on magnet size
- Terminal velocity for dipole
- Dependence of terminal velocity on material conductivity
- Equation of motion

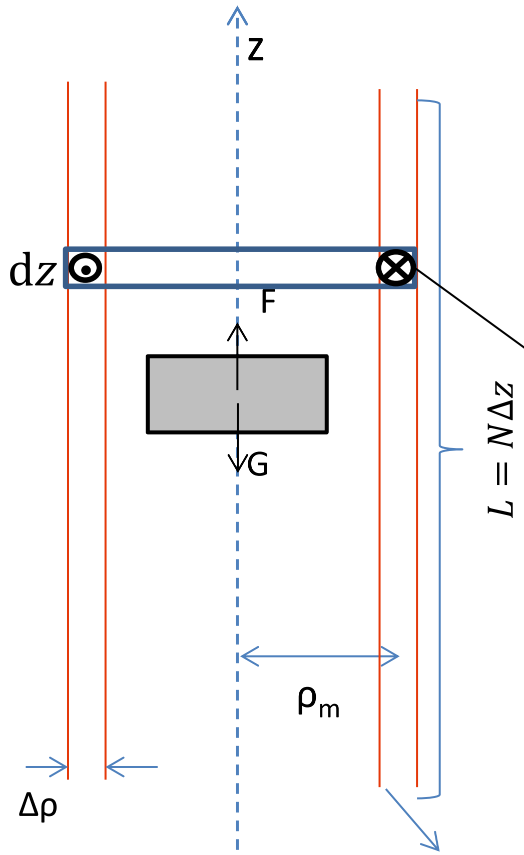
What causes the force?

- Magnet falls down under the influence of gravity
- Magnetic flux is changing in space
- Current is induced in the tube
- Currents act on magnet by force that reduces the speed of magnetic flux change (reducing the magnet speed)



$\vec{m}$ – *magnetic moment*
$\vec{v}$ –*velocity of a magnet*

Force

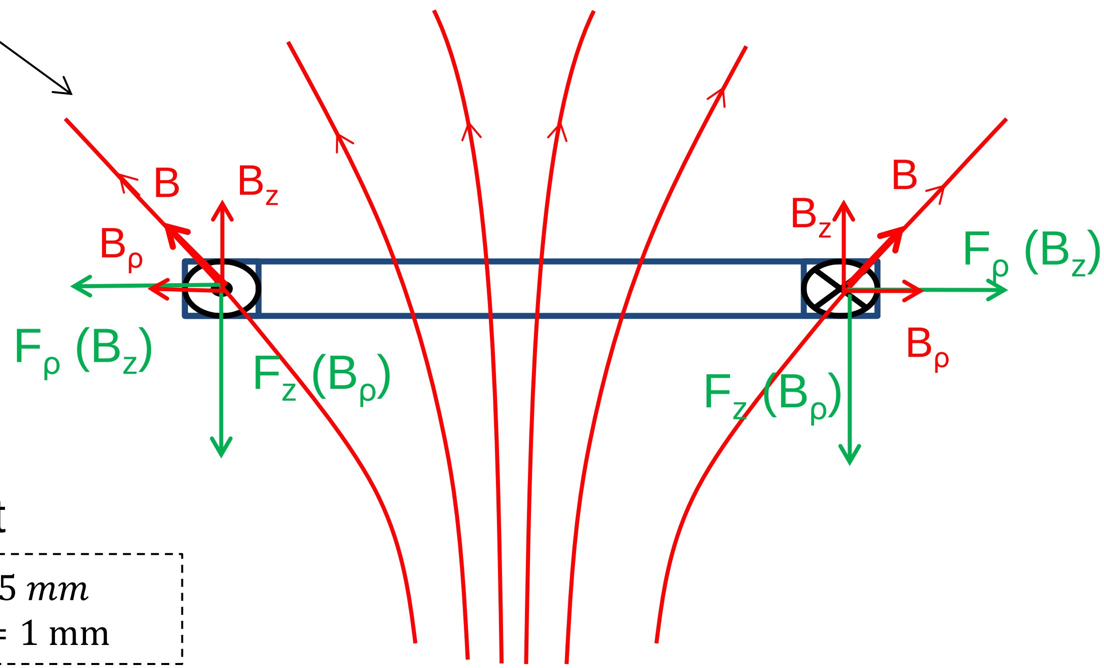


- According to Newton III. law:

$$\overrightarrow{F_m} = -\overrightarrow{F_c}$$

- Because we know what is the force on a current loop we can write

$$d\vec{F}_m = -dI\vec{l} \times \vec{B}$$



F_m – force on magnet
F_c –force on current
$\Delta\rho$ –tickness of the tube
dz – lenght of a loop
ρ_m –mean radius of the pipe

Assumption:
no skin effect

$\delta = 38.3\ mm > 5.5\ mm$
Cu: $v = 0.1 ms^{-1}; \Delta\rho = 1$ mm

Force

- Magnetic field component B_ρ is causing the force in z direction so the force can be written as

$$\boldsymbol{dF_m = -dI \cdot 2\pi\rho_m \cdot B_\rho}$$

$$dI = \frac{\sigma\Delta\rho\varepsilon}{2\pi\rho_m}dz$$

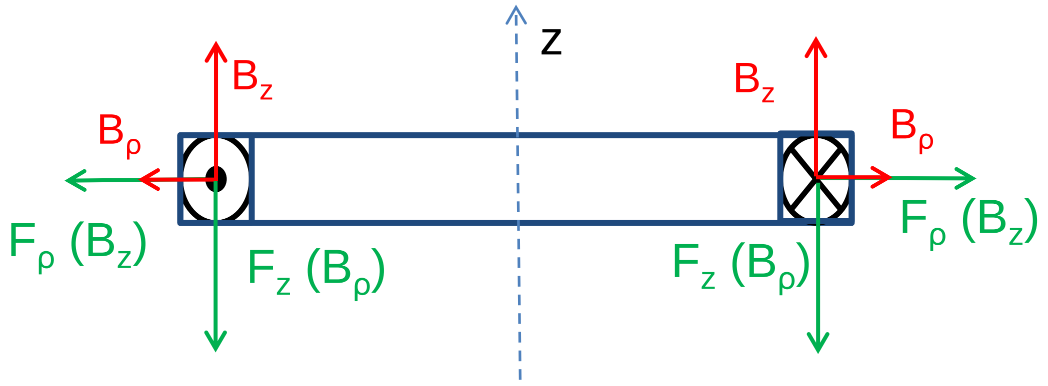
$$\varepsilon = \frac{d\phi}{dz}v$$

$$\boldsymbol{dI = \frac{\Delta\rho\sigma v}{2\pi\rho_m}\frac{d\phi}{dz}dz}$$

Using Maxwell equation $\vec{\nabla}\vec{B} = 0$ we derive expresion for B_ρ

$$\boldsymbol{B_\rho = -\frac{1}{2\pi\rho_m}\frac{d\phi}{dz}}$$

$\frac{d\phi}{dz}$ – change of magnetic flux in z direction
ε – induced voltage
σ – material conductivity
R – restistance of the ring



Force

- After rearranging the term we obtain total force on a magnet caused by induced current

$$F_m = \frac{\sigma \Delta \rho v}{2\pi \rho_m} \int_{z_1}^{z_2} \left(\frac{d\phi}{dz}\right)^2 dz$$

z_1, z_2 – coordinates of the tube edges

Equation of motion

- II. Newton law for magnet states

$$m\frac{dv}{dt} = mg - F_m$$

- After solving for v we obtain ($v(0) = 0$)

$$v(t) = \tau g\left(1 - e^{-\frac{t}{\tau}}\right)$$

- Function is in form of exponential decay
- Terminal velocity can be expressed as

$$v = \tau g$$

Substitution:

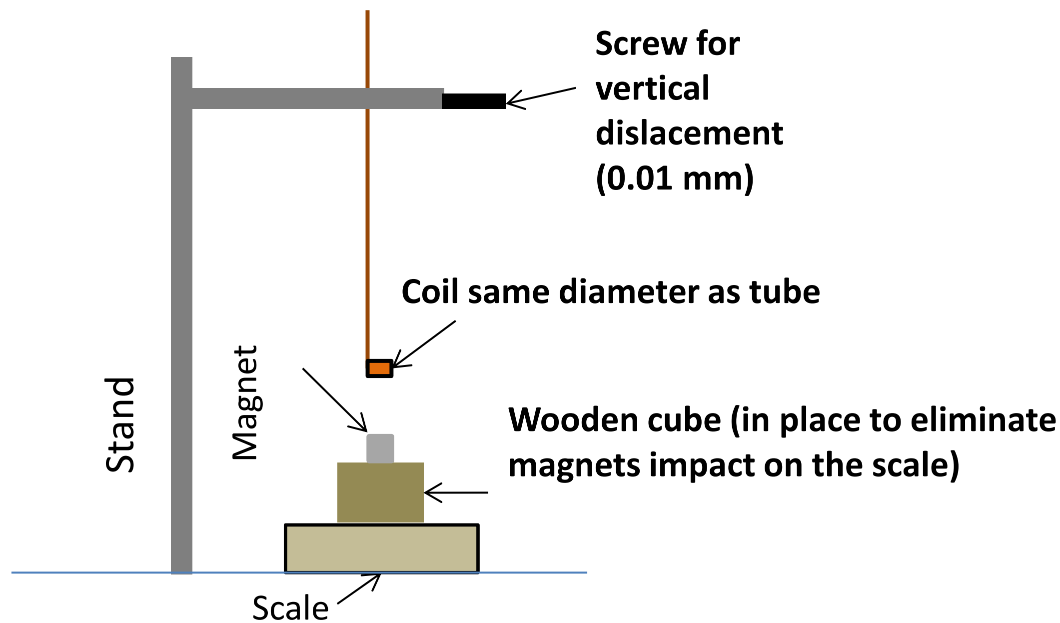
$$\tau = \frac{2\pi\rho_m m_{mag}}{\sigma\Delta\rho}\frac{1}{\int_{z_1}^{z_2}\left(\frac{d\phi}{dz}\right)^2 dz}$$

Determining magnetic flux change

- Because $\frac{d\phi}{dz}$ can be difficult to calculate we decided to experimentaly determine it

$$F = \mathrm{NI} \cdot 2\pi\rho \cdot B_\rho$$

$$F = NI \cdot \frac{d\phi}{dz}$$

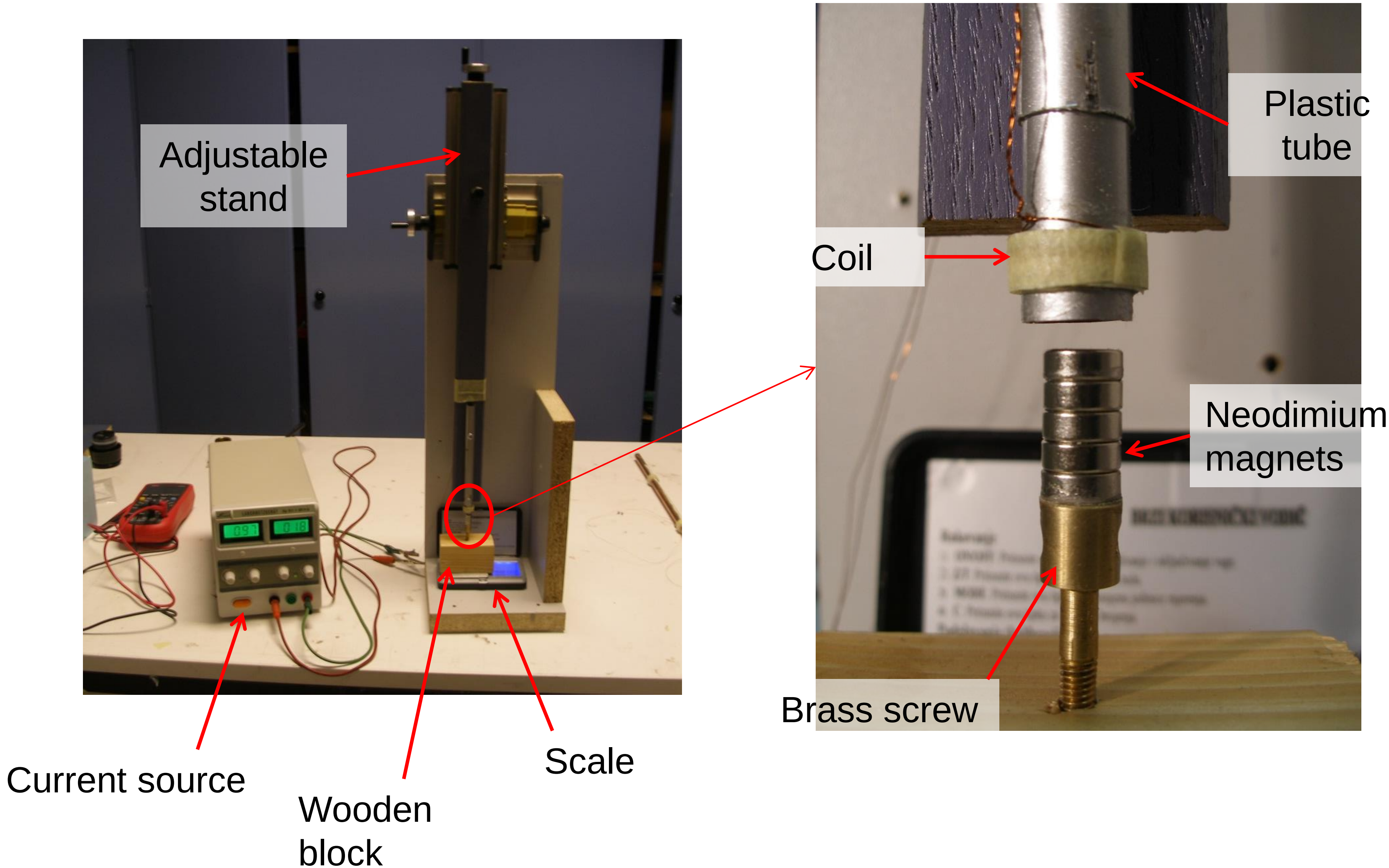


Measured by scale

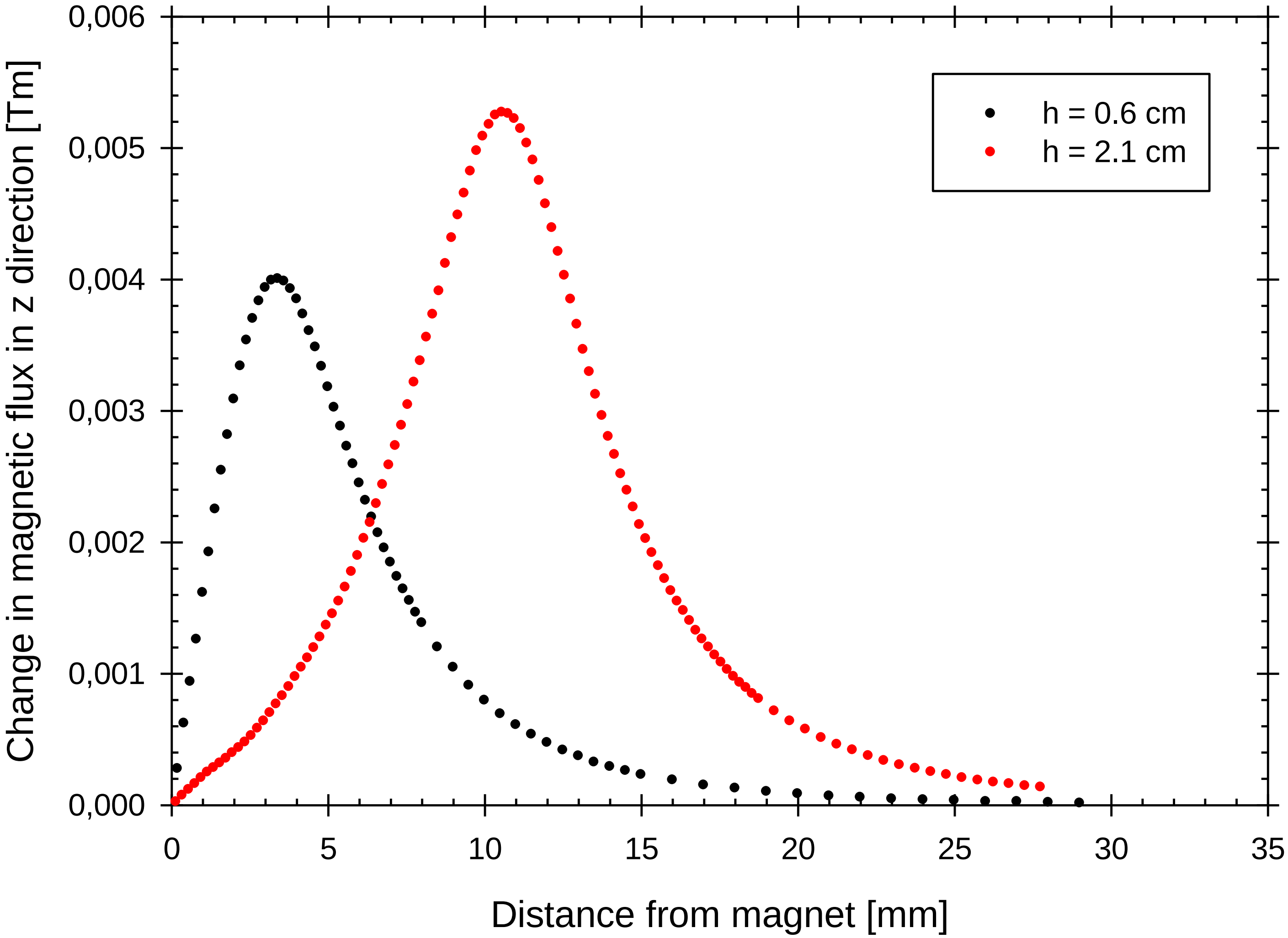
$$\frac{d\phi}{dz} = \frac{F}{NI}$$

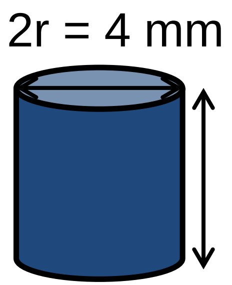
Property of coil $\mathrm{N} = 40$

Adjusted on current source I = 1A (DC)

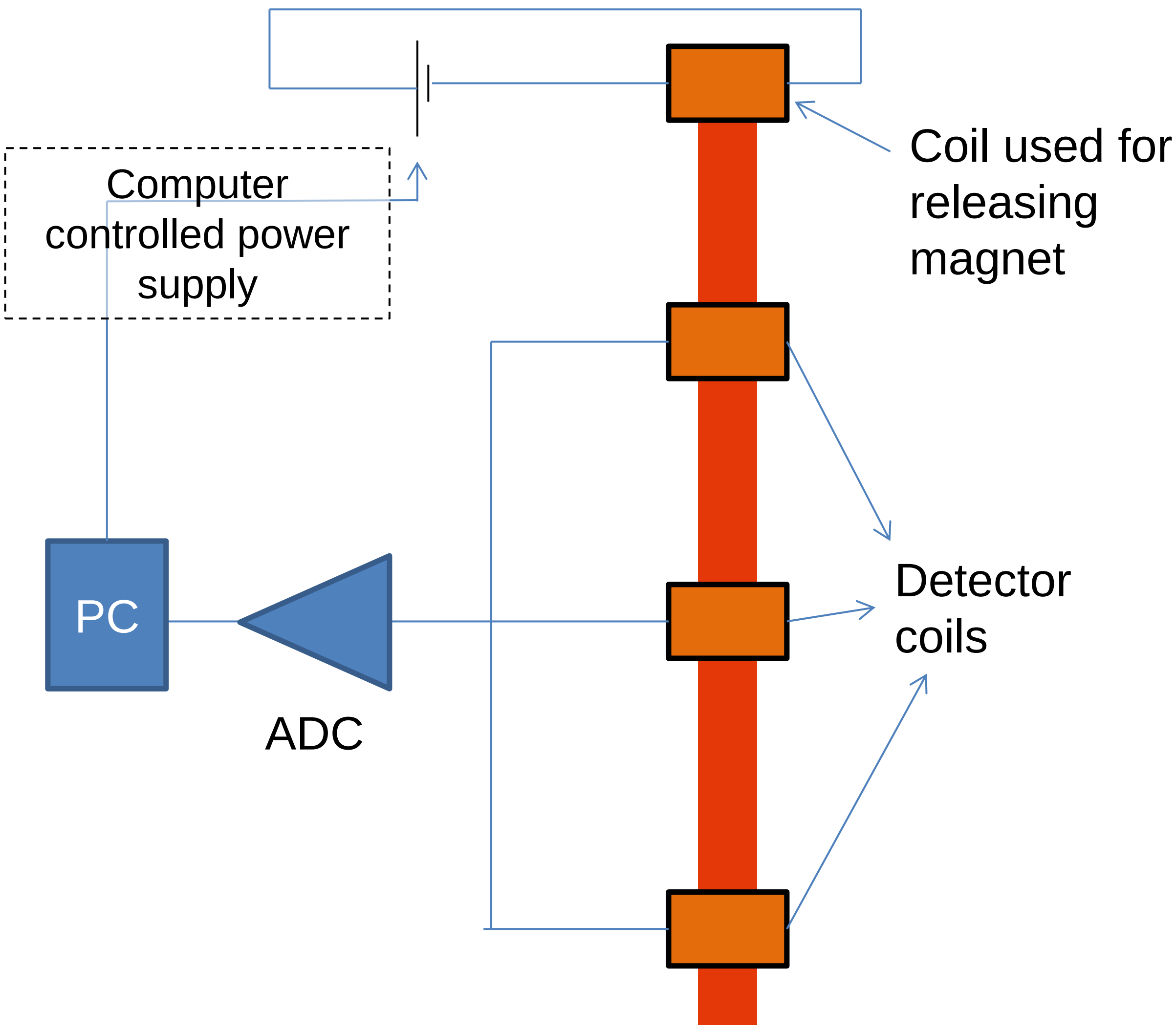

Adjustable stand
Current source
Wooden block
Scale
Plastic tube
Coil
Neodimium magnets
Brass screw

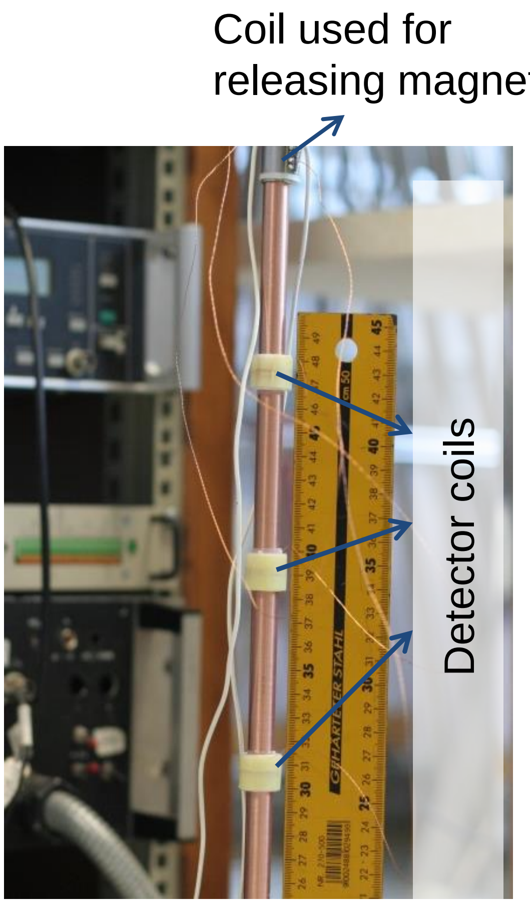
Example of obtained data - magnetic flux change



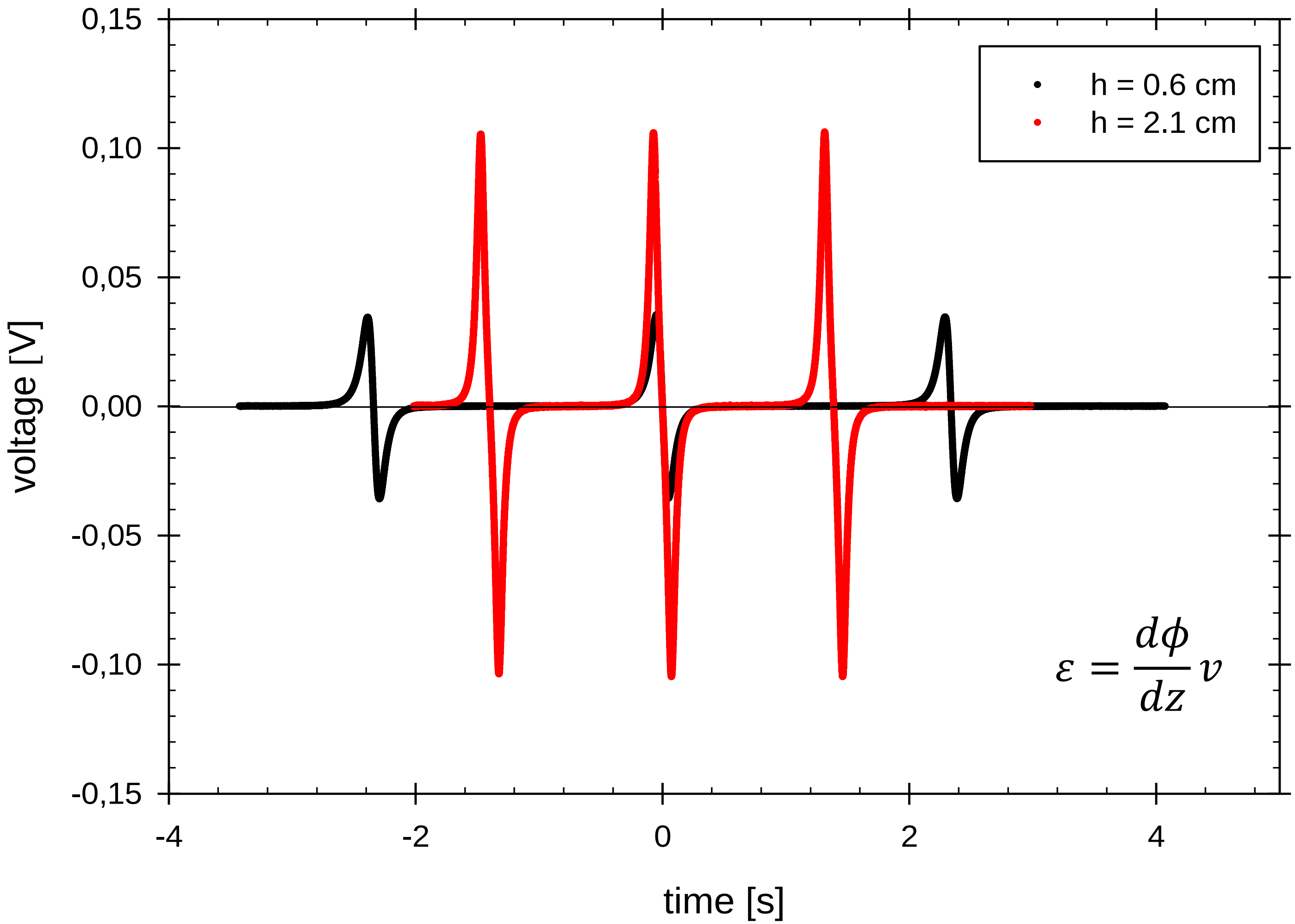


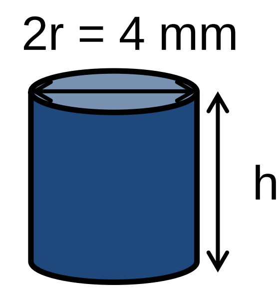
Velocity measurement





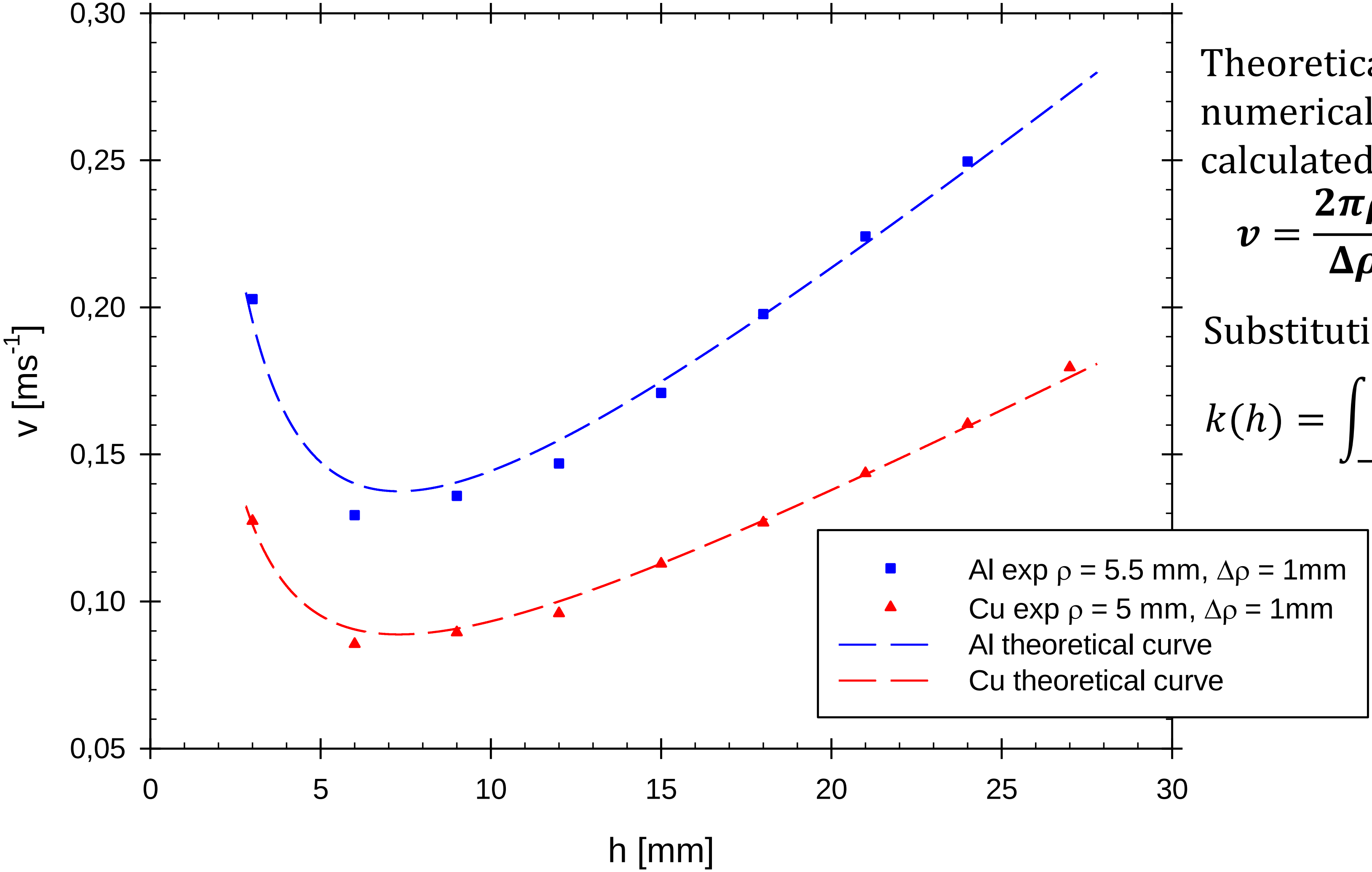
- Velocity measurement – coils detect passing magnet due to induction:





Copper
3 solenoids
$\rho = 5.5\ mm$
$\Delta\rho = 1\ mm$

Dependence of terminal velocity on cylindrical magnets height – Cu and Al

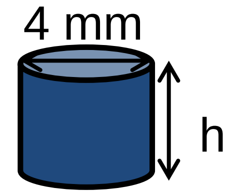


Theoretical values numericaly calculated from:

$$\boldsymbol{v} = \frac{\boldsymbol{2\pi\rho_m m_m g}}{\boldsymbol{\Delta\rho\sigma\, k(h)}}$$

Substitution:

$$k(h) = \int_{-\infty}^{\infty} \left(\frac{d\phi}{dz}\right)^2 dz$$



Dipole

- Dipole magnetic field in z direction (measured from the magnet):

$$B_z(z,\rho) = \frac{\mu_0 m}{4\pi}\left(\frac{2z^2 - {\rho_m}^2}{\left(\sqrt{z^2 + {\rho_m}^2}\right)^5}\right)$$

- If the magnet is far enough from the edges of the tube we can write

$$\boldsymbol{F_m} = \frac{\mathbf{45(\mu_0 MV)^2 \sigma \Delta\rho \cdot v}}{\mathbf{1024\pi {\rho_m}^4}}$$

Terminal velocity - dipole

- Spherical magnet – radius 2.5 mm
- Aluminium tube $\rho = 5\,mm, \Delta\rho = 1\,mm$

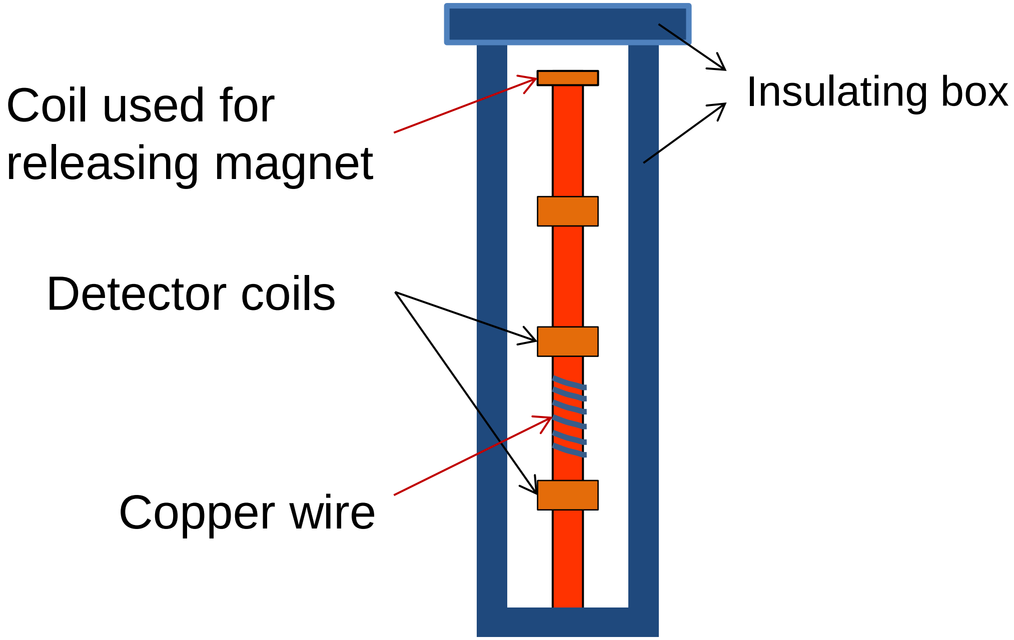
Theoretical prediction	Experimental result
$0.458\,ms^{-1}$	$(0.452 \pm 0.012)ms^{-1}$

$$v_t = \frac{256dg}{15\pi\sigma(\mu_0 M)^2} \cdot \left(\frac{\rho_m}{r}\right)^3 \left(\frac{\rho_m}{\Delta\rho}\right)$$

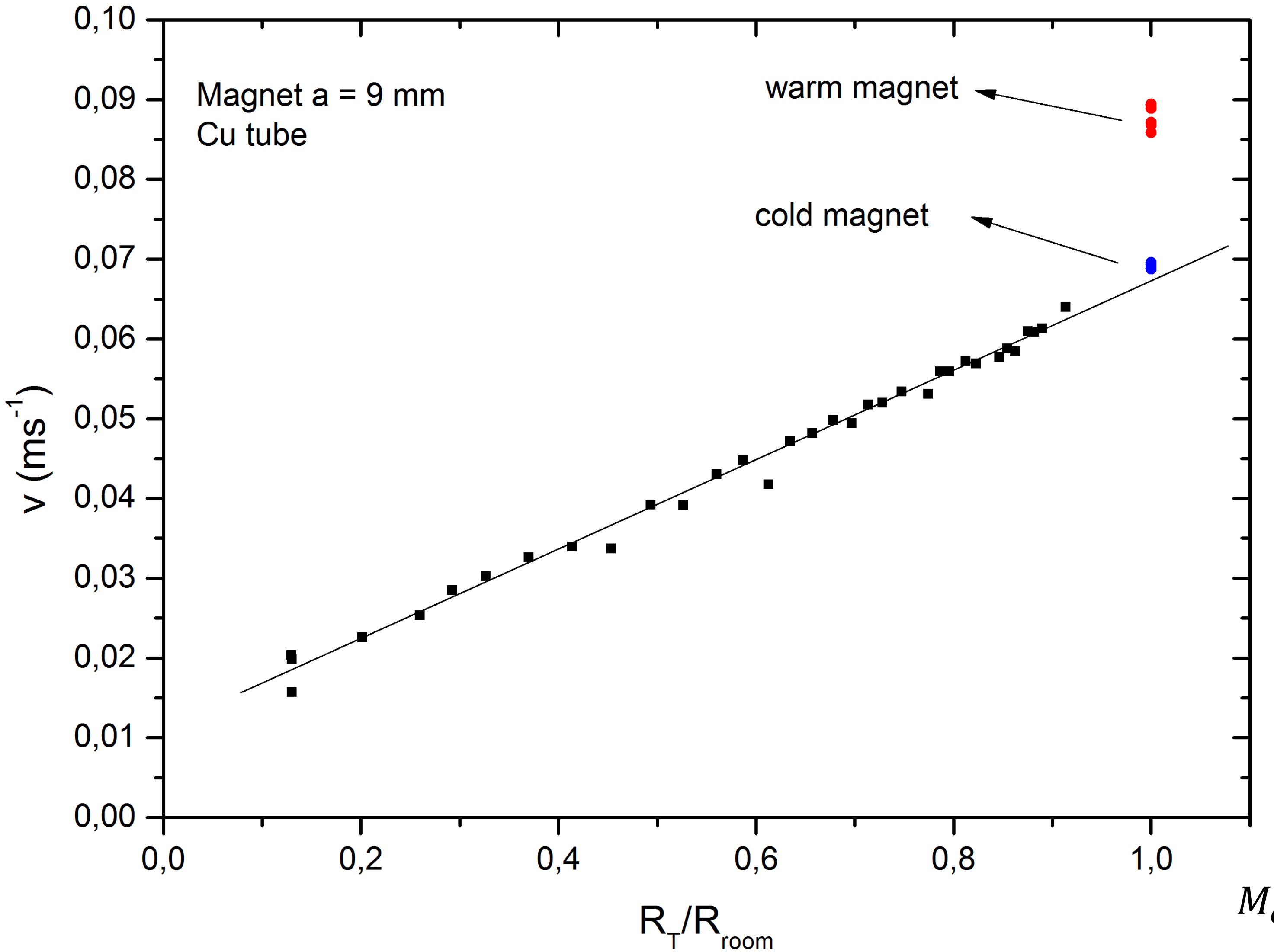
d – magnet's density
r – radius of a magnet
v_t – theoretical velocity

Velocity - conductivity

- Copper tube coled down to 77K in insulating box with liquid N2
- As tube warms up magnet is released and velocity measured
- Magnet was constantly kept on liquid nitrogen temperature ≈ 77K (constant magnetization $M_{cold} = 1.13 M_{warm}$)
- Conductivity measured directly - resistance of wire attached to tube



Dependence of final speed on material conductivity

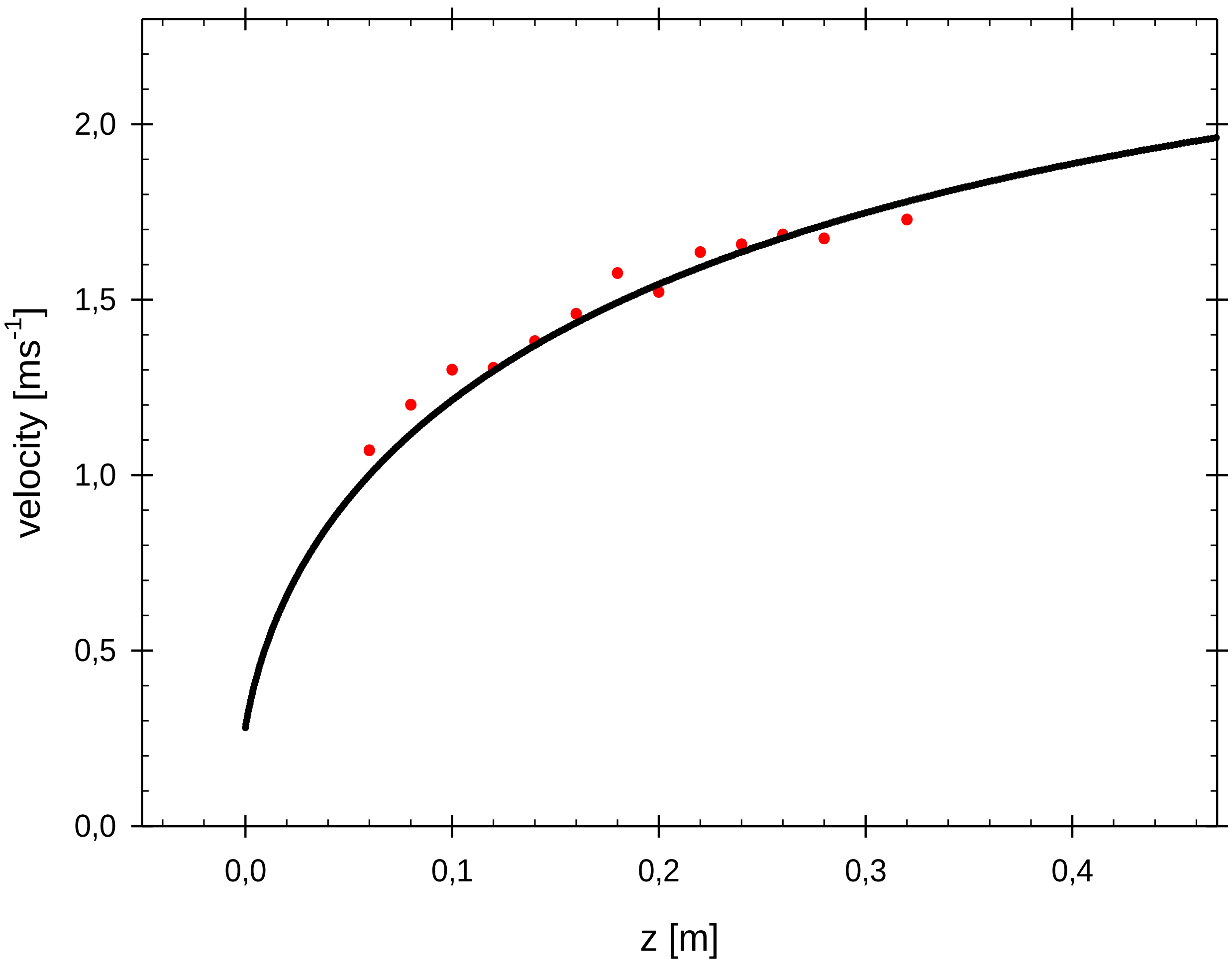


$$v \propto \frac{1}{\sigma}$$

$$R \propto \frac{1}{\sigma}$$

$$M_{cold} = 1.13 M_{warm}$$

Dependence of velocity on distance traveled



- Theoretical values calculated numericaly from $$v\frac{dv}{dz}+\frac{v}{\tau m}-g=0$$
- Initial speed determined experimentally
- System of cylindrical magnet ($h = 3\ mm$) and weight
- Velocity experimentaly determined by using $\varepsilon = \frac{d\phi}{dz}v$ relation

Conclusion

- Theoretical model developed
  - Quantitaive
    - No free parameters
  - Very good correlation with experiment
  - Assumptions
    - No skin effect
    - Constant direction and magnitude of magnetization of magnet
    - Rotational symmetry of the tube

- Experiment
  - Parameters changed
    - Size and shape of magnet
    - Conductivity of material

Reference

1. Valery S Cherkassky, Boris A Knyazev, Igor A Kotelnikov, Alexander A Tyutm; Eur. J. Phys.; Breaking of magnetic dipole moving through whole and cut conducting pipes

2. Edward M. Purcell; Electricity and Magnetism Berkley physics course – volume 2, Second Edition; McGraw-Hill book company

IYPT 2014
TEAM OF CROATIA

THANK YOU

Reporter: Domagoj Pluščec

Current

$$I = \frac{\varepsilon}{R}$$

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d\phi}{dt}\frac{dz_0}{dz_0} = \frac{d\phi}{dz}v$$

$$R = \frac{1}{\sigma}\frac{l}{A} = \frac{1}{\sigma}\frac{2\pi\rho_m}{\Delta\rho \cdot dz}$$

$$\varepsilon = \frac{d\phi}{dz}v$$

$$\boldsymbol{dI = \frac{\Delta\rho\sigma v}{2\pi\rho_m}\frac{d\phi}{dz}dz}$$

$\frac{d\phi}{dt}$ - can be expresed as space change because magnetic flux around the magnet doesn't depend on time

v – magnets speed
$\frac{d\phi}{dt}$ *– time change of magnetic flux*
ε – induced voltage
σ – material conductivity
l – ring perimeter
A – area of ring cross section
R – restistance of the ring
z_0 *- magnets position*

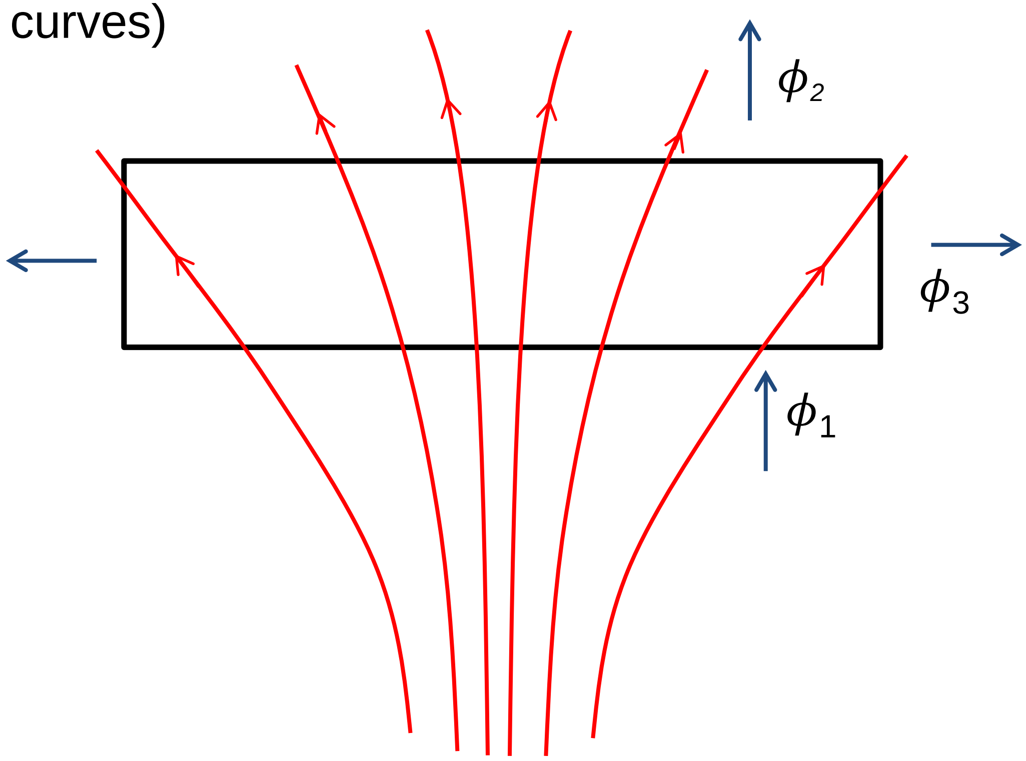
Determing B_ρ

- Maxwell equation $\vec{\nabla}\vec{B} = 0$
  (magnetic field lines are closed curves)

$$\phi_1 = \phi_2 + \phi_3$$

$$\phi_3 = -d\phi$$
$$\phi_3 = 2\pi\rho_m B_\rho dz$$

$$\boldsymbol{B_\rho = -\frac{1}{2\pi\rho_m}\frac{d\phi}{dz}}$$



ϕ – magnetic flux

Schematic representation of the magnetic flux through a closed surface (disc shape)

Terminal velocity

- Terminal velocity is reached when gravitational force is equal to magnetic force

$$F_g = F_m$$

$$m_m g = \frac{\Delta\rho\sigma v_0}{2\pi\rho_m}\int_{-\infty}^{\infty}\left(\frac{d\phi}{dz}\right)^2 dz$$

Supstitution

$$k = \int_{-\infty}^{\infty}\left(\frac{d\phi}{dz}\right)^2 dz$$

$$\boldsymbol{v_0 = \frac{2\pi\rho_m m_m g}{\Delta\rho\sigma k}}$$

Assumptions used in derivation of model

- Tube is rotational symetrical in z direction
- Thin tube and the magnet is falling with low velocities (model does not take into consideration skin effect)

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}} \gg \rho_m; \;\; \omega \sim \frac{v}{\rho_m}$$

$\delta = 38.3\ mm > 5.5\ mm$
Cu; $v = 0.1ms^{-1}$; $\Delta\rho = 1$ mm

- We didn't take into account case in which magnet rotates in such a way that direction of its magnetic moment changes

δ – *skin depth*
ω – *characteristic frequency*

Detailed dipole model derivation 1/2

- In theoretical modeling part we obtained

$$\boldsymbol{F_{mag}} = \frac{\boldsymbol{\sigma \Delta \rho v_o}}{\boldsymbol{2\pi \rho_m}} \int_{\boldsymbol{z_1}}^{\boldsymbol{z_2}} \left(\frac{\boldsymbol{d\phi}}{\boldsymbol{dz}}\right)^{\boldsymbol{2}} \boldsymbol{dz}$$

in this equation we need to find $\frac{d\phi}{dz}$ for dipole

- Magnetic flux trought cross section of tube can be expressed as

$$\phi(z) = 2\pi \int_0^{\rho_m} B_z \rho \, d\rho$$

- And by using expresion for a field of dipole in z direction $B_z(z,\rho) = \frac{\mu_0 m}{4\pi}\left(\frac{2z^2 - {\rho_m}^2}{\left(\sqrt{z^2 + {\rho_m}^2}\right)^5}\right)$ we obtain magnetic flux in z direction

Detailed dipole model derivation 2/2

- Magnetic flux in z direction

$$\phi(z) = \frac{\mu_0 m}{2} \frac{\rho_m^2}{(z^2 + \rho_m^2)^{\frac{3}{2}}}$$

- From that we derive magnetic flux change

$$\frac{d\phi(z)}{dz} = -\frac{3\rho_m z}{(z^2 + \rho_m^2)^{\frac{5}{2}}}$$

- By knowing magnetic flux change we can obtain force

$$F_{mag} = \frac{9(\mu_0 m)2\sigma\Delta\rho \cdot \rho m^3 \cdot v_0}{8\pi} \int_{z1}^{z2} \frac{z^2}{(z^2 + \rho_m{}^2)^5} dz$$

- If the magnet is far enough from the edges of the tube we can write

$$\boldsymbol{F_{mag}} = \frac{\mathbf{45}(\boldsymbol{\mu_0 MV})^2 \boldsymbol{\sigma\Delta\rho \cdot v_0}}{\mathbf{1024}\boldsymbol{\pi\rho_m}^4}$$

Equation of motion with initial velocity

$$m\frac{dv}{dt} = mg - F_{mag}, \mathrm{v}(0) = v_0$$

After solving for v we obtain:

$$v(t) = v_0 \cdot e^{-\frac{t}{\tau m}} + mg\tau(1 - e^{-\frac{t}{\tau m}})$$

Supstitution:

$$\tau = \frac{2\pi\rho_m m_{mag}}{\sigma\Delta\rho} \frac{1}{\int_{\boldsymbol{z_1}}^{\boldsymbol{z_2}} \left(\frac{\boldsymbol{d\phi}}{\boldsymbol{dz}}\right)^{\boldsymbol{2}} \boldsymbol{dz}}$$

Terminal velocity derived from energy conservation

$$P_{current} = P_{gravity}$$

$$P_c = I^2 R$$

After substituing terms for current and resistance we obtain

$$P_c = \frac{\Delta\rho\sigma v_0^2}{2\pi\rho_m} k$$

$$P_g = mgv_0$$

$$\boldsymbol{v = \frac{2\pi\rho_m m_m g}{\Delta\rho\sigma k}}$$

Supstitution

$$k = \int_{-\infty}^{\infty} \left(\frac{d\phi}{dz}\right)^2 dz$$

Apparatus properties - tubes

	Copper	Aluminium
ρ_m	5.5 mm	5 mm
$\Delta\rho$	1 mm	1 mm
σ	$5.96 \cdot 10^7 Sm^{-1}$	$3.5 \cdot 10^7 Sm^{-1}$
μ_r	0.999994	1.000022

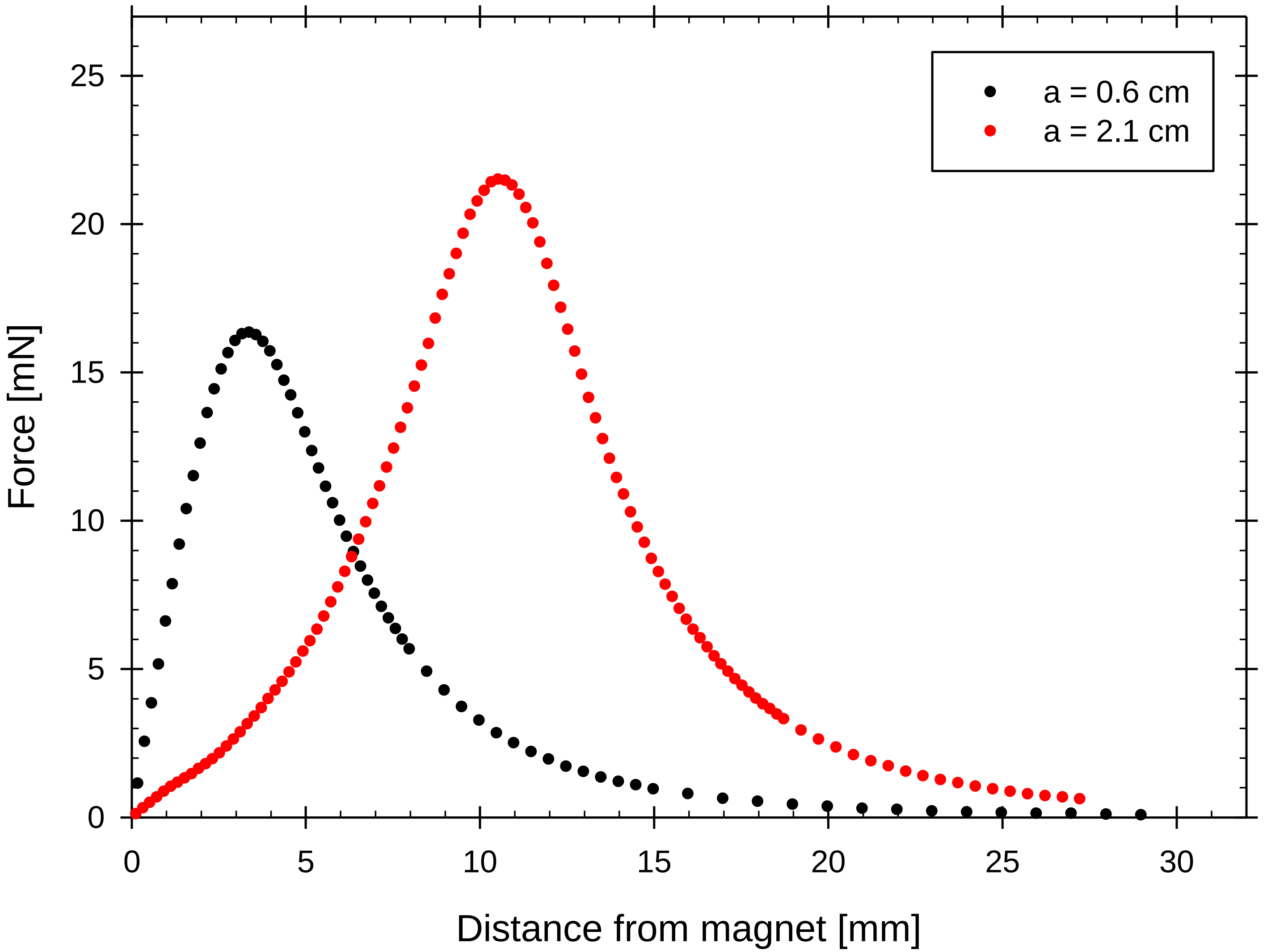
Apparatus properties - magnets

- Neodimium magnets
  - Magnetic permeability $\mu_r = 1.05$
  - Density $d = 7.5\ gcm^{-3}$
  - $\mu_0 M \approx 1.1T$

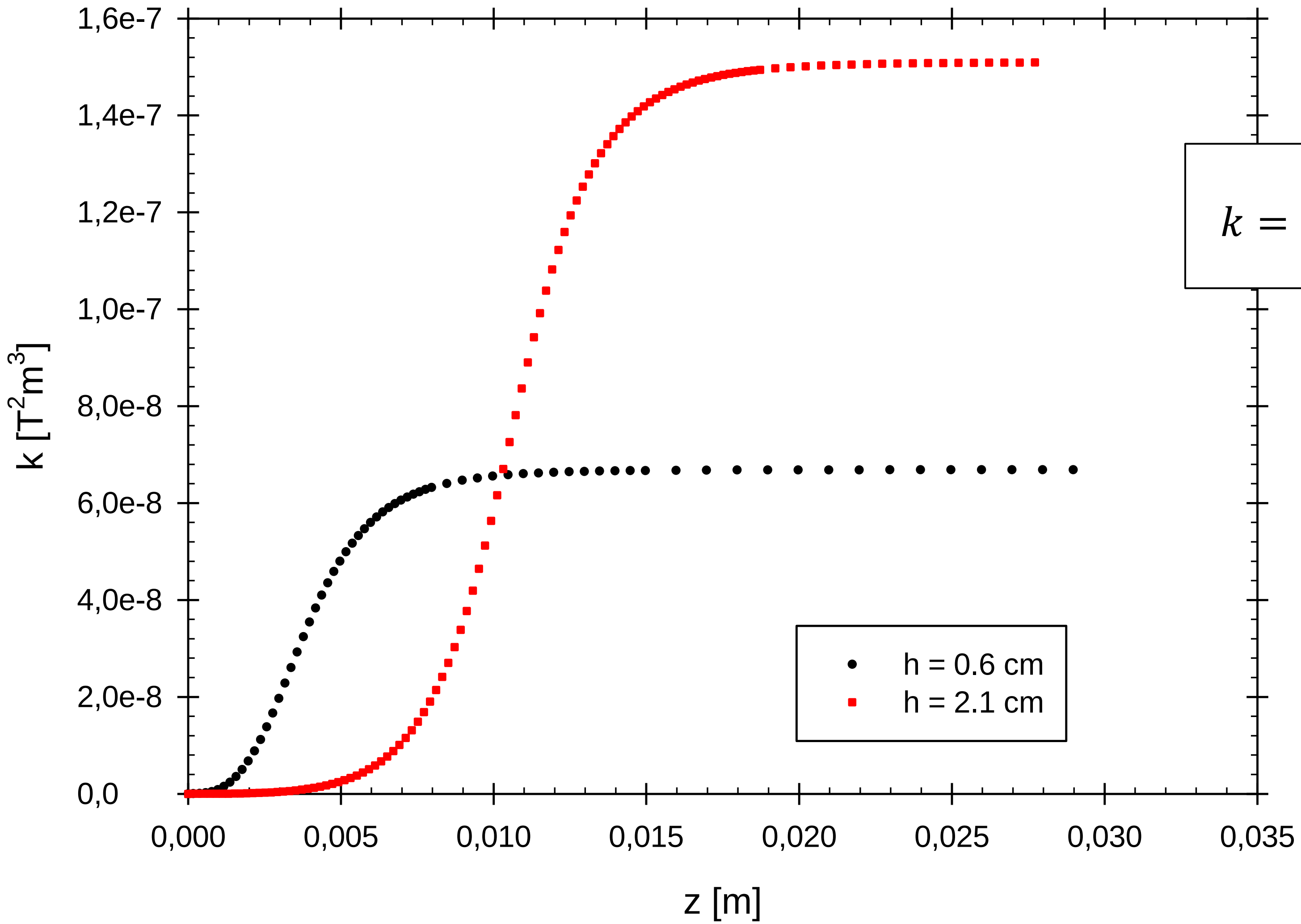
➢ Cylindrical magnets
  - $m = 1.13\ g$
  - $2r = 4mm; h = 3mm$

➢ Spherical magnets
  - $m = 0.50\ g$
  - $r = 2.5\ mm$

Example of obtained data - force

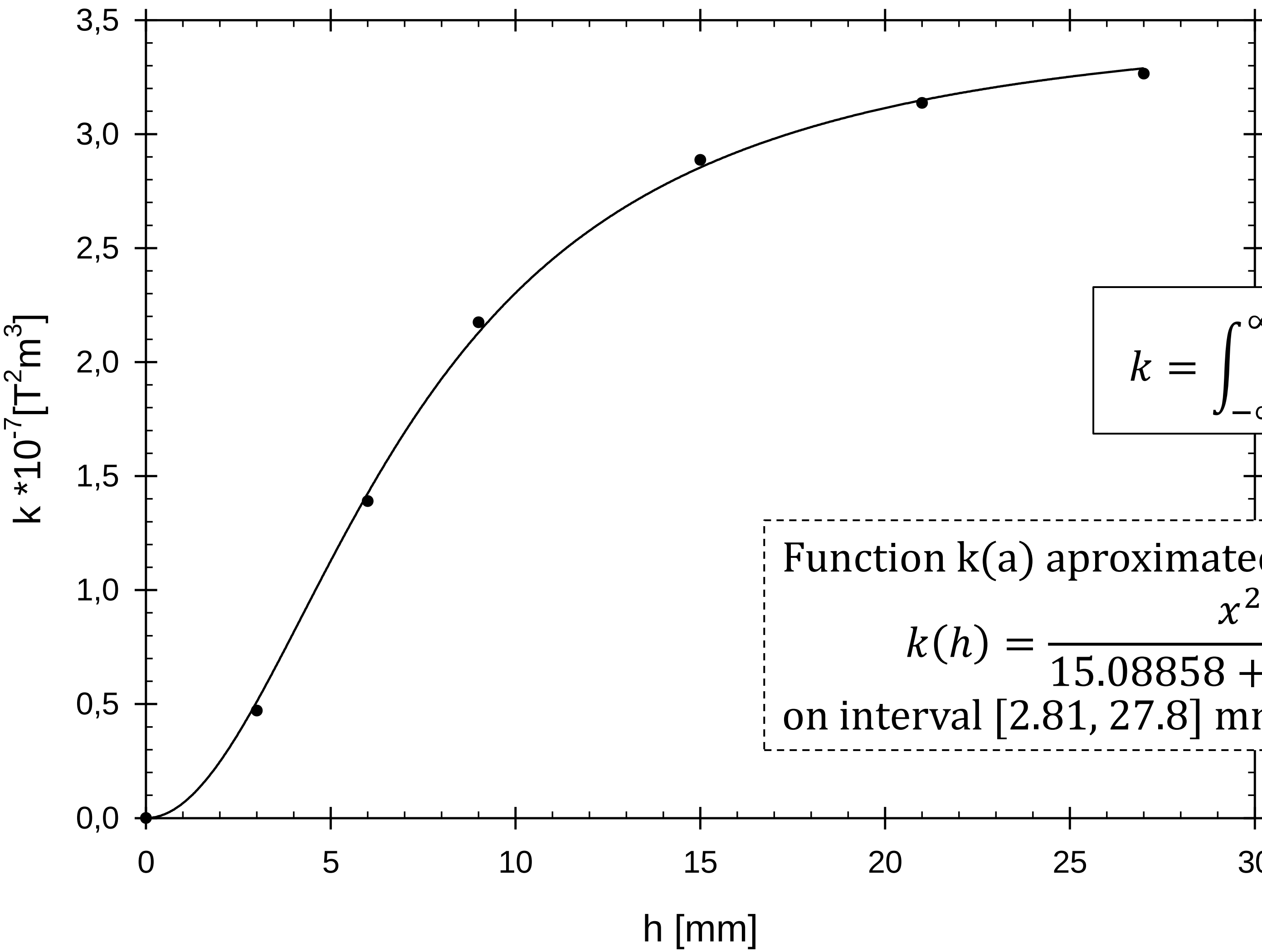


Determing magnetic flux change 2



$$k = \int_{-\infty}^{\infty} \left(\frac{d\phi}{dz}\right)^2 dz$$

Determing magnetic flux change 3



$$k = \int_{-\infty}^{\infty} \left(\frac{d\phi}{dz}\right)^2 dz$$

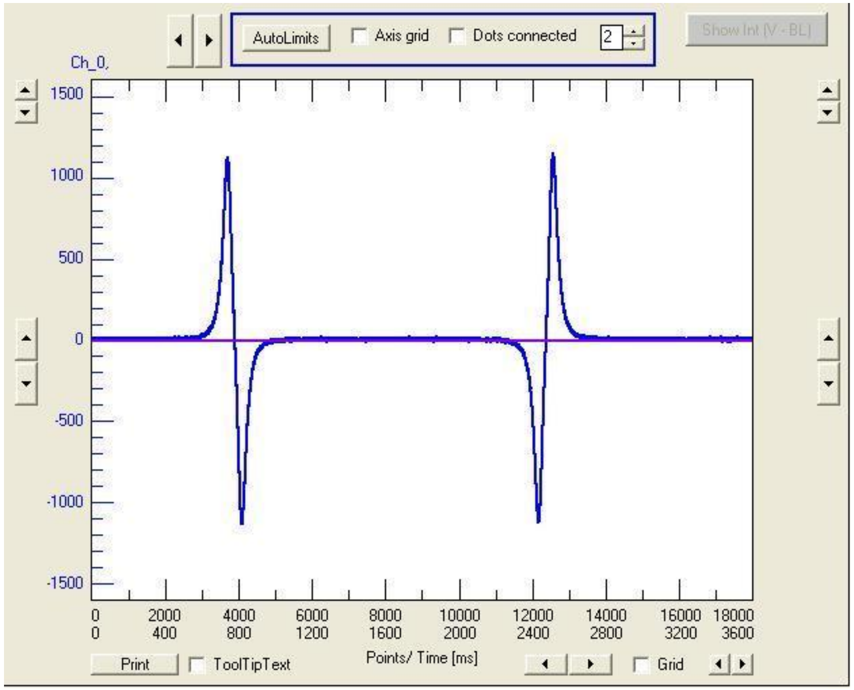
Function k(a) aproximated by

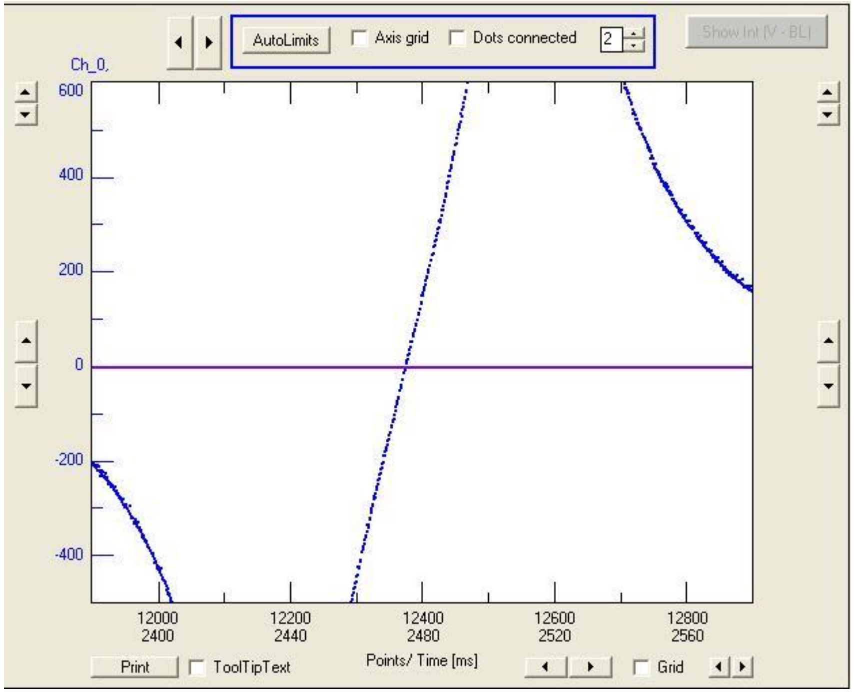
$$k(h) = \frac{x^2}{15.08858 + 0.28338}$$

on interval [2.81, 27.8] mm

Velocity measurement

- Coils detect passing magnet due to induction:

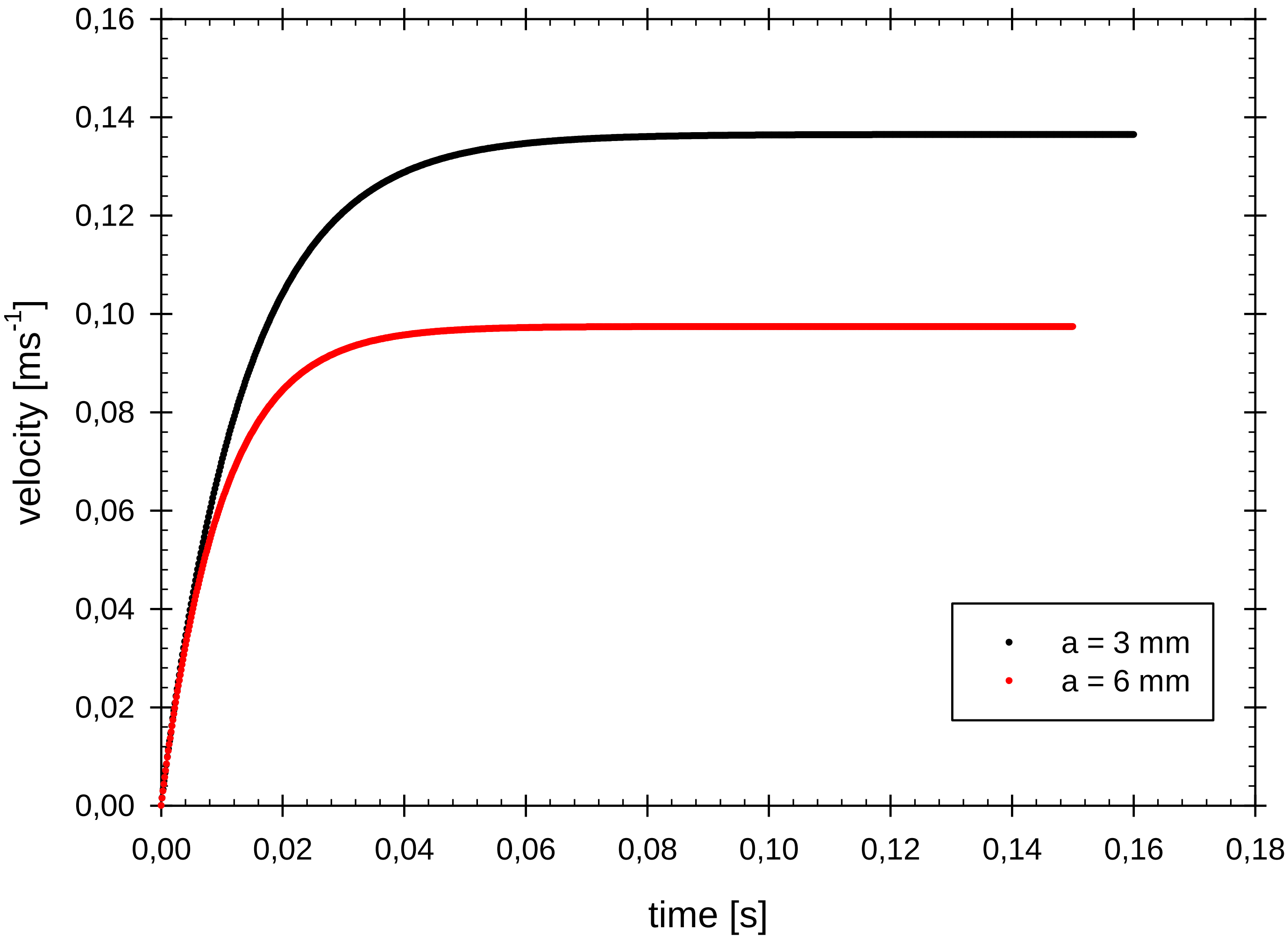




Aluminium, 2 solenoids

$$\varepsilon = \frac{d\phi}{dz} v$$

Theoretical change of velocity



$$v(t) = \tau g \left(1 - e^{-\frac{t}{\tau}}\right)$$

Supstitution:

$$\tau = \frac{2\pi \rho_m m_{mag}}{\sigma \Delta \rho k}$$

Cooling - change of the magnetization of the magnet

$$v \propto \frac{1}{M^2}$$

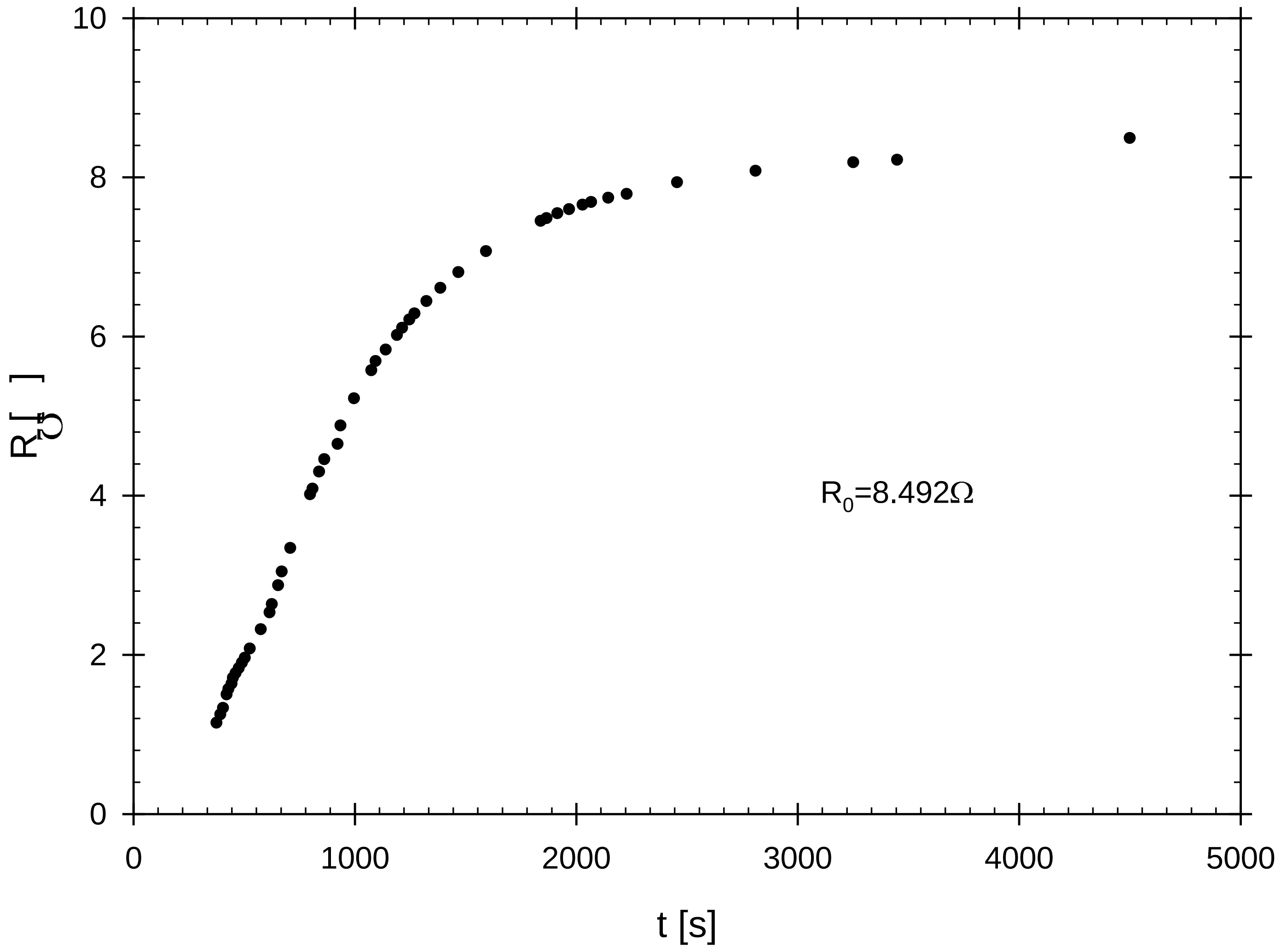
$$\frac{v_{warm}}{v_{cold}} = \frac{M_c^2}{M_w^2}$$

$$M_c = M_w \sqrt{\frac{v_{warm}}{v_{cold}}}$$

For our case:

$$M_c = 1.13 M_w$$

Heating of cooled tube



Cooling - change of tube dimesion

- Change of radius of the pipe

$$A = A_0(1 + 2\alpha\Delta T)$$
$$\frac{\rho_1}{\rho_0} = \sqrt{1 + 2\alpha\Delta T}$$

- $\frac{\rho_1}{\rho_0} = \sqrt{1 + 2 \cdot 16.6 \cdot 10^{-6} K^{-1} \cdot 216K} = 1.00357919468$

$\alpha_{cu} = 16.6 \cdot 10^{-6} K^{-1}$

Tube with slit vs tube without slit

	Tube	Tube with slit	Without tube
Time	$(2.61 \pm 0.05)s$	$(1.68 \pm 0.19)s$	$0.45\ s$

1 m long aluminium tube
$\rho = 1.25\ cm$; $\Delta\rho = 2\ mm$
Magnet properties

- Cylindrical magnet
- $2r = 1\ cm; h = 1cm$

Superconductor

Skin effect

- $J = J_s e^{-\frac{d}{\delta}}$