SLOVAKIA
IYPT '14

4

Ball Sound

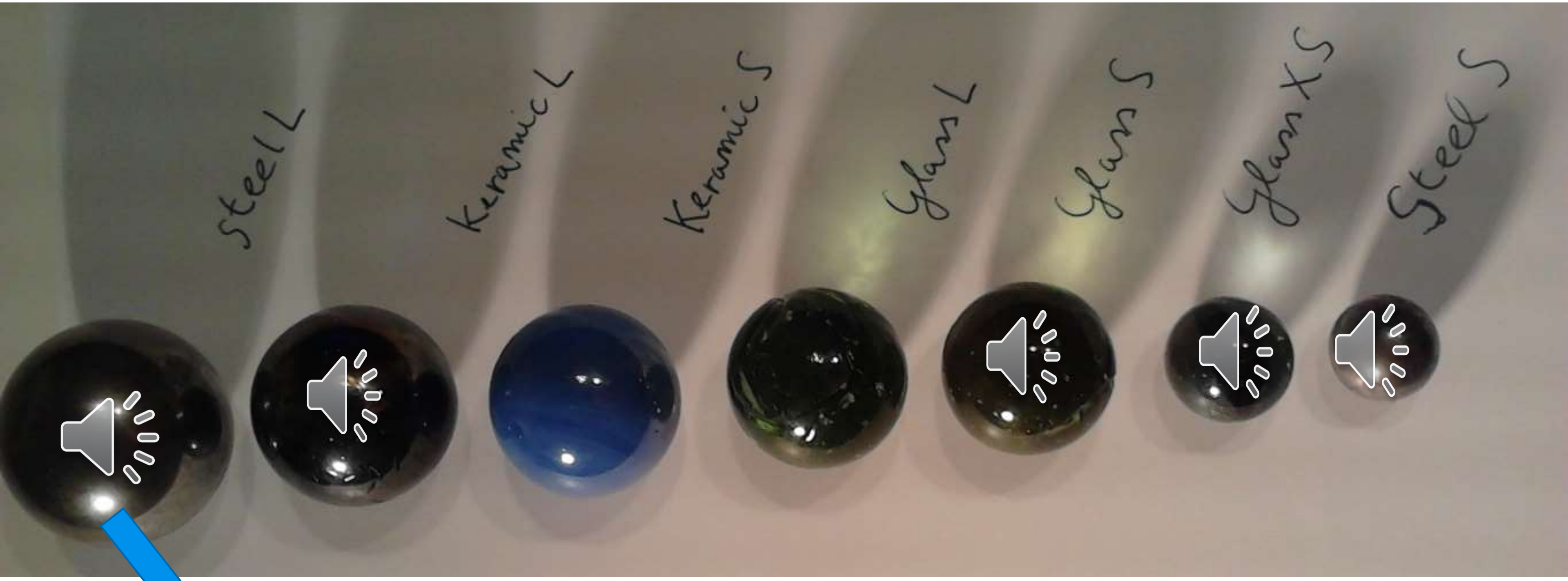
Jakub Chudík

Task

When two hard steel balls, or similar, are brought gently into contact with each other, an unusual **'chirping' sound** may be produced.

Investigate and explain the nature of the sound.

Different Balls



Collisions between different balls and one large steel ball

Chosen for the longest duration and the lowest prevalent frequency

In our presentation

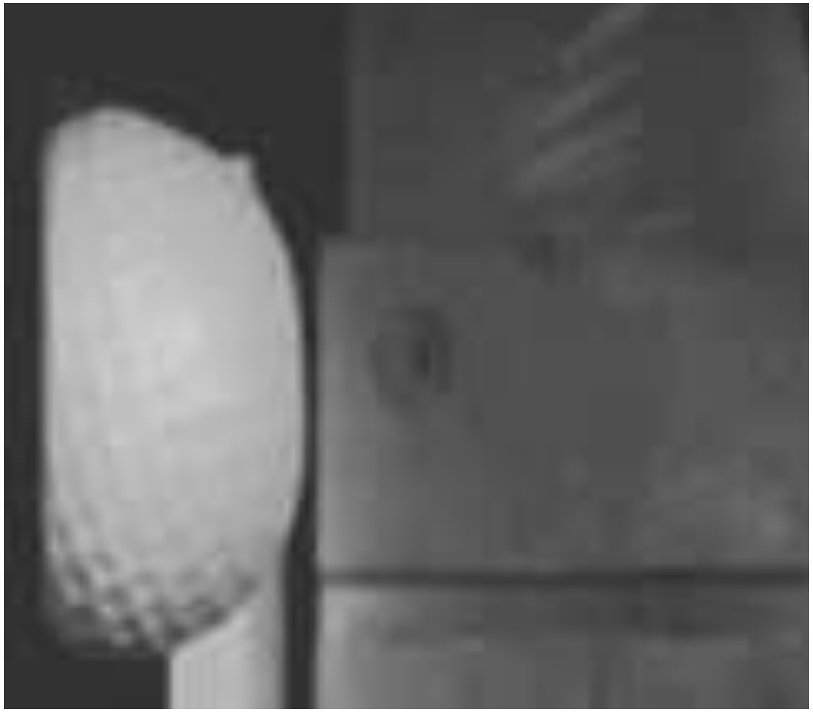
Nature of the sound

Examine the sound

Recreate the sound

Possible sources

Vibration of the balls



Impulsive translational acceleration

Vibrations for steel balls of 5cm diam.

Mode number	Natural frequency (kHz)
1	51.8
2	68.3
3	77[illegible]
4	93.1
[illegible]	97[illegible]
6	98.5
7	118.7
8	128.2
9	136.7
10	138.3

Lowest frequency is 51,8 kHz

[illegible] frequency for our [illegible] kHz

Vibrations of the balls certainly
do not *contribute to audible sound!*

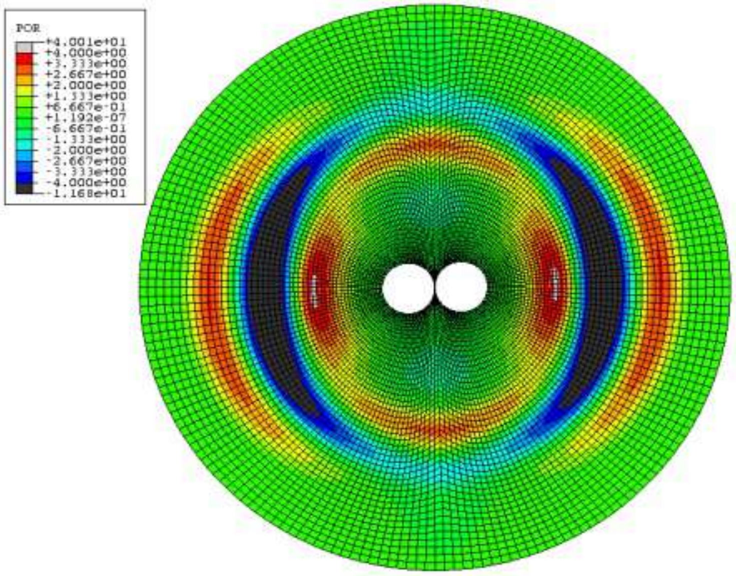
Limit of human ear is 18-20 kHz

K. Mehraby et al. „*Impact noise radiated by collision of two spheres*“, Journal of Mechanical Science and Technology 25 (7) (2011)

Impulsive translational acceleration

One oscillating sphere

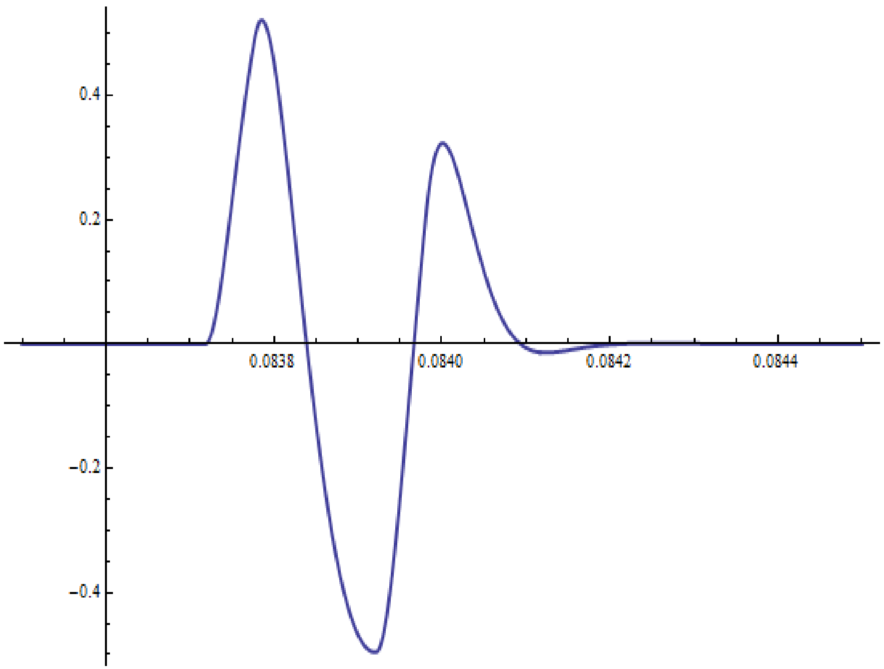
Two colliding spheres



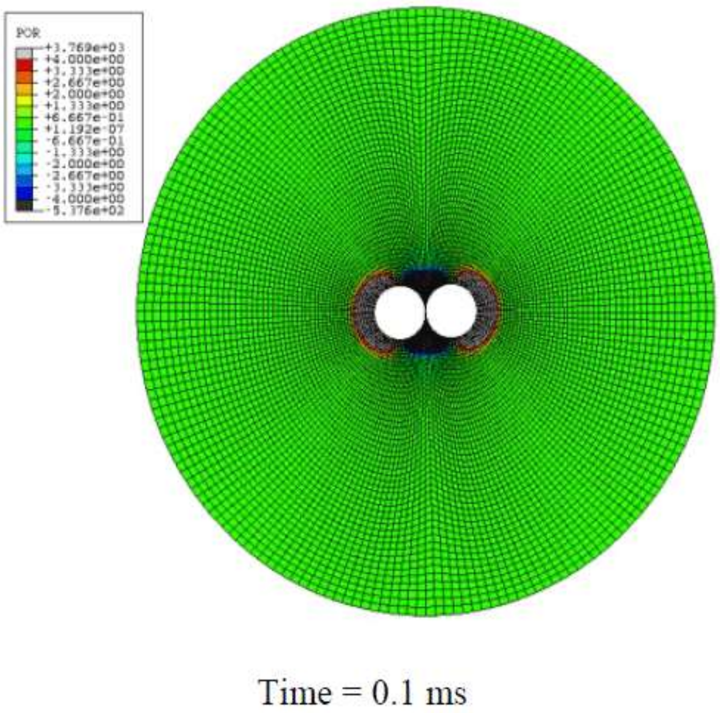
K. Mehraby et al. *„Impact noise radiated by collision of two spheres“*, Journal of Mechanical Science and Technology 25 (7) (2011)

Impulsive translational acceleration

One collision



Two colliding spheres



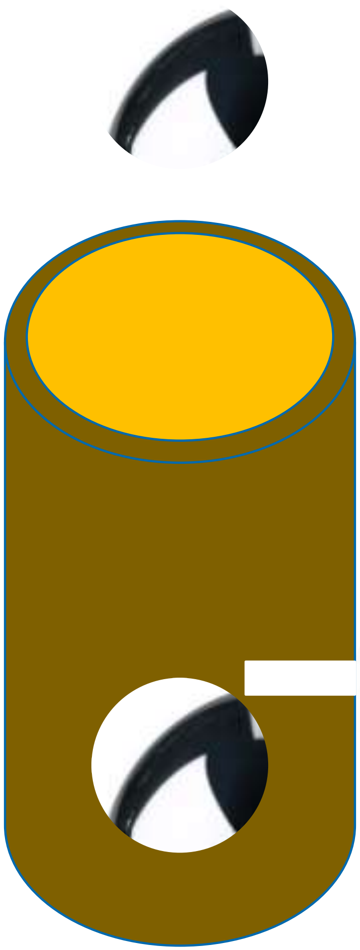
K. Mehraby et al. *„Impact noise radiated by collision of two spheres“*, Journal of Mechanical Science and Technology 25 (7) (2011)

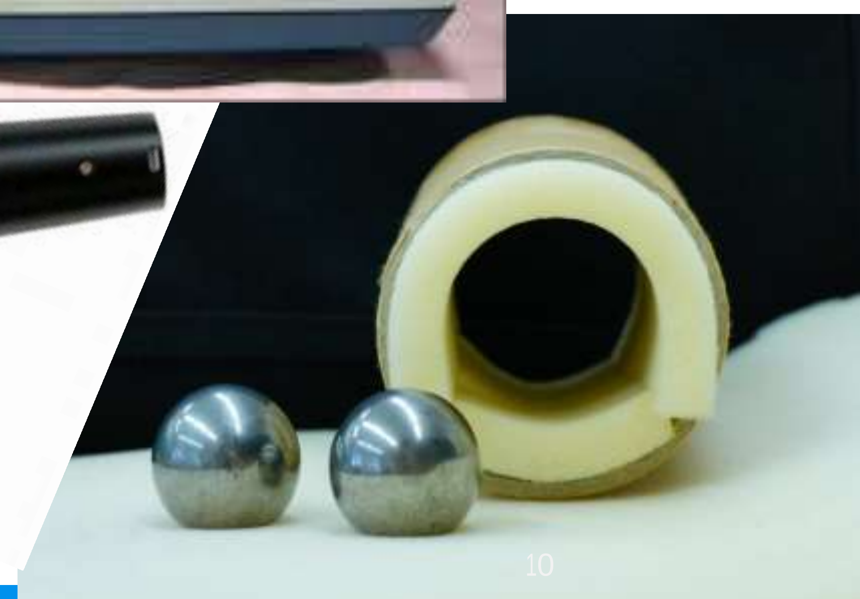
Nature of the sound

Examine the sound

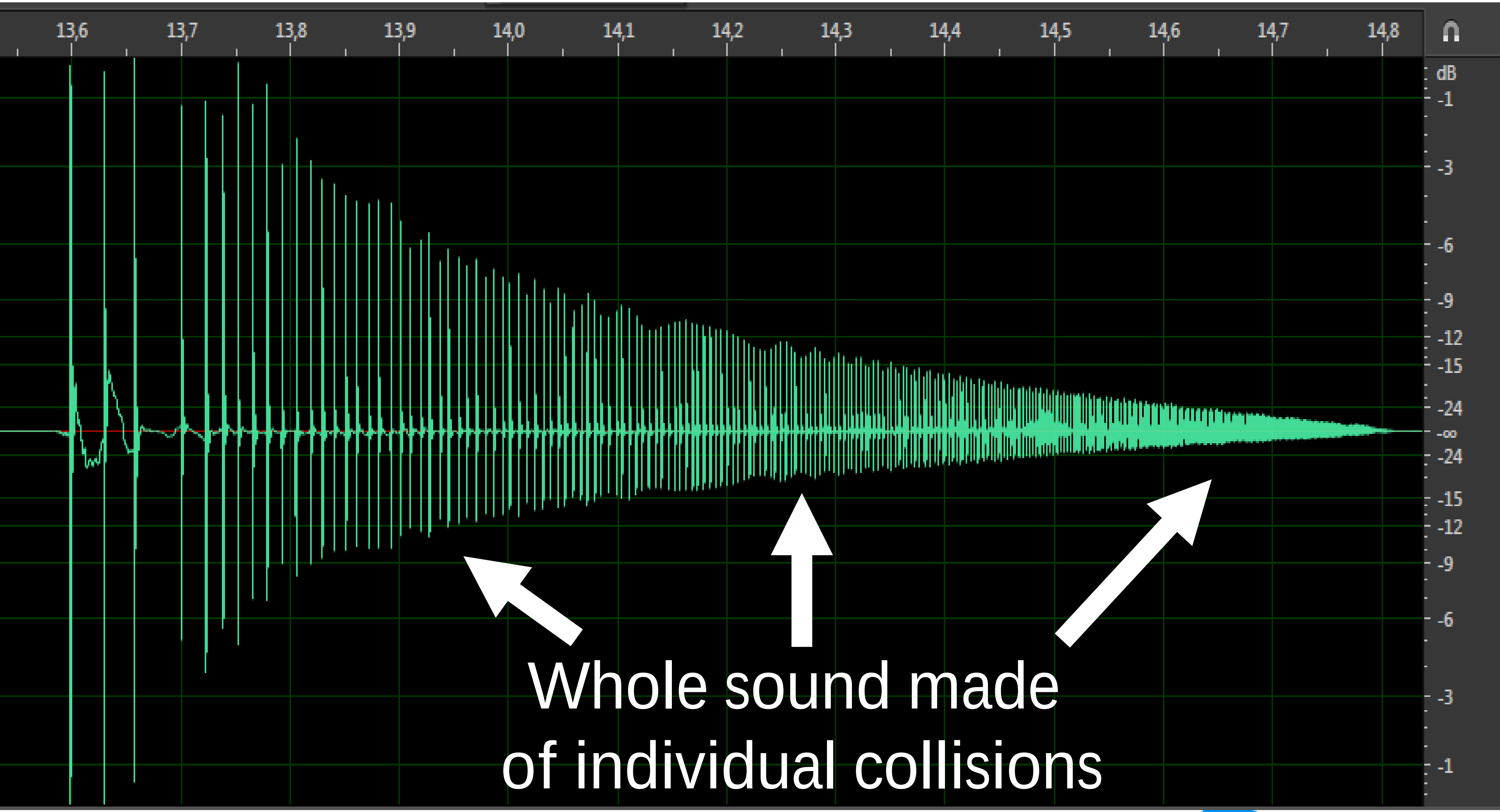
Recreate the sound

Our apparatus

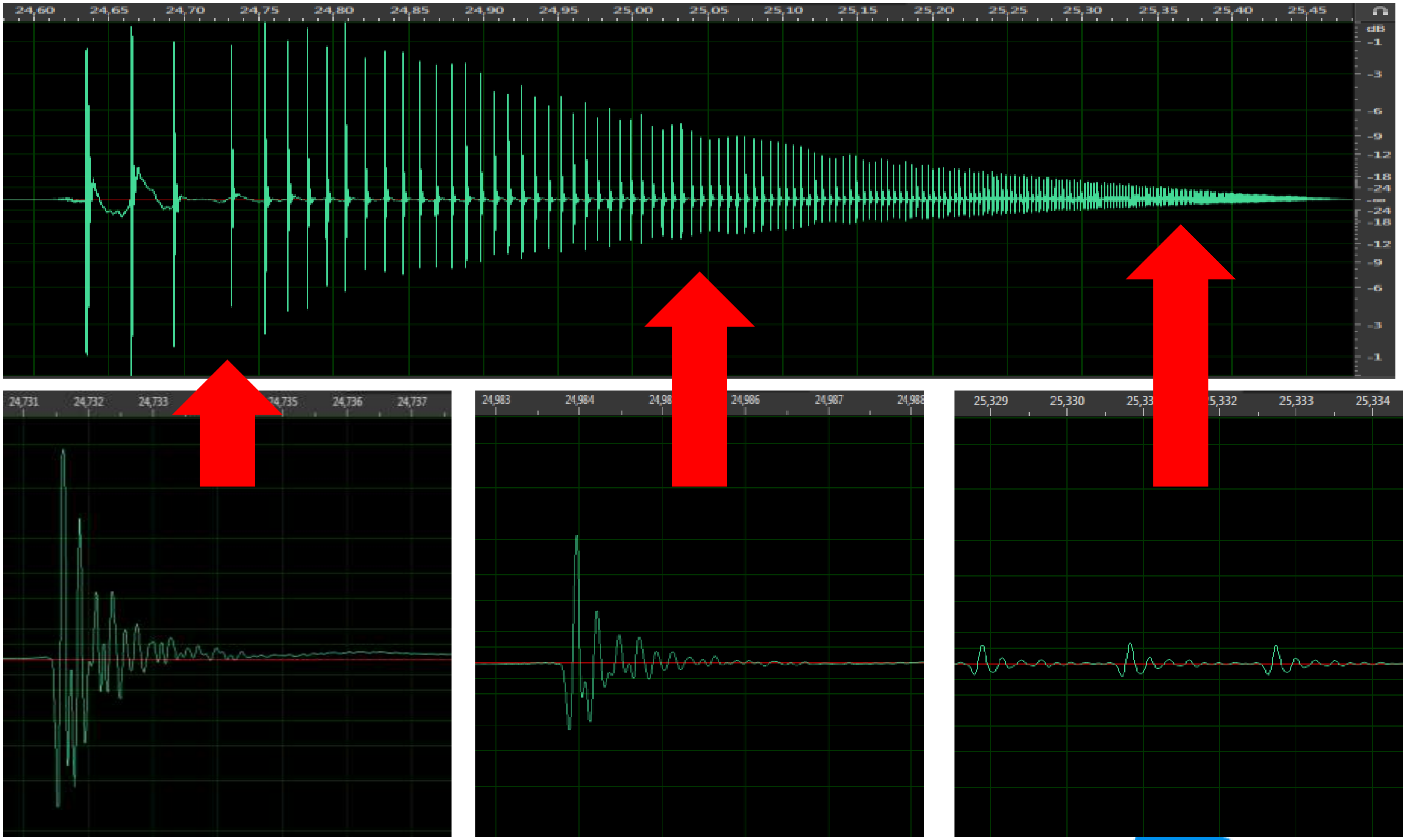




Sound analysis



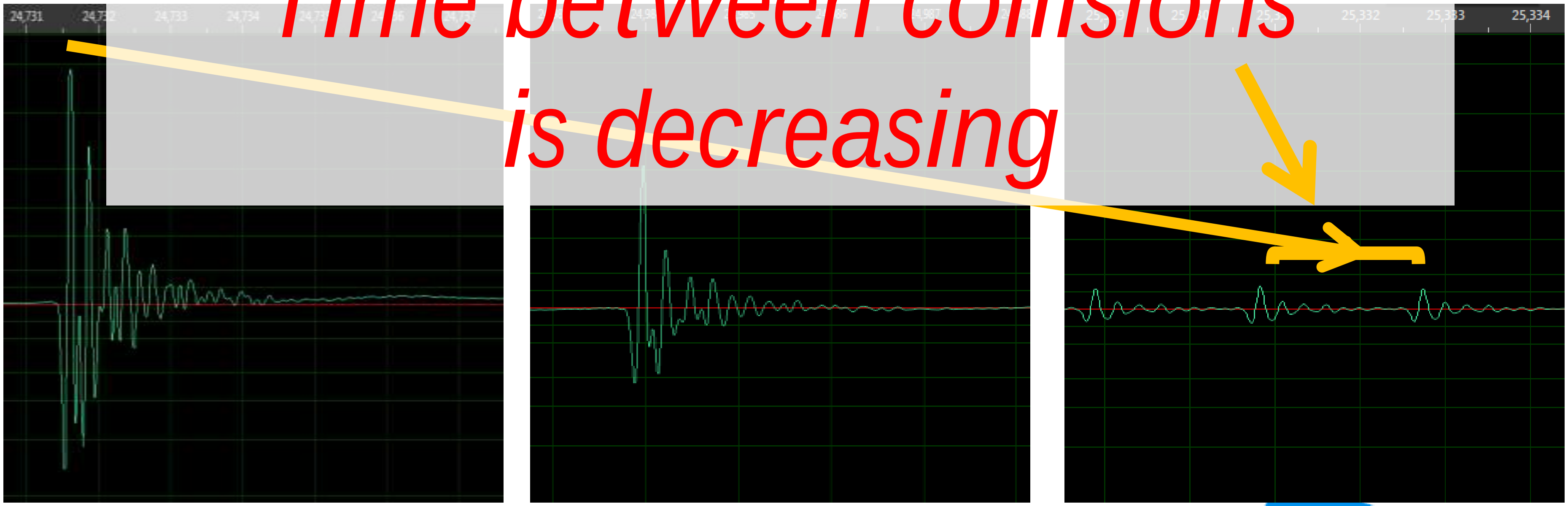
Sound of individual collisions



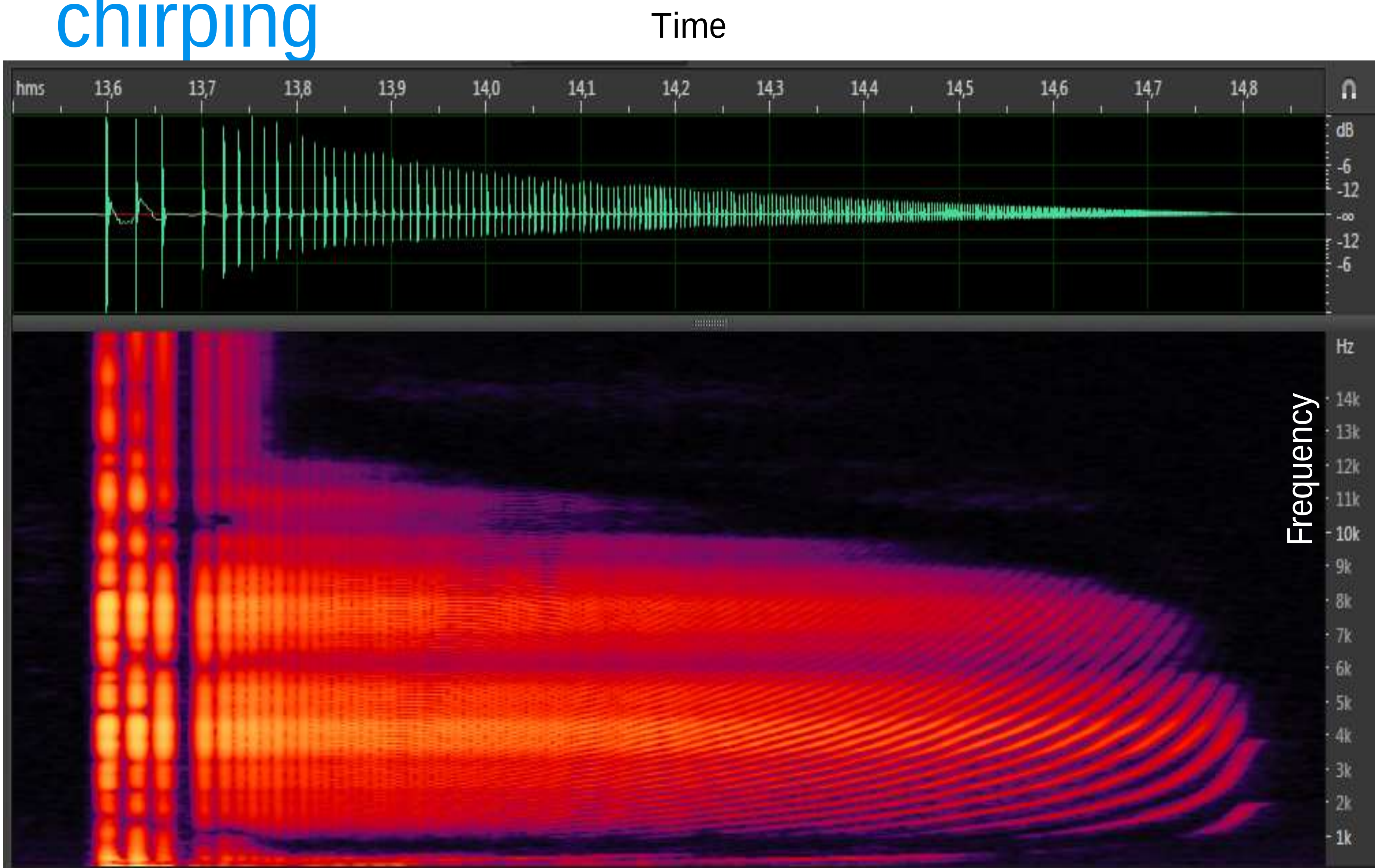
Sound of individual collisions

- Decreasing amplitude
- Individual collisions are similar
- But the chirping has a rising tone

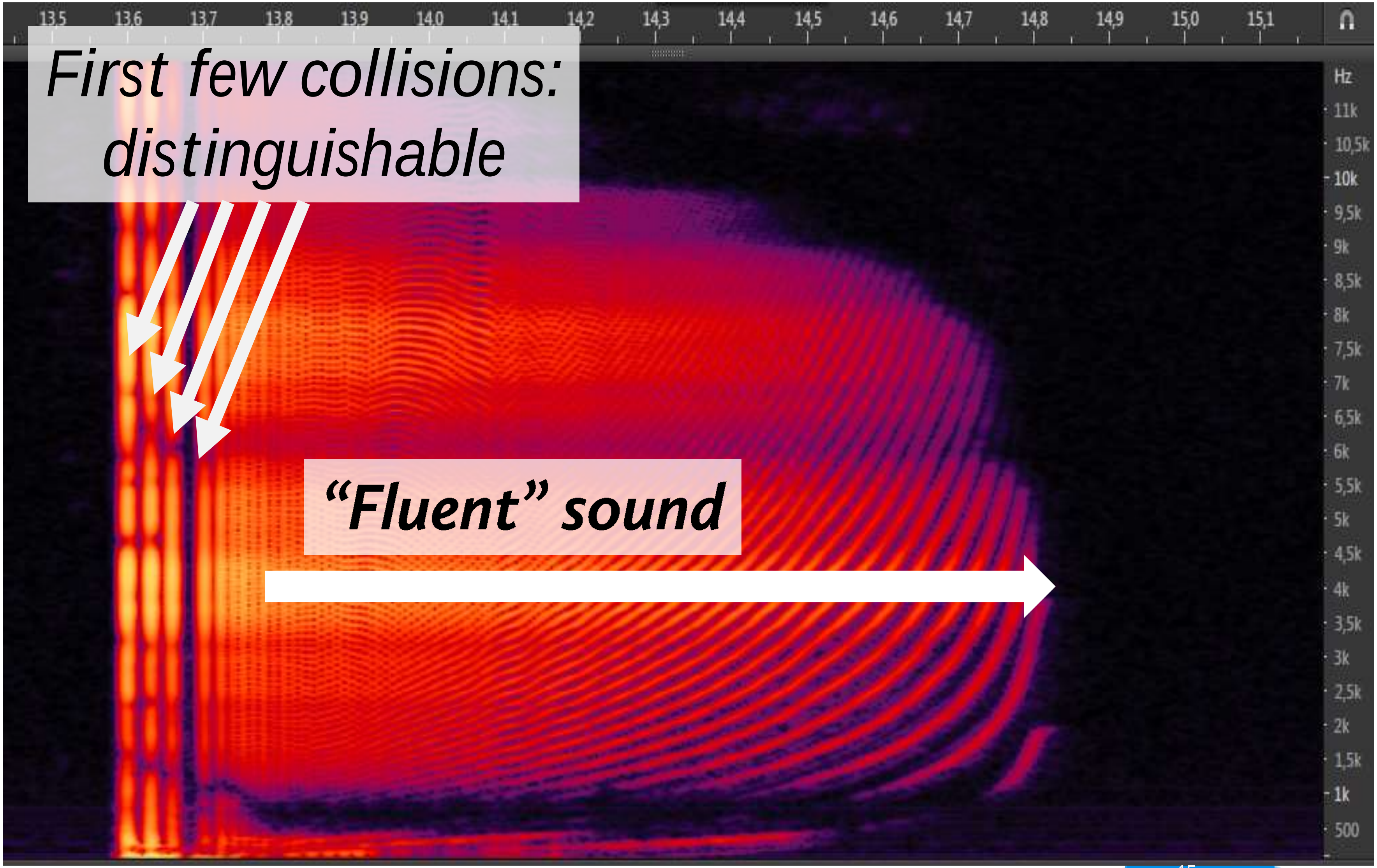
Time between collisions is decreasing



Spectral analysis of our measured chirping



Spectrum of chirping sound (FFT)



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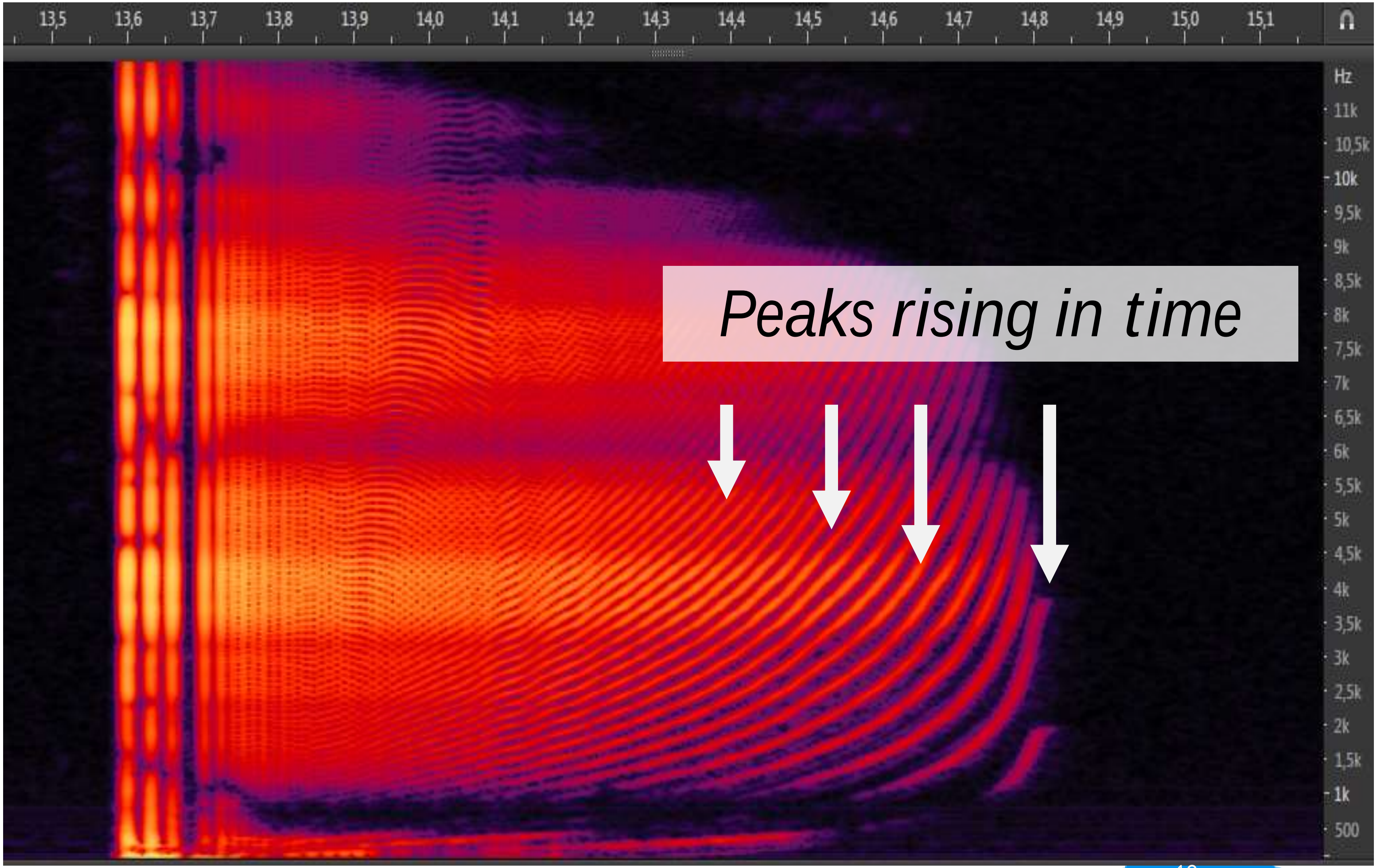
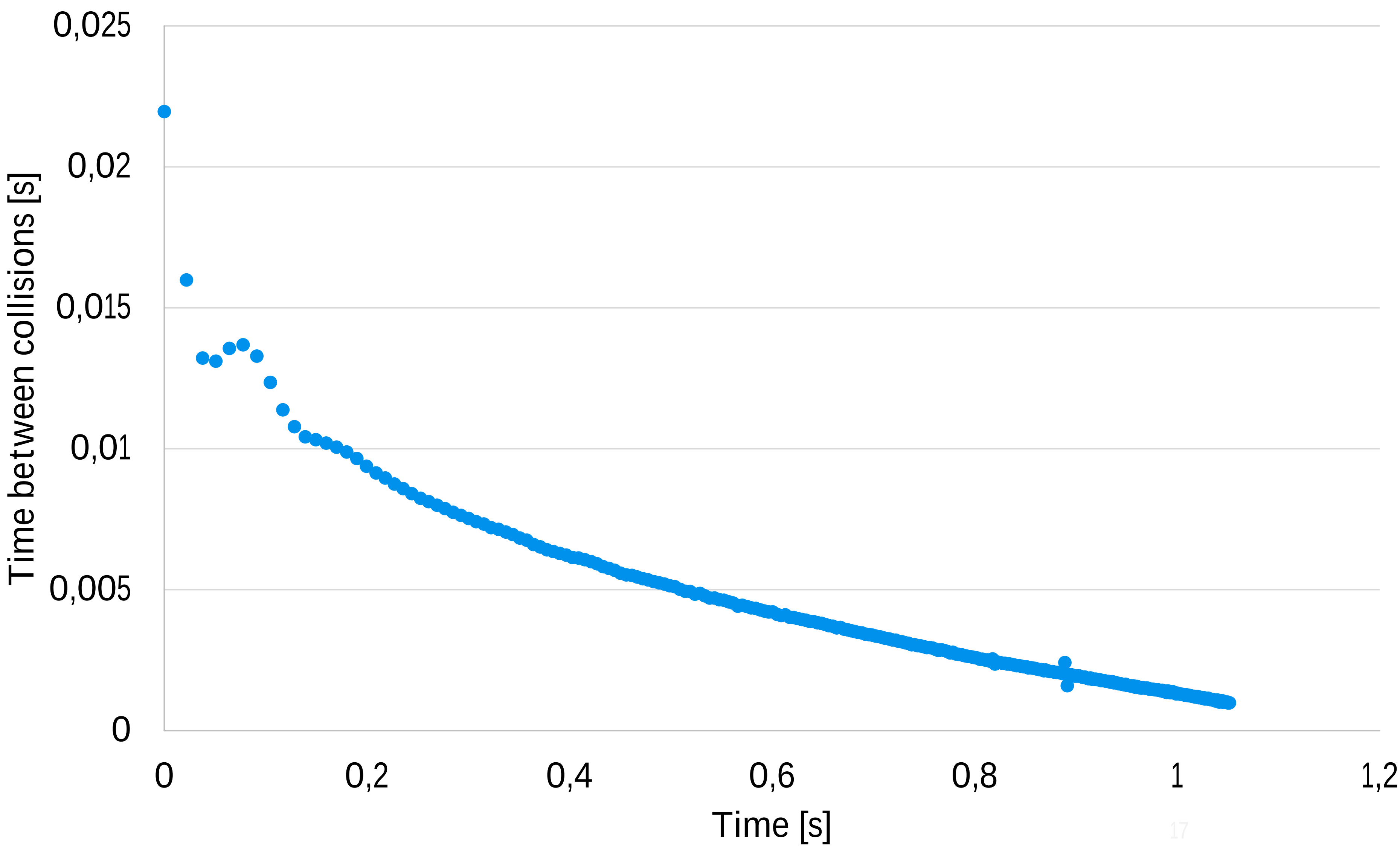
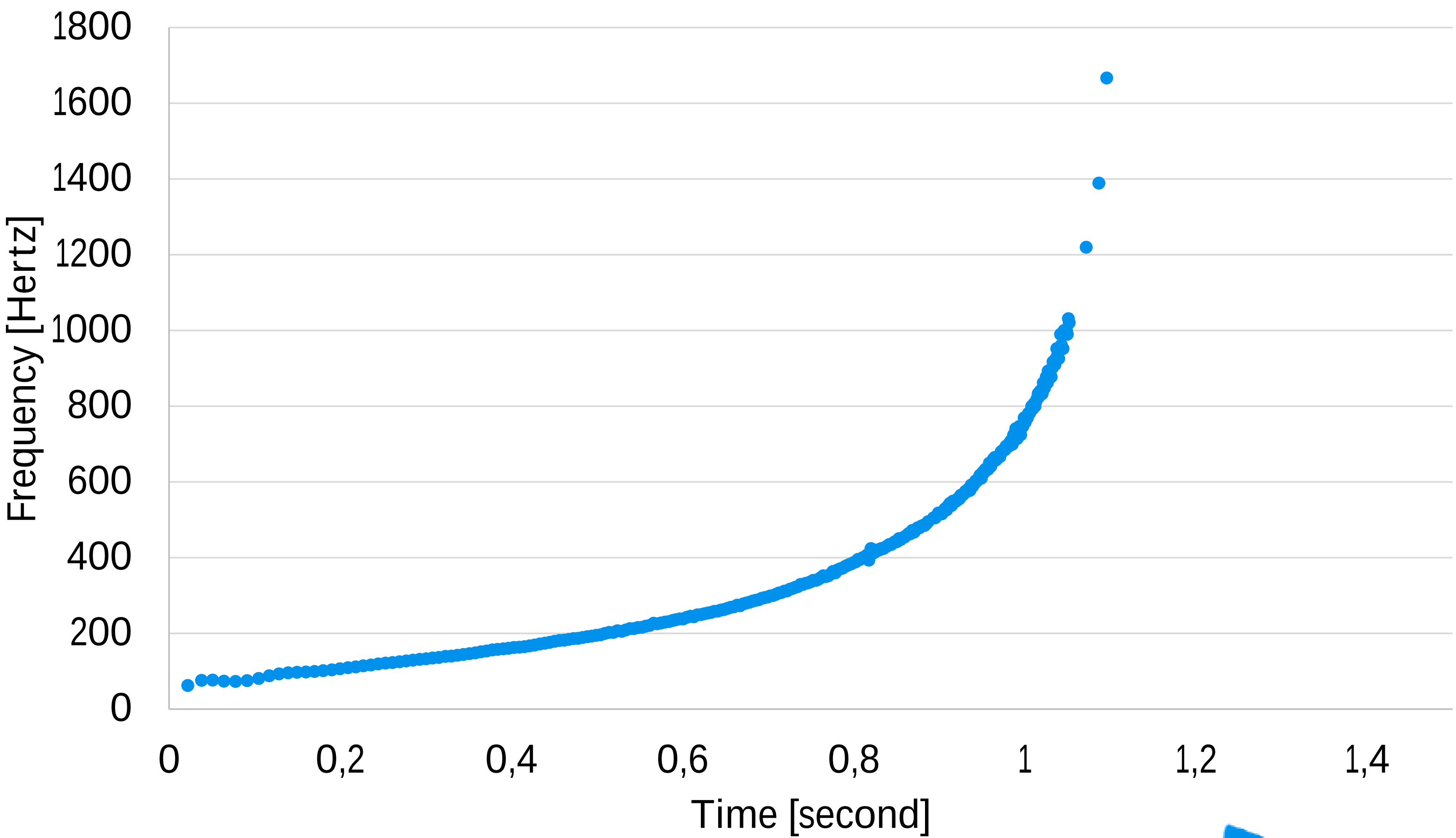


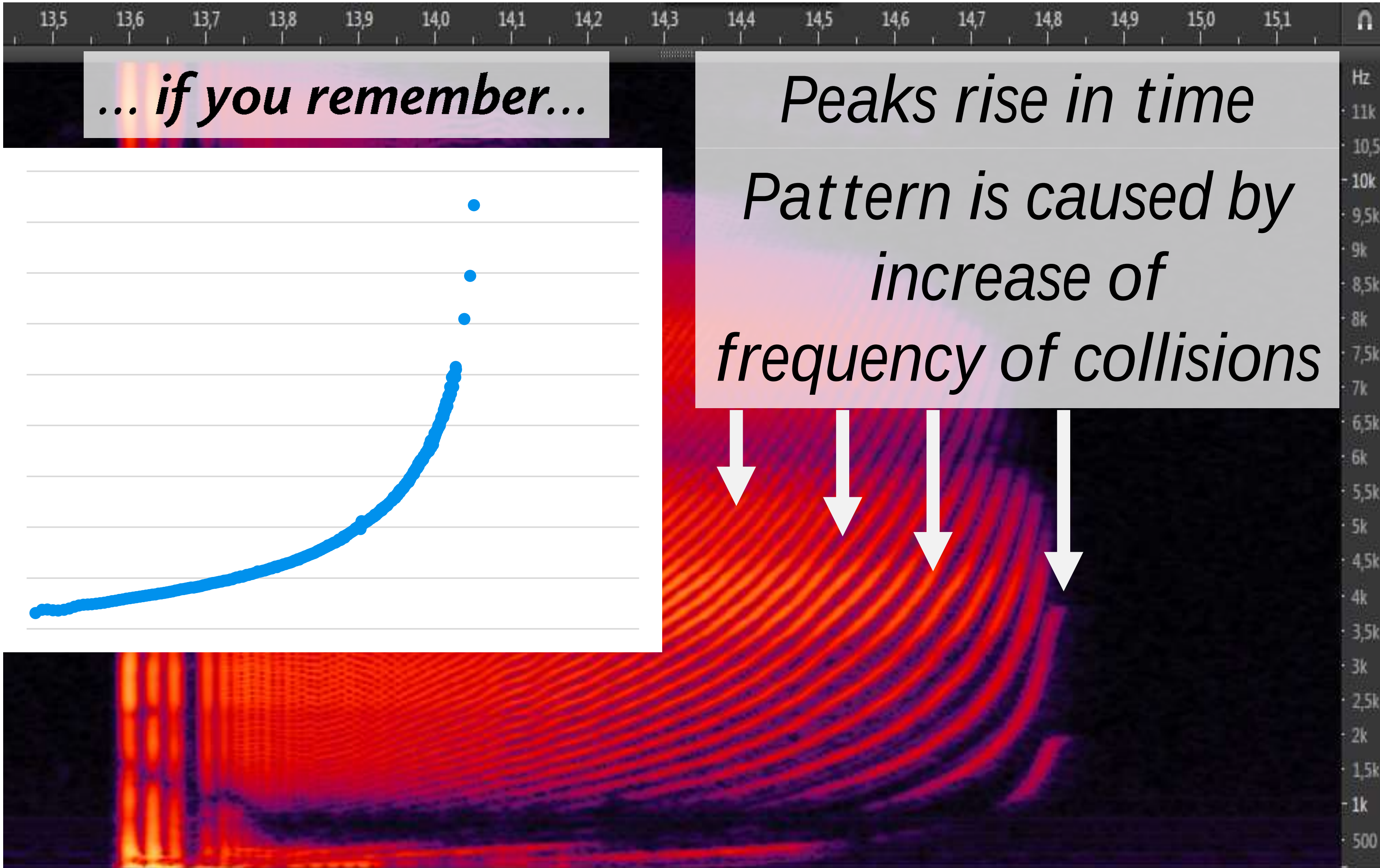
Time between collisions

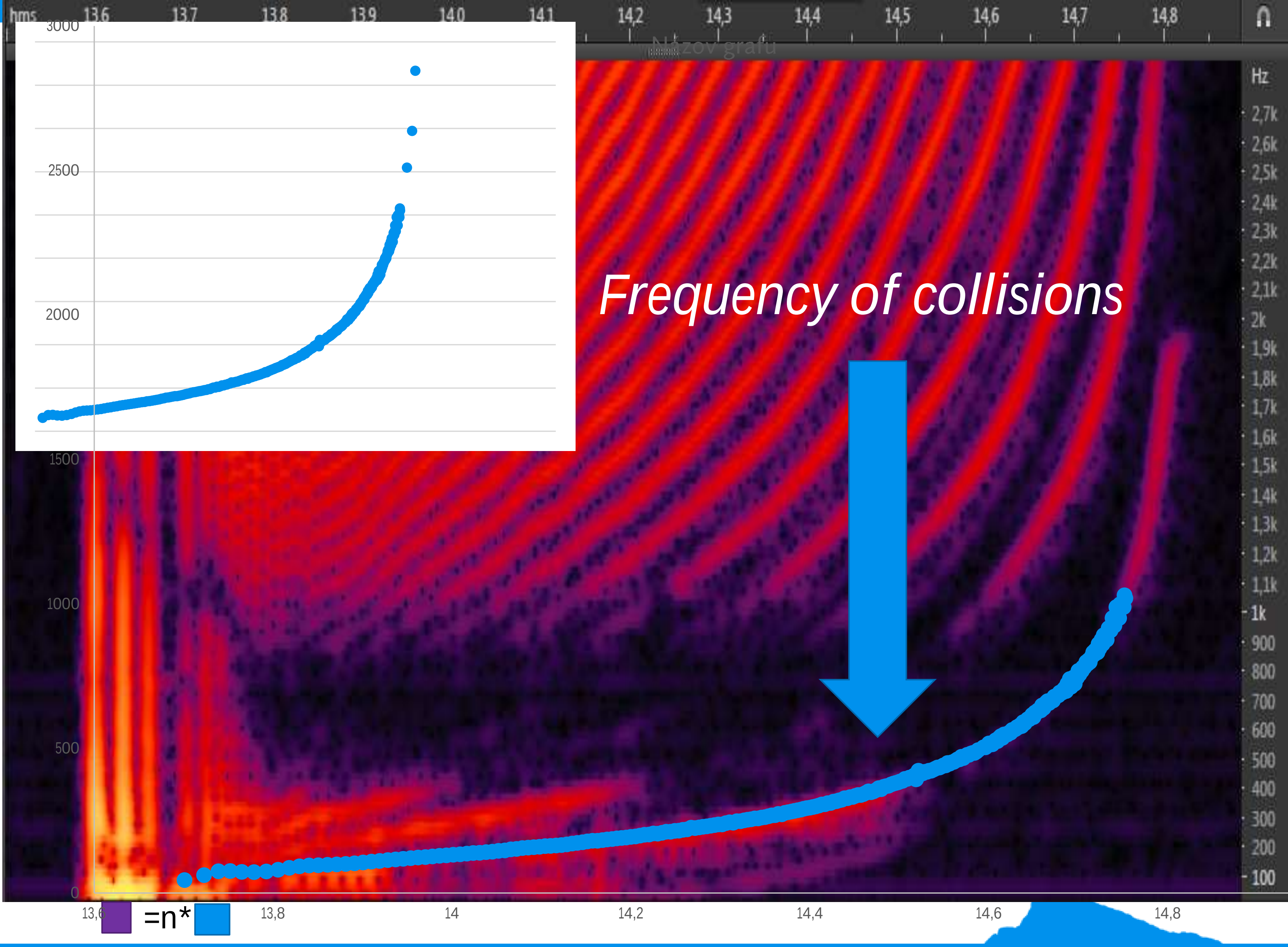


Frequency of collisions in time

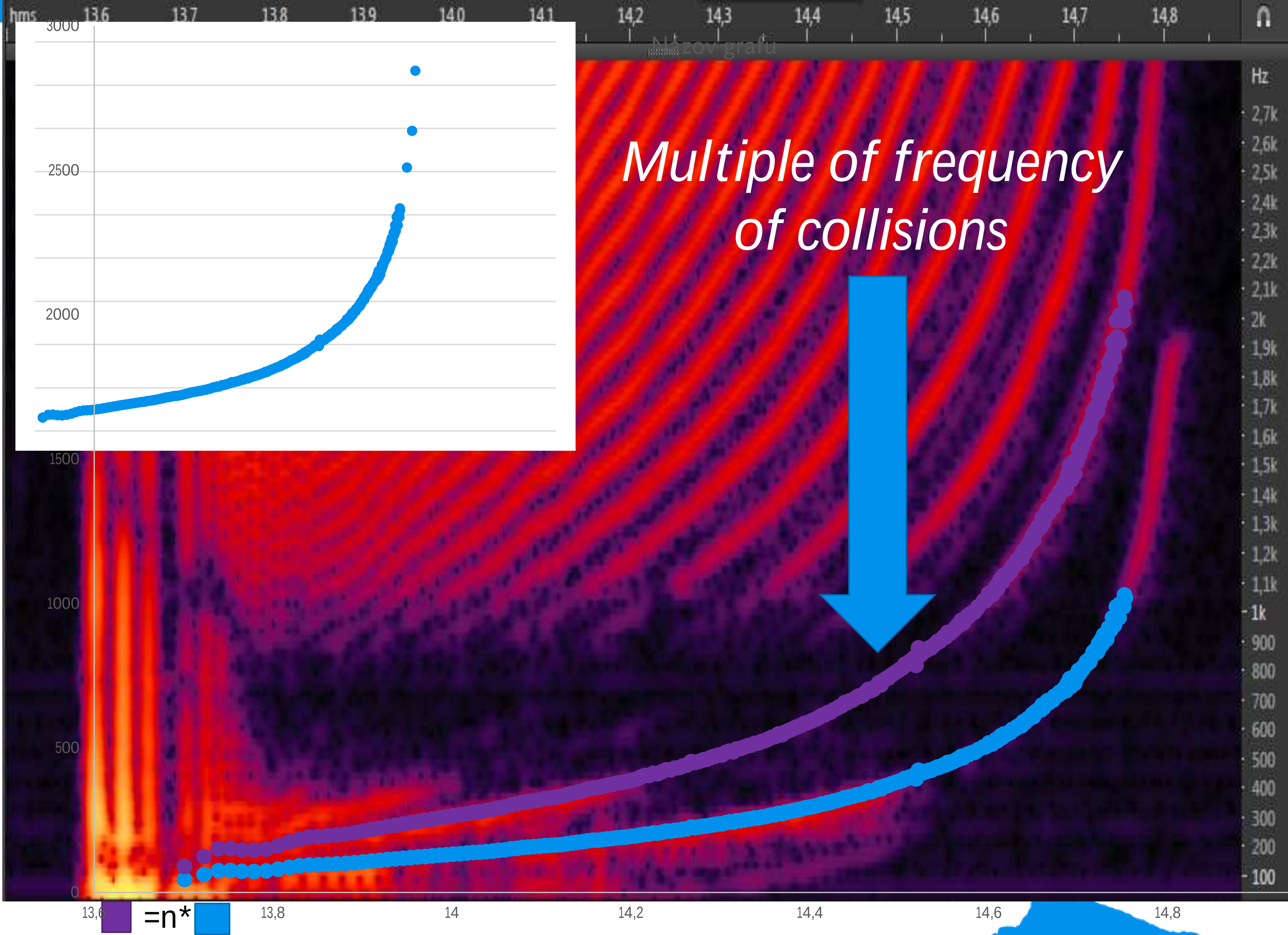


Spectrum of chirping sound (FFT)

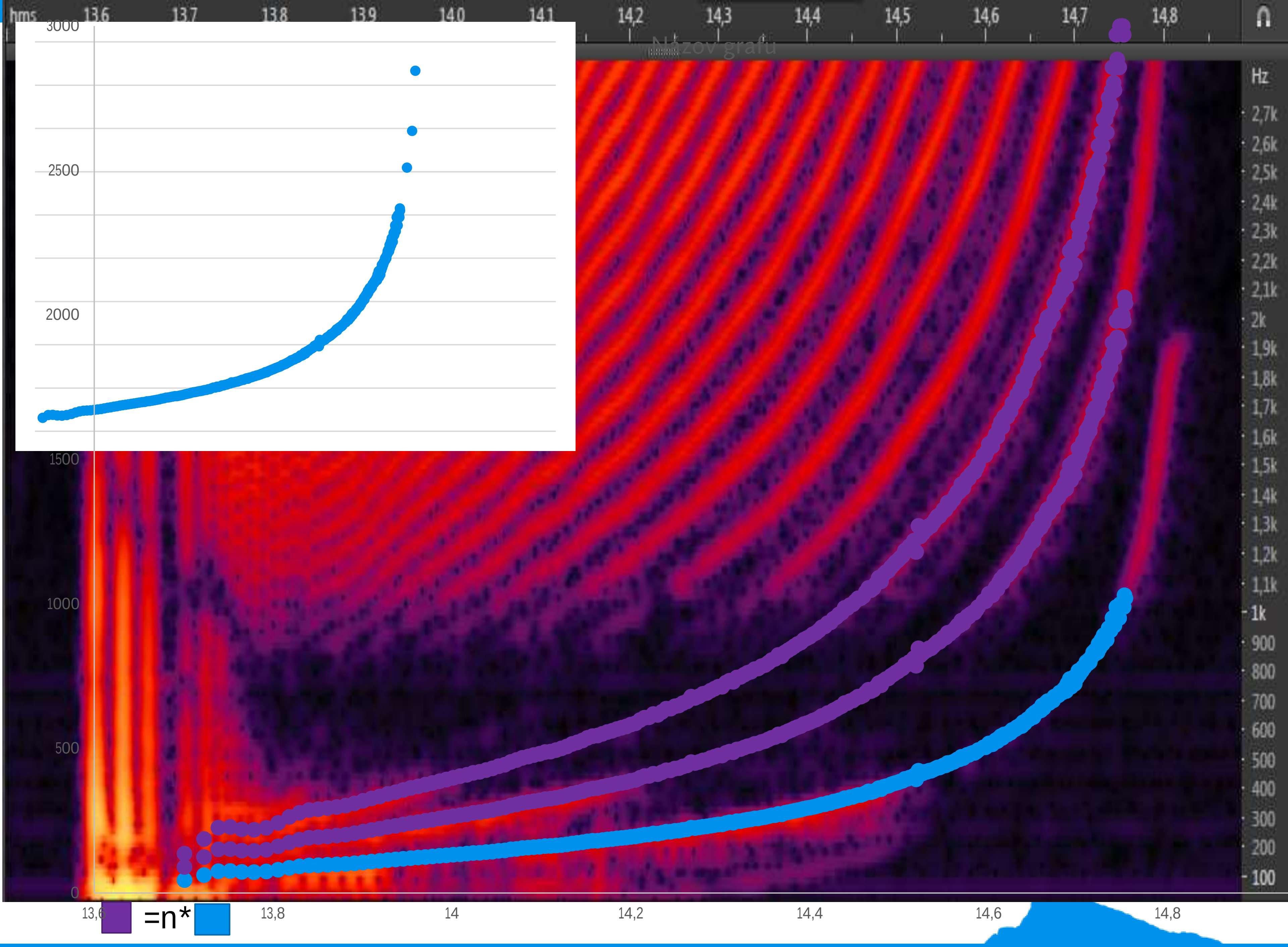




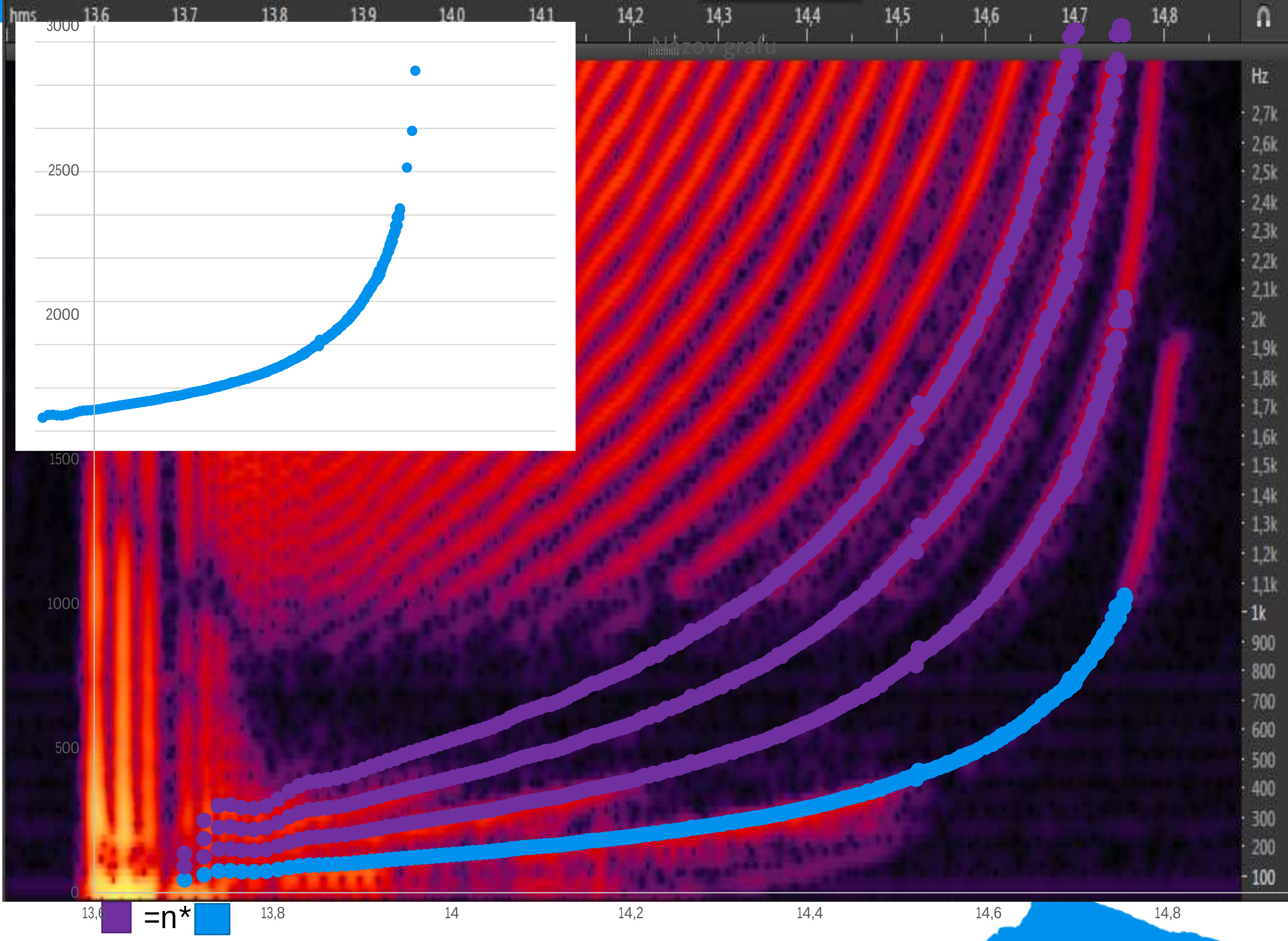
Frequency of collisions
=n*
Názov grafu
hms
Hz



Multiple of frequency of collisions
hms
13,6
13,7
13,8
13,9
14,0
14,1
14,2
14,3
14,4
14,5
14,6
14,7
14,8
Názov grafu
Hz
2,7k
2,6k
2,5k
2,4k
2,3k
2,2k
2,1k
2k
1,9k
1,8k
1,7k
1,6k
1,5k
1,4k
1,3k
1,2k
1,1k
1k
900
800
700
600
500
400
300
200
100
3000
2500
2000
1500
1000
500
0
13,6
13,8
14
14,2
14,4
14,6
14,8
=n*



hms
13,6
13,7
13,8
13,9
14,0
14,1
14,2
14,3
14,4
14,5
14,6
14,7
14,8
Hz
2,7k
2,6k
2,5k
2,4k
2,3k
2,2k
2,1k
2k
1,9k
1,8k
1,7k
1,6k
1,5k
1,4k
1,3k
1,2k
1,1k
1k
900
800
700
600
500
400
300
200
100
3000
2500
2000
1500
1000
500
0
14
14,2
14,4
14,6
14,8
=n*



hms
13,6
13,7
13,8
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14,0
14,1
14,2
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14,4
14,5
14,6
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Názov grafu
Hz
2,7k
2,6k
2,5k
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2,3k
2,2k
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1,9k
1,8k
1,7k
1,6k
1,5k
1,4k
1,3k
1,2k
1,1k
1k
900
800
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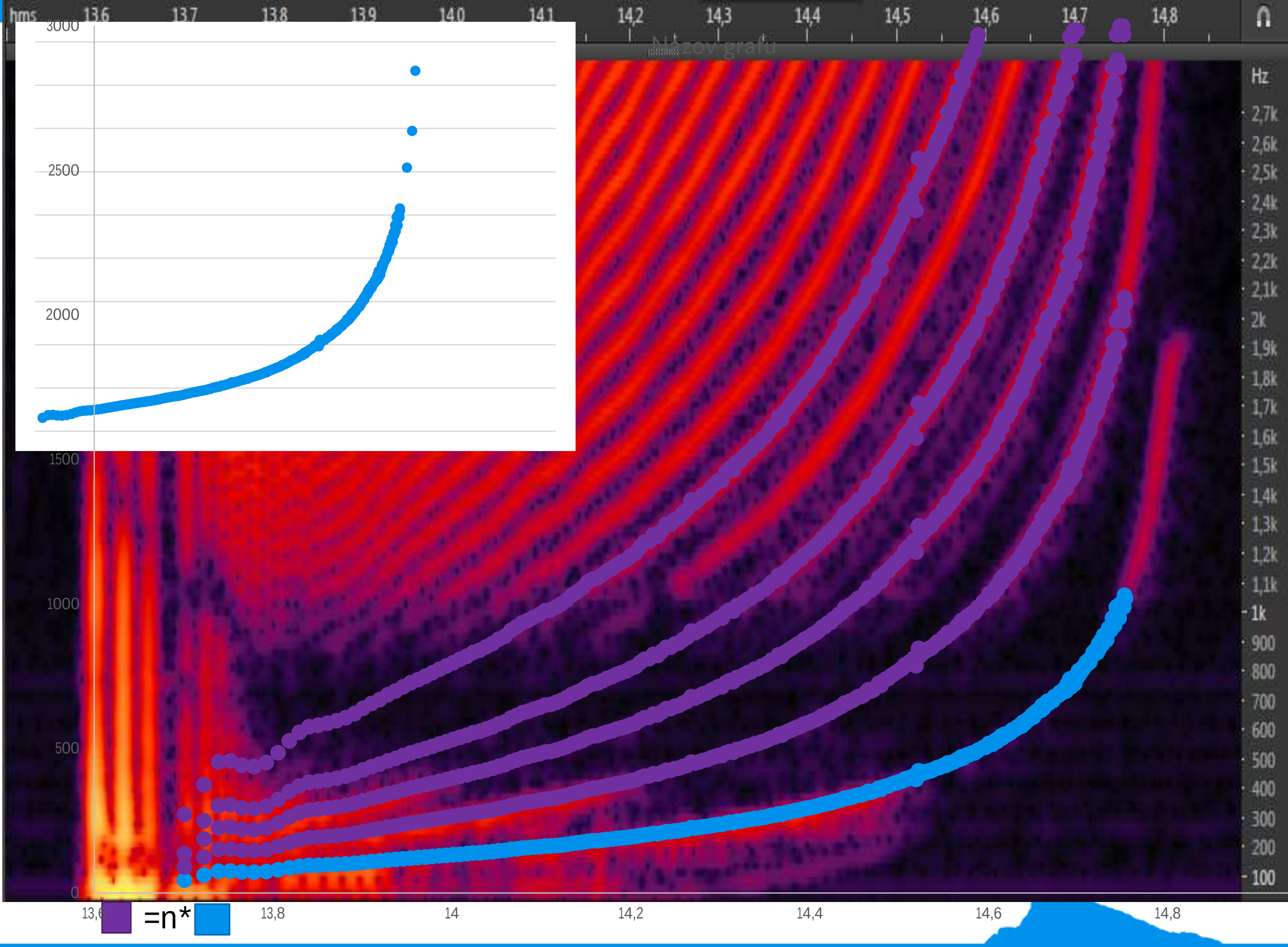
1500
1000
500
0
13,6
13,8
14
14,2
14,4
14,6
14,8
=n*



hms
13,6
13,7
13,8
13,9
14,0
14,1
14,2
14,3
14,4
14,5
14,6
14,7
14,8
Hz
2,7k
2,6k
2,5k
2,4k
2,3k
2,2k
2,1k
2k
1,9k
1,8k
1,7k
1,6k
1,5k
1,4k
1,3k
1,2k
1,1k
1k
900
800
700
600
500
400
300
200
100
3000
2500
2000
1500
1000
500
0
13,6
13,8
14
14,2
14,4
14,6
14,8
=n*

Nature of the sound

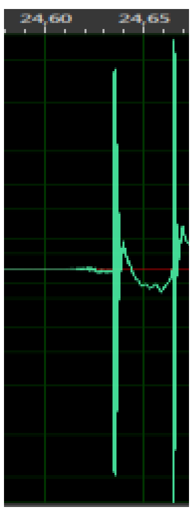
Examine the sound

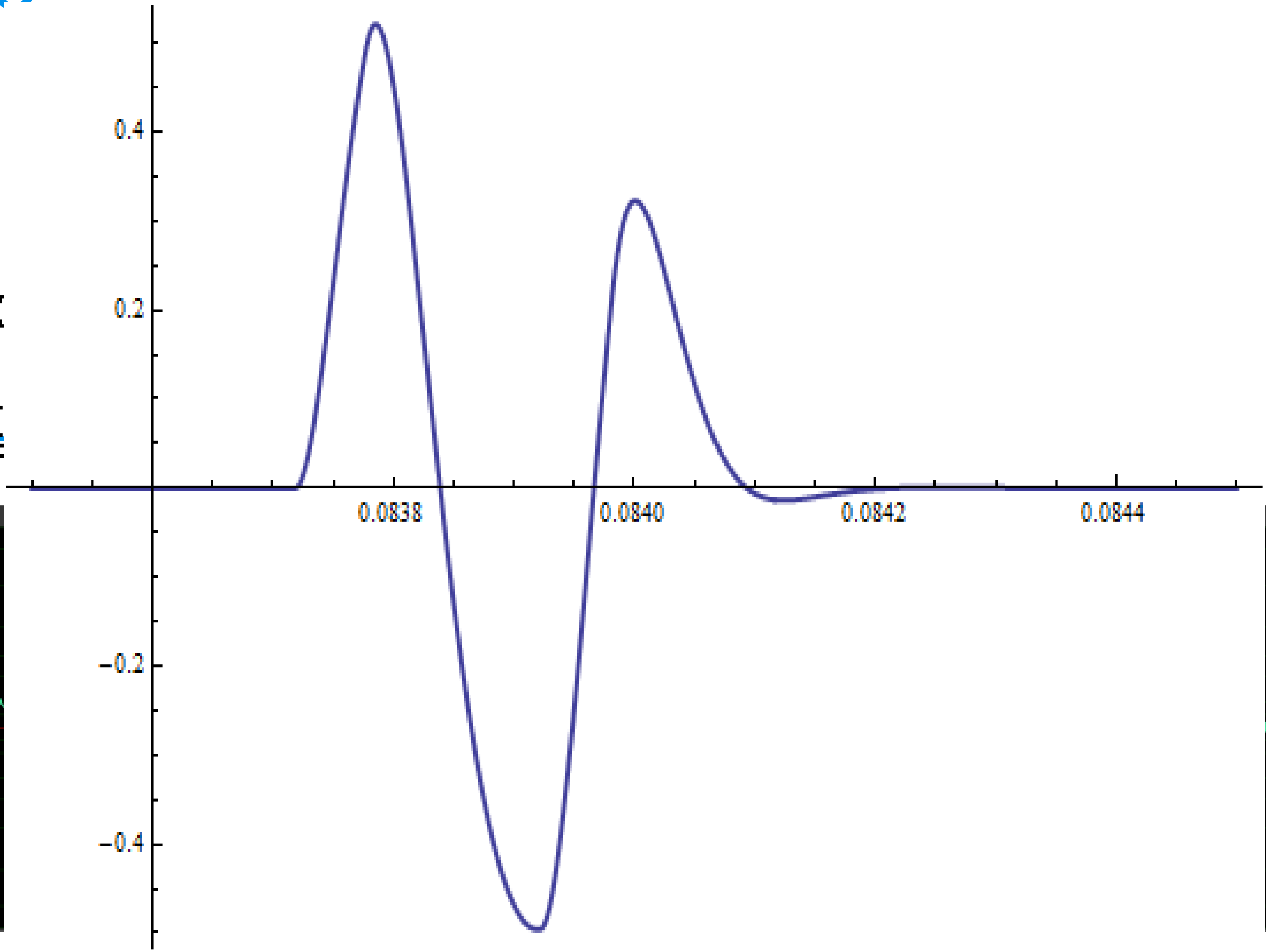
Recreate the sound

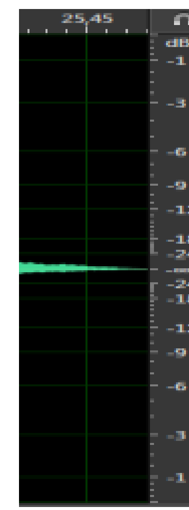
Ingredients for an artificial sound

1.

2.







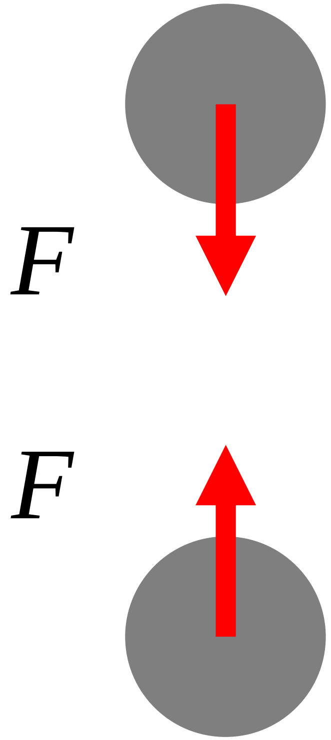
Simplified model of collisions

- Constant attractive force

Time between 2 collisions: $\Delta t = \frac{2mv}{F}$

m mass

v speed after the last collision



- Coefficient of restitution:

Impact speed change: $\Delta v = kv - v$

Simplified model of collisions

- Impact speed decrease in time:

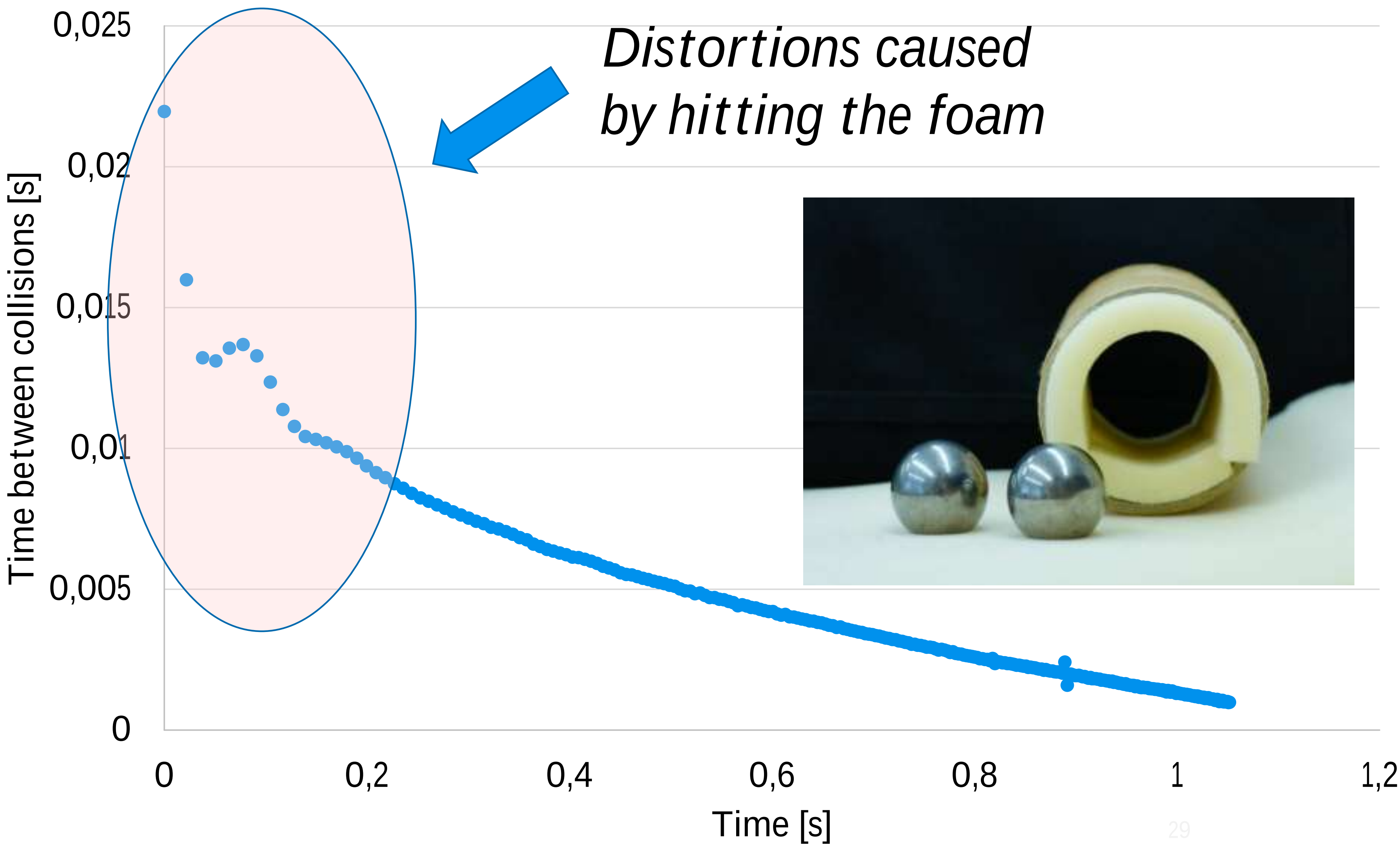
$$\frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{k-1}{2m} F = const$$

  – Suitable for fast chirping

- Speed before collisions: $v = v_0 - \frac{(1-k)F}{2m} t$

- Time between collisions: $\Delta t = \frac{2mv_0}{F} - (1-k)t$

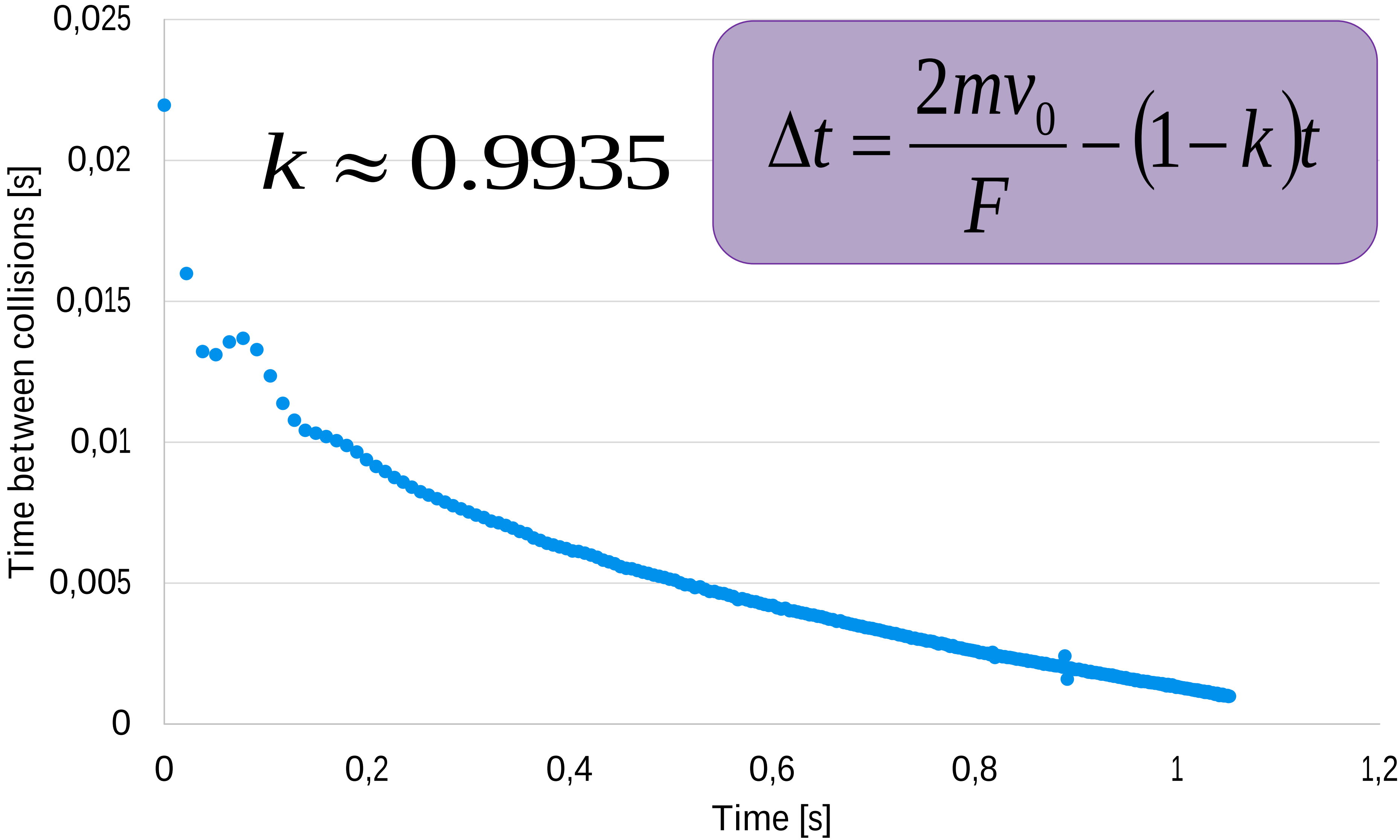
Reality check



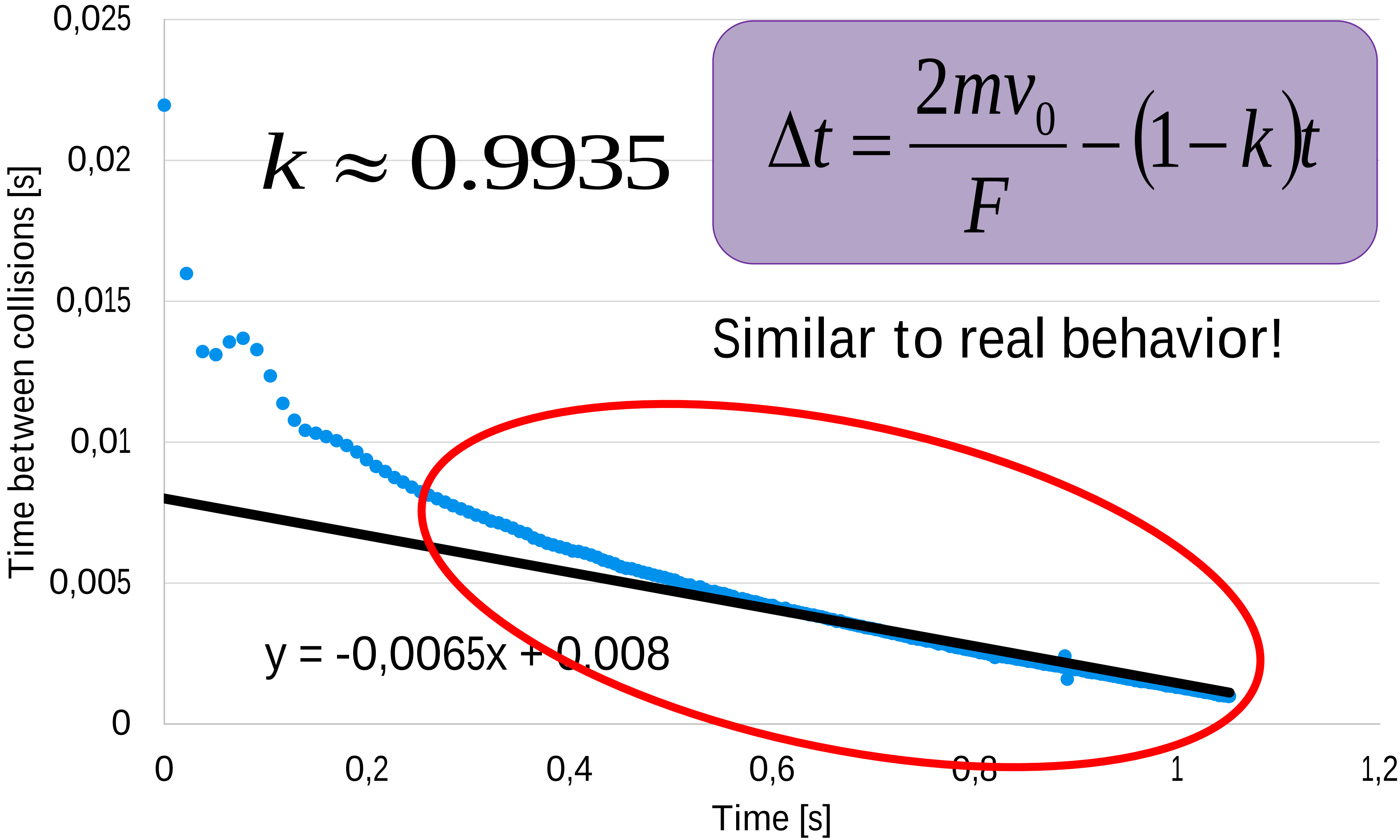
Reality check

$$k \approx 0.9935$$

$$\Delta t = \frac{2mv_0}{F} - (1-k)t$$



Reality check



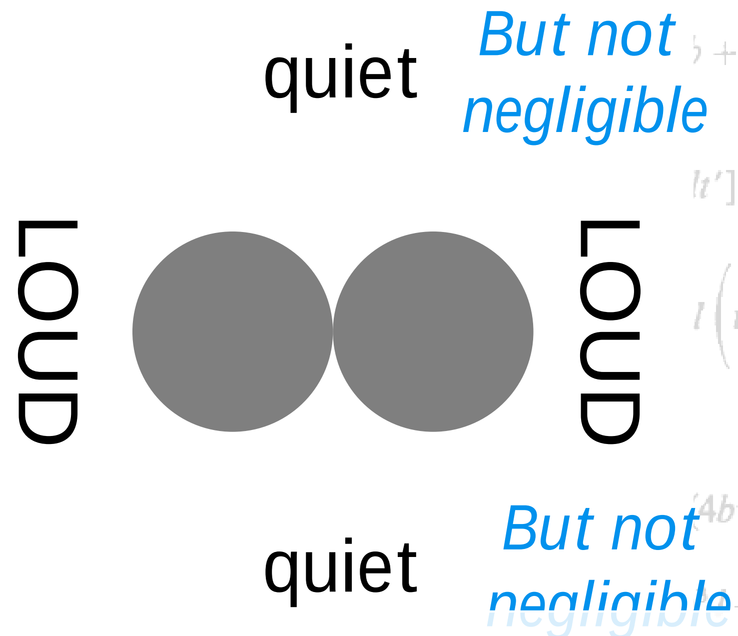
$$k \approx 0.9935$$

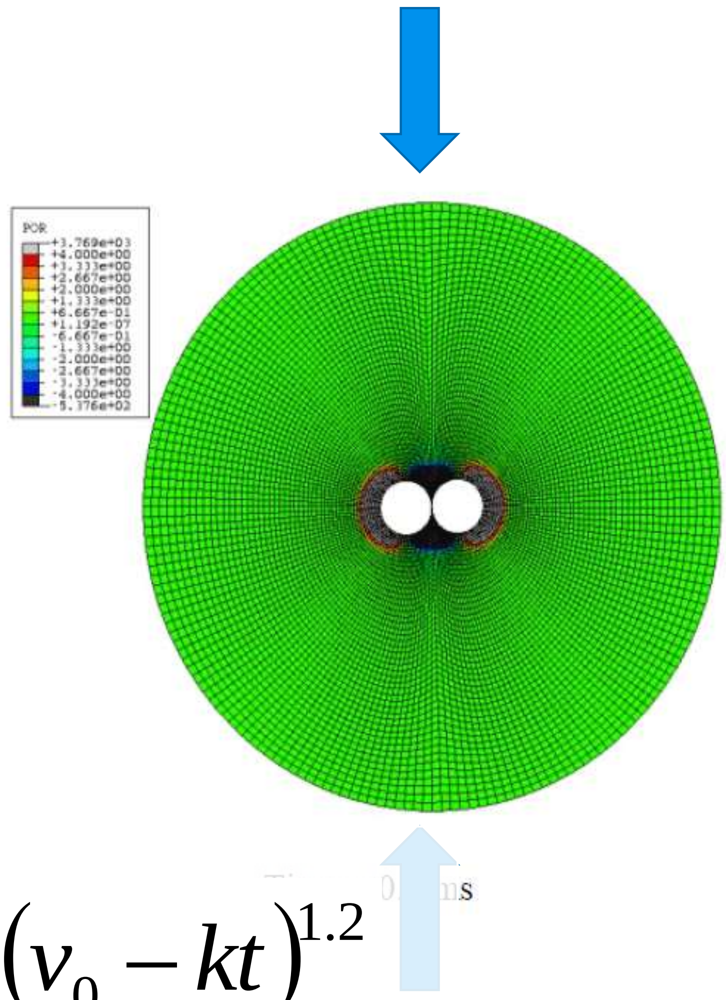
$$\Delta t = \frac{2mv_0}{F} - (1-k)t$$

Similar to real behavior!

Notable conclusions

- Loudness of the sound

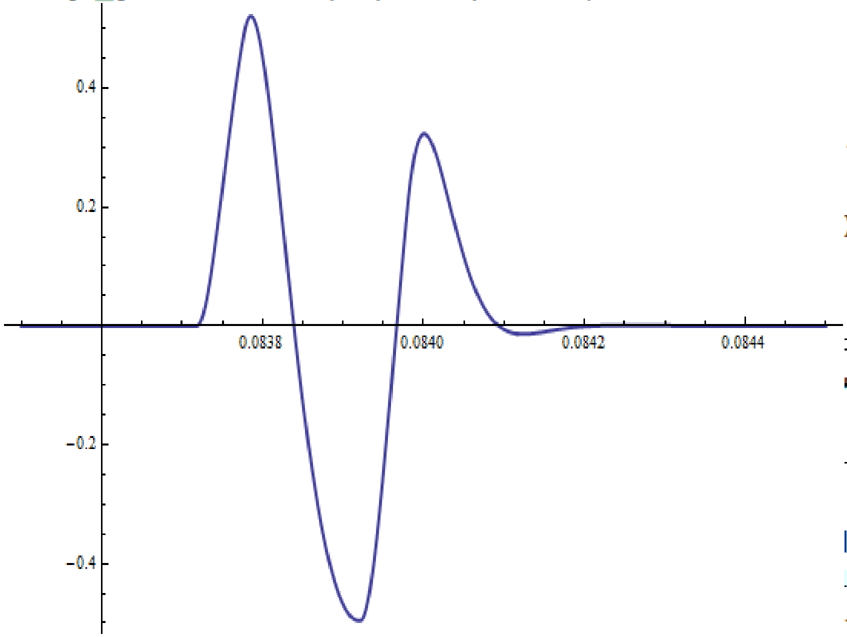




$$p \propto a_m \cos\theta \propto v^{1.2} \cos\theta \propto \left(v_0 - kt\right)^{1.2}$$

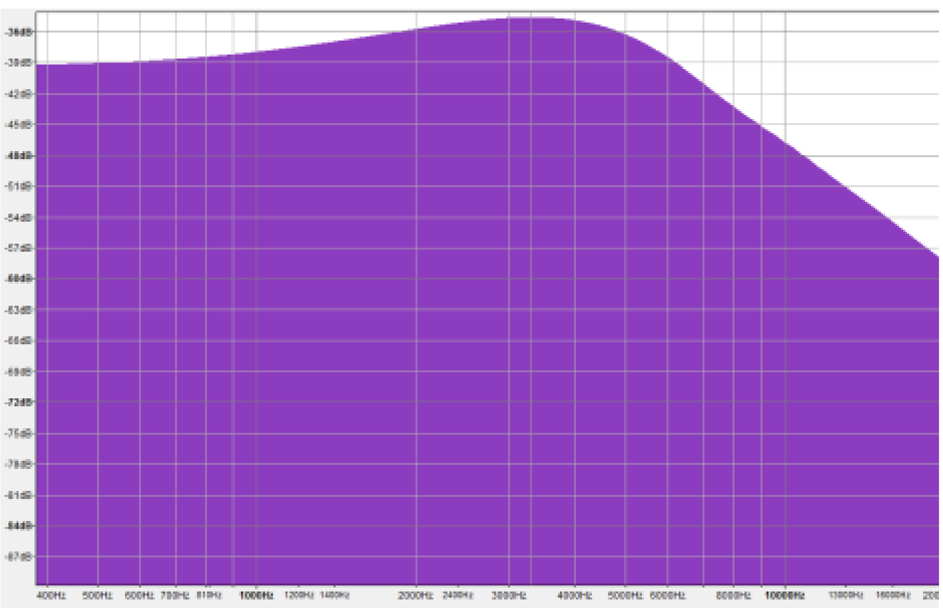
Sound creation in Mathematica

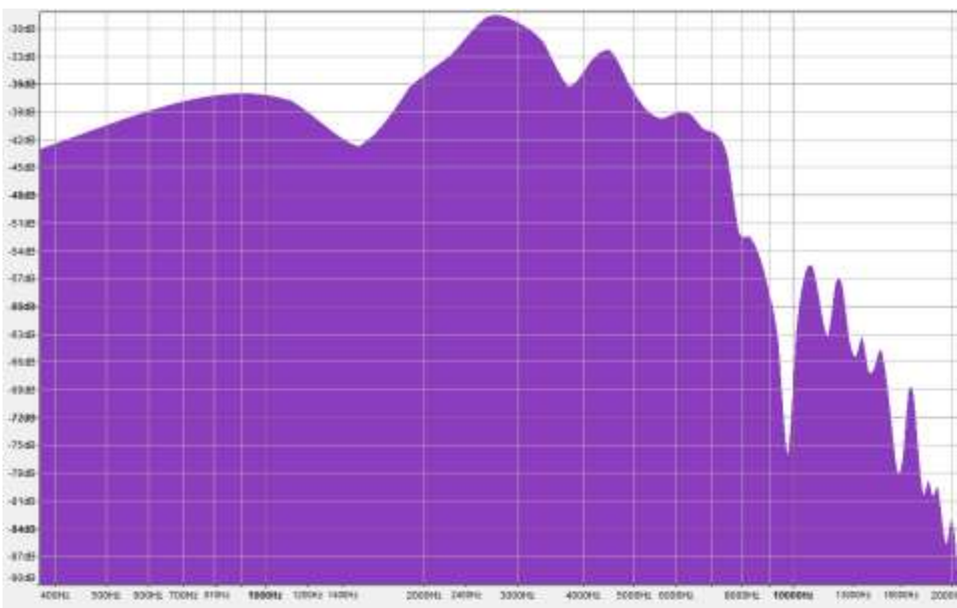
```
v[T_] = v0 - (1 - k) * F / 2 / m * T;
z[T_] = 2 * m * v0 / F - (1 - k) * T;

h[v_, r_, cth_] = 1.2 * a ^ 3 * cth / 2 / r ^ 2 * (k2 * (5 * v ^ 2 / 4 / k2 / k1) ^ 0.6) / m;
d[v_] = 1.13 * 10 ^ (-4) * v ^ (-0.200);
```




```
* Cos[b[v] * t] + 8 * b[v] ^ 2 * l ^ 2 * Sin[b[v] * t]) -
* l * b[v] ^ 3) * Cos[b[v] * t] +
) * Cos[l * t] - (8 * b[v] * l ^ 3 + 4 * b[v] ^ 3 * l) Sin[l * t]) *

Pi / 2 / l)] -
- Pi / 2 / l)]) * Exp[-l * t]) + h[v, r, cth] * Sin[b[v] * t];
l ^ 4) *

- d[v])] - (8 * b[v] * l ^ 3 + 4 * b[v] ^ 3 * l) * Sin[l * (t - d[v])]) *

] - (4 * b[v] ^ 3 * l + 8 * b[v] * l ^ 3) * Sin[l * t]) * Exp[-l * t]) -
[v] - Pi / 2 / l)] +
- d[v] - Pi / 2 / l)]) * (Exp[-l * (t - d[v])] + Exp[-l * t]));
s[t_, v_, r_, cth_] = Piecewise[{{x[t, v, r, cth], t ≤ d[v]}, {y[t, v, r, cth], t > d[v]}}];

t[T_] = T - Floor[T, z[T]];

p[T_] = s[t[T - dT], v[T - dT], r1, cth1] + s[t[T], v[T], r2, cth2];
chirping = Play[p[T], {T, 0, 1.17}, SampleRate → 196000, PlayRange → {-2, 2}]
Export["Chirping.wav", chirping]
```

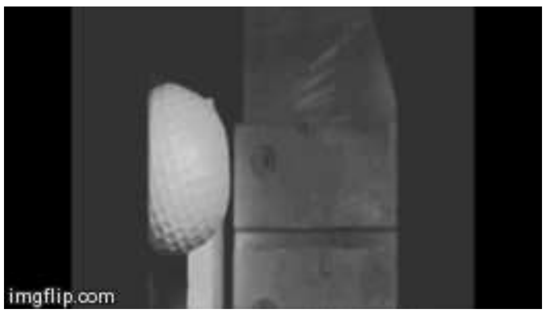
Artificial vs. Real





Conclusion – Nature of sound

- Two options
  - Vibration of balls



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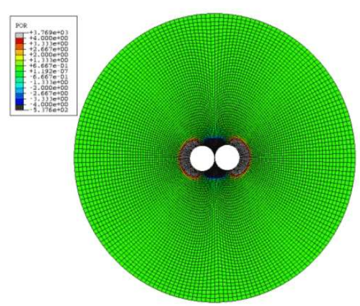
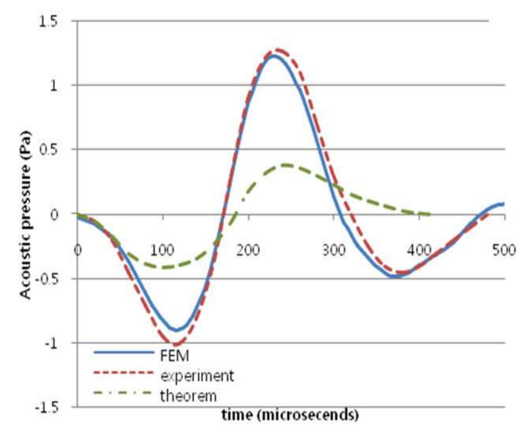
Lowest frequency of our balls 113,5 kHz

  - Impulsive translational acceleration

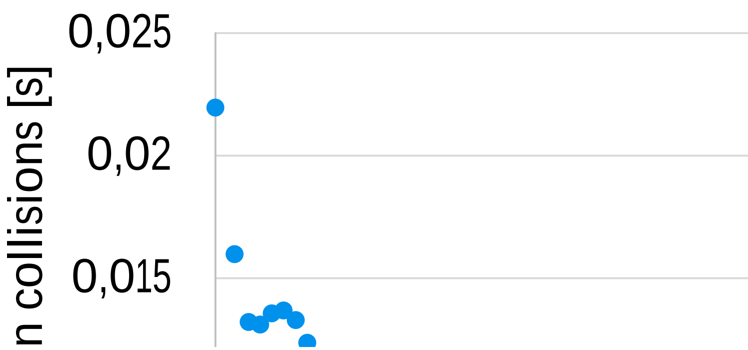


Time = 0.1 ms



Conclusion – Analysis and Recreation

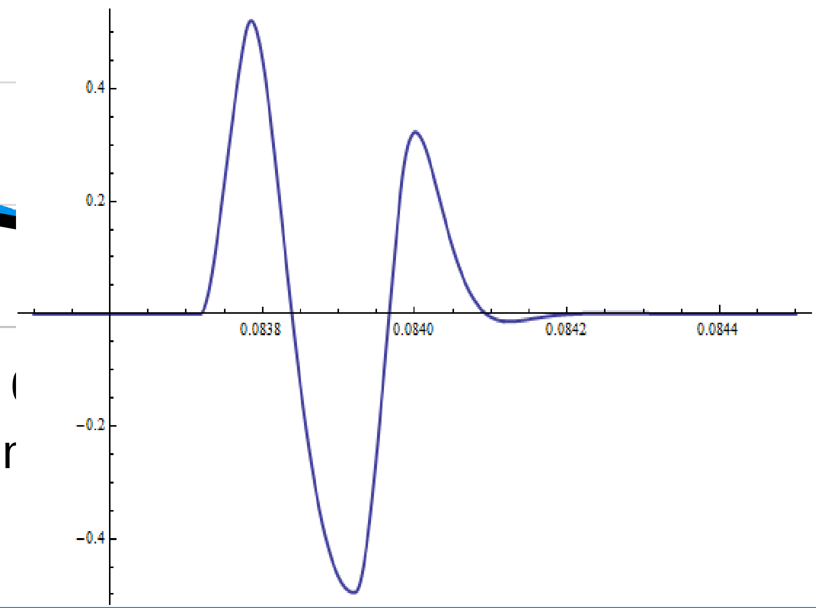
- Simplified model of collision
- Ad
  - |
- Ge



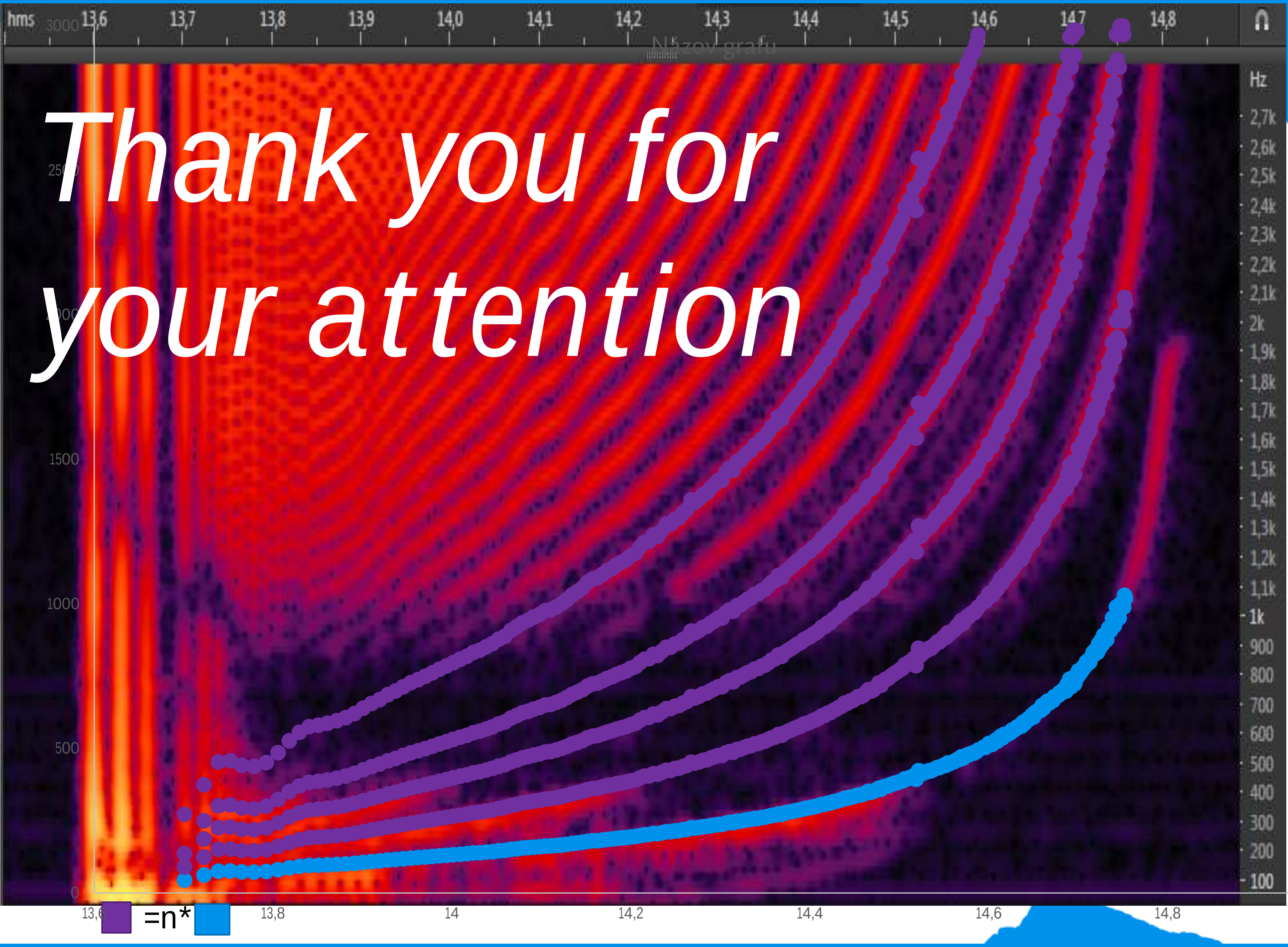
$$\Delta t = \frac{2mv_0}{F} - (1-k)t$$

```
v[T_] = v0 - (1 - k) * F / 2 / m * T;
z[T_] = 2 * m * v0 / F - (1 - k) * T;

h[v_, r_, cth_] = 1.2 * a^3 * cth / 2 / r^2 * (k2 * (5 * v^2 / 4 / k2 / k1) ^0.6) / m;
d[v_] = 1.13 * 10^ (-4) * v^ (-0.200);
b[v_] = Pi / d[v];
x[t_, v_, r_, ctheta_] =
  h[v, r, cth] / 4 / (b[v] ^4 + 4 * l^4) *
    ((2 * r / a - 1) * ((8 * l^3 * b[v] - 4 * l * b[v] ^3) * Cos[b[v] * t] + 8 * b[v] ^2 * l^2 * Sin[b[v] * t]) -
      4 * b[v] ^4 * Sin[b[v] * t] - (8 * l^3 * b[v] + 4 * l * b[v] ^3) * Cos[b[v] * t] +
      (2 * r / a - 1) * ((4 * b[v] ^3 * l - 8 * b[v] * l^3) * Cos[l * t] - (8 * b[v] * l^3 + 4 * b[v] ^3 * l) Sin[l * t]) *
       Exp[-l * t] +
      ((4 * b[v] ^3 * l - 8 * b[v] * l^3) * Cos[l * (t - Pi / 2 / l)] -
         (8 * b[v] * l^3 + 4 * b[v] ^3 * l) * Sin[l * (t - Pi / 2 / l)]) * Exp[-l * t]) + h[v, r, cth] * Sin[b[v] * t];
y[t_, v_, r_, cth_] = h[v, r, cth] / 4 / (b[v] ^4 + 4 * l^4) *
   ((2 * r / a - 1) *
      (((4 * b[v] ^3 * l - 8 * b[v] * l^3) * Cos[l * (t - d[v])] - (8 * b[v] * l^3 + 4 * b[v] ^3 * l) * Sin[l * (t - d[v])]) *
         Exp[-l (t - d[v])] +
        ((4 * b[v] ^3 * l - 8 * b[v] * l^3) * Cos[l * t] - (4 * b[v] ^3 * l + 8 * b[v] * l^3) * Sin[l * t]) * Exp[-l * t]) -
     ((8 * b[v] * l^3 - 4 * b[v] ^3 * l) * Cos[l * (t - d[v] - Pi / 2 / l)] +
        (8 * b[v] * l^3 + 4 * b[v] ^3 * l) * Sin[l * (t - d[v] - Pi / 2 / l)]) * (Exp[-l * (t - d[v])] + Exp[-l * t]));
s[t_, v_, r_, cth_] = Piecewise[{{x[t, v, r, cth], t ≤ d[v]}, {y[t, v, r, cth], t > d[v]}}];

t[T_] = T - Floor[T, z[T]];

p[T_] = s[t[T - dT], v[T - dT], r1, cth1] + s[t[T], v[T], r2, cth2];
chirping = Play[p[T], {T, 0, 1.17}, SampleRate → 196000, PlayRange → {-2, 2}]
Export["Chirping.wav", chirping]
```

Tir



Thank you for your attention



Sound of single collision

K. Mehraby et al:

"Impact noise radiated by collision of two spheres"
Journal of Mechanical Science and Technology, 2011

- Finite elements method simulation
- "Perfect" agreement with experiment

L. L. Koss, R. J. Alfredson:

"Transient sound radiated by spheres undergoing an elastic ***collision"***
Journal of Sound and Vibration, 1972

- Fully theoretical & analytical solution
- Underestimates the loudness for theta=90°
- Otherwise good correlation

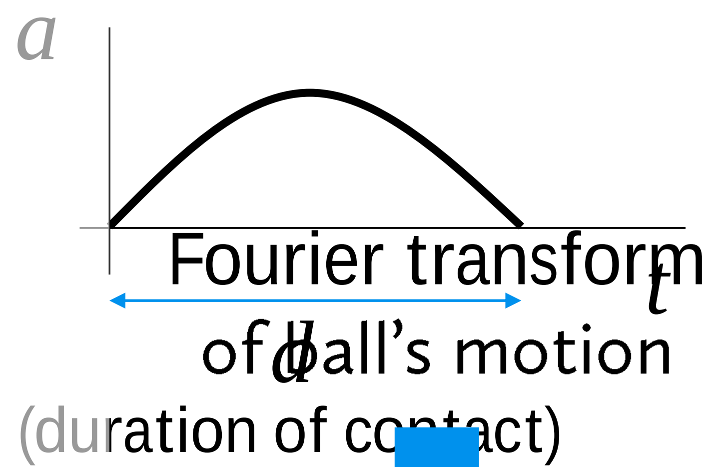
Koss & Alfredson: Theory basis

- Interaction of balls:

$$F=-kx^{\frac{3}{2}}$$

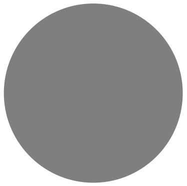
  - Acceleration approximation:

$$a\approx a_m\sin\frac{\pi}{d}t$$



Fourier transform of ball's motion

Velocity potential for colliding balls



- Velocity potential for an oscillating sphere*:

$$\Phi(r,\theta,t)=\frac{a^3v_1}{r^2}-$$

*I. Malecki: *Physical Foundations of Technical Acoustics*

Koss & Alfredson: results

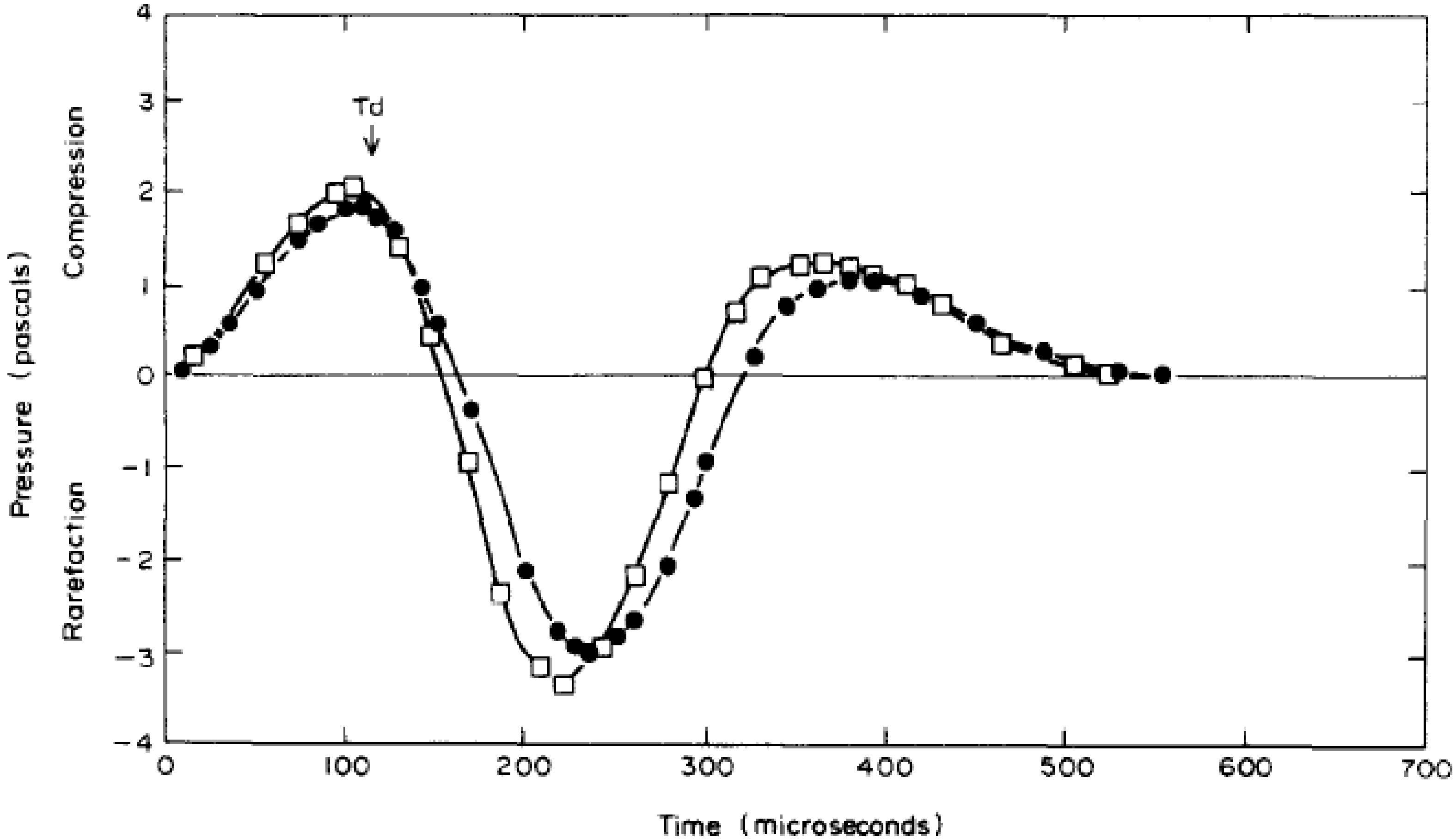
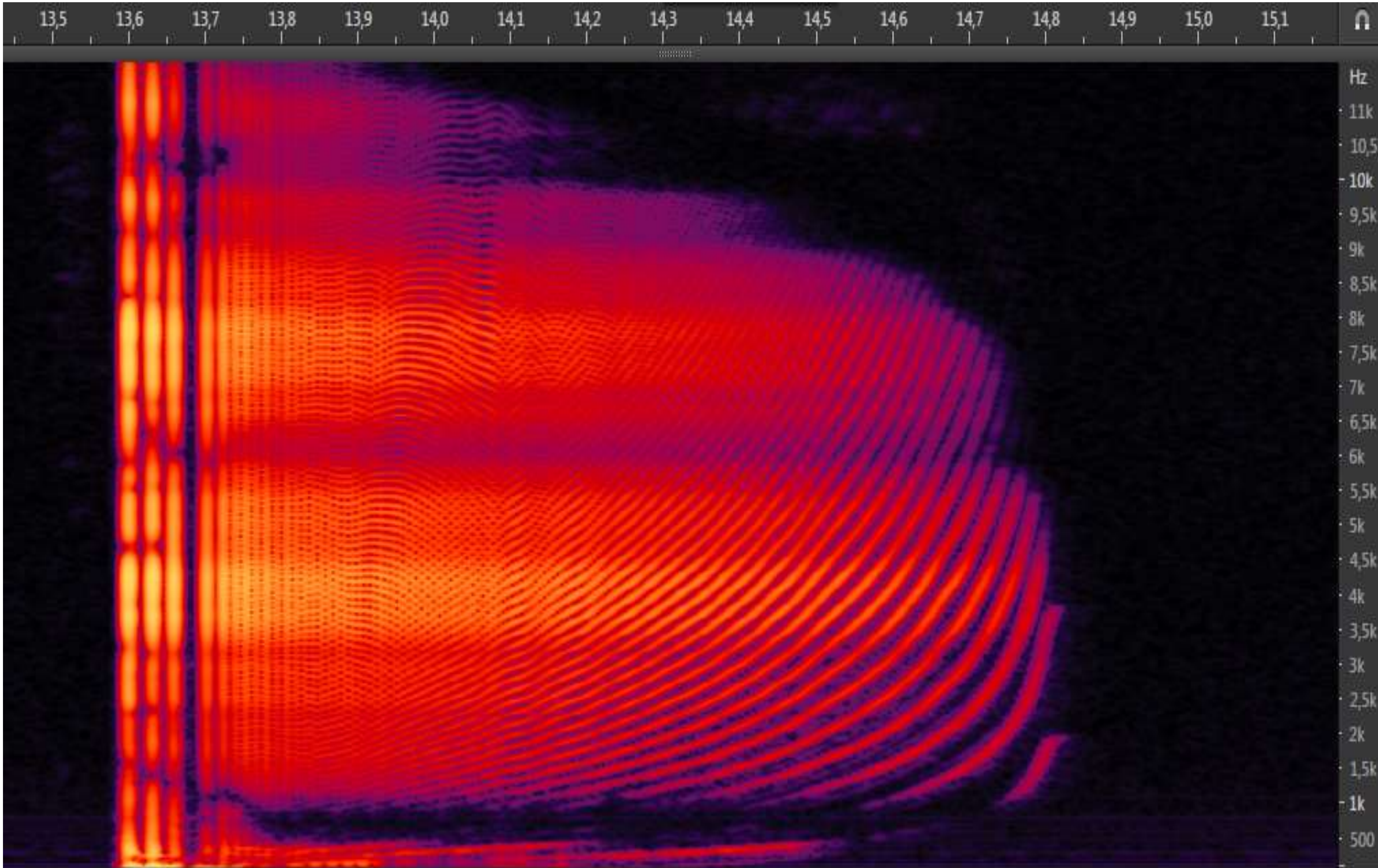


Figure 10. Pressure–time trace comparison for 2-inch spheres; $V_0 = 0{\cdot}3$ m/s, $r = 0{\cdot}285$ m, $\theta = 40°$. ●, Equation (22); □, ¼-inch microphone grazing orientation.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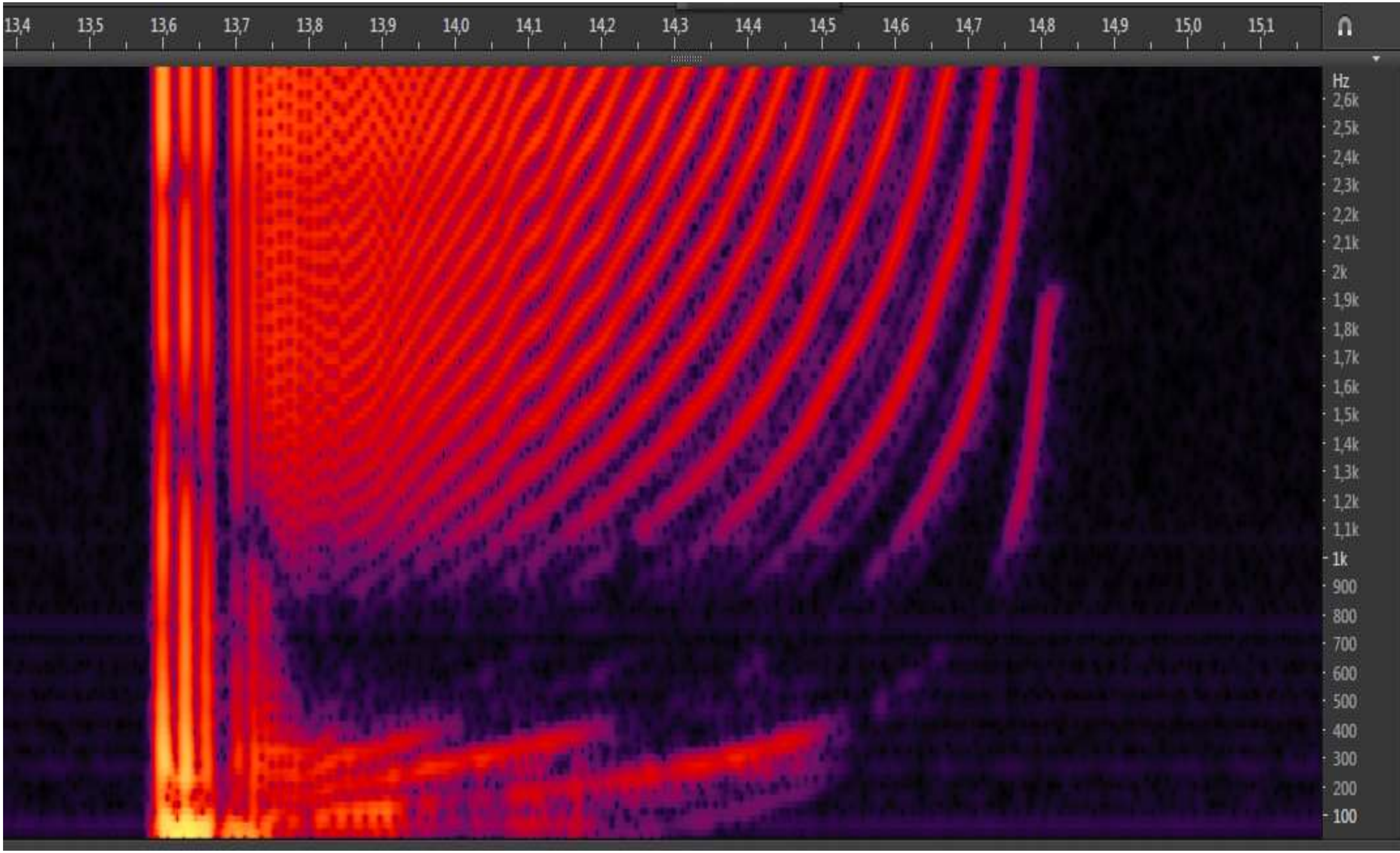
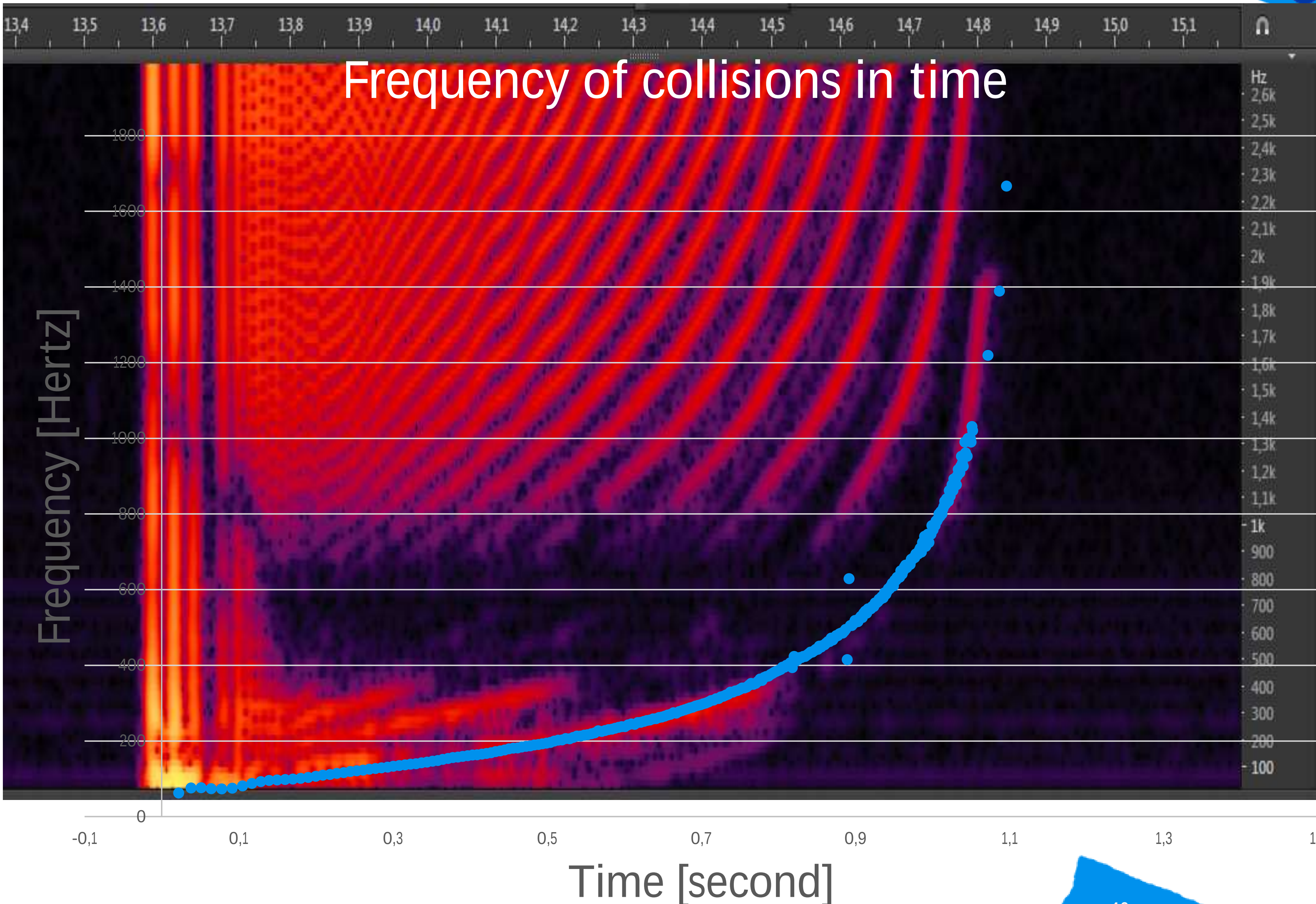
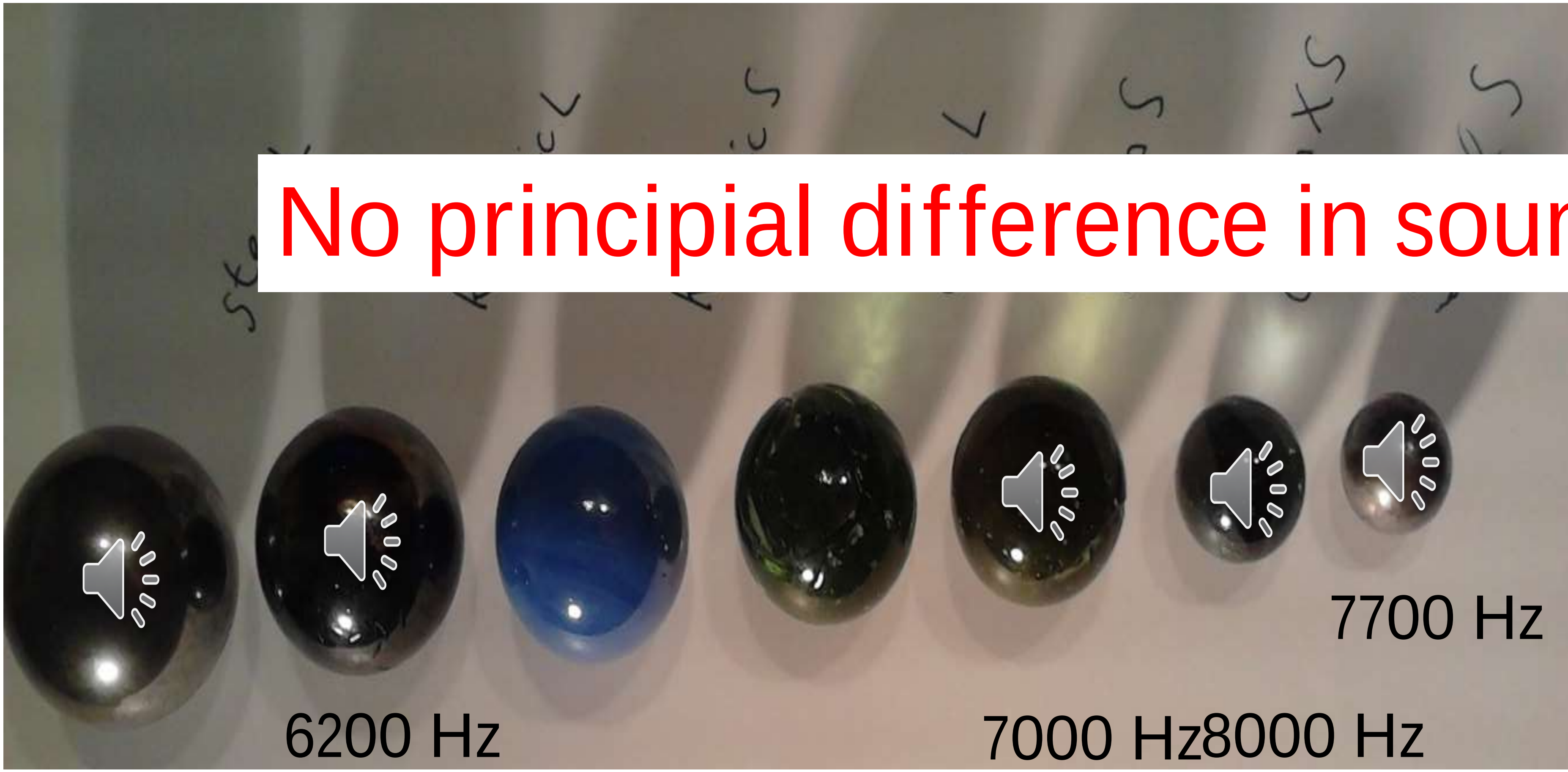
13,6
13,7
13,8
13,9
14,0
14,1
14,2
14,3
14,4
14,5
14,6
14,7
14,8
14,9
15,0
15,1
Hz
11k
10,5k
10k
9,5k
9k
8,5k
8k
7,5k
7k
6,5k
6k
5,5k
5k
4,5k
4k
3,5k
3k
2,5k
2k
1,5k
1k
500


13,4
13,5
13,6
13,7
13,8
13,9
14,0
14,1
14,2
14,3
14,4
14,5
14,6
14,7
14,8
14,9
15,0
15,1
Hz
2,6k
2,5k
2,4k
2,3k
2,2k
2,1k
2k
1,9k
1,8k
1,7k
1,6k
1,5k
1,4k
1,3k
1,2k
1,1k
1k
900
800
700
600
500
400
300
200
100

Frequency of collisions in time



Different Balls



No principial difference in sound

6200 Hz

7700 Hz

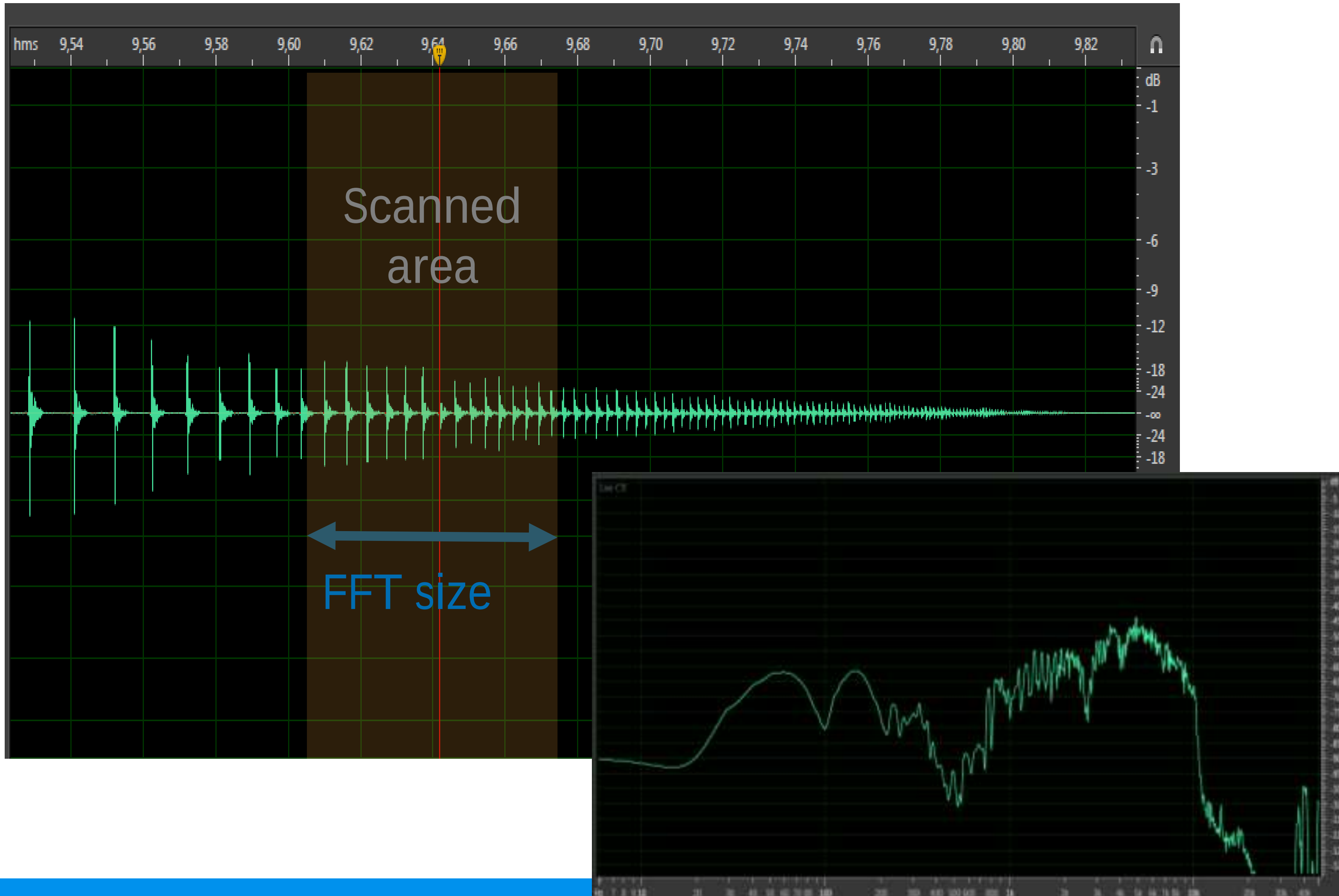
7000 Hz 8000 Hz

5000 Hz

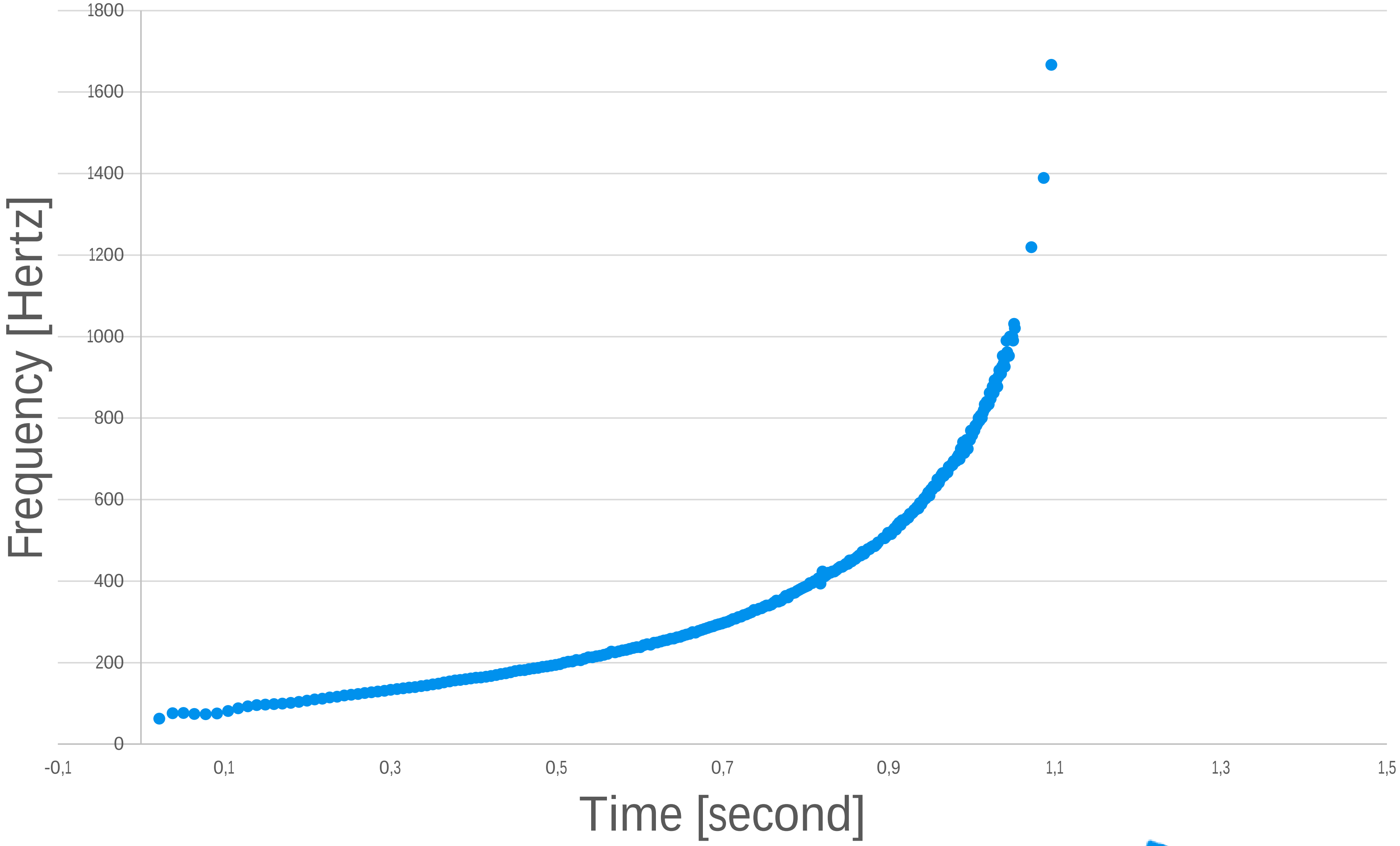
Sound differs in frequency and duration

Sound analysis

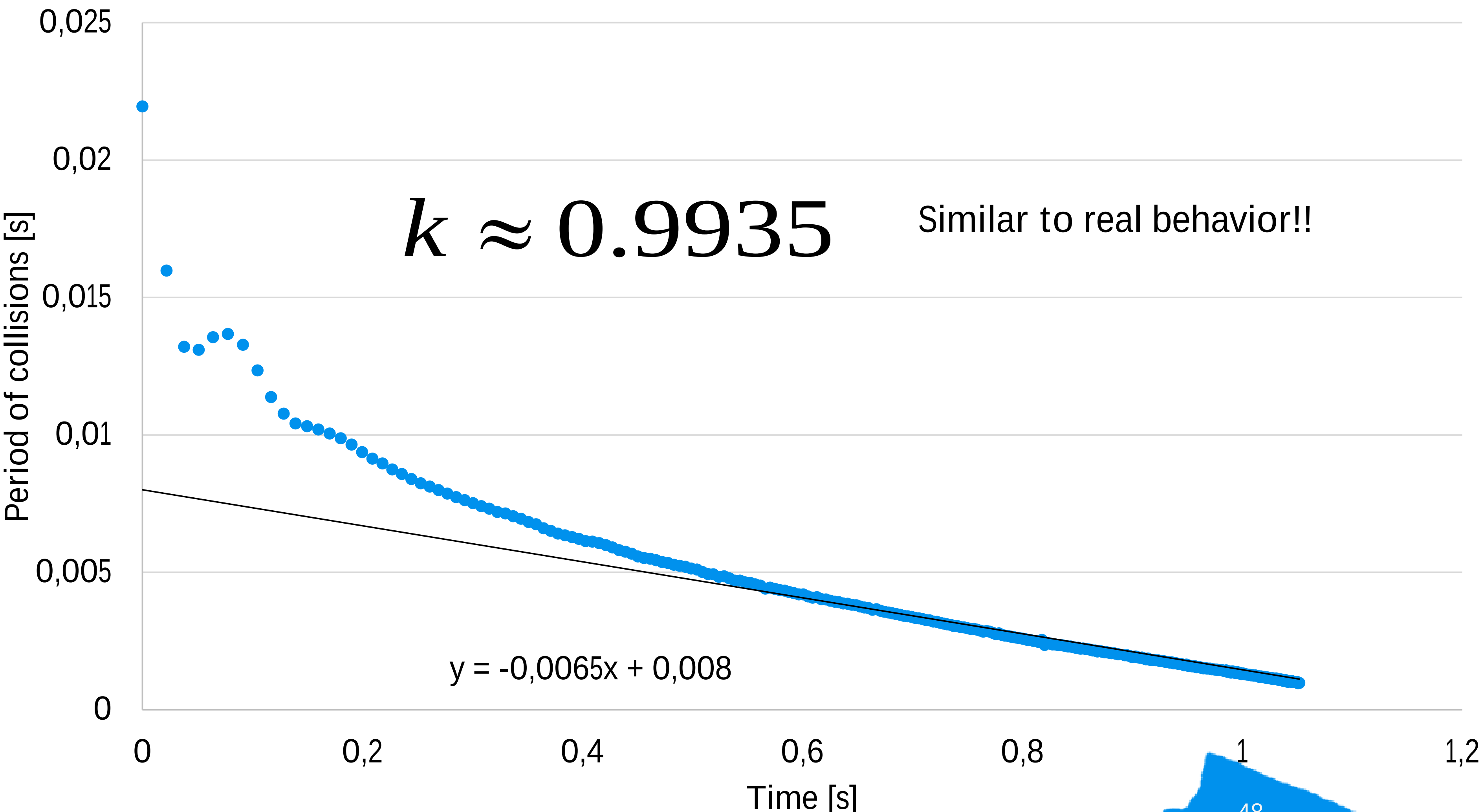
Procedure of Fourier transform



Frequency of collisions in time



Period of collisions



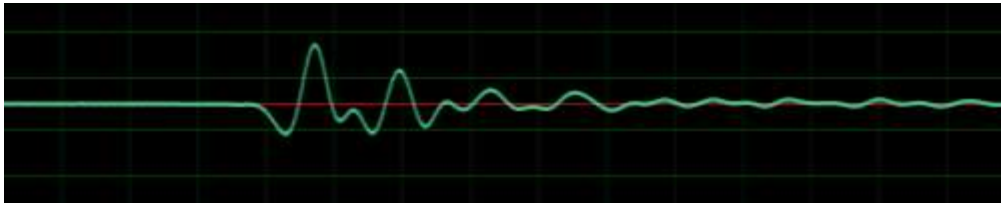
$$k \approx 0.9935$$

Similar to real behavior!!

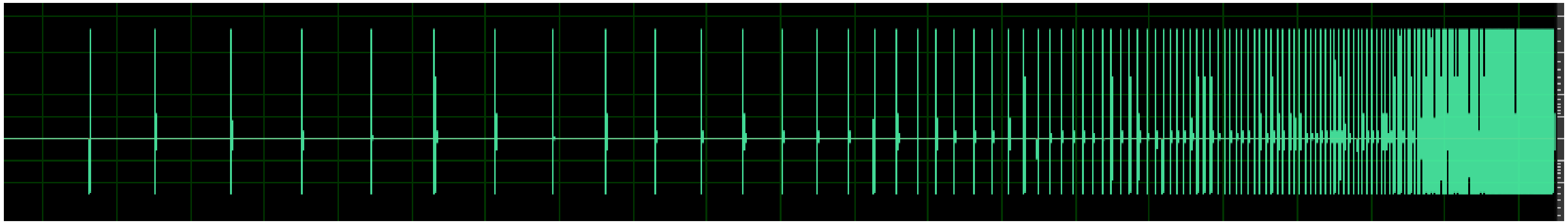
If all collisions are the same only frequency of the collisions increases we can generate sound

Generating fake chirping sound

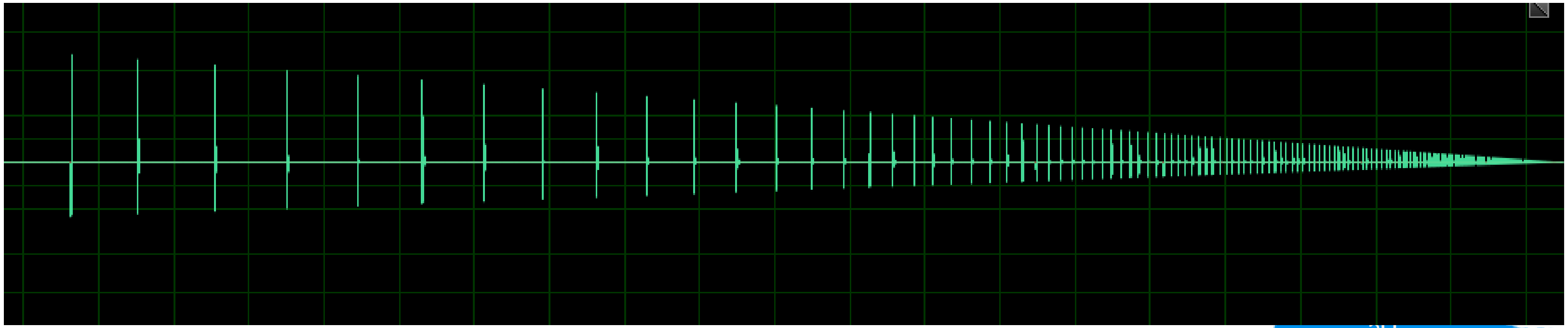
Take 1 collision



Paste 185 times with increasing frequency

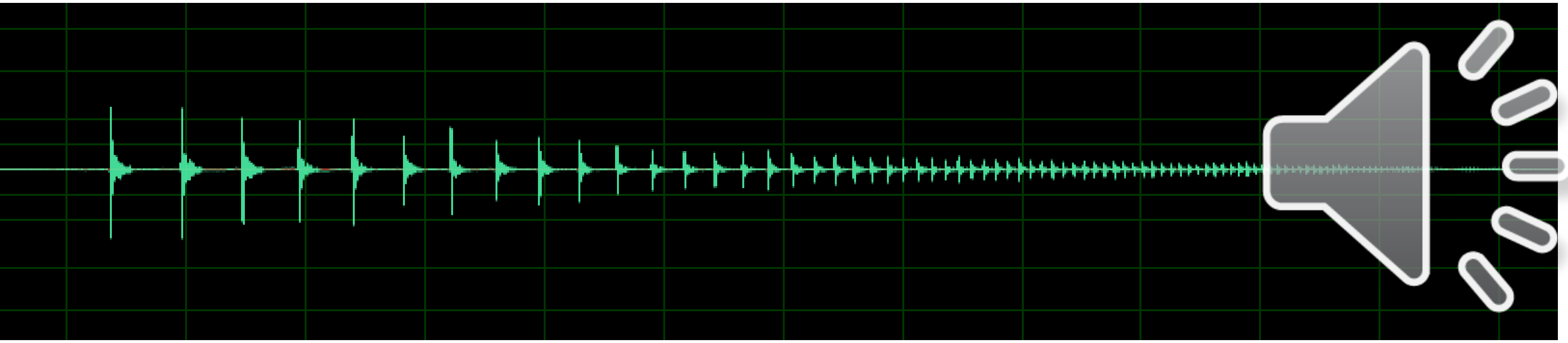


And decreasing amplitude

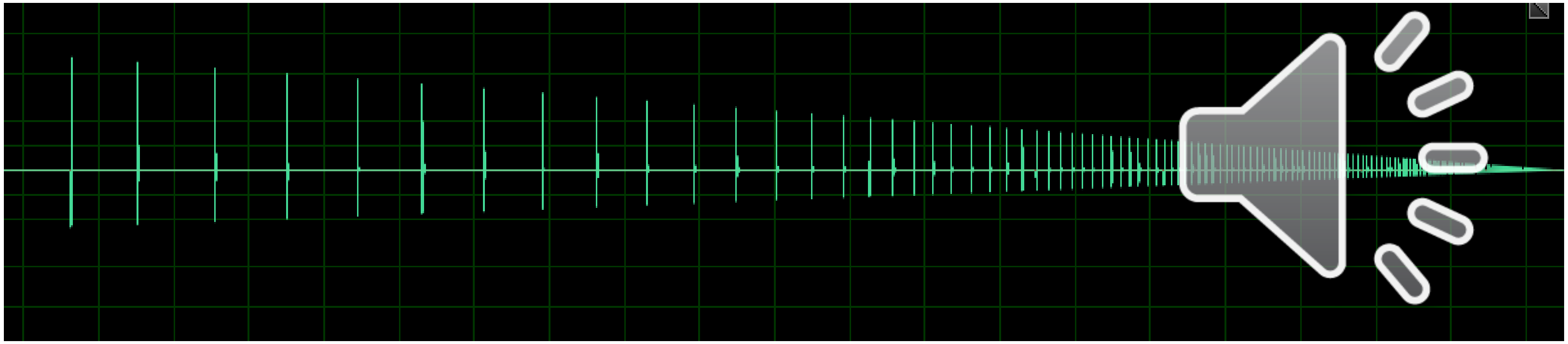


Reality check

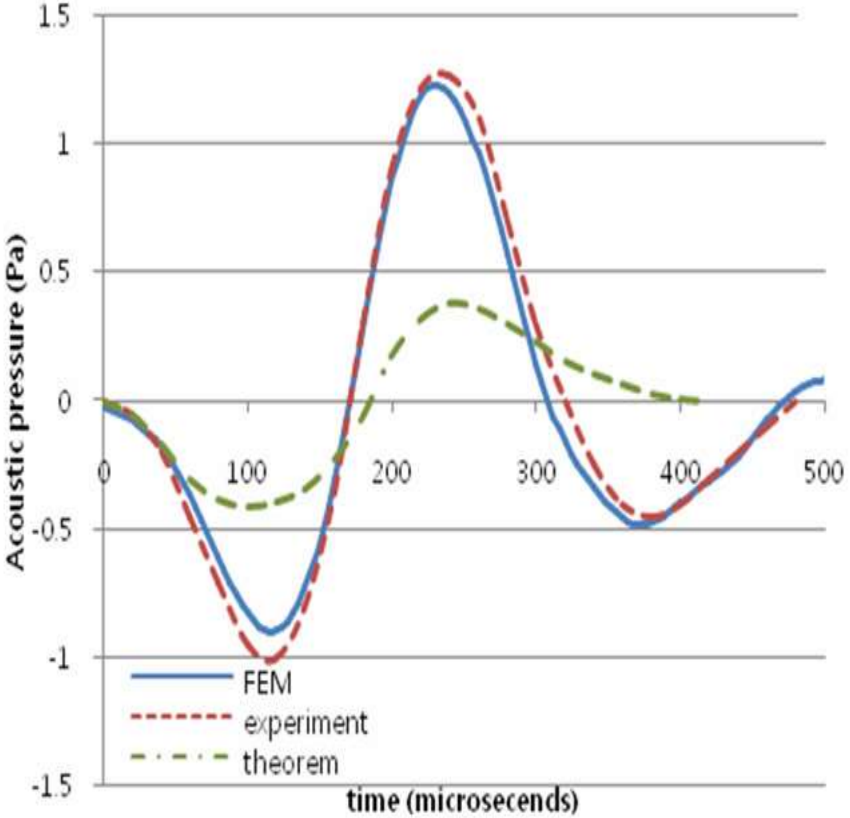
Measured



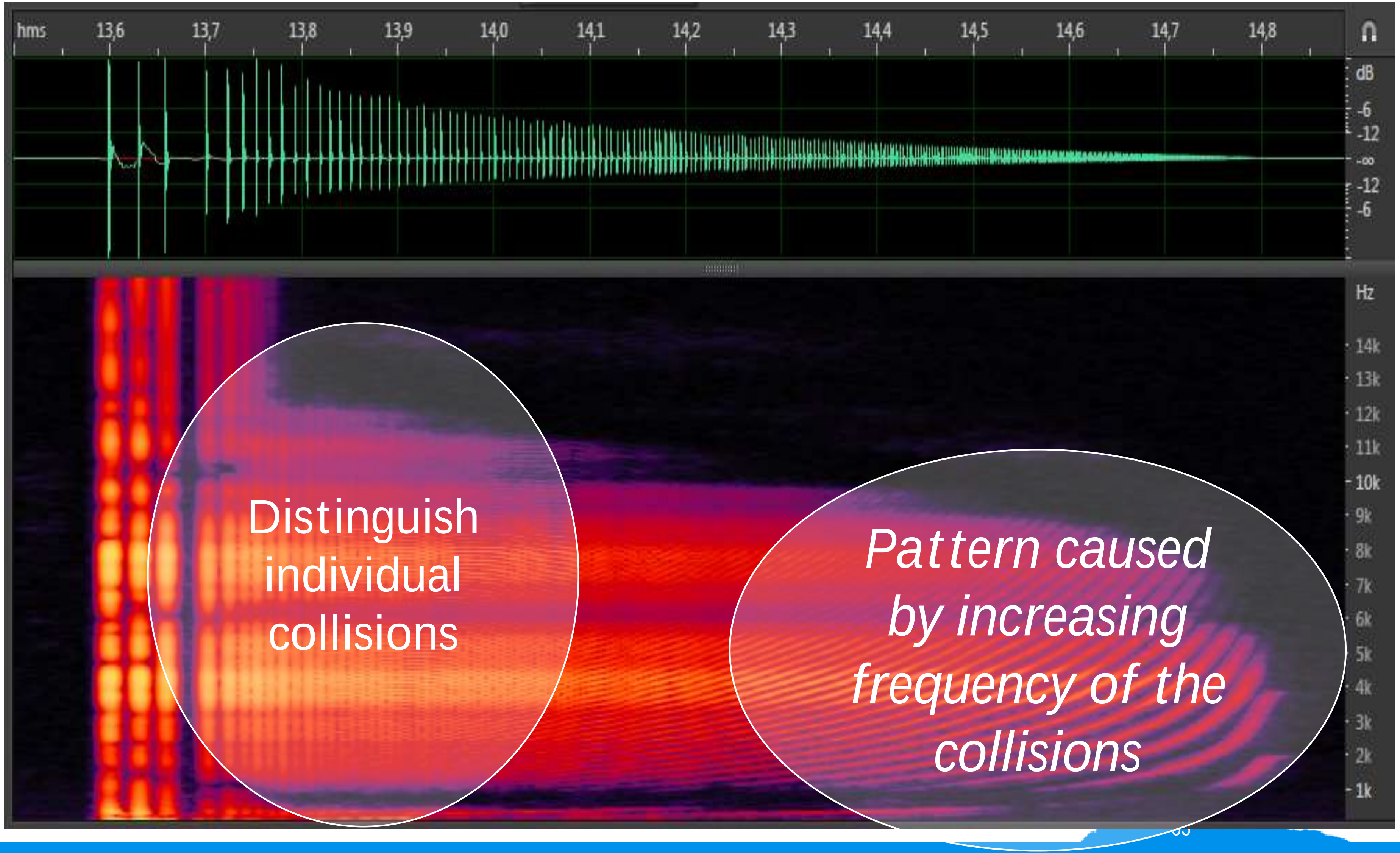
Generated



Single collision – our measurement vs theory



Spectral analysis



Our apparatus

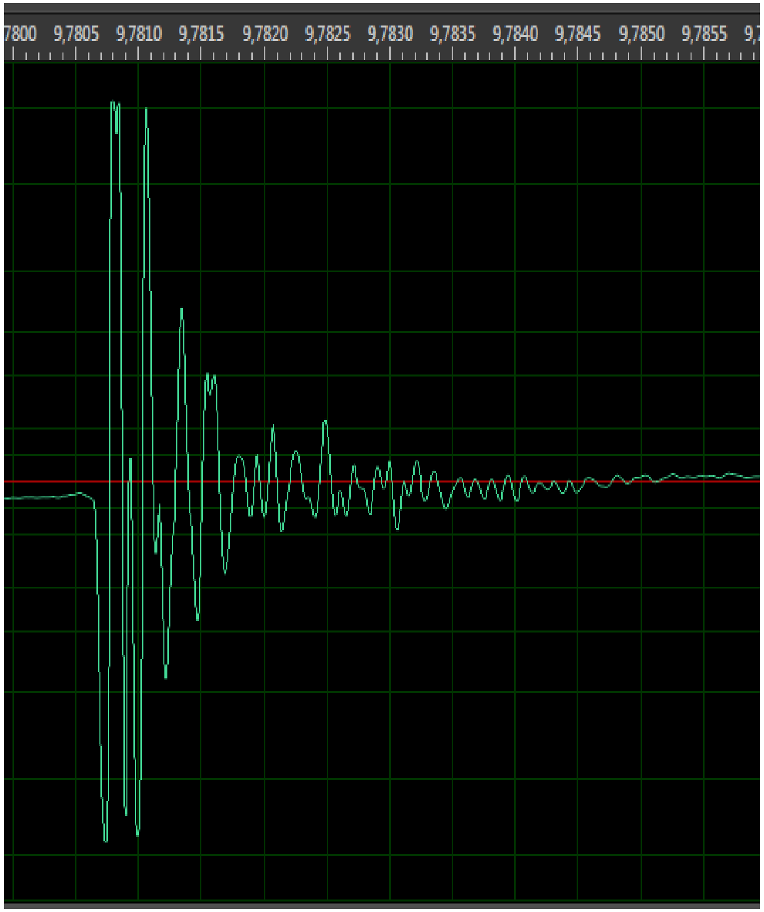
Steel Balls:

Radius: 1.43 cm
Mass: 0.095 kg

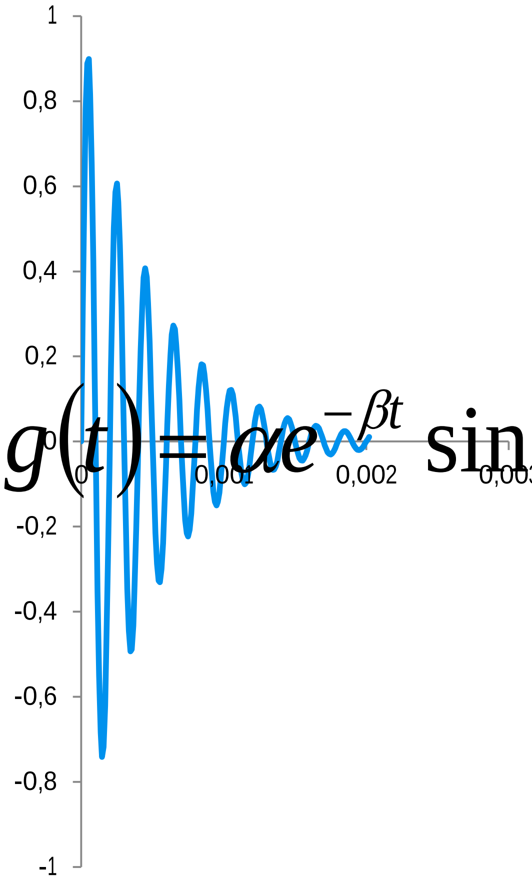
Colliding on soft foam

- Little energy losses due to damping

Single collision noise approximation

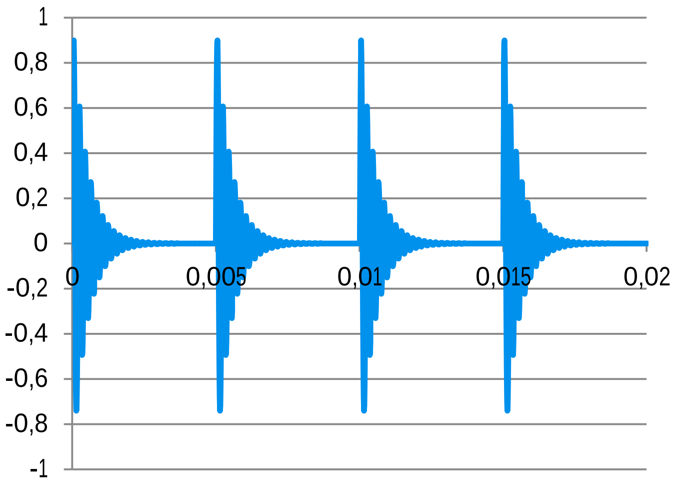


Real waveform



$$g(t) = \alpha e^{-\beta t} \sin(\omega t)$$

Chirping sound

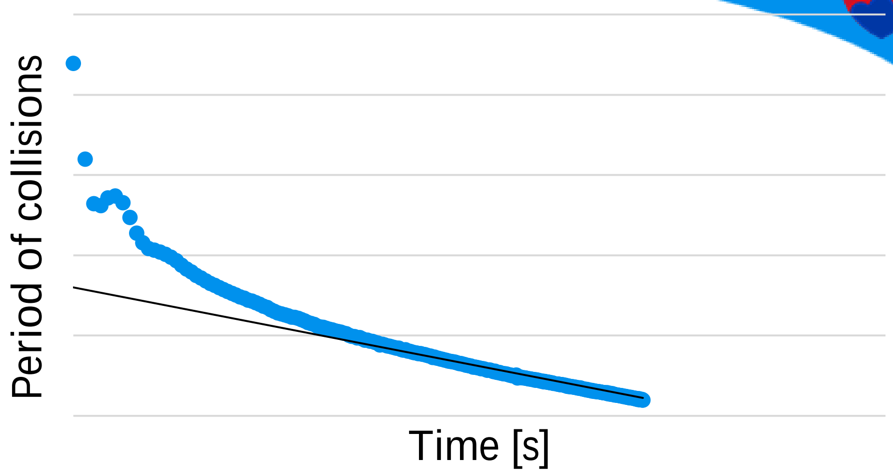


- REAL chirping sound

- Subsequent Identical sounds

Second verification method

- We can generate chirping sound by pasting the same single-collision-waveform one after another



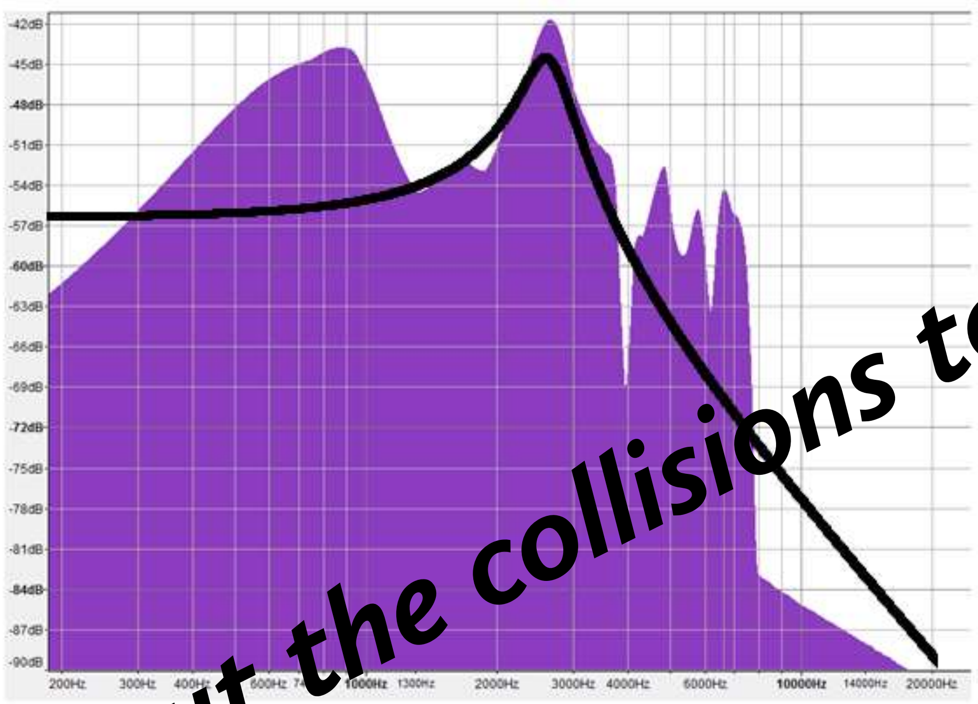
Now we know the motion of the balls

Motion ✓

Origin ✓

Why do we hear the chirping?

Spectrum of one collision



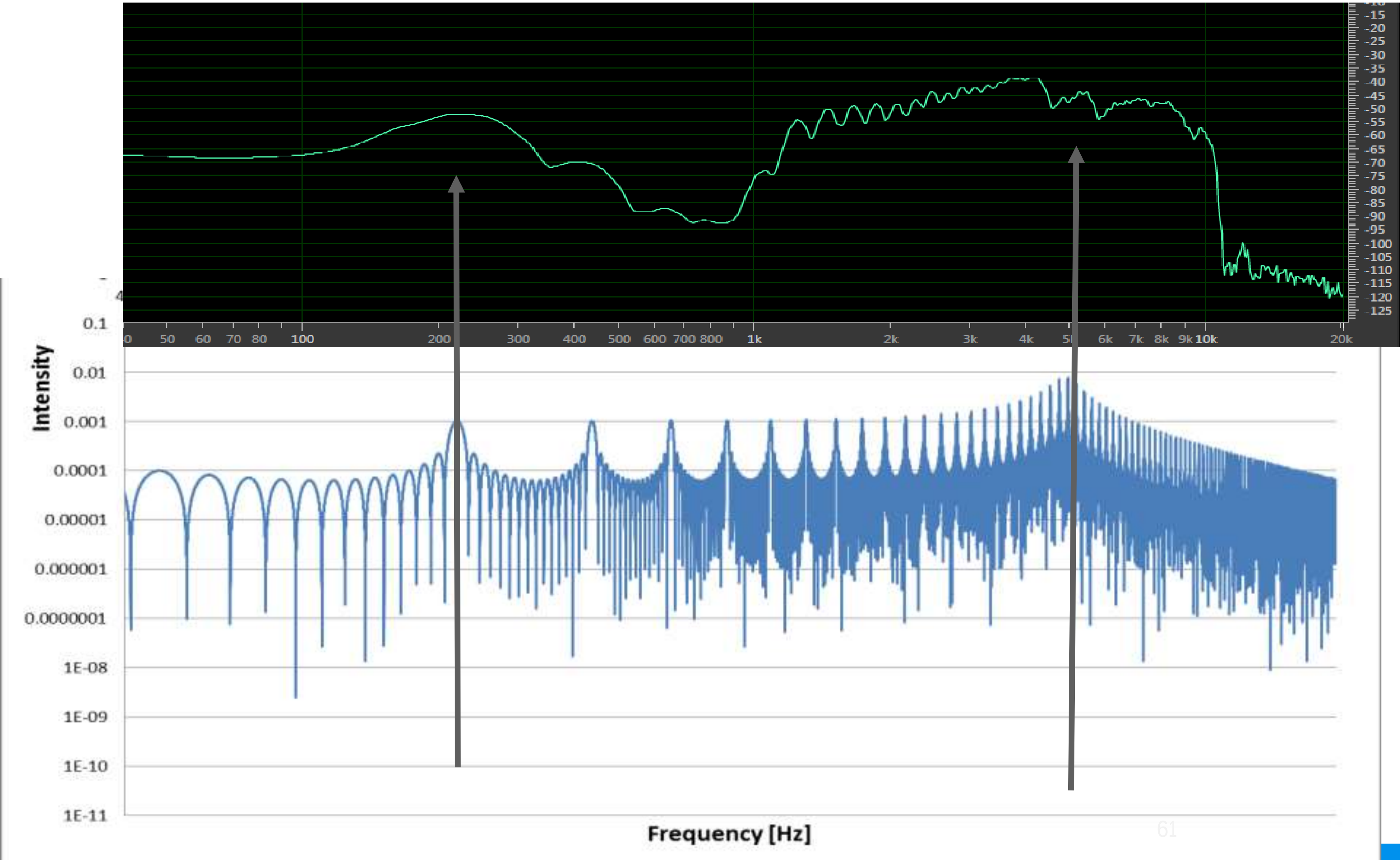
Our model

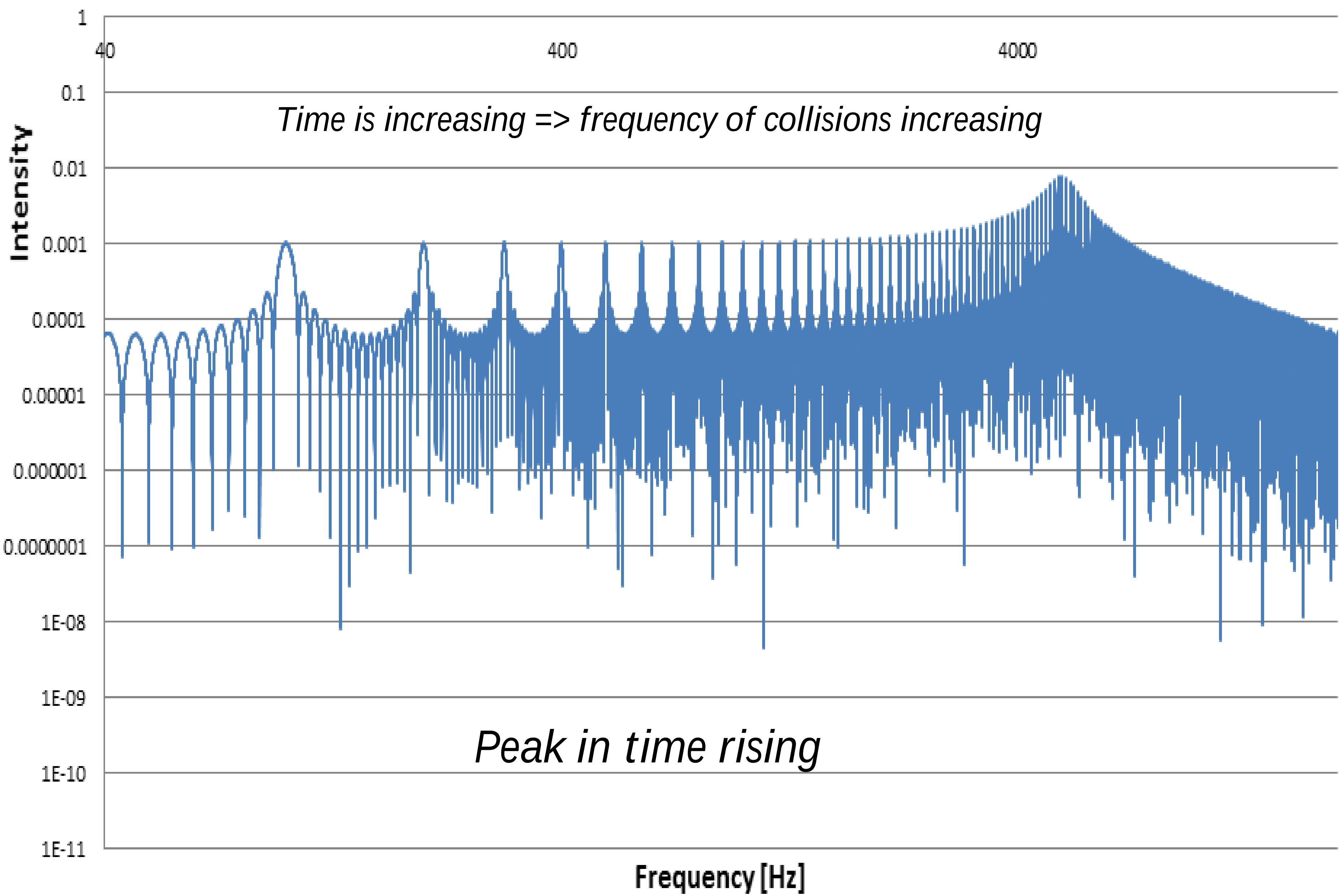
Let's put the collisions together!

Fourier transform

$$\mathcal{F}(g(t)) = G(f) = \int_{-\infty}^{\infty} g(t)\, e^{i2\pi ft}\, dt \qquad A(f) = |G(f)|$$

Spectrum of one moment of chirping




1
0.1
0.01
0.001
0.0001
0.00001
0.000001
0.0000001
1E-08
1E-09
1E-10
1E-11
Intensity
40
400
4000
Time is increasing => frequency of collisions increasing
Peak in time rising
Frequency [Hz]