8

Freezing droplets

Jakub Chudík

Task

Place a water droplet on a plate cooled down to around -20 °C.

As it freezes, the shape of the droplet may become cone-like with a sharp top. Investigate this effect.

Equipment

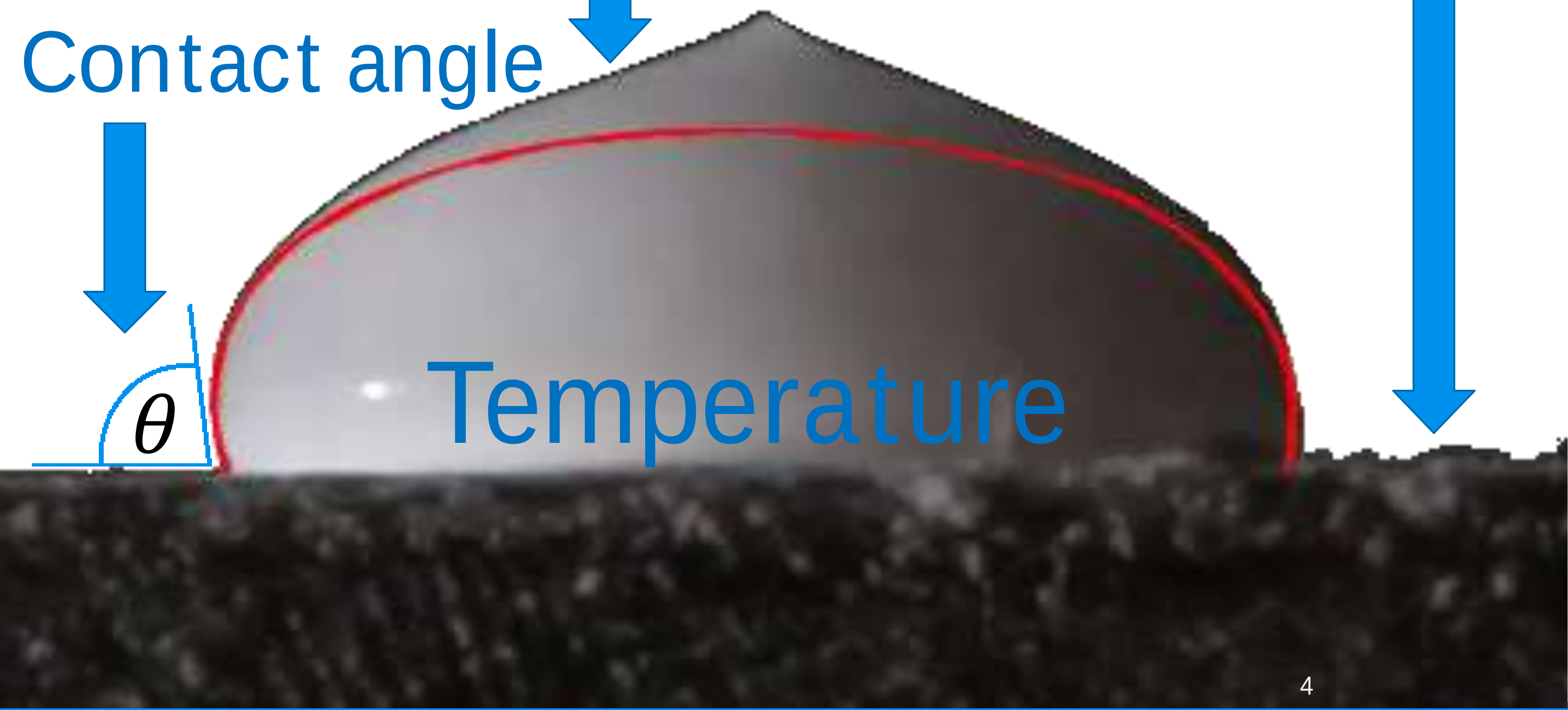
Droplet

- Volume

Surface

- Inclination
- Curvature
- Material

Contact angle

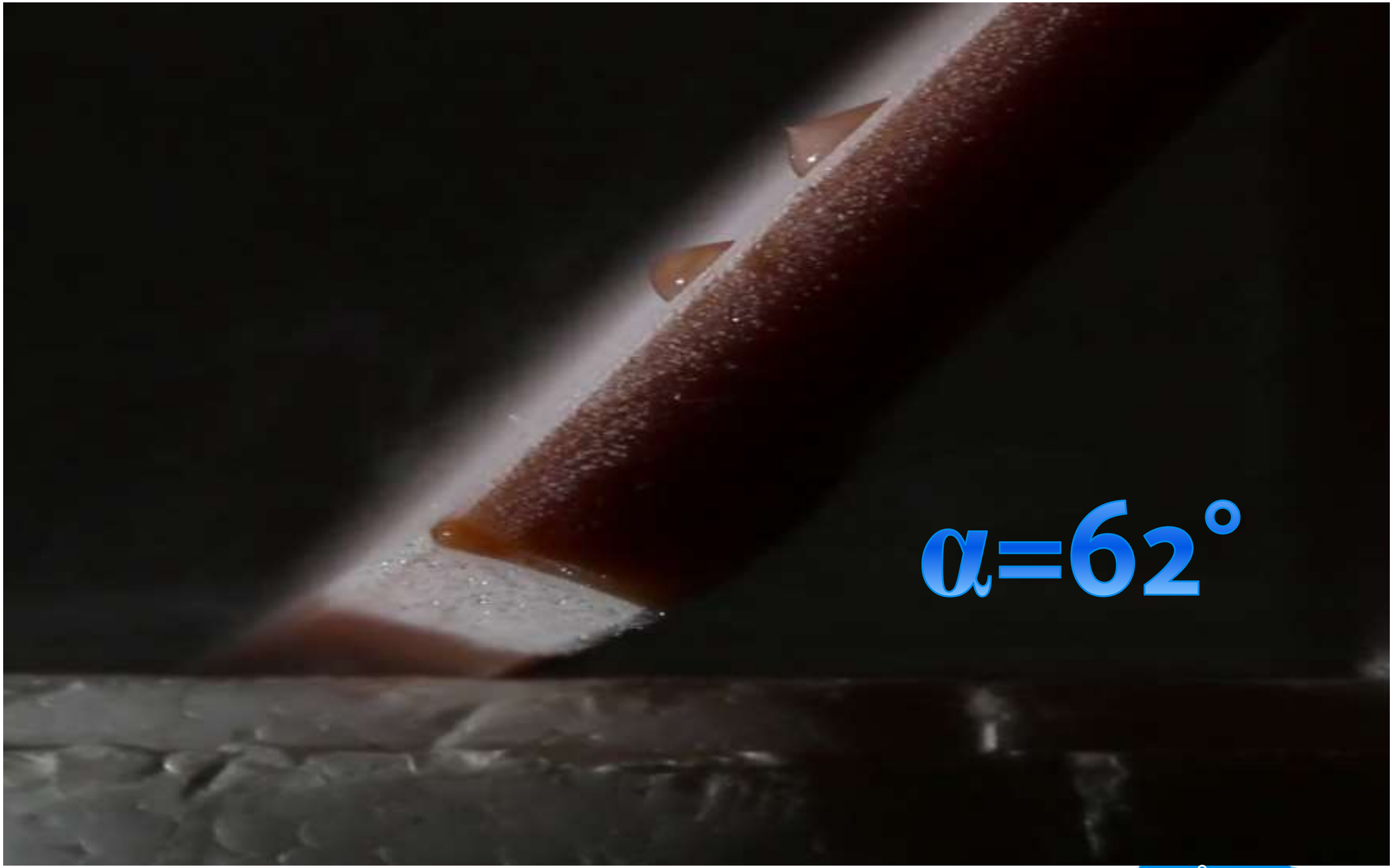


Temperature

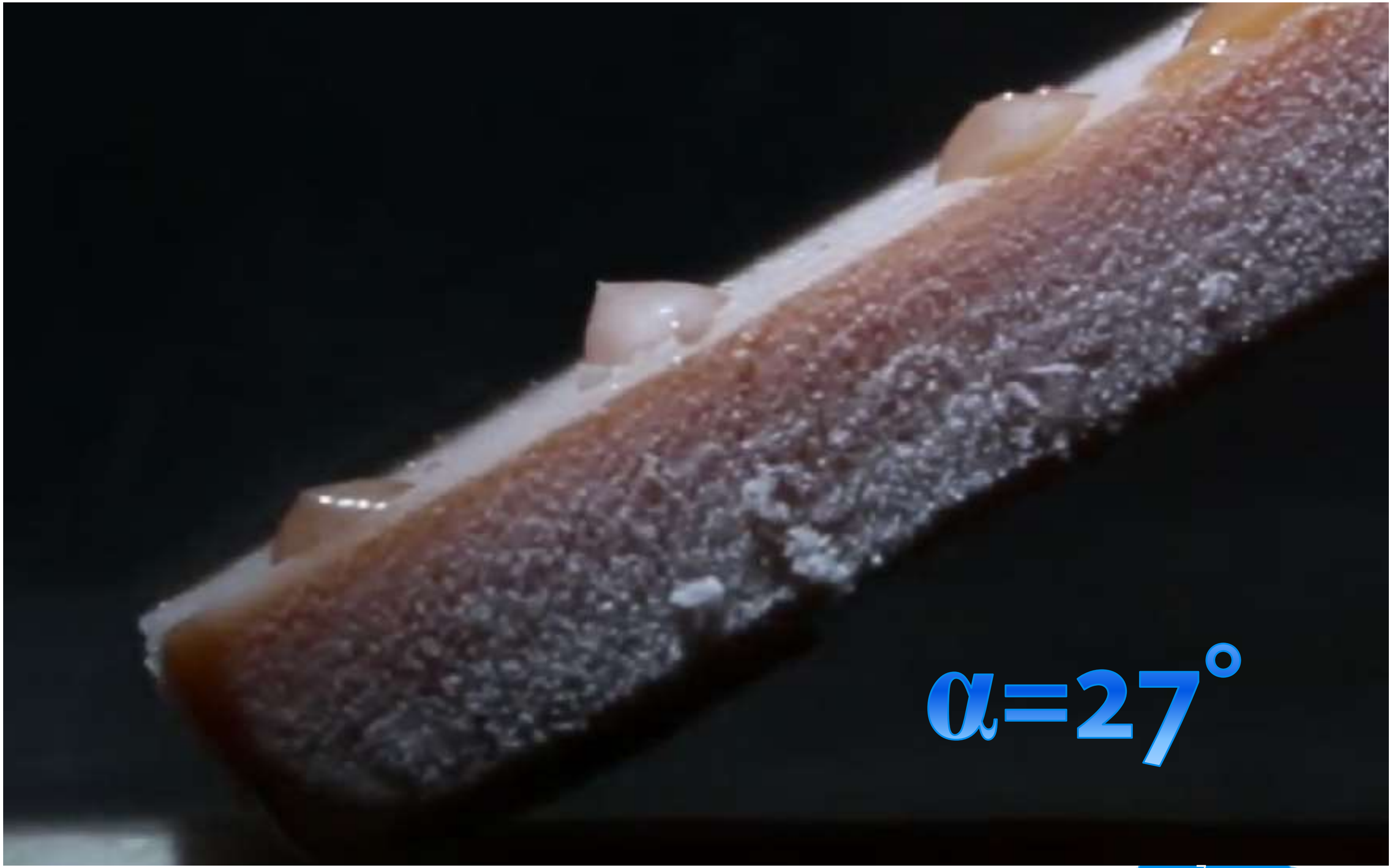
Dependence on the volume of the droplet

Essentially the same

Inclination of the surface



Inclination of the surface

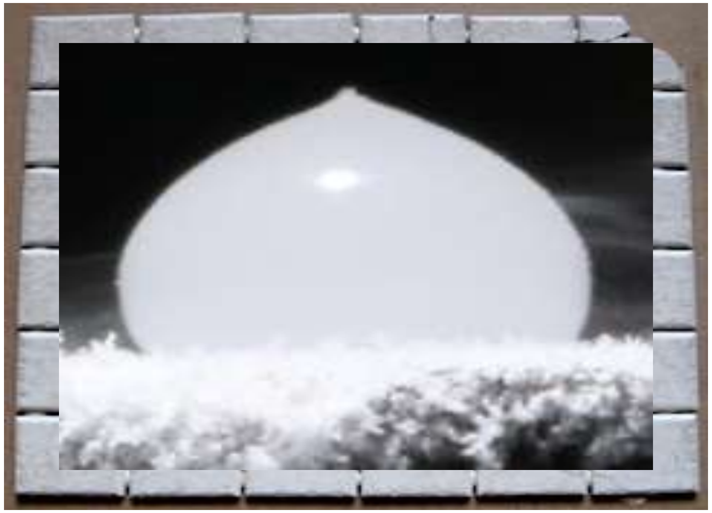


Curvature of the surface

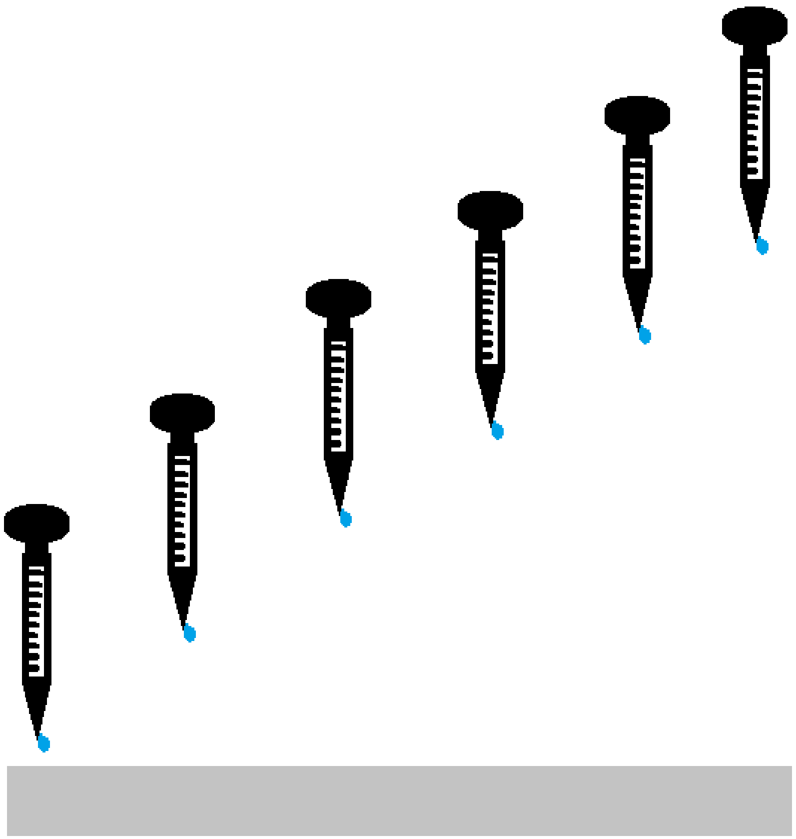
Curvature of the surface

Curvature of the surface

Material of the surface



Contact angle of the droplet



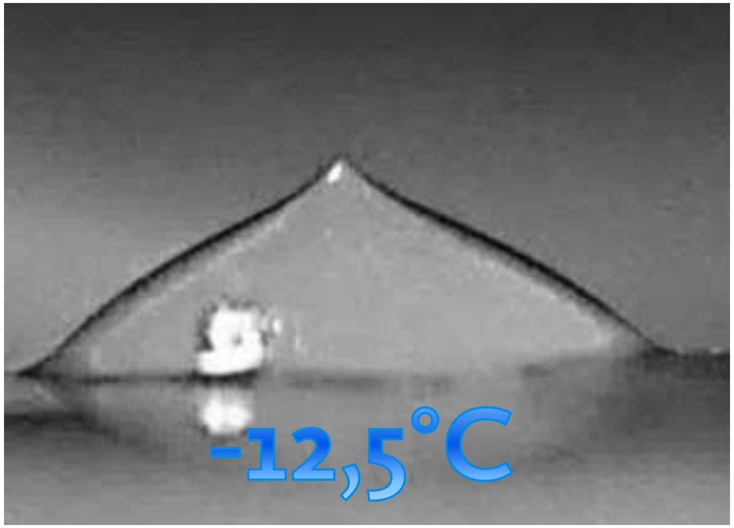
+

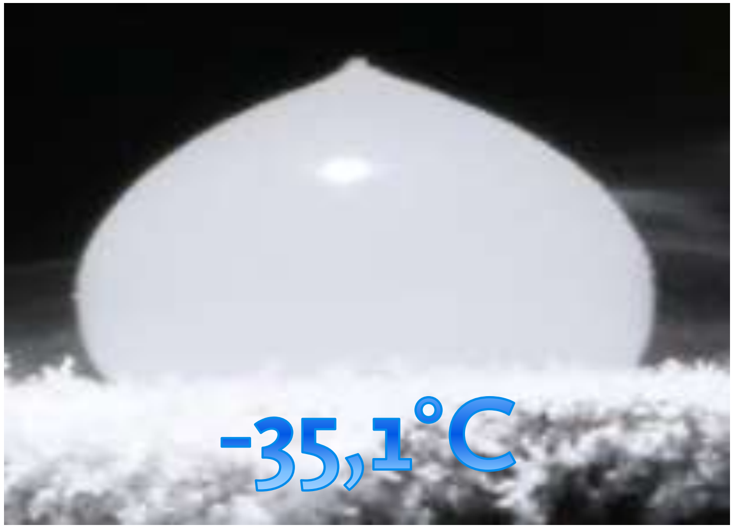
Different heights

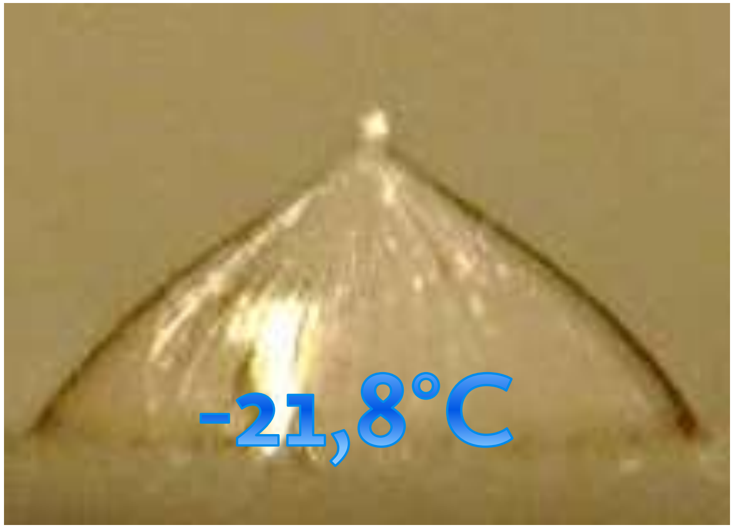
Droplets of the same volume

Different contact angles

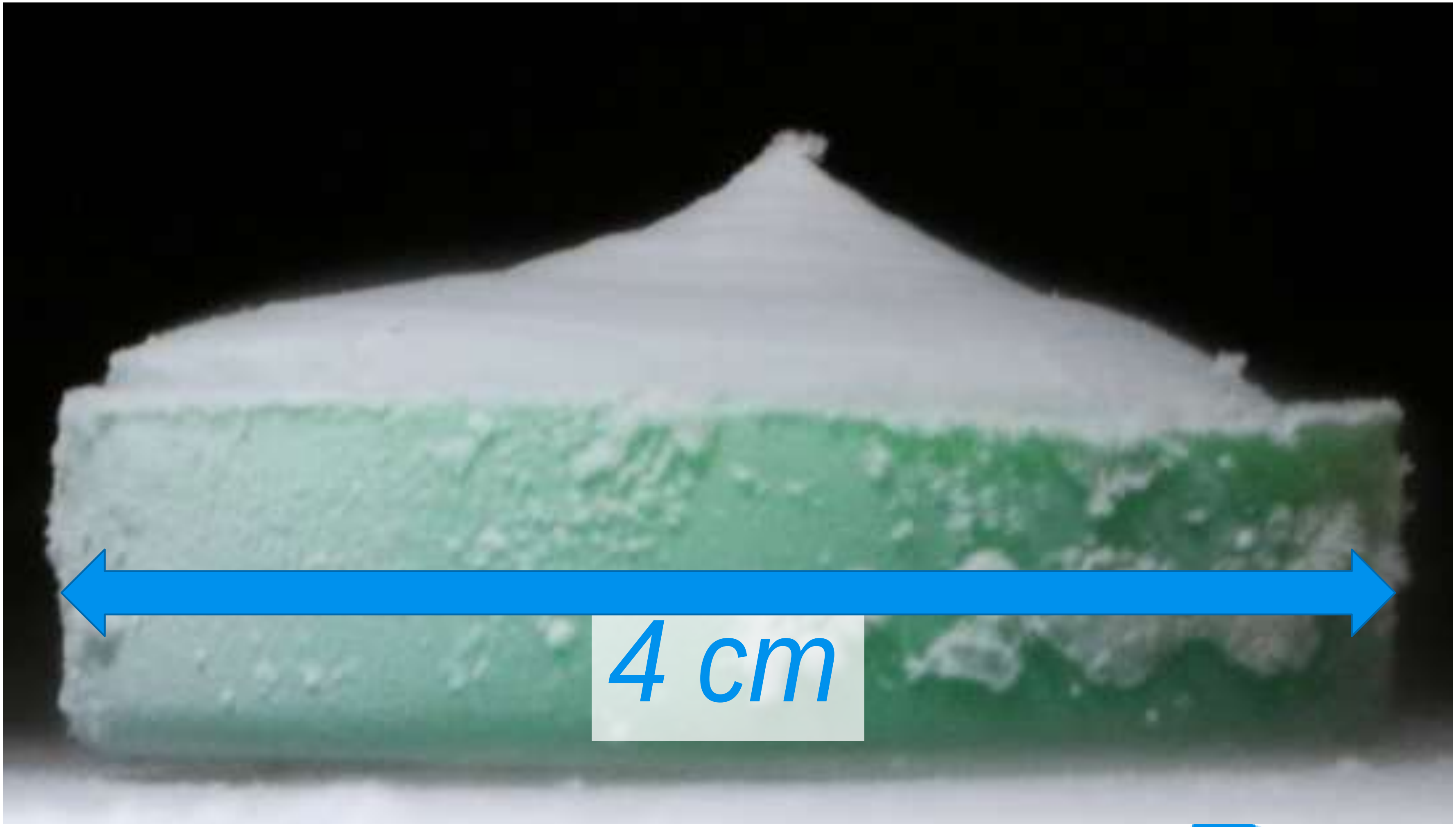
Temperature







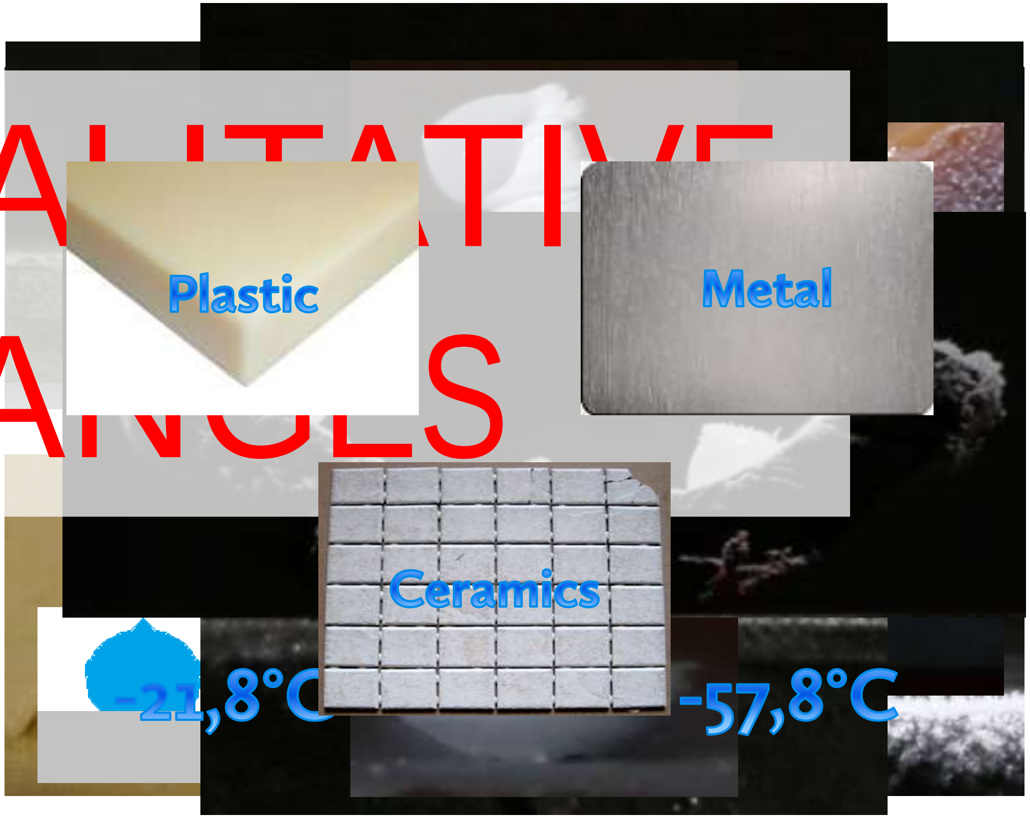
Extreme volume of the droplet



Summary of preliminary experiments

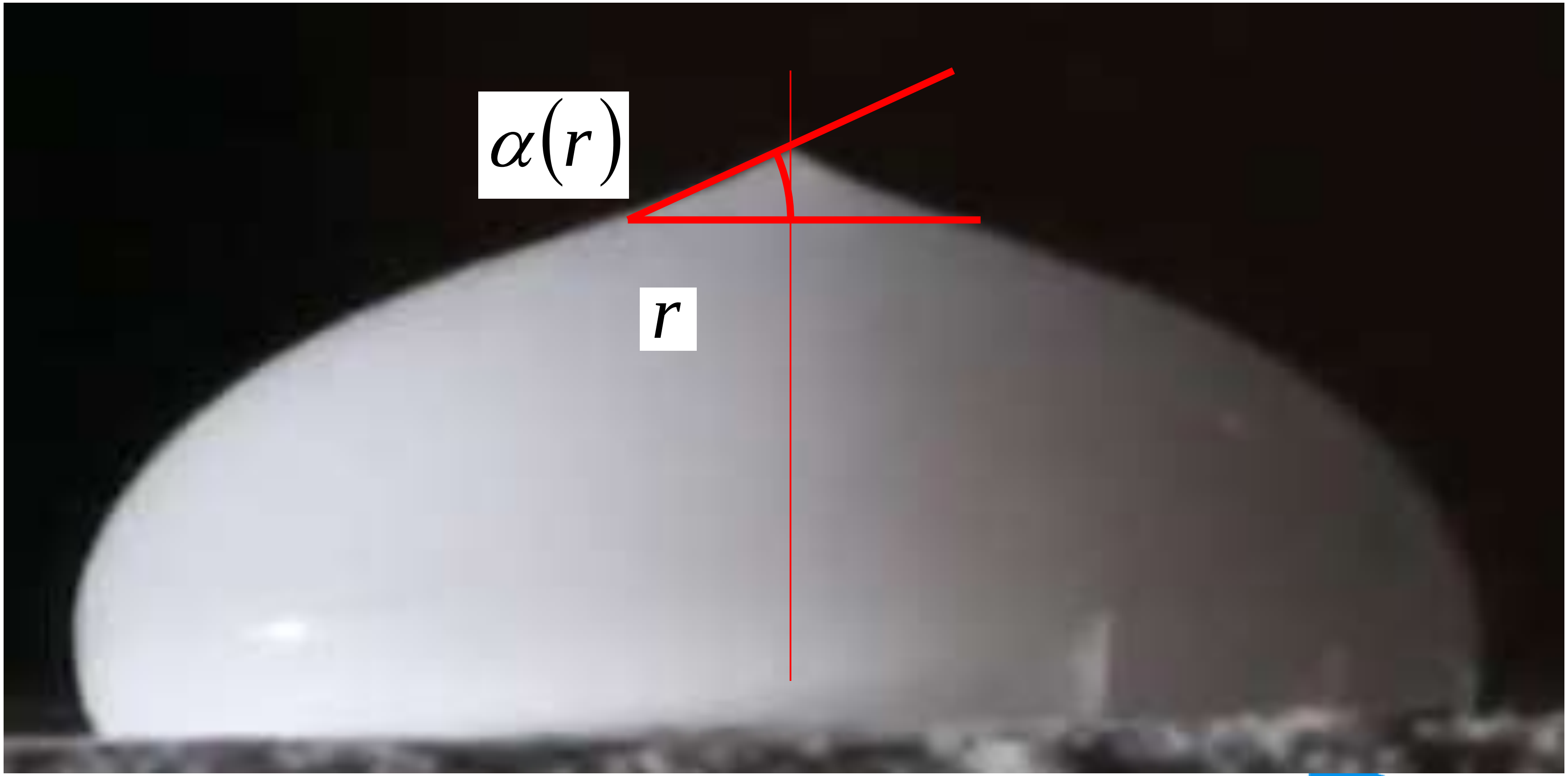
- Changeable parameters
  - Droplet
    - Volume
  - Surface
    - Inclination
    - Curvature
    - Material
  - Contact angle
  - Temperature

NO QUALITATIVE CHANGES



What is the shape of the peak?

- Described by $\alpha(r)$

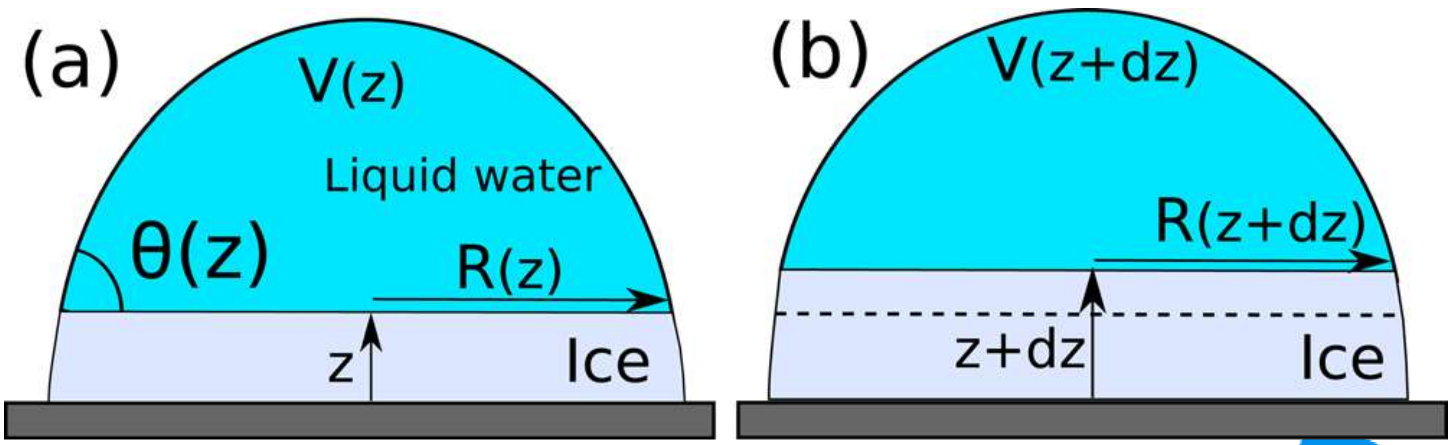


Existing literature

*"**Pointy** ice-drops: How water freezes into a singular shape"*

J.H. Snoeijer and P. Brunet, Am. J. Phys. 80, 764 (2012)

- Heat conduction equations solved
- ~~Assumption: Planar freezing~~



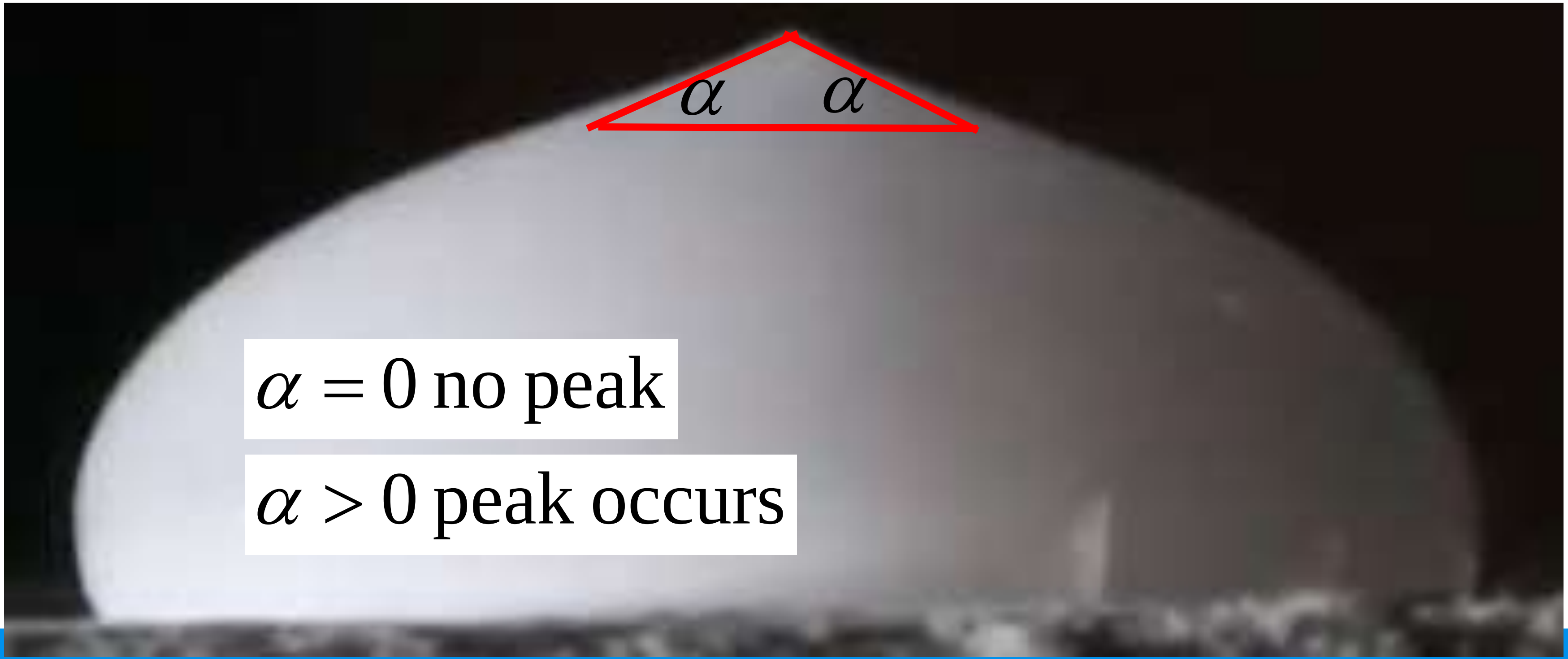
Existence of reservoir

Existence of reservoir

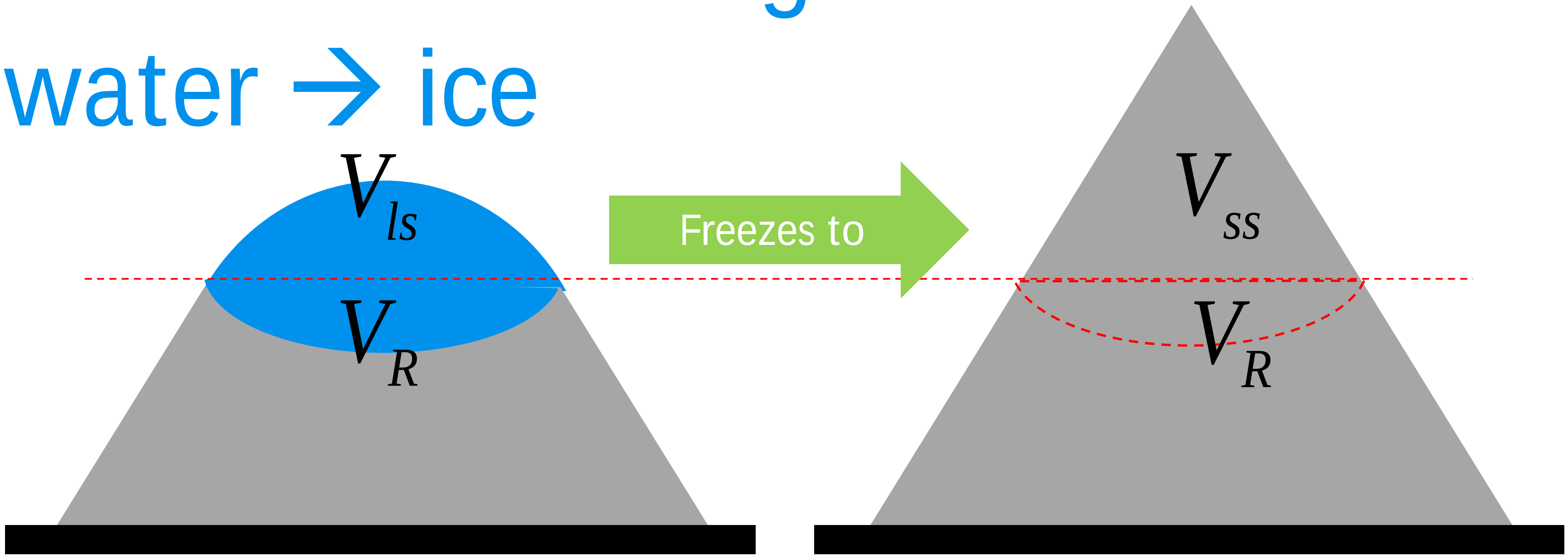
Existence of reservoir

The peak: An alternative approach

- Shape as we approach the top: *always* a cone
  - 1st term of Taylor's expansion of $\alpha(r)$: $\alpha(r) \approx \alpha$
  - "Stabilized freezing"



Stabilized freezing: water → ice



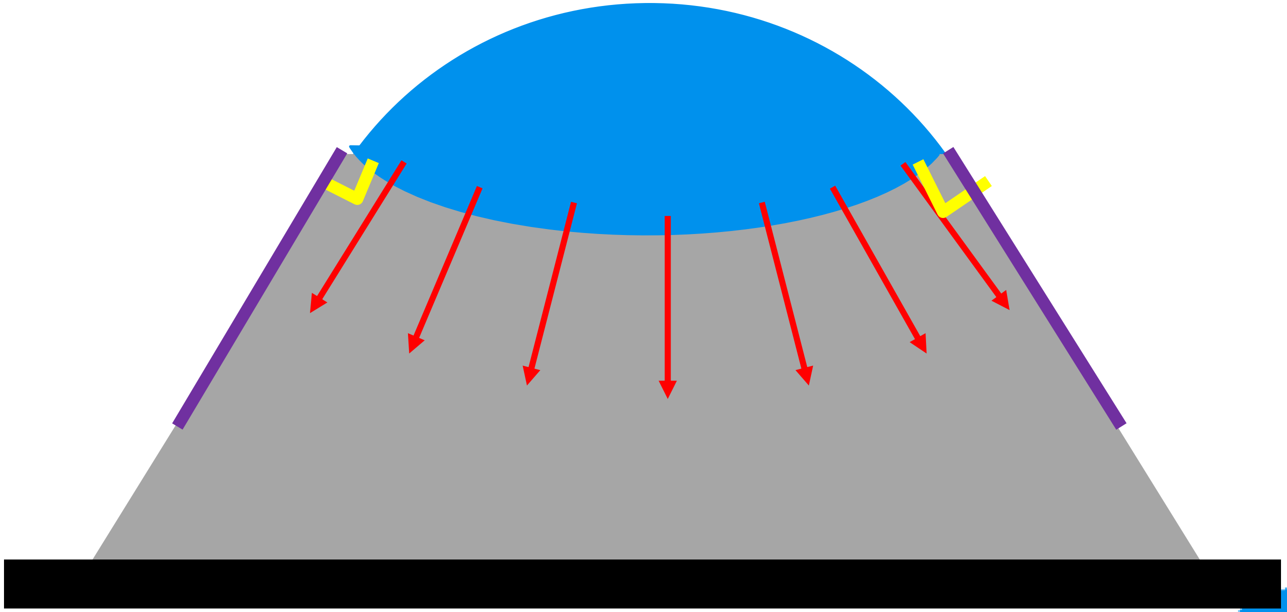
$$\left(V_{ls}+V_{R}\right)\rho_{water}=\left(V_{ss}+V_{R}\right)\rho_{ice}$$

Equation for α

But we need to know the shape of reservoir

Heat flow on the water-ice interface

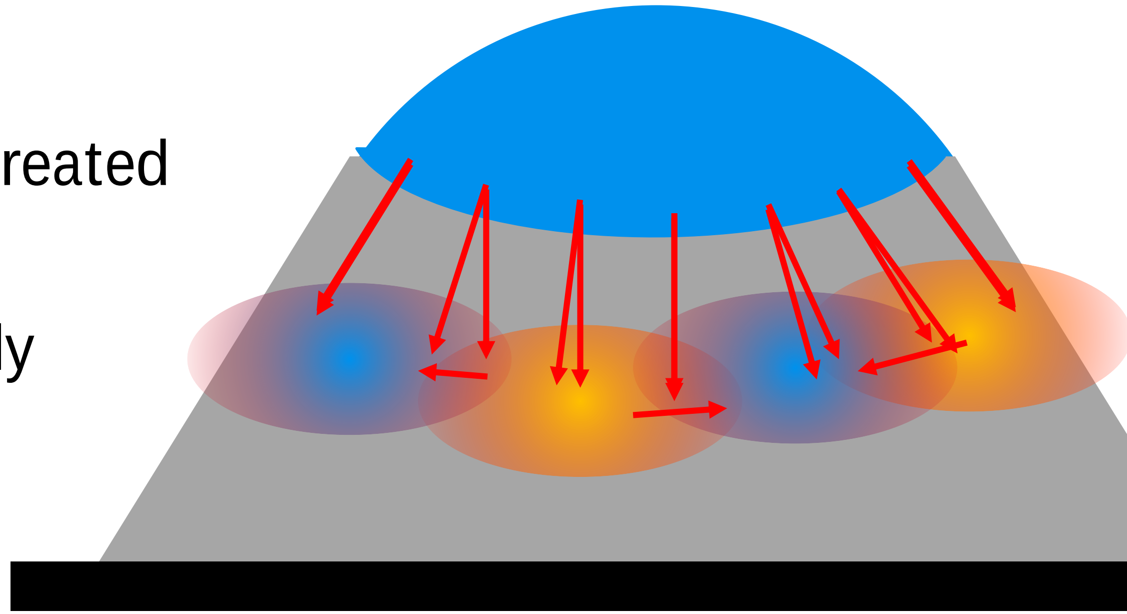
- Heat flow:
  - perpendicular to the water-ice interface
  - near the surface: parallel to the surface
- Interface is perpendicular to the surface



Shape of the reservoir

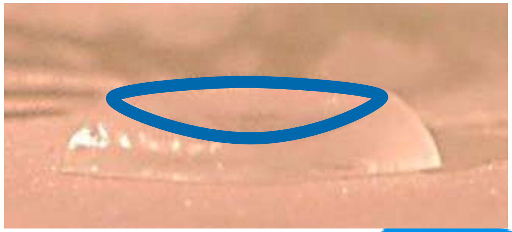
Heat flowing unevenly:

- Hotter/colder areas are created
- Heat flows to the colder
- Heat flow becomes evenly distributed

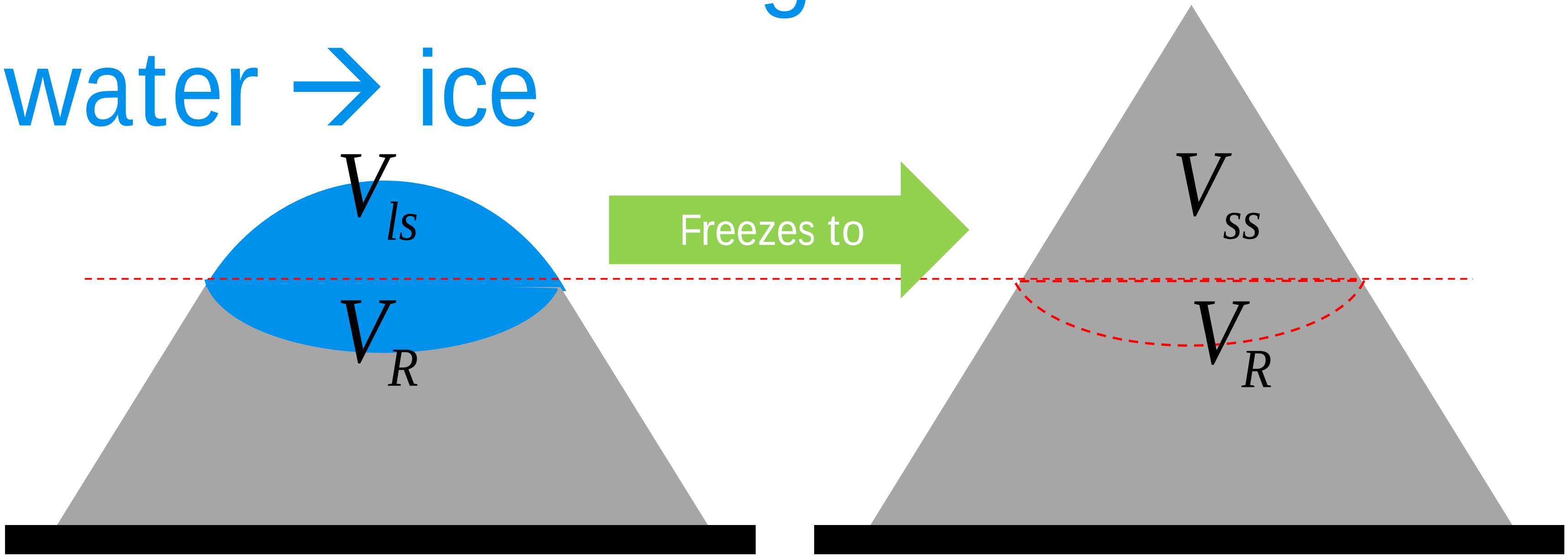


Stabilized freezing:

- Heat flow evenly distributed; perpendicular to interface
- Reservoir: spherical cap



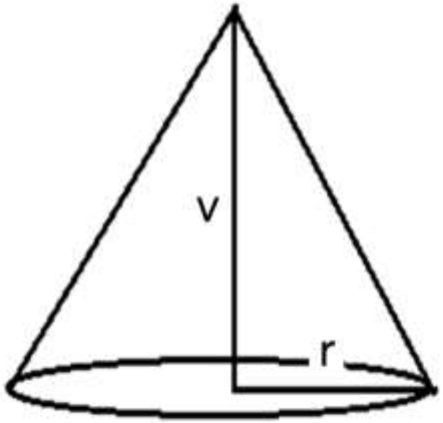
Stabilized freezing: water → ice



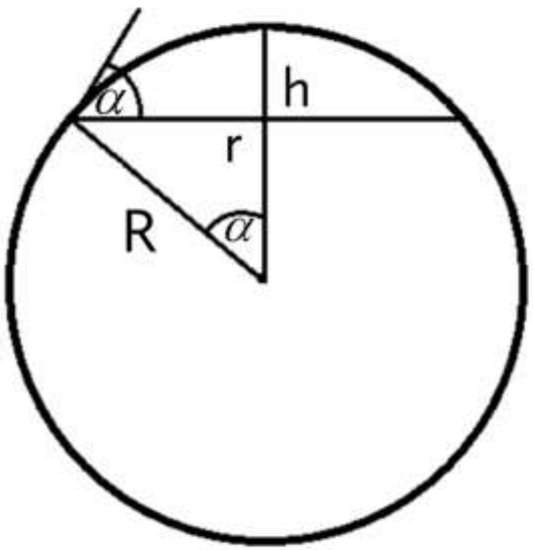
$$\left(V_{ls} + V_R\right)\rho_{water} = \left(V_{ss} + V_R\right)\rho_{ice}$$

Substitute for V_{ls}, V_{ss}, V_R

Volume of liquid and solid



$$V_{ss} = \frac{\pi}{3} r^{3} \tan \alpha$$



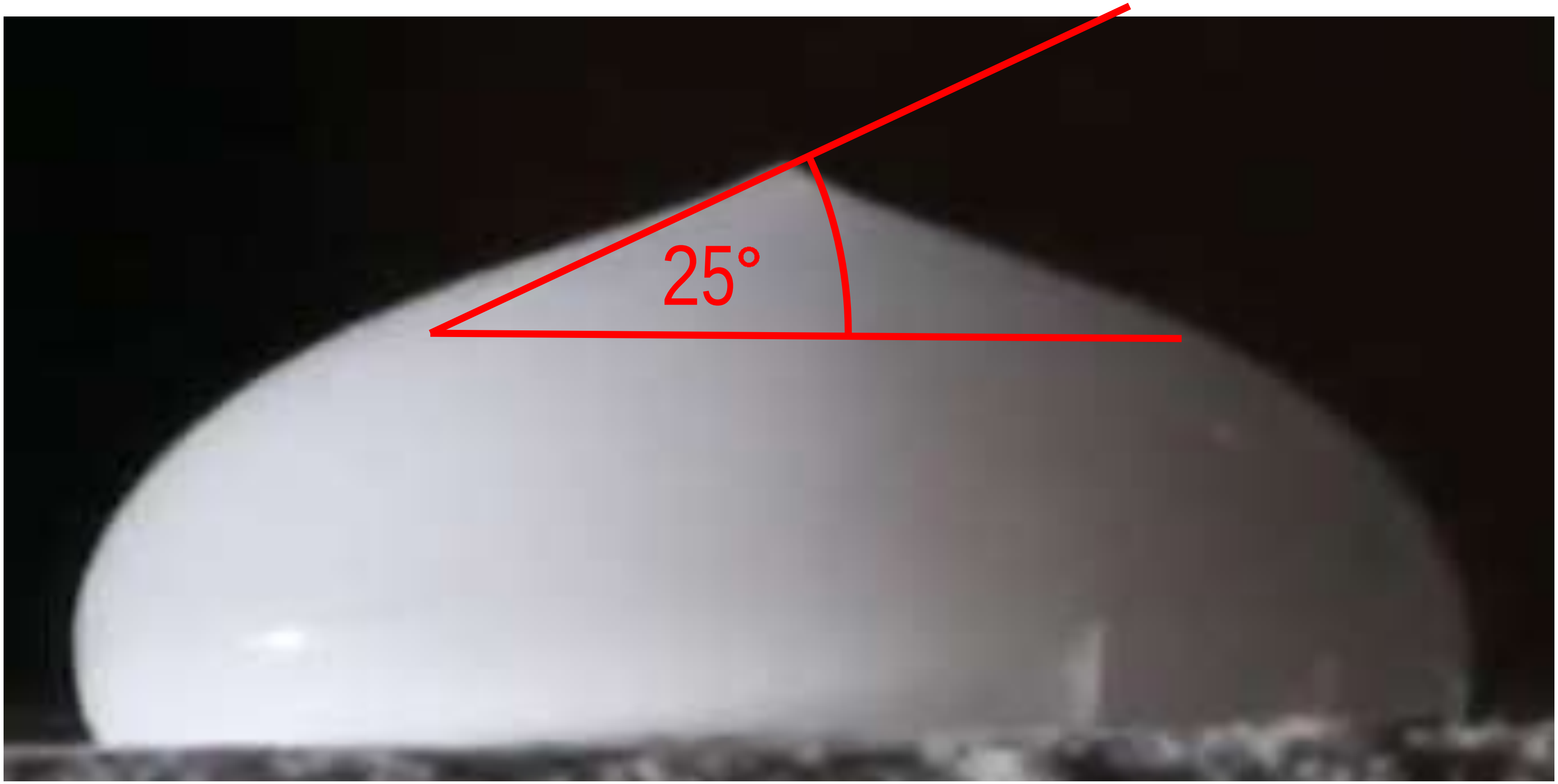
$$V_{ls} = \frac{\pi r^{3}}{3} \left(\frac{1 - \cos \alpha}{\sin \alpha} \right)^{2} \left(\frac{2 + \cos \alpha}{\sin \alpha} \right)$$

Equation for alpha

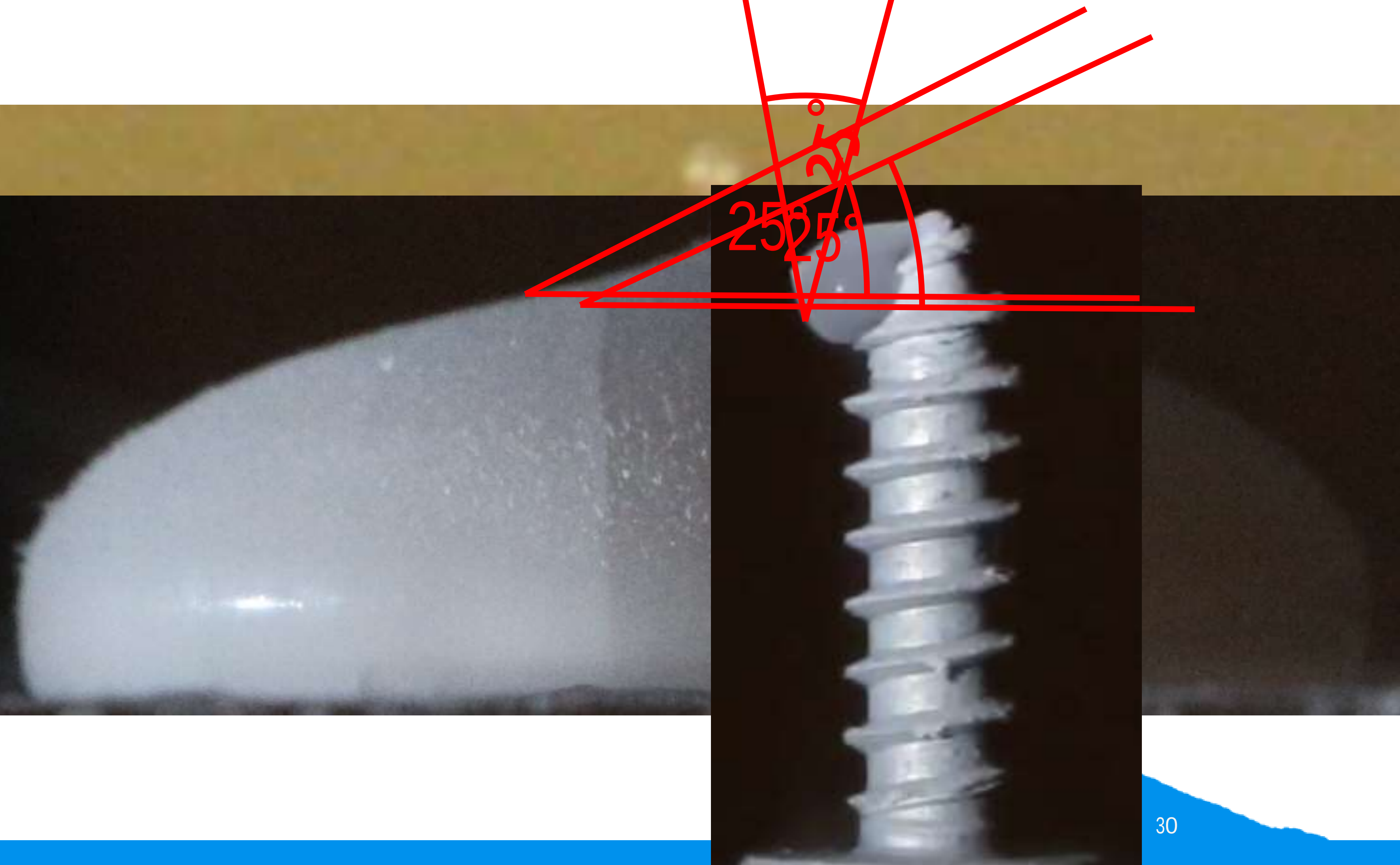
$$\frac{\rho_{ice}}{\rho_{water}} = \frac{\left(\frac{1-\cos\alpha}{\sin\alpha}\right)^2\left(\frac{2+\cos\alpha}{\sin\alpha}\right)+\left(\frac{1-\sin\alpha}{\cos\alpha}\right)^2\left(\frac{2+\sin\alpha}{\cos\alpha}\right)}{\tan\alpha+\left(\frac{1-\sin\alpha}{\cos\alpha}\right)^2\left(\frac{2+\sin\alpha}{\cos\alpha}\right)}$$

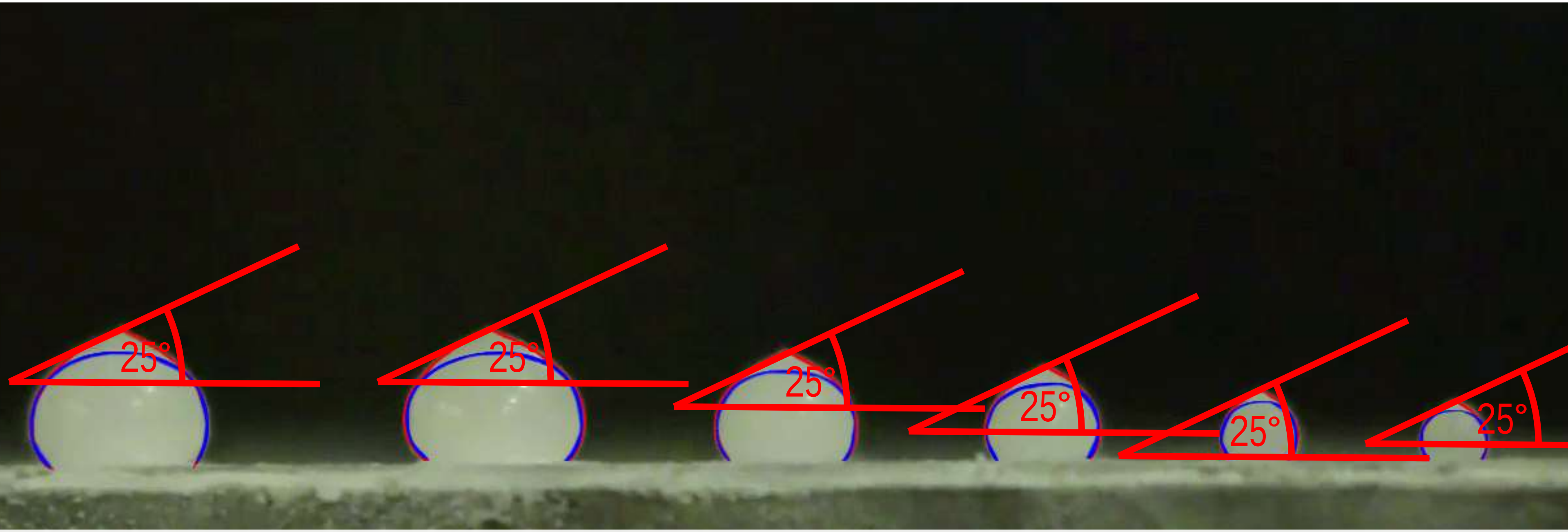
Solution: $\alpha \approx 25°$

Measured angle of approximately 25° on our droplet



Seemingly different droplets





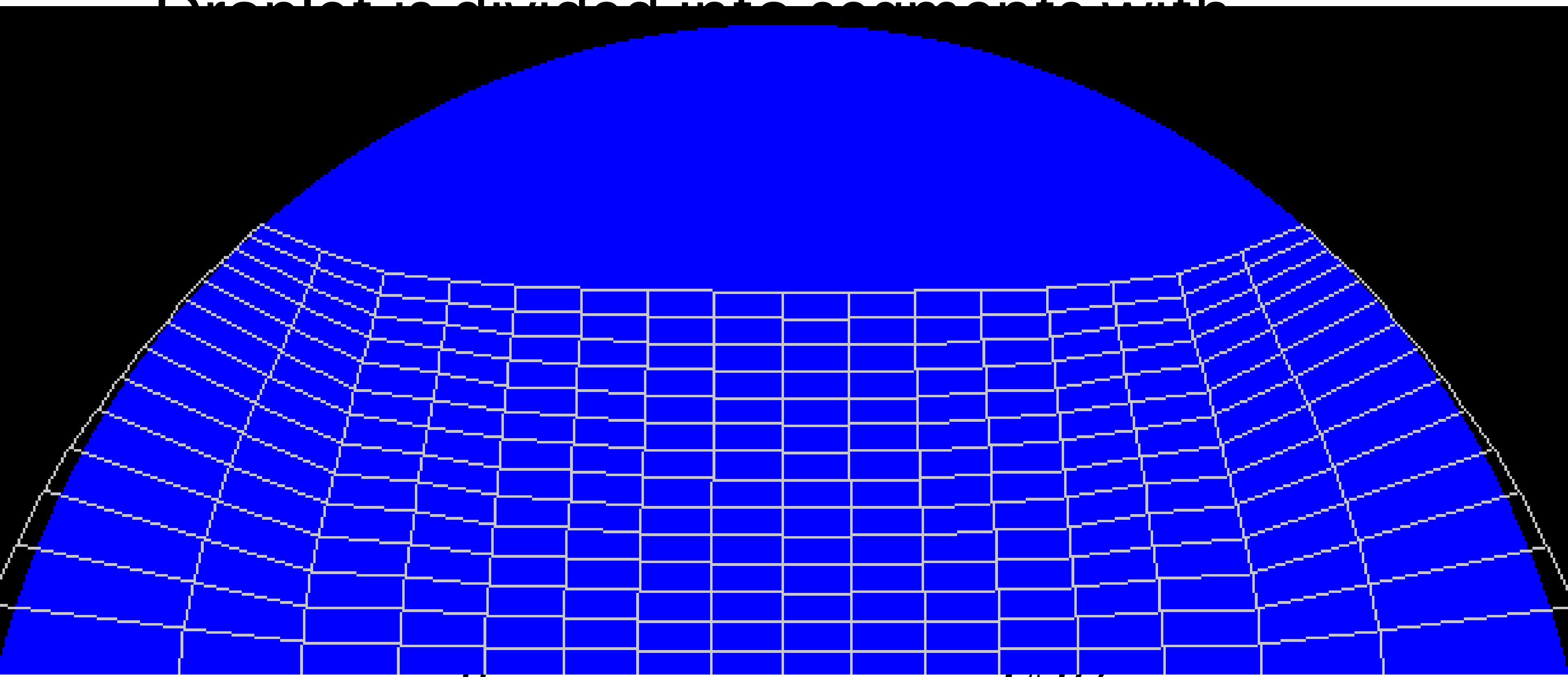
Different droplets but we still measured approximately 25° on each one

Simulation

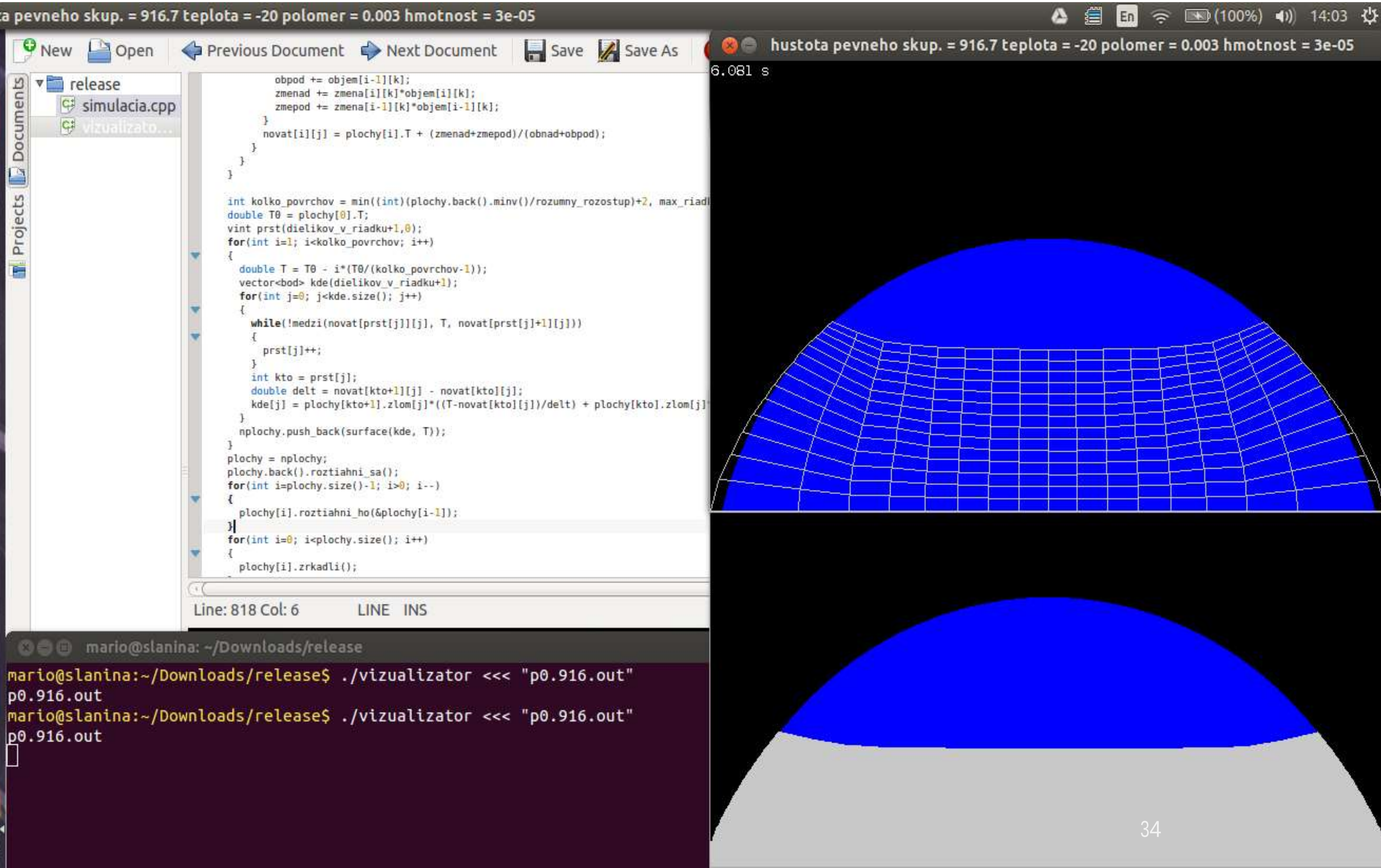
- Heat conduction
- Rotational symmetry
- No heat transfer to the air
- Plate: constant temperature

Simulation time increment

Droplet is divided into segments with



Simulation


a pevneho skup. = 916.7 teplota = -20 polomer = 0.003 hmotnost = 3e-05
En (100%) 14:03
New Open Previous Document Next Document Save Save As
Documents Projects
release
simulacia.cpp
vizualizato...
obpod += objem[i-1][k];
zmenad += zmena[i][k]*objem[i][k];
zmepod += zmena[i-1][k]*objem[i-1][k];
}
novat[i][j] = plochy[i].T + (zmenad+zmepod)/(obnad+obpod);
}
}
}
int kolko_povrchov = min((int)(plochy.back().minv()/rozumny_rozostup)+2, max_riad
double T0 = plochy[0].T;
vint prst(dielikov_v_riadku+1,0);
for(int i=1; i<kolko_povrchov; i++)
{
double T = T0 - i*(T0/(kolko_povrchov-1));
vector<bod> kde(dielikov_v_riadku+1);
for(int j=0; j<kde.size(); j++)
{
while(!medzi(novat[prst[j]][j], T, novat[prst[j]+1][j]))
{
prst[j]++;
}
int kto = prst[j];
double delt = novat[kto+1][j] - novat[kto][j];
kde[j] = plochy[kto+1].zlom[j]*((T-novat[kto][j])/delt) + plochy[kto].zlom[j]
}
nplochy.push_back(surface(kde, T));
}
plochy = nplochy;
plochy.back().roztiahni_sa();
for(int i=plochy.size()-1; i>0; i--)
{
plochy[i].roztiahni_ho(&plochy[i-1]);
}
for(int i=0; i<plochy.size(); i++)
{
plochy[i].zrkadli();
Line: 818 Col: 6 LINE INS
mario@slanina: ~/Downloads/release
mario@slanina:~/Downloads/release$ ./vizualizator <<< "p0.916.out"
p0.916.out
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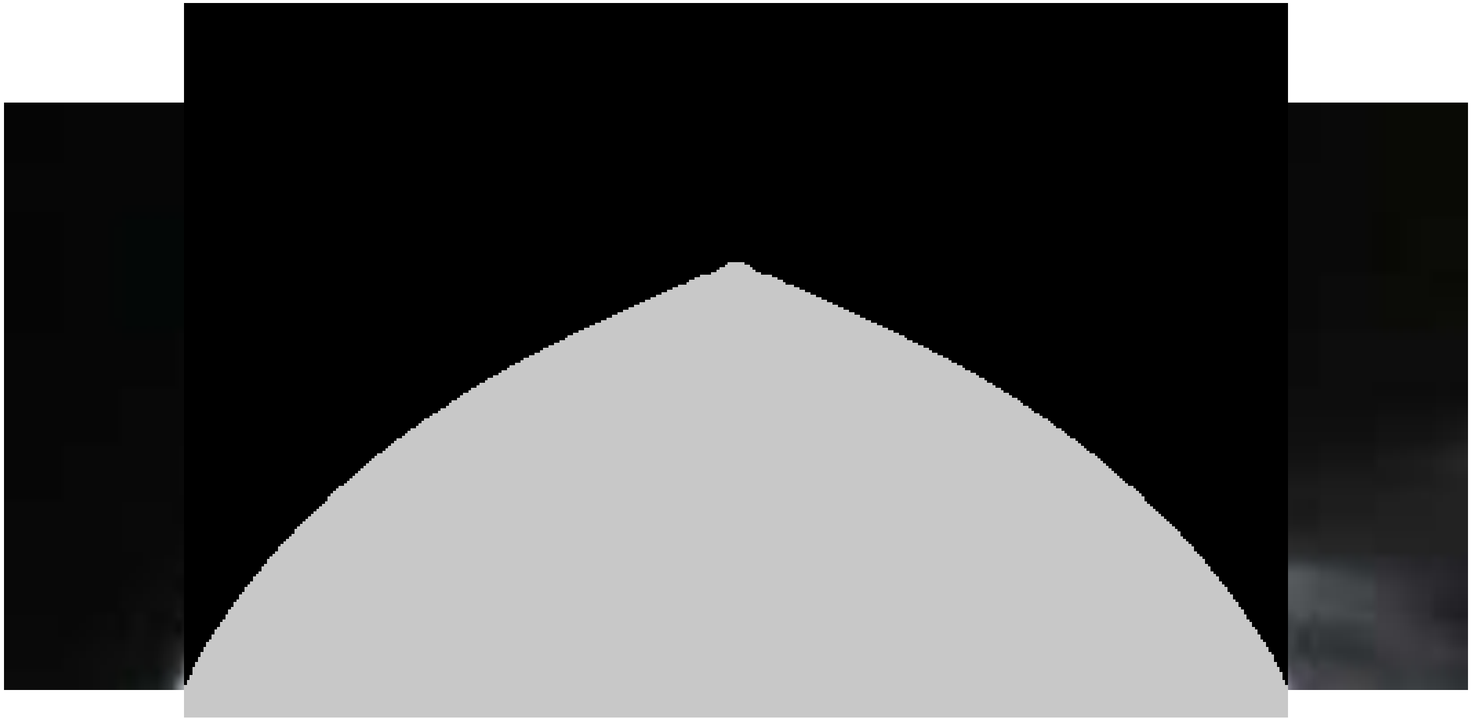
p0.916.out
hustota pevneho skup. = 916.7 teplota = -20 polomer = 0.003 hmotnost = 3e-05
6.081 s

Real droplet vs simulated one



Freezing: Simulation vs. Experiment

Freezing: Simulation vs. Experiment

Summary of preliminary experiments

- Changeable parameters
  - Droplet
    - Volume
  - Surface
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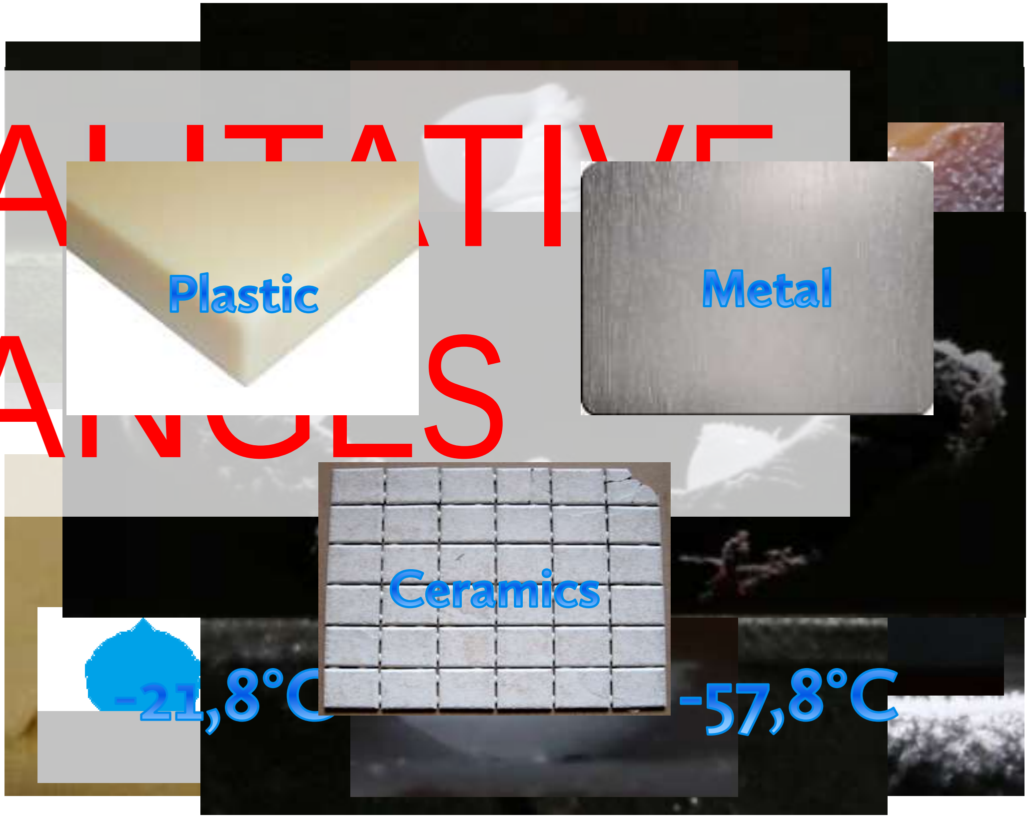
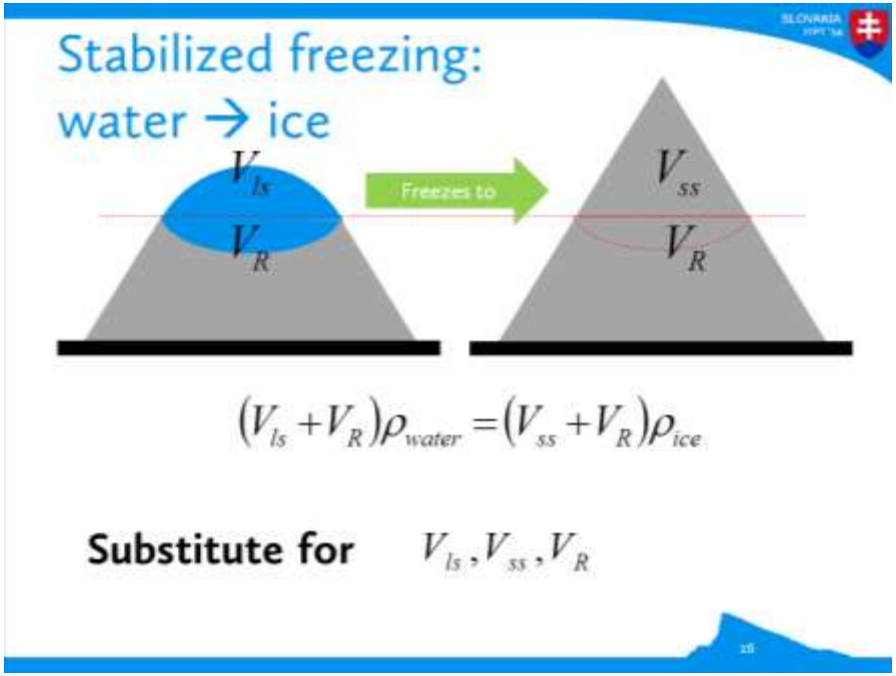
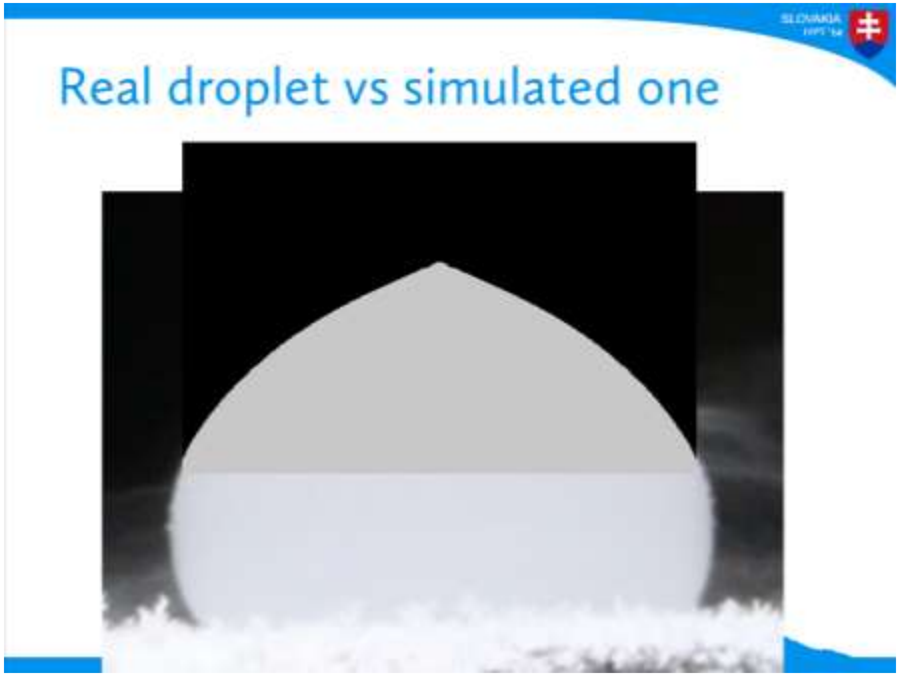
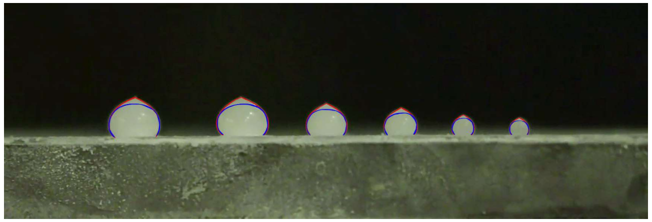
NO QUALITATIVE CHANGES



Conclusion


Stabilized freezing:
water → ice
V_{ls}
V_R
Freezes to
V_{ss}
V_R
$(V_{ls}+V_R)\rho_{water}=(V_{ss}+V_R)\rho_{ice}$
Substitute for V_{ls}, V_{ss}, V_R


Real droplet vs simulated one




Thank you for your attention
Yes the snowman is made from frozen droplets r=cca 3mm

Apendix

Hairiness

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Hairy droplets – known problem

091102-2 Enríquez *et al.* Phys. Fluids **24**, 091102 (2012)

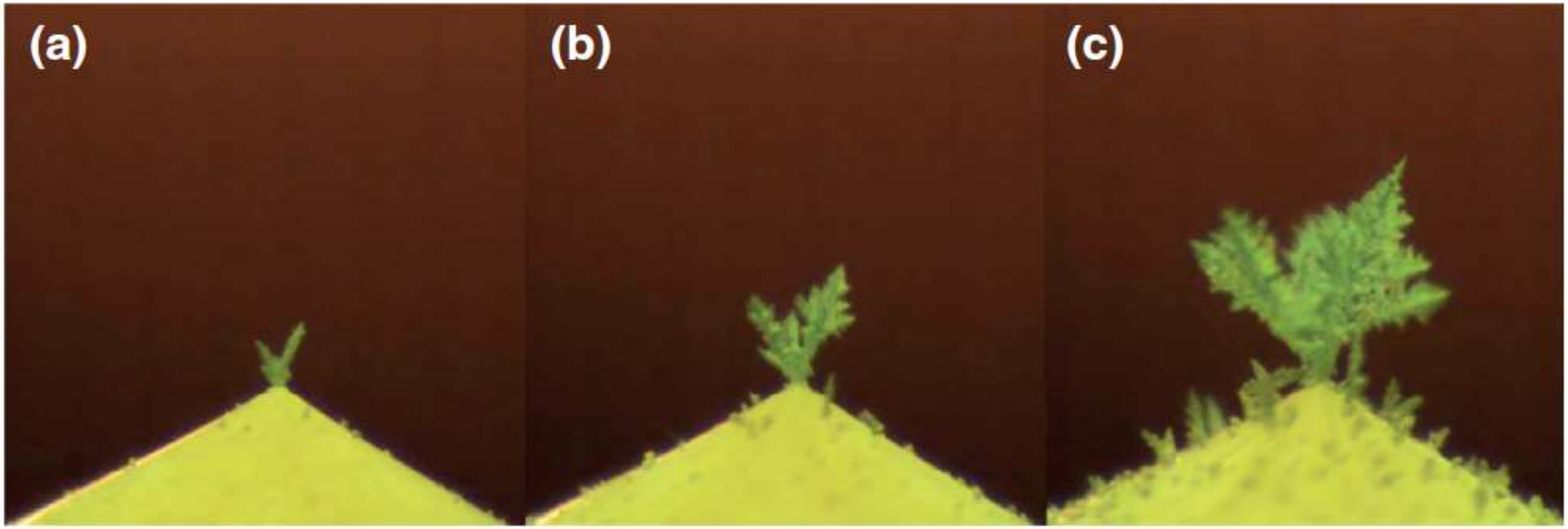
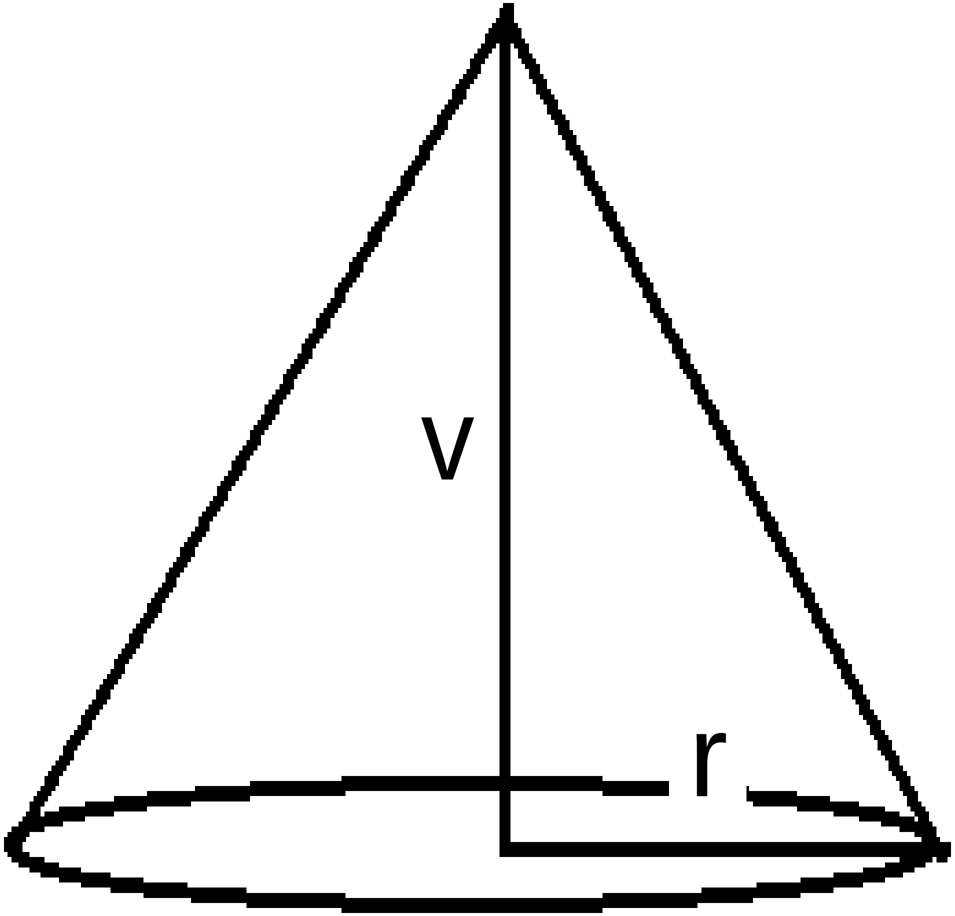


FIG. 2. Three snapshots of the "frozen tree" formation after the water drop has completely solidified. The singularity acts as a preferential site for deposition of water vapor from the surrounding air, and ice crystals grow at the tip of the ice drop. The width of each snapshot is approximately 1.5 mm. The times between frames (a) and (b) is 12 s, and between (b) and (c) is 27 s.

Volume of solid



$$V_s = \frac{\pi}{3} r^2 v$$

$$\tan\alpha = \frac{v}{r}$$

$$v = r\tan\alpha$$

$$V_s = \frac{\pi}{3} r^3 \tan\alpha$$

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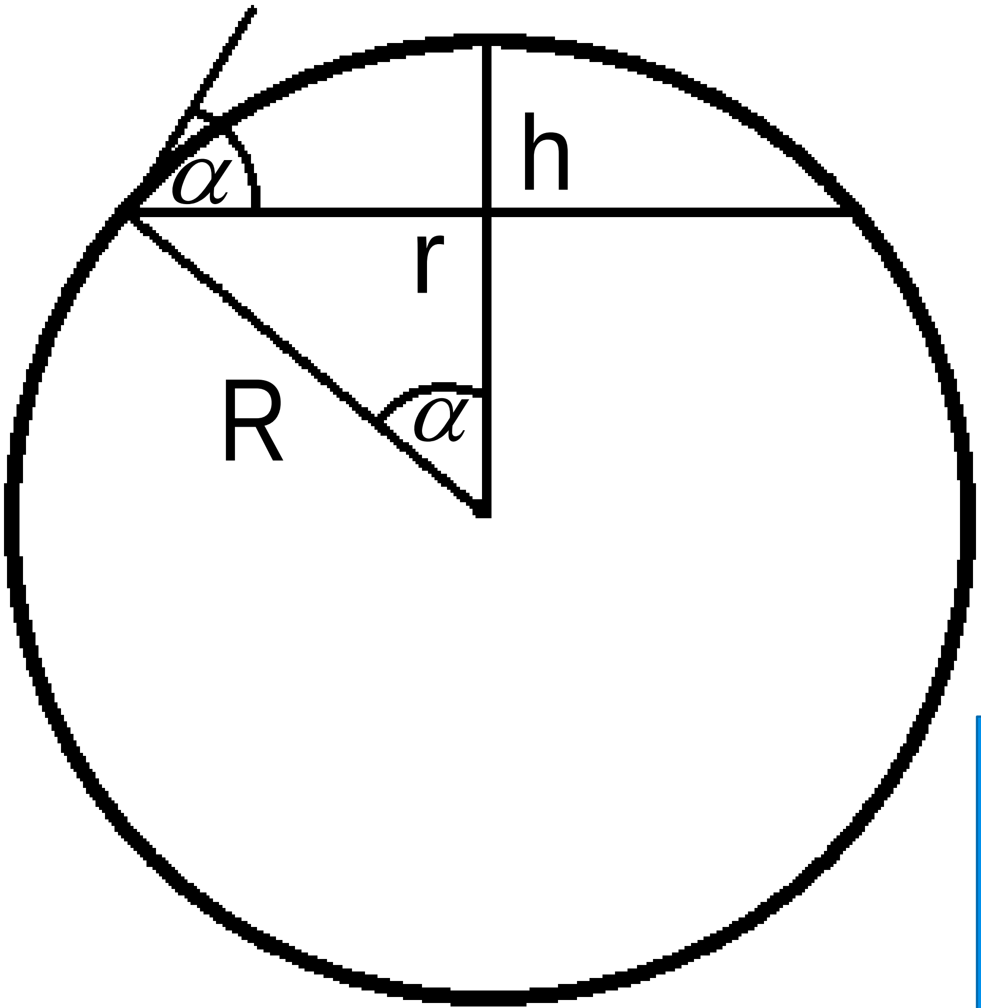
Volume of liquid



$$V_l = \frac{\pi h^2}{3}(3R - h)$$

$$\tan\alpha = \frac{r}{R - h}$$

$$h = R - \frac{r}{\tan\alpha}$$

$$\sin\alpha = \frac{r}{R}$$

$$R = \frac{r}{\sin\alpha}$$

$$V_l = \frac{\pi r^3}{3}\left(\frac{1 - \cos\alpha}{\sin\alpha}\right)^2\left(\frac{2 + \cos\alpha}{\sin\alpha}\right)$$

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Spherical shape of the droplet

$$\mathrm{Bo} = \frac{\rho g R^2}{\gamma}$$

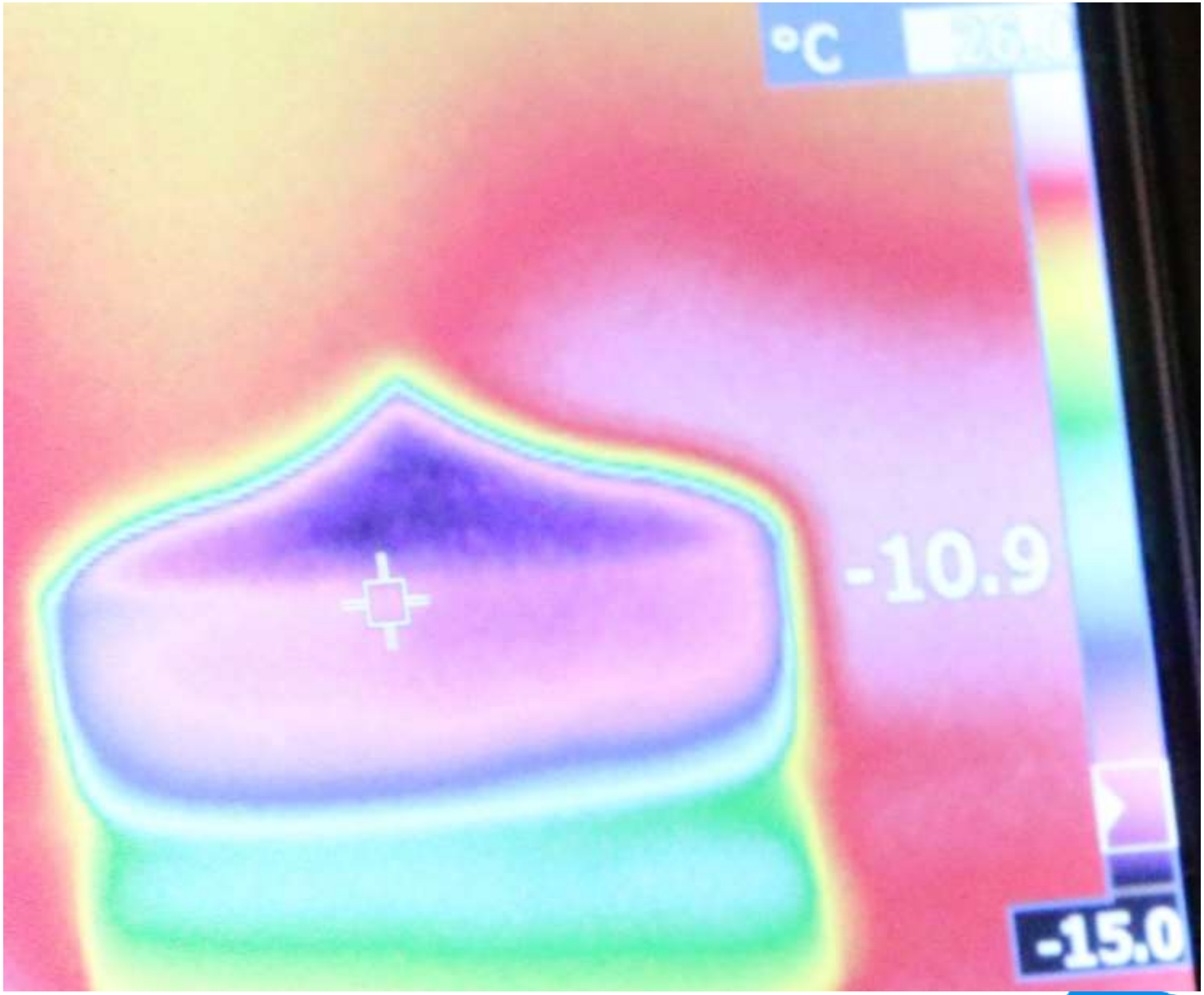
ρ : liquid density

g : gravity acceleration

R : diameter of perfect sphere with the same volume

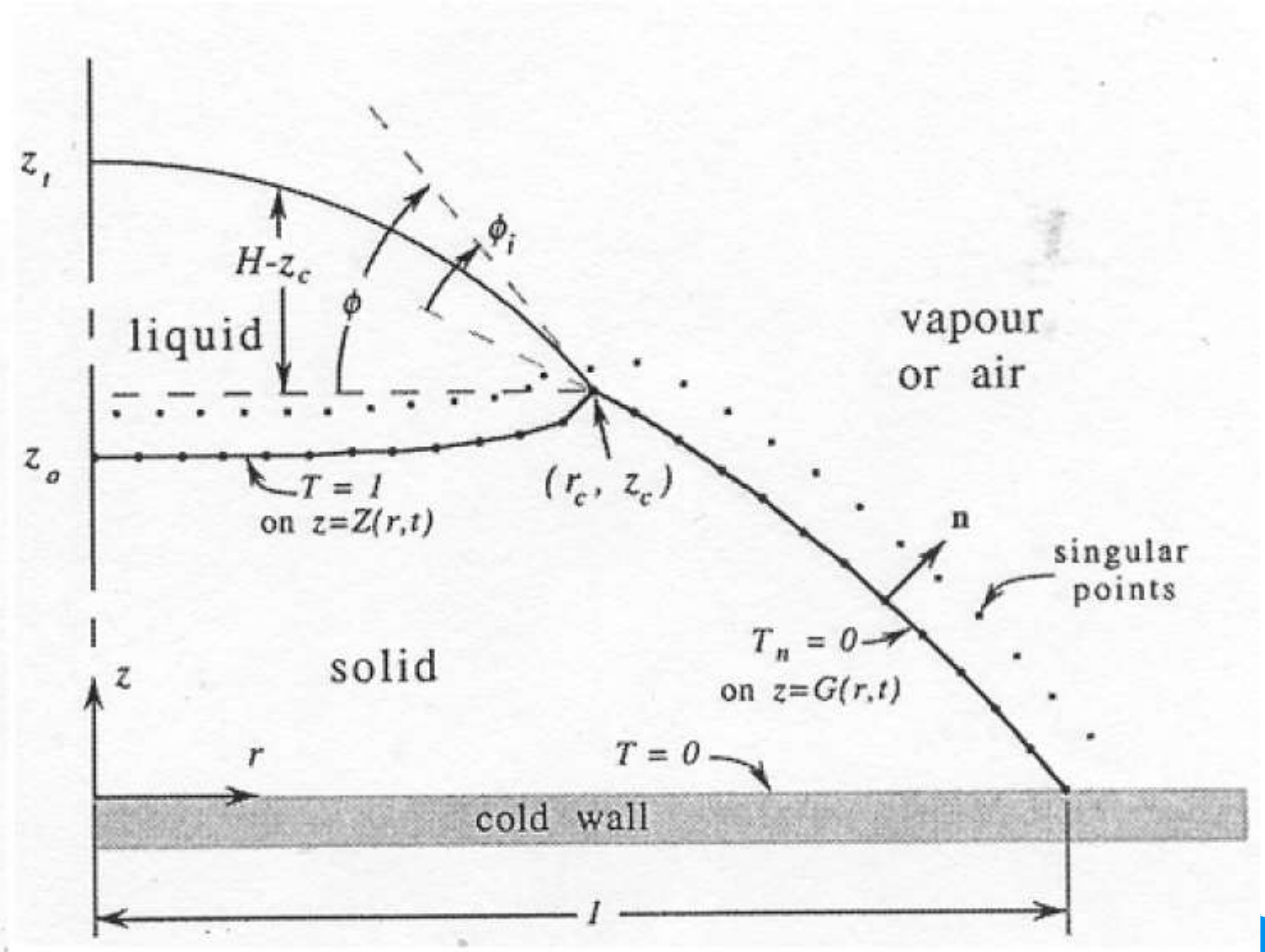
γ : surface tension

46

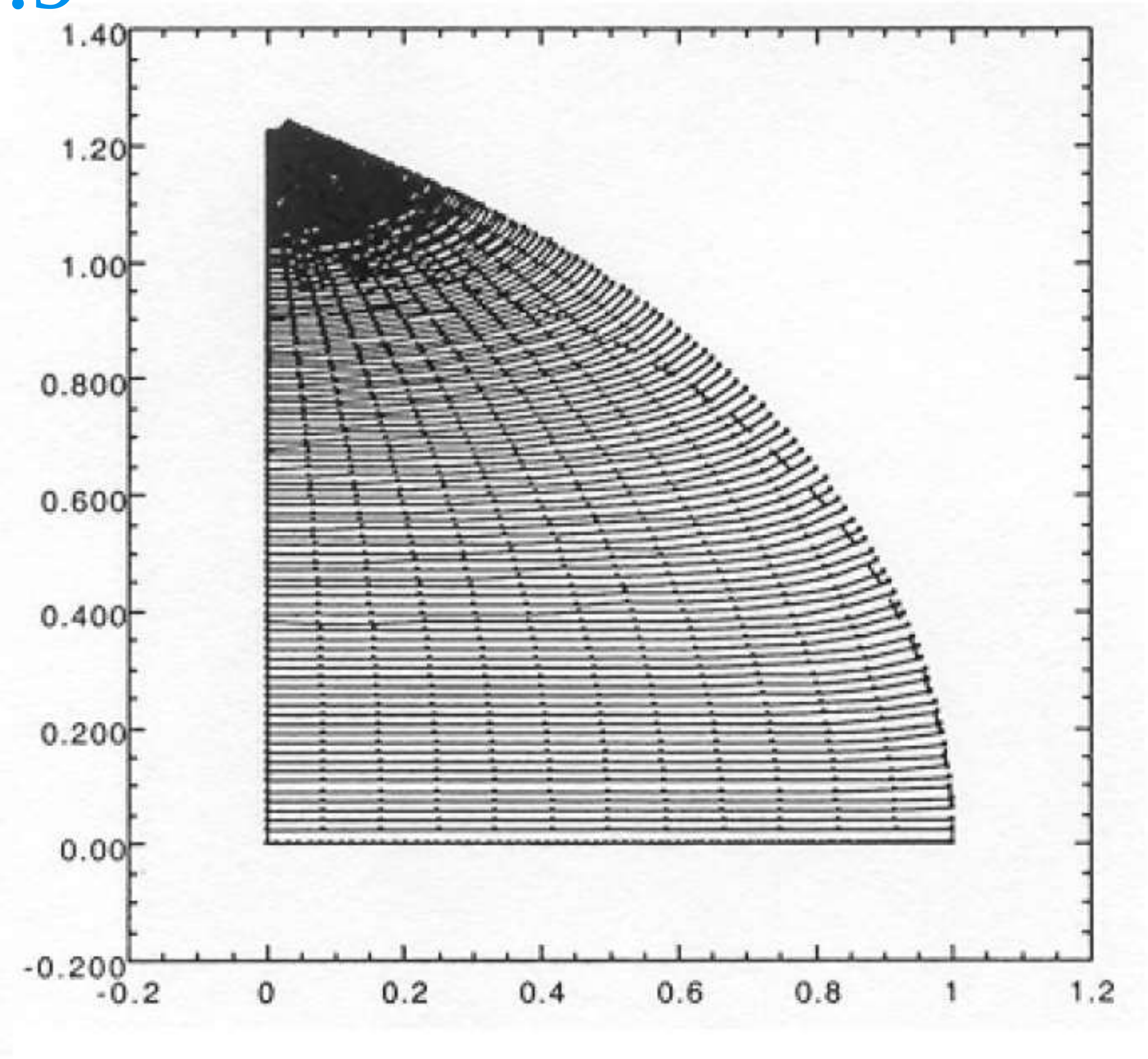

°C
-10.9
-15.0

Solidifying sessile water droplets

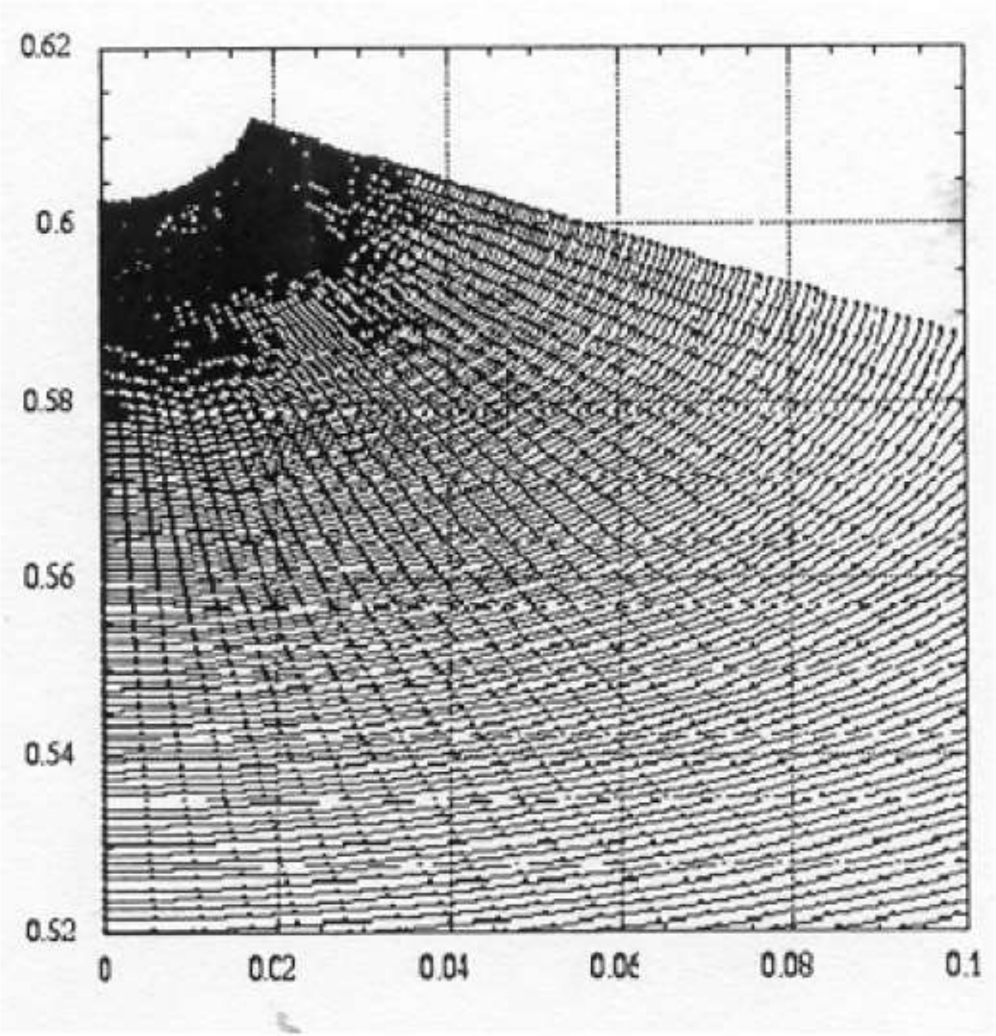
W. W. Schultz, M. G. Worster, D. M. Anderson

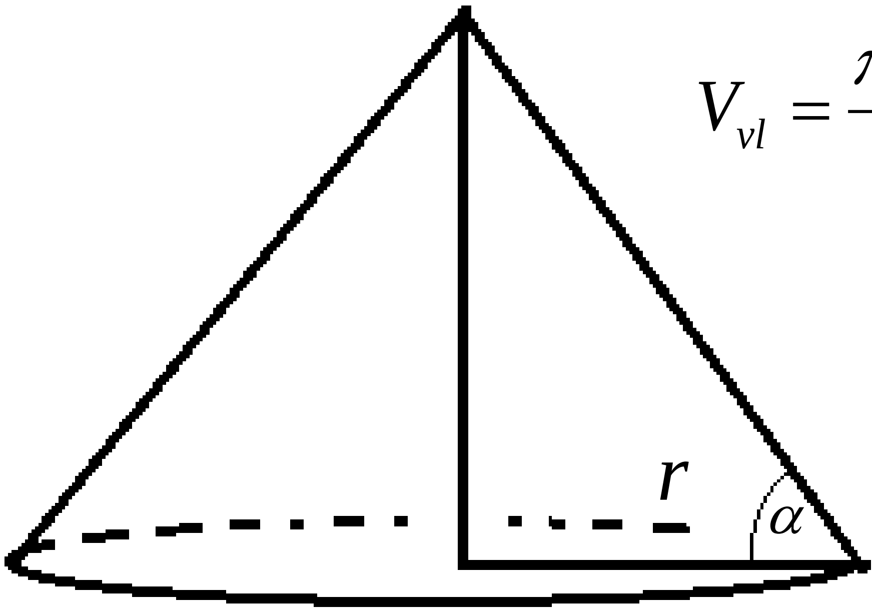


The shape of solidifying droplet when $\rho = 0.9$

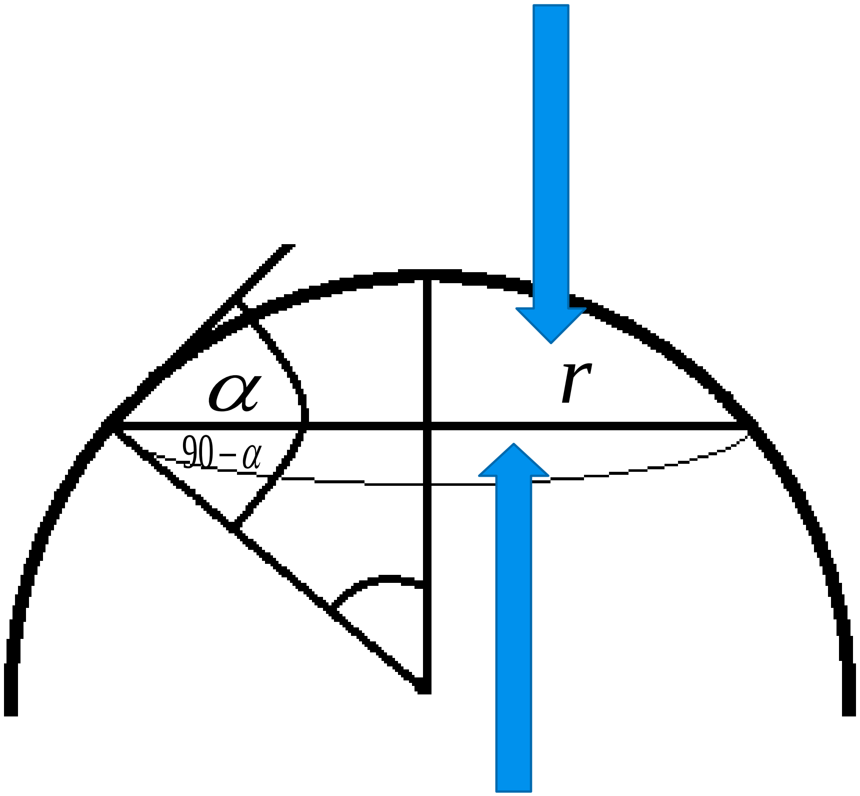


Close ...





$$V_{vl} = \frac{\pi r^3}{3}\left(\frac{1-\cos\alpha}{\sin\alpha}\right)^2\left(\frac{2+\cos\alpha}{\sin\alpha}\right)$$



$$V_{vs} = \frac{\pi}{3} r^3 \tan\alpha$$

$$P = \frac{V_{vl} + V_R}{V_{vs} + V_R}$$

$$V_R = \frac{\pi r^3}{3}\left(\frac{1-\sin\alpha}{\cos\alpha}\right)^2\left(\frac{2+\sin\alpha}{\cos\alpha}\right)$$

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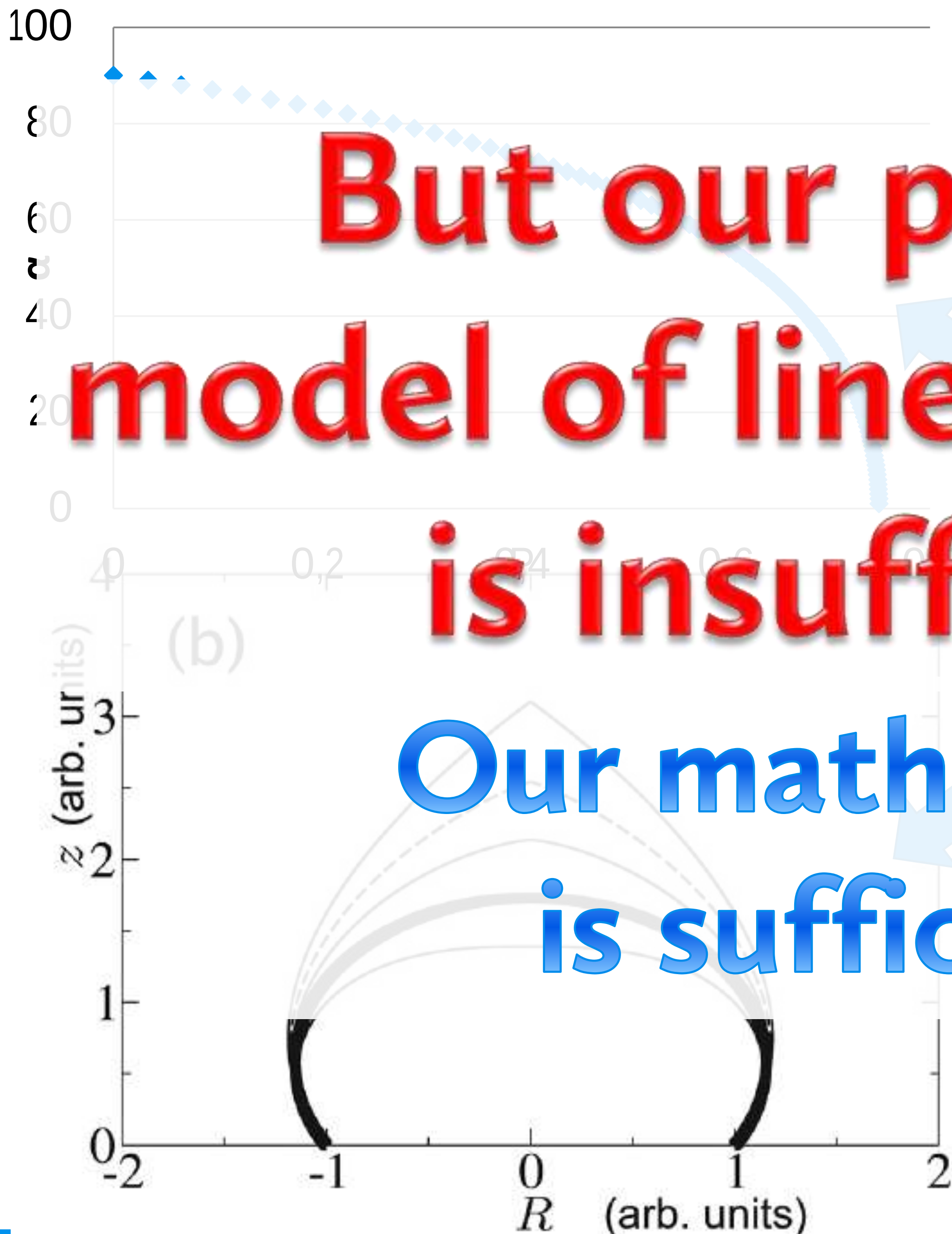
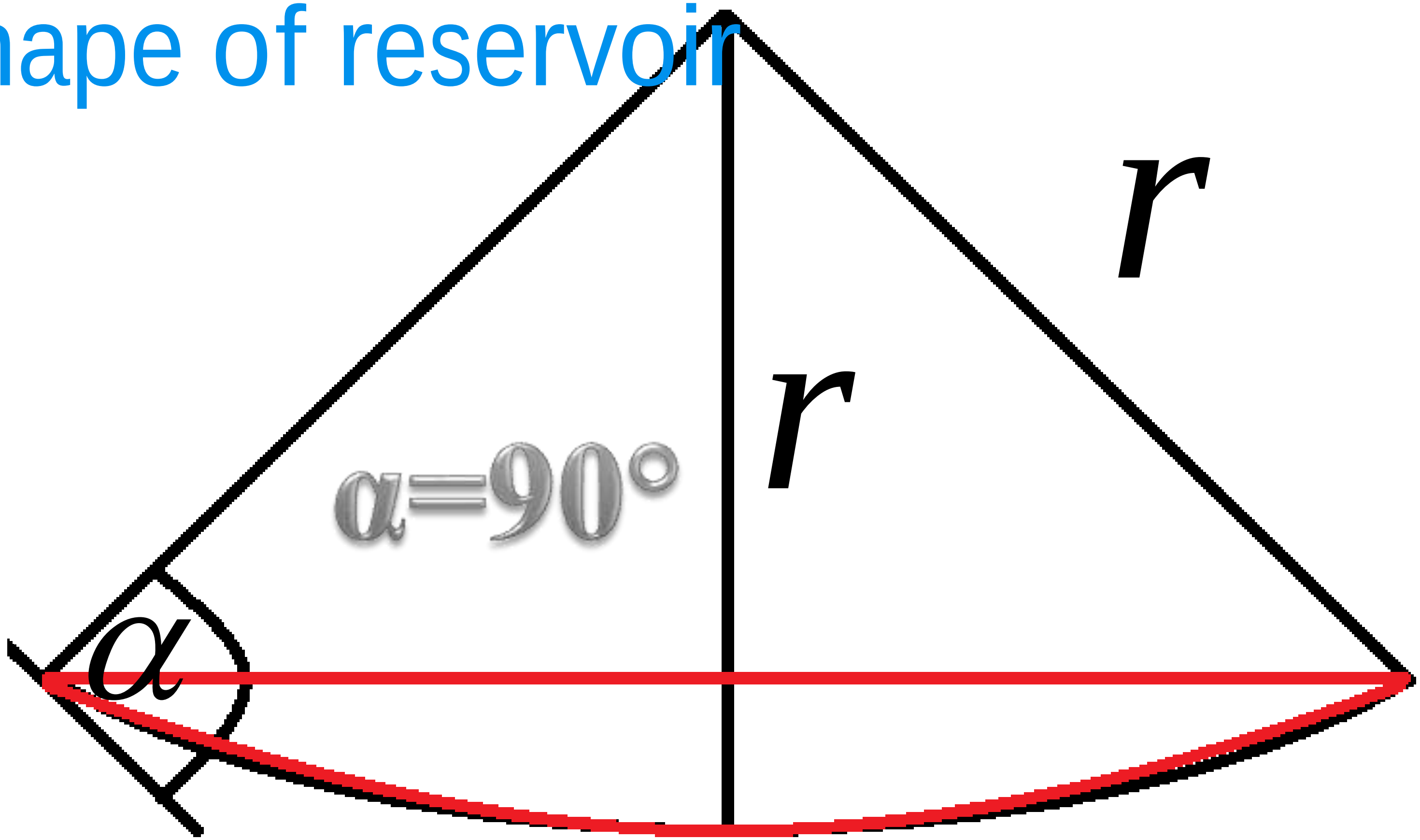
But our physical model of linear freezing is insufficient

Our math model is sufficient

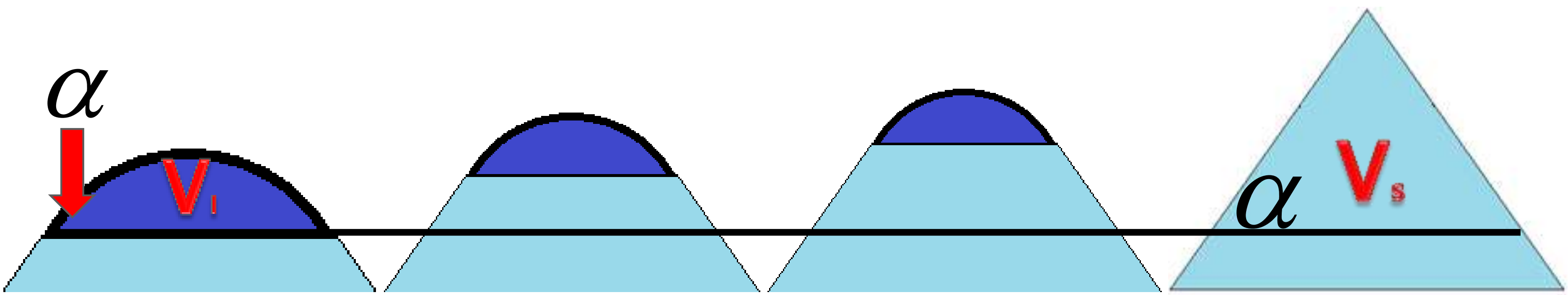


Shape of reservoir



Its a spherical cap!

Planar freezing



$$V_l = PV_s$$

V_l = volume of liqiud

V_s = volume of solid

$$P = \frac{\rho_s}{\rho_l}$$

Using known formulas for volumes

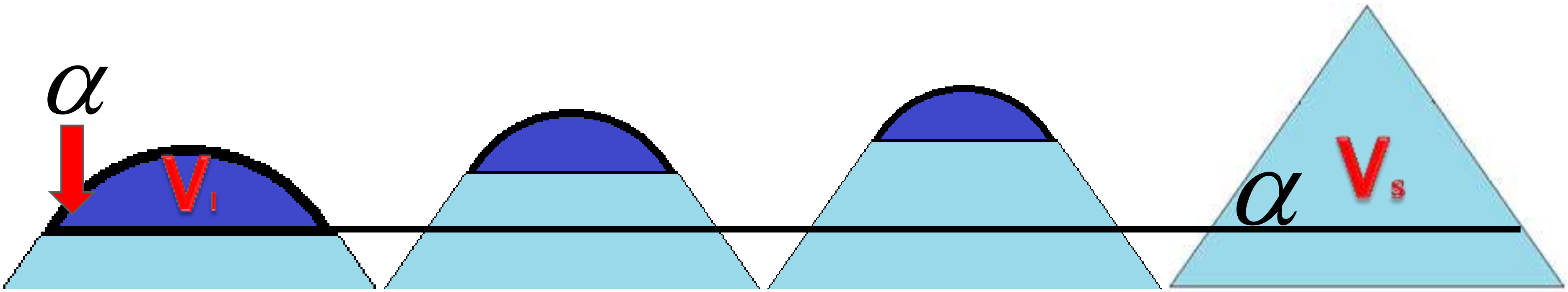
Spherical cap

$$V_l = \frac{\pi}{3} r^3 \left(\frac{1-\cos_\alpha}{\sin_\alpha}\right)^2 \left(\frac{2+\cos_\alpha}{\sin_\alpha}\right)$$

Cone

$$V_s = \frac{\pi}{3} r^3 \tan_\alpha$$

Planar freezing



$$V_l = PV_S$$

$V_l = volume\ of\ liqiud$

$V_s = volume\ of\ solid$

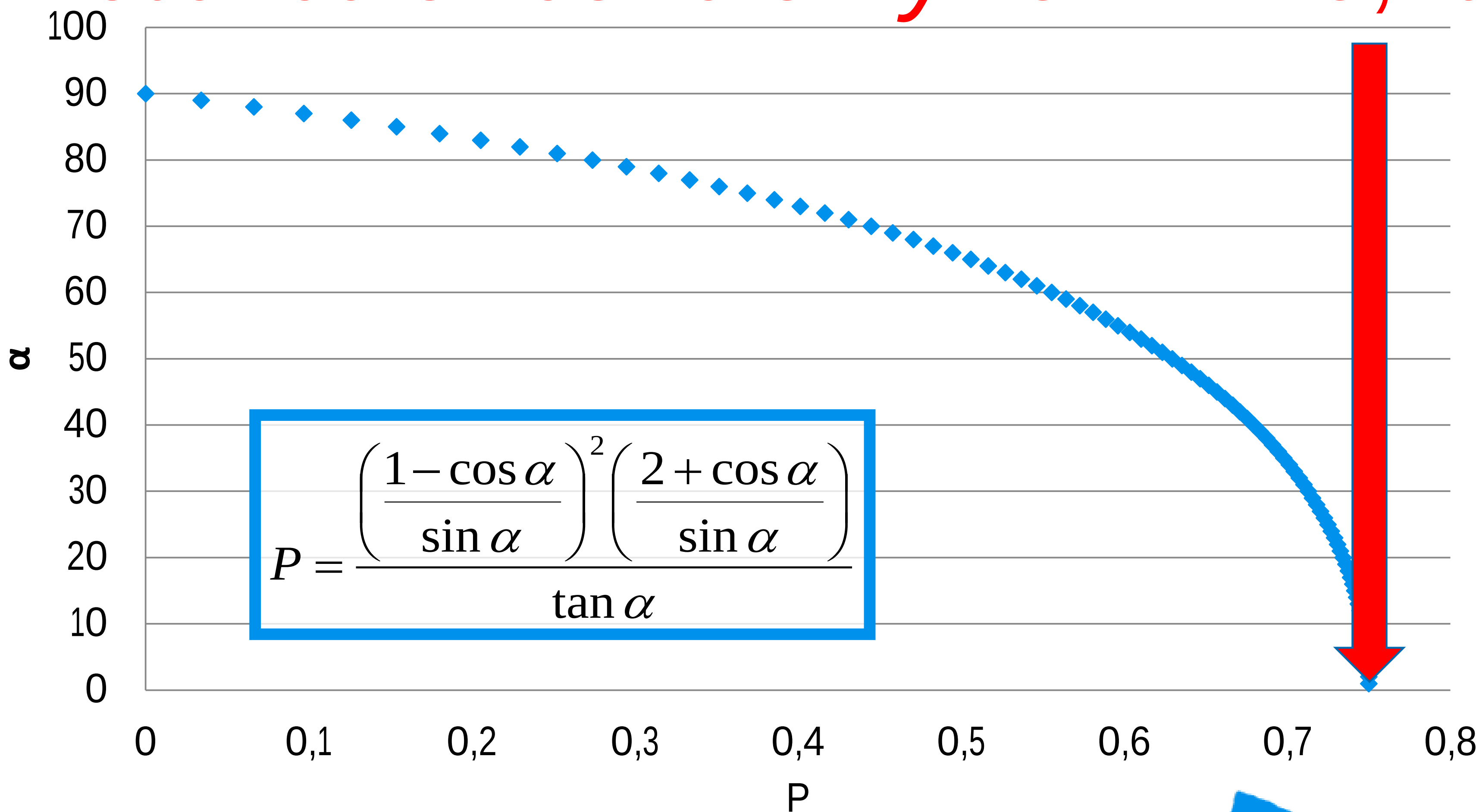
$$P = \frac{\rho_s}{\rho_l}$$

We can calculate density/pike angle relation

$$P = \frac{\left(\frac{1-\cos\alpha}{\sin\alpha}\right)^2 \left(\frac{2+\cos\alpha}{\sin\alpha}\right)}{\tan\alpha}$$

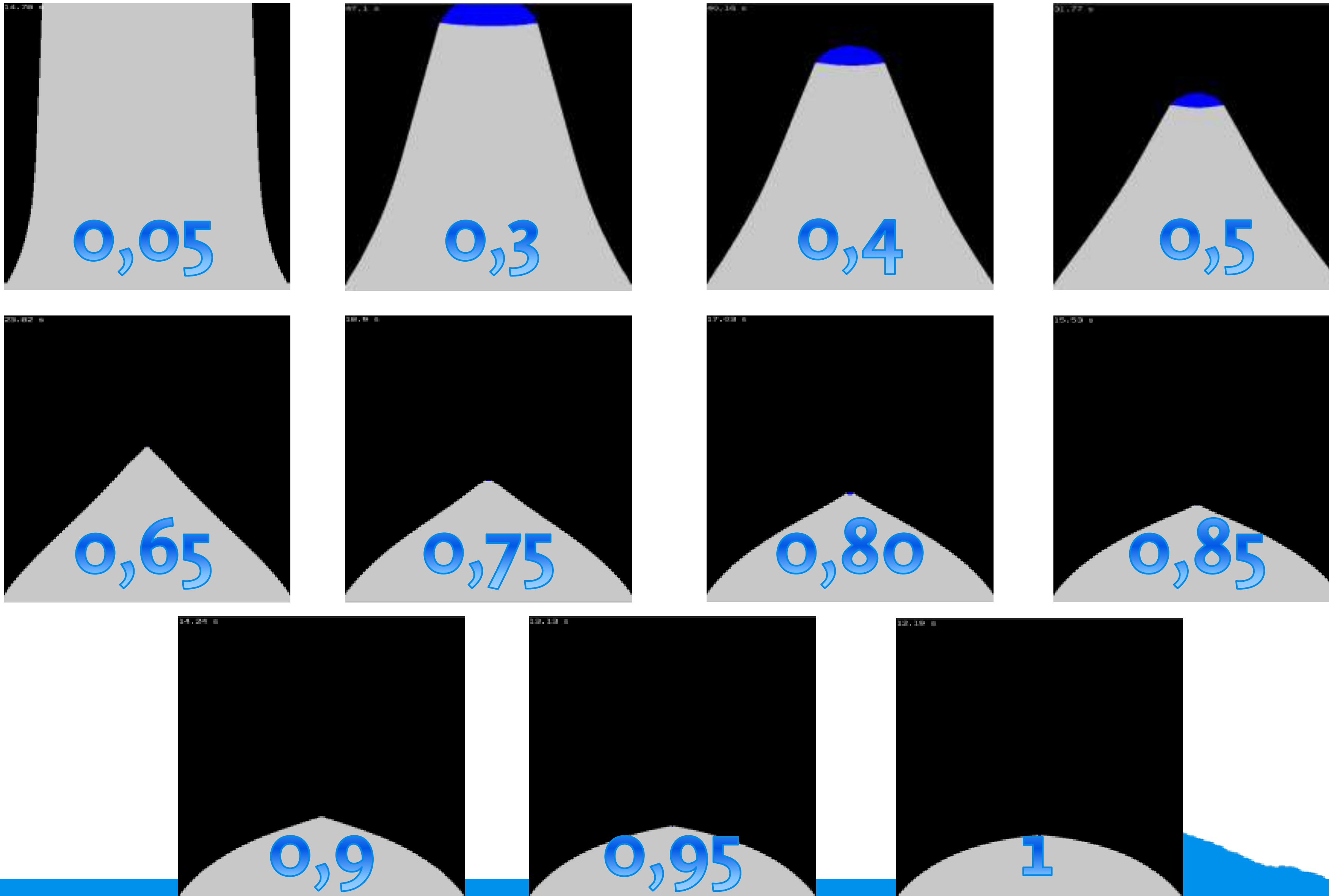
Angle of cone vs ratio of densities

Existence of cone only for P < 0,75

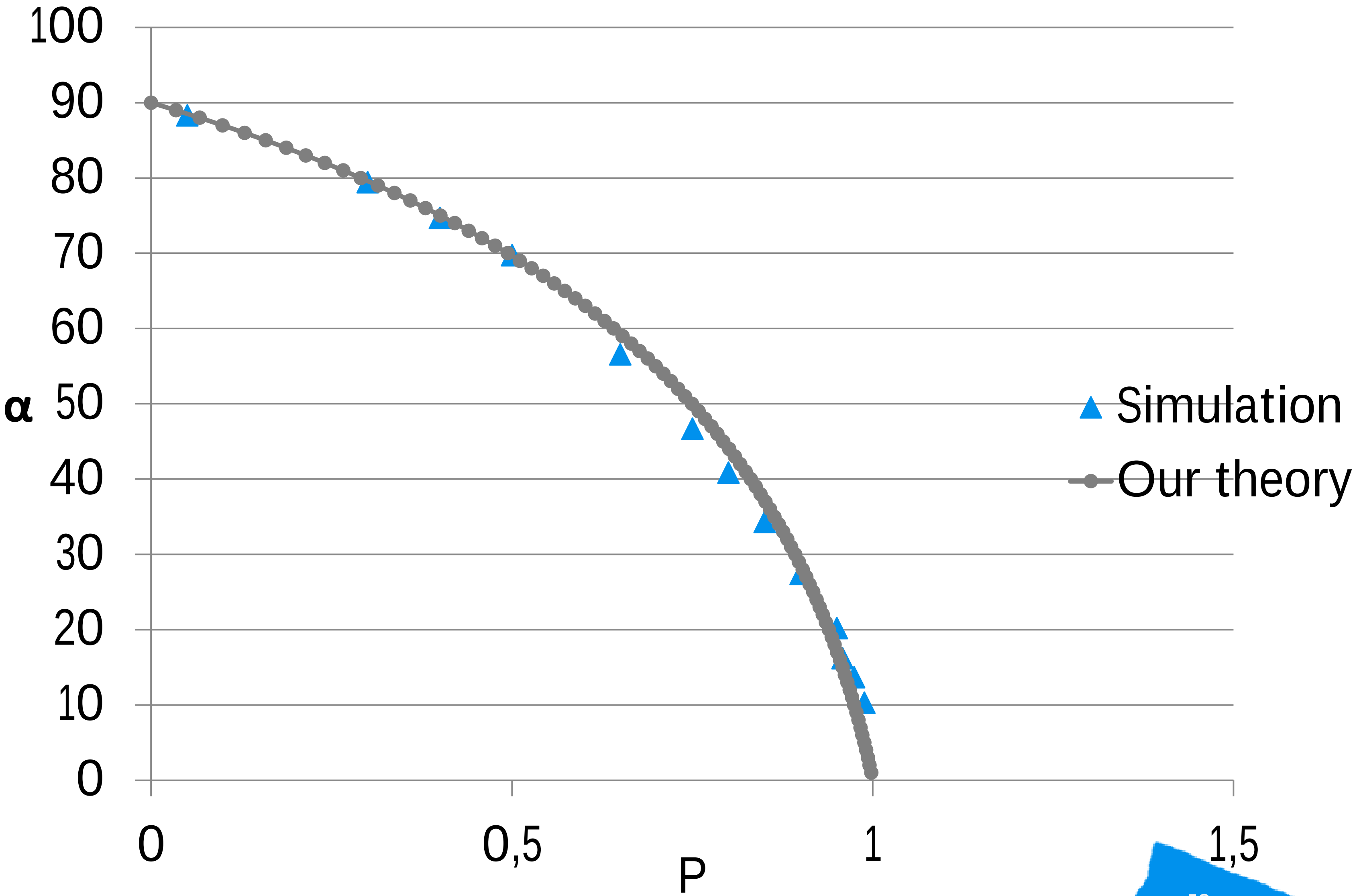


$$P = \frac{\left(\frac{1-\cos\alpha}{\sin\alpha}\right)^2\left(\frac{2+\cos\alpha}{\sin\alpha}\right)}{\tan\alpha}$$

Simulated droplets with different P

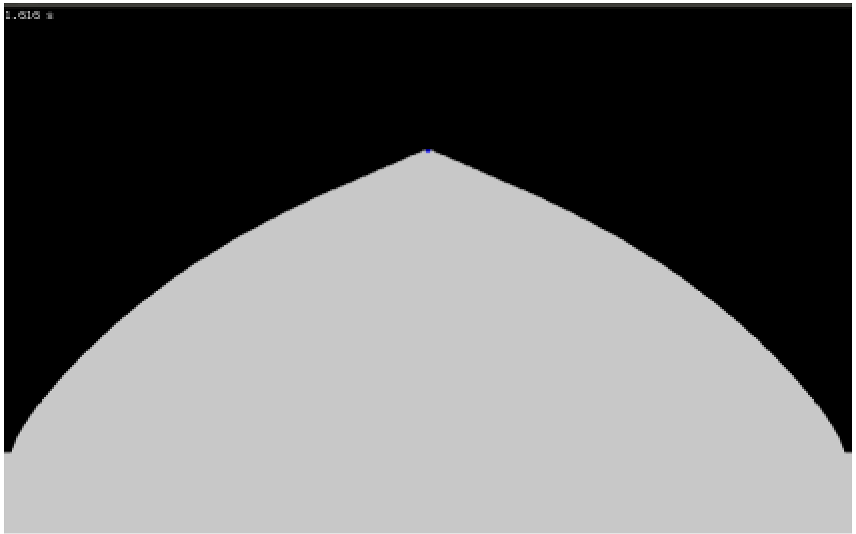


Simulation vs theory

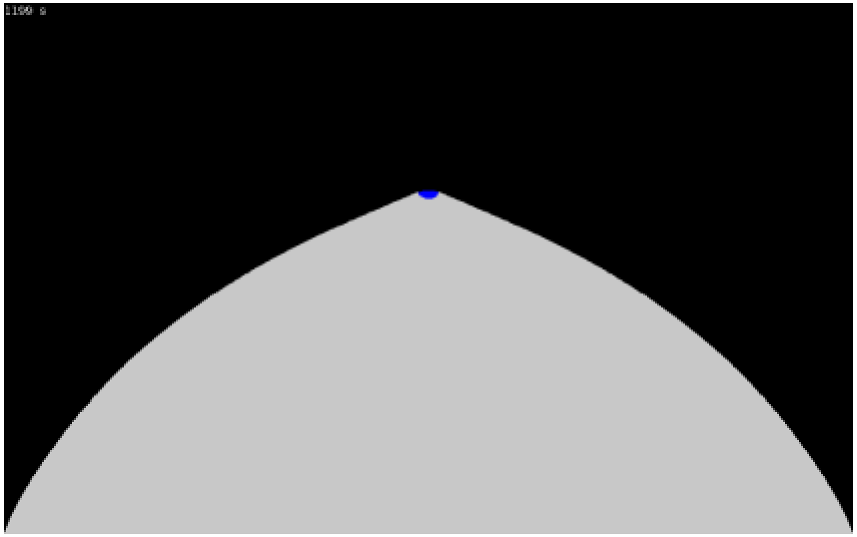


Simulated extremes - size

3 times smaller



10 times larger



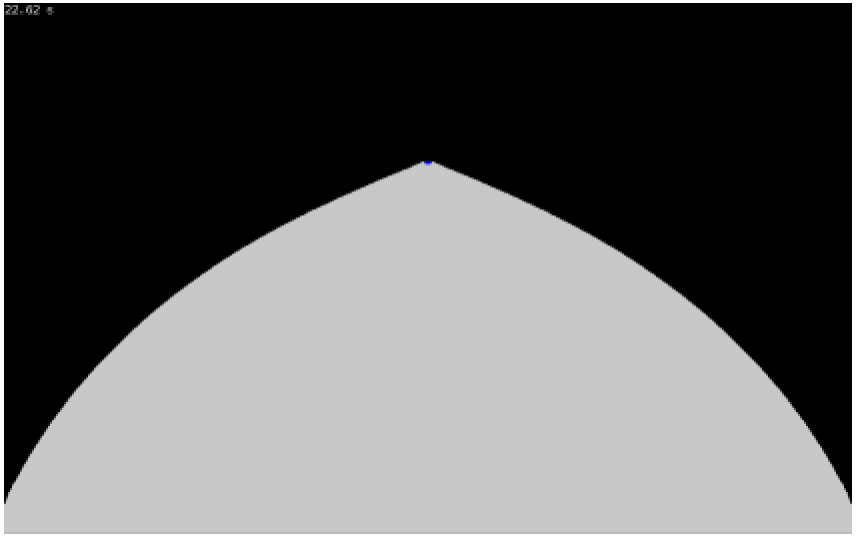
Change of peek angle is negligible

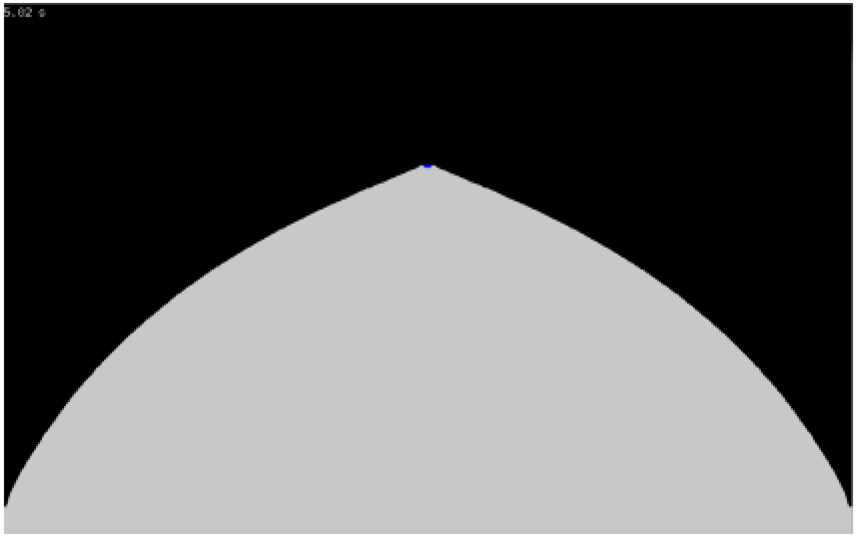
Simulated extremes - temperature

-12°C

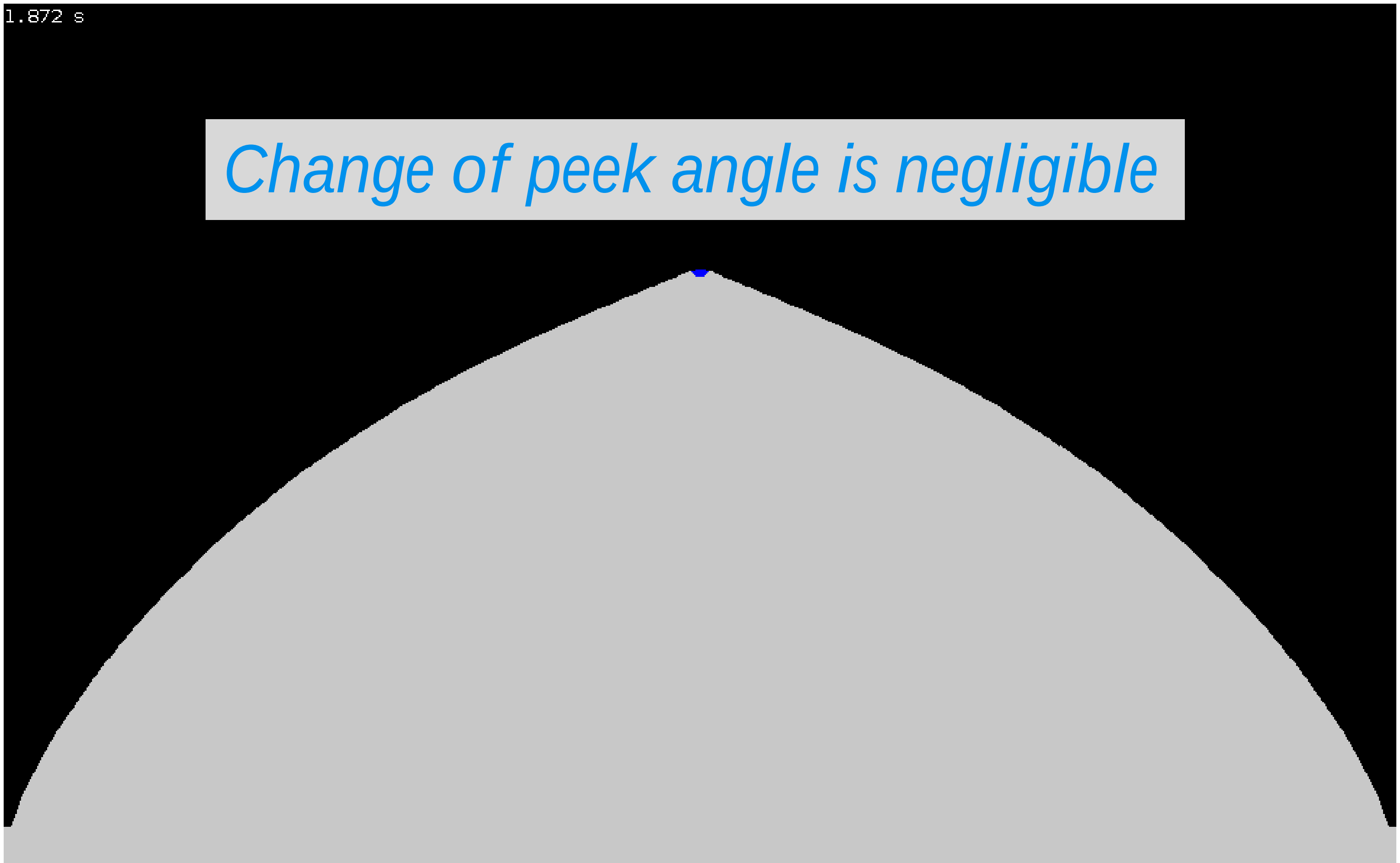
-60°C

Change of peek angle is negligible





Simulated extremes – heat capacity 34kJ



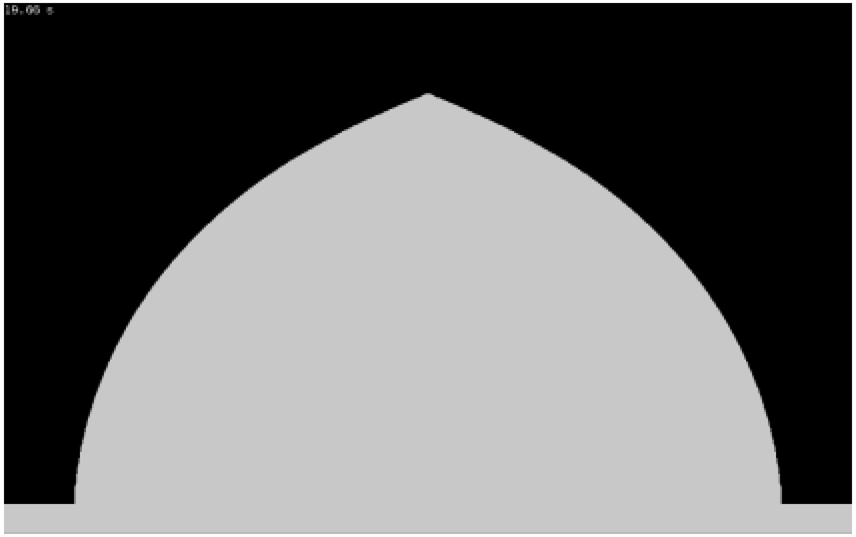
Change of peek angle is negligible

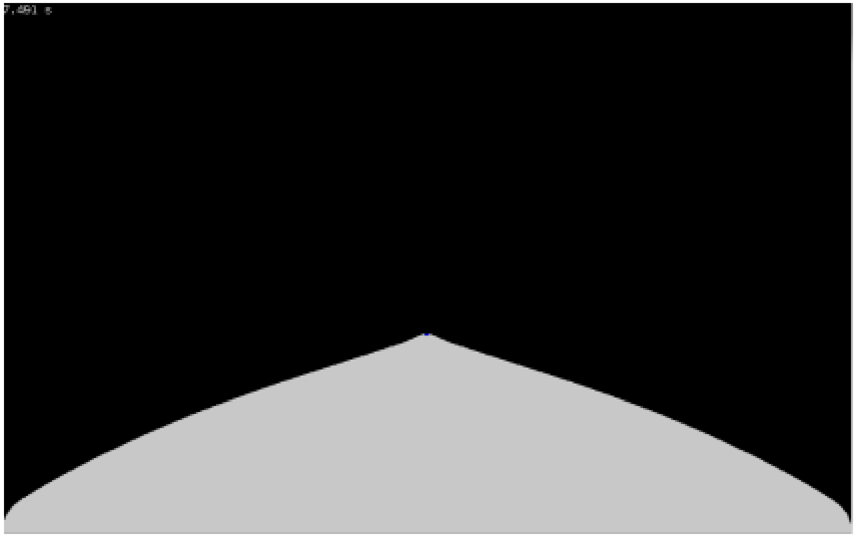
Simulated extremes – radius of contact area

r=2,5 mm

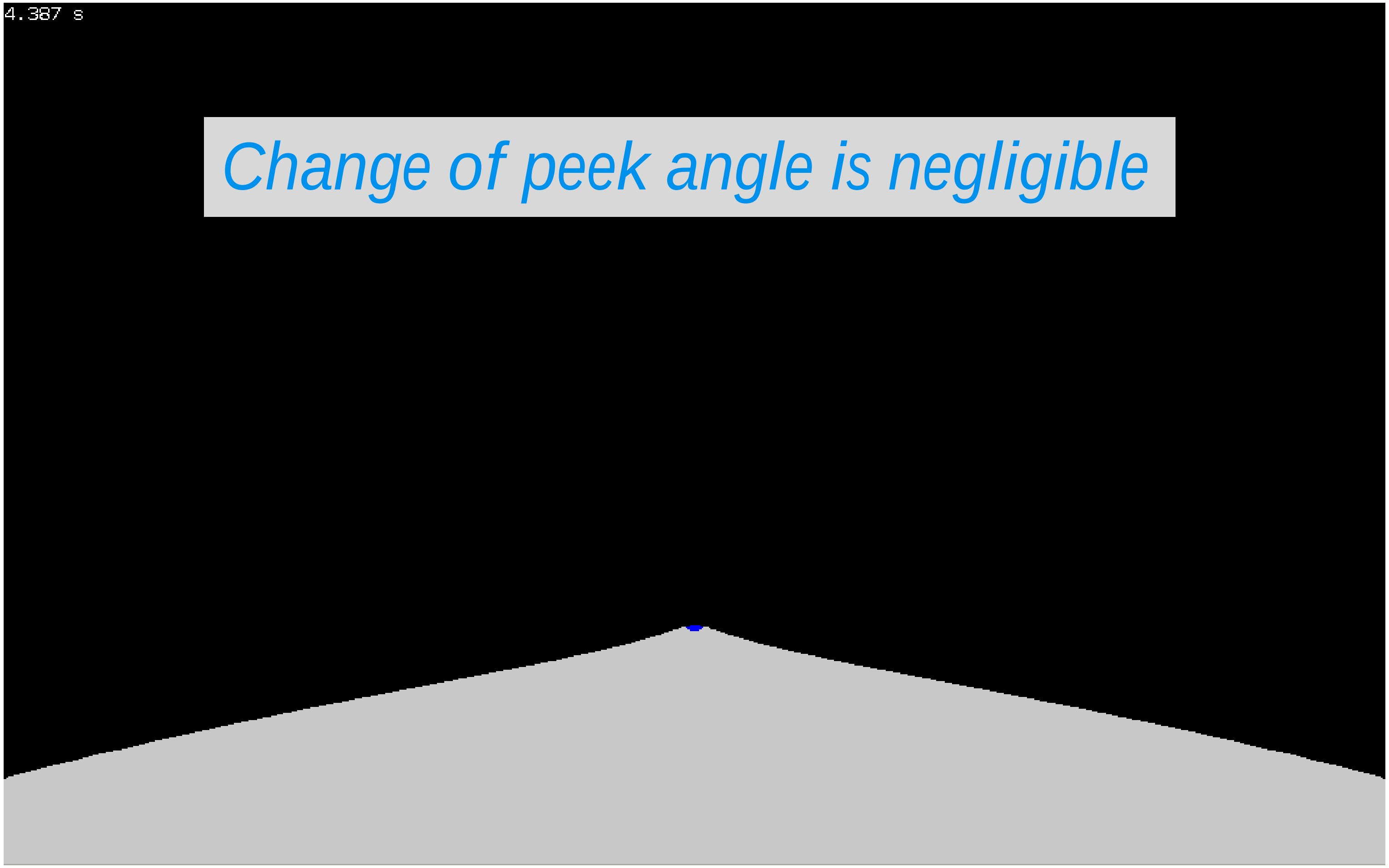
r=4 mm

Change of peek angle is negligible

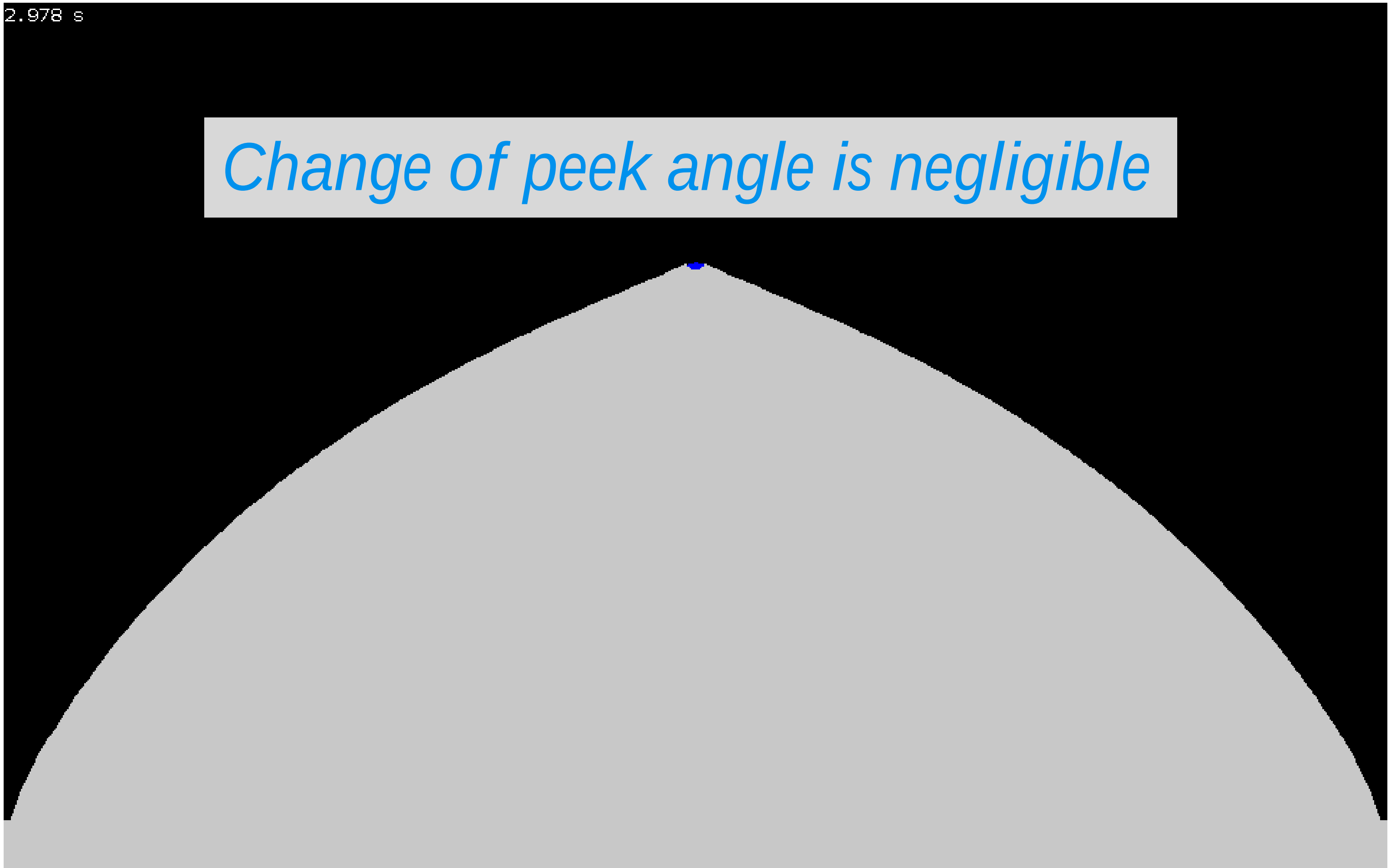




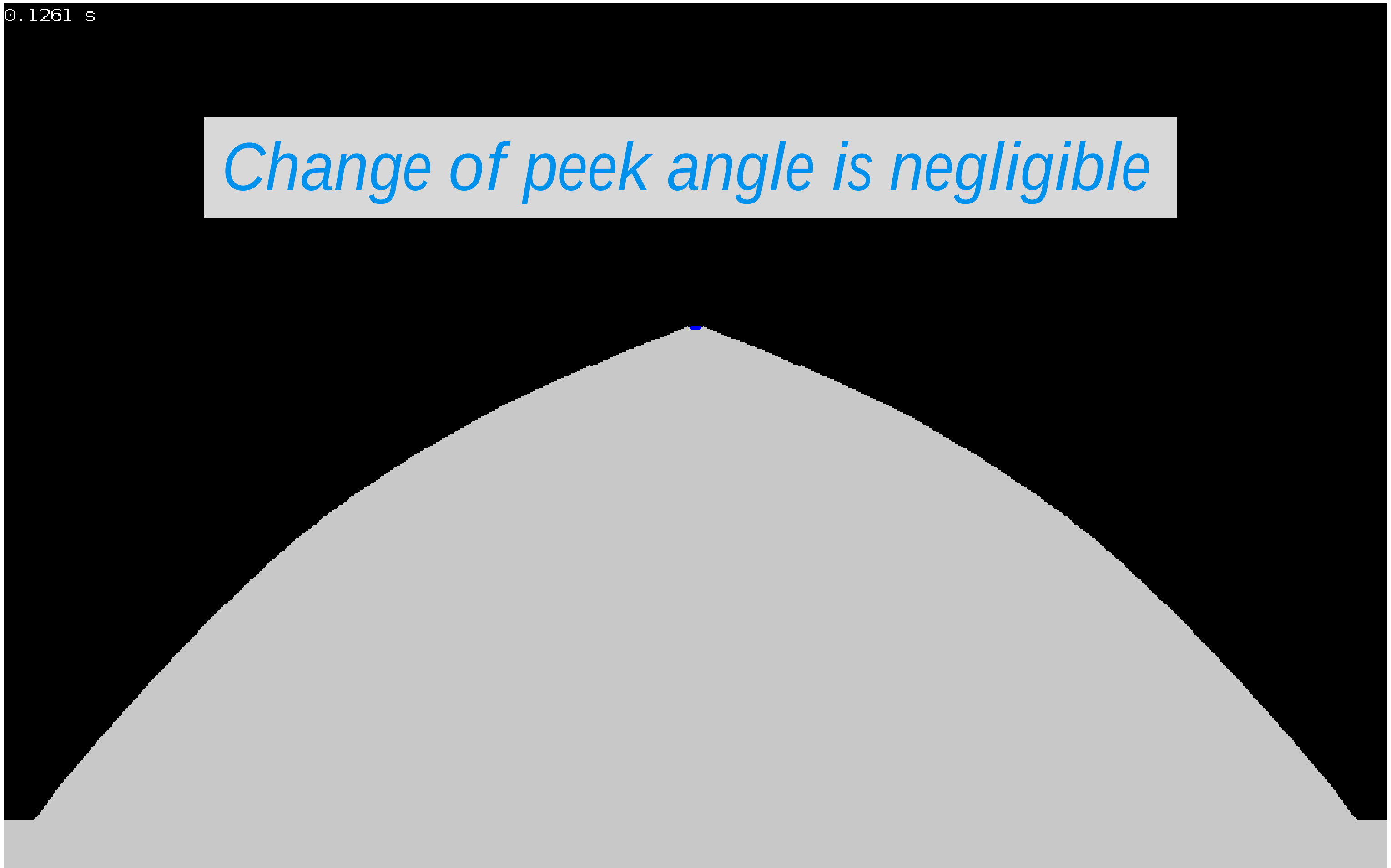
Simulated extremes – radius of contact area r=5 mm



Simulated extremes – conductivity 4 times larger

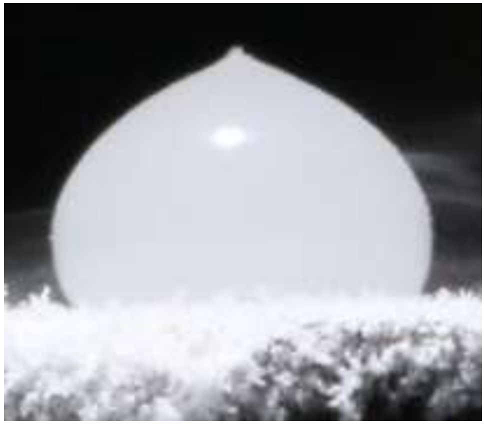


Simulated extremes – all together



Density ratio

$$\frac{\rho_{solid}}{\rho_{liquid}} < 1 \Rightarrow \textit{it works}$$



Bismuth (Bi)
Antimony (Sb)
Silicon (Si)
Germanium (Ge)

High temperature of melting

Gallium (Ga)

Toxic

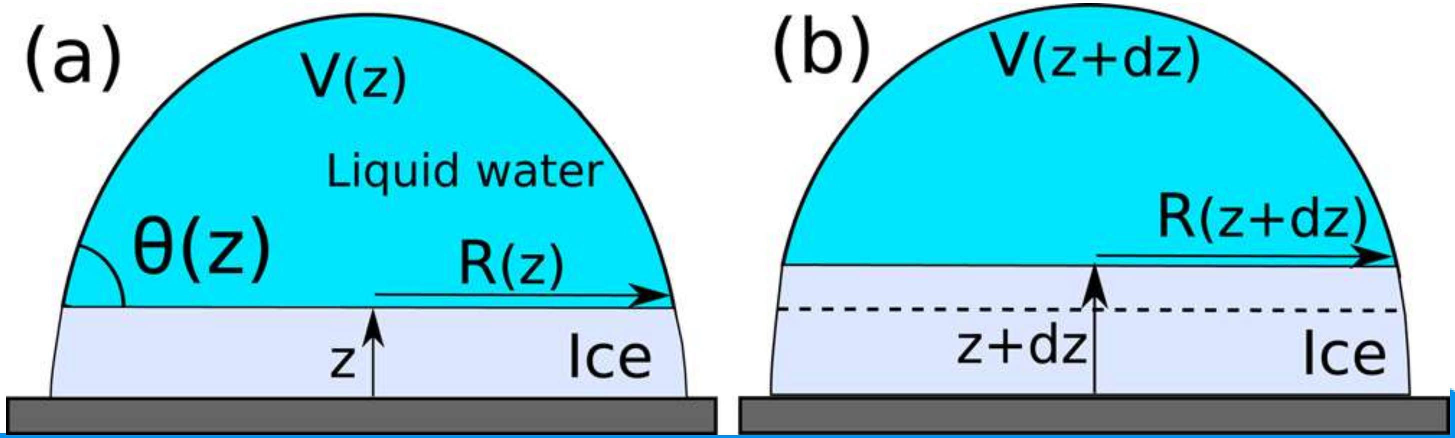
Water

STARE SLIDY

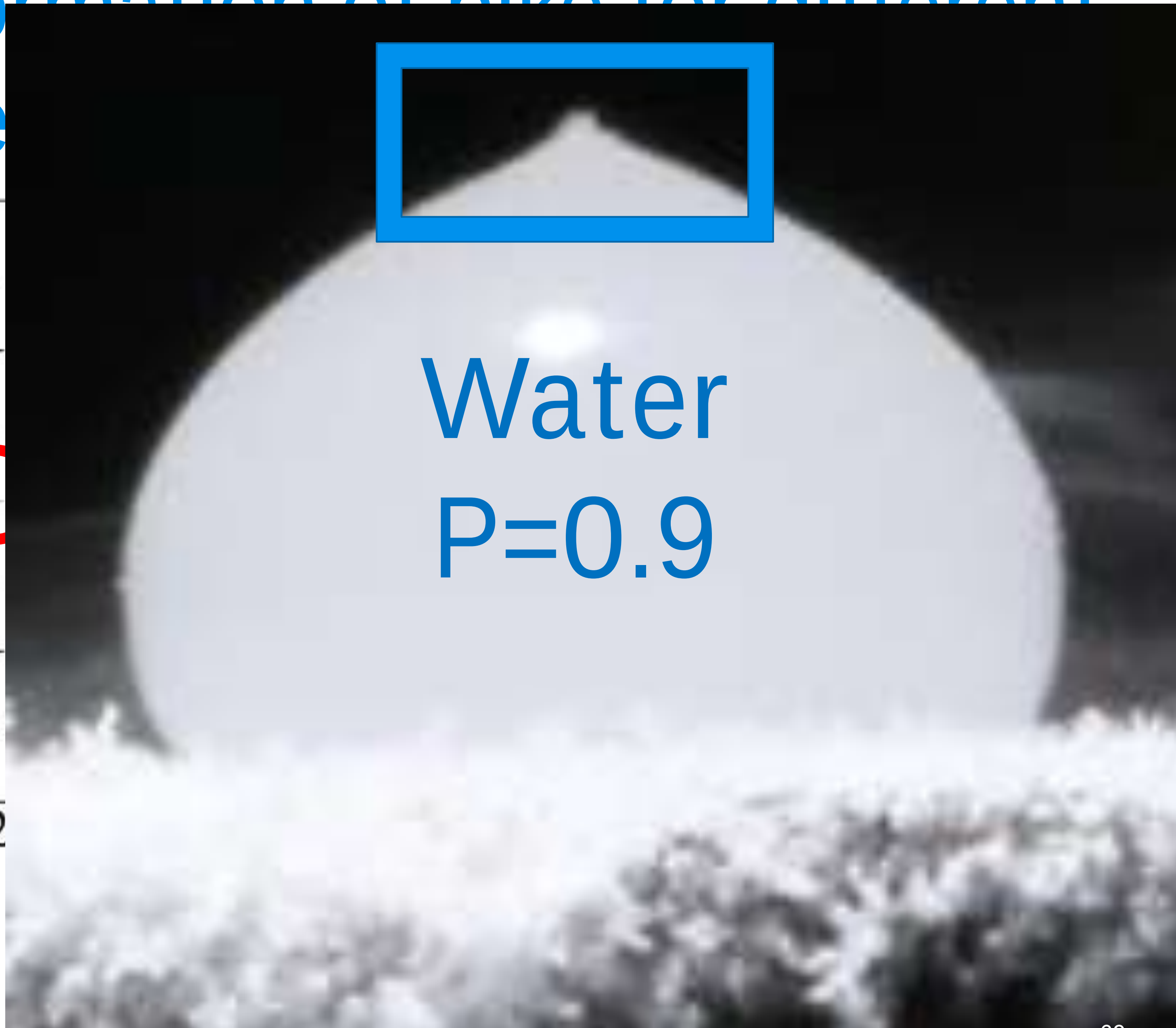
Existing literature

*"**Pointy** ice-drops: How water freezes into a singular shape"* J.H. Snoeijer and P. Brunet Am. J. Phys. 80, 764 (2012)

- Planar freezing
- Ratio of densities
  - Critical ratio of densities to create convex pike is ¾
  - Water is not predicted to create convex pike



Formation of pike for different de

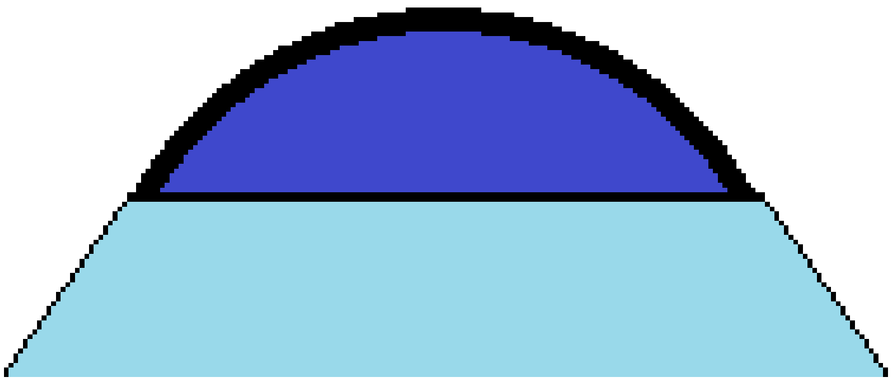


$\frac{\rho_{solid}}{\rho_{liquid}}$

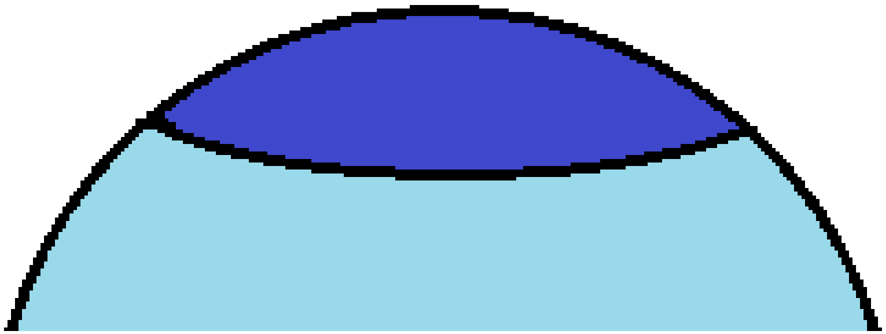
G

Middle section of a freezing droplet

Assumed shape



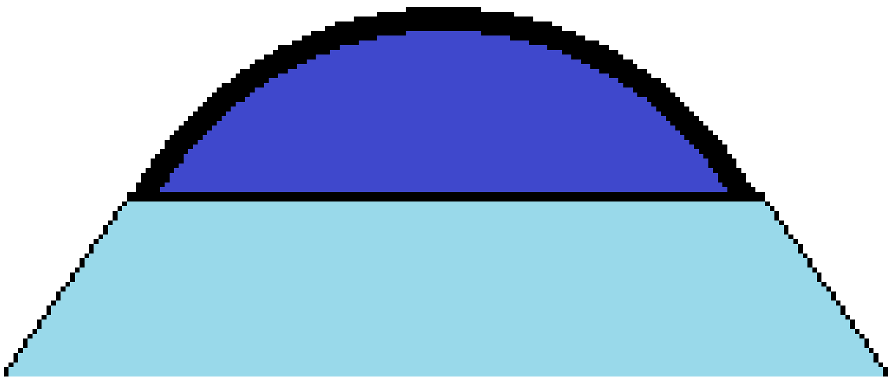
Real shape



Exact shape of reservoir?

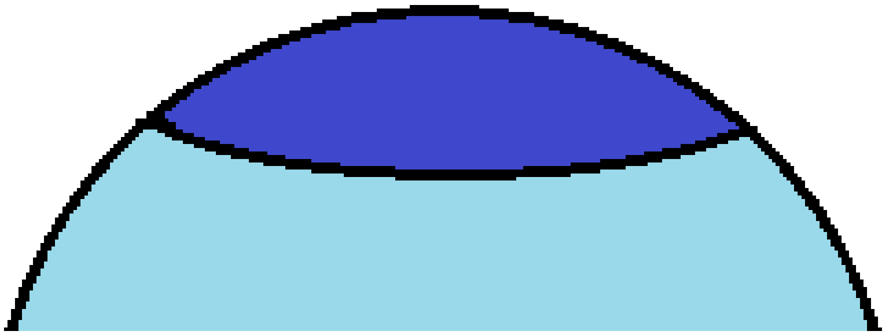
Middle section of freezing droplet

Assumed volume



$$V_l = PV_S$$

Real volume



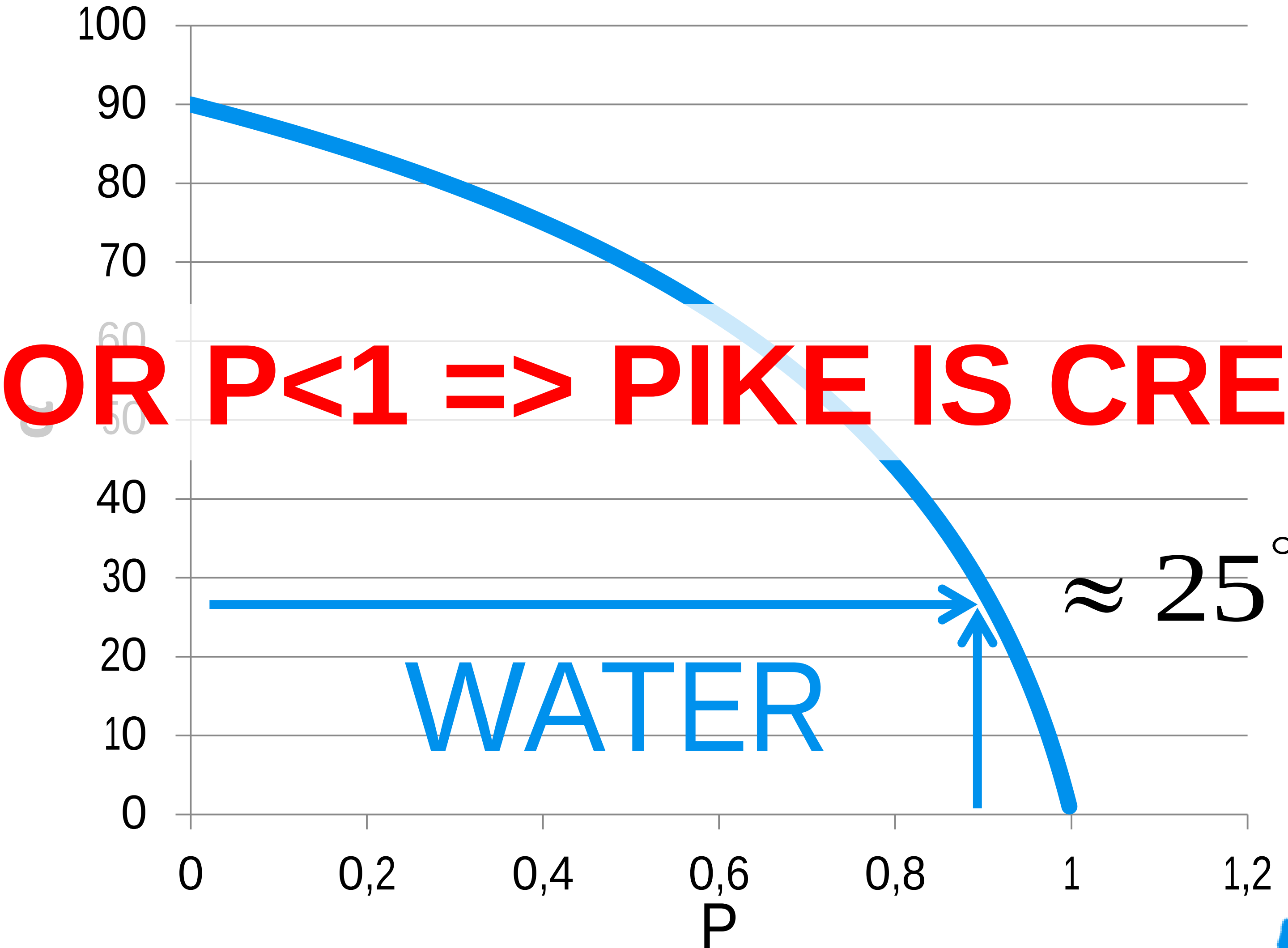
$$V_{ls} + V_R = P\left(V_{ss} + V_R\right)$$

$V_{ls} = volume\ of\ seen\ liquid$

$V_{ss} = volume\ of\ seen\ solid$

$V_R = volume\ of\ reservoir$

Angle/density ratio



FOR P<1 => PIKE IS CREATED

Simulation of the process of freezing

- Heat convection
- Changeable parameters
  - Density ratio
  - Volume of droplet
  - Contact area
  - Temperature of plate
  - Heat capacity
  - Heat conductivity