5

Loaded hoop

Mário Lipovský

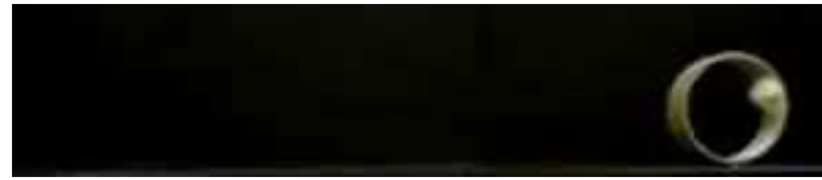
Task

Fasten a small weight to the inside of a hoop and set the hoop in motion by giving it an initial push.

Investigate the **hoop's motion**.

Types of hoop motion

1) Rolling $v = \omega r$



2) Spinning $v < \omega r$

Skidding $v > \omega r$

3) Jump $N = 0$

1) Rolling

m_{weight} = 22g
m_{hoop} = 28g
r_{hoop} = 41 mm

1) Rolling

- Conservation of mechanical energy

$$E = mgy + \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2}I_G\omega^2$$

Potential energy of the system

Translational kinetic energy of the system

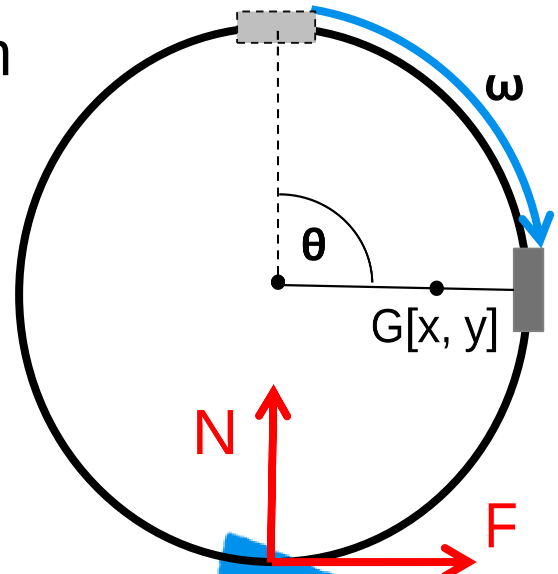
Rotational kinetic energy of the system

- Rolling condition

$F \leq fN$ Friction force is strong enough to prevent slipping

$\rightarrow v = \omega r$

$\omega(t)$ can be calculated from $\omega(0)$



2) Spinning/Skidding

m_{weight} = 322g
m_{hoop} = 28g
r_{hoop} = 41 mm

2) Spinning/Skidding

Newton's laws of motion

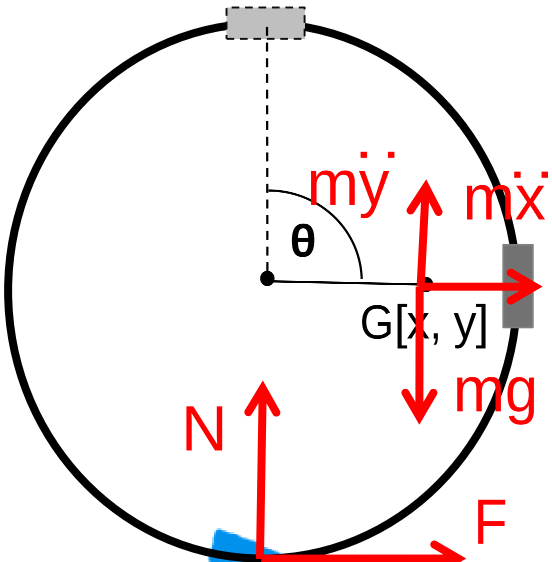
$$m\ddot{x} = F$$

$$m\ddot{y} = mg - N$$

$$N r_G \sin\theta - F y = I\varepsilon$$ Torque equation

$$|F| = fN$$ Friction force reaches it's maximum

ω can be calculated by numerical integration of ε



3) Jump

m_{weight} = 220g
m_{hoop} = 28g
r_{hoop} = 41 mm

3) Conditions needed to jump

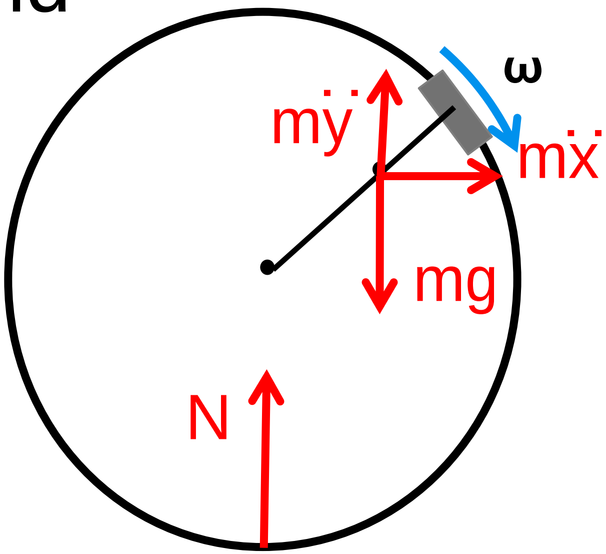
- Forces
  - gravity mg
  - centrifugal $F_c = m\ddot{x} + m\ddot{y} = m\omega^2 r_G$
- Condition of leaving the ground

$$N = 0$$

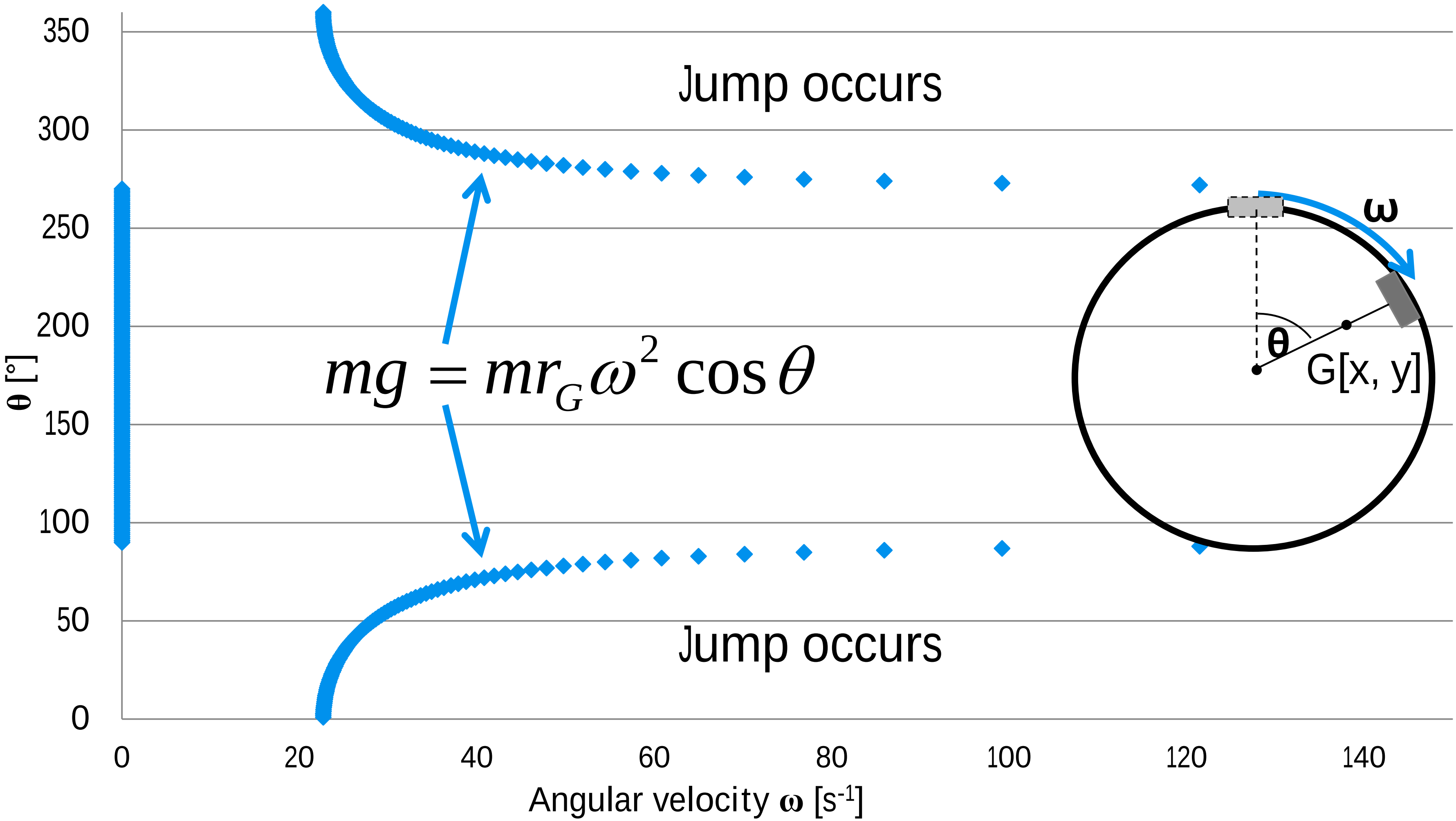
$$m\ddot{y} = mg - N$$

$$m\ddot{y} = mg$$

$$mg = m\omega^2 r_G \cos\theta$$

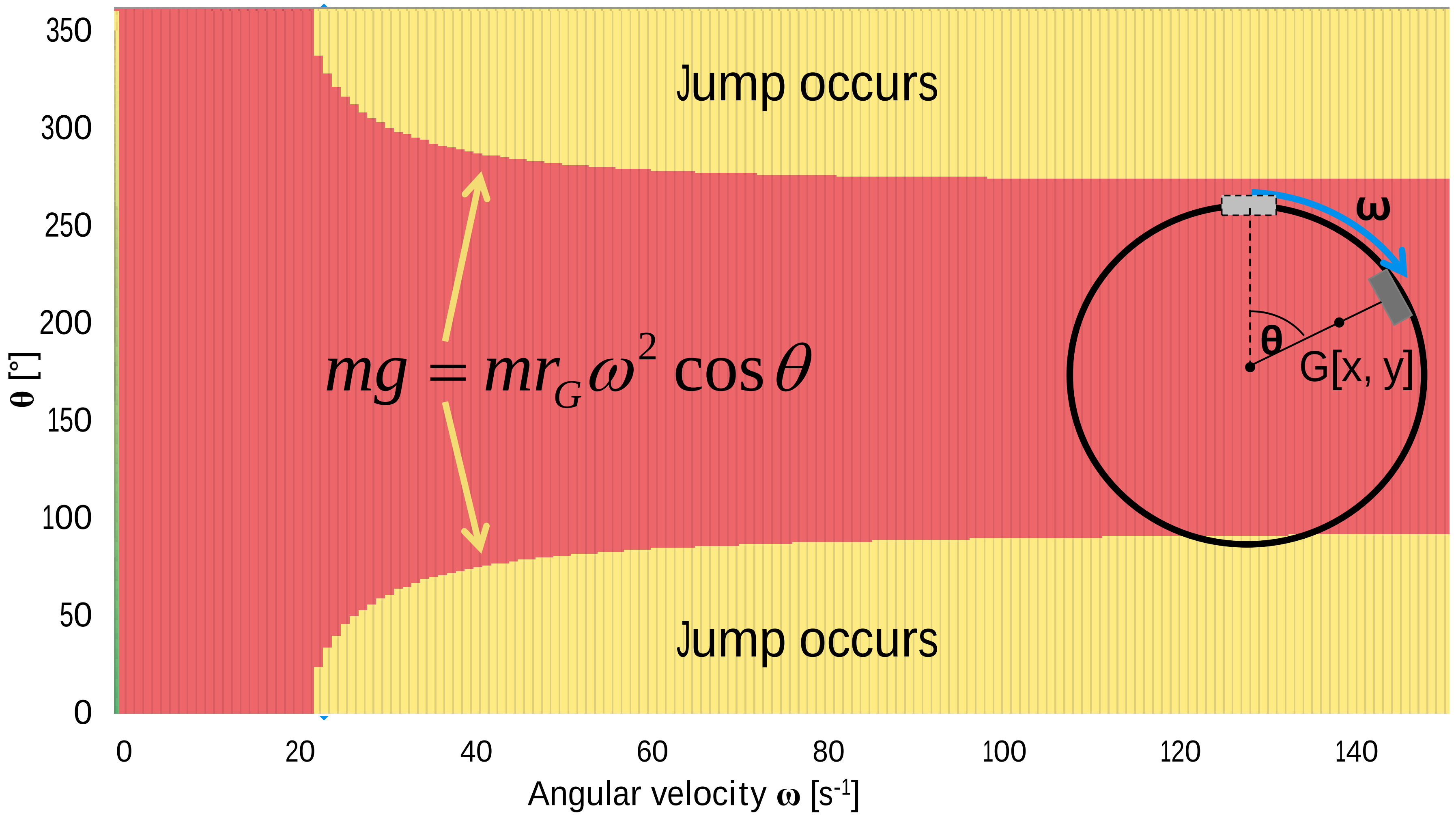


3) When does hoop jump?



$$mg = mr_G\omega^2\cos\theta$$

3) When does hoop jump?



3) Jump

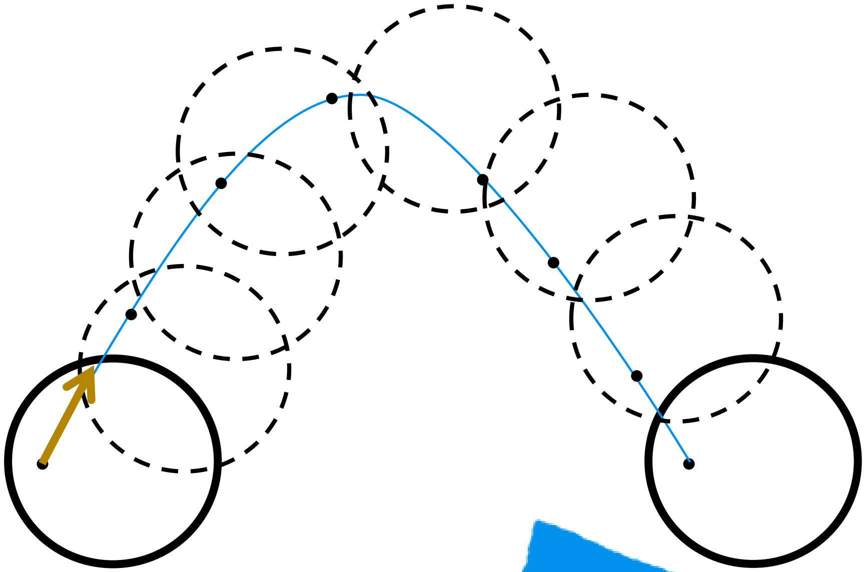
- Hoop – Constant ω around centre of gravity

$$\omega = const.$$

- Centre of gravity – projectile motion

$$x = v_0 t \cos\alpha$$

$$y = v_o t \sin\alpha - \frac{1}{2} g t^2$$



Types of hoop motion

1) Rolling

$$\Delta E = 0$$
$$v = \omega r$$

2) Spinning

$$|F| = fN$$

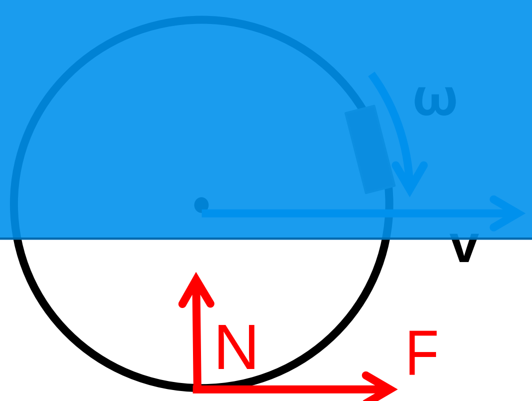
Skidding

3) Jump

$$mg = m\omega^2 r \cos\alpha$$

$$x = v_0 t \cos\alpha$$

$$y = v_o t \sin\alpha - \frac{1}{2} g t^2$$



$$\omega = const.$$

How to determine changes of motion types?

What has been done

Analysis of Rolling Motion of Loaded Hoops

[W.F.D. Theron]

- 4 types of motion
  - Combined into 36 patterns
- Numerical simulation
  - Observed only 1 x 360° rotation
  - Stopped right after the hoop leaves ground

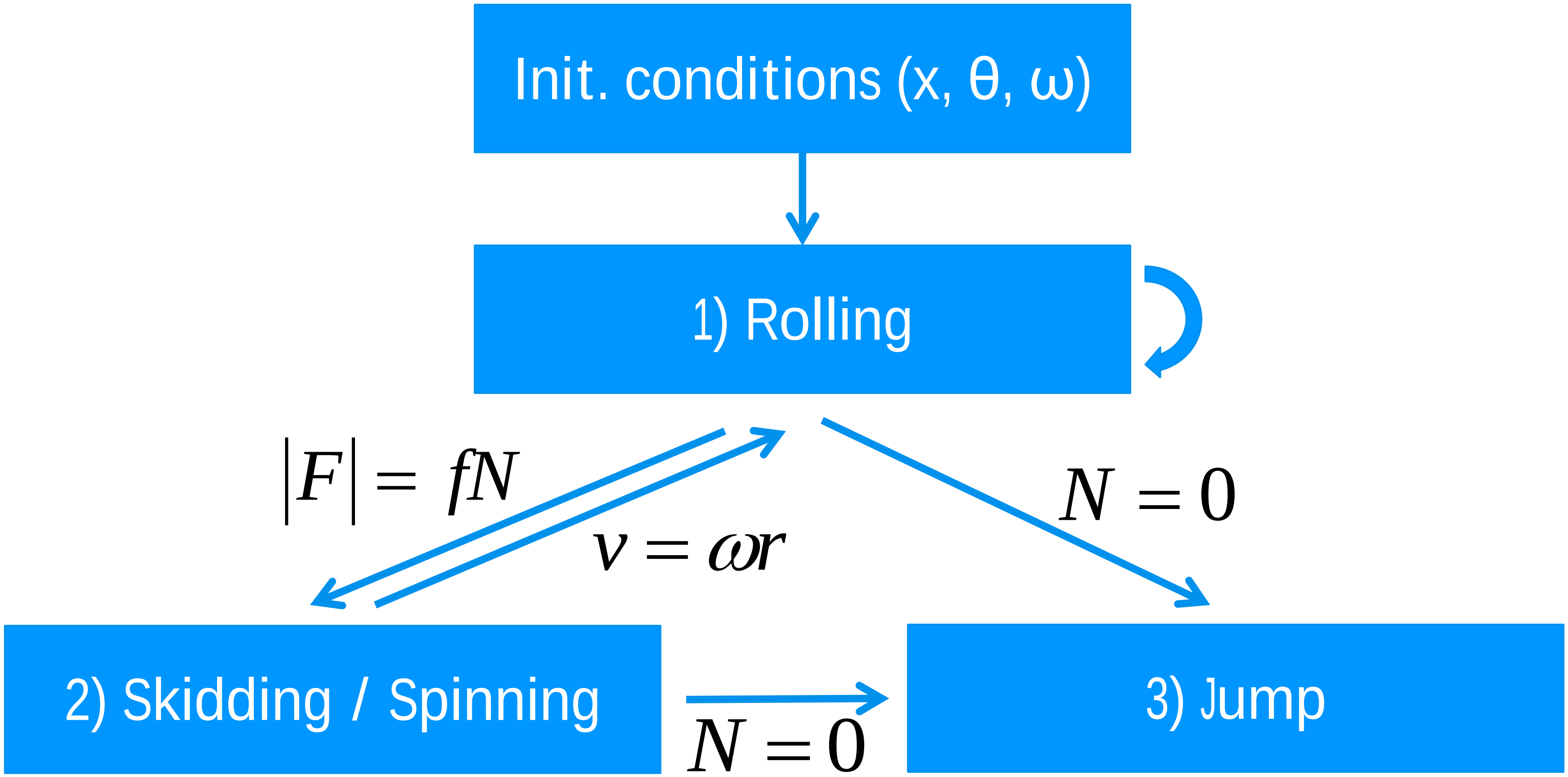
Our own simulation

- Compare with our experiments
- Simulate jump

List of patterns

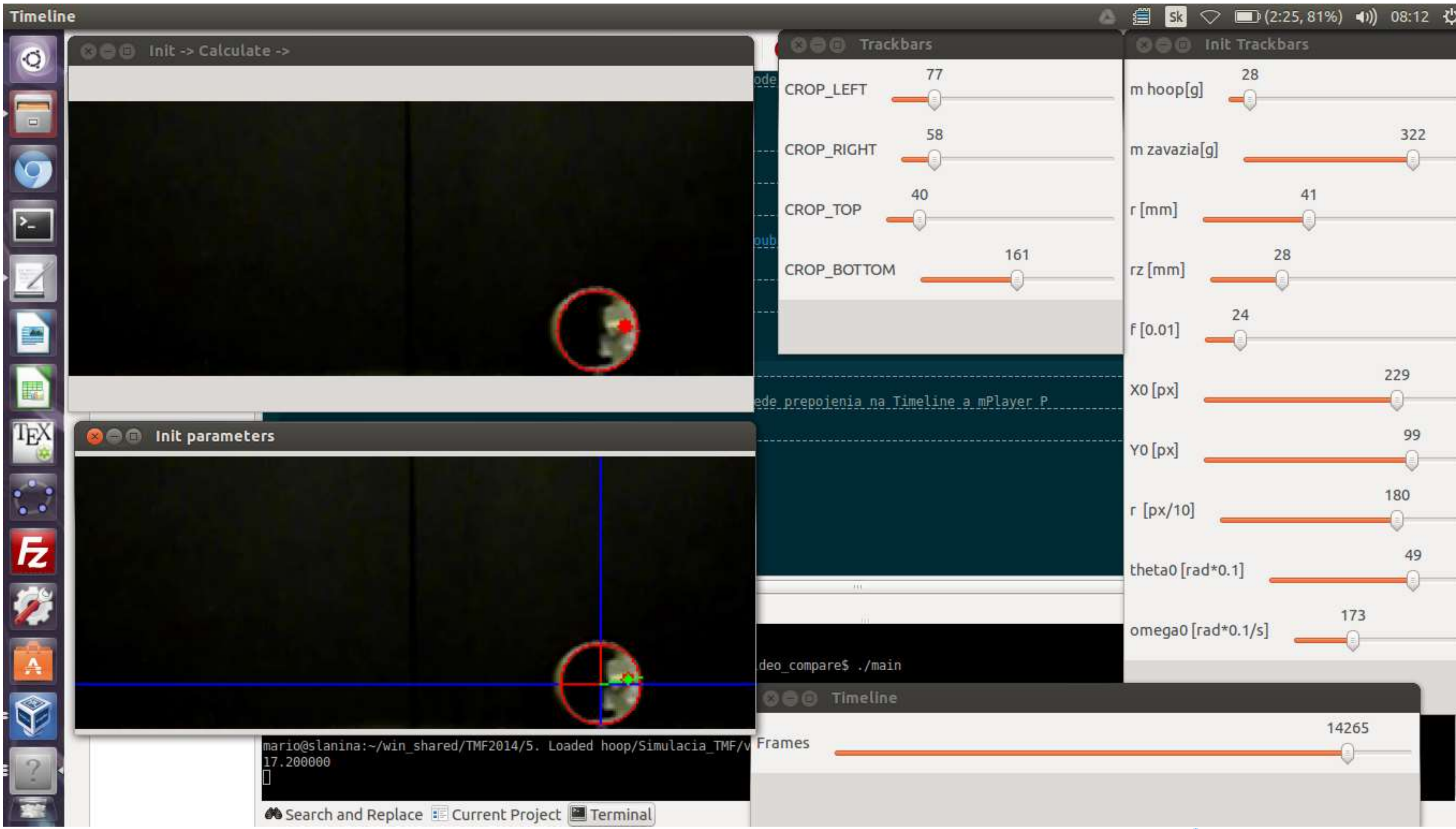
Type	Ref.	Pattern
HF	1	R T
MF	2	RDRSRDR T
	3	RDRS-DR T
	4	RDRSR-- T
	5	RDRSR-- Z
	6	RDRS--- H
	7	RDRS--- T
	8	RD-SR-- T
	9	RD-S--- H
	10	RD-S--- T
LF	11	RSRDRSRDR T
	12	RSRDRSRDR Z
	13	RSRDRSRD- T
	14	RSRDRSRD- Z
	15	RSRDRSR-- T
	16	RSRDRS-DR T
	17	RSRDRS--- T
	18	RSRD-S--- H
	19	RSRD-S--- T
	20	RSR--S-D- Z
	21	RSR----DR T
	22	RSR----DR Z
	23	RSR----D- T
	24	RSR----D- Z
	25	RS-DRSRDR Z
	26	RS-DRSRD- Z
	27	RS-DRS-DR Z
	28	RS-DRS-DR T
	29	RS-DRS-D- Z
	30	RS-D-S--- T
	31	RS-D-SR-- T
	32	RS-D-S-DR T
	33	RS-D-S-DR Z
	34	RS-D-S-D- T
	35	RS-D-S-D- Z
	36	RS-----D- Z

Simulation



Numerical solution, using method of small increments (t)

Simulation


Timeline
(2:25, 81%) 08:12
Init -> Calculate ->
Trackbars
CROP_LEFT 77
CROP_RIGHT 58
CROP_TOP 40
CROP_BOTTOM 161
Init Trackbars
m hoop[g] 28
m zavazia[g] 322
r [mm] 41
rz [mm] 28
f [0.01] 24
X0 [px] 229
Y0 [px] 99
r [px/10] 180
theta0 [rad*0.1] 49
omega0 [rad*0.1/s] 173
Init parameters
ede prepojenia na Timeline a mPlayer P
deo_compare$ ./main
Timeline
Frames 14265
mario@slanina:~/win_shared/TMF2014/5. Loaded hoop/Simulacia_TMF/v
17.200000
Search and Replace
Current Project
Terminal

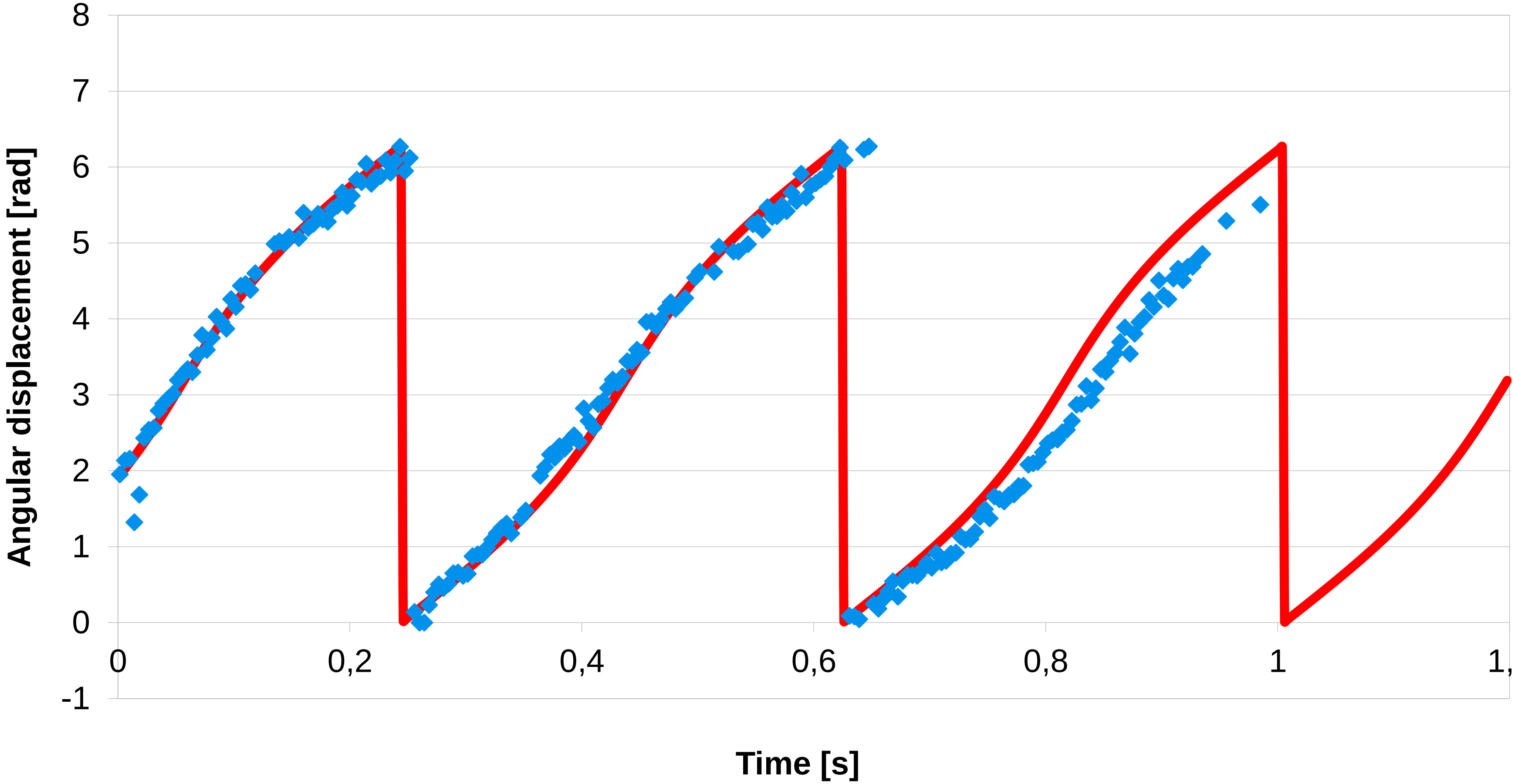
Experiments

- Constant parameters
  - r – hoop radius
  - hoop material (rigid)
  - f – coefficient of friction (ground)
- Changing parameters
  - ω – initial velocity
  - m – mass of weight

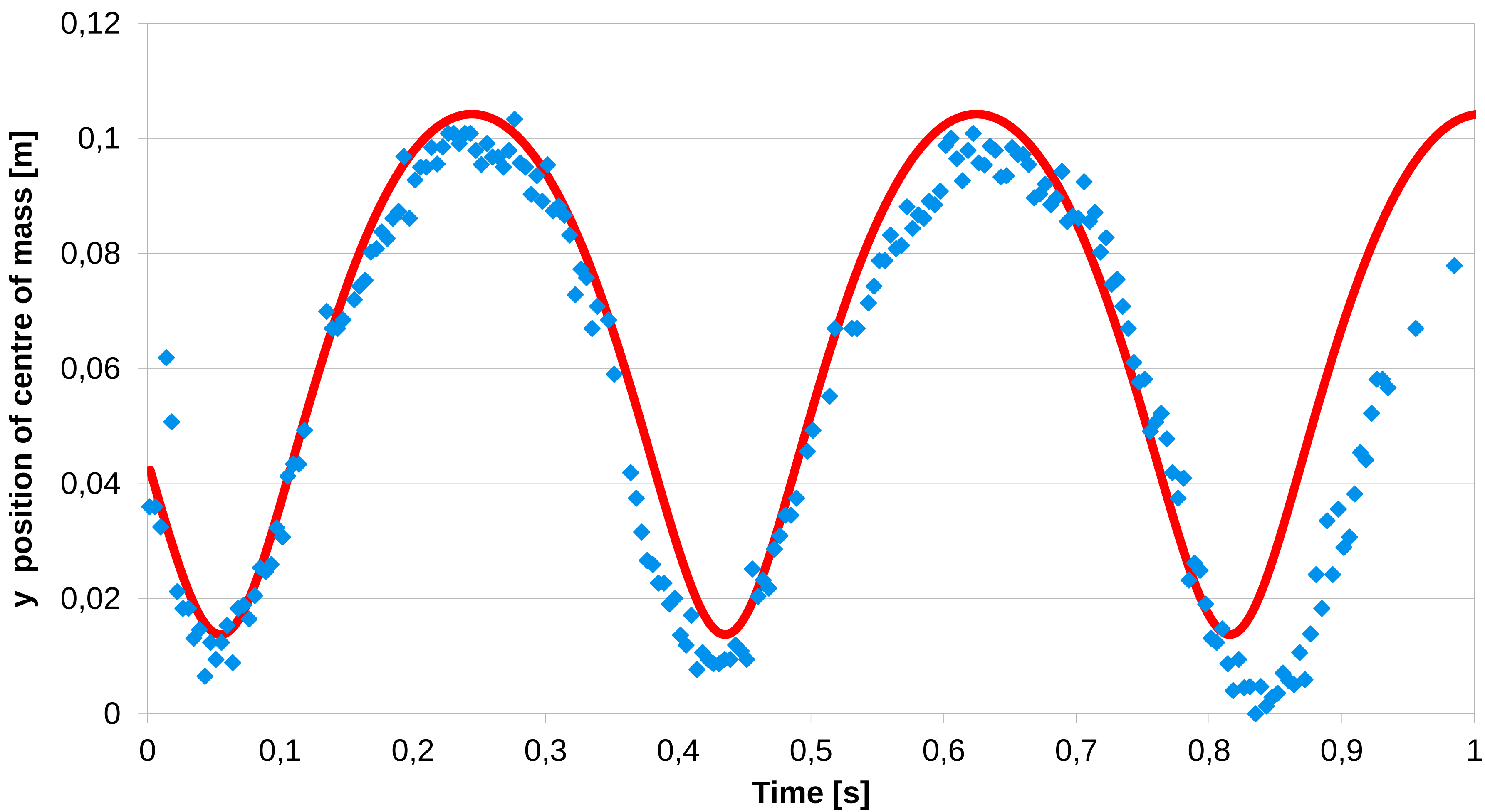
1) Rolling

$m_{weight} = 22g$
$m_{hoop} = 28g$
$r_{hoop} = 41\ mm$

Angular displacement in time θ(t)



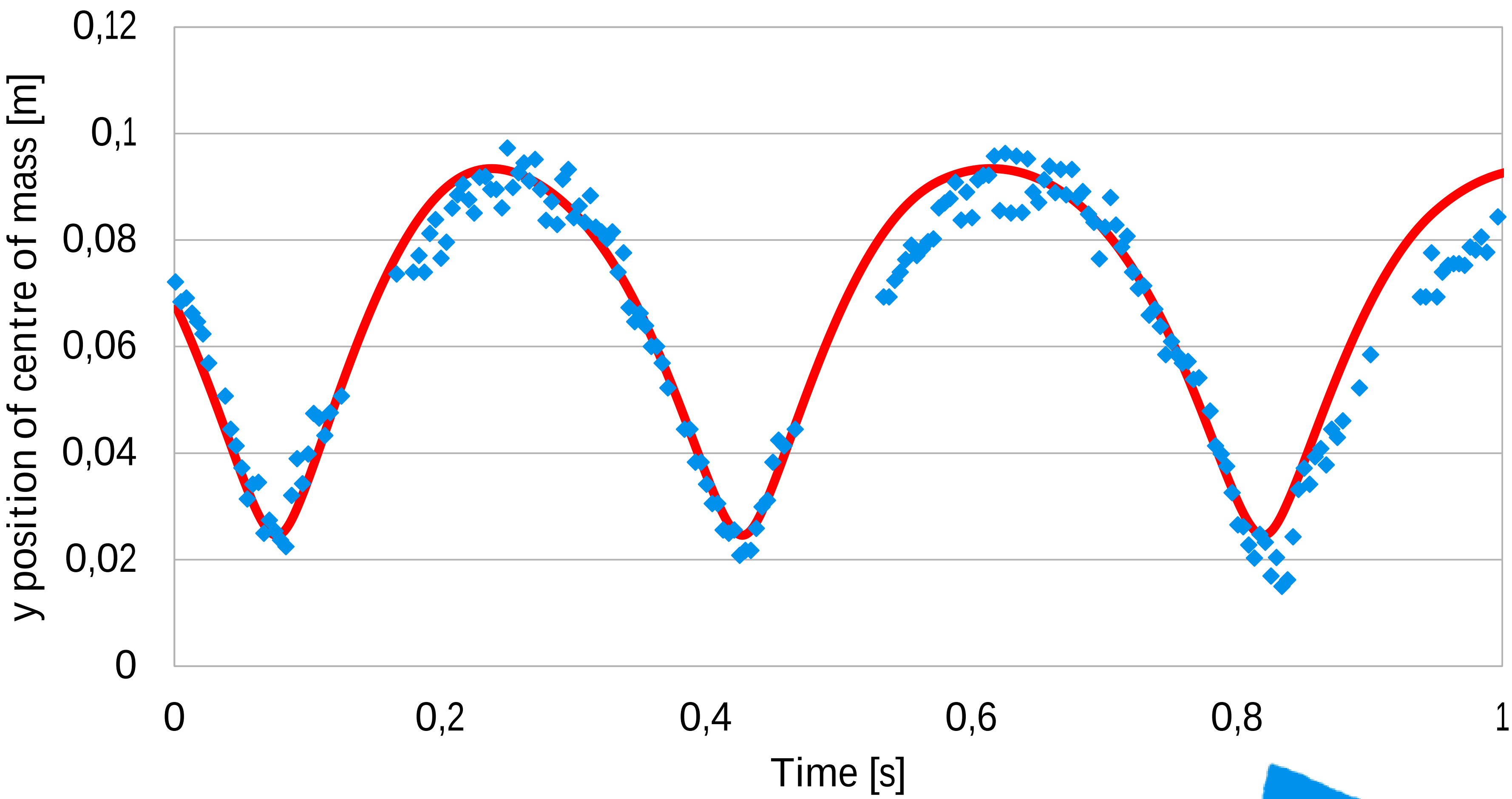
Position of centre of mass y(t)



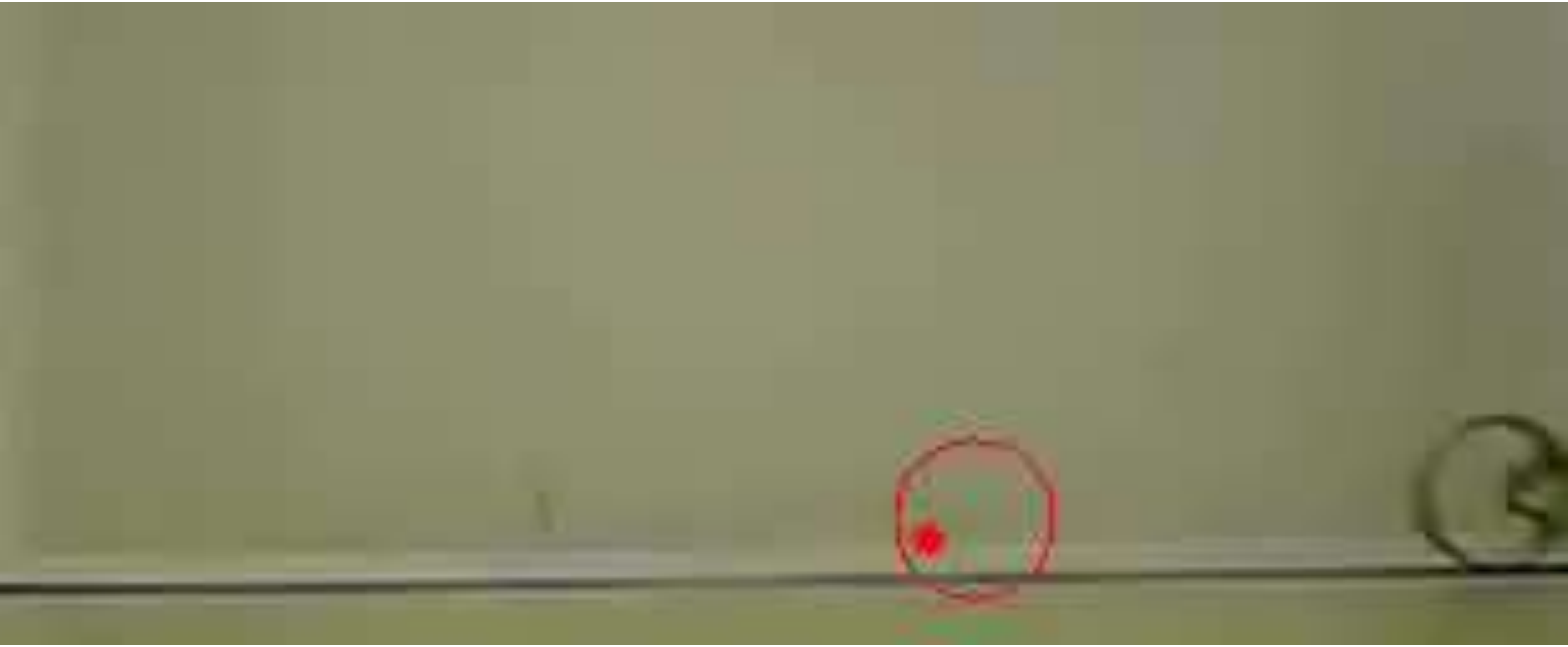
2) Spinning/skidding

m_{weight} = 322g
m_{hoop} = 28g
r_{hoop} = 41 mm

Position of centre of mass y(t)



3) Jump



m_{weight} = 220g
m_{hoop} = 28g
r_{hoop} = 41 mm

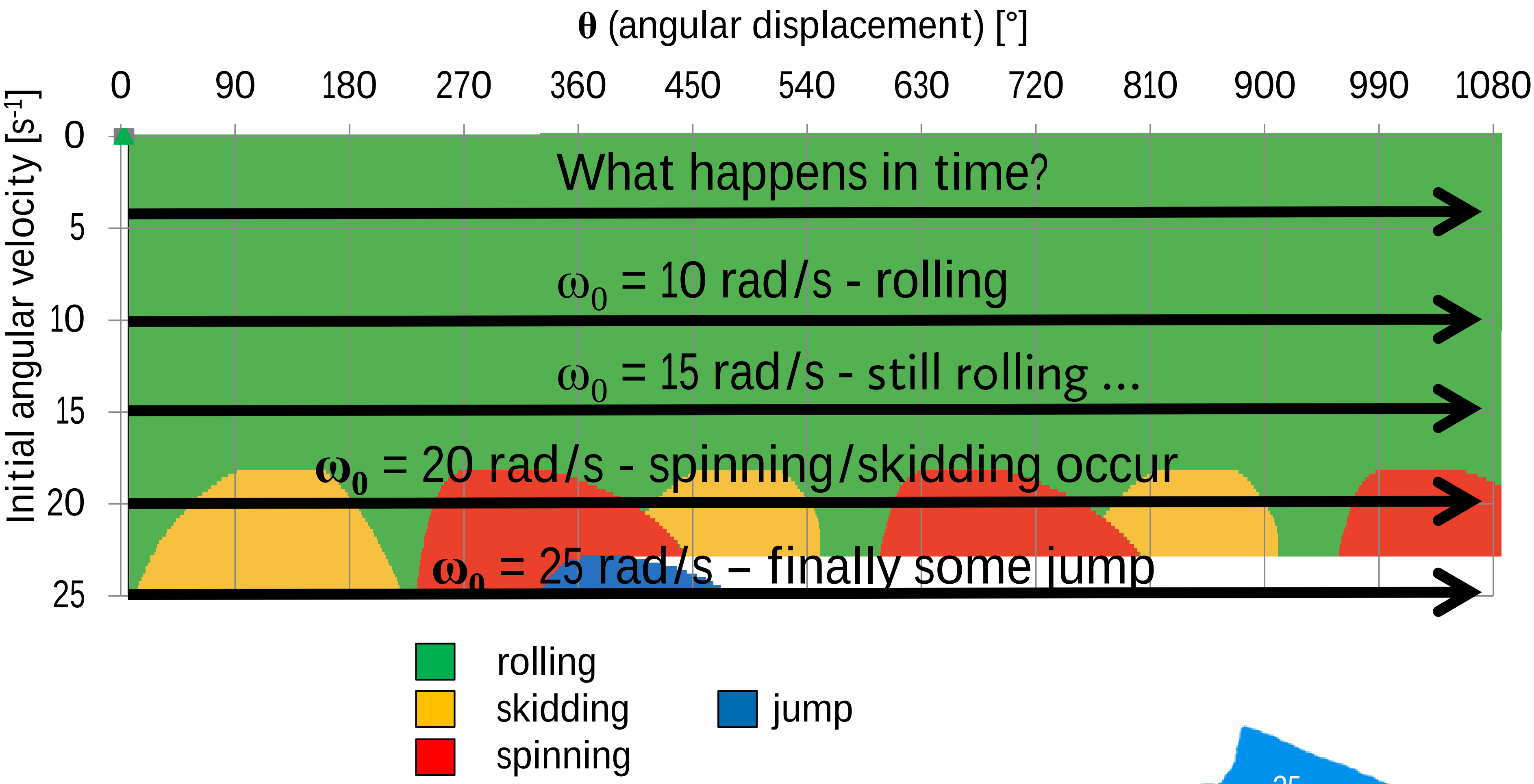
4) Oscillation

$m_{weight} = 3{,}6g$
$m_{hoop} = 3{,}9g$
$r_{hoop} = 20{,}4\ mm$

m_{weight} = 22g = small eccentricity

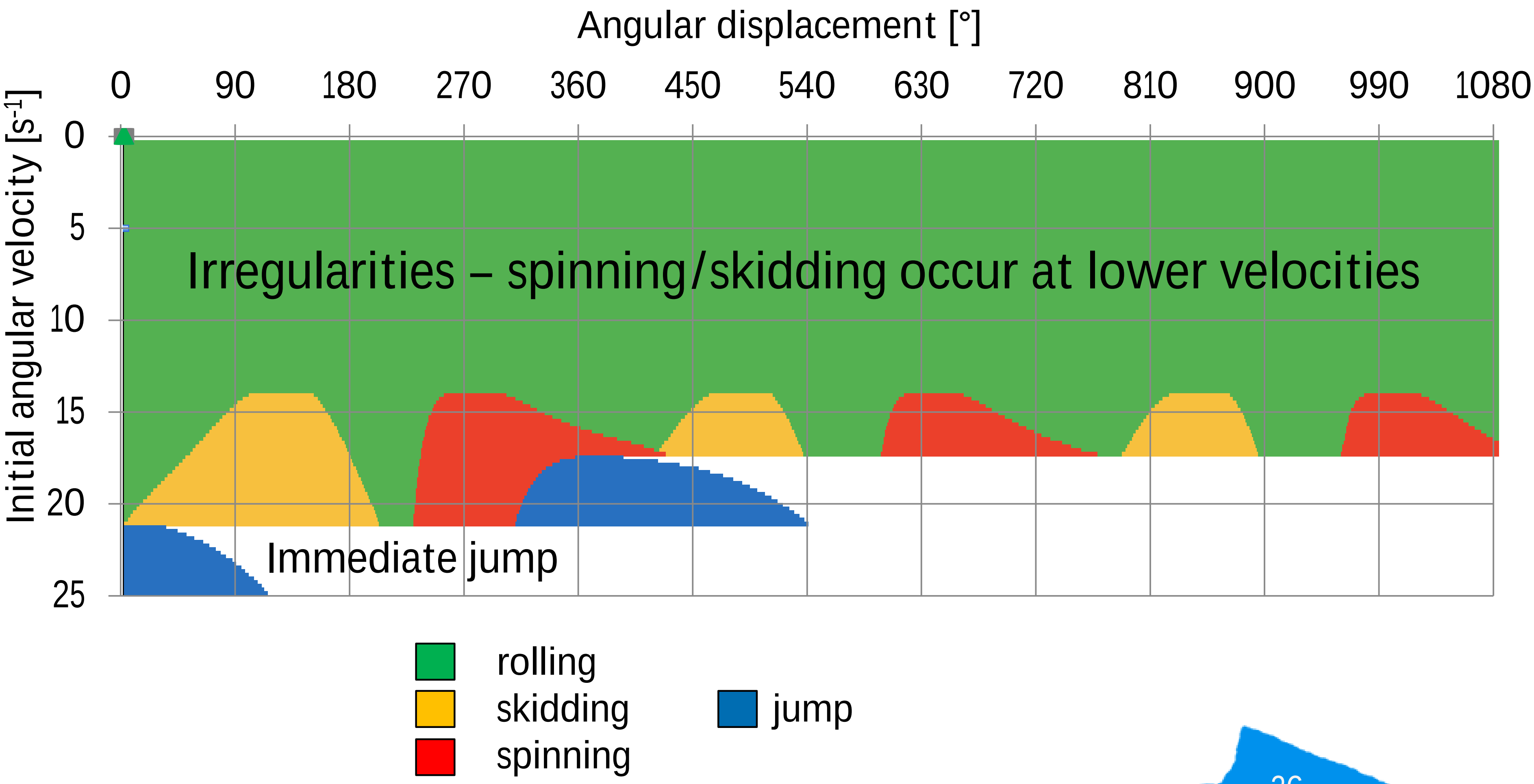
mostly boring rolling

Phase diagram of movement



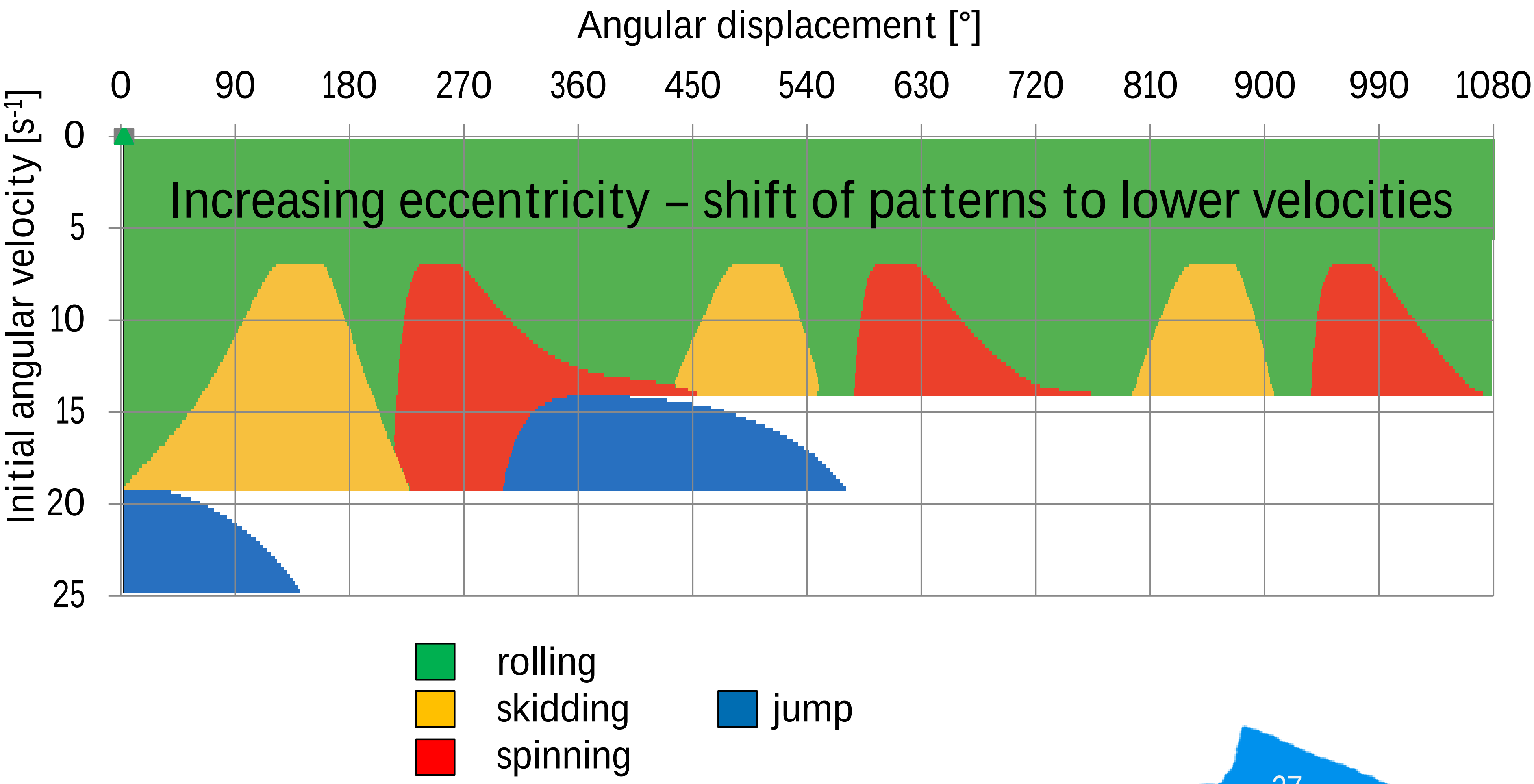
$m_{weight} = 214g$ = higher eccentricity

Phase diagram of movement



$m_{weight} = 322g$

Phase diagram of movement



Conclusion

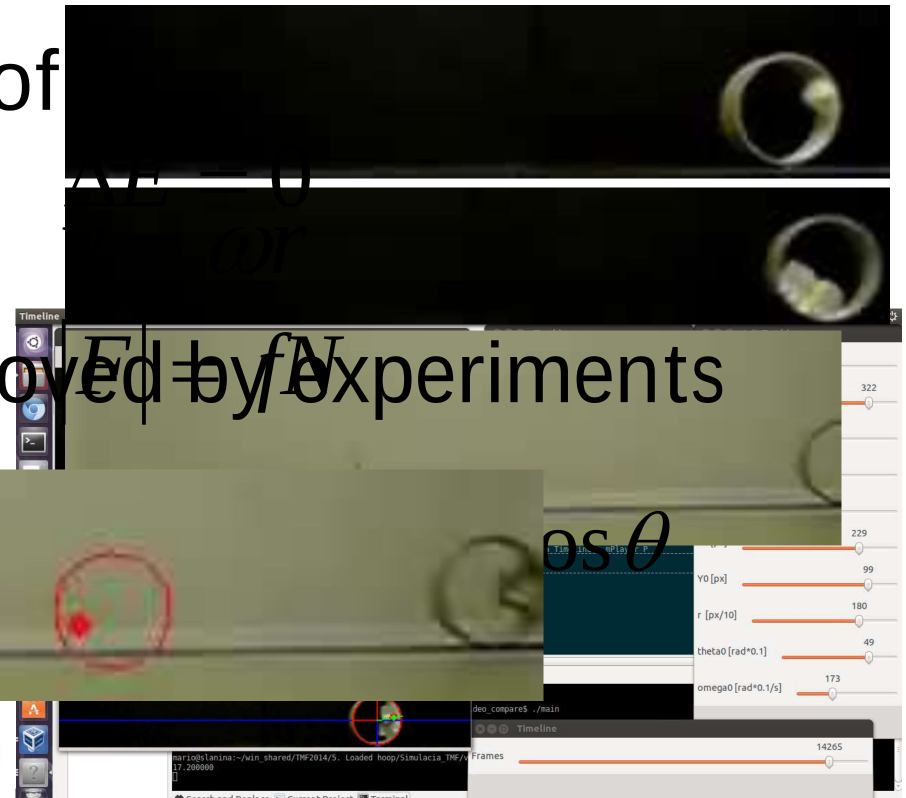
- Description of types of motion
- Theoretical prediction of
  1) Rolling
  2) Spinning/Skidding
- Numerical simulation
- Theory & simulation proved by experiments

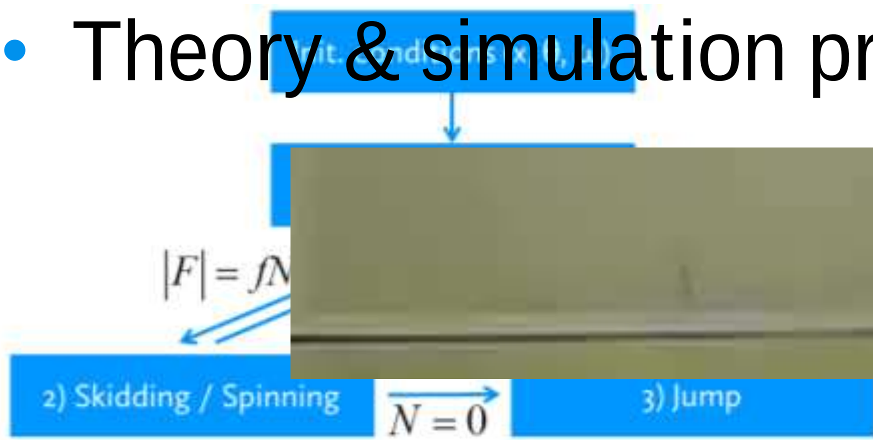
$\Delta E = 0$

$v = \omega r$

$|F| = fN$

$\cos\theta$



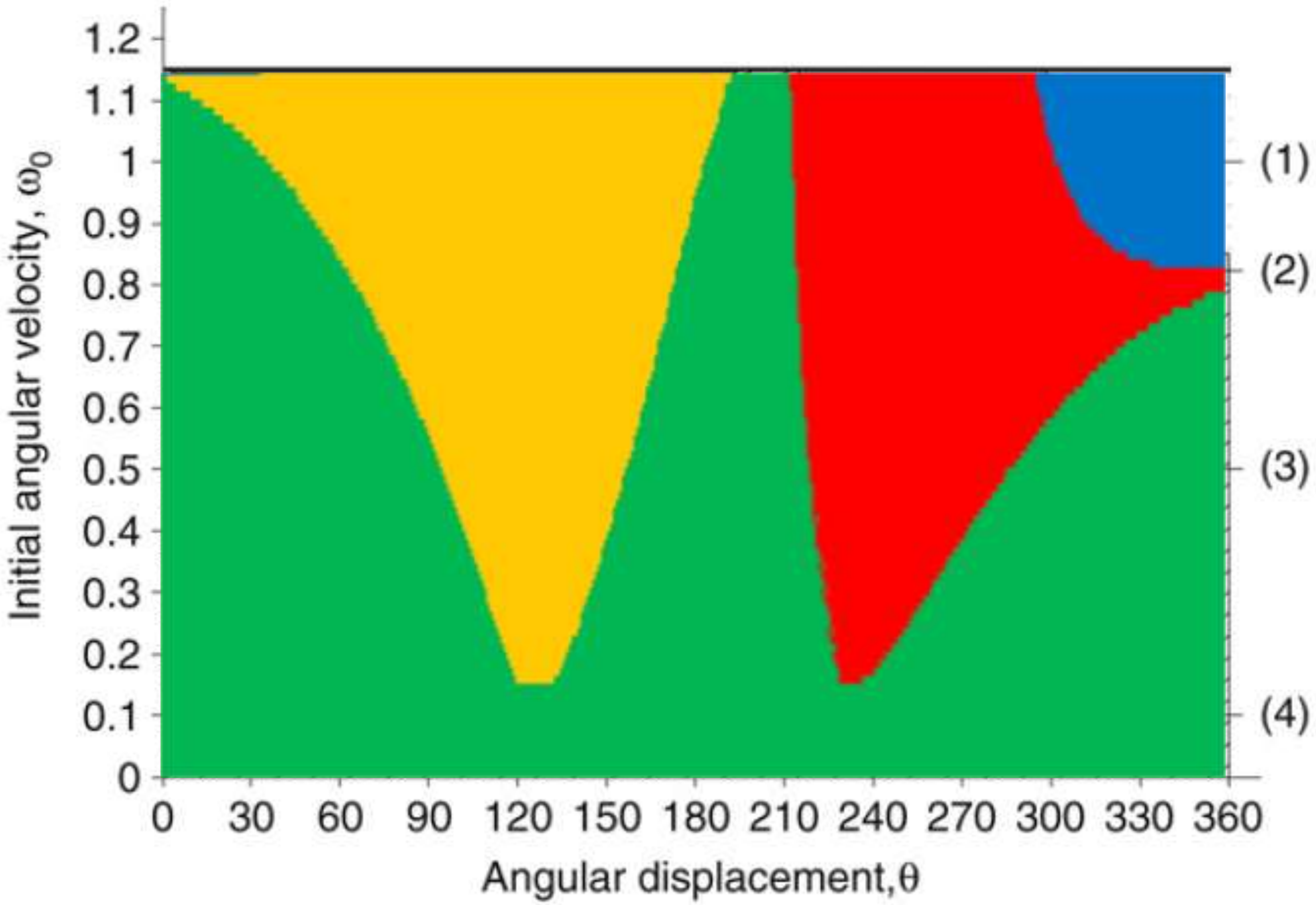


Thank you for your attention

THANK YOU FOR YOUR ATTENTION

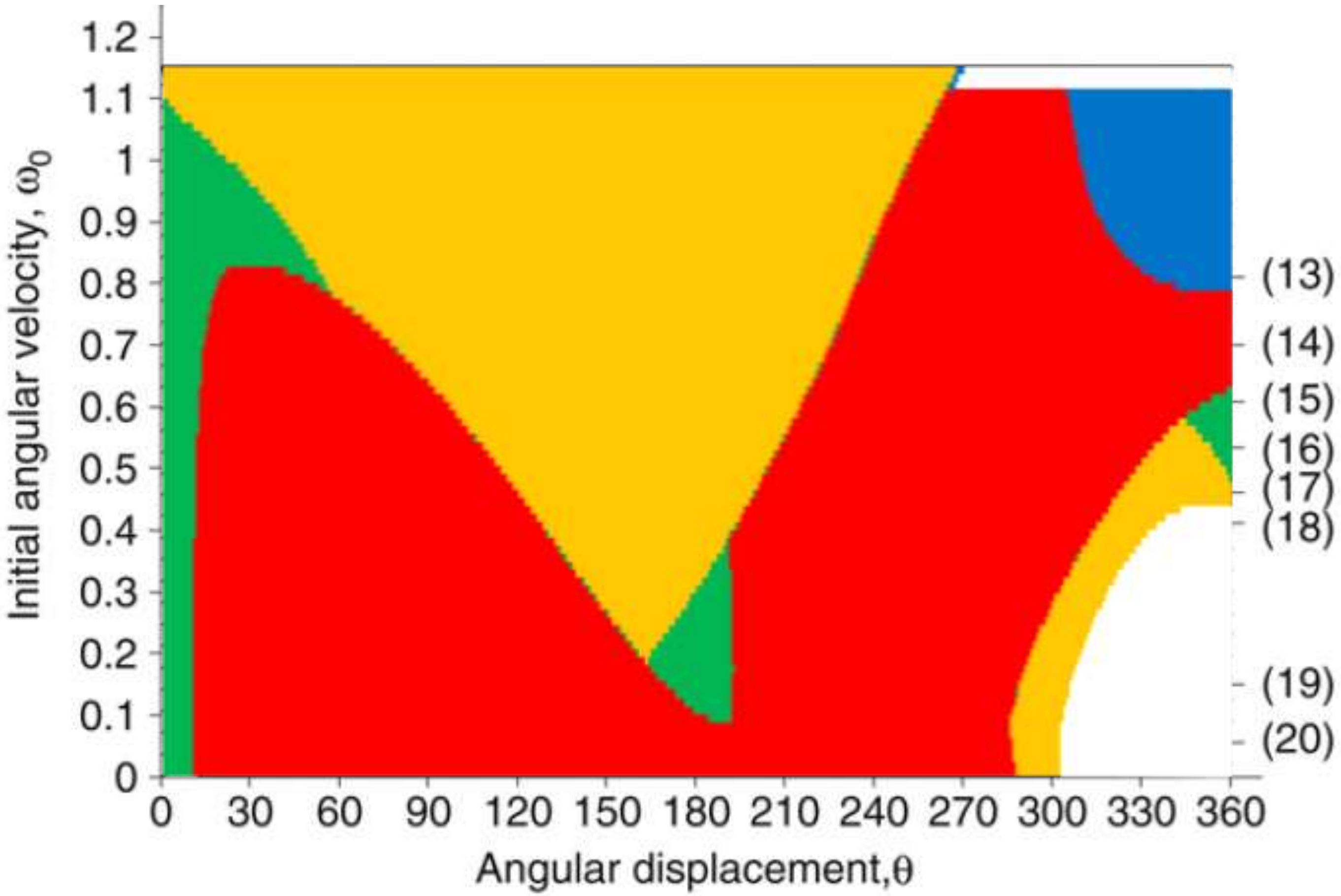
APPENDICES

Comparison to original paper [W.F.D. Theron]



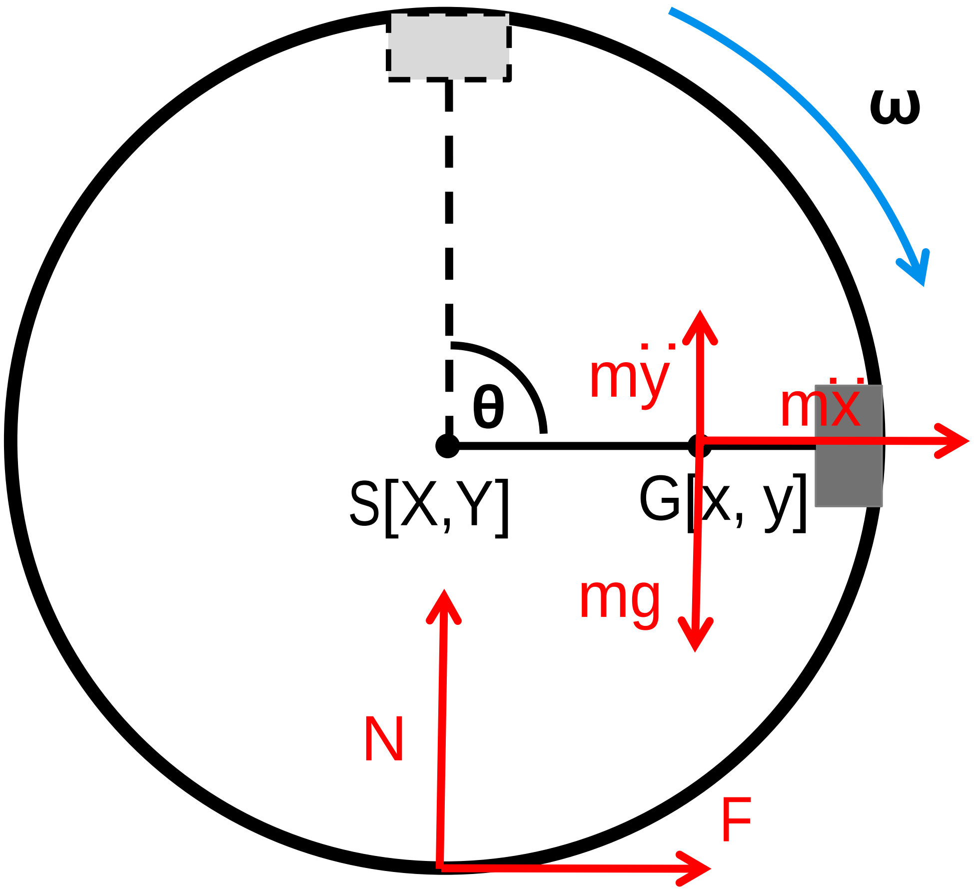
$\gamma = 3/4$
$f = 0{,}4$

Comparison to original paper [W.F.D. Theron]



$\gamma = 3/4$
$f = 0{,}09$

Description of the system



Centre of hoop S [X, Y]

Angular displacement θ

Centre of gravity G [x, y]

$$x = X + R_G \sin\theta$$

$$y = R + R_G \cos\theta$$

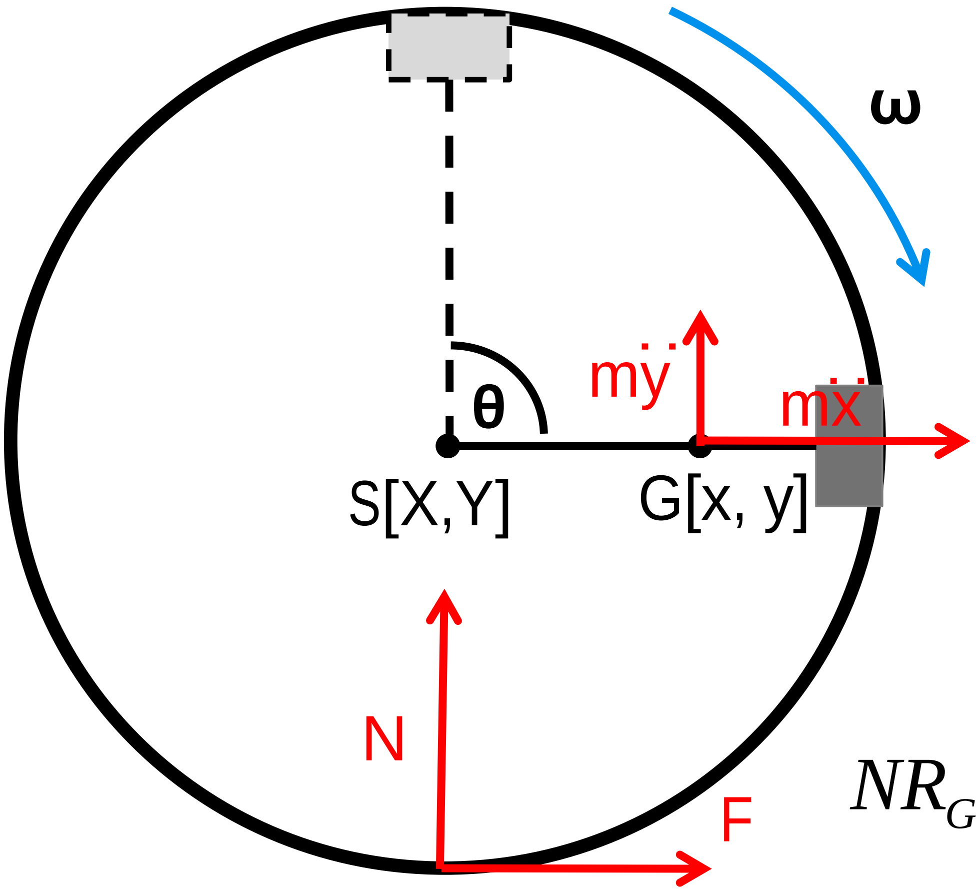
Normal force

$$m\ddot{y} = mg - N$$

Friction force

$$m\ddot{x} = F$$

Description of the system



$$x = X + R_G \sin\theta$$
$$y = Y + R_G \cos\theta$$

$$\dot{x} = \dot{X} + R_G \dot{\theta} \cos\theta$$
$$\dot{y} = -R_G \dot{\theta} \sin\theta$$

$$\ddot{x} = \ddot{X} - R_G \dot{\theta}^2 \sin\theta + R_G \ddot{\theta} \cos\theta$$
$$\ddot{y} = -R_G \dot{\theta}^2 \cos\theta - R_G \ddot{\theta} \sin\theta$$

$$mg - m\ddot{y} = N$$
$$m\ddot{x} = F$$
$$N R_G \sin\theta - F y = I_G \ddot{\theta}$$

Elastic hoop
– obvious difference in motion