13. ROTATING SADDLE

Reporter: Natália Ružičková

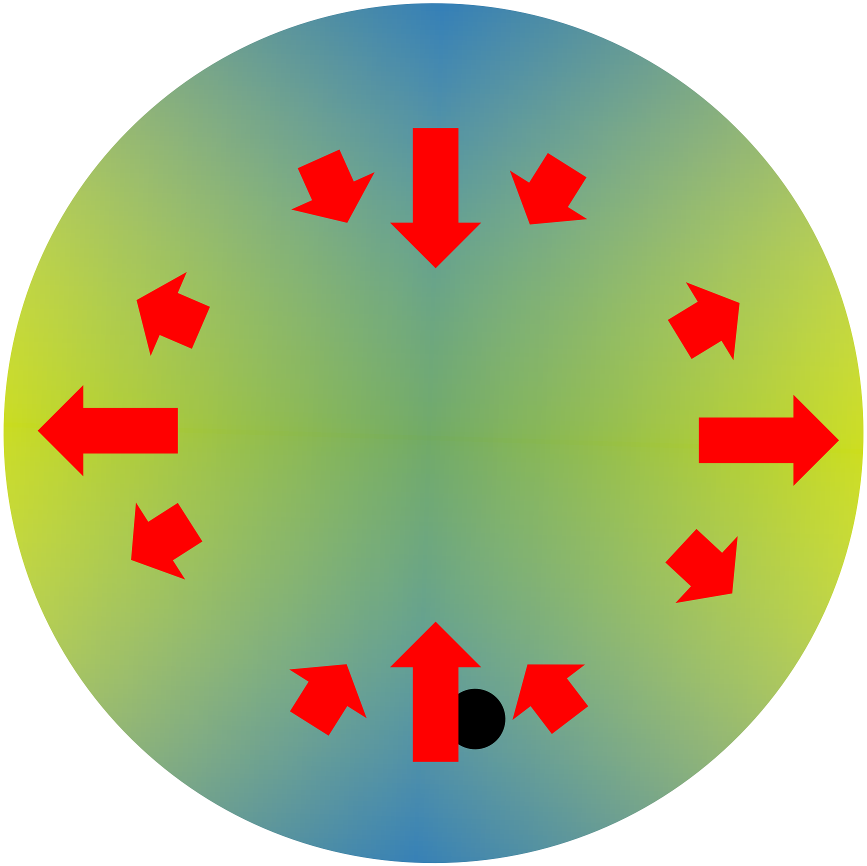
Task

A ball is placed in the middle of a rotating saddle.

Investigate it's dynamics and explain the conditions under which the ball does not fall off the saddle.

How can the rotation *help* the stability?

- Static saddle:
  just rolls off
- Rotating saddle:
  rolls around the saddle
  → effect of slopes
  cancels



Ball's motion (laboratory ref. frame)

1. Goes around with the saddle

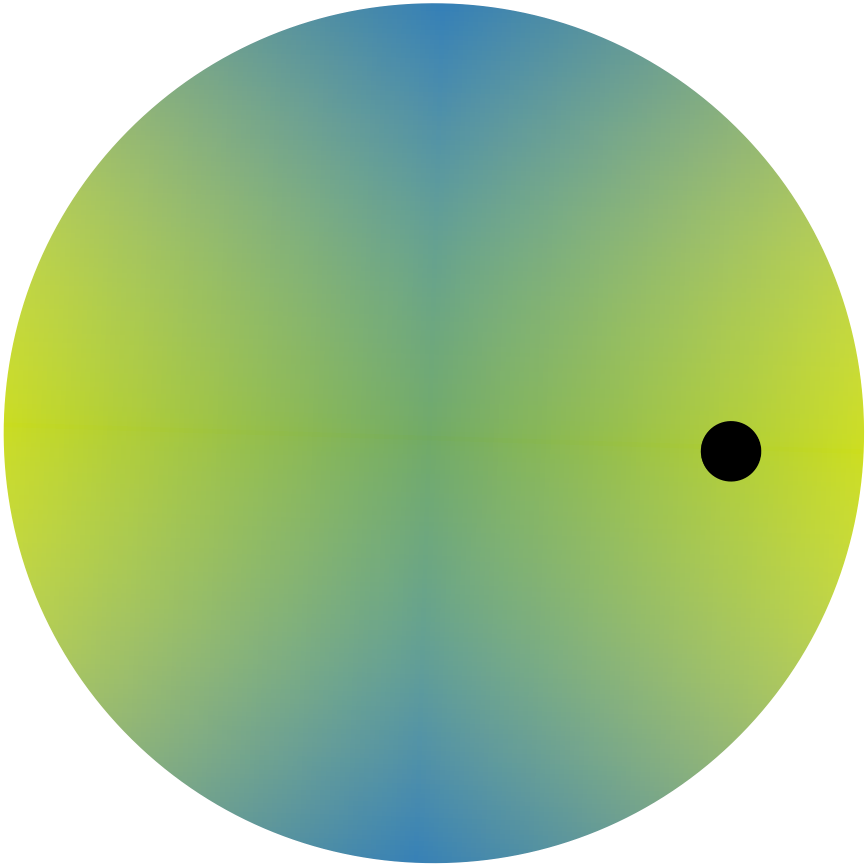
   Centripetal force needed

   → unstable

2. Remains stationary

   (Rolls back quickly enough)

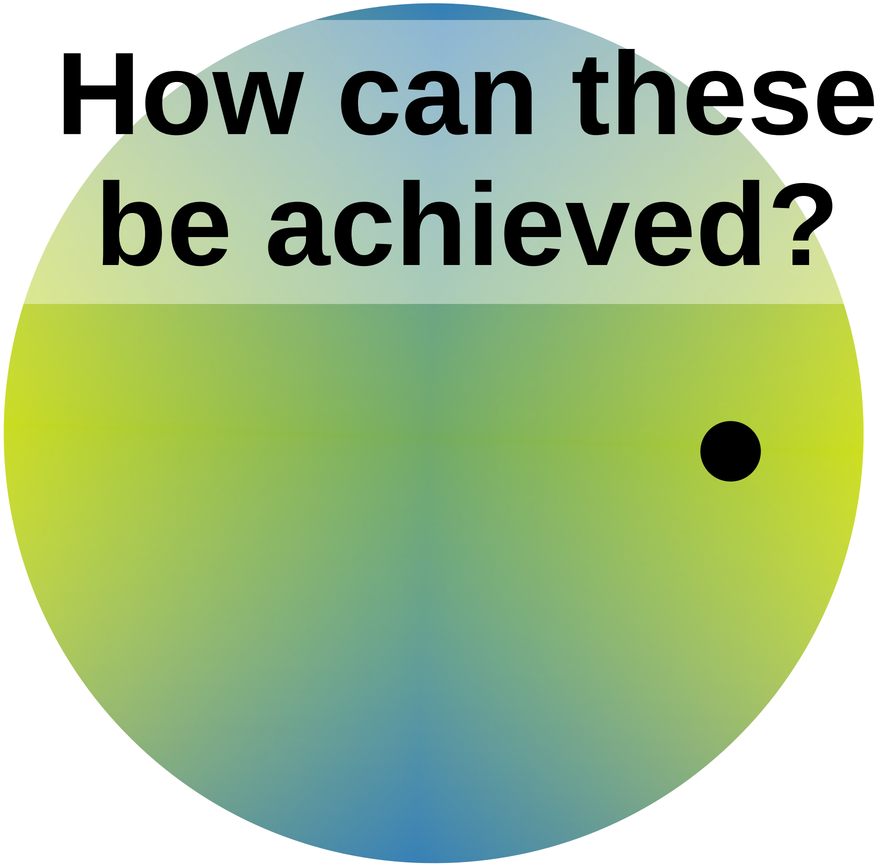
   → stable



Ball's stability requirements

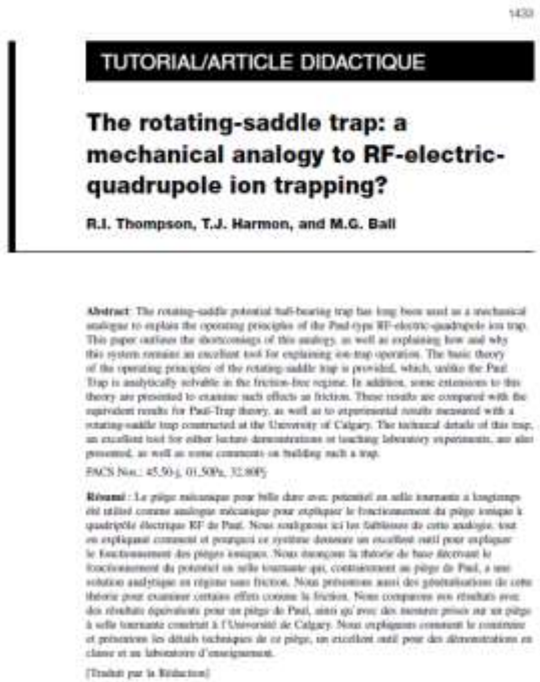
1. Sufficient saddle rotation
   – Cancels the effect of slopes

2. Rolling backwards
   – Avoids centrifugal force



Existing theory

- **Thompson:** *The rotating-saddle trap: a mechanical analogy to RF-electricquadrupoleion trapping?*
  - Canadian journal of Physics, Vol. 80, 2002
- **Koch:** *Konzeption und Aufbau einer mobilen Experimentiereinheit fur Schuleräprsentationen zum Thema Teilchenfallen*
  - *Universität Stuttgard, 2004*



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TUTORIAL/ARTICLE DIDACTIQUE

The rotating-saddle trap: a mechanical analogy to RF-electric-quadrupole ion trapping?

R.I. Thompson, T.J. Harmon, and M.G. Ball

Abstract: The rotating-saddle potential ball-bearing trap has long been used as a mechanical analogue to explain the operating principles of the Paul-type RF-electric-quadrupole ion trap. This paper outlines the shortcomings of this analogy, as well as explaining how and why this system remains an excellent tool for explaining ion-trap operation. The basic theory of the operating principles of the rotating-saddle trap is provided, which, unlike the Paul Trap is analytically solvable in the friction-free regime. In addition, some extensions to this theory are presented to examine such effects as friction. These results are compared with the equivalent results for Paul-Trap theory, as well as to experimental results measured with a rotating-saddle trap constructed at the University of Calgary. The technical details of this trap, an excellent tool for either lecture demonstrations or teaching laboratory experiments, are also presented, as well as some comments on building such a trap.

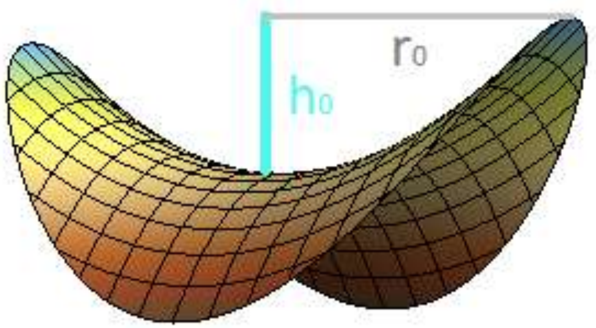
PACS Nos.: 45.50.-j, 01.50.Pa, 32.80.Pj

Résumé : Le piège mécanique pour bille dure avec potentiel en selle tournante a longtemps été utilisé comme analogue mécanique pour expliquer le fonctionnement du piège ionique à quadripôle électrique RF de Paul. Nous soulignons ici les faiblesses de cette analogie, tout en expliquant comment et pourquoi ce système demeure un excellent outil pour expliquer le fonctionnement des pièges ioniques. Nous énonçons la théorie de base décrivant le fonctionnement du potentiel en selle tournante qui, contrairement au piège de Paul, a une solution analytique en régime sans friction. Nous présentons aussi des généralisations de cette théorie pour examiner certains effets comme la friction. Nous comparons nos résultats avec des résultats équivalents pour un piège de Paul, ainsi qu'avec des mesures prises sur un piège à selle tournante construit à l'Université de Calgary. Nous expliquons comment le construire et présentons les détails techniques de ce piège, un excellent outil pour des démonstrations en classe et au laboratoire d'enseignement.

[Traduit par la Rédaction]

- Point mass in gravitational potential
  - Constrained to saddle's surface

$$U(x',y') = \frac{mgh_0}{r_0^2}\left(x'^2 - y'^2\right) \qquad F = -\nabla U$$



$$z = k(x^2 - y^2)$$

- Mathematical trick:
  - coordinates in complex plane $z = x + iy$

ball's POSITION

Solution

$$z(\tau) = (Ae^{+\beta_+\Omega t} + Be^{-\beta_+\Omega t} + Ce^{+\beta_-\Omega t} + De^{-\beta_-\Omega t})e^{i\tau}$$

The only requirement for stability:

f > fc

$$\frac{gh_0}{r_0^2\Omega^2} \leq 0.5$$

$$f \geq \frac{\sqrt{2gh_0}}{2\pi r_0} = f_{CRITICAL}$$

EXPERIMENTAL VERIFICATION

Apparatus: Saddles

h_0 = 1,5cm

r_0 = 8cm

- **f_c = 1,08 Hz**

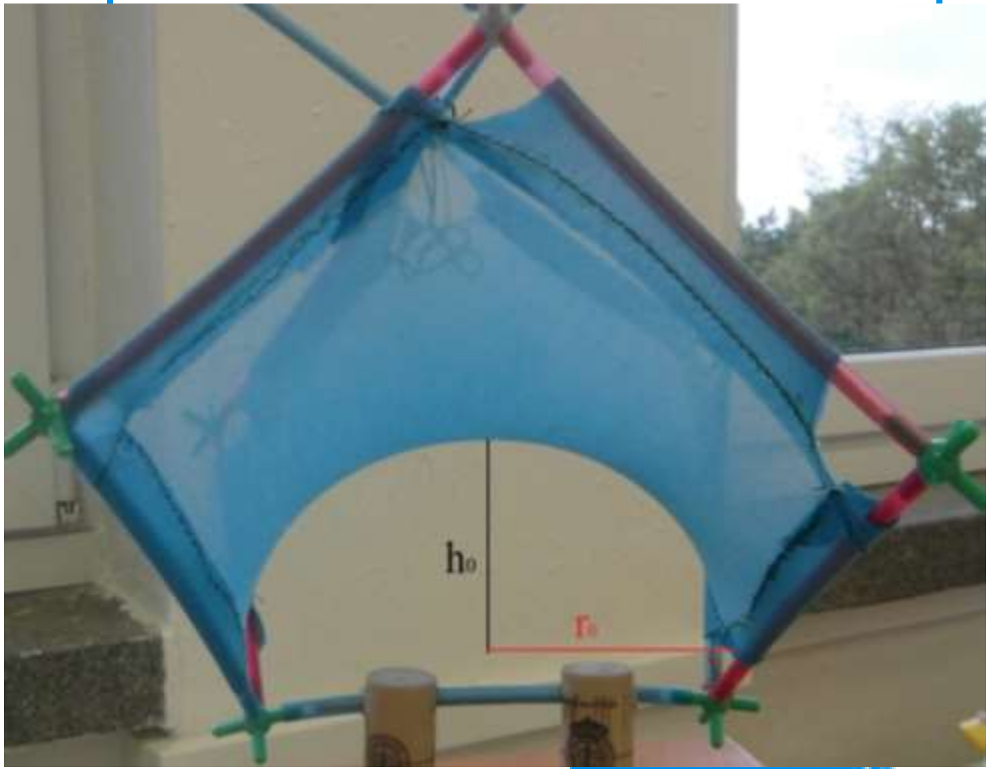
material = plastic

h_0 = 6,5cm

r_0 = 8cm

- **f_c = 2,25 Hz**

material = nylons



Apparatus: Balls

solid

Radius range:

0,63 cm – 3,26 cm

Mass range:

8,39 g – 35,79 g

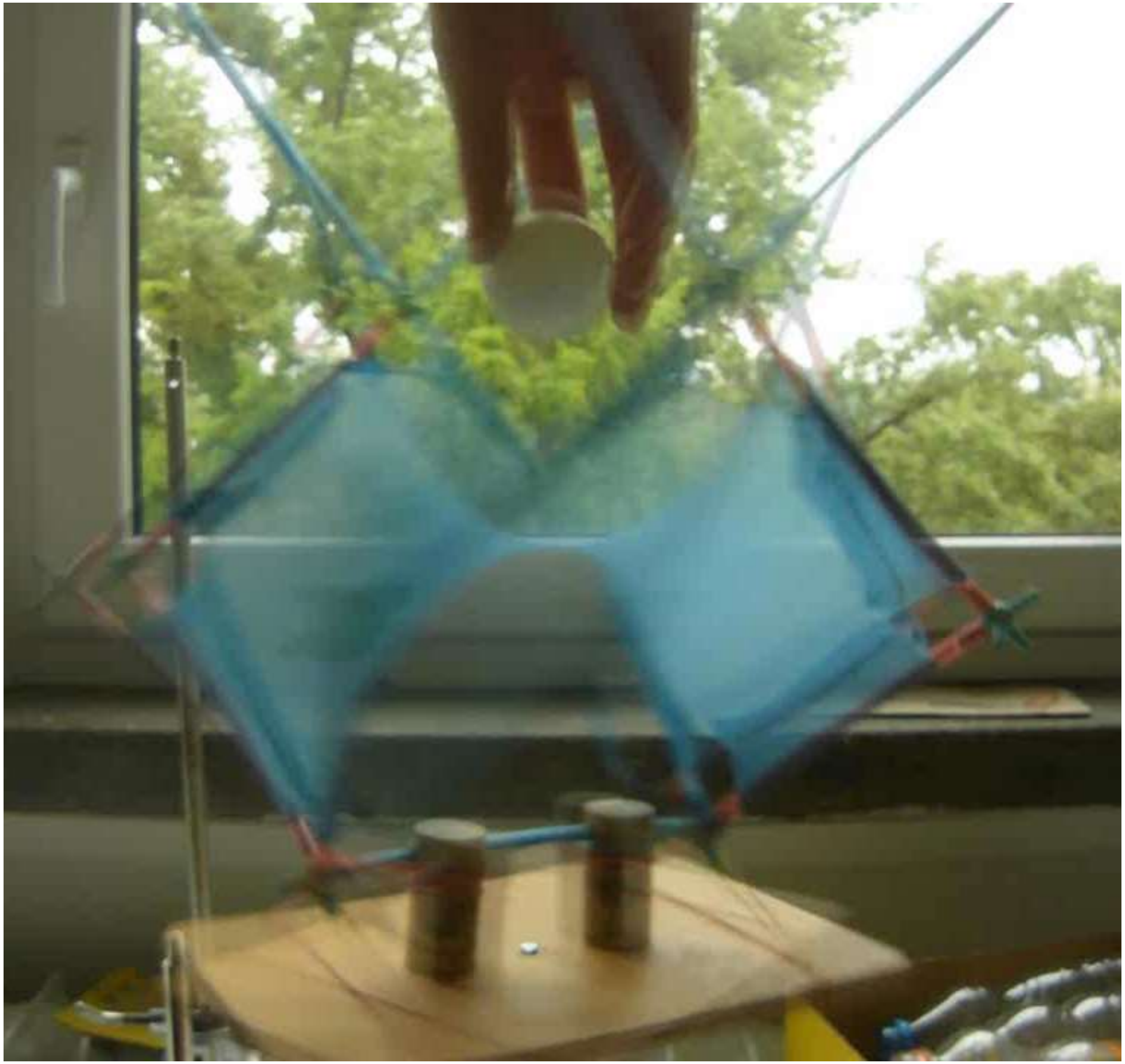
hollow

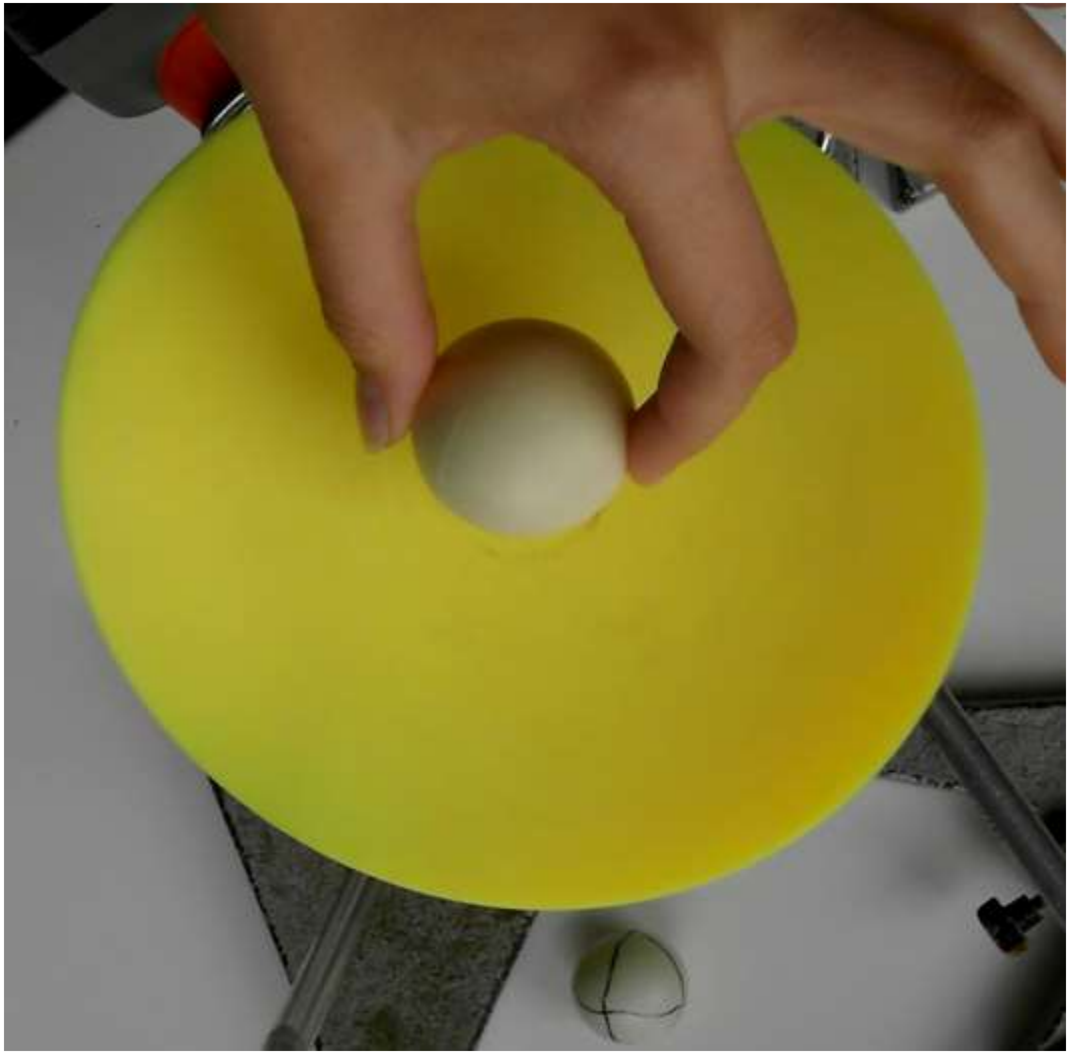
Radius range:

1,88 cm – 5,0 cm

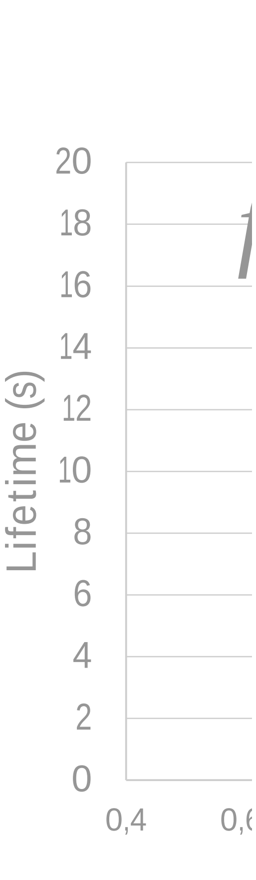
Mass range:

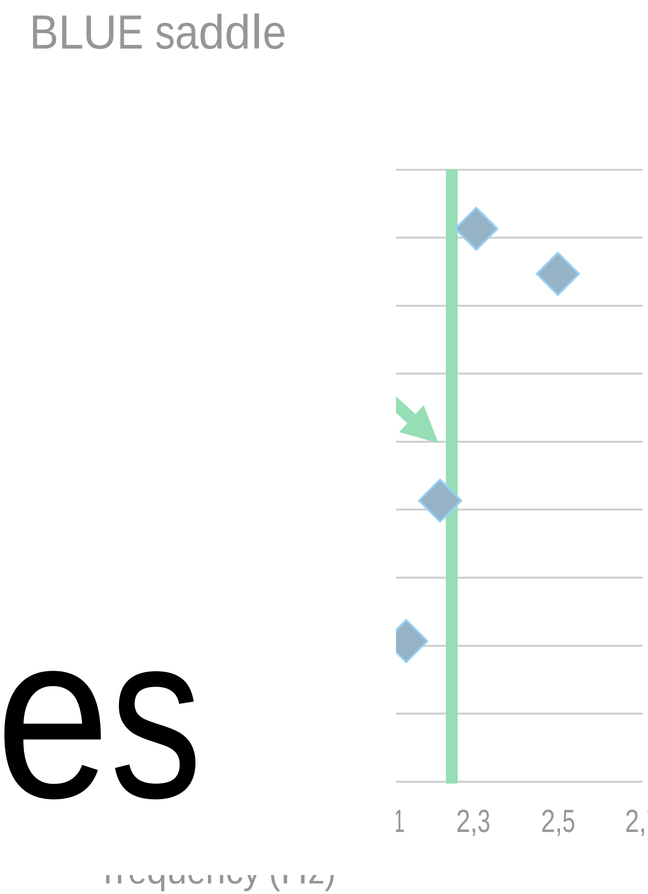
2,46 g – 26,56 g





Stability vs. Frequency





WHY?

… drag forces

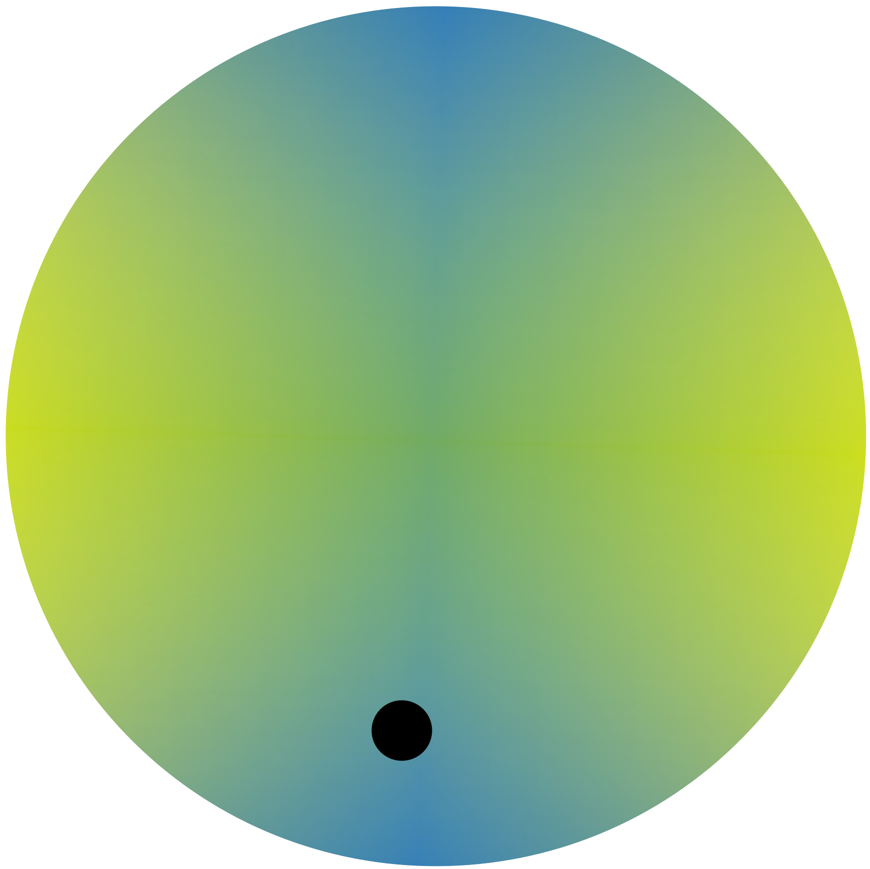
$f \geq f_C$ → Significant increase in lifetime but clearly not infinite

Second condition:
Must roll back fast enough

Friction/rolling resistance:

Drags the ball
to rotate with the saddle

Ball becomes unstable



Literature: Effect of friction

- Thompson: $\vec{F}_{Friction} = -k\vec{v}$
  - Analytical solution; always diverges

- Koch: $\vec{F}_{Friction} = -k\frac{\vec{v}}{|\vec{v}|}$
  - Numerical solution; no record of stability

~~Stability~~

Maximal lifetime

Parametres affecting the lifetime

1. Drag forces & Friction
2. Frequency
3. Ball ≠ point mass
4. Initial position

?

1. DRAG FORCES & FRICTION

Effect of friction

Thompson:

$$T_L = \frac{1}{\sigma \Omega} \ln\left(\frac{r_0}{R}\right)$$

- TL = trapping lifetime
- σ ~ friction coefficient
- R = initial distance from the center
- r_0= trap's radius

Higher friction → lower lifetime

1. Lifetime vs. Friction: Experiment

DRY NYLON SADDLE		NYLON SADDLE SOAKED WITH WATER
μ=0,25	>	μ=0,09
MAXIMUM LIFETIME: 2,25s	<	6,33s

PREDICTION CONFIRMED

Not so simple

<u>Koch's article:</u>

	Teflonspray (lower friction)	Clean saddle (higher friction)
Lifetime	10,1s	54,7s

HIGHER FRICTION

⬇

HIGHER LIFETIME

?

What if the ball does NOT slip?

SUFFICIENT FRICTION

⇩

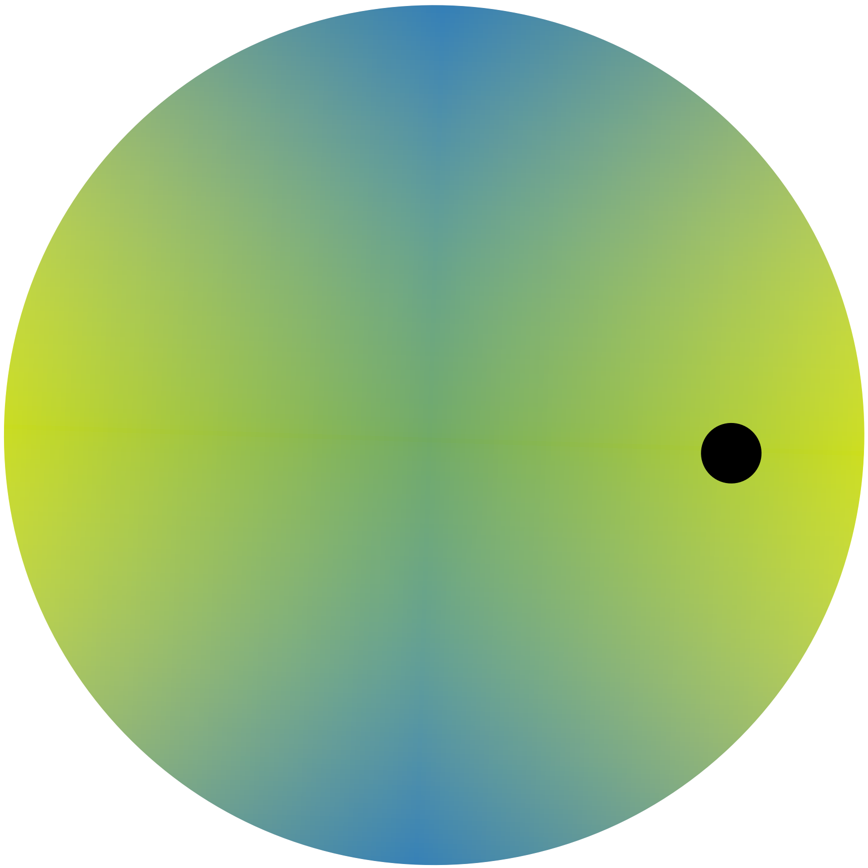
Avoids slipping

⇩

Causes ball's rotation

Sufficient friction: no slipping

Similar to zero friction (no slip)



Dragging effect:

only rolling resistance
(much lower than
dynamic friction)

Relatively stable:

- Zero friction
- Sufficient friction

Measurement:

Slipping vs. Rolling

SUFFICIENT FRICTION

=

HIGHER LIFETIME

2,7 s ± 0,4 s

8,8 s ± 2,6 s

Parameters affecting lifetime

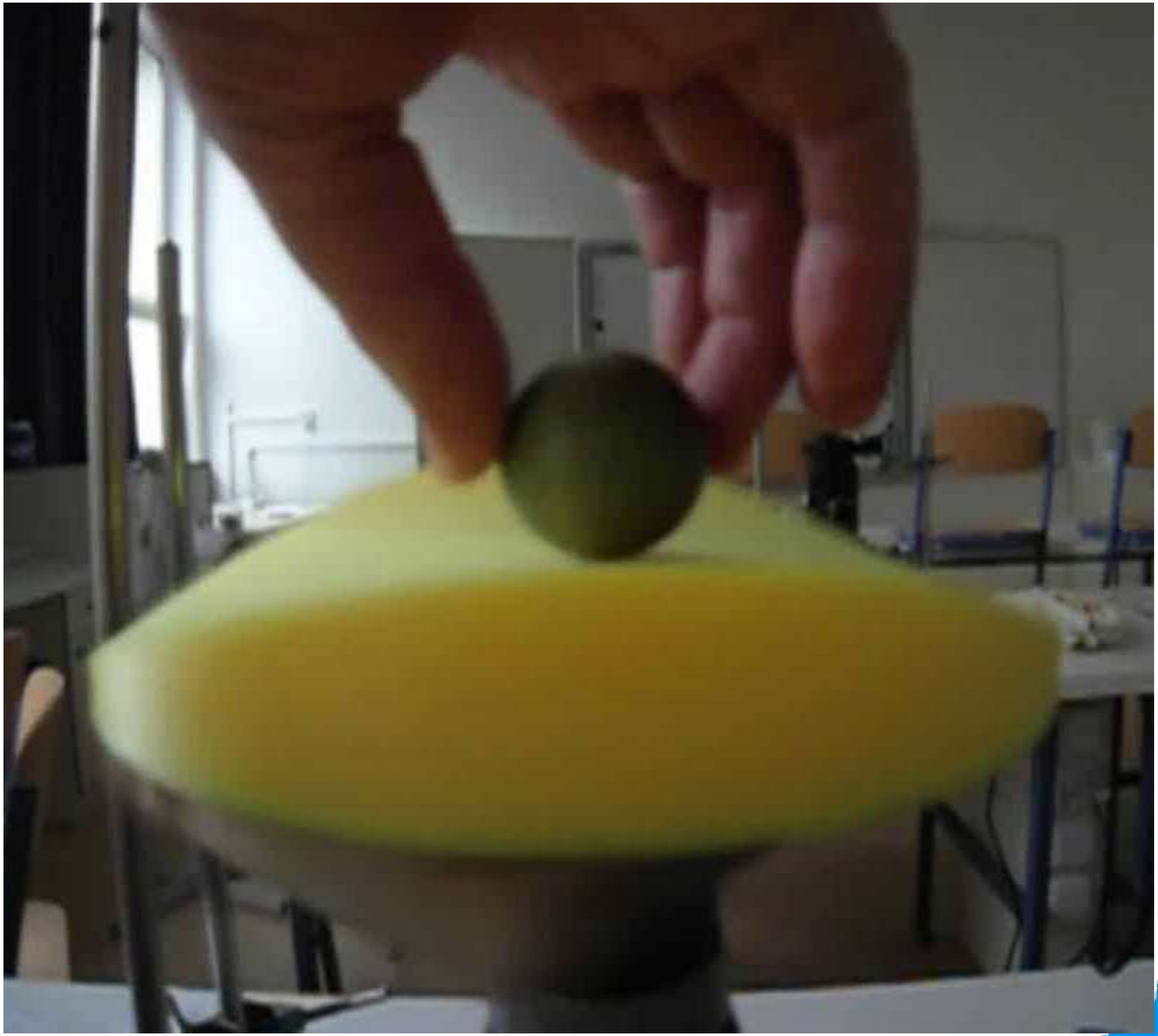
1. Friction
   - dynamic: the lower, the longer lifetime
   - static: fulfills *'no slipping'* condition

?

2. Frequency
3. Moment of inertia
4. Initial position

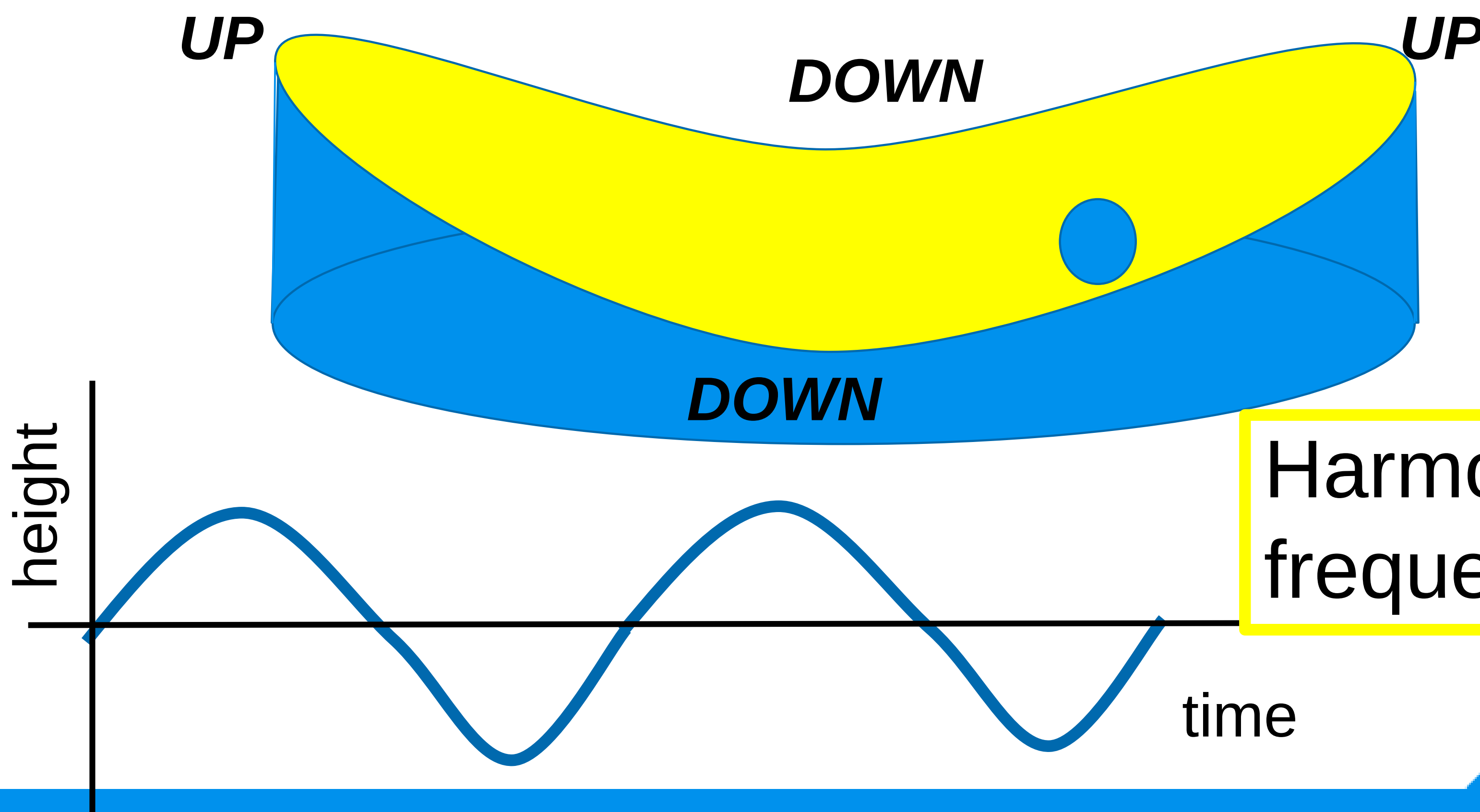
2. FREQUENCY

JUMPING



1. Ball free to move upwards

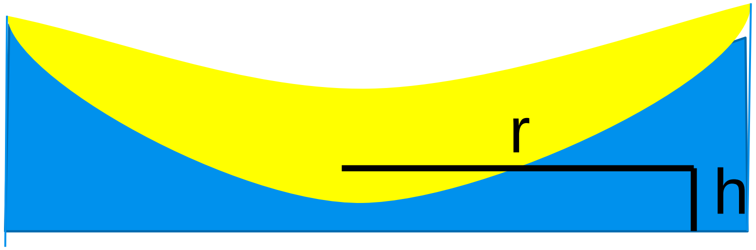
- Very fast rotation:
  insufficient g to follow the saddle's curvature
- Height changes harmonically



Harmonic
frequency = 2Ω

Condition of jumping

- Saddle shape in polar coordinates $h = kr^2 \cos(2\Omega t)$
- Vertical acceleration: $a = -(2\Omega)^2 kr^2 \cos(2\Omega t)$



$a \leq g$ Constrained to surface

$a > g$ Jumps

Critical frequency for jumping:

$$f > f_{jump} = \frac{1}{4\pi r}\sqrt{\frac{g}{k}}$$

Est

- W ... urred
an

f ... ,14Hz

- Me

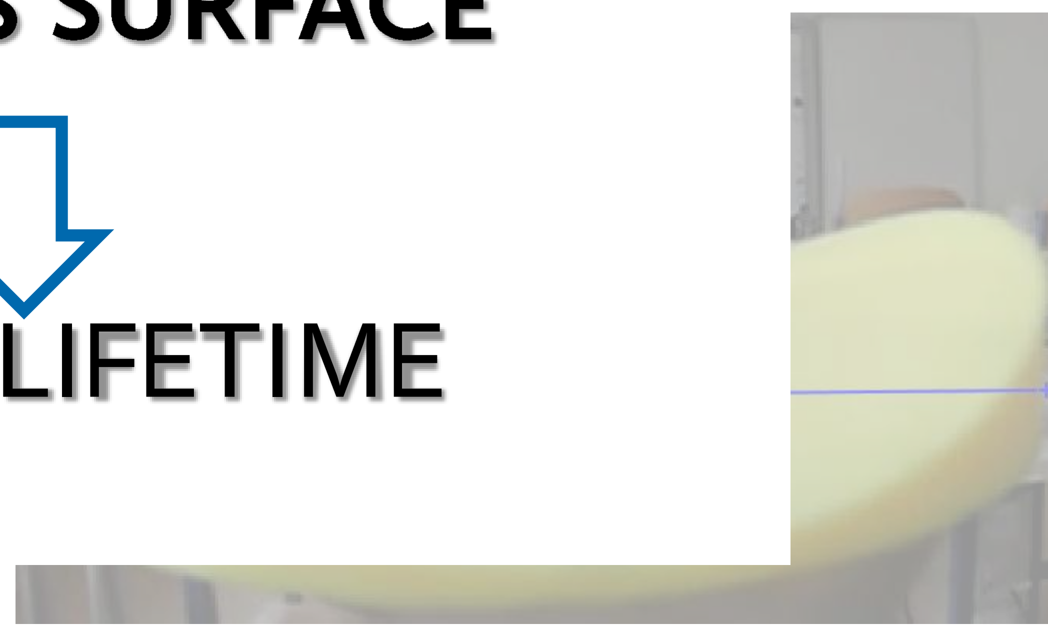
TOO HIGH FREQUENCY

⇩

NO MORE CONSTRAINED TO SADDLE'S SURFACE

⇩

LOWER LIFETIME



Parameters affecting lifetime

✓ ✓

1. Friction
2. Frequency
   – Lower limit: rise of lifetime
   – Upper limit: jumping

?

3. Moment of inertia
4. Initial position

3. MOMENT OF INERTIA

Hollow VS. Solid Ball

Greater moment of inertia

More energy needed for rolling $(M = J\varepsilon)$

Lower speed

Longer lifetime

$$J = \frac{2}{3}mR^2$$

Should have
longer lifetime than

$$J = \frac{2}{5}mR^2$$

EXPERIMENT

Greater moment of inertia

=

longer lifetime

2

IME:

ts)

8,96 s ± 1,74 s

2,11 s ± 0,34 s

Parameters affecting lifetime

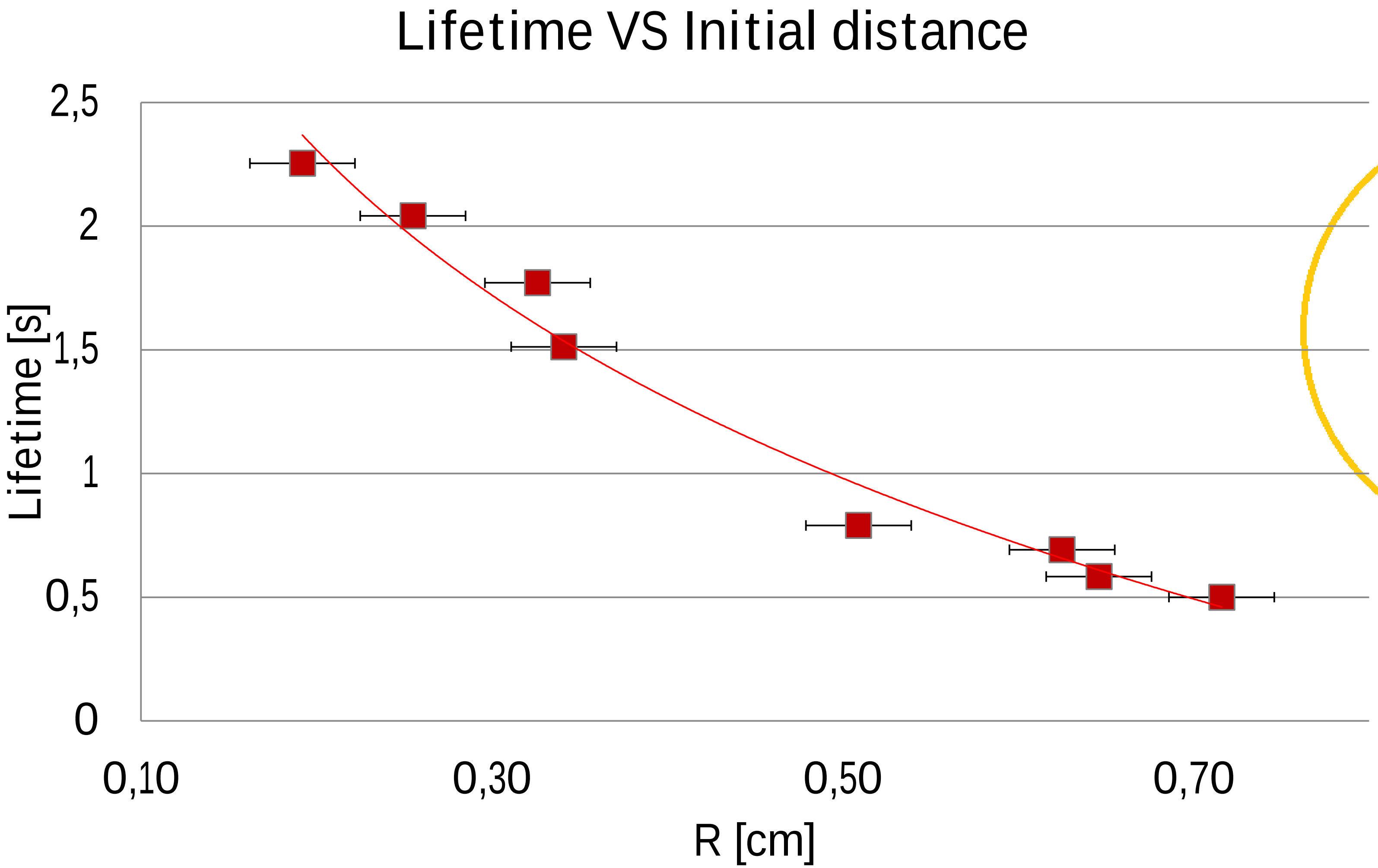
✓ 1. Friction
2. Frequency

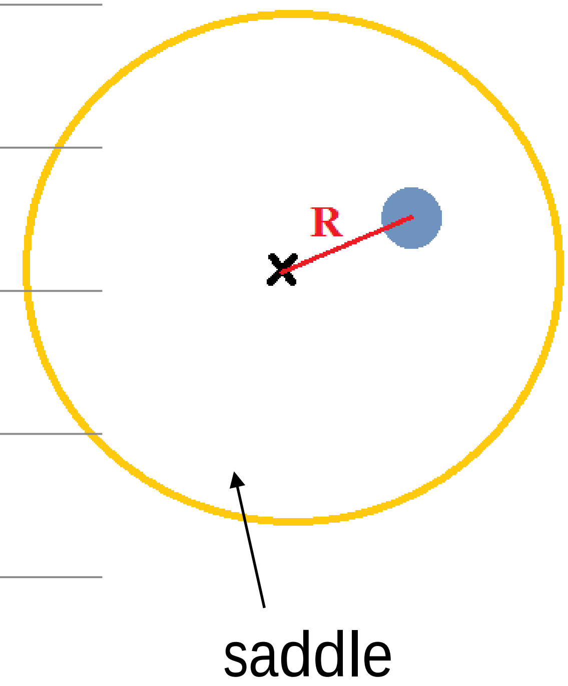
✓ 3. Moment of inertia
   – Higher moment of inertia = longer lifetime

? 4. Initial position

4. INITIAL POSITION

INITIAL POSITION

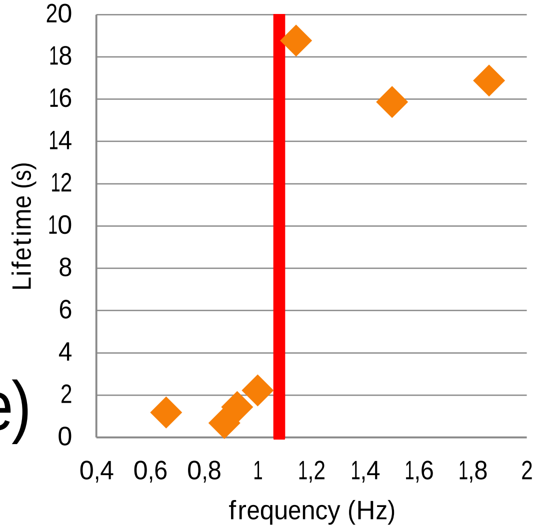




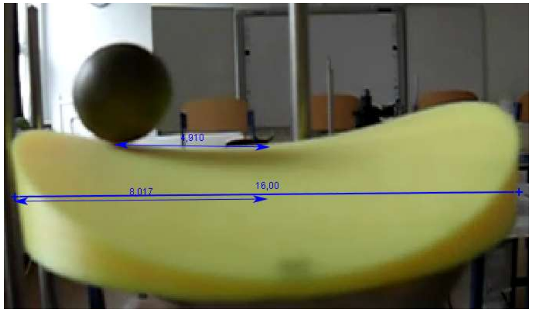
The further from the center we place the ball, the sooner it falls off

Conclusions

- Conditions under which the ball should be stable
  - Sufficient saddle rotation
    - Theory: Critical frequency fc
    - Experiment – never stable (rise od lifetime)
  - Avoiding centripetal force
    - Theory: No or low drag force
    - Our contribution: by backward rotation

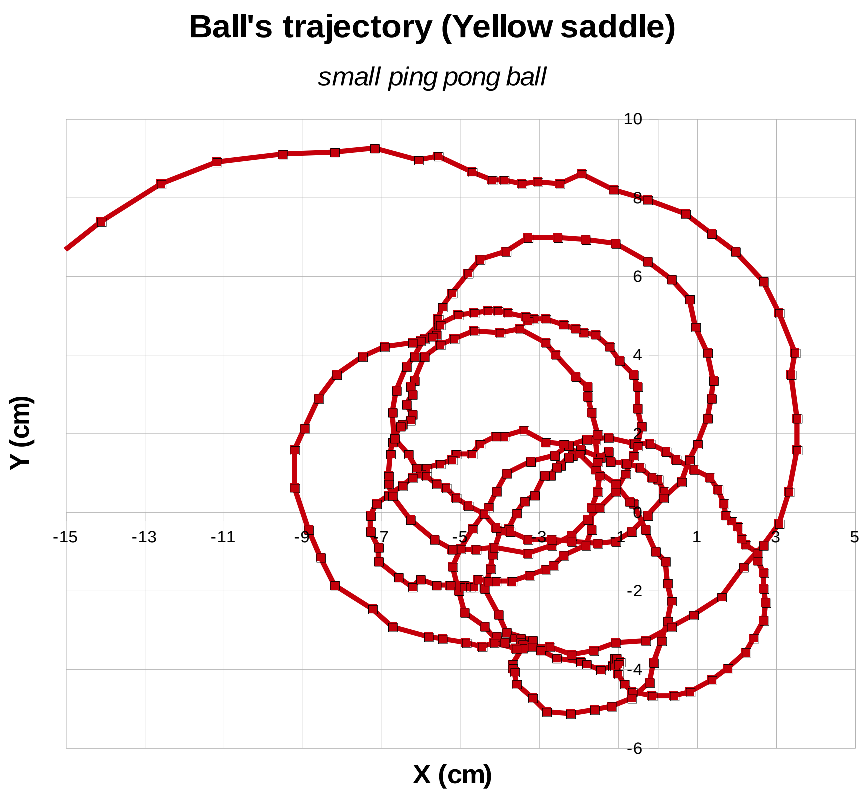


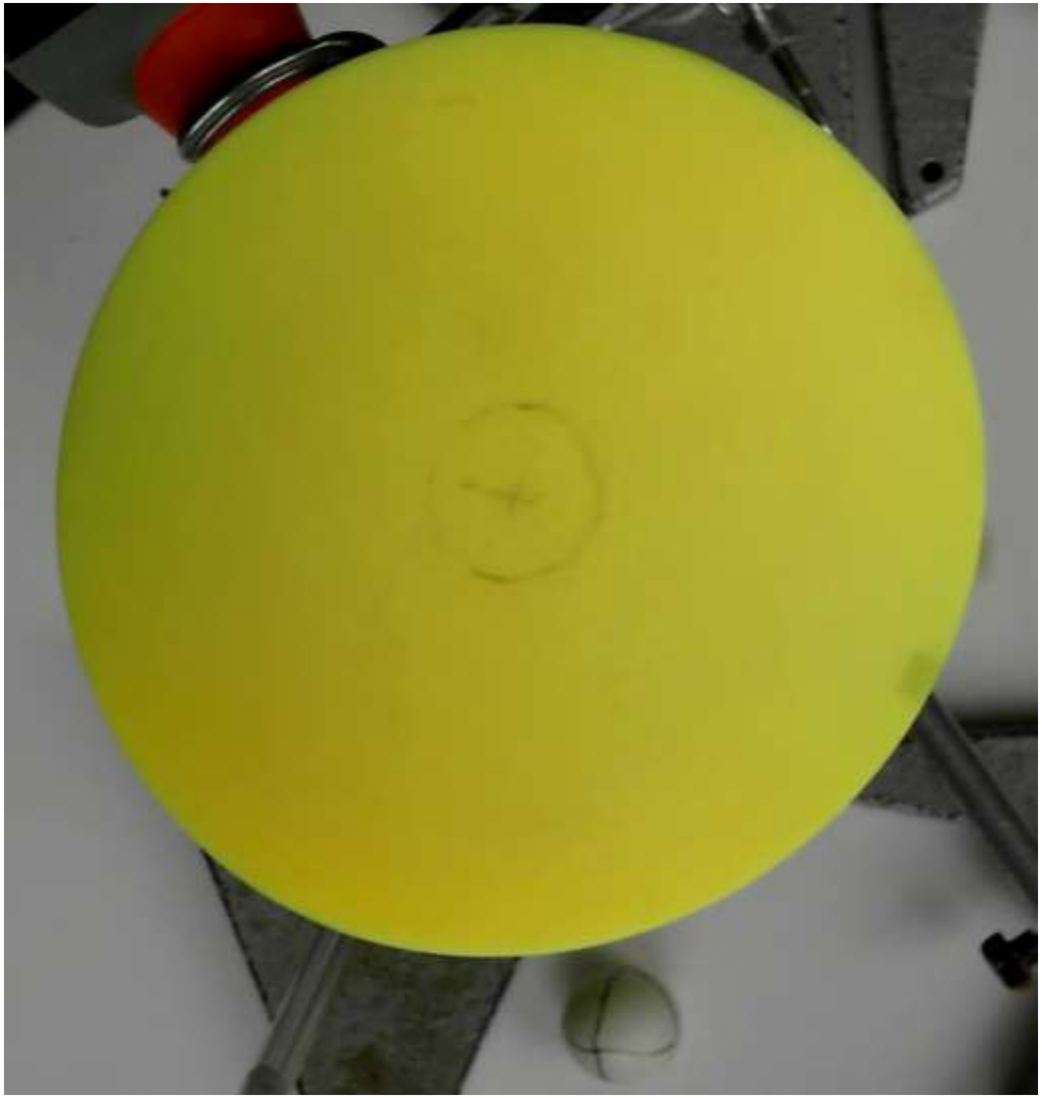
Conclusions



- Examined 'minuses' of the theory
  - Friction:
    - Theory: solution only for specific case
    - Our contribution: sufficient friction = more stable
  - Jumping (not mentioned in theory)
    - upper limit for frequency exists + estimation
  - Rotation of the ball (not mentioned)
    - Dependence on the moment of inertia

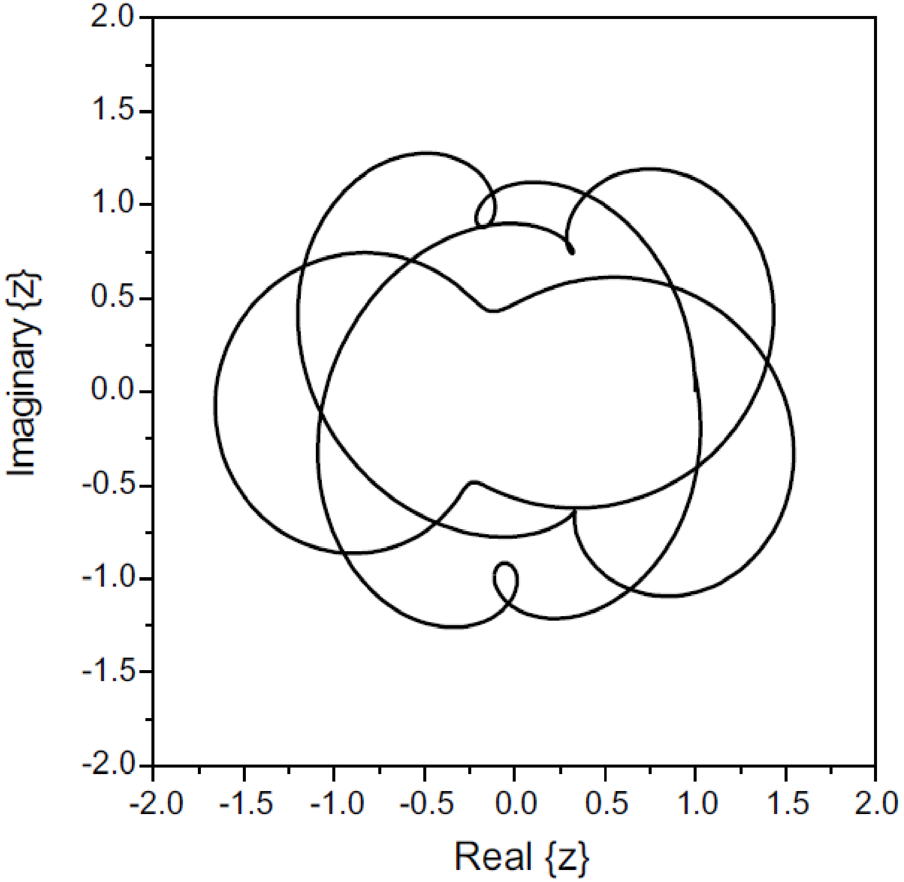
Thank you for your attention



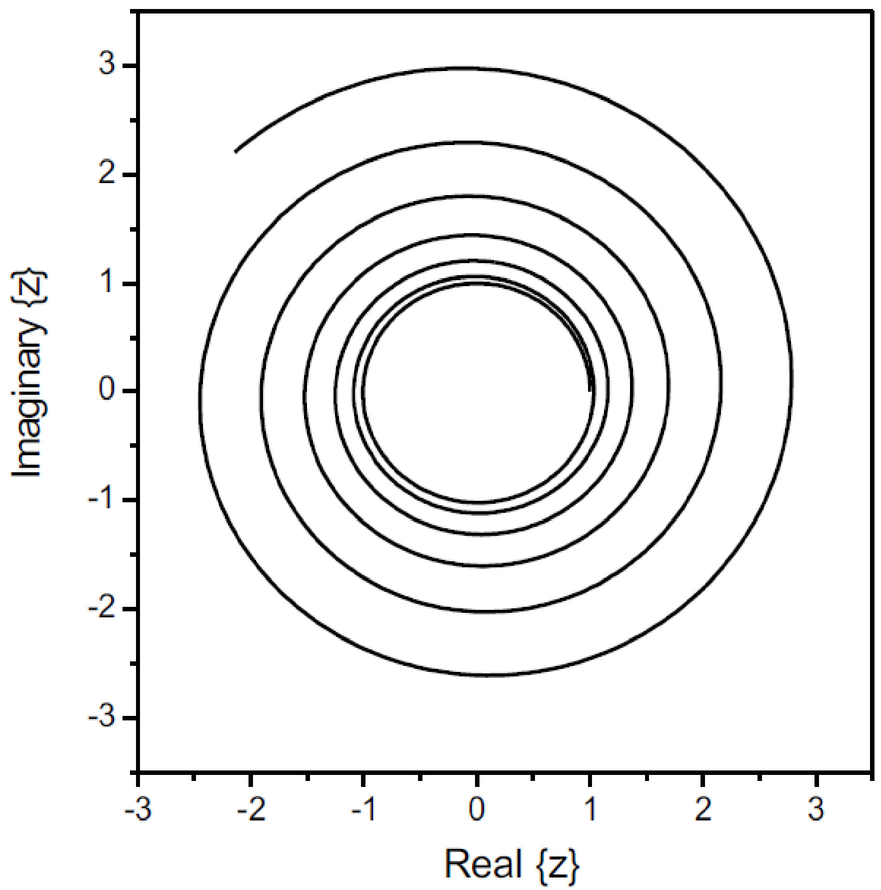


APPENDICES

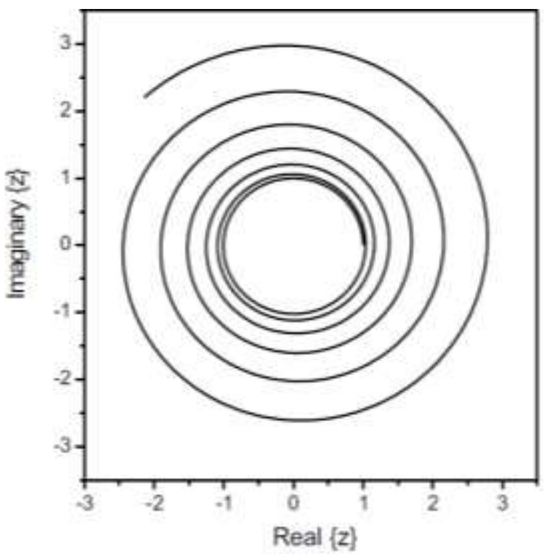
Prediction: Stability vs. Frequency



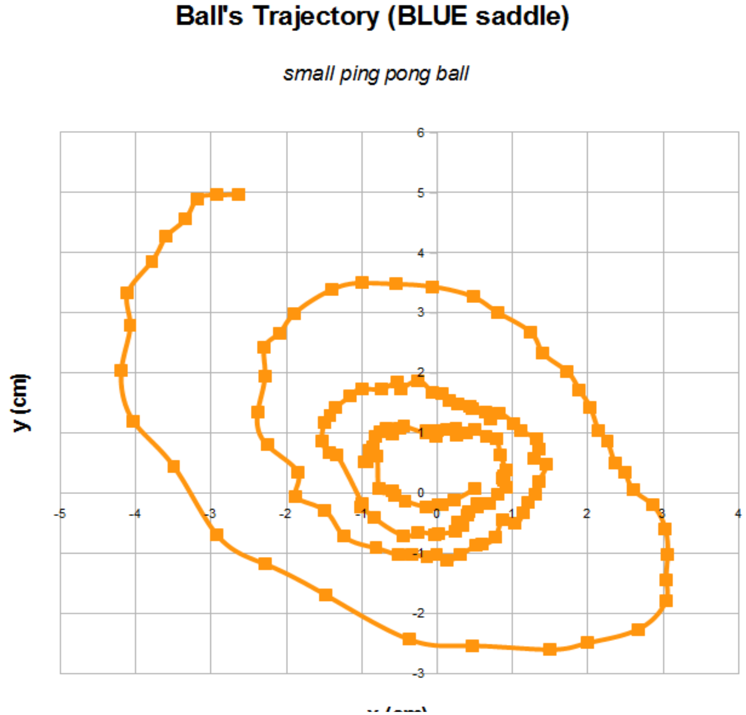
$$f \geq f_{critical}$$



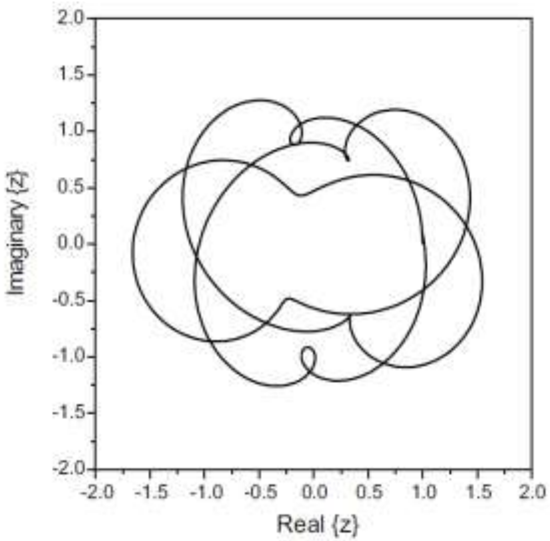
$$f < f_{critical}$$



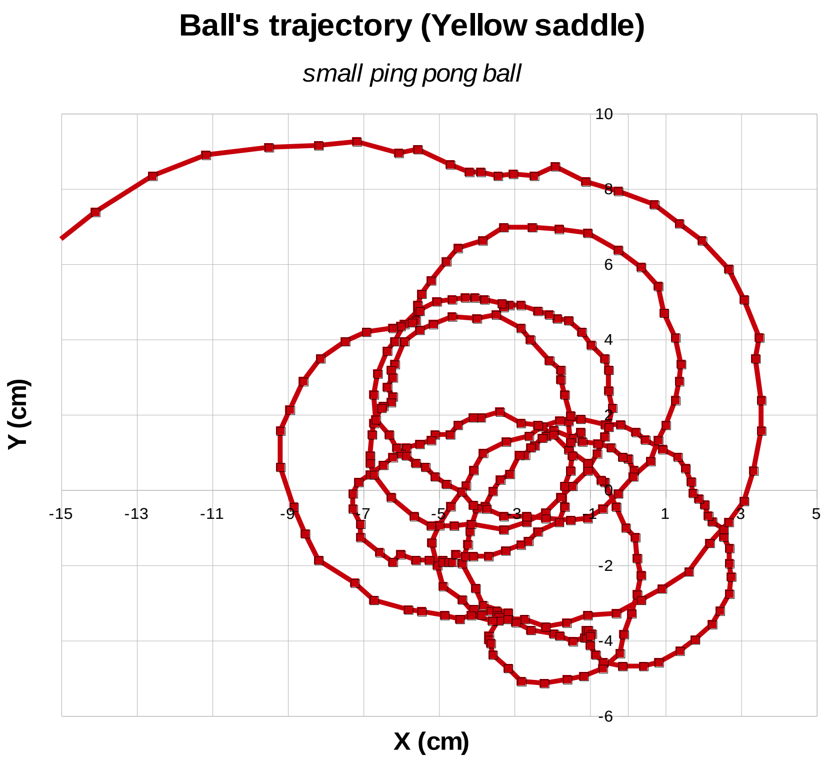
$f < f_C$



Similar to predicted behaviour



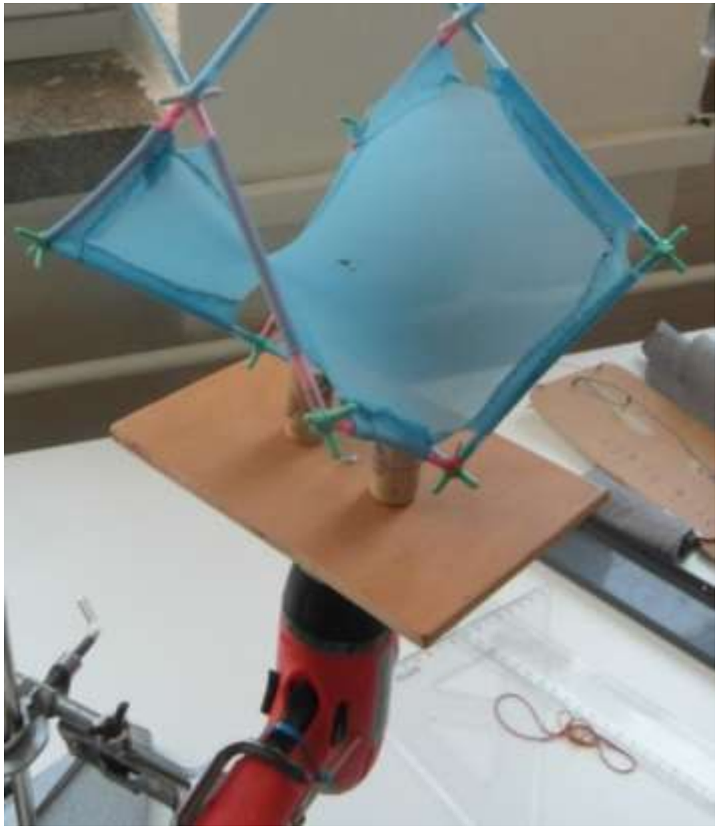
$f \geq f_C$



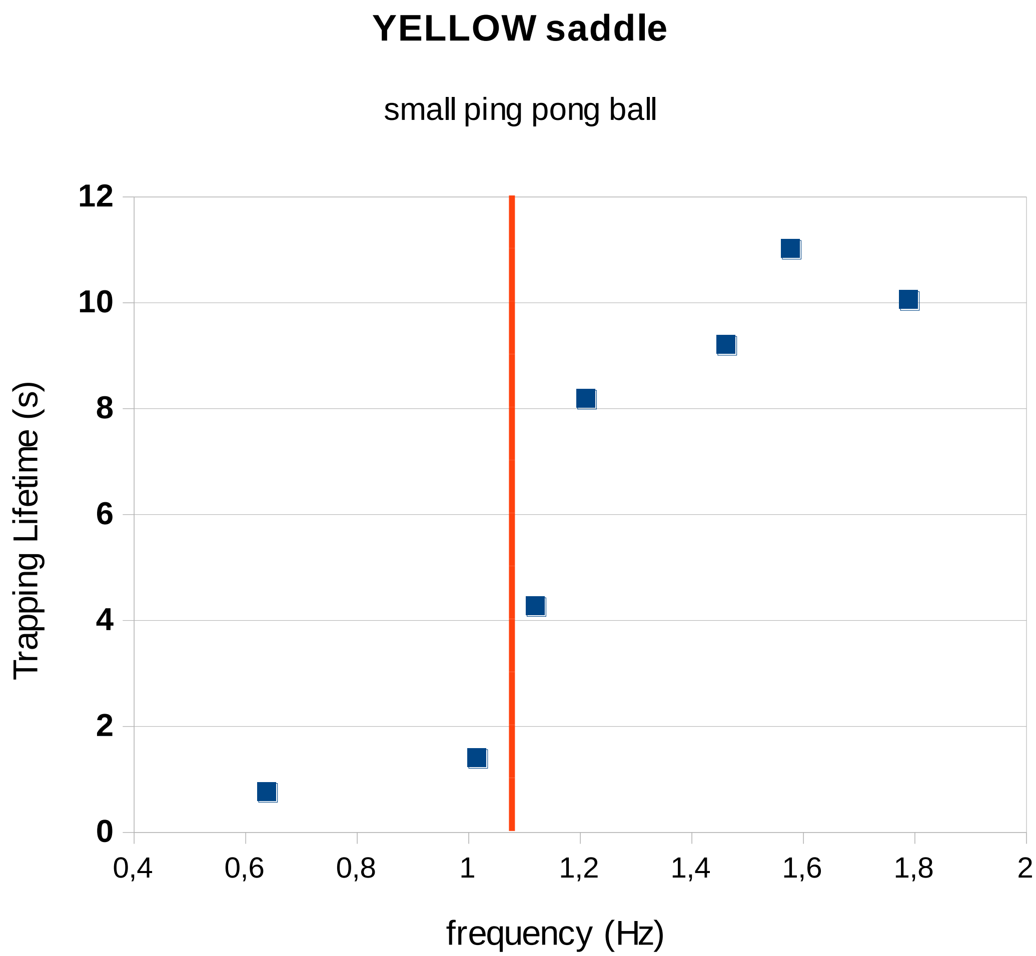
But always limited lifetime

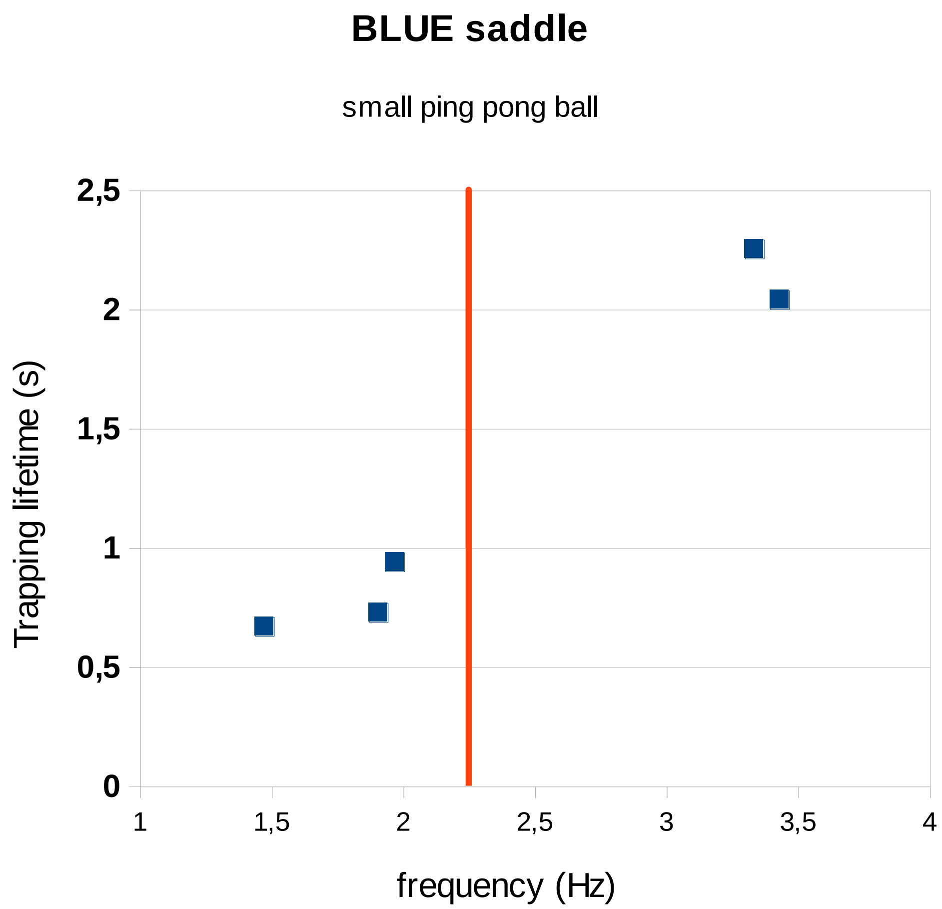
Apparatus: Rotation

- Rotation: driller
  - Frequency range: 0.6Hz – 3.7Hz

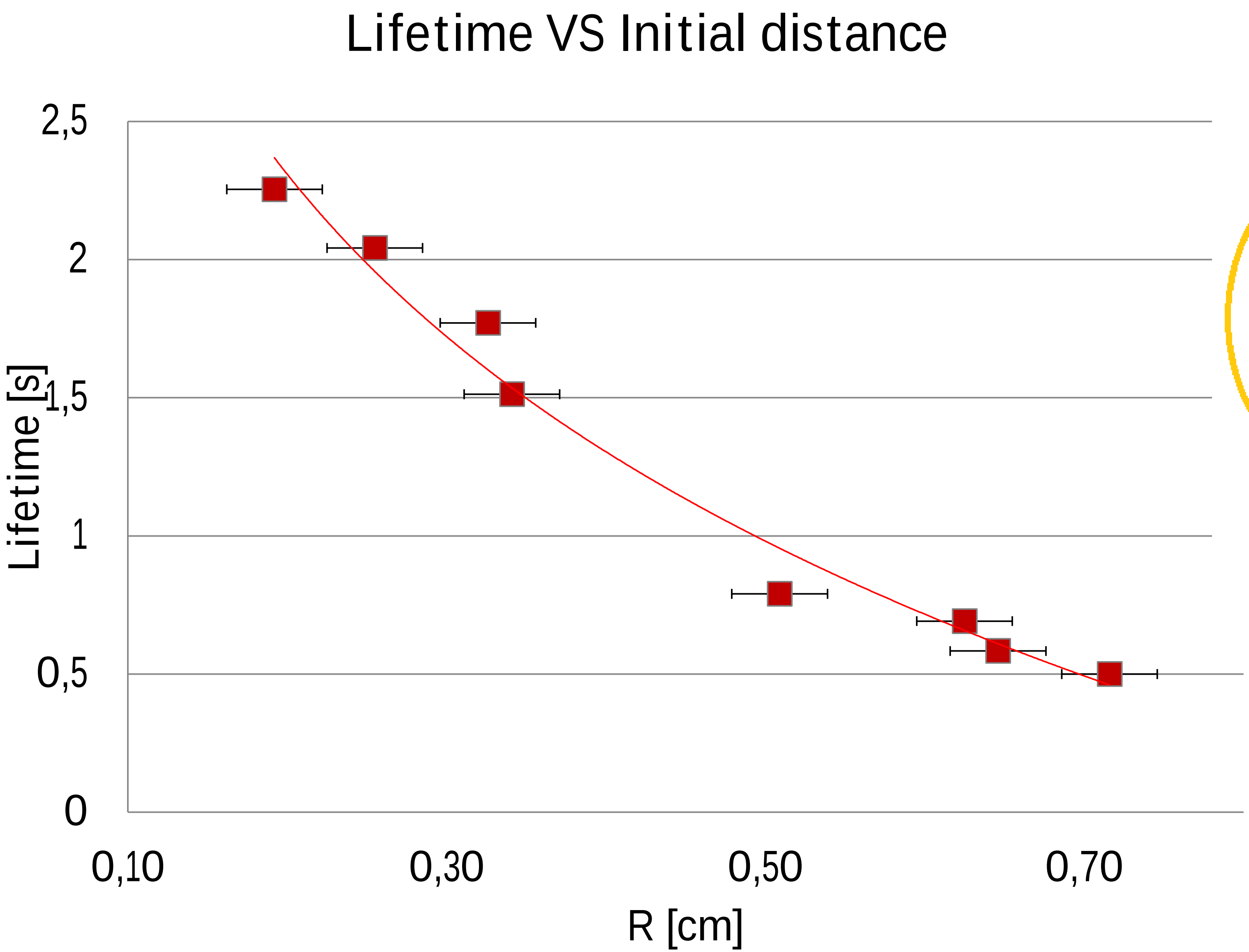


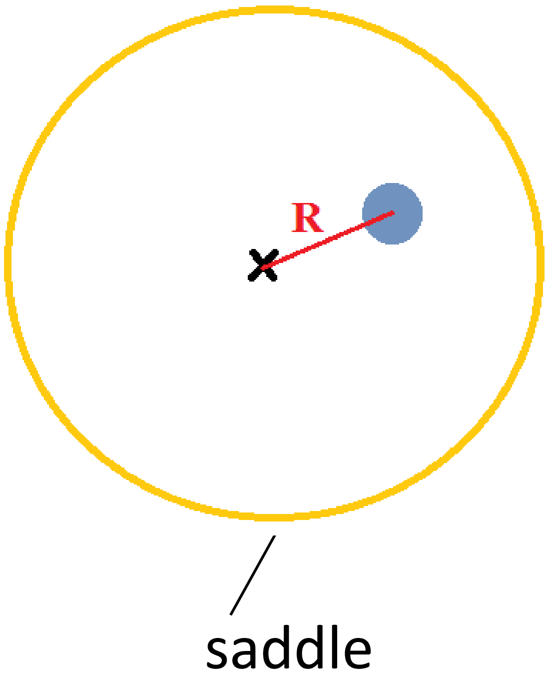
Small ping pong ball





INITIAL POSITION





Hollow balls' parameters

white ball:

r=2,9cm

m=46,79g

big red-yellow ball:

r=5,0cm

m=26,56g

ping-pong ball:

d= 3,94 cm

m= 2,46 g

α (friction coef.) = 0,19

small ping pong ball:

d= 3,76 cm

m= 3,16 g

big orange ball:

r= 3,26 cm

m= 6,93g

α (friction coef.) = 0,25

Hollow balls' parameters

Big green ball:

r= 3,26 cm

m= 13,30g

α (friction coef.) = sufficient

small green ball:

r= 1,77 cm

m= 17,72g

Small metal ball:

r=0,63cm

m=8,39g

Big metal ball:

r= 1,01 cm

m= 35,79g

Sufficient friction (no slipping)

No friction

- 2.

$$m\vec{a} = m\vec{g} + \vec{N} + \vec{I}$$

Sufficient friction

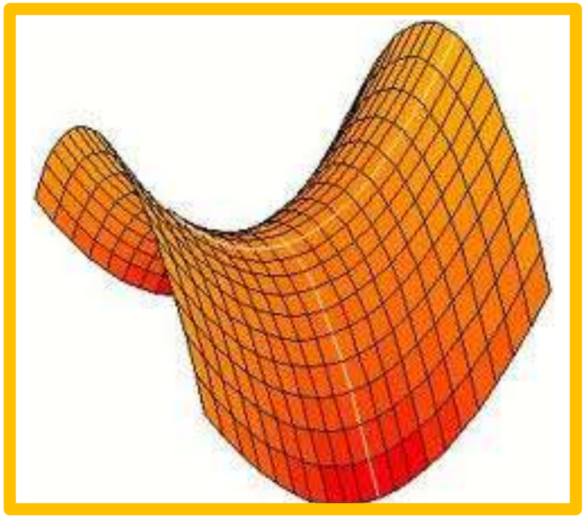
$$m\vec{a} = m\vec{g} + \vec{N} + \vec{I} + \vec{F}$$

$$J\vec{\varepsilon} = R\vec{F} \times \vec{n}$$

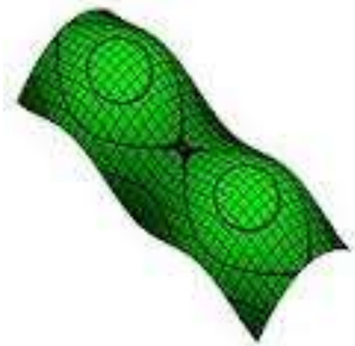
$$\vec{\omega} = \frac{\vec{n} \times \vec{v}}{R}$$

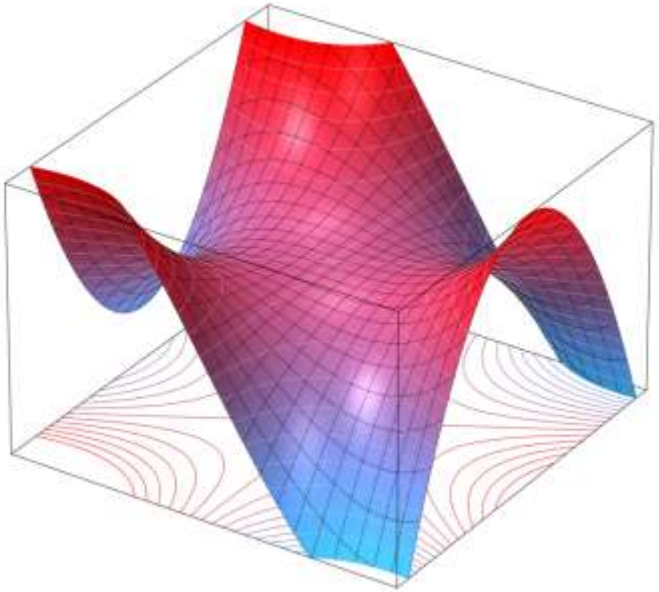
Saddle

- Convex in one direction; concave in the other
- Various saddle types:



$$z = k(x^2 - y^2)$$

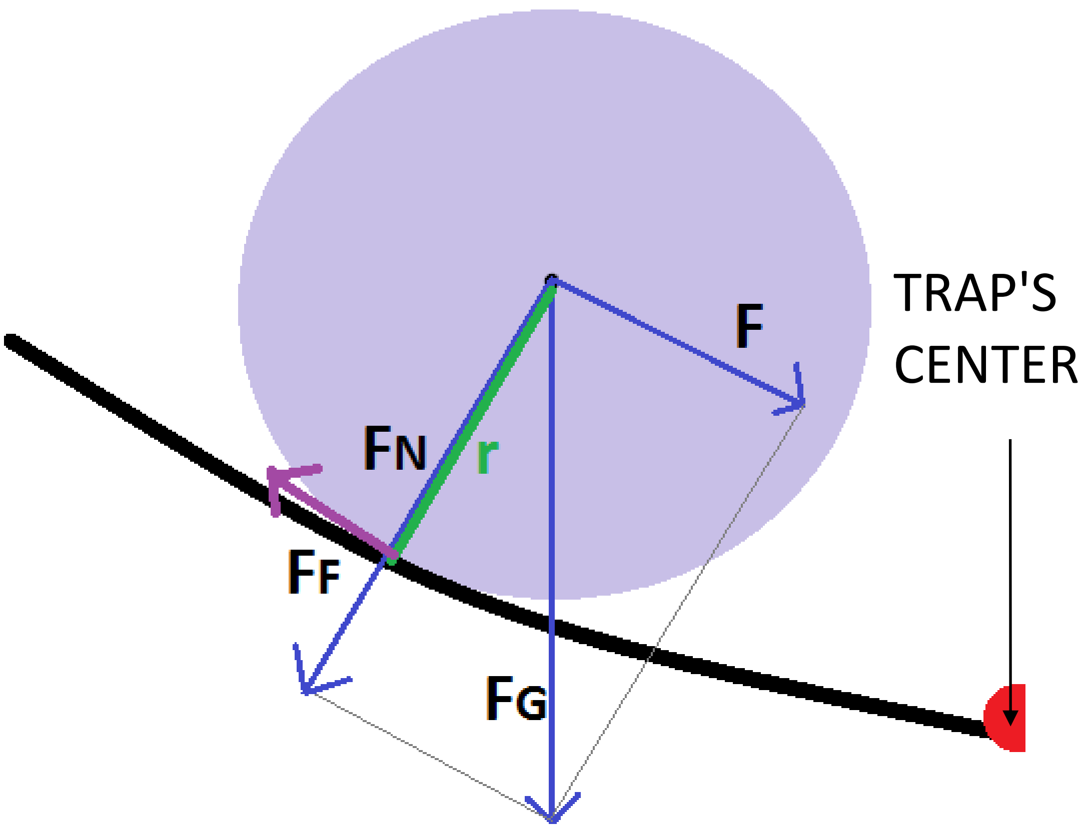




- Convenient for mathematical description

POINT MASS vs BALL

$$M = Fr = J\,\epsilon$$



DEEPER SADDLE

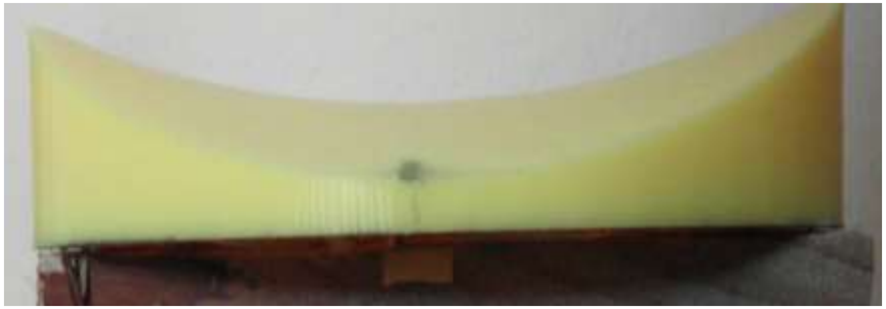
↓

THE **BIGGER** COMPONENT OF $\mathbf{F_G}$ CAUSES THE **TORQUE**

↓

THE **LESS** IT RESEMBLES POINT MASS

'DEEP' vs 'SHALLOW' saddle



MAXIMUM LIFETIME:

4,07 s

MAXIMUM LIFETIME:

23,91 s

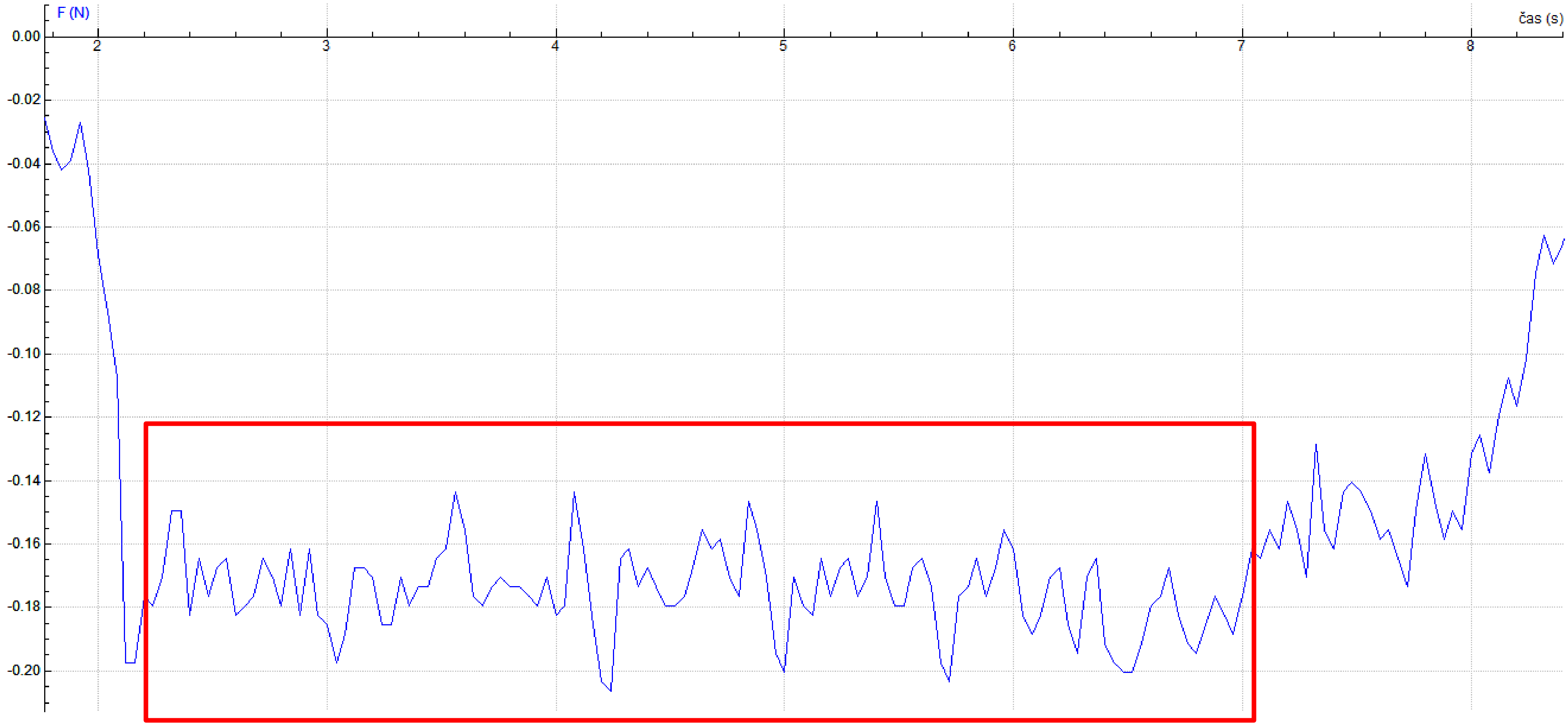
THE **SMALLER** THE SADDLE'S **HEIGHT** IS, THE **MORE STABLE** THE BALL IS

FRICTION

Friction

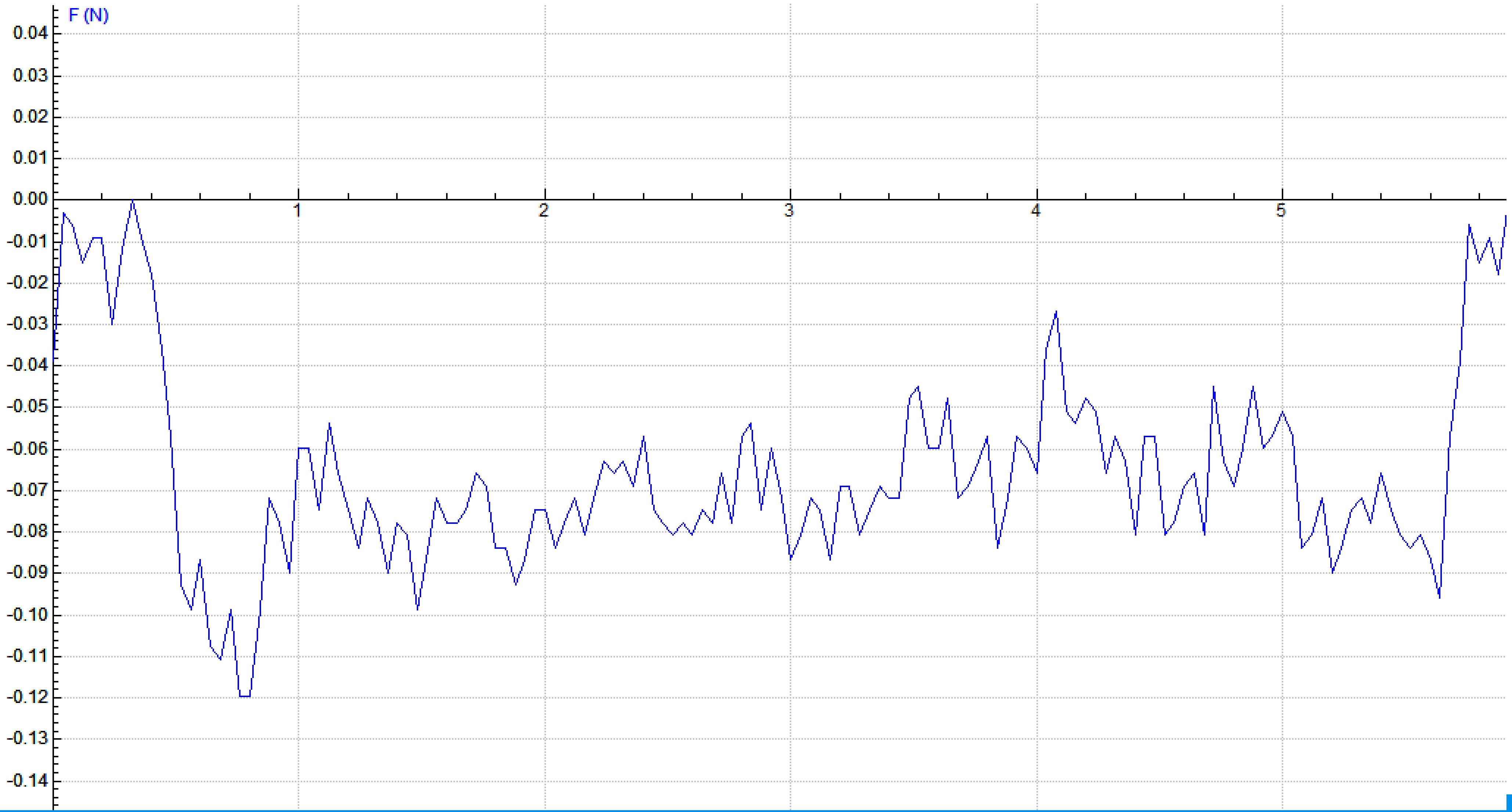
– blue saddle

$$f = \frac{\overline{F_{friction}}}{F_N}$$

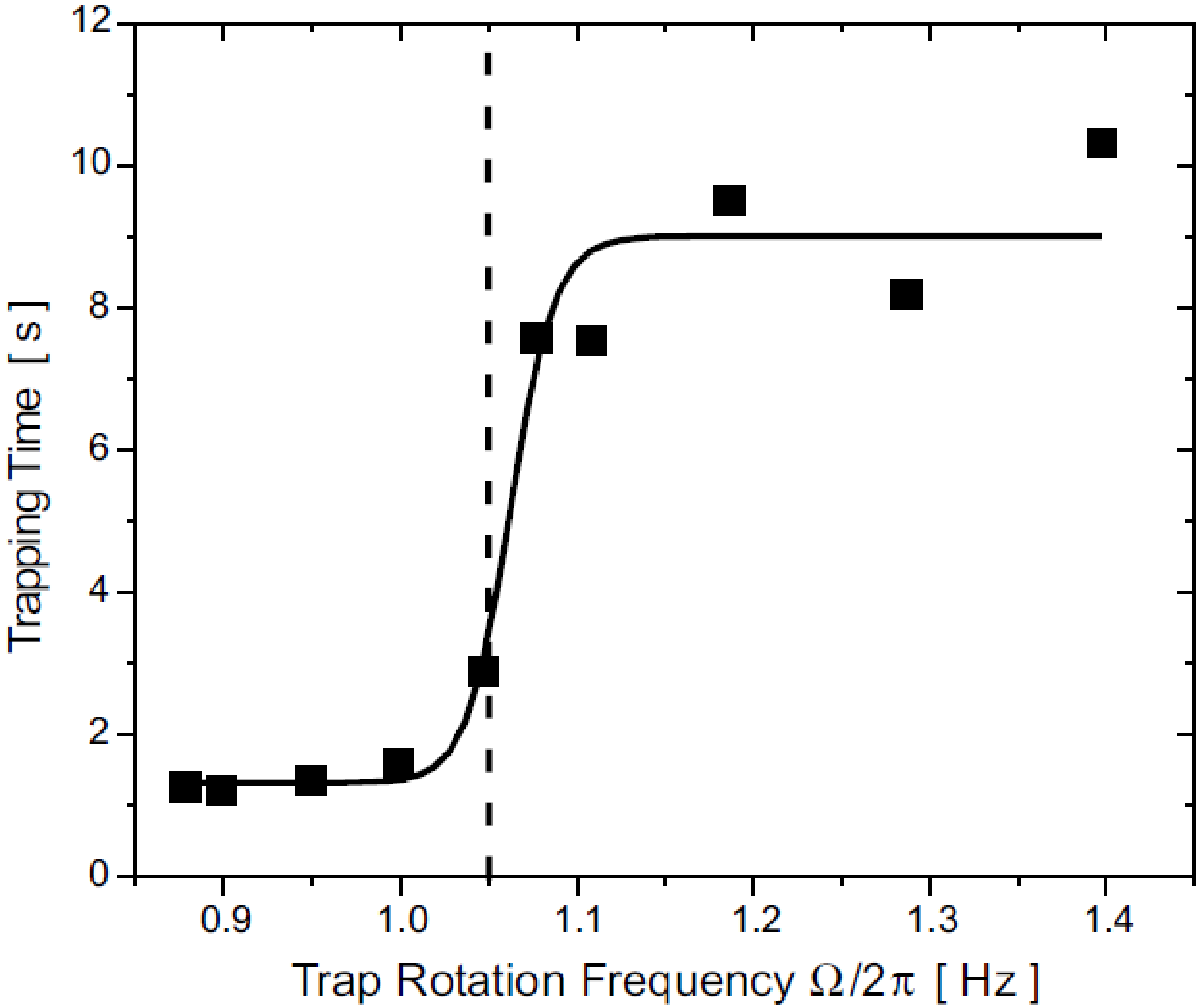


Friction

–blue saddle+ WATER



TRAPPING LIFETIME



THOMPSON'S ARTICLE

THEORY

Theory

Gravitational potential:

- assigned to the rotating frame (fixed to U)

$$U(x',y')=\frac{mgh_0}{r_0^2}\left(x'^2-y'^2\right)$$

- converted to the laboratory frame:

$$U(x,y)=\frac{mgh_0}{r_0^2}[(x^2-y^2)\cos(2\Omega t)+2xy\sin(2\Omega t)]$$

using the following formula $F = -\nabla U$
yields

$$\frac{\partial^2 x}{\partial t^2} = \frac{2\mathrm{mgh}_0}{r_0^2}[-x\cos(2\Omega t) - y\sin(2\Omega t)]$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{2\mathrm{mgh}_0}{r_0^2}[y\cos(2\Omega t) - x\sin(2\Omega t)]$$

using dimensionless parameters $\tau = \Omega t$ and $q = \frac{gh_0}{r_0^2 \Omega^2}$

converting to the complex plane $(z = x + iy)$,

the 2 equations are reduced into:

$$\frac{\partial^2 z}{\partial \tau^2} + 2\mathrm{q} * e^{i2\tau} = 0$$

Applying another substitution $z(\tau) = f(\tau)e^{i\tau}$ *yields the solution:*

$$f(\tau) = Ae^{+\beta_+\tau} + Be^{-\beta_+\tau} + Ce^{+\beta_-\tau} + De^{-\beta_-\tau}$$

where A,B,C,D are real parameters depending on initial conditions and

$$\beta_\pm = \sqrt{\pm 2|q| - 1}$$

β± ∈ R - {0} => result will diverge in any case => particle is trapped only if β± ∈ I, thus

$$2|q| \le 1 \quad \Rightarrow \quad q \le 0{,}5$$

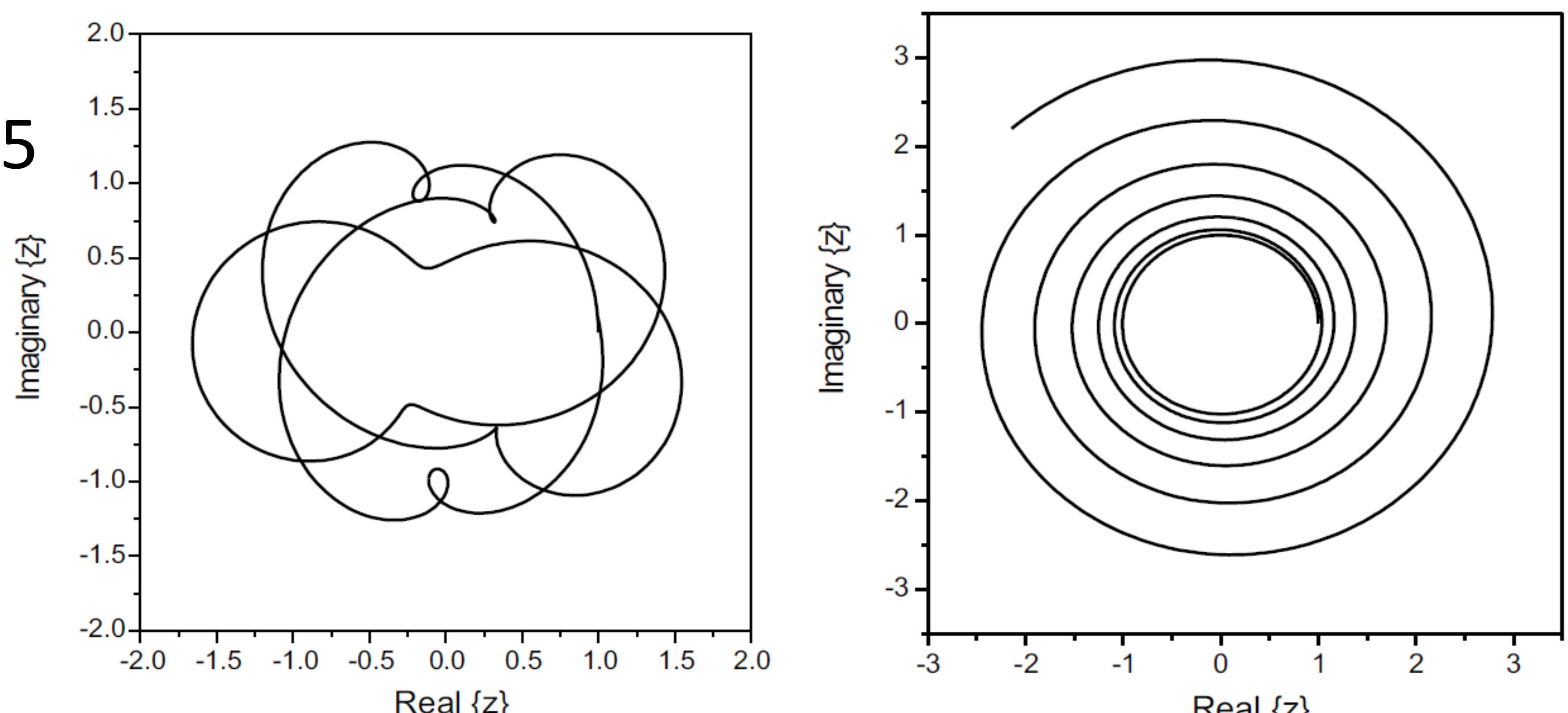
$$q = \frac{gh_0}{\Omega^2 r_0^2} \le 0{,}5$$

- **The condition for stability is:**
- $$\Omega \geq \frac{\sqrt{2gh_0}}{r_0} \longrightarrow f \geq \frac{\sqrt{2gh_0}}{2\pi r_0}$$

regardless of initial position of the ball

q>0,5

q≤0,5



Limited trapping lifetime

1) unstable trapping parameters (q>0,5)

$$T_L = \frac{1}{\beta_+ \Omega} \ln\left(\frac{r_0}{R}\right)$$

$$T_L = \frac{1}{\sqrt{2q-1}} \ln\left(\frac{r_0}{R}\right)$$

2) friction (q≤0,5)

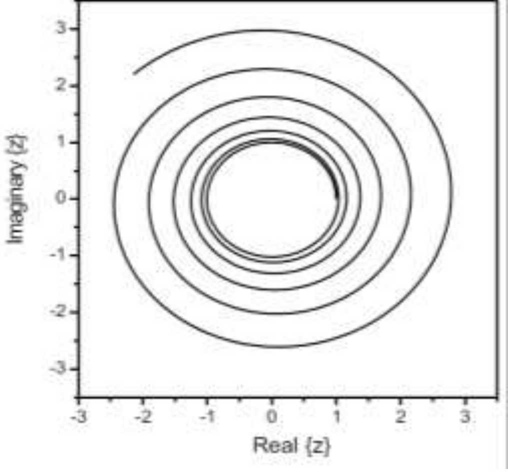
$$T_L = \frac{1}{\beta\Omega} \ln\left(\frac{r_0}{R}\right)$$

– β ~ friction coefficient

BALL'S PATH

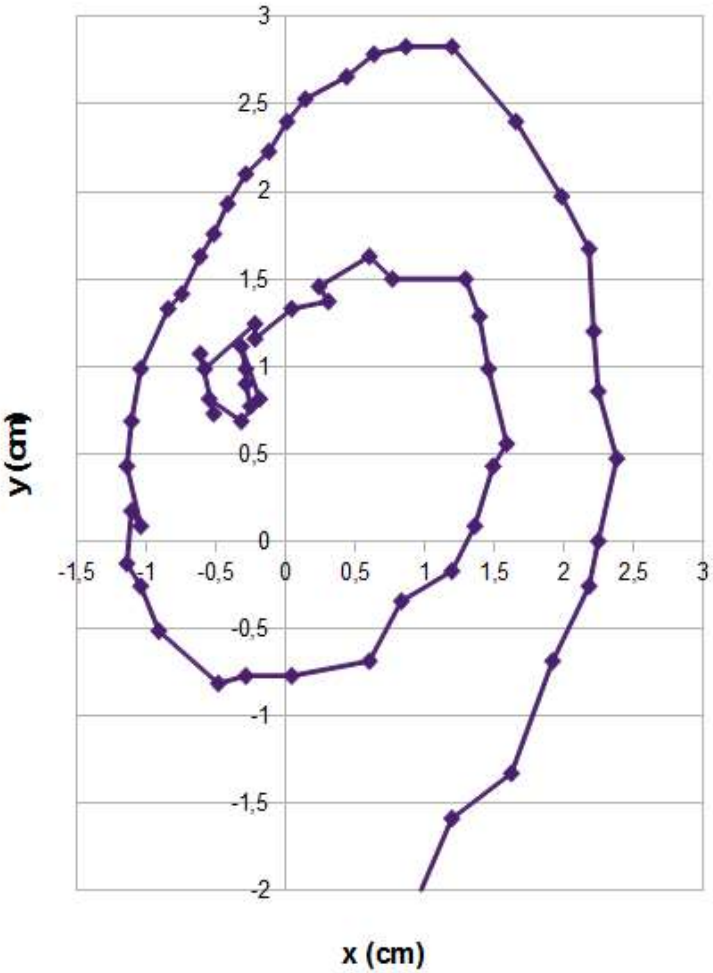
BALL'S PATH

– short trapping lifetimes



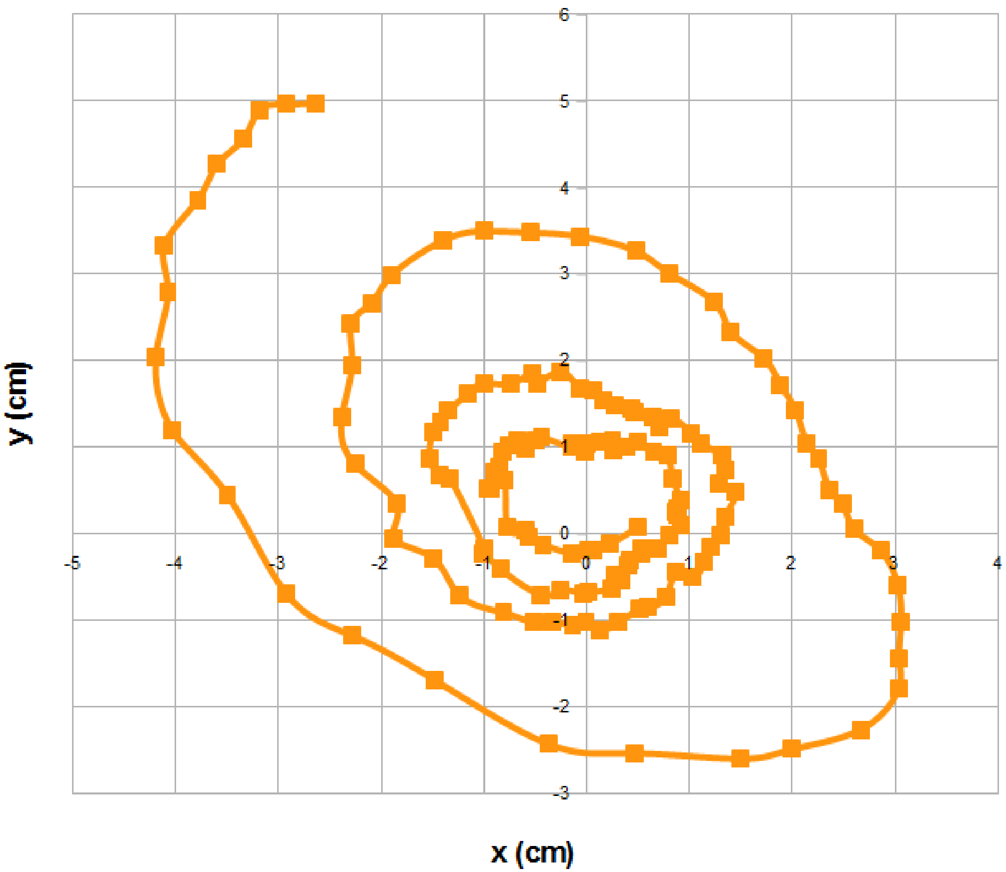
Ball's path

big purple ball



Ball's Trajectory (BLUE saddle)

small ping pong ball



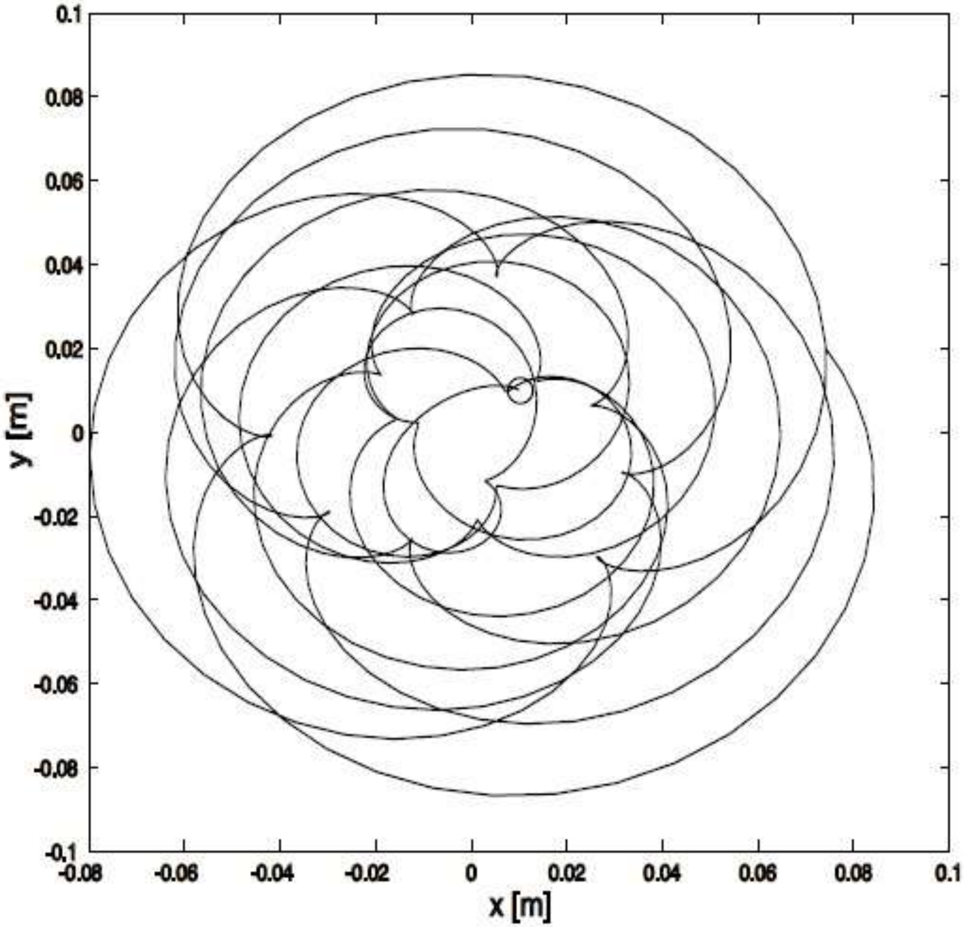
video

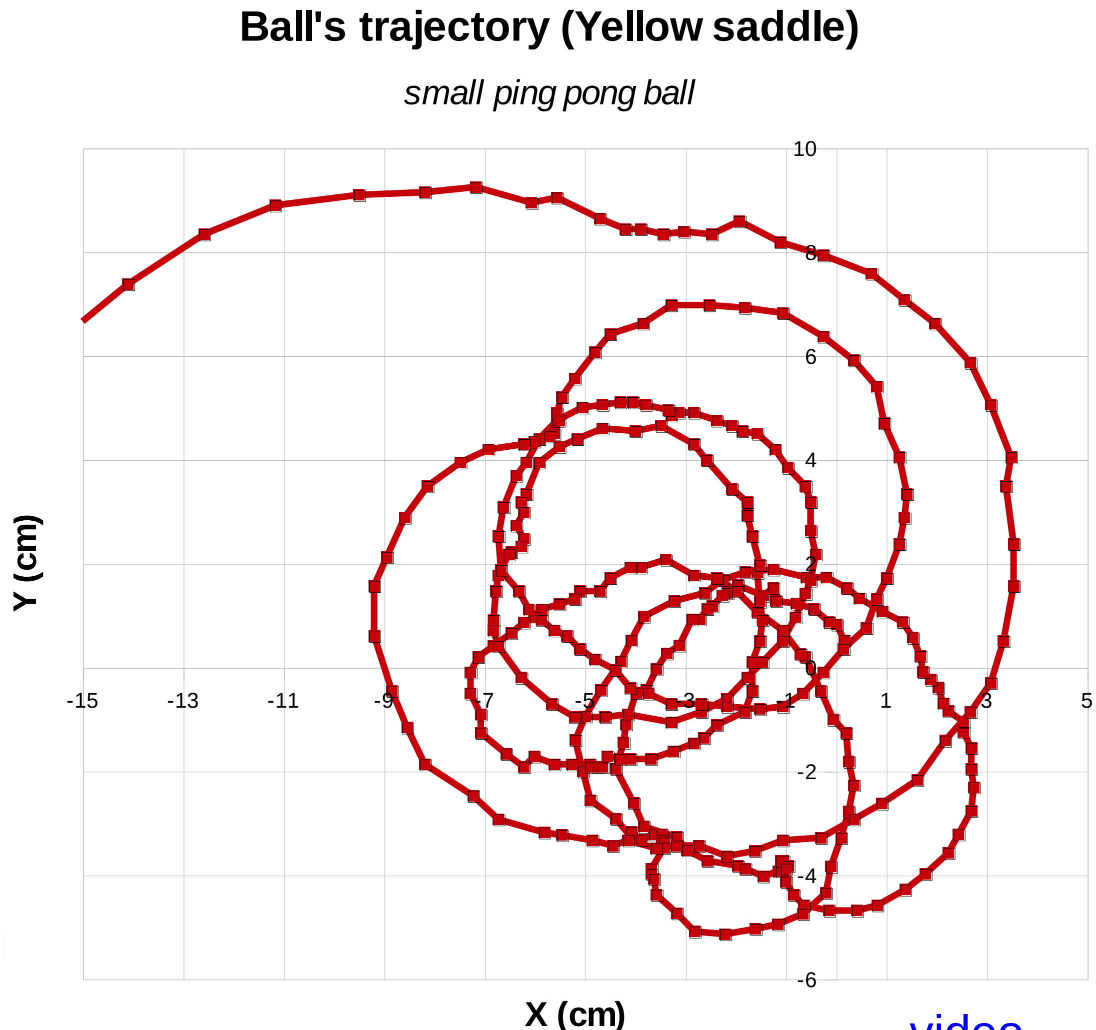
BALL'S MOTION

– <u>longer</u> trapping lifetimes

OUR DIAGRAM

THEORETICAL DIAGRAM





video

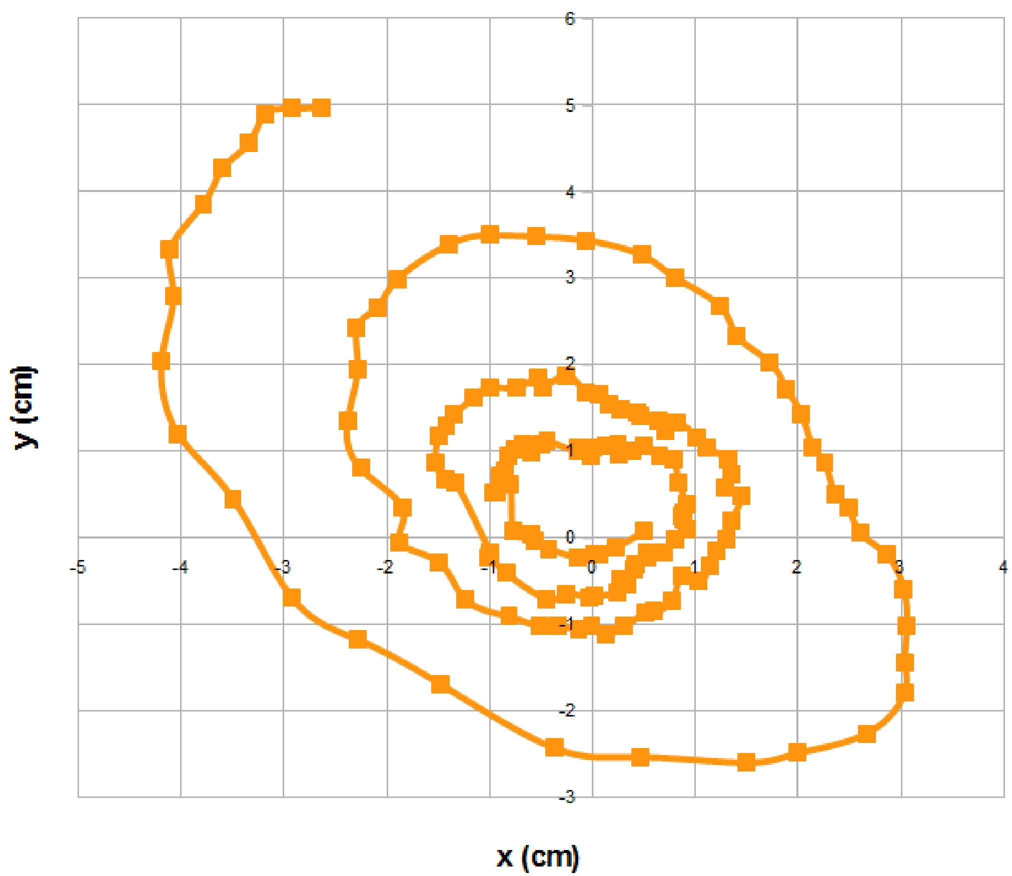
KOCH'S ARTICLE

11:35

STABLE TRAPPING PARAMETERS

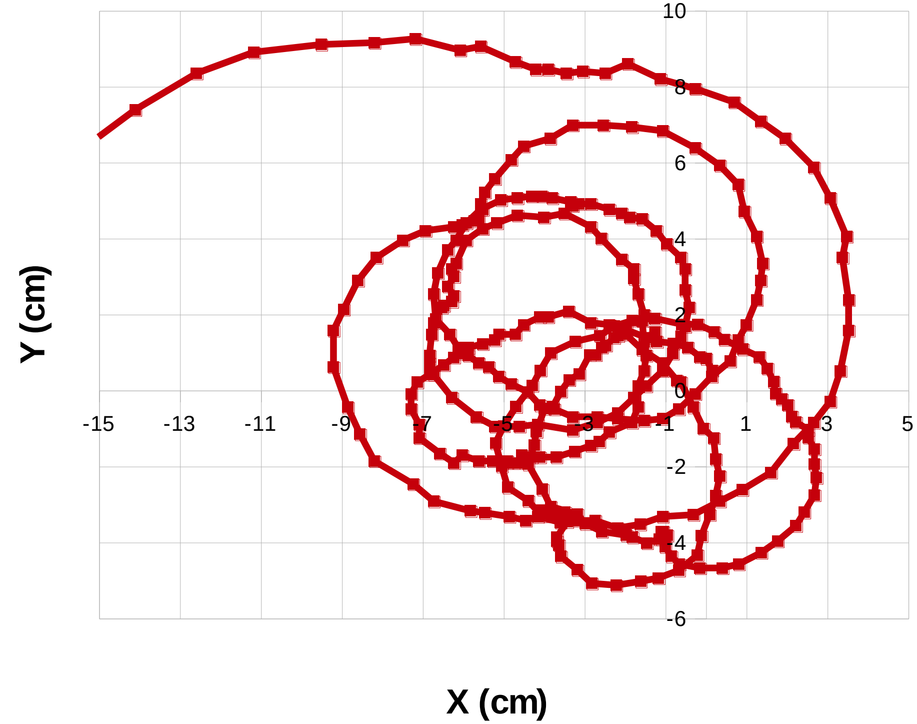
Ball's Trajectory (BLUE saddle)

small ping pong ball

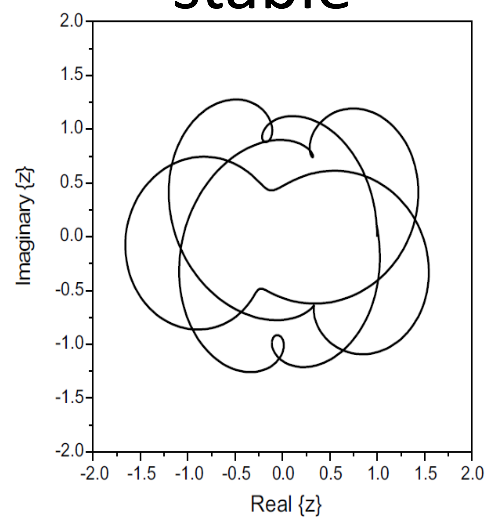


Ball's trajectory (Yellow saddle)

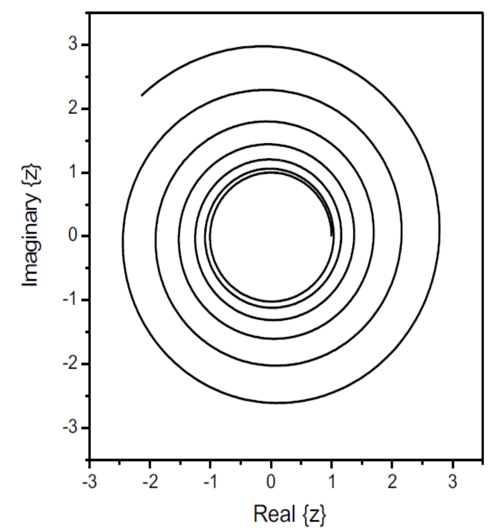
small ping pong ball



stable

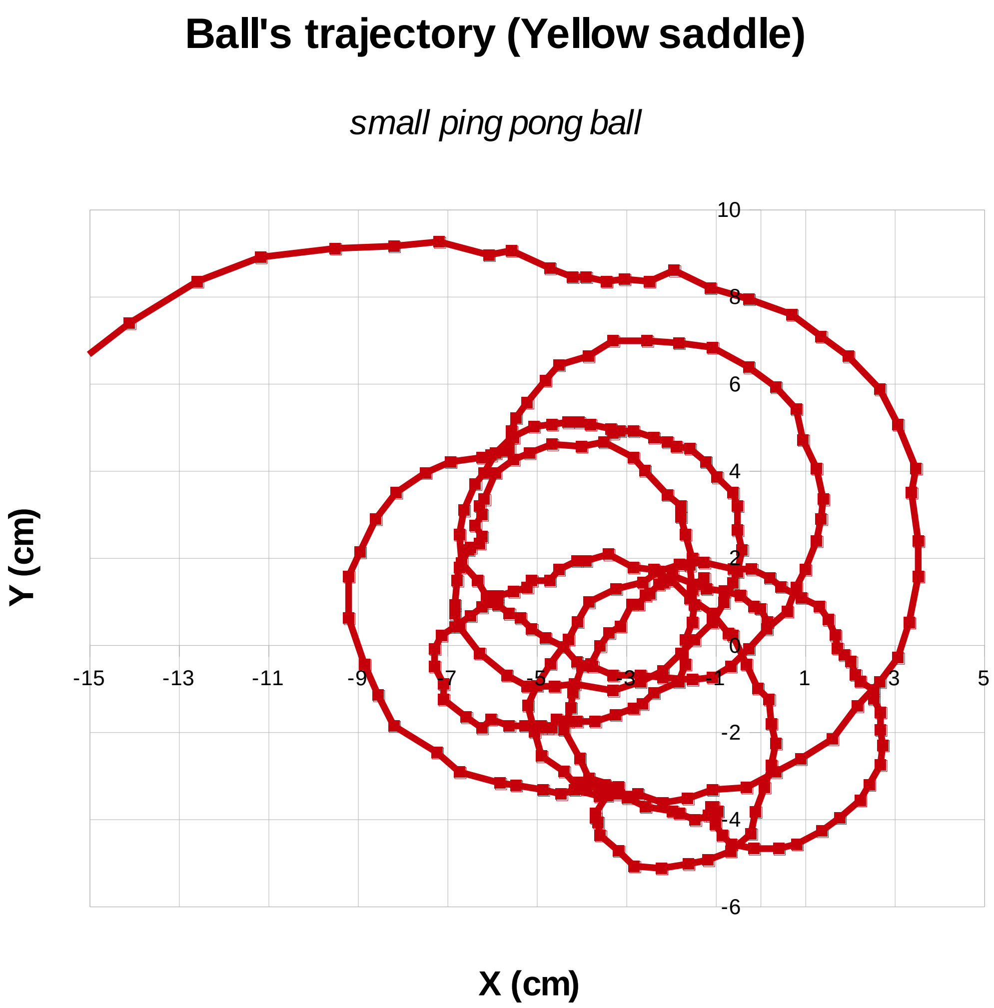


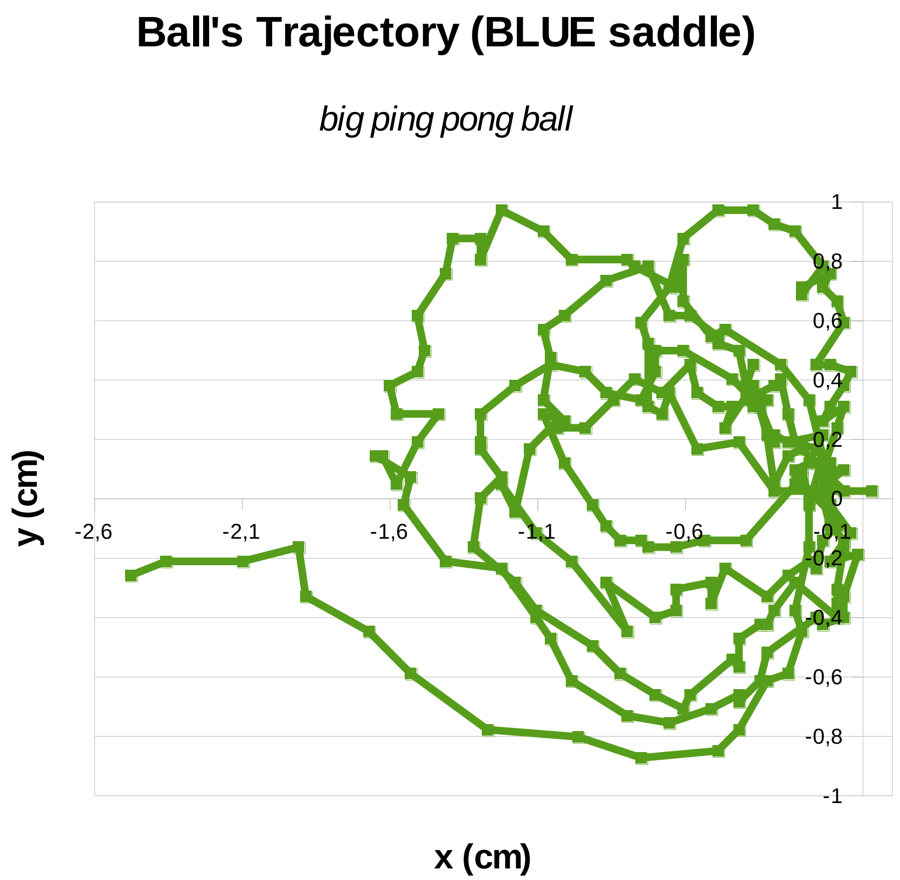
unstable



BALL'S MOTION

– longer trapping lifetimes

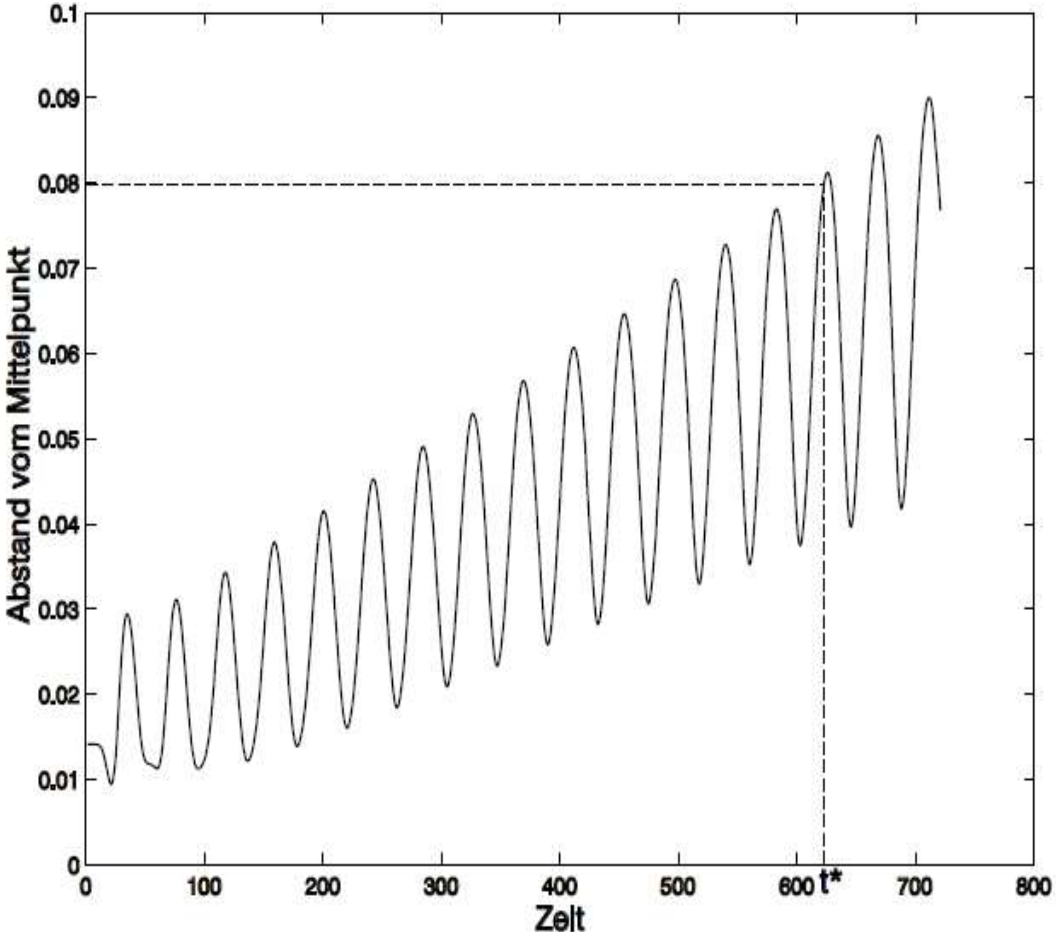




BALL'S PATH

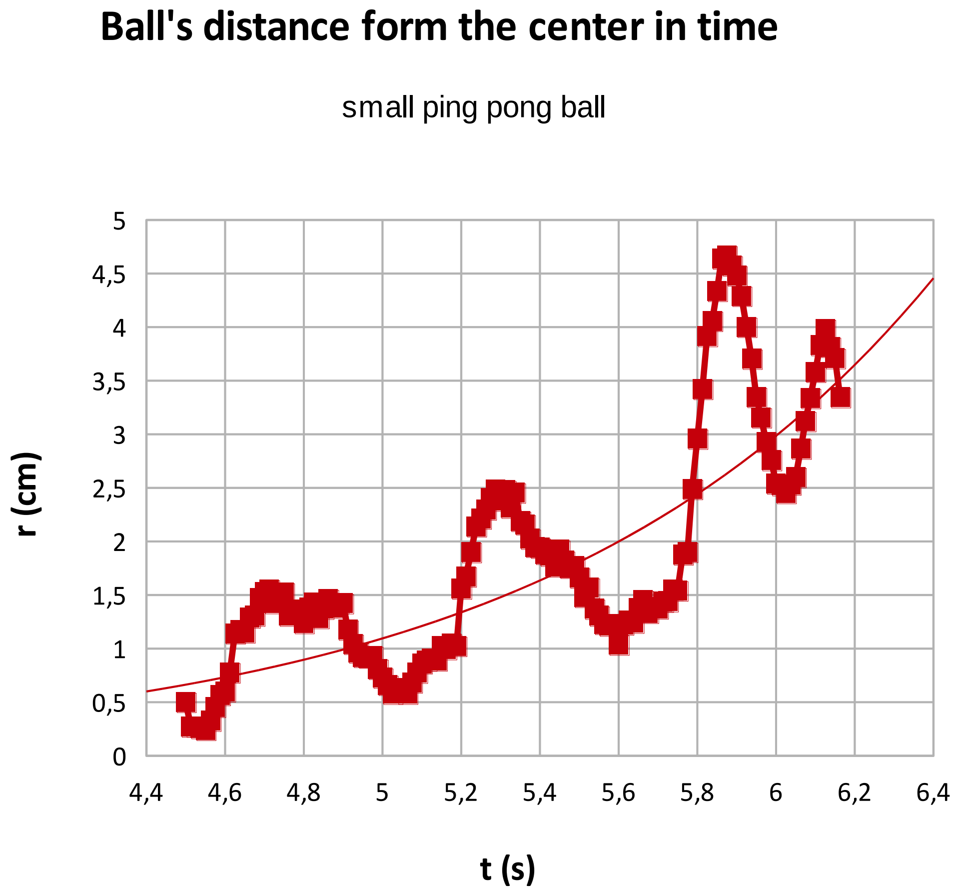
– <u>longer</u> trapping lifetimes

THEORETICAL DIAGRAM

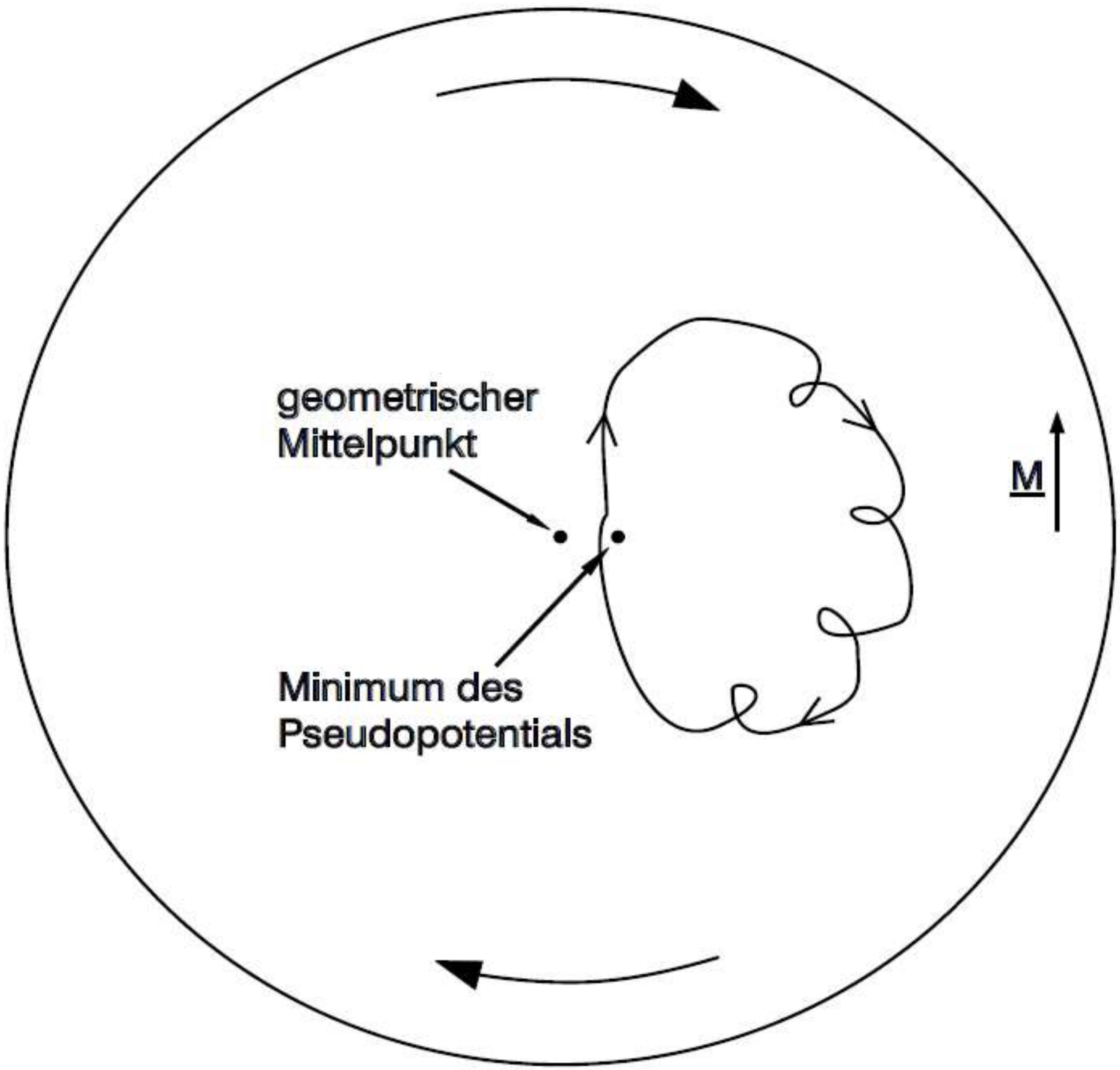


KOCH'S ARTICLE

OUR DIAGRAM



ANGLE


geometrischer
Mittelpunkt
Minimum des
Pseudopotentials
M

$$\beta_{\pm} = \sqrt{\pm 2|q| - 1}$$

$\beta \pm \in R - \{0\}$ => *result will diverge in any case =>*

particle is trapped only if $\beta \pm \in I$, *thus*

$$2|q| \leq 1 \quad => \quad q \leq 0{,}5$$

$$q = \frac{gh_0}{\Omega^2 r_0^2} \leq 0{,}5$$

The condition for stability is:

$$\Omega \geq \frac{\sqrt{2gh_0}}{r_0} \longrightarrow f \geq \frac{\sqrt{2gh_0}}{2\pi r_0}$$

Theory (Thompson's article)

Gravitational potential:

- assigned to the rotating frame (fixed to U)

$$U(x', y') = \frac{mgh_0}{r_0^2}(x'^2 - y'^2)$$

- converted to the laboratory frame:

$$U(x, y) = \frac{mgh_0}{r_0^2}[(x^2 - y^2)\cos(2\Omega t) + 2xy\sin(2\Omega t)]$$

$$\beta_{\pm} = \sqrt{\pm 2|q| - 1}$$

$\beta_{\pm} \in R - \{0\}$ => result will diverge in any case => particle is trapped only if $\beta_{\pm} \in I$, thus

$$q = \frac{gh_0}{\Omega^2 r_0^2} \leq 0{,}5$$

<u>The condition for stability is:</u>

$$\Omega \geq \frac{\sqrt{2gh_0}}{r_0} \longrightarrow f \geq \frac{\sqrt{2gh_0}}{2\pi r_0}$$

SOURCES

R.I. Thompson, T.J. Harmon, and M.G. Ball:
The rotating-saddle trap: a mechanical analogy to RF-electricquadrupoleion trapping?
(Can. J. Phys. Vol. 80, 2002)

Wolfgang Rueckner, Justin Georgi, Douglass Goodale, Daniel Rosenberg, David Tavilla:
Rotating saddle Paul trap
(American Journal of Physics 63, 186 (1995); doi: 10.1119/1.17983)

A. K. Johnson and J. A. Rabchuk:
A bead on a hoop rotating about a horizontal axis: A one-dimensional ponderomotive trap
(Citation: American Journal of Physics 77, 1039 (2009); doi: 10.1119/1.3167216)

Tobias Koch:
Konzeption und Aufbau einer mobilen Experimentiereinheit fur Schuleräprsentationen zum Thema Teilchenfallen

Stara prezentacia

Friction & Initial conditions

friction in equations → exponential growth

↓

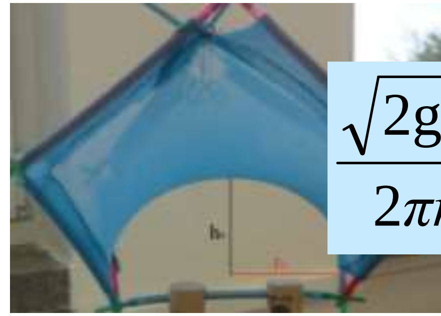
limited trapping lifetimetime T_L

2 reasons for lifetime limitation:

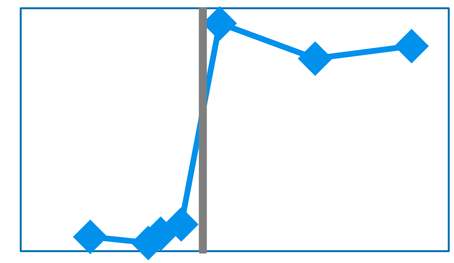
1) unstable trapping parameters $f < f_{critical}$

2) **friction**

Conclusion

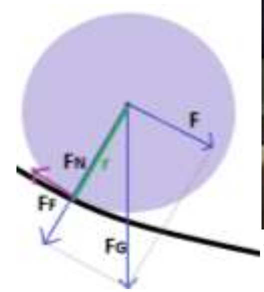


$$\frac{\sqrt{2gh_0}}{2\pi r_0} = f_{CRITICAL}$$



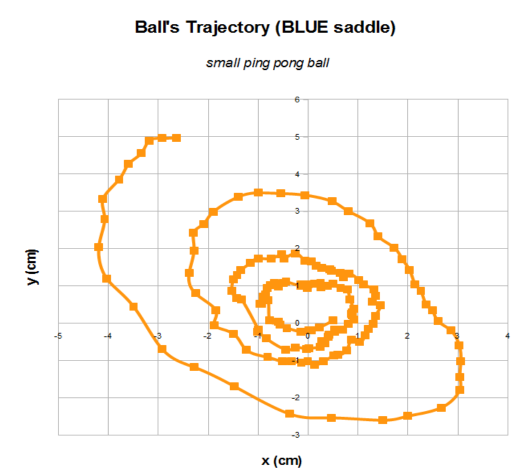
- Conditions under which the ball should be stable
  - Critical frequency
- We found its limitations and examined the effects
  - Jumping
  - Friction, initial position
  - Rotation of the ball

+VERIFICATION



nt

- We constructed 2 saddle traps with parameters, used different types of balls
- Analysis of ball's **path** and motion while being trapped <u>+comparison with theory</u>



Conclusions

1. Stability

Theory's assumptions

I. Calculation of critical frequency ☺

II. Infinite trapping lifetime for

$$f > f_c$$

✗

Our contribution

I. Experimental verification ☺

II. Experimentally confuted

- Lifetime rise for

$$f > f_c$$

Conclusions

1. Friction
   i. Dynamic
   ii. Static
2. Jumping
3. Rotation

Theory's approach

I. Solved for special case only
II. not mentioned ✗

2) not mentioned ✗

3) not mentioned ✗

Our contribution

I. Experimental verification ☺
II. Correlates best with point-mass theory- optimal case ☺
III. upper limit for frequency + estimation ☺
IV. Dependence on moment of inertia + verification ☺