3

# Twisted Rope

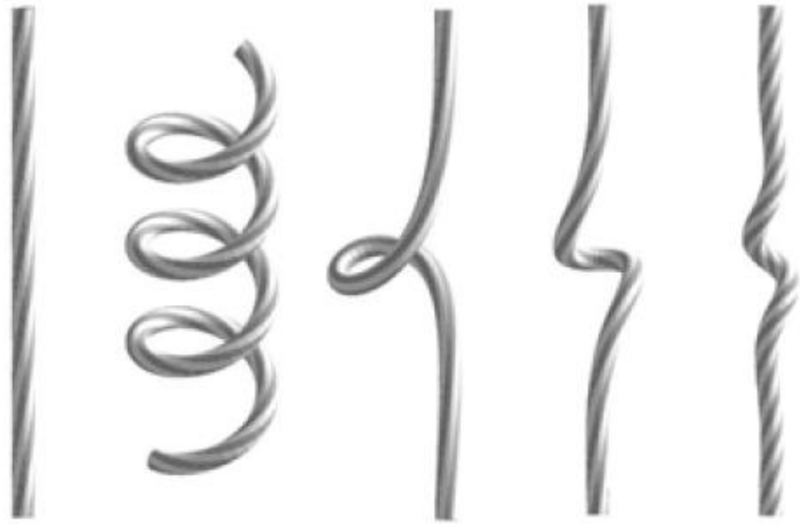
Matej Badin

# Problem

Hold a rope and **twist one end** of it.

At some point the rope will form a **helix or a loop.**

**Investigate and explain** the phenomenon.



# Apparatus

**Rope suspension** –holder

**Handle**
*("Twist one end of it")*

**Weight**
*(Changeable tension)*

# We twist one end. What happens?

- Loops/helices are formed

**Why does it occur?**

# Why do the do the formations appear?

Energetically favorable state.

Potential energy rises upon deformation:

- **Torsional deformation**

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- **Bending (loops)**

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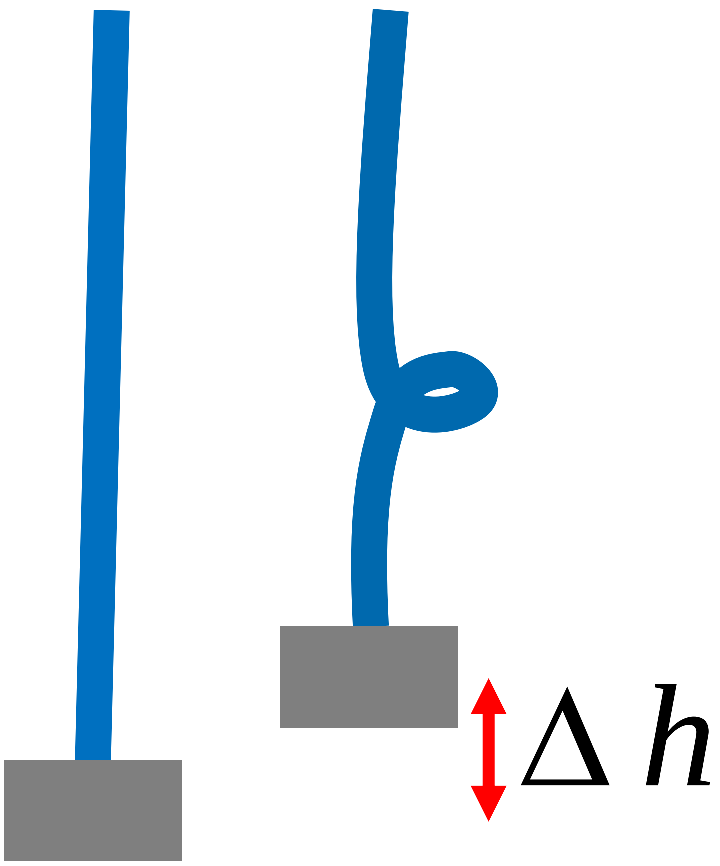
- **Potential of tension force** (Modeled by gravity force of weight)



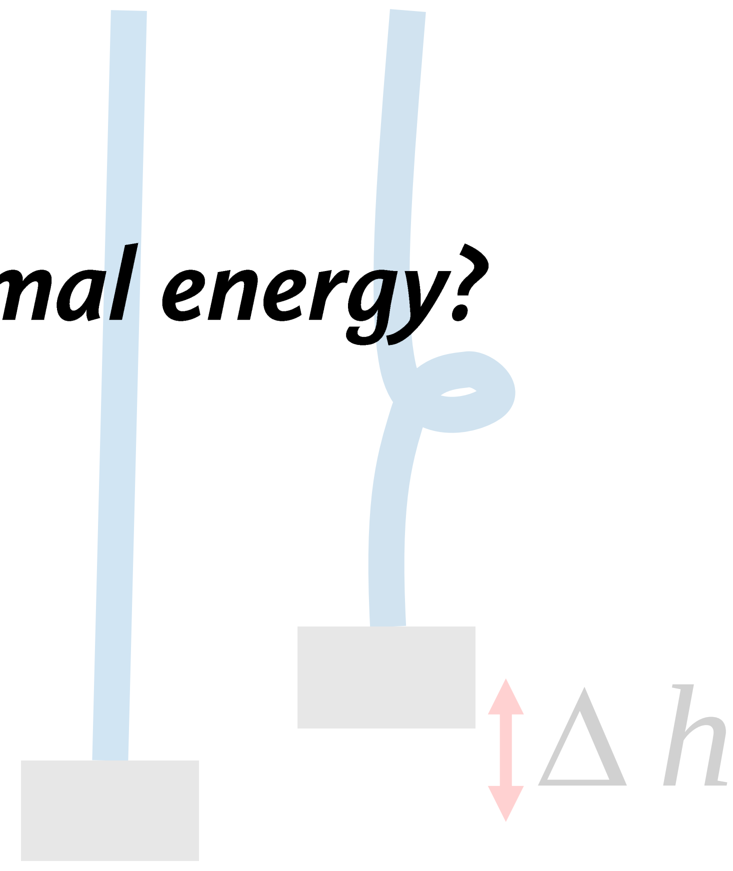
# Why do the do the formations appear?

Energetically favorable state.

Potential energy rises upon deformation:

- **Torsional deformation**
- **Bending (loops)**
- **Potential of tension force** (Modeled by gravity force of weight)

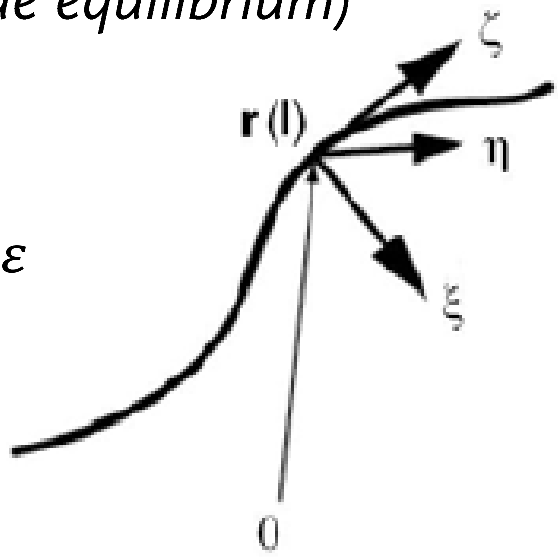
***How to achieve the minimal energy?***



# Existing theoretical model

Euler-Kirchhoff's equations *(Force and torque equilibrium)*

$$EI\dot{\kappa}_1 + (C - EI)\kappa_2\tau = F \cdot e_\eta$$
$$EI\dot{\kappa}_2 + (EI - C)\kappa_1\tau = -F \cdot e_\varepsilon$$
$$C\dot{\tau} = 0$$




**<u>Limitations & Drawbacks</u>:**

- Works only for ideal rods
- Several solutions with different energy
- No general analytical solutions is known
  - → Solutions only for simple cases
  - → Numerical modeling

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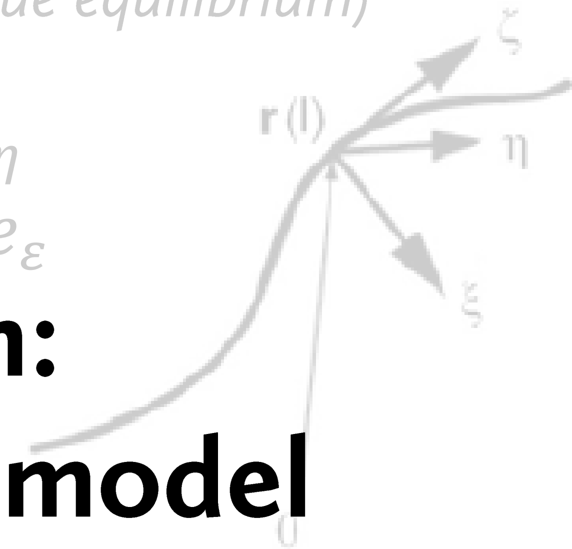


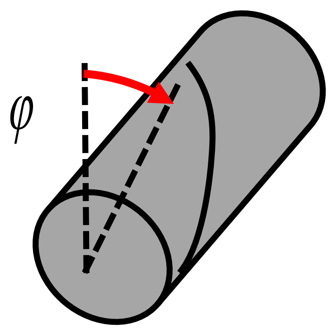

**Our approach:**
**simple qualitative model**

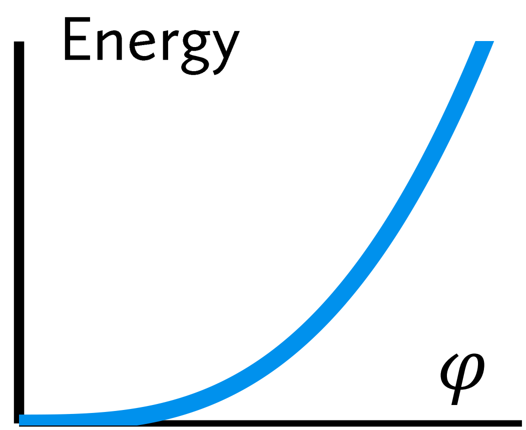
<u>Limitations & Drawbacks:</u>

- Works only for ideal rods
- Several solutions with different energy
- No general analytical solutions is known
  → Solutions only for simple cases
  → Numerical modeling

# Energy of the rope

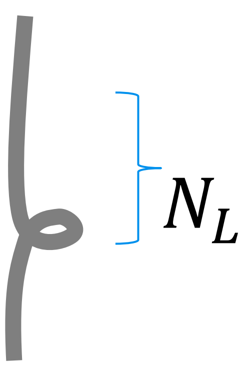
- Twist angle

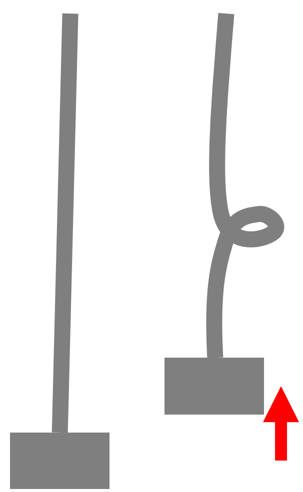



***$E_T(\varphi)$: Rises faster than linearly***

- Number of loops



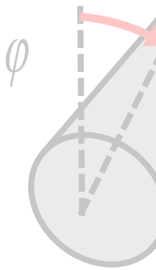
$E_{Bending}$

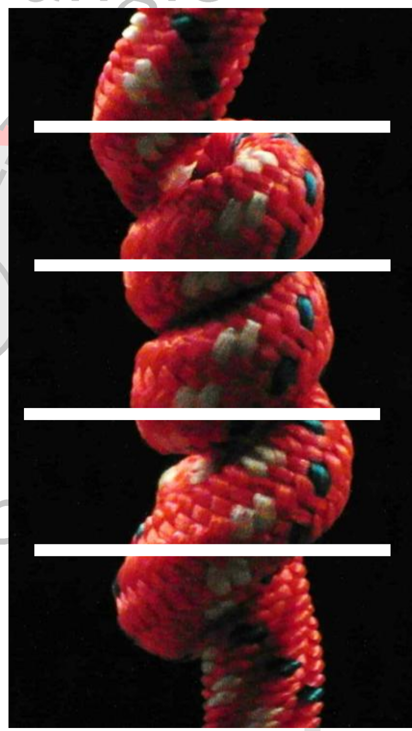
$F_s \Delta s = mg\Delta h$

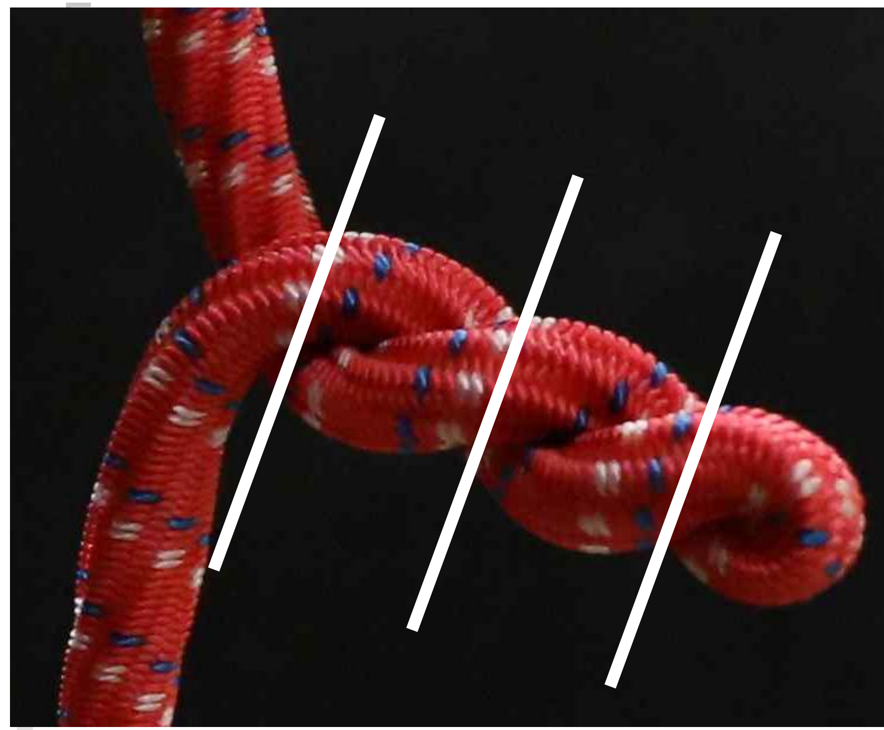
*Loops are similar:*
***Constant energy increment $\Delta E_L$ per loop***

# Energy of the rope

- Twist angle





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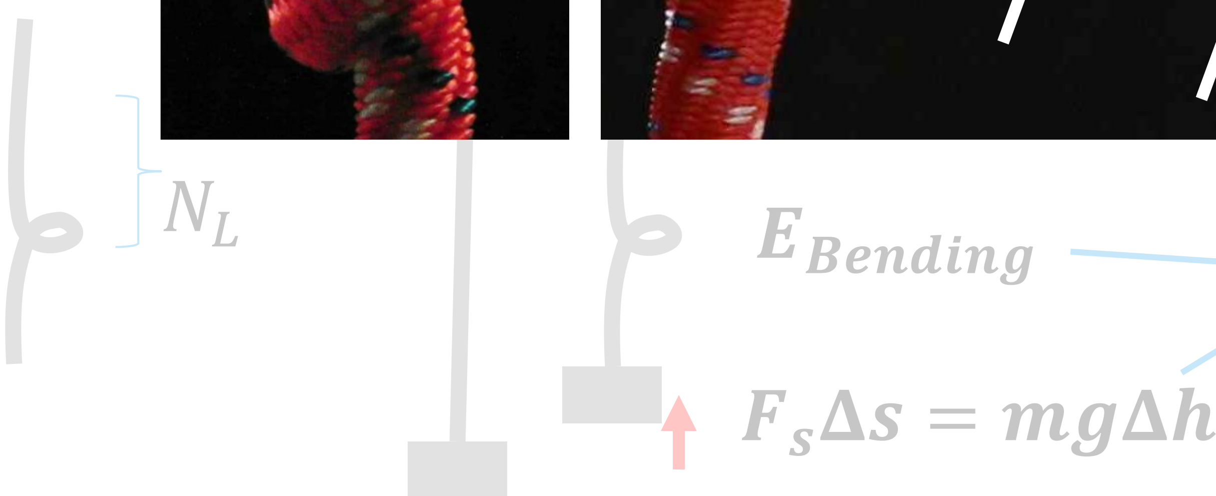
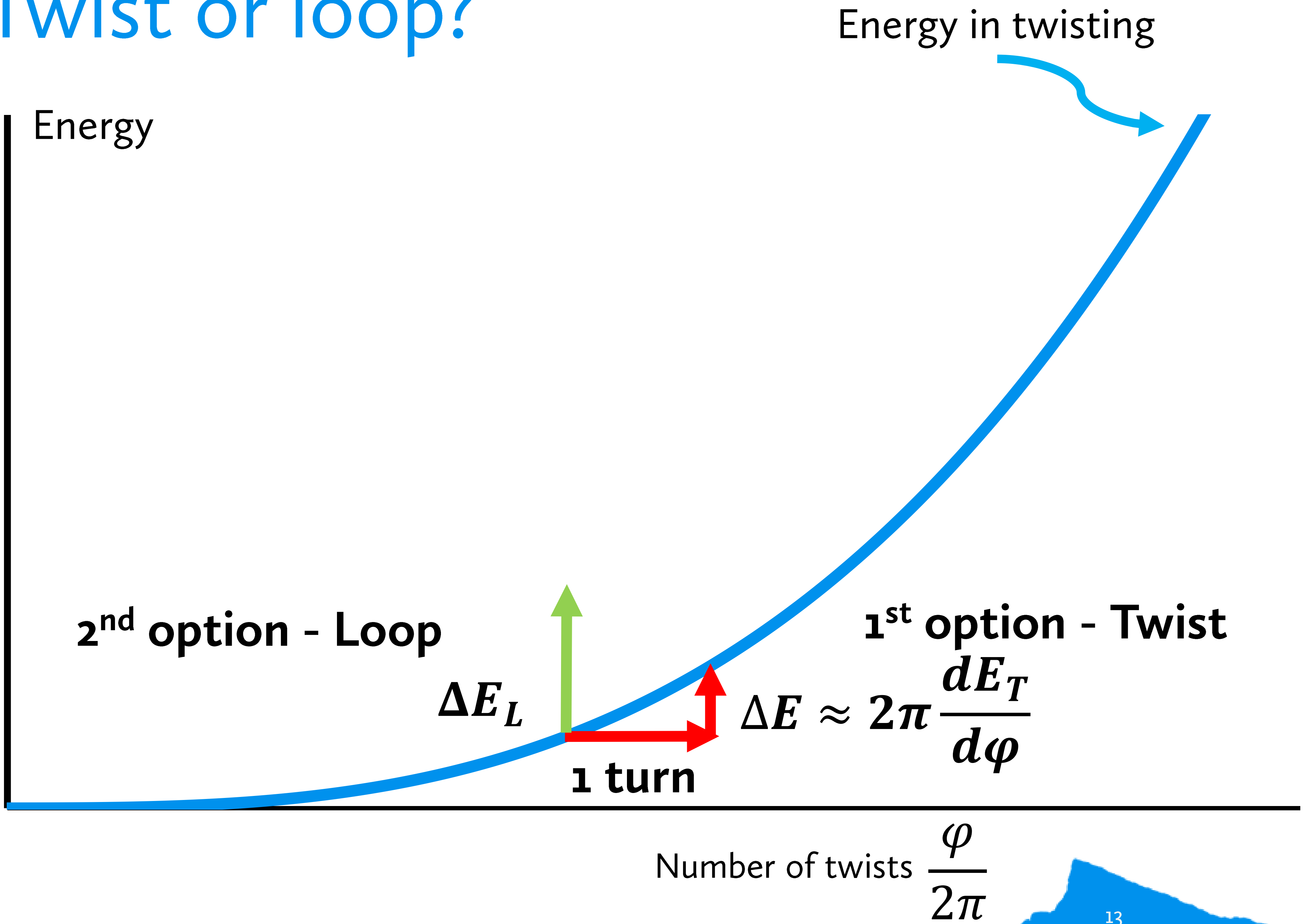


# Twist or loop?



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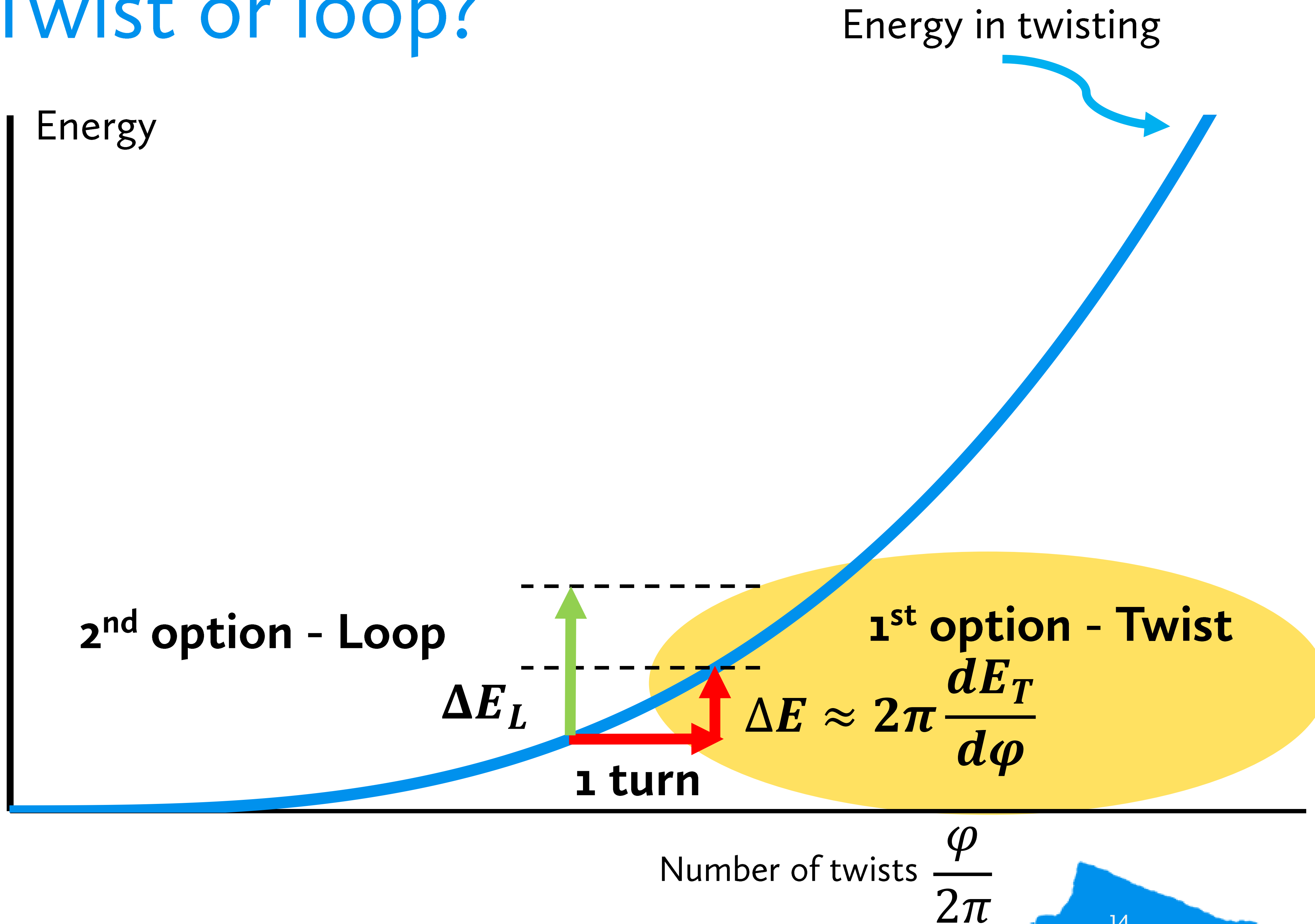

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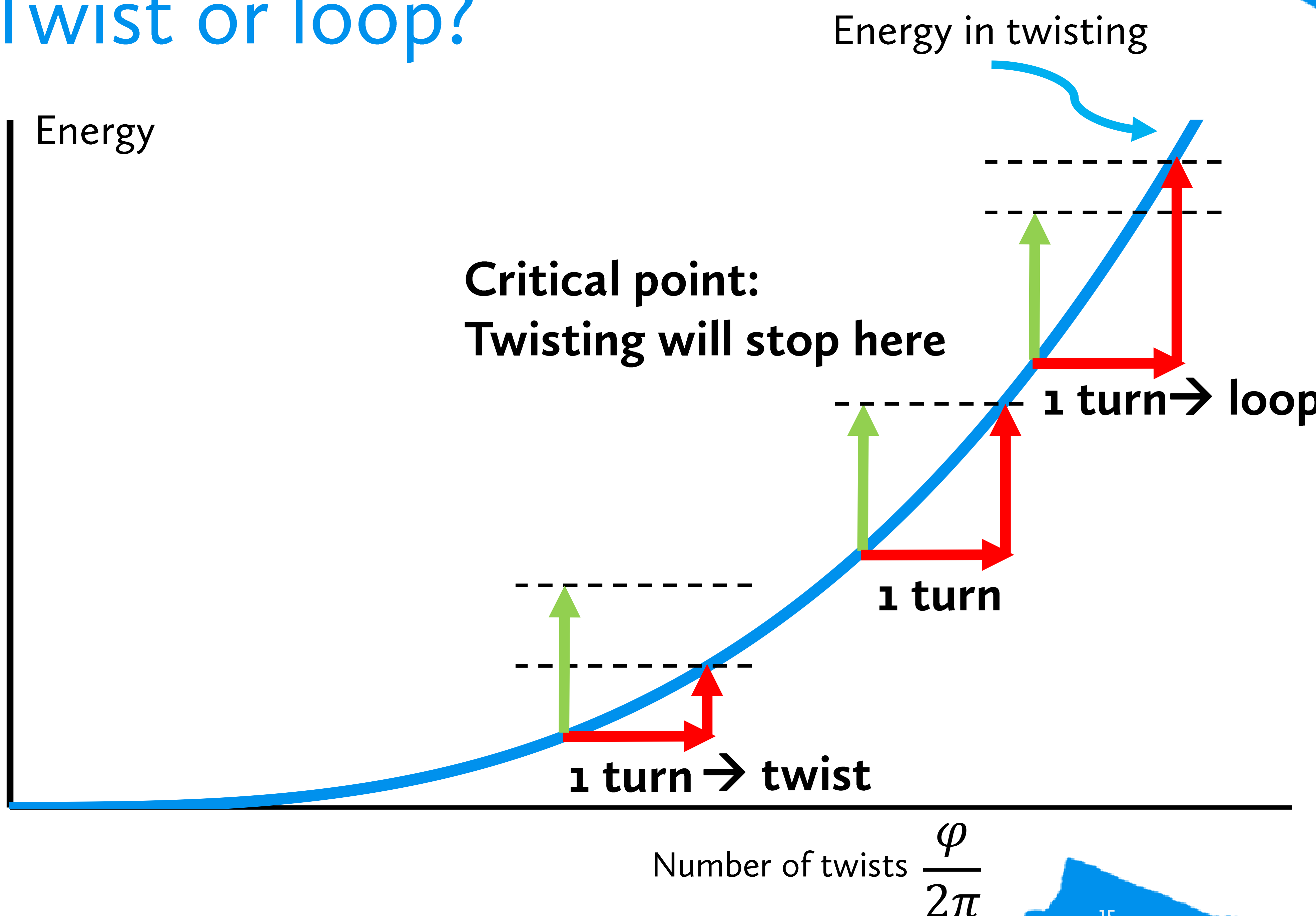


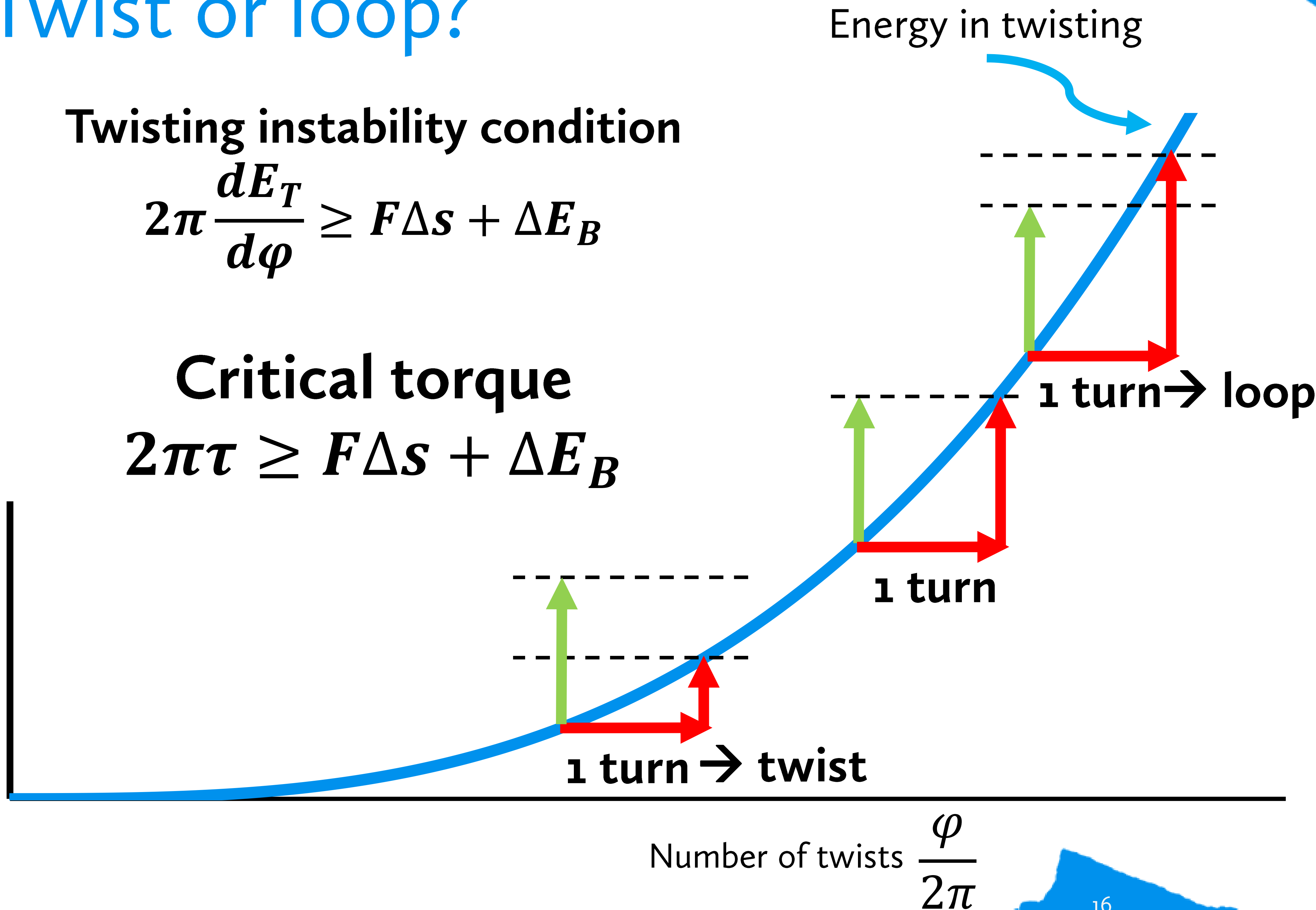
# Twist or loop?

**Twisting instability condition**

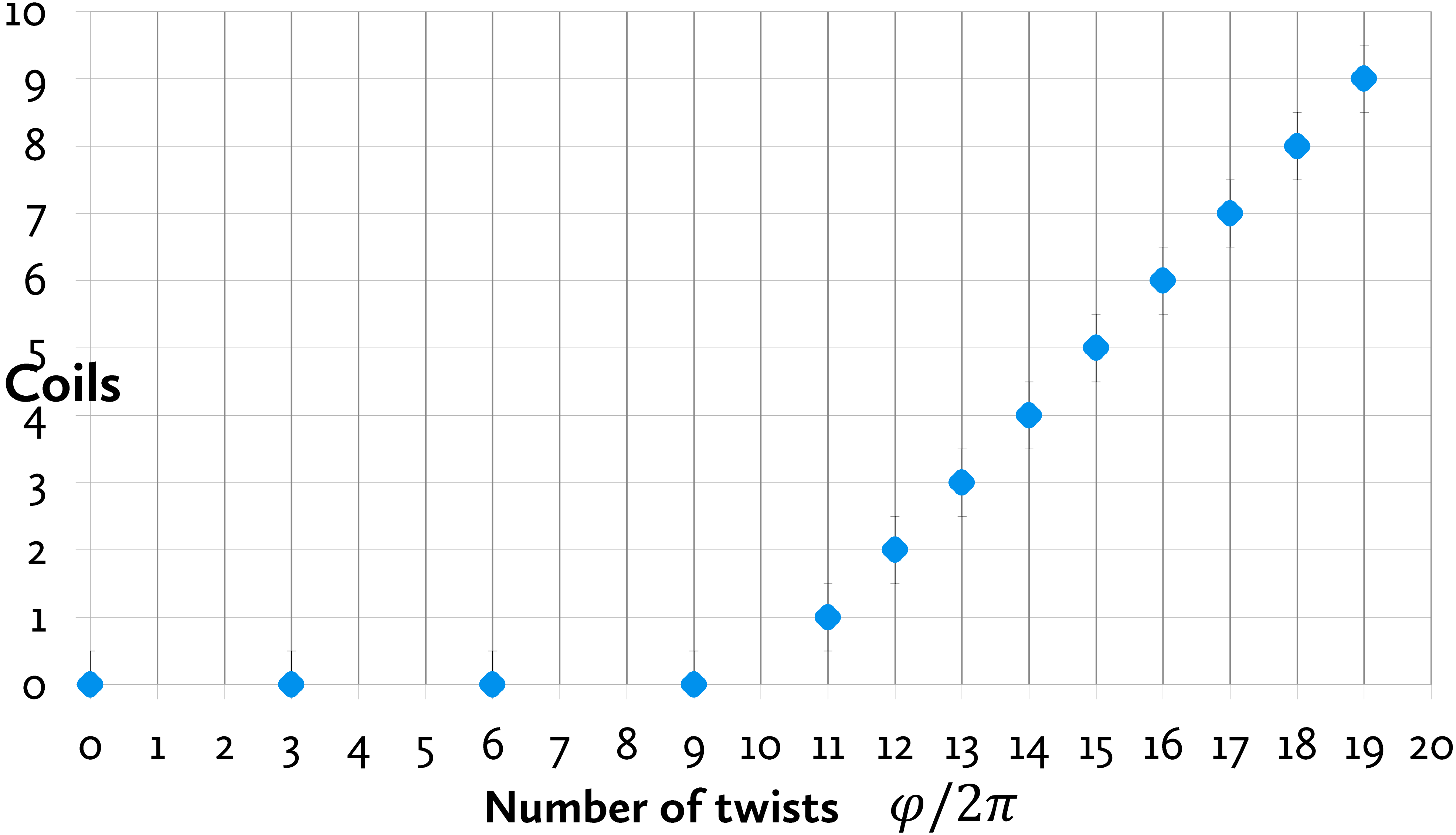
$$2\pi \frac{dE_T}{d\varphi} \geq F\Delta s + \Delta E_B$$

## Critical torque

$$2\pi\tau \geq F\Delta s + \Delta E_B$$



# Reaching the torsional instability



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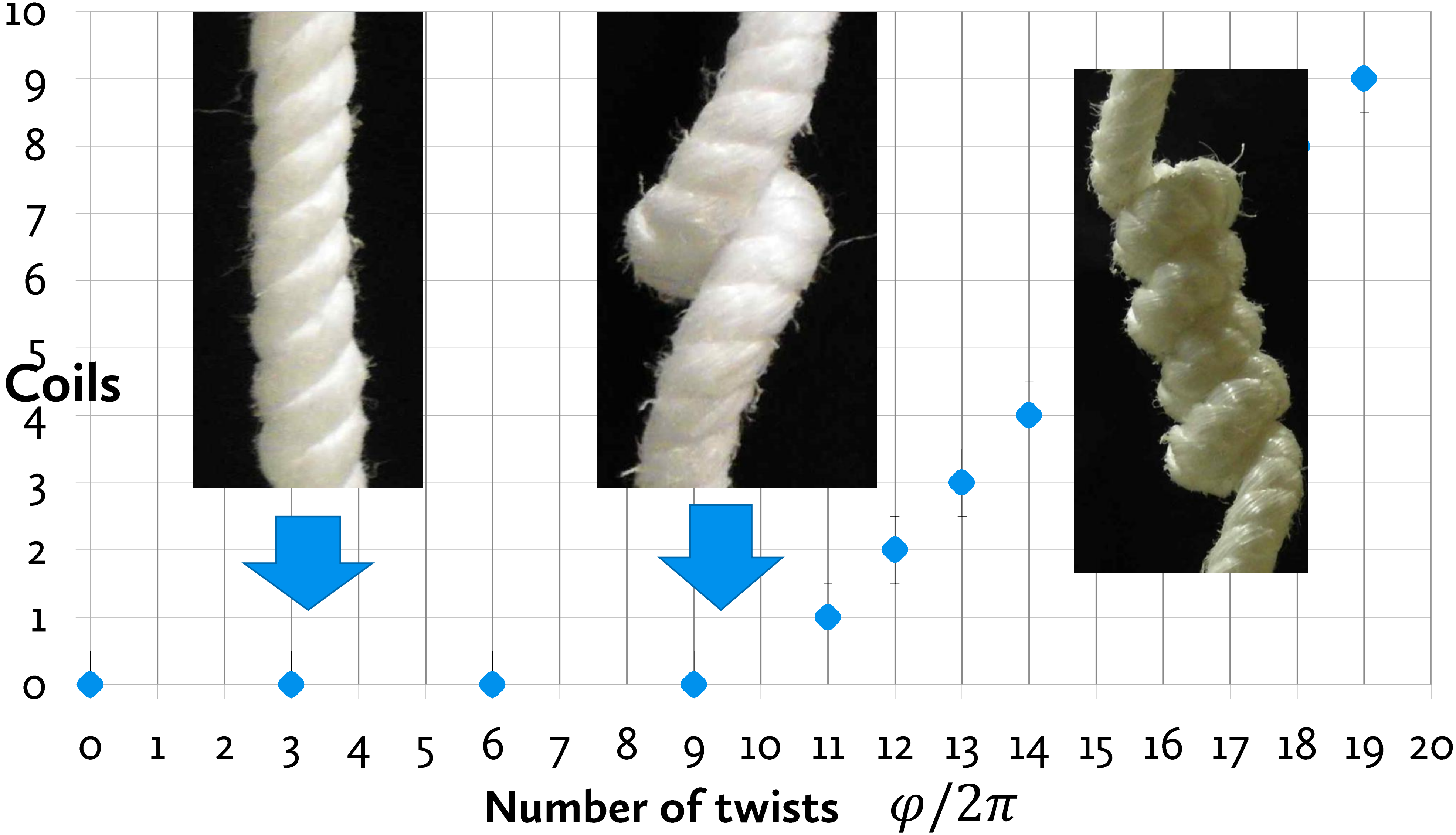

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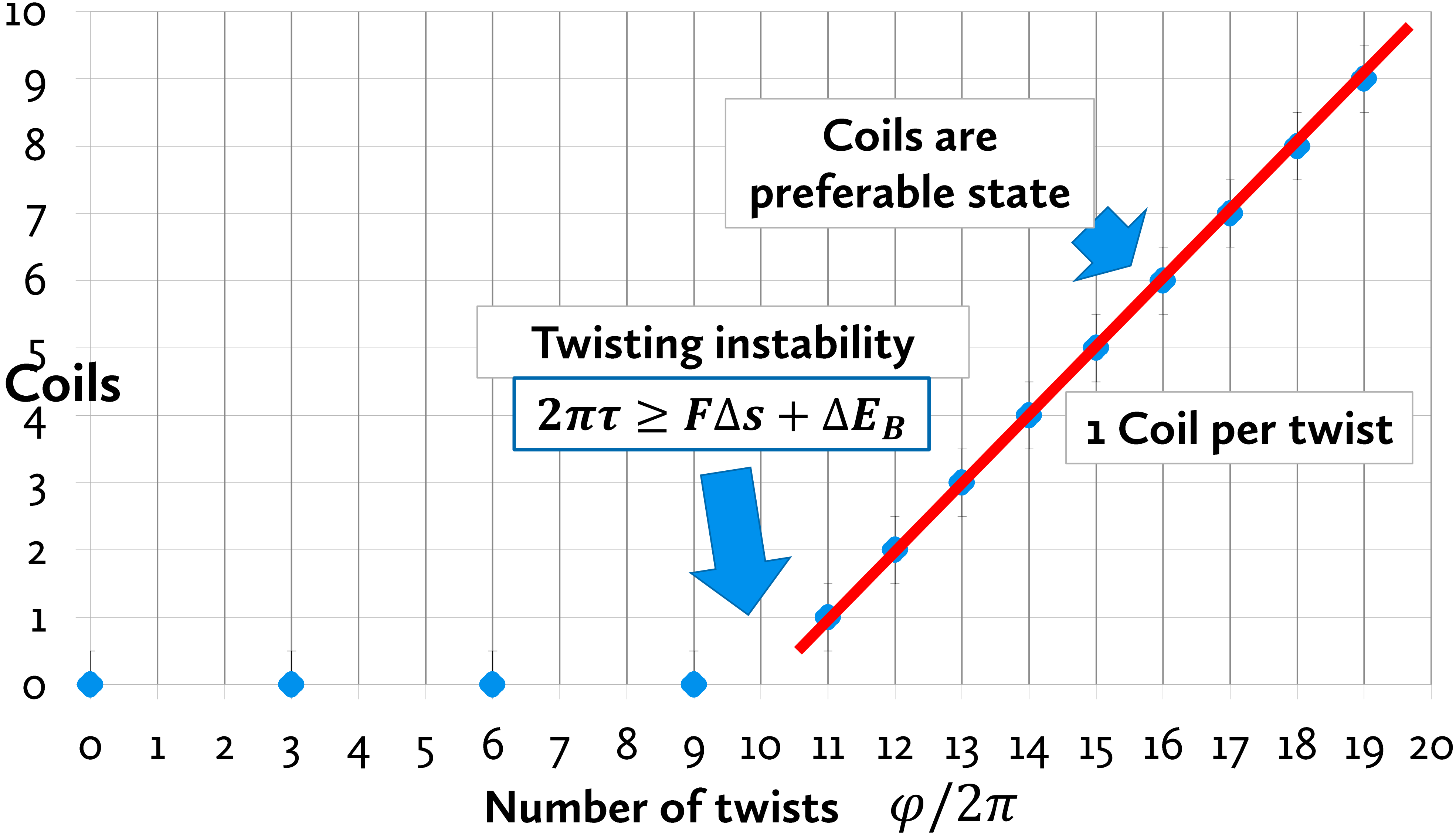
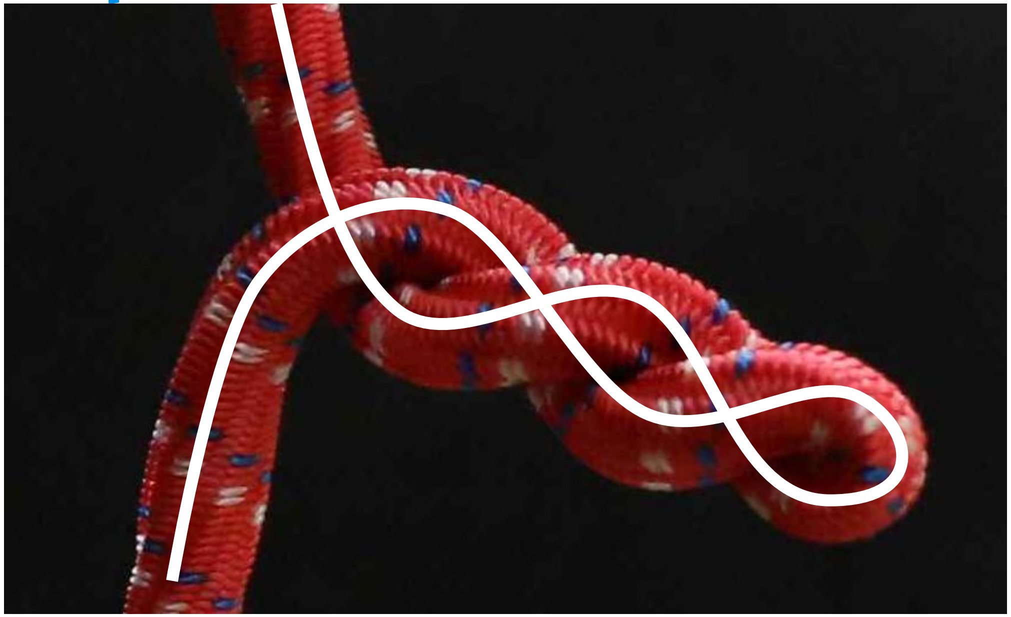


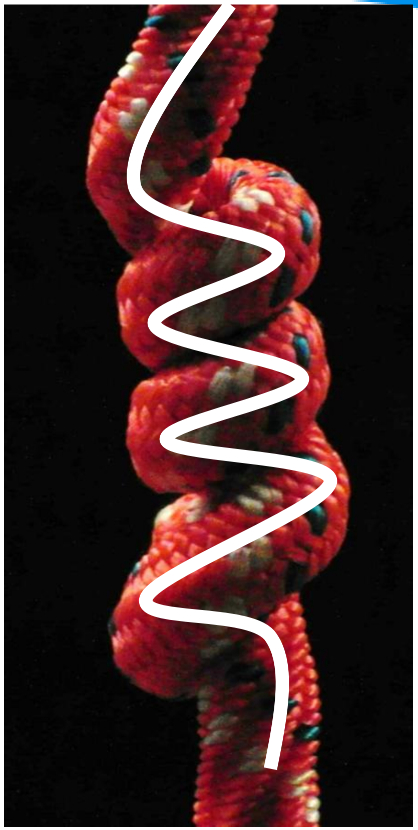
# Formations:

## Spirals

## Coils

# Spirals vs. coils





Smaller curvature ($\Delta E_B$)
Greater impact on length ($\Delta s$)

Greater curvature
Smaller impact on length

$$2\pi\tau \geq F\Delta s + \Delta E_B$$

Prevail under **low tension**

Prevail under **high tension**

# Spirals

# Coils

# Points of investigation

1 Torque & Length during creation

2 Thickness and tension of rope

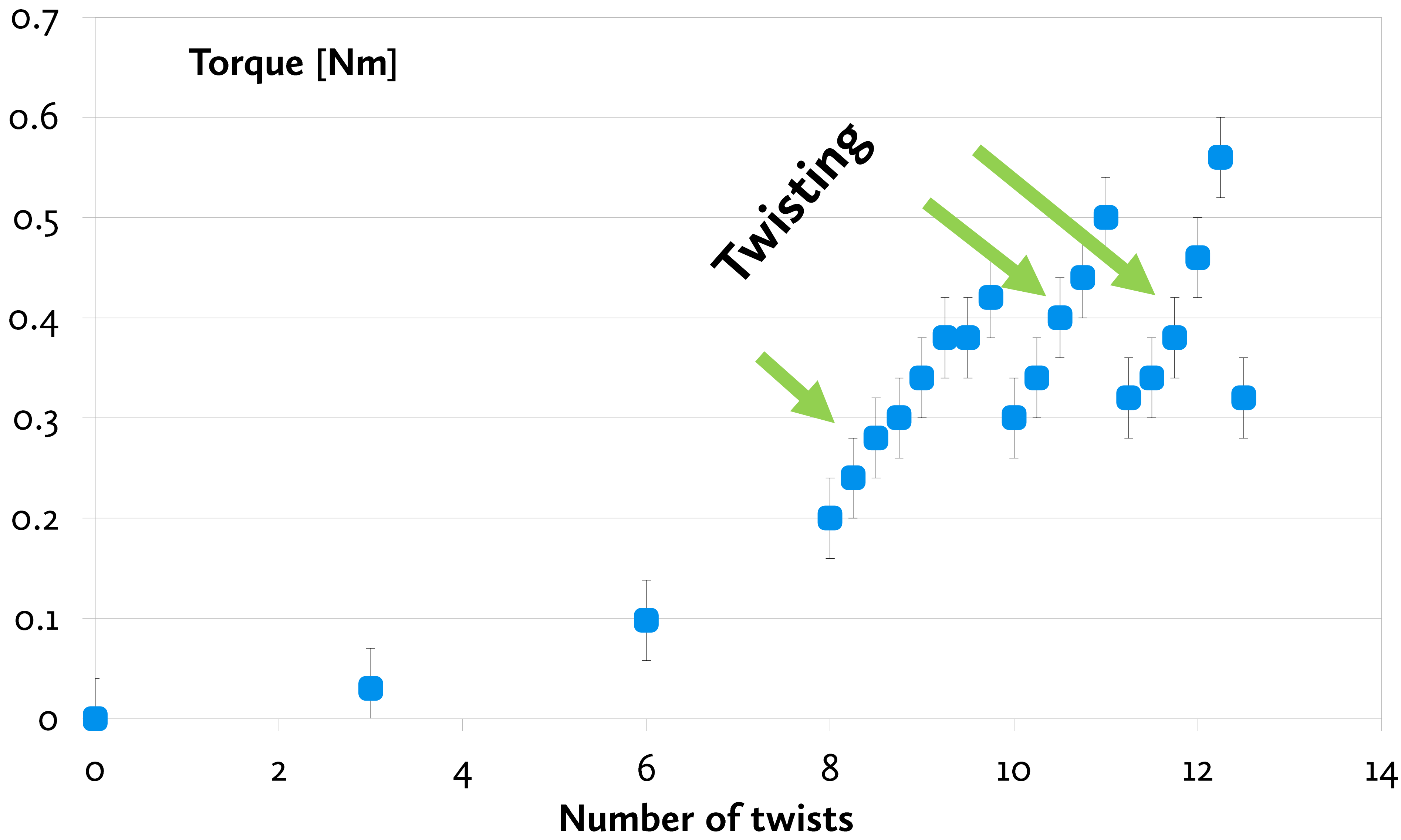
# 1. **Torque and length** during the twisting

- Depending on number of twists
- 2 materials:

Sisal

Polypropylene

# Torque characteristics – Sisal rope



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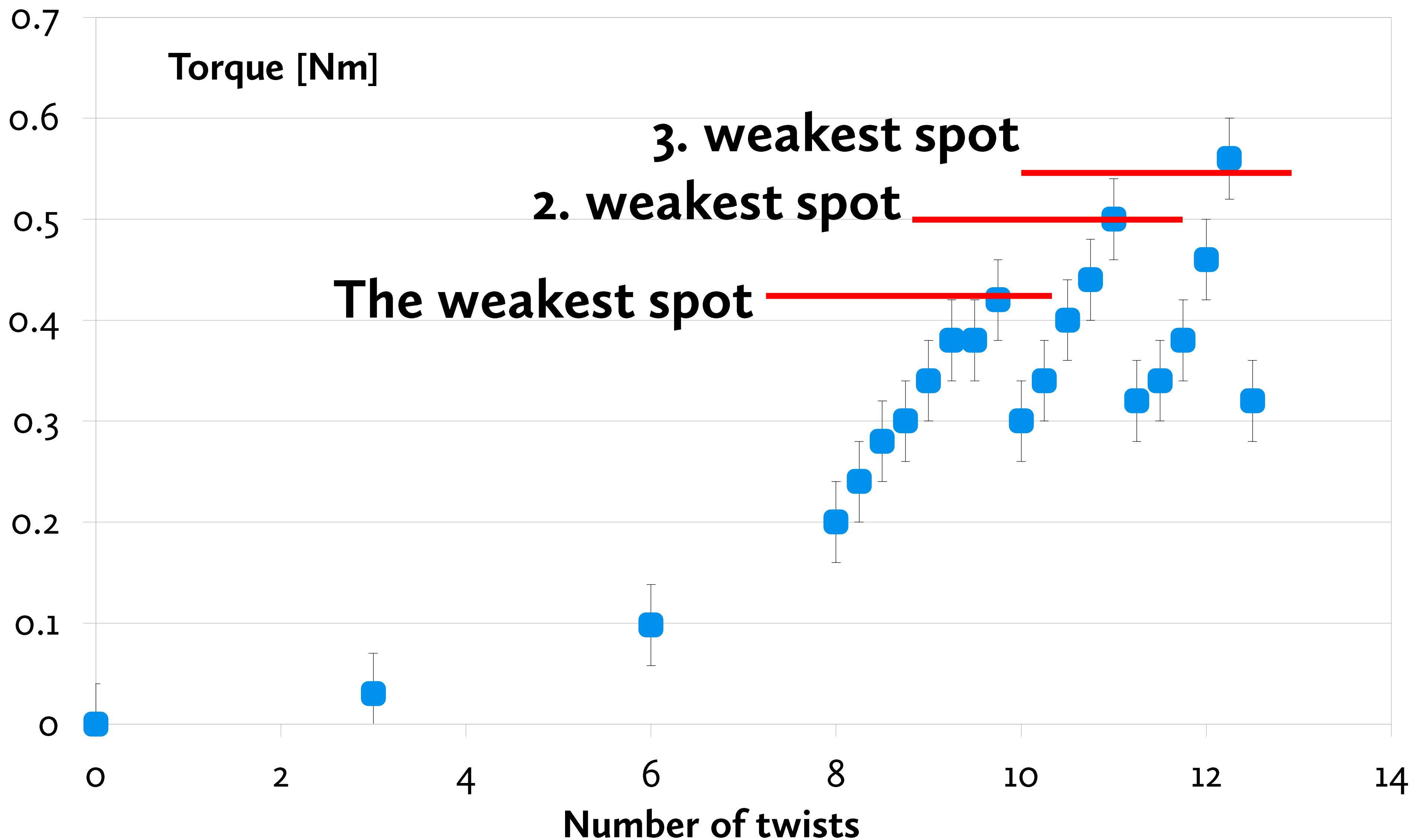

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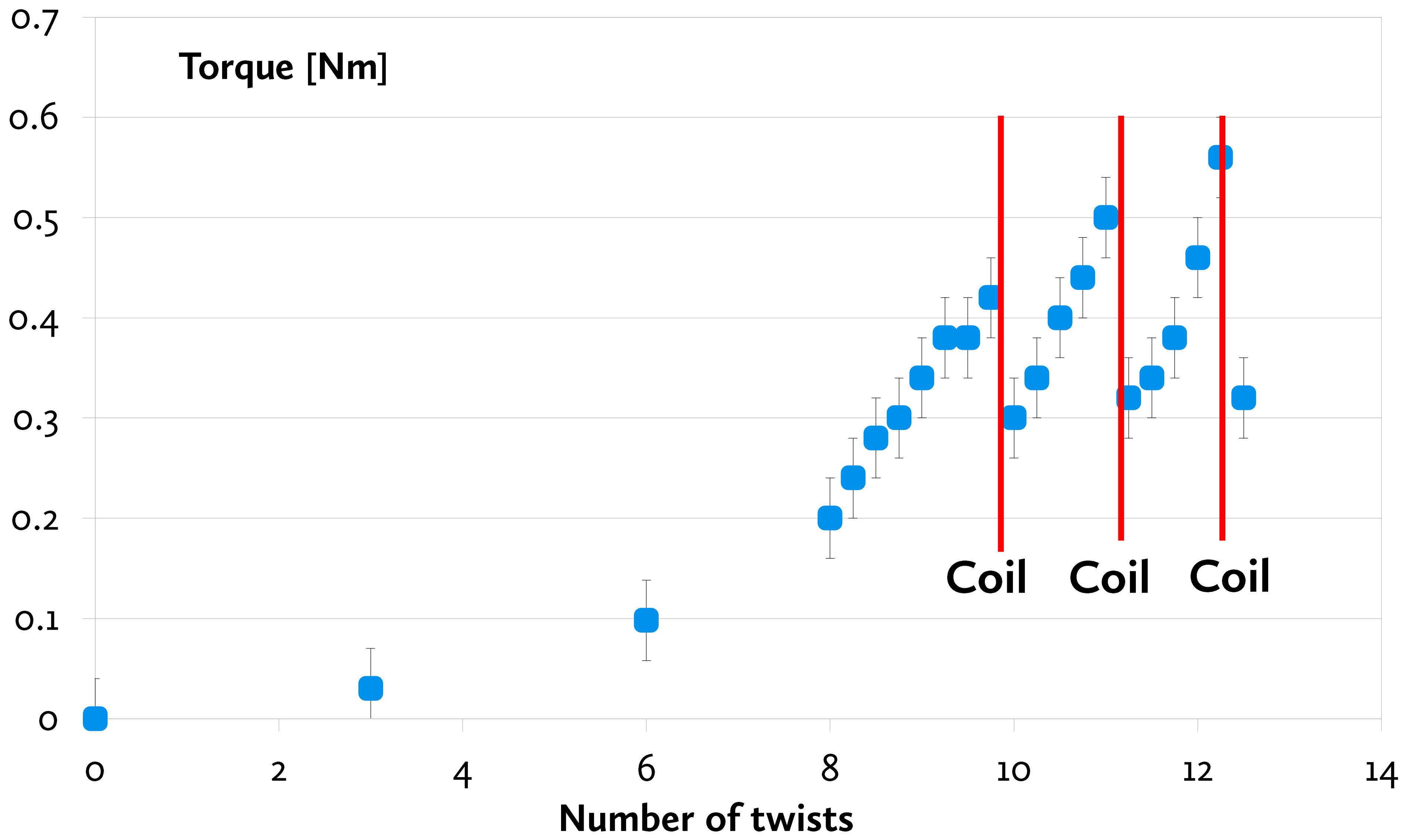

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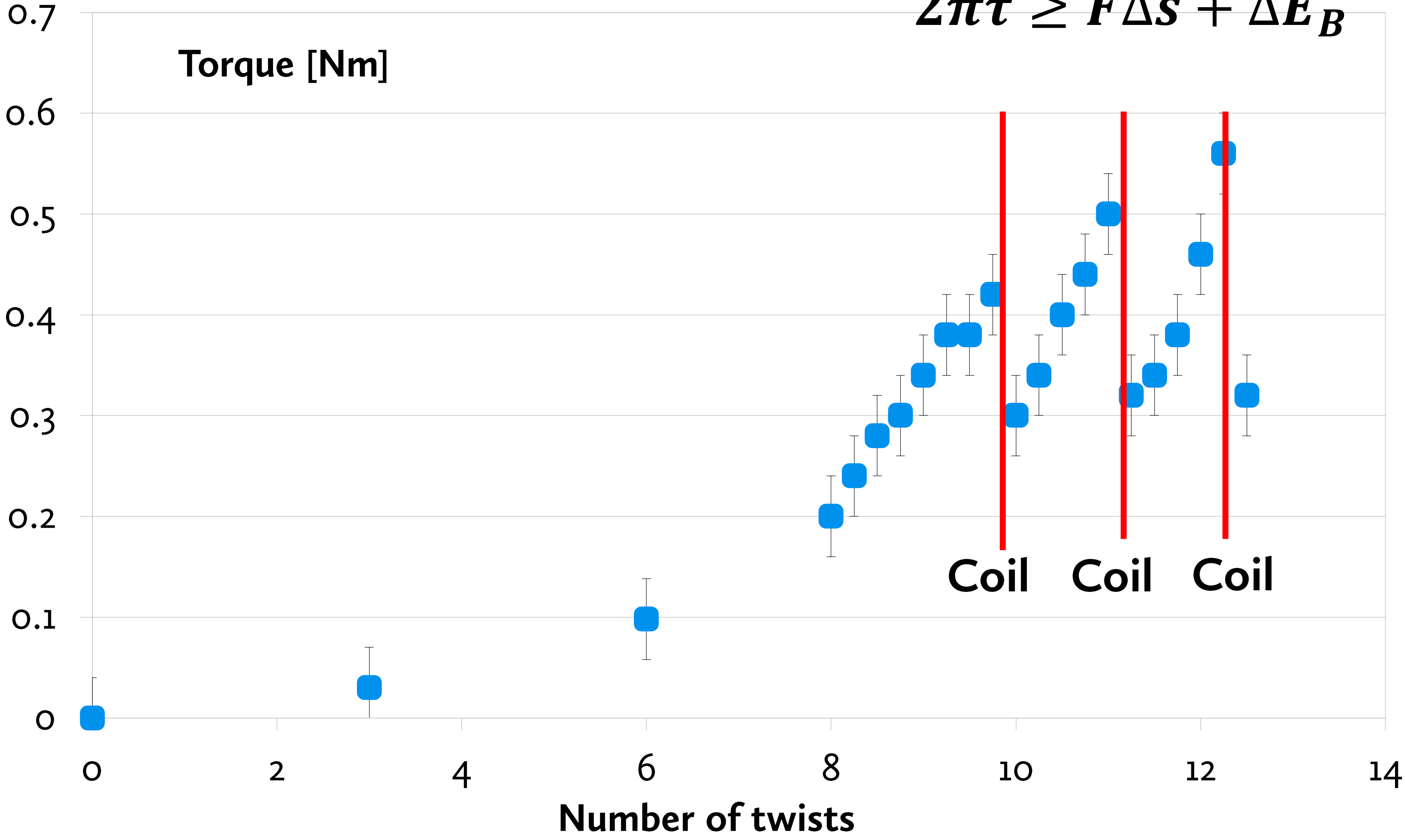
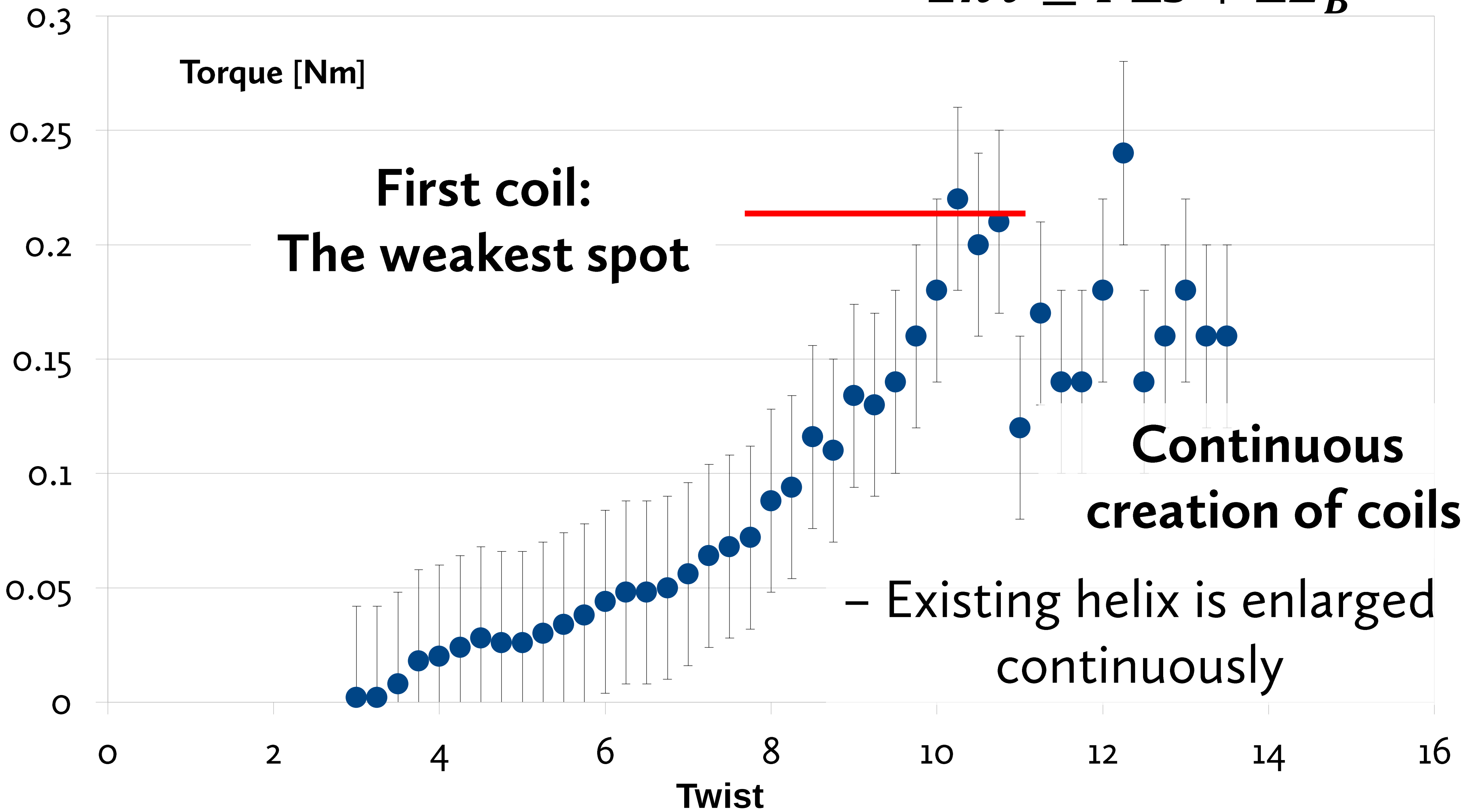


# Torque characteristics - Polypropylene

$$2\pi\tau \geq F\Delta s + \Delta E_B$$



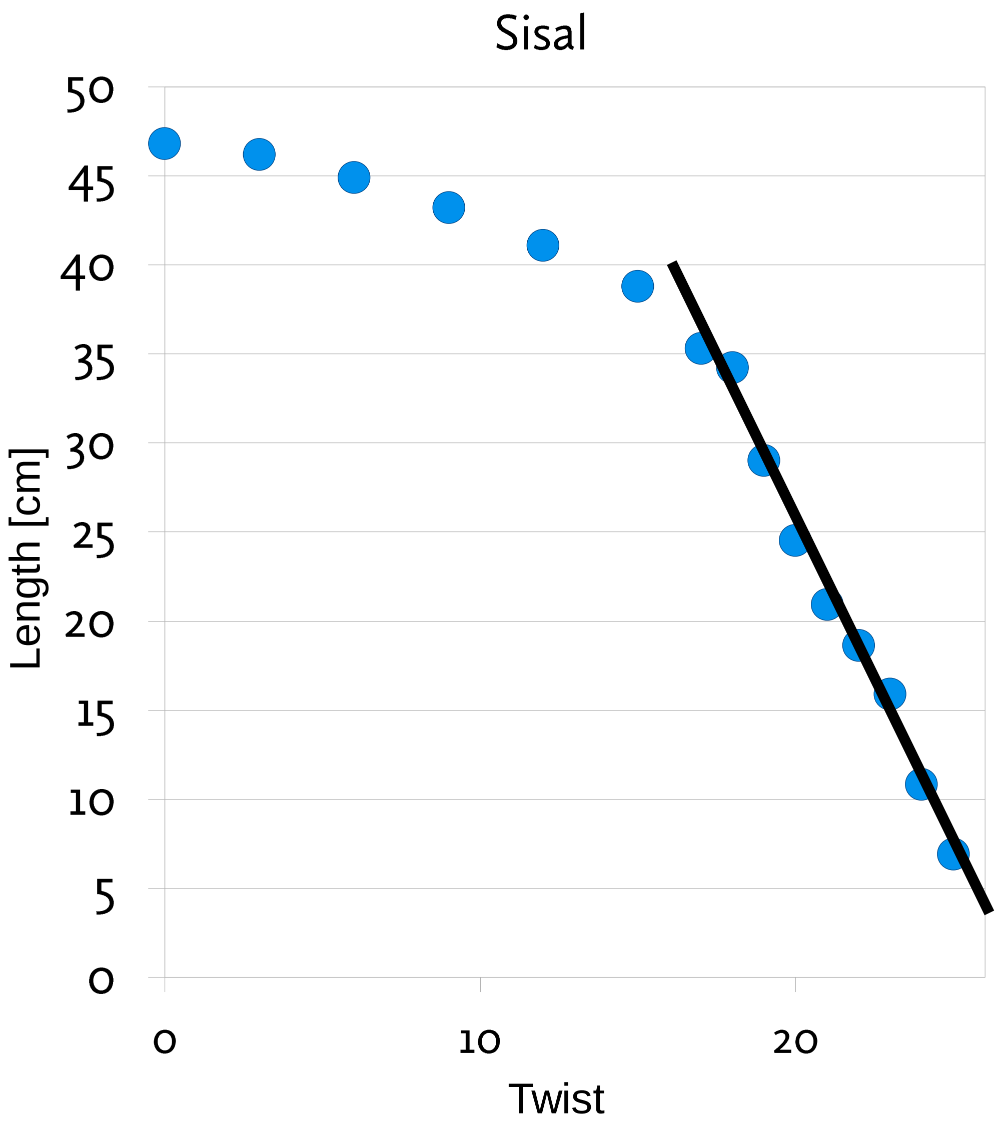
# Length of rope during coil creation

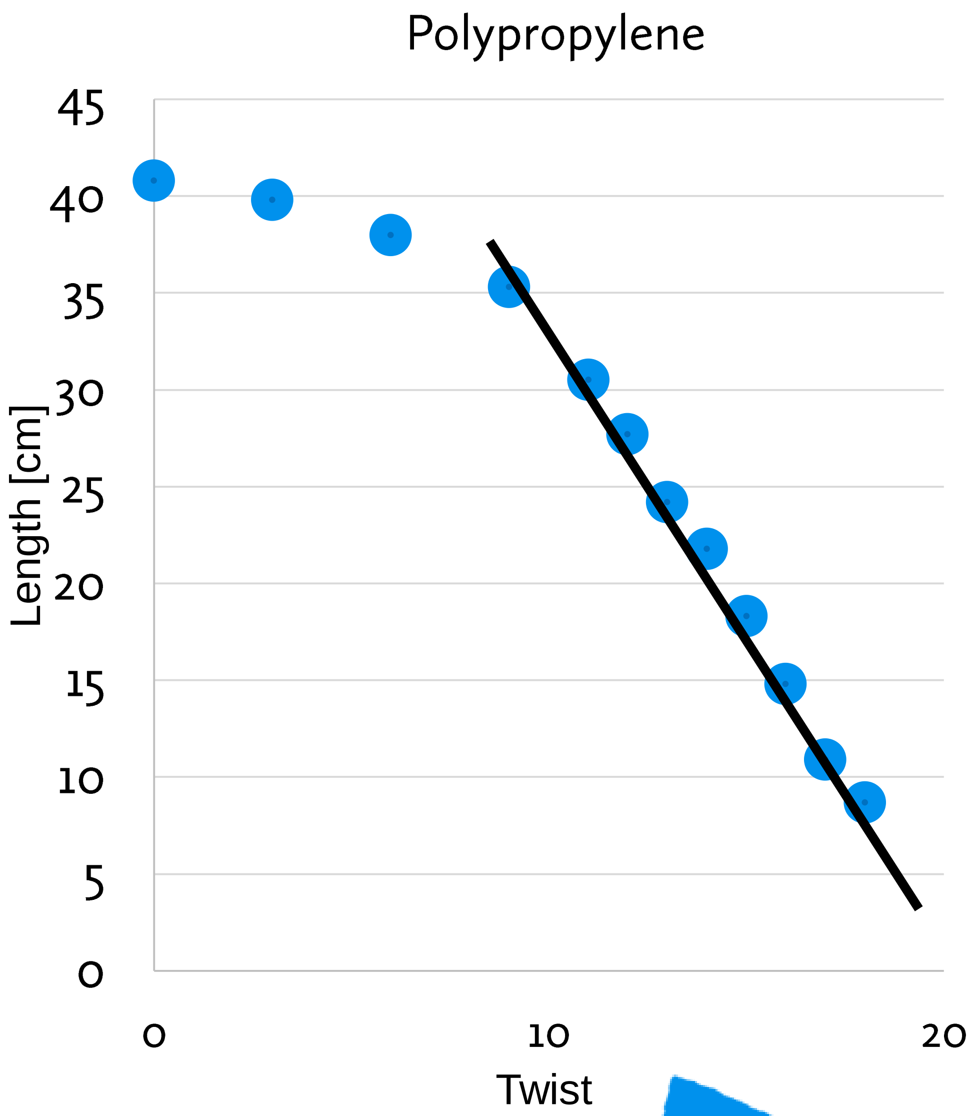
- 1 twist → 1 coil
- Each coil takes the same length of rope

**=> Approx. linear decrease in length with number of twists**

- **Sudden jumps** for sisal rope, **continuous** line for polypropylene

# Coils effect on length



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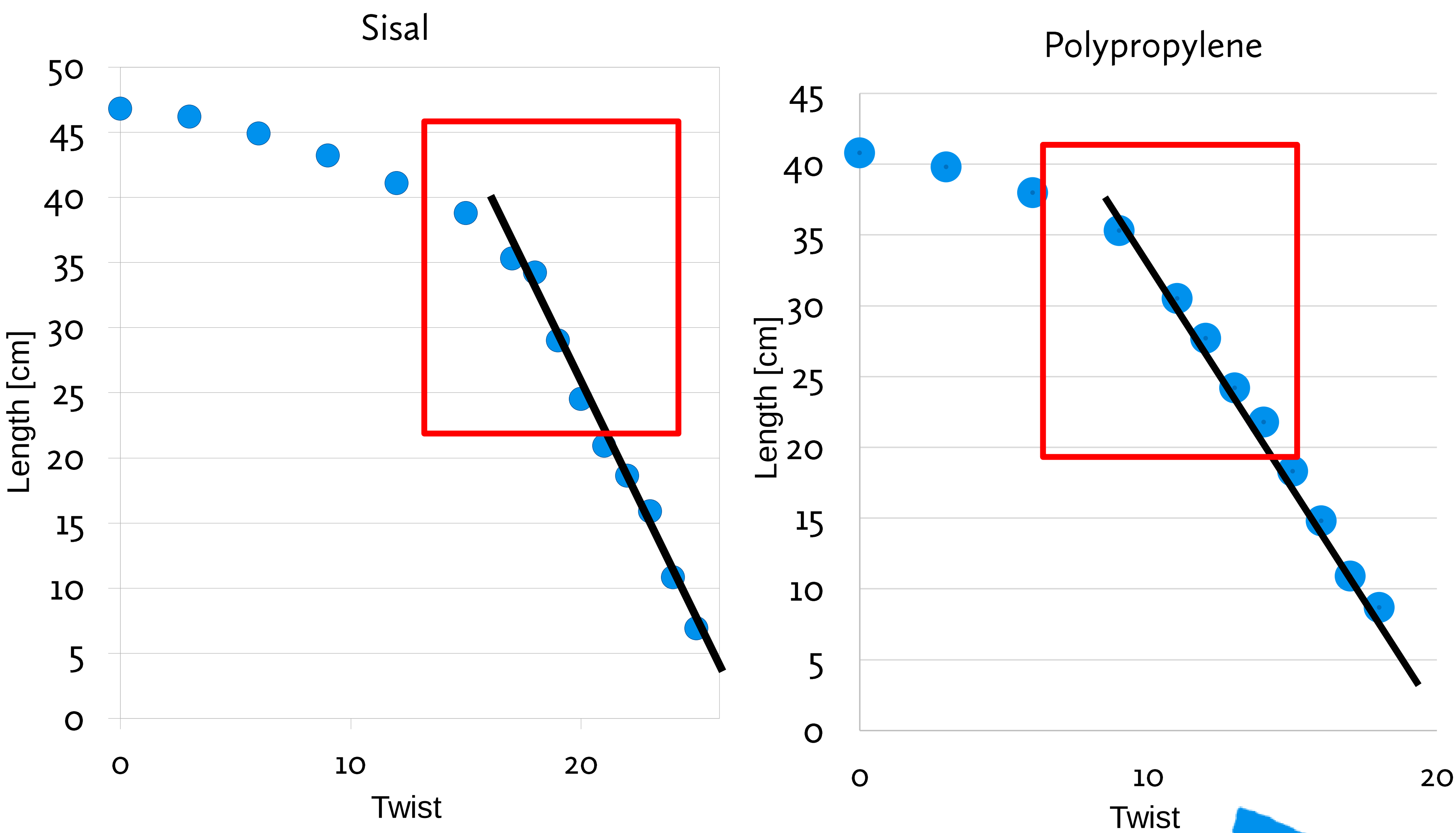

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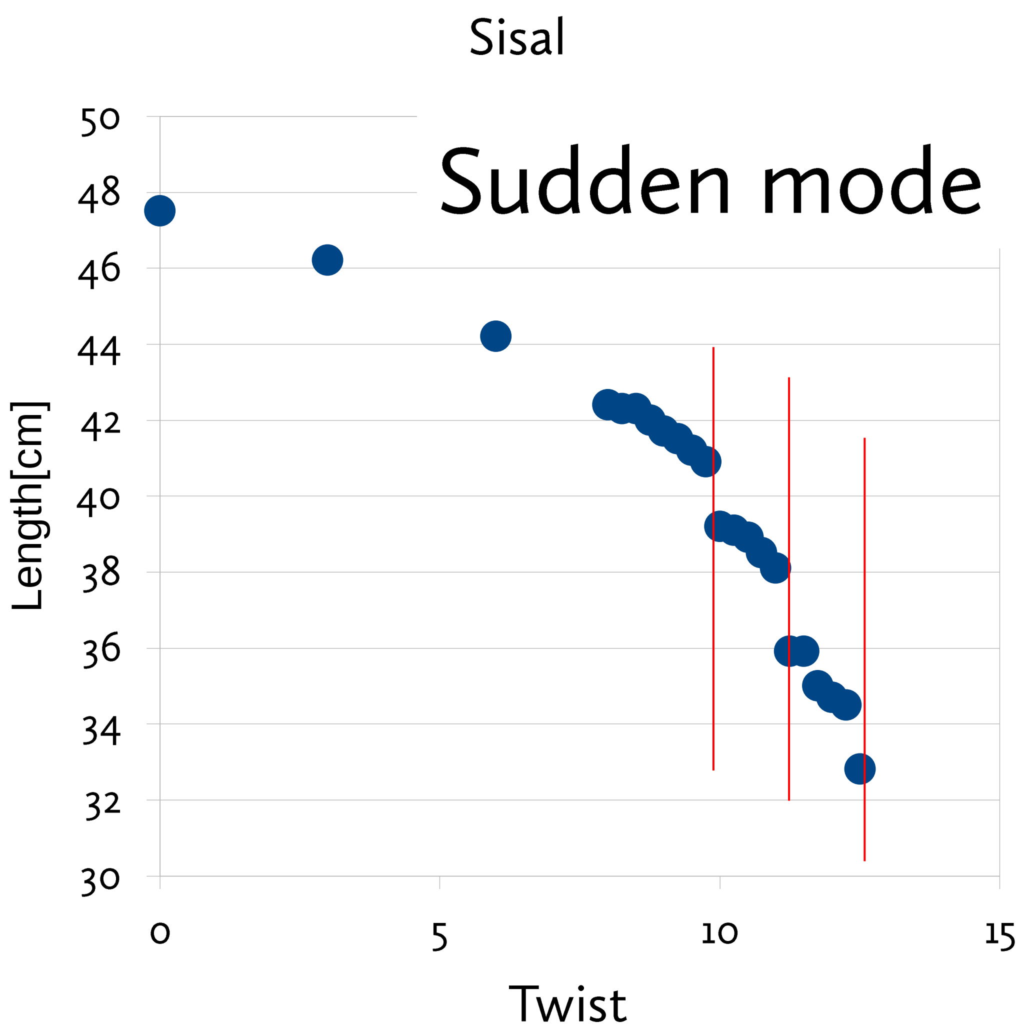
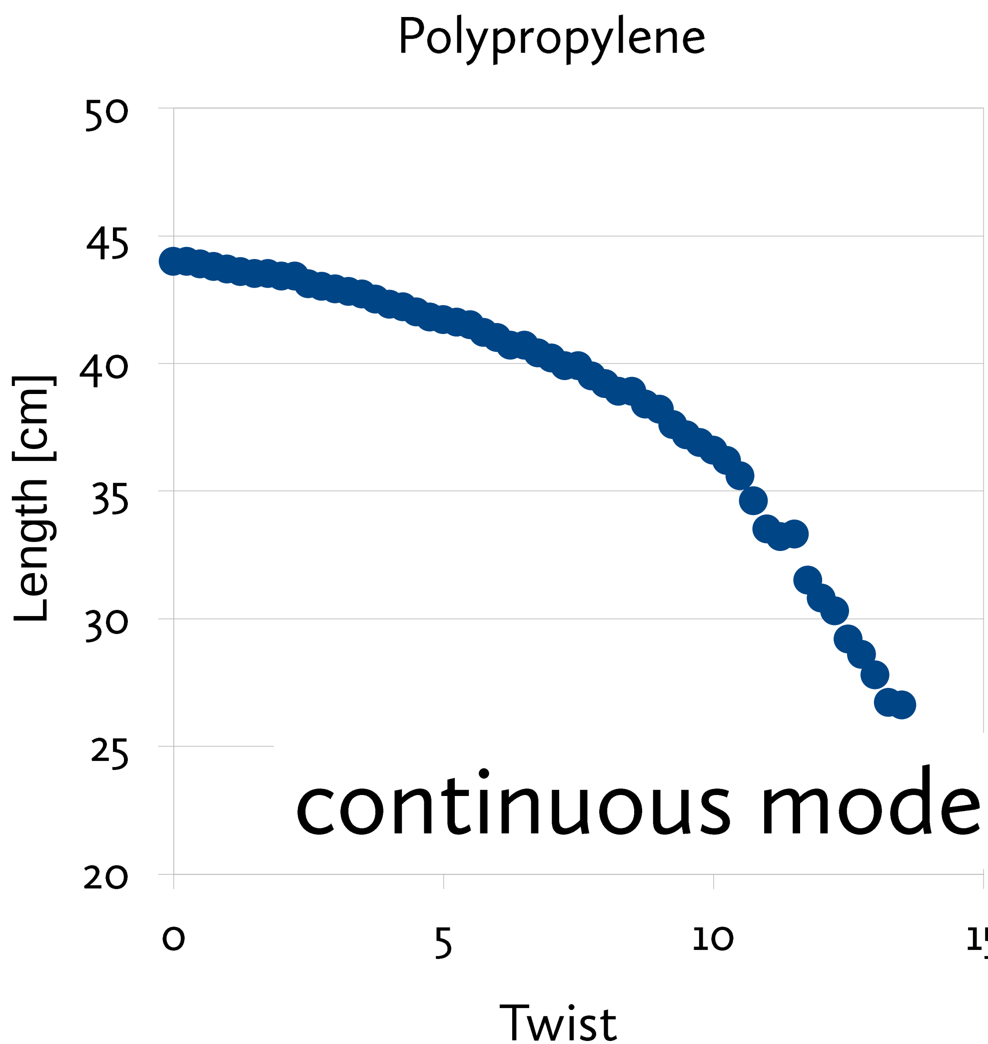
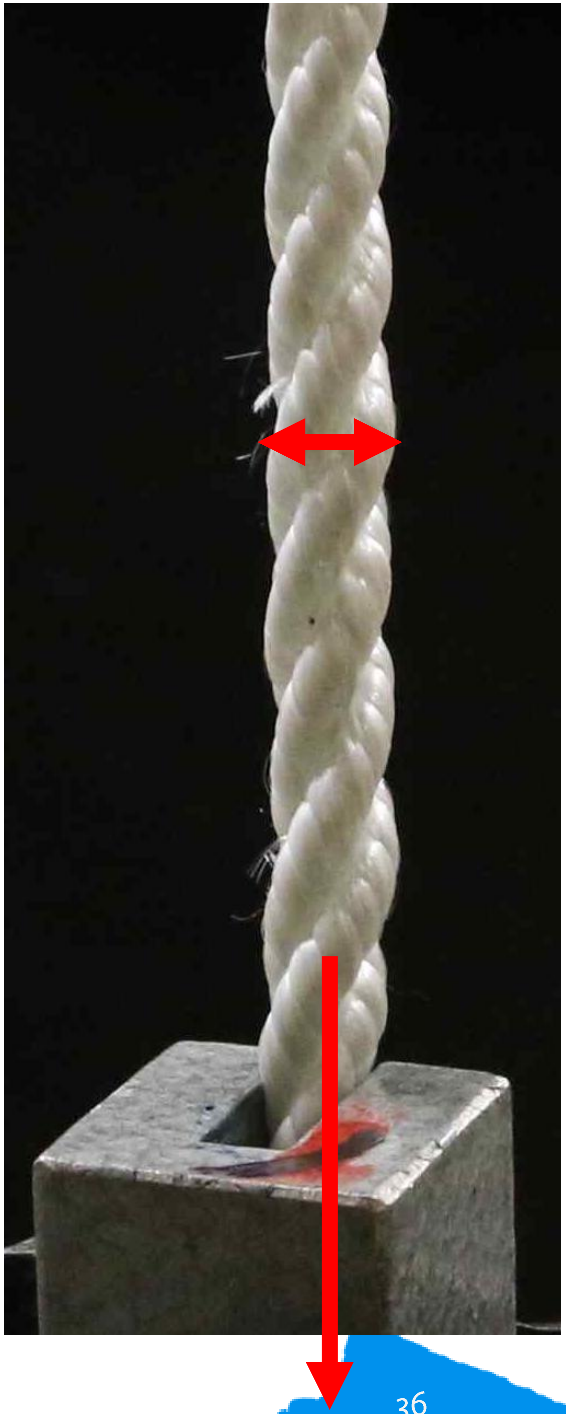


# Conclusion part 1:

- Coils form **1 per twist**
- ...after the **critical torque** is reached
- And result in **linear decrease in length**
- **Discrete or continuous** coil creation

# 2. Influence of tension and diameter



Changed:

**tension** (different weights)

Measured:

**number of twists for the 1st coil**

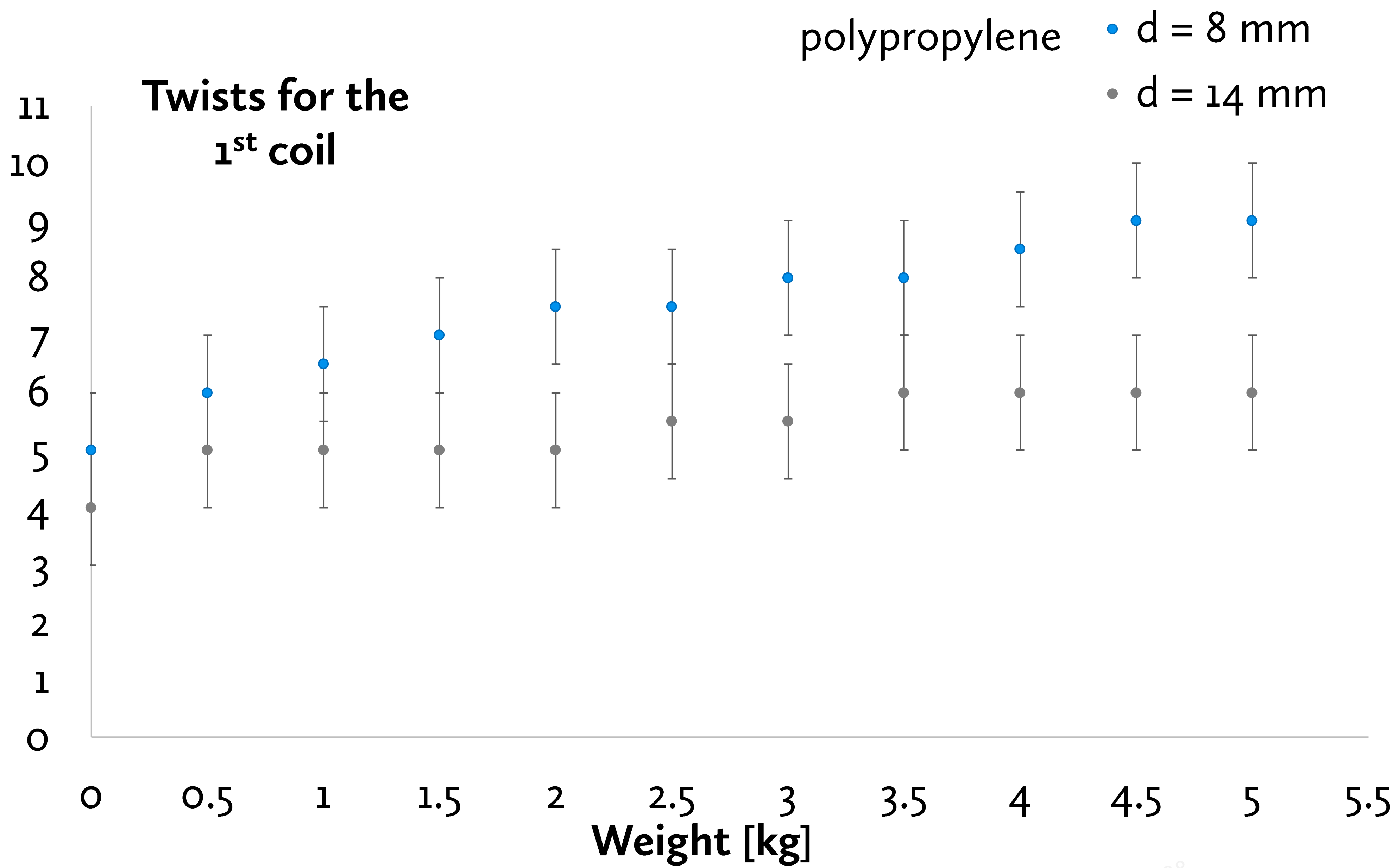
$$2\pi\tau \geq F\Delta s + \Delta E_B$$

**Greater tension**

→ coils need greater torque

→ **will form later**

# Number of twists for the 1st coil



Changed:

**diameter** (different ropes)

Measured:

**number of twists for the 1st coil**

Prediction:

**Scaling...**

# Scaling

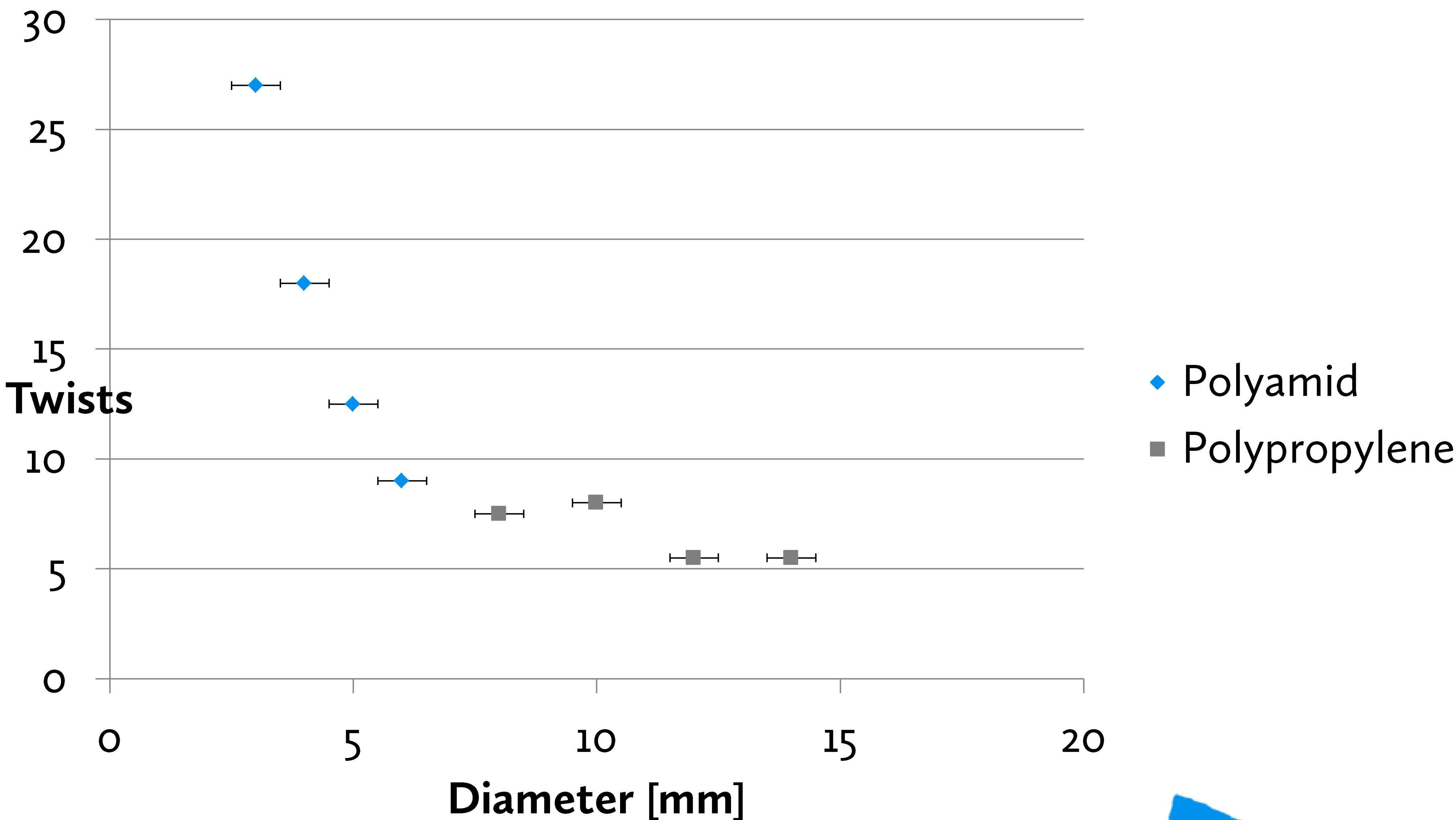
Piece of rope:
N twists needed

Half as thick:
**Similar to twice as long**
**2N twists needed**

Twice as long:
2N twists needed

Dependence on diameter: $N_{Twists} \propto \frac{1}{diameter}$

# Twists vs. Diameter



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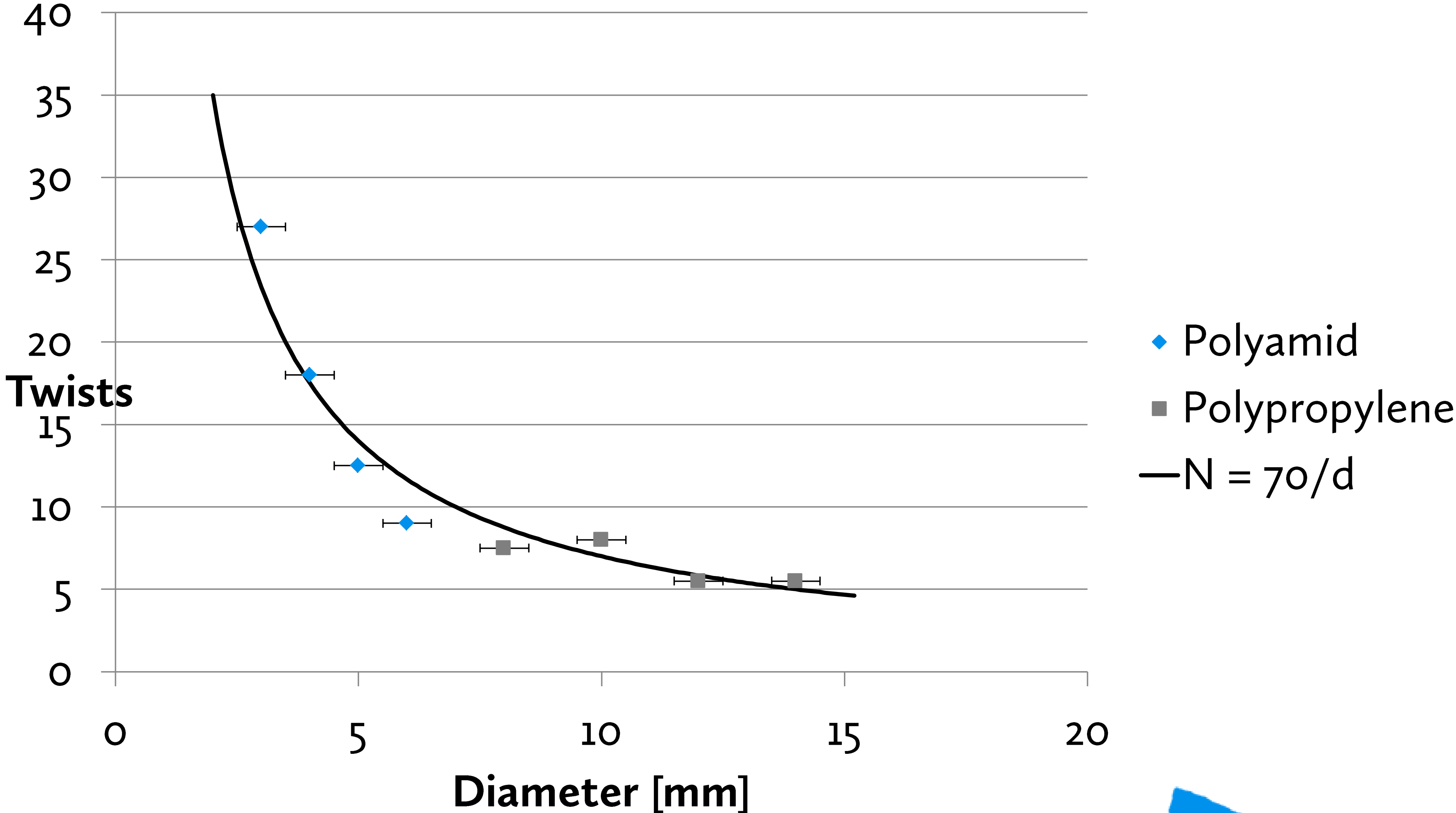


Changed:

**Diameter** (different ropes)

Measured:

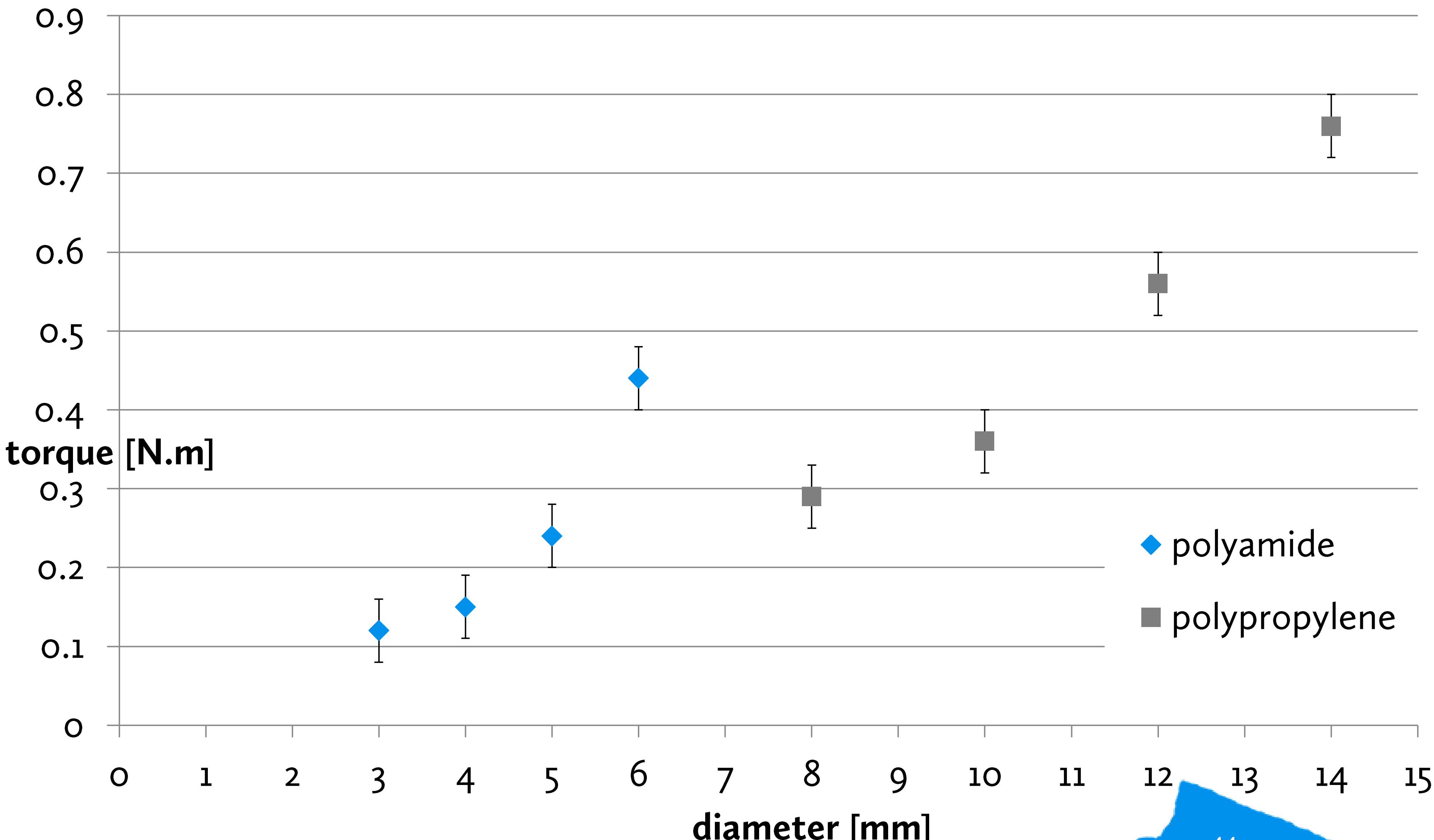
**Critical torque**

$$2\pi\tau \geq F\Delta s + \Delta E_B$$

Grows with diameter    Grows with diameter

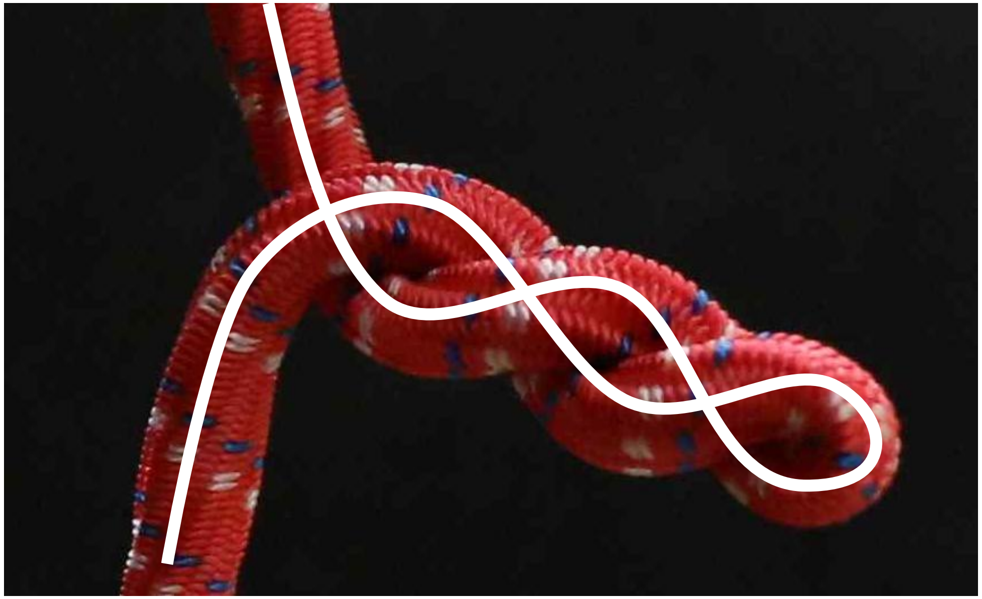
→ **Greater torque needed**

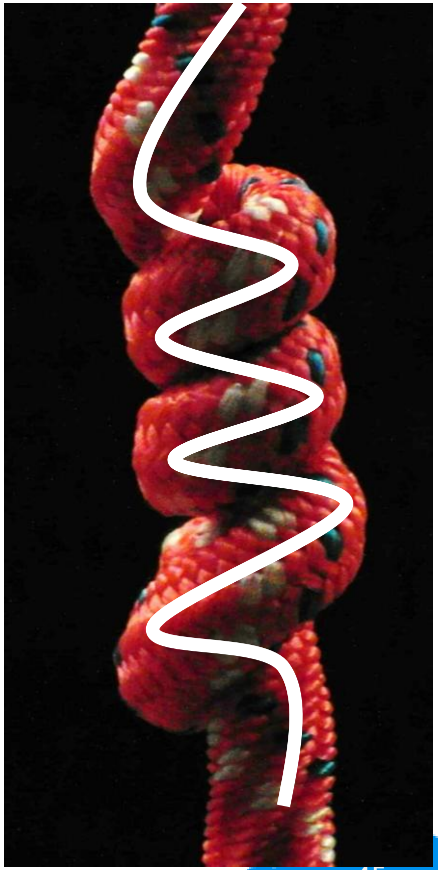
# Max torque dep. on diameter



# Conclusion

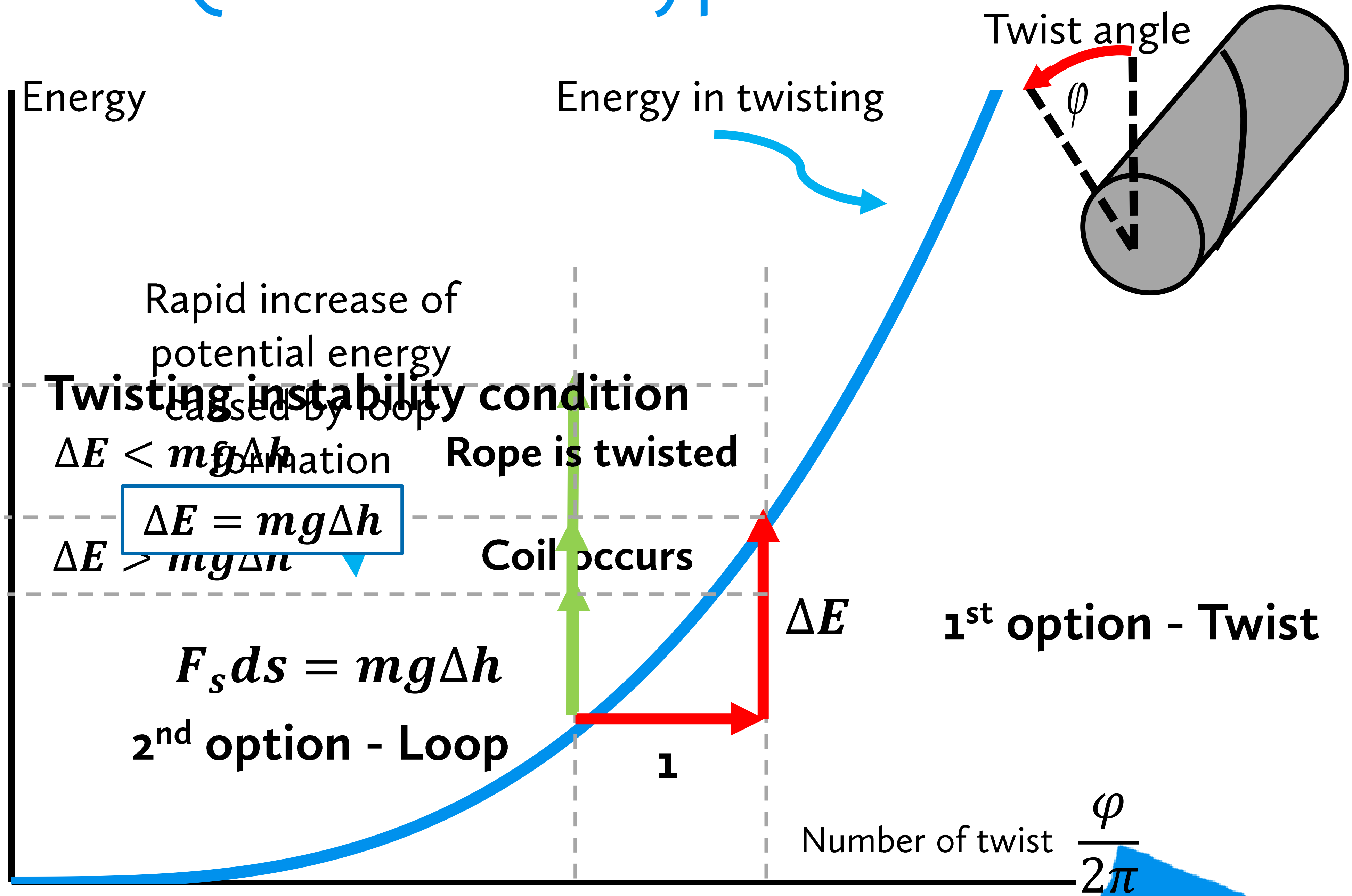
- Reason to form:
  - **Energetically favorable for large twists**
- Spirals vs. coil occurrence:
  - **small vs. large tension**

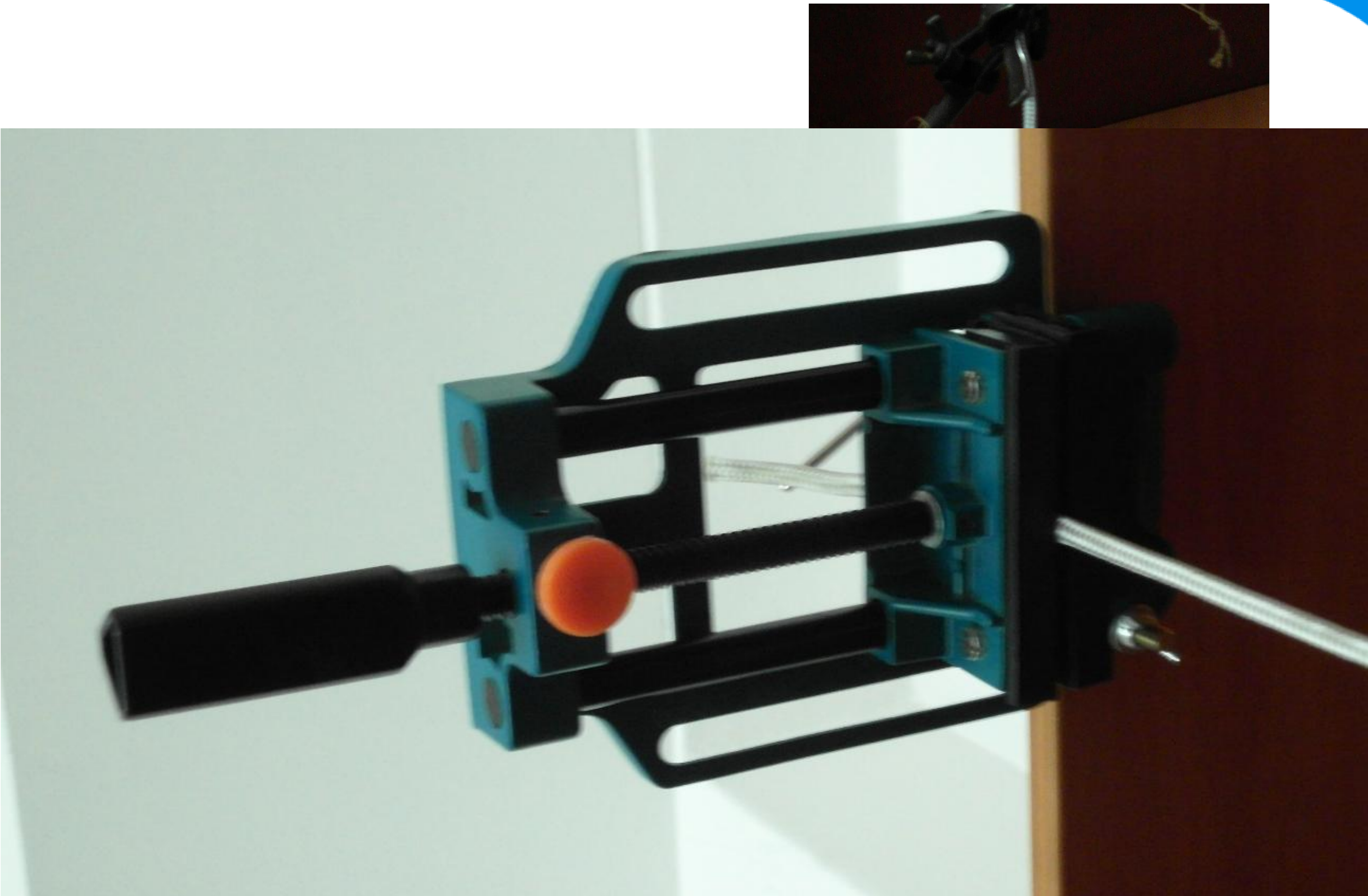


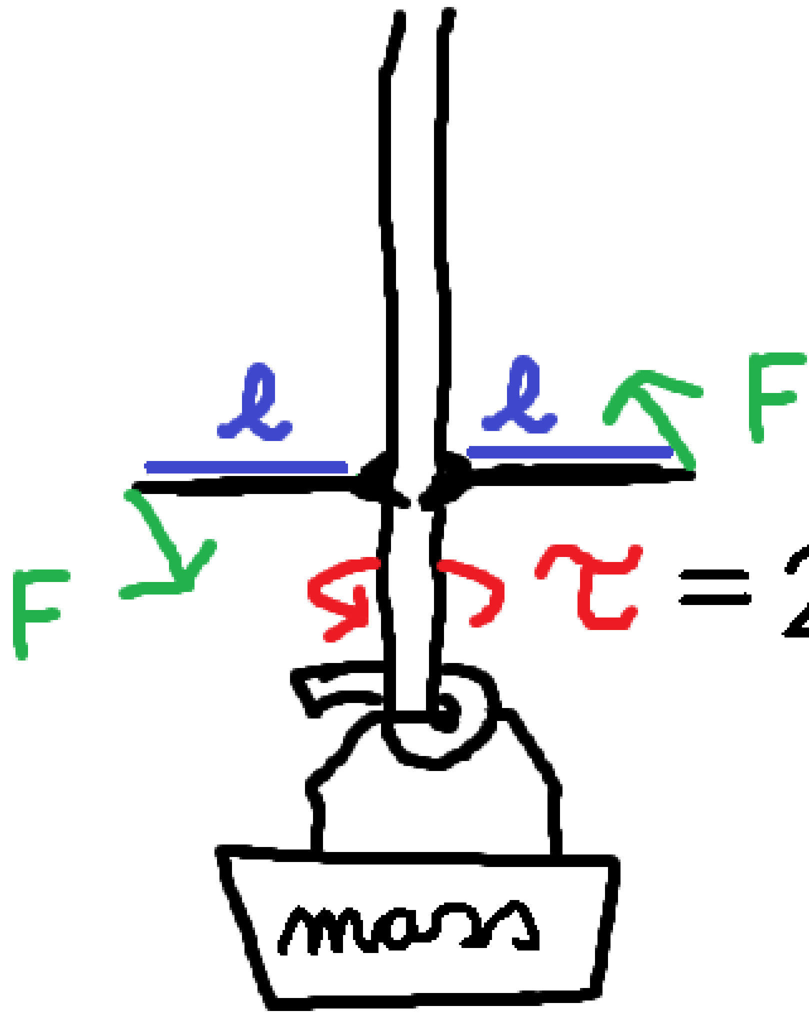


# APPENDIX

# Our Qualitative Hypothesis

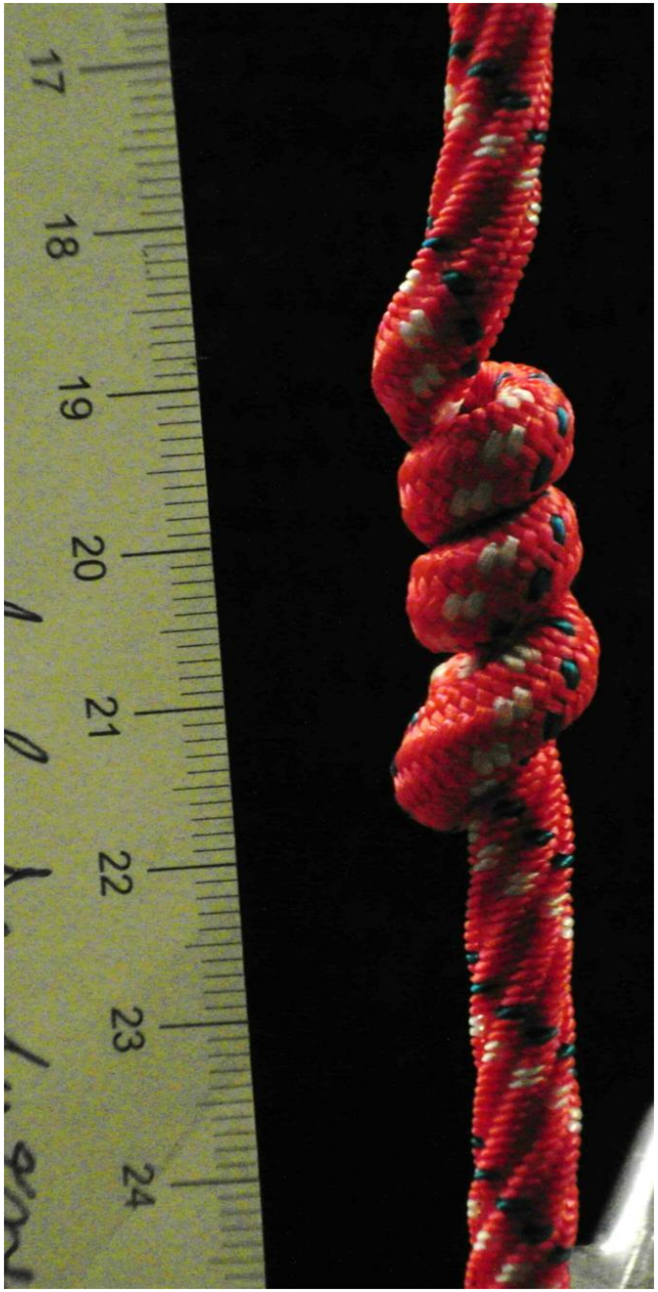







$$\tau = 2 \cdot F \cdot l$$

# Formations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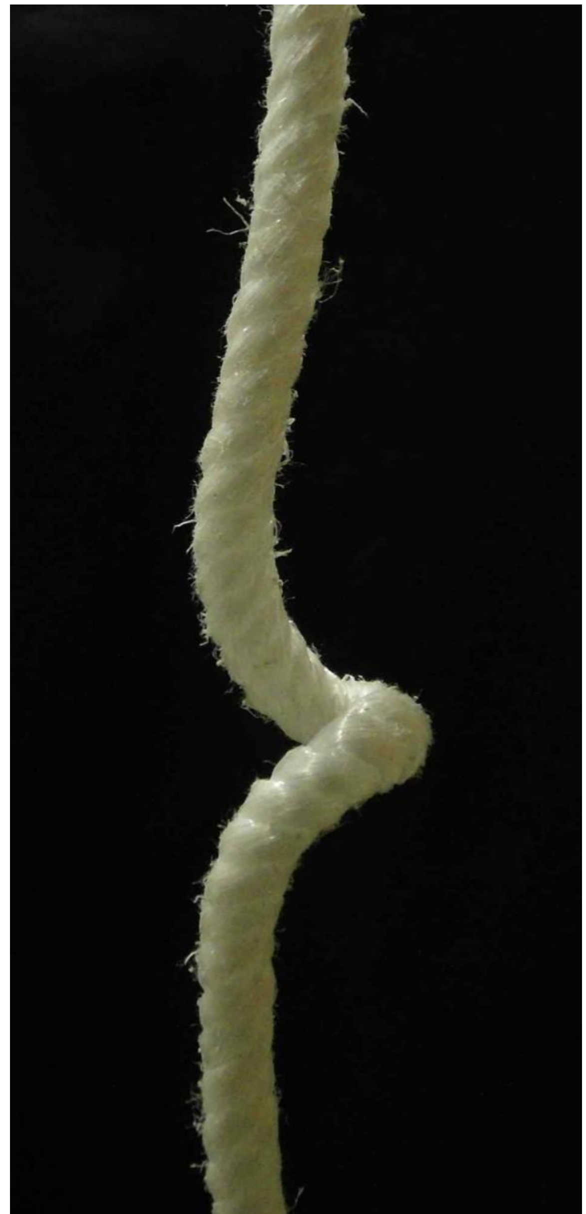
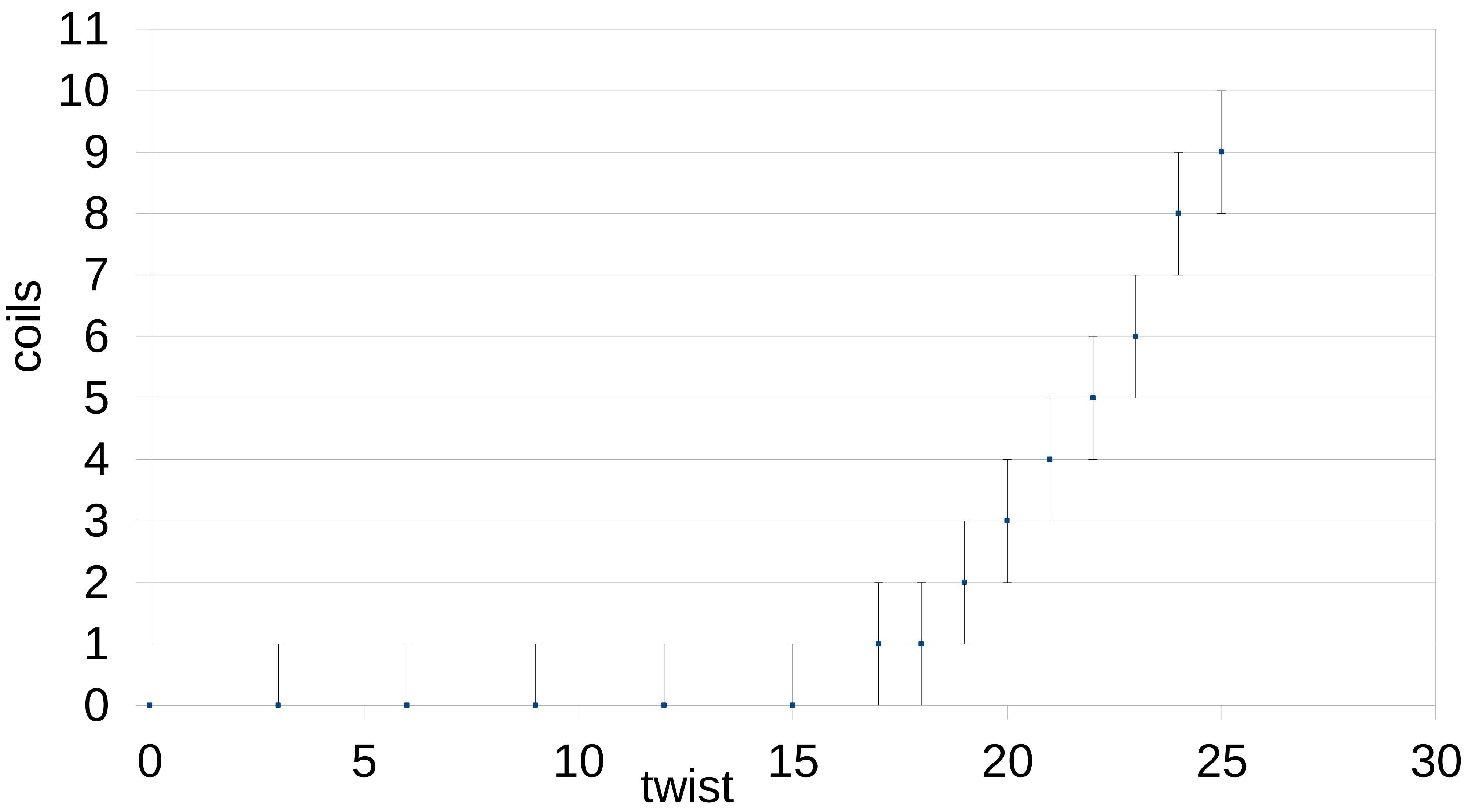
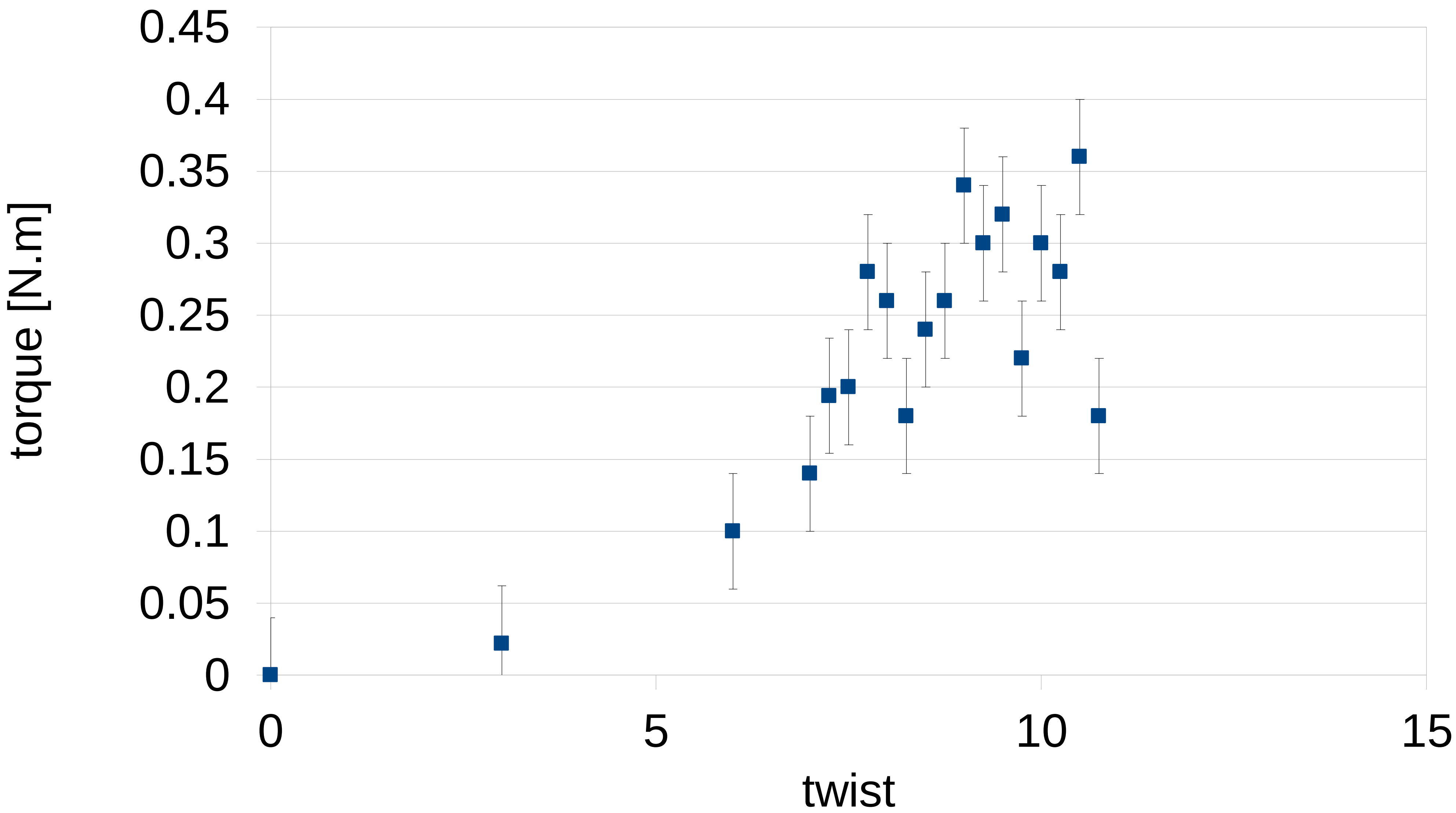


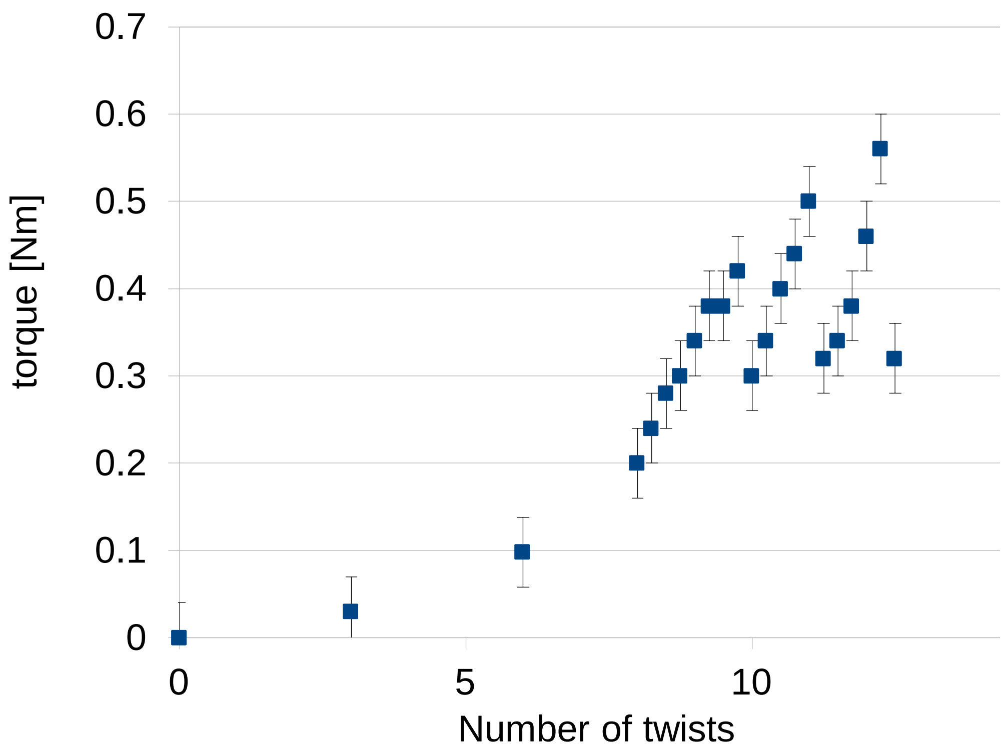
# Sisal number of coils

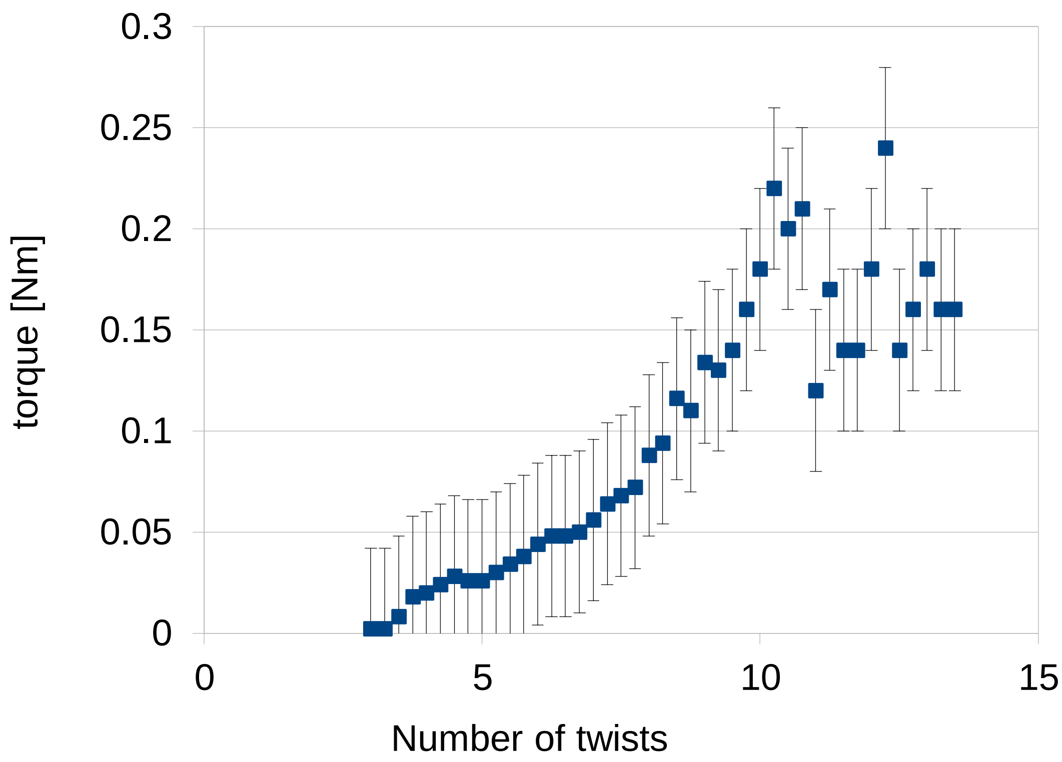


# Sisal torque



# Sudden vs. continuous



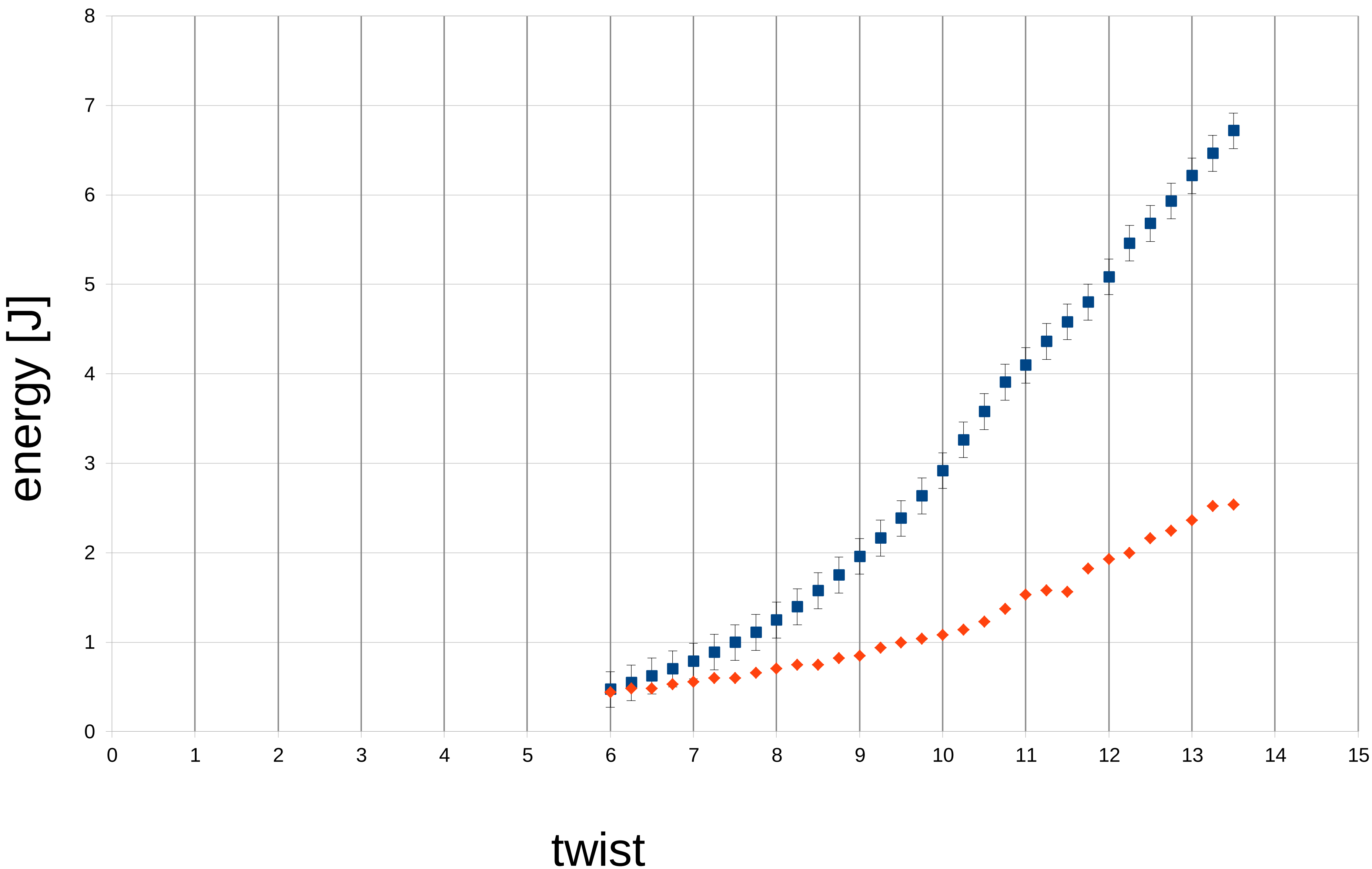



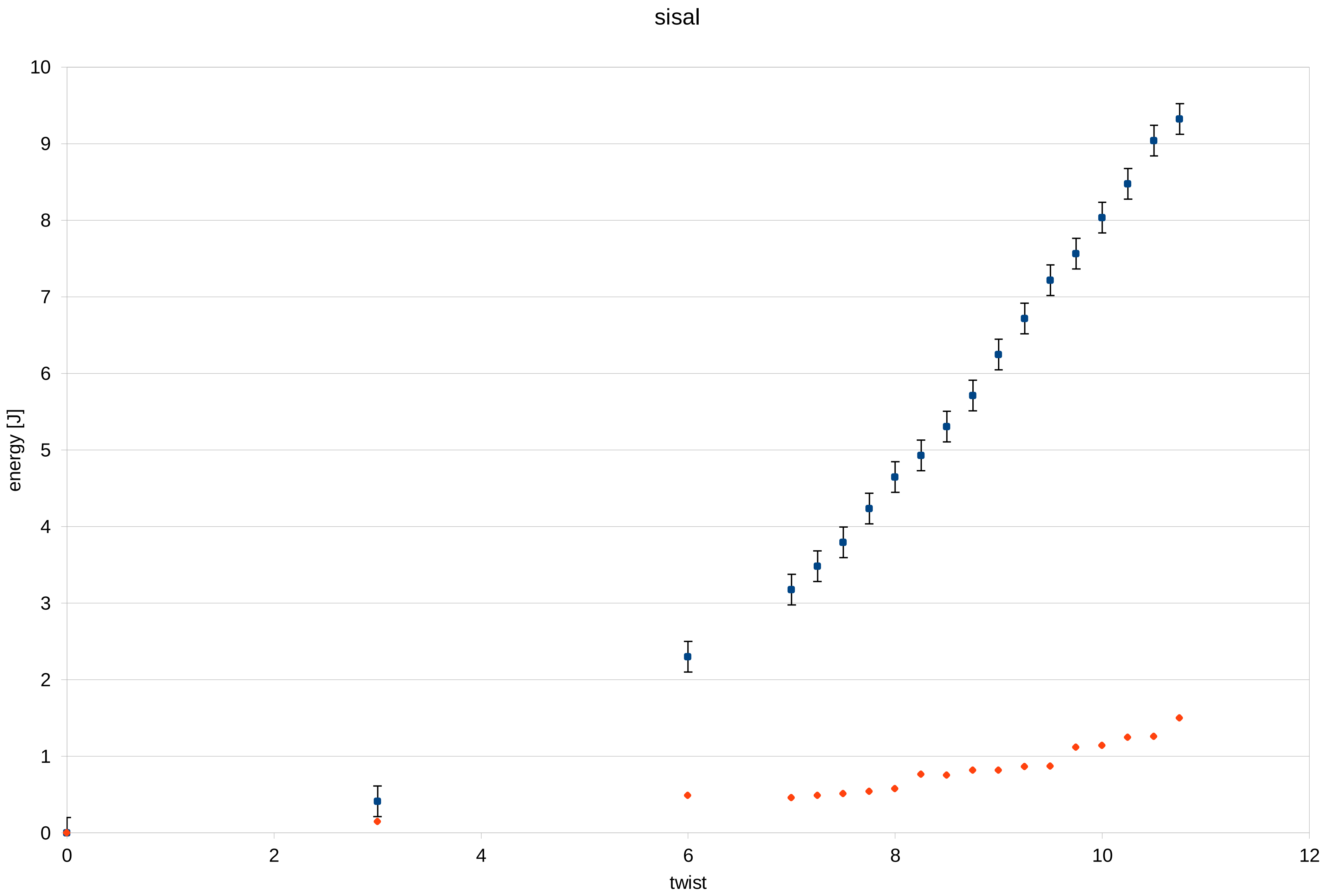

**Hypothesis:**

**Sisal – higher internal friction, delays coil creation**

**Also visible on length!**

# Work and potential energy

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# Length

- Effect of coil creation on length.