15

Contactless calliper

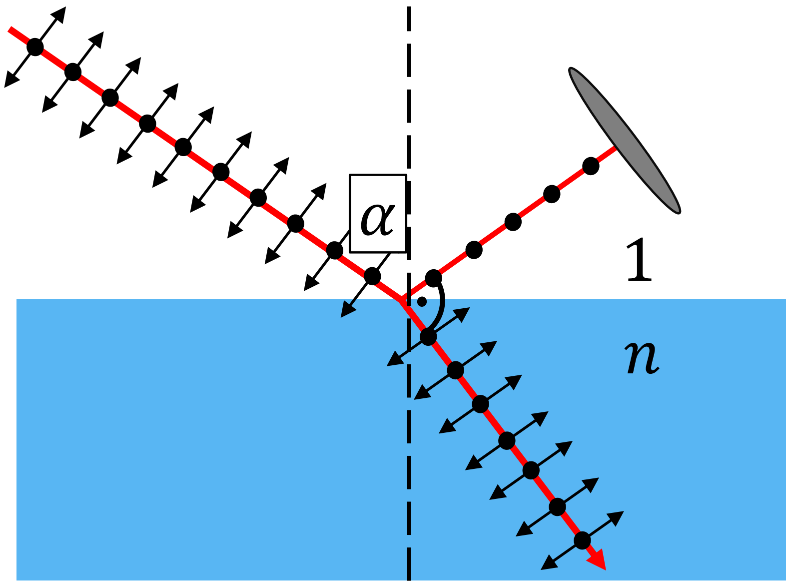
Martin Marek

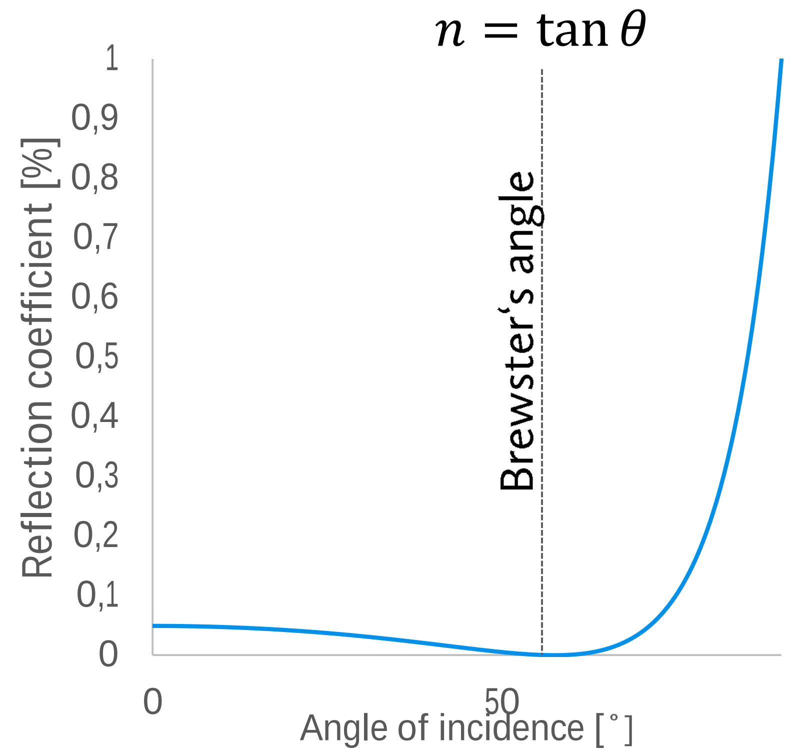
Problem definition

Invent and construct an optical device that uses a laser pointer and allows contactless determination of thickness, refractive index, and other properties of a glass sheet.

Measuring refractive index using **Brewster's angle**

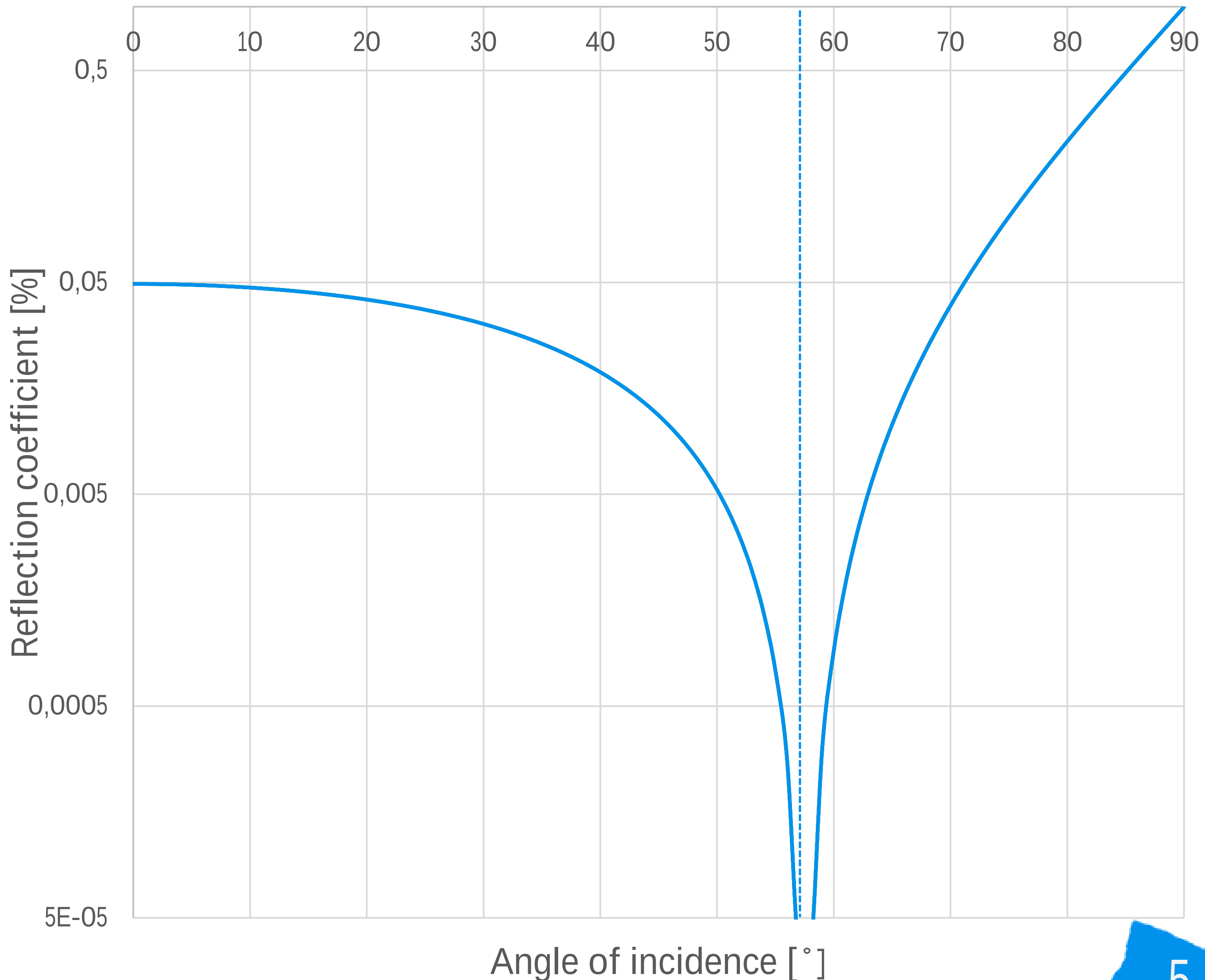
Measuring the index of refraction:
Brewster's angle



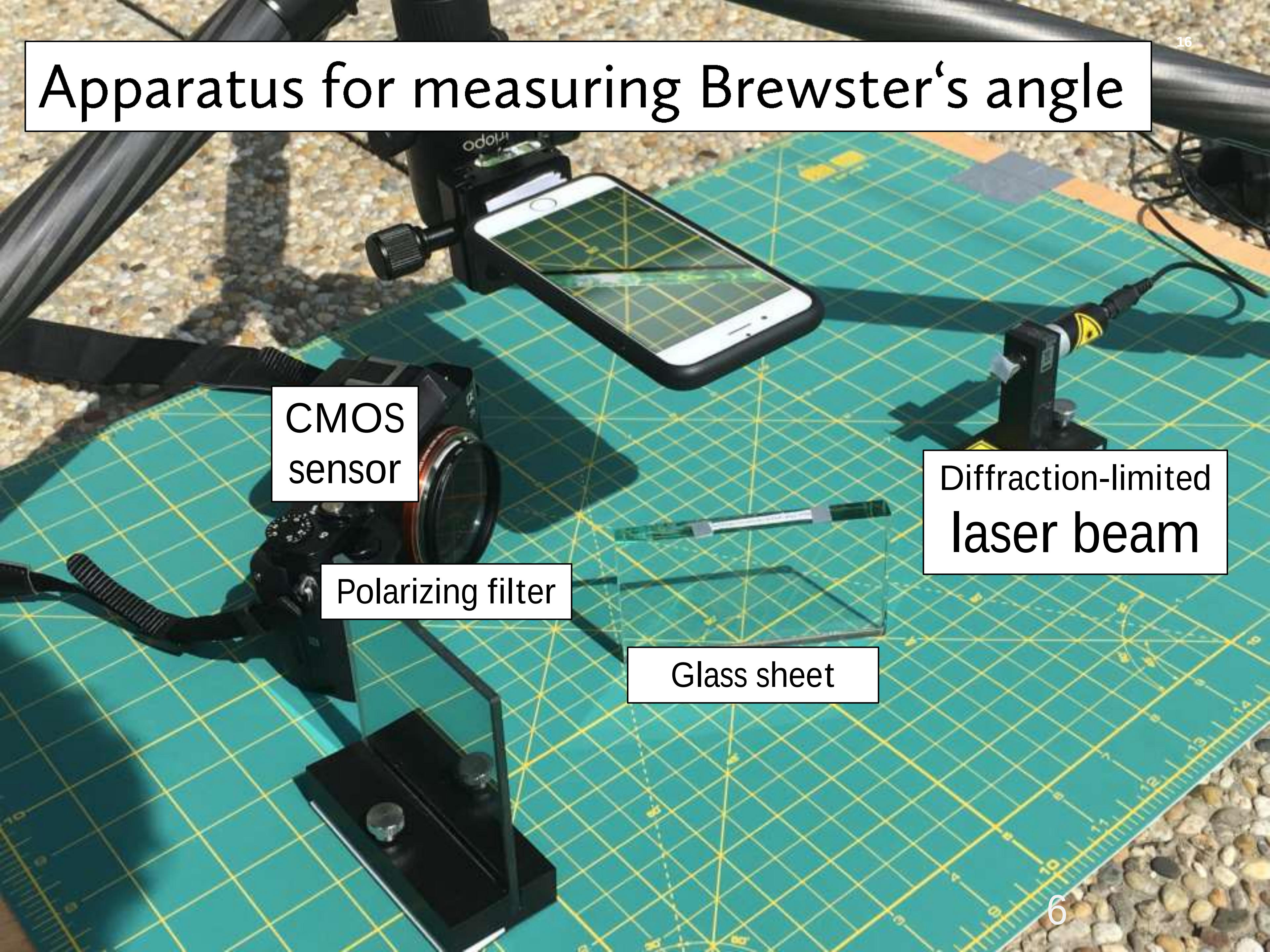


Fresnel equations → **Brewster's angle**
(reflected beam is completely *s-polarized*)

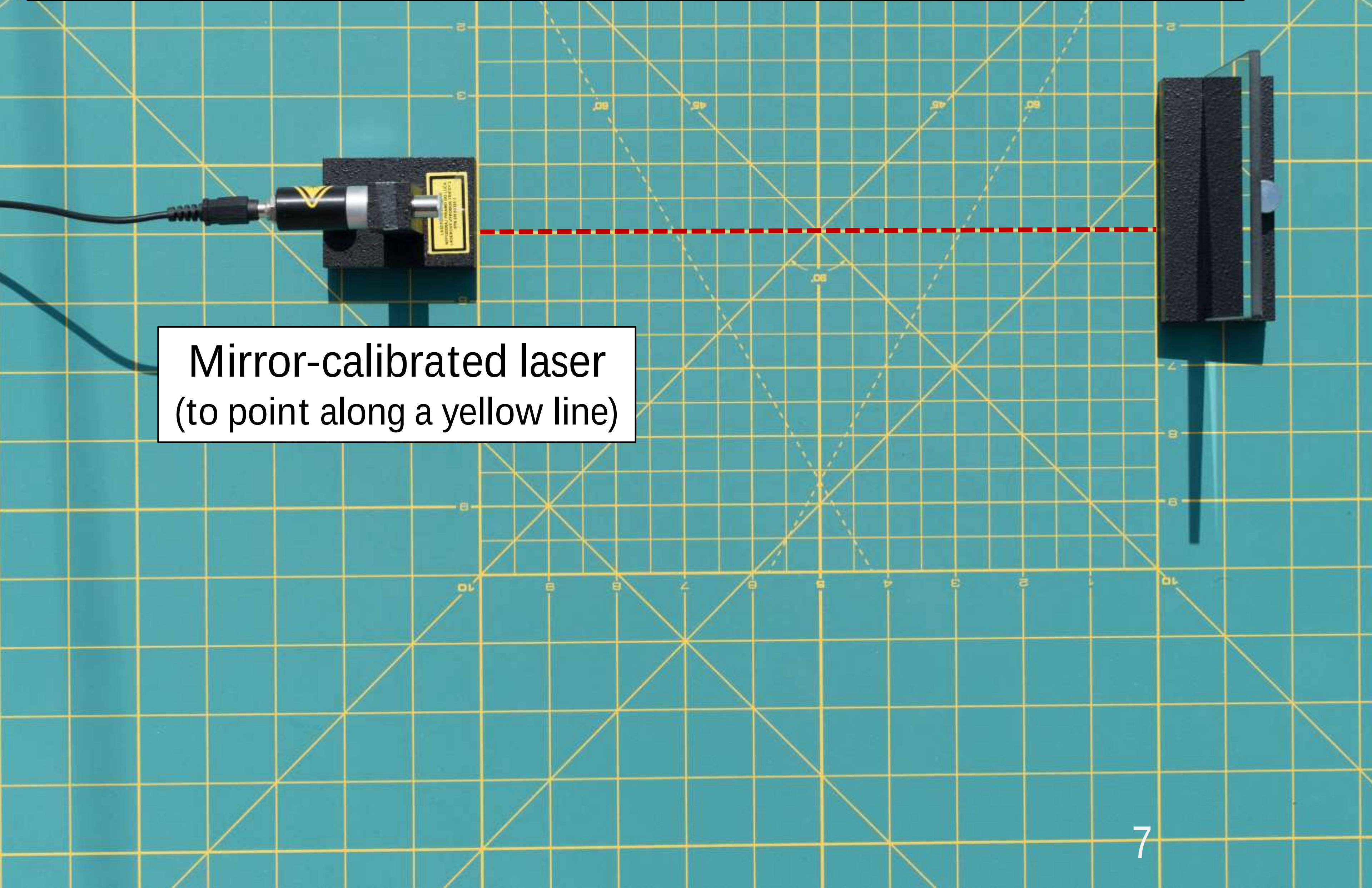
Brewster's angle in logarithmic scale



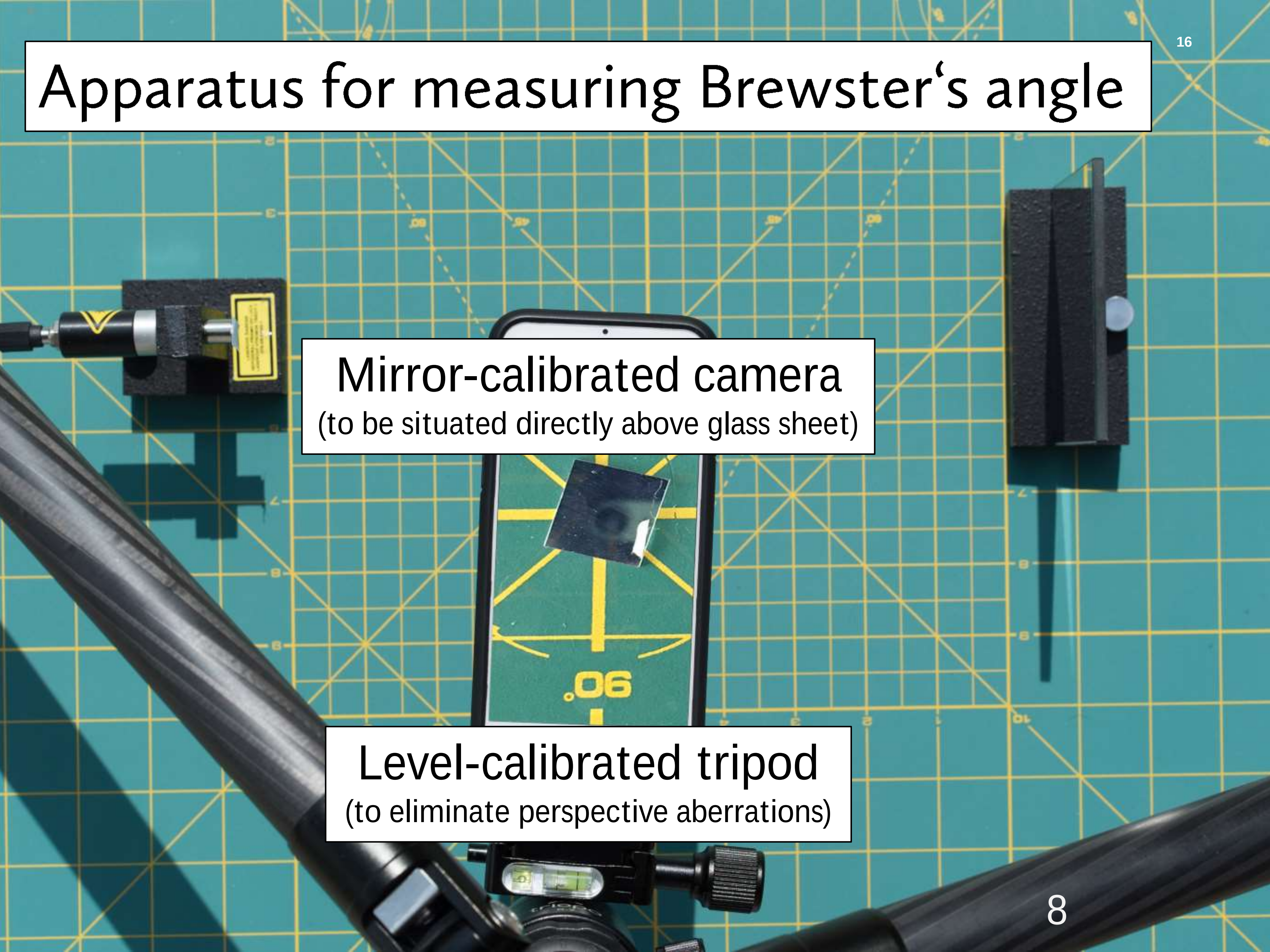
Apparatus for measuring Brewster's angle



Apparatus for measuring Brewster‘s angle



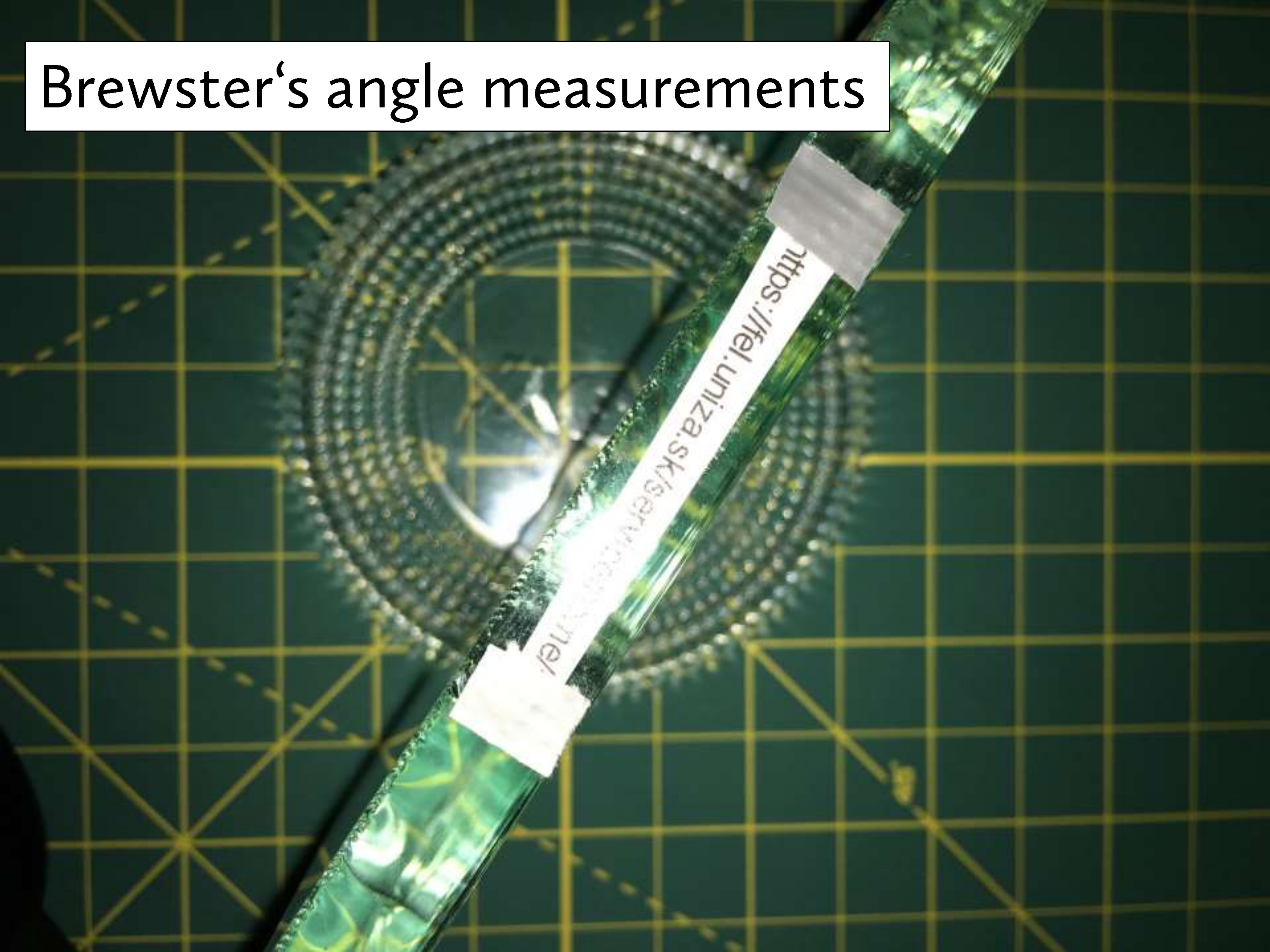
Apparatus for measuring Brewster‘s angle



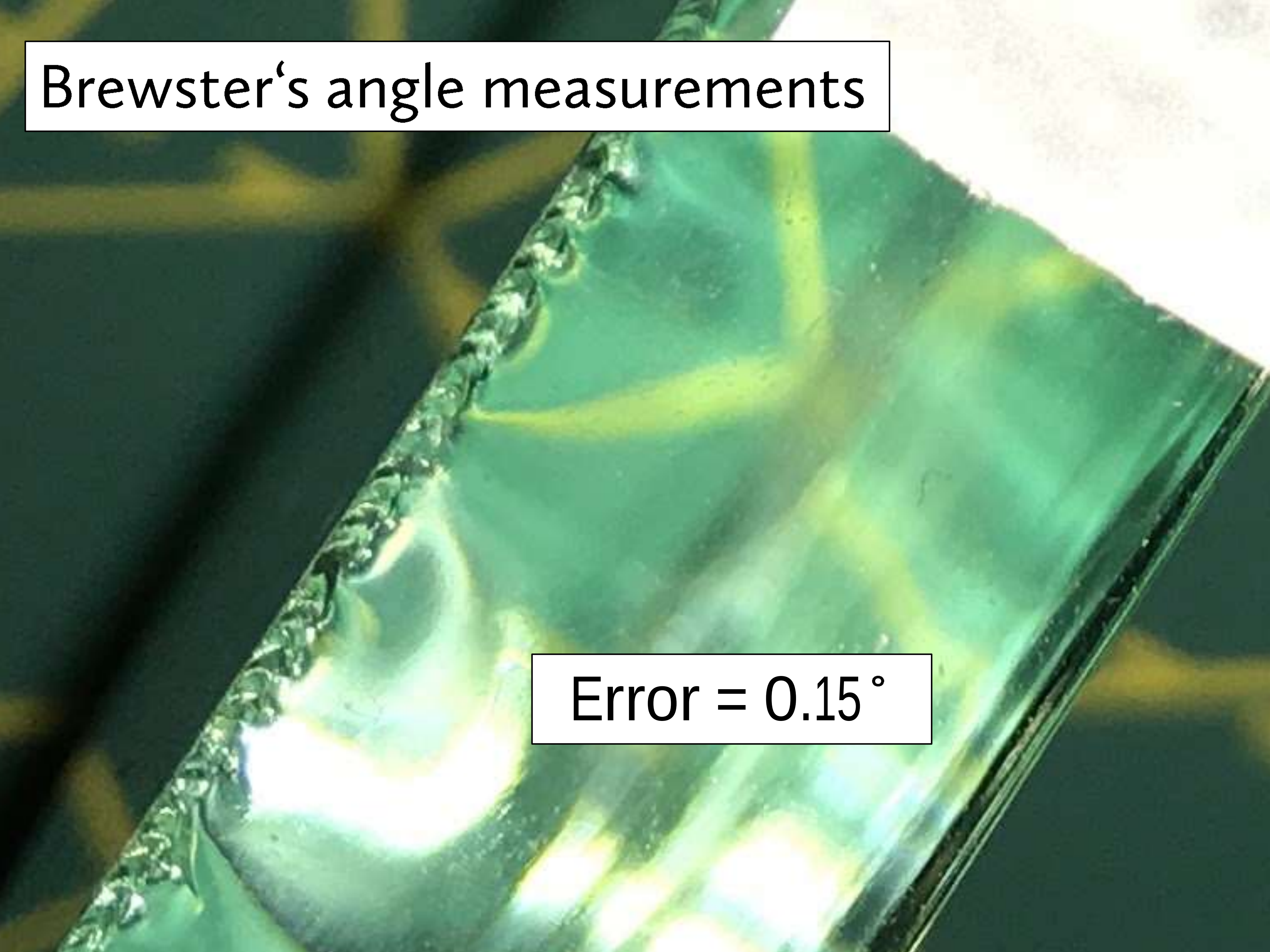
Mirror-calibrated camera
(to be situated directly above glass sheet)

Level-calibrated tripod
(to eliminate perspective aberrations)

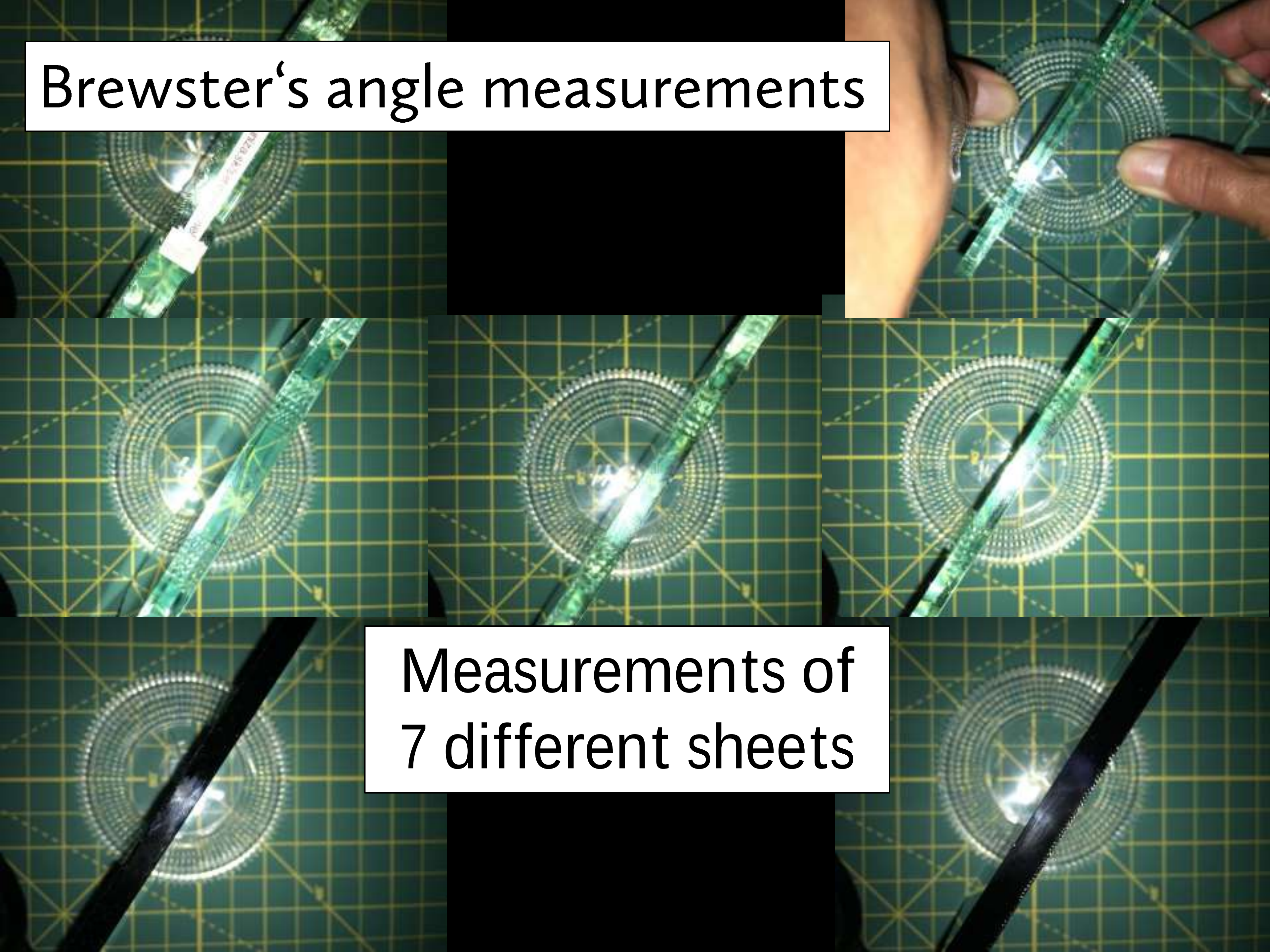
Brewster's angle measurements



Brewster's angle measurements

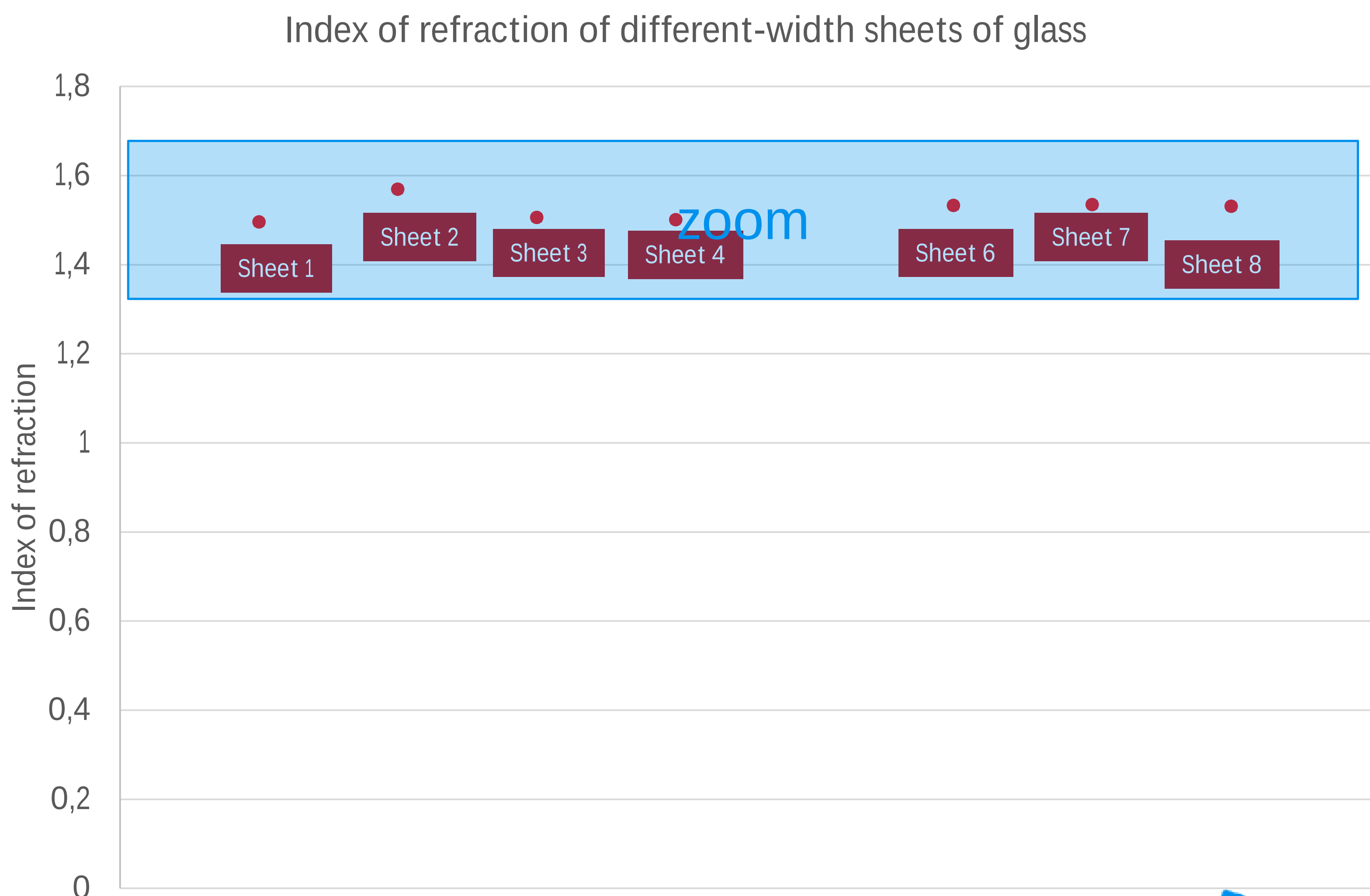


Brewster‘s angle measurements

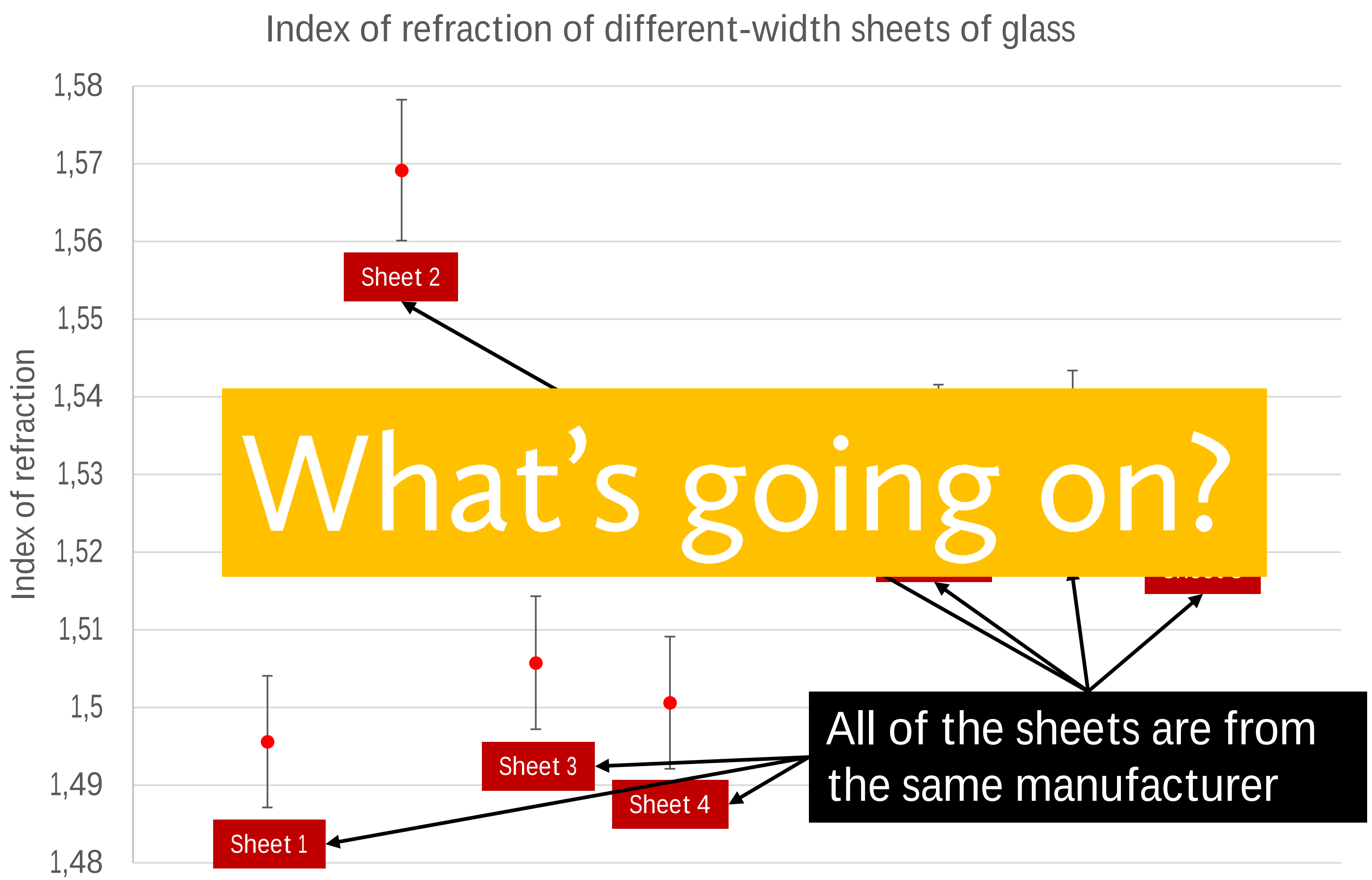


Measurements of
7 different sheets

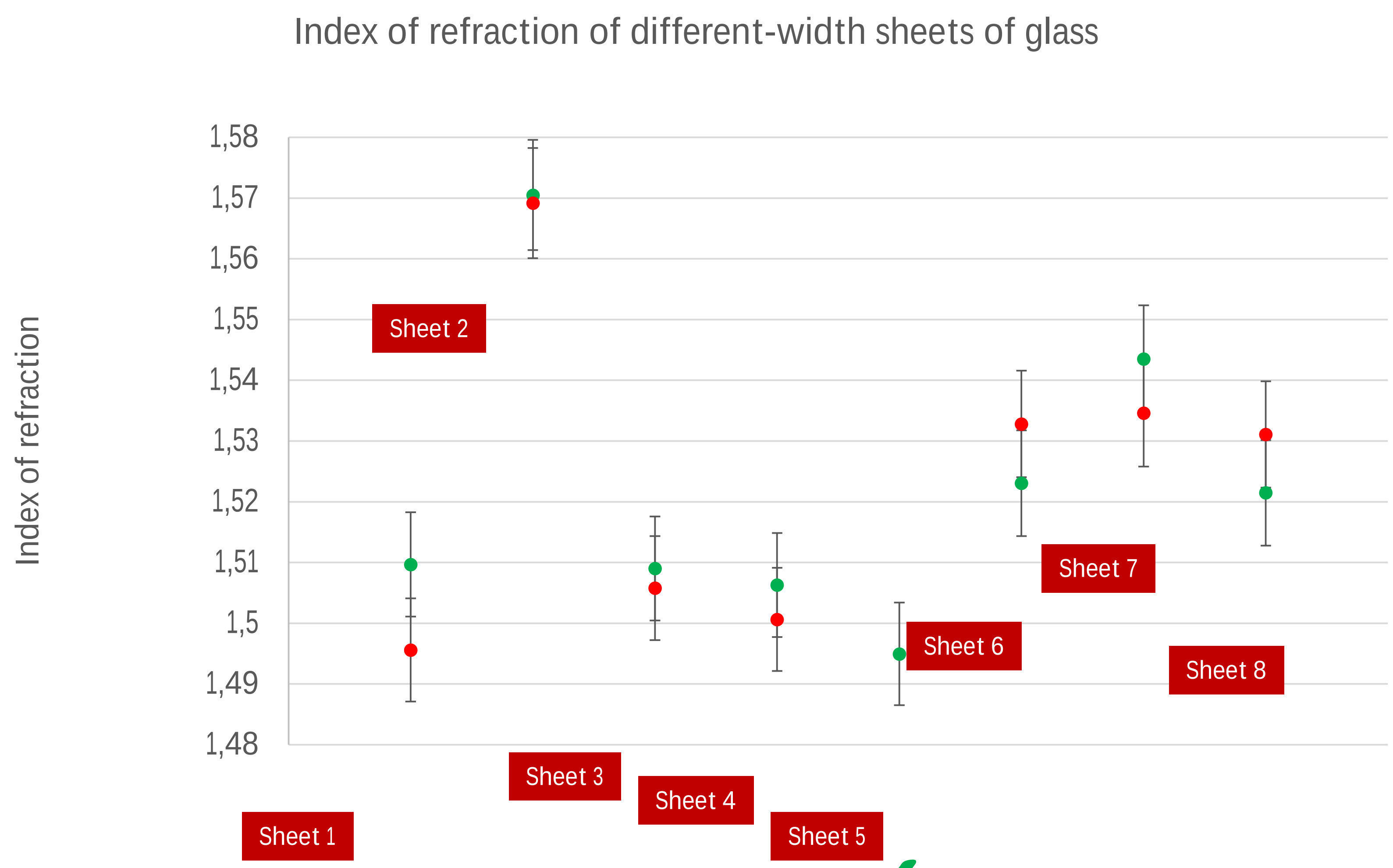
Brewster‘s angle measurements



Brewster's angle measurements

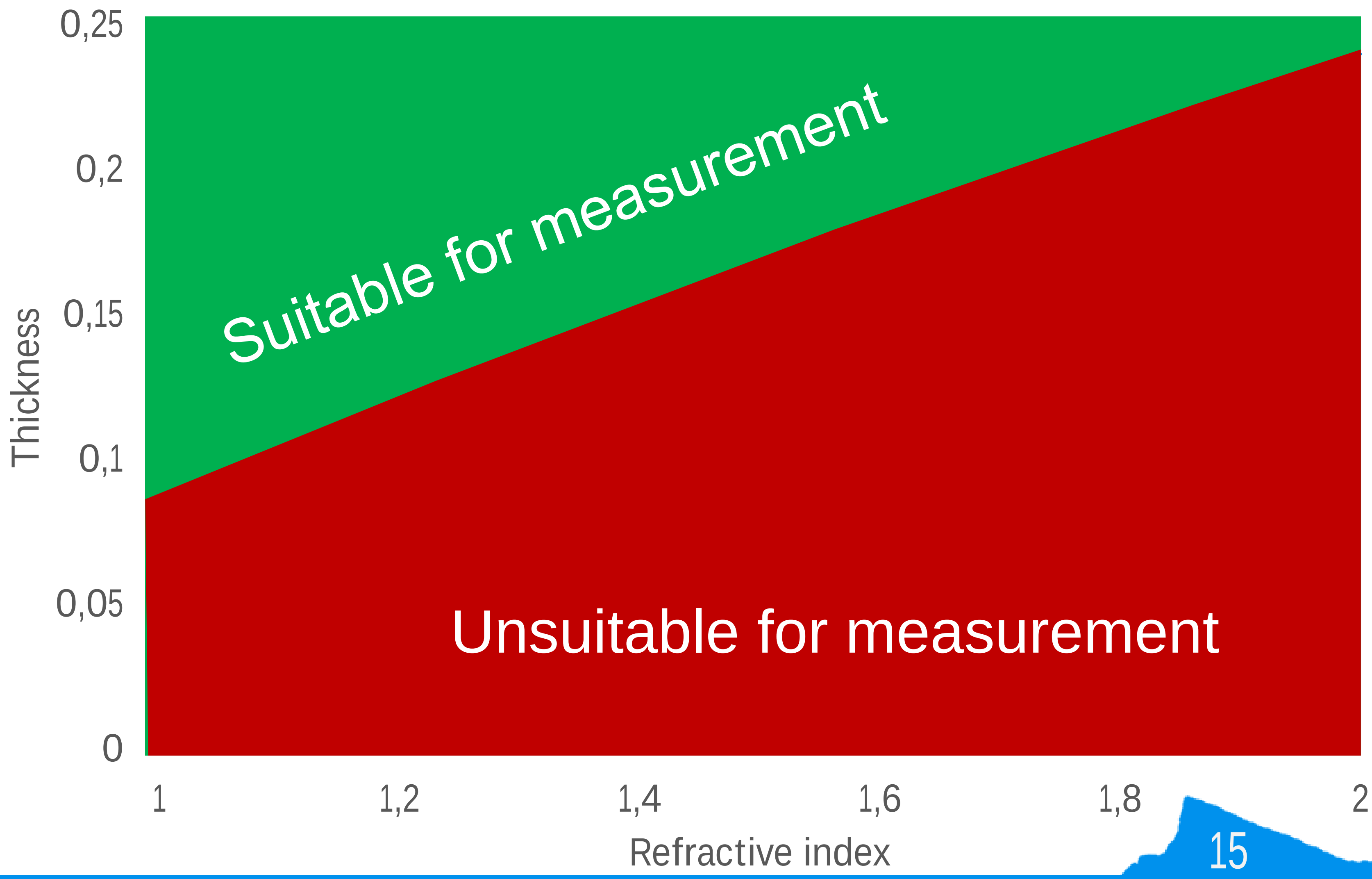


Brewster's angle: 2nd (verification) measurements



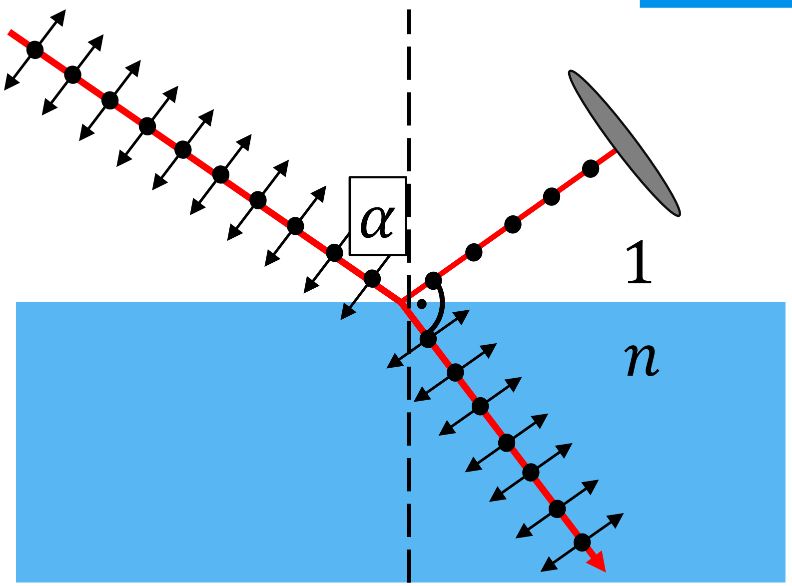
n error ≈ 0.008 ✓

Limitations of reflections method: phase diagram



Brewster's angle: conclusion

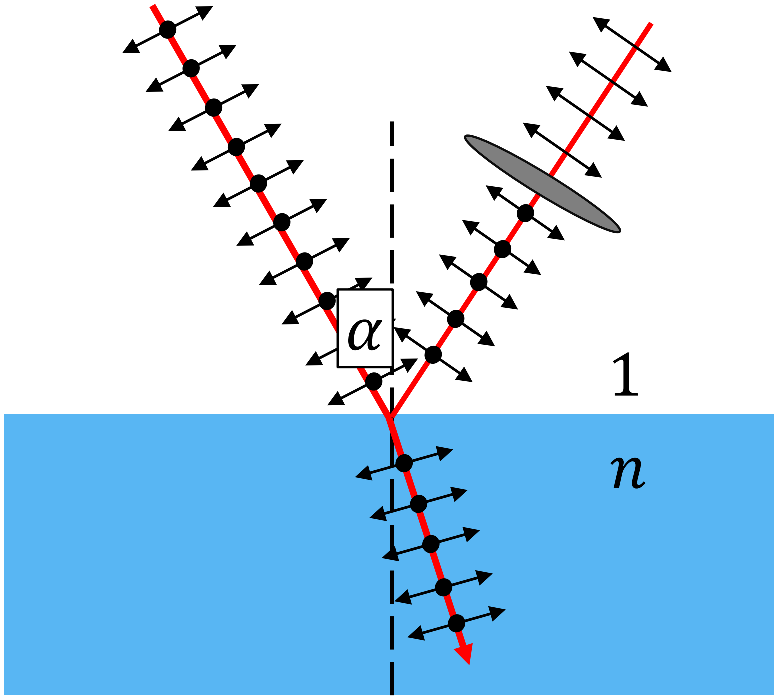
Usage:	Refractive index
Error:	± 0.08
Limitation (n = 1.5):	~0.16mm

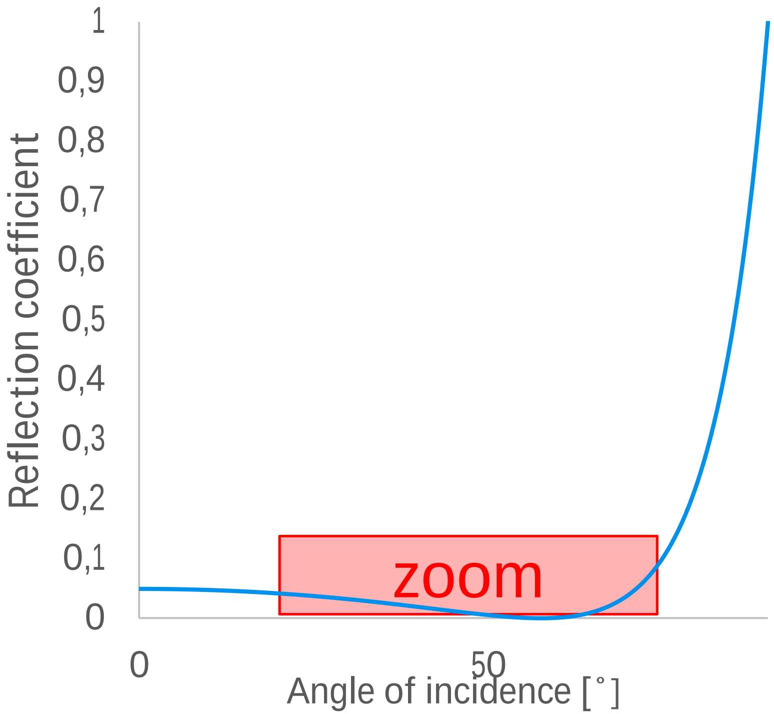


Measuring refractive index using Fresnel equations

2nd method of measuring the index of refraction: Intensity of *p-polarized* reflected light

$$R_p = \left| \frac{n_1 cos\theta_t \ - n_2 \ cos\theta_i}{n_1 cos\theta_t + n_2 \ cos\theta_i} \right|^2$$





2nd method of measuring the index of refraction: Intensity of *p-polarized* reflected light

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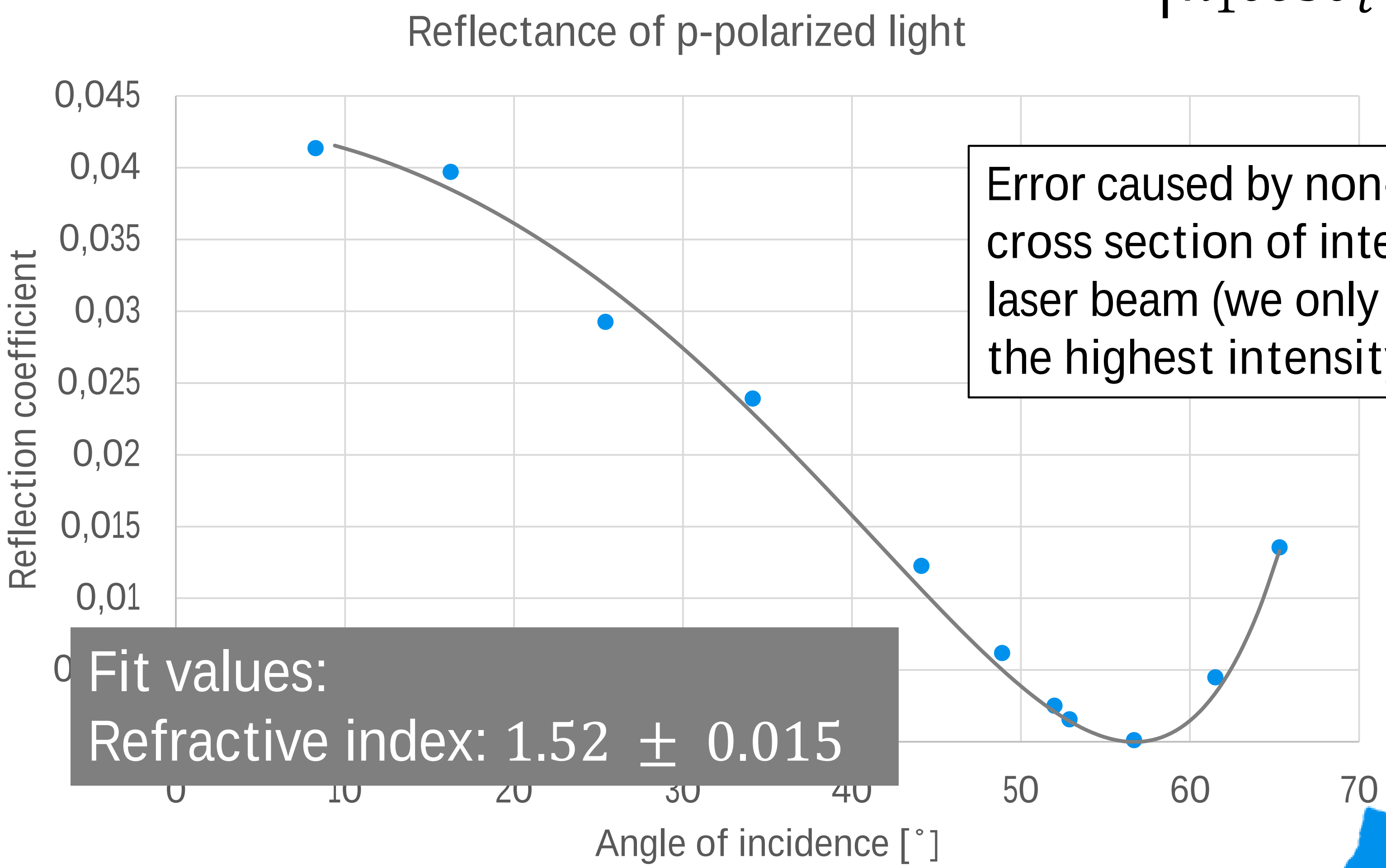
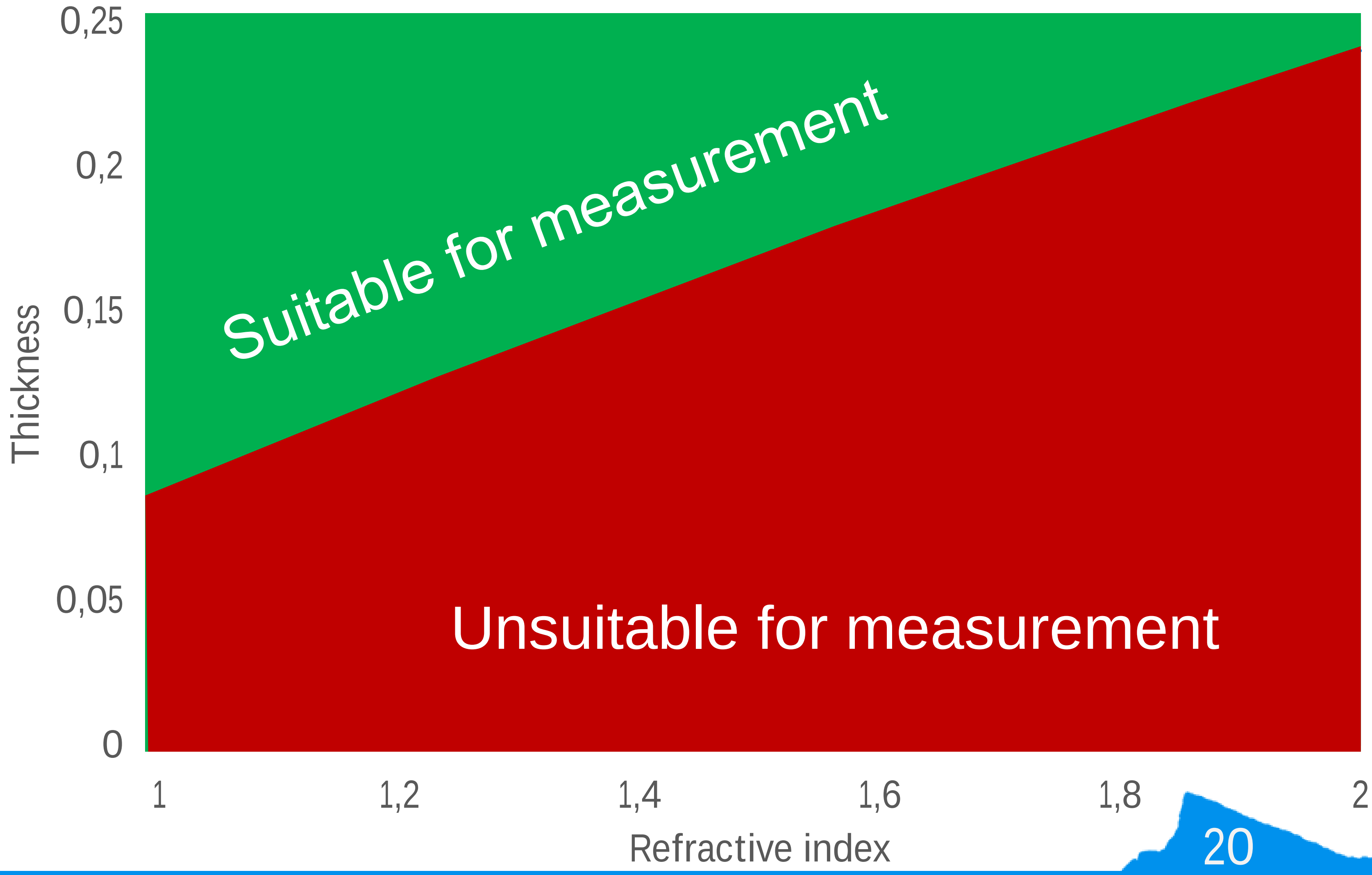


Error caused by non-uniform cross section of intensity of laser beam (we only consider the highest intensity pixel)

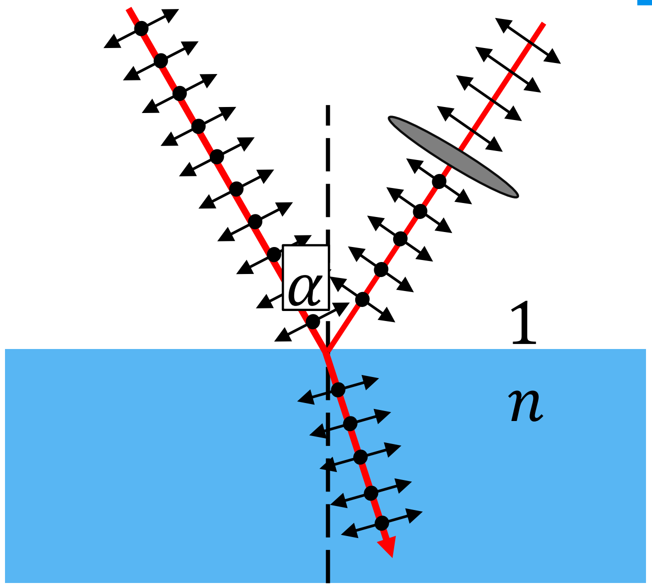
Fit values:
Refractive index: 1.52 ± 0.015

Limitations of reflections method: phase diagram



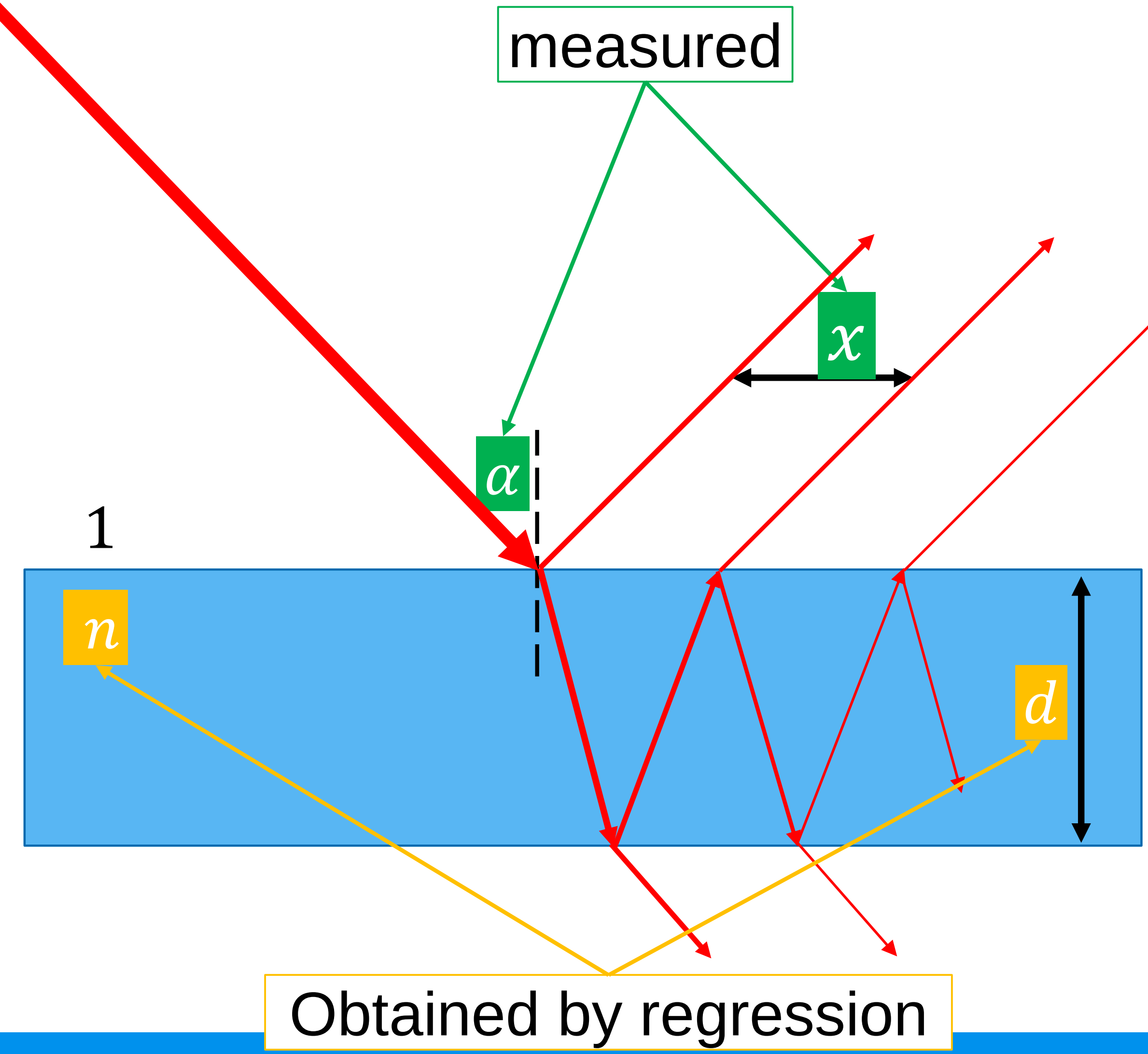
Fresnel equations: conclusion

Usage:	Refractive index
Error:	± 0.015
Limitation (n = 1.5):	0.16mm



Measuring thickness using internal reflections

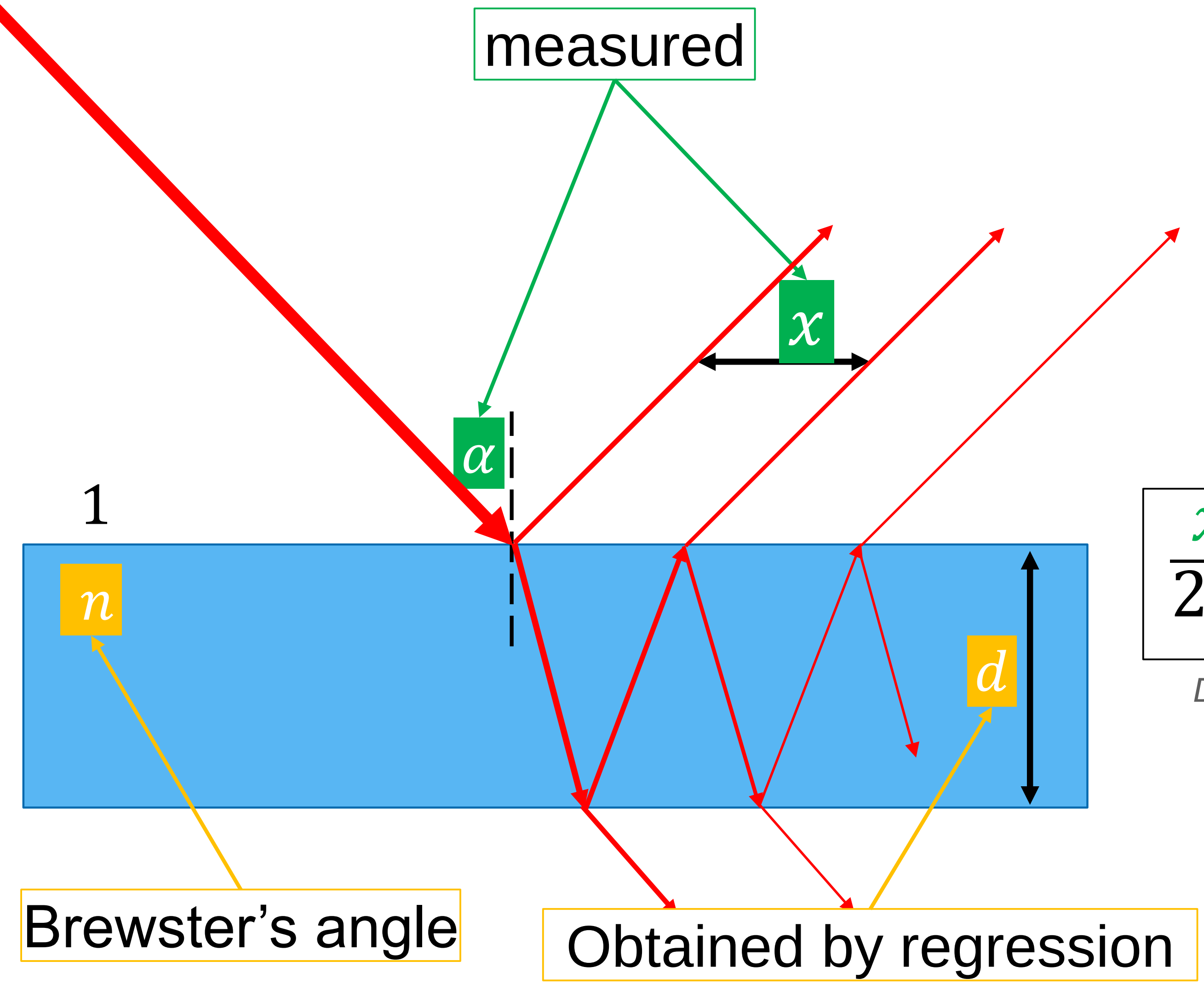
Thickness and index of refraction: internal reflection



$$\frac{x}{2d} = \frac{\sin\alpha}{\sqrt{n^2 - \sin^2\alpha}}$$

Derived from geometrical optics

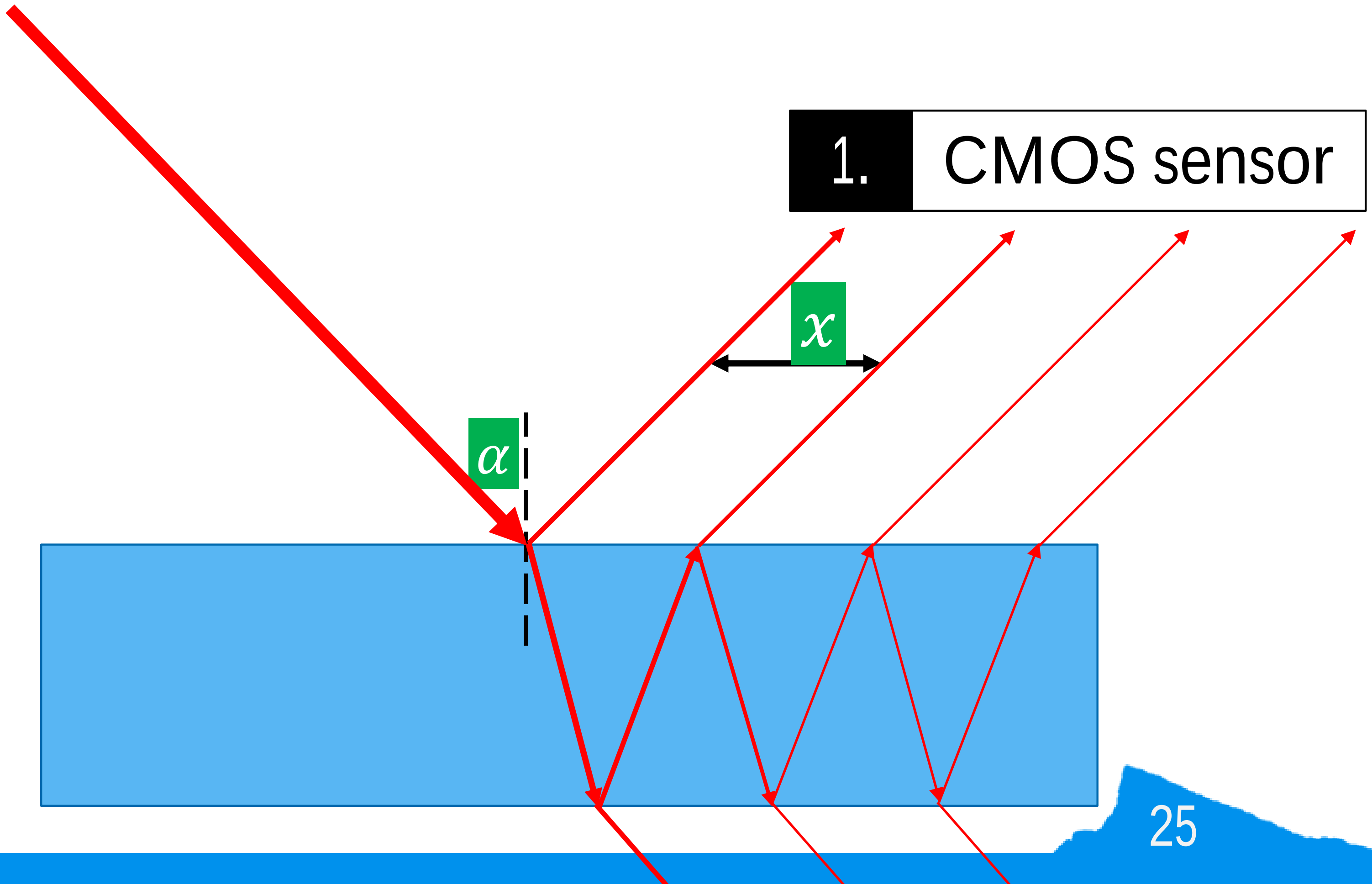
Thickness and index of refraction: internal reflection



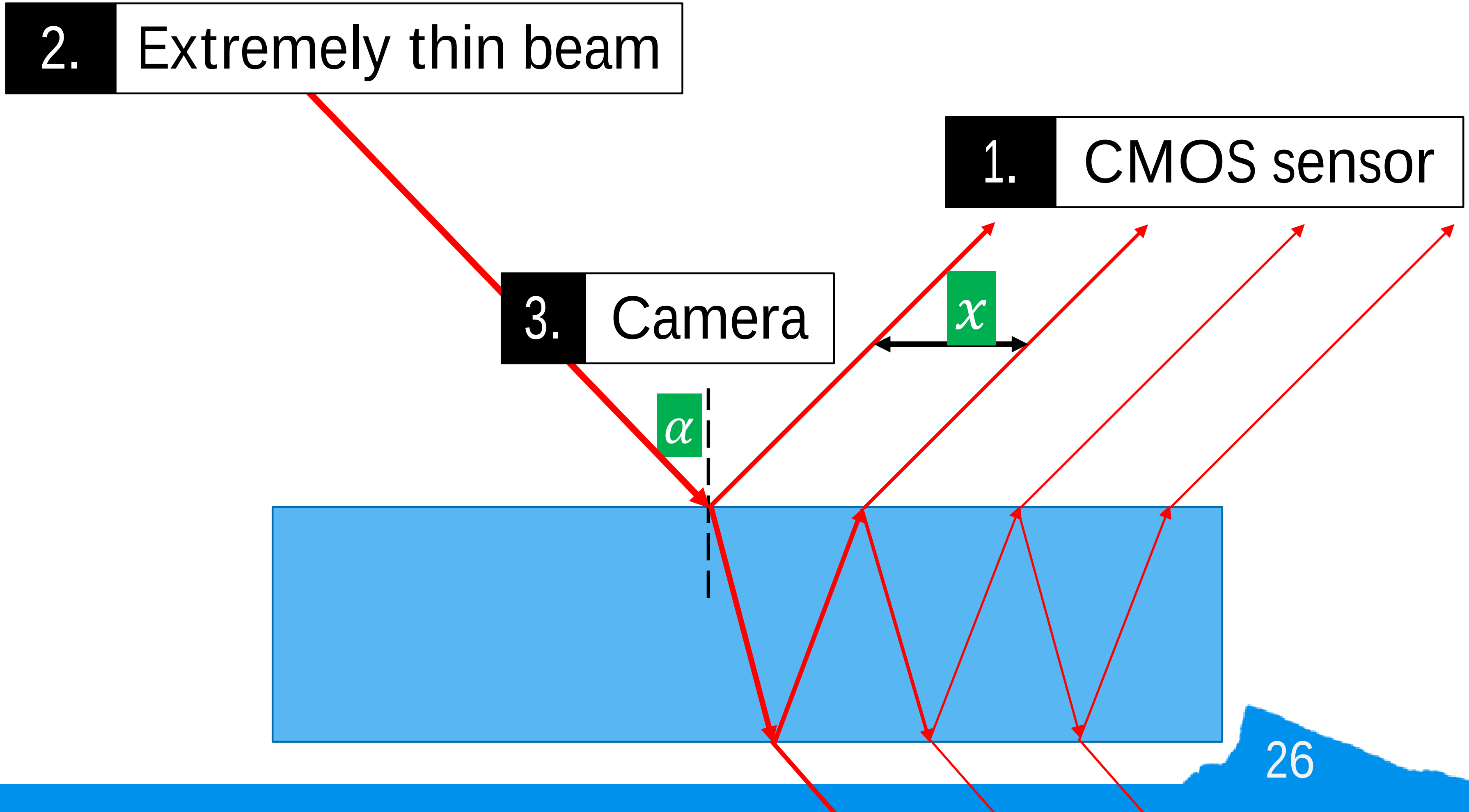
$$\frac{x}{2d} = \frac{\sin \alpha}{\sqrt{n^2 - \sin^2 \alpha}}$$

Derived from geometrical optics

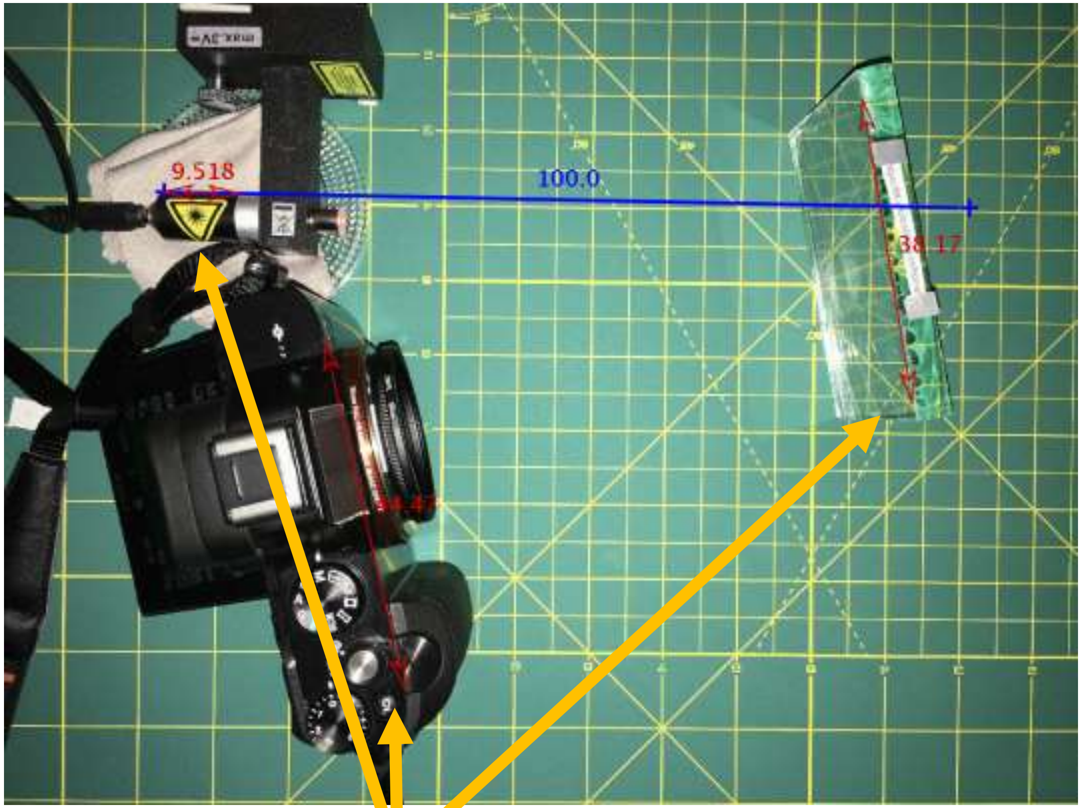
Thickness and index of refraction: internal reflection



Thickness and index of refraction: internal reflection

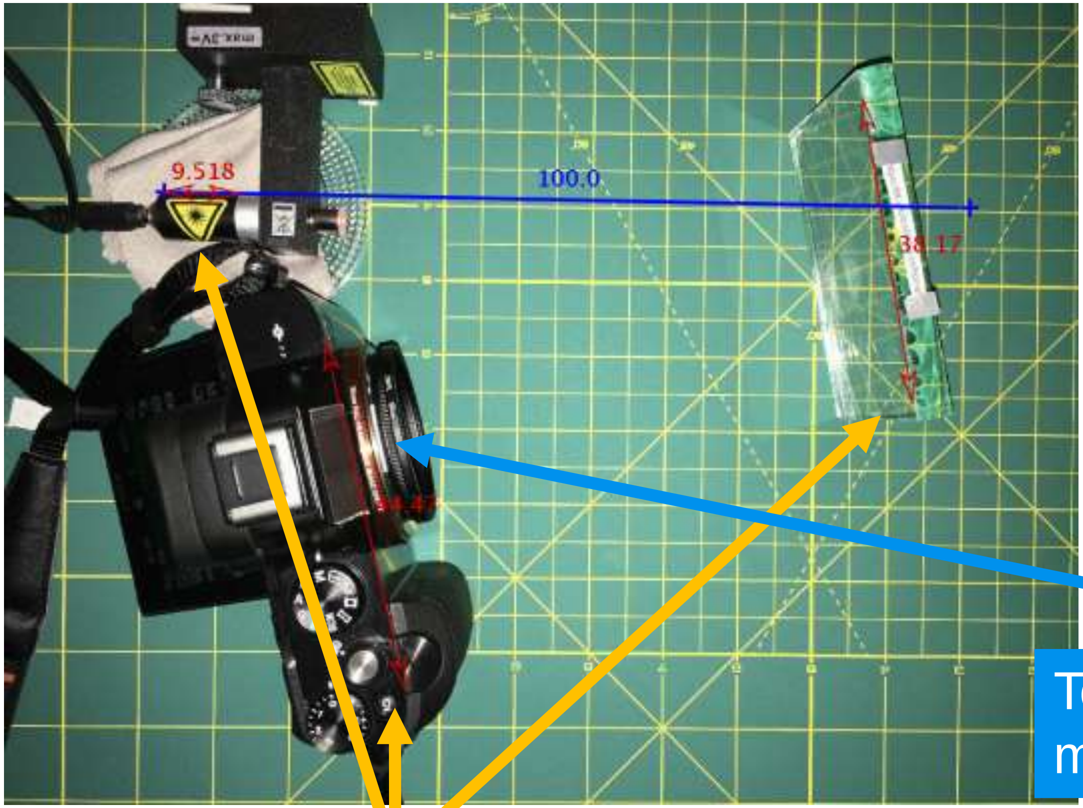


Digitally measuring the distance and angle



Took a picture from the top to measure the angles

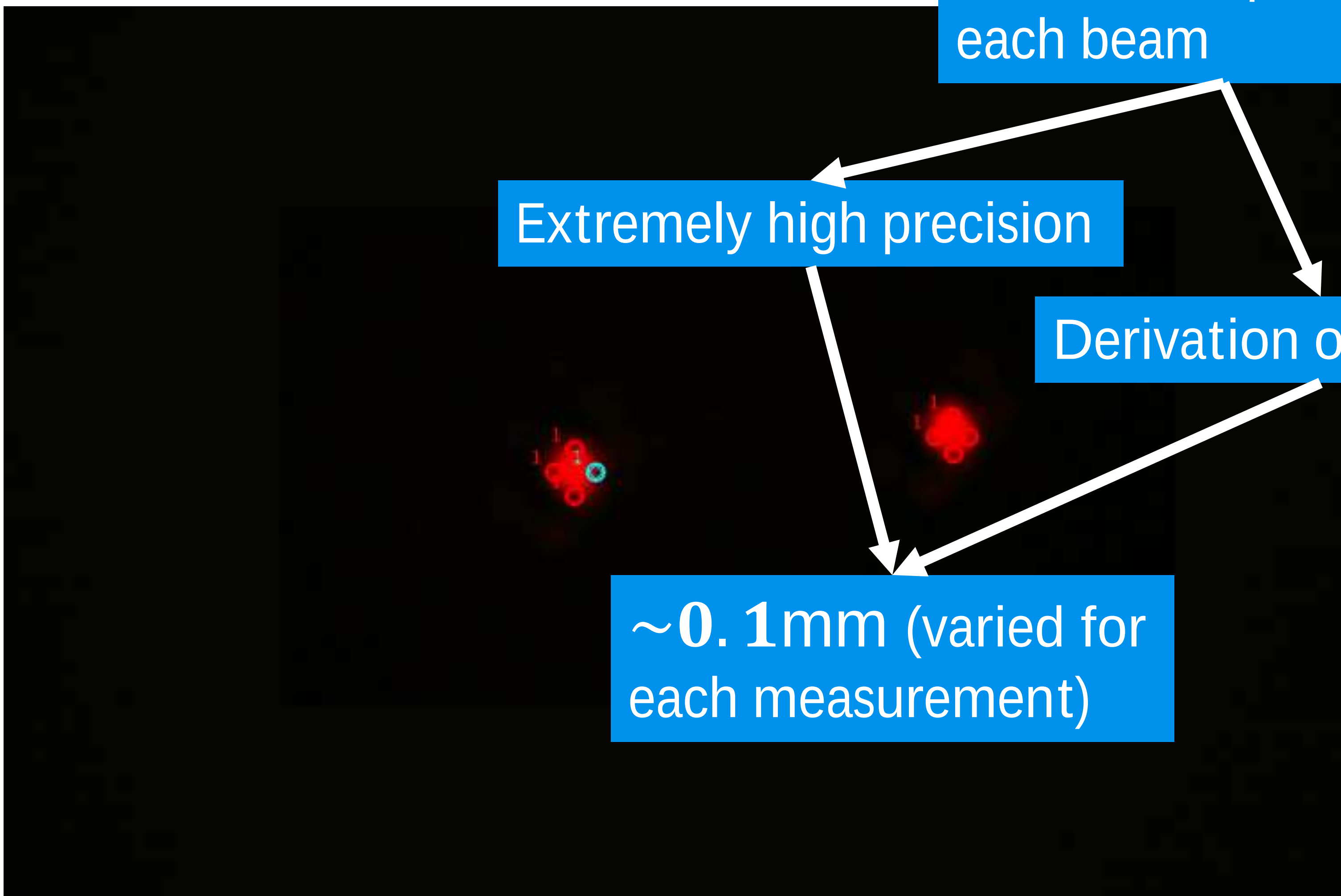
Digitally measuring the distance and angle



Took a picture on the camera to measure the distance

Took a picture from the top to measure the angles

Digitally measuring the distance and angle



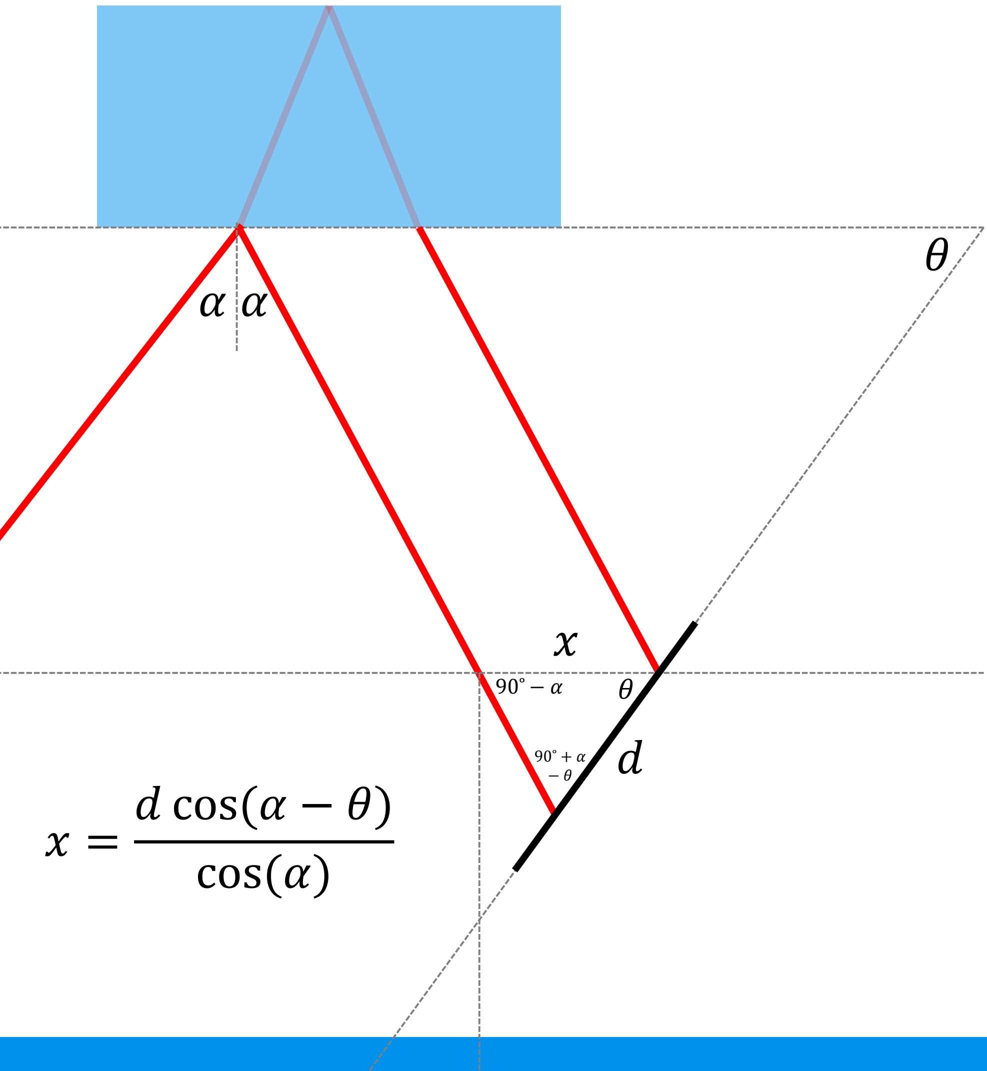
Measured 4 points for each beam

Extremely high precision

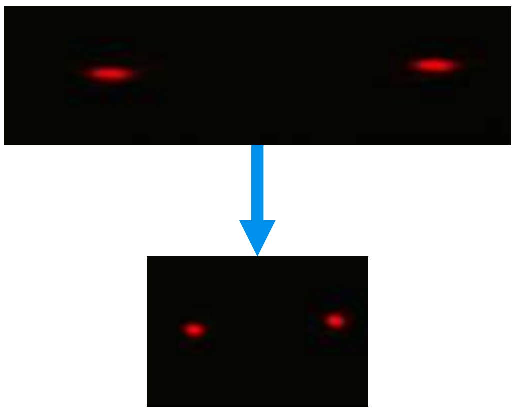
Derivation of error

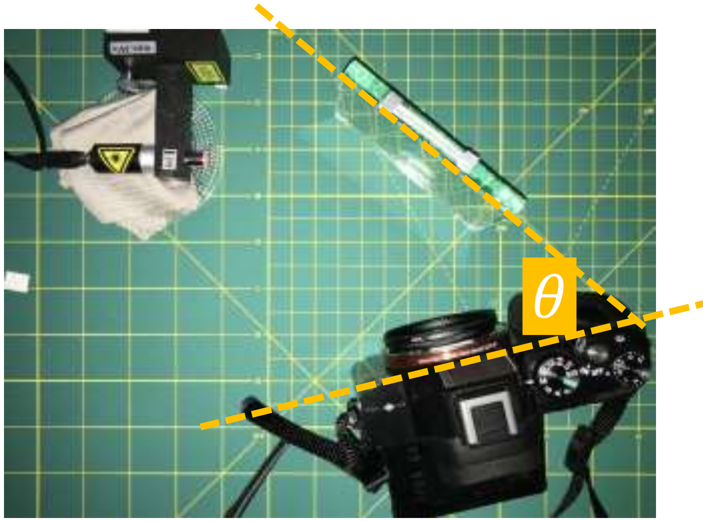
$\sim\mathbf{0.1}$mm (varied for each measurement)

Inclining the sensor to prevent a ”stretched” image

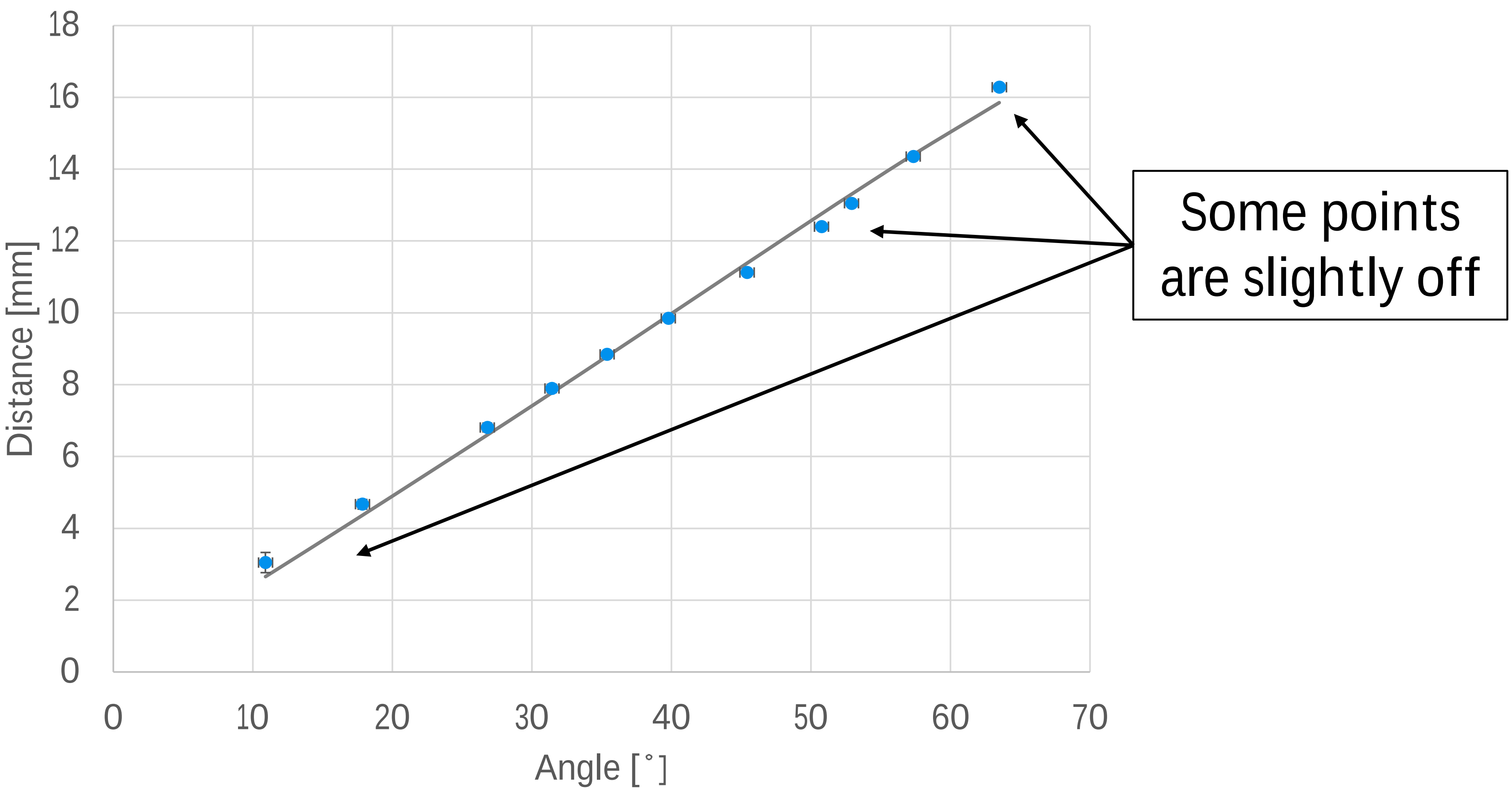


$$x = \frac{d\cos(\alpha - \theta)}{\cos(\alpha)}$$





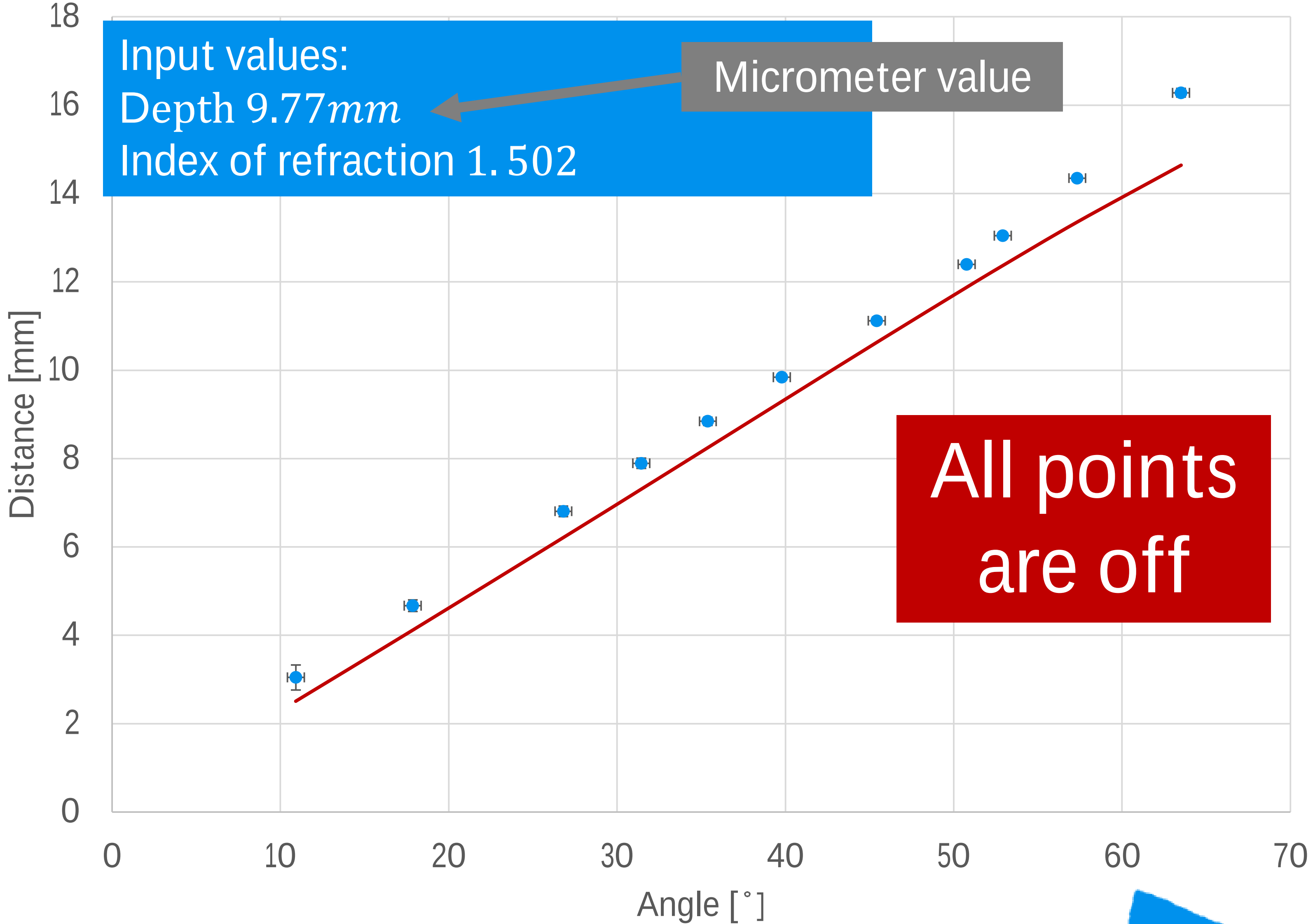
Fitting depth, using refractive index from Brewster's angle



Input values:
Index of refraction: (1.44 ± 0.073)

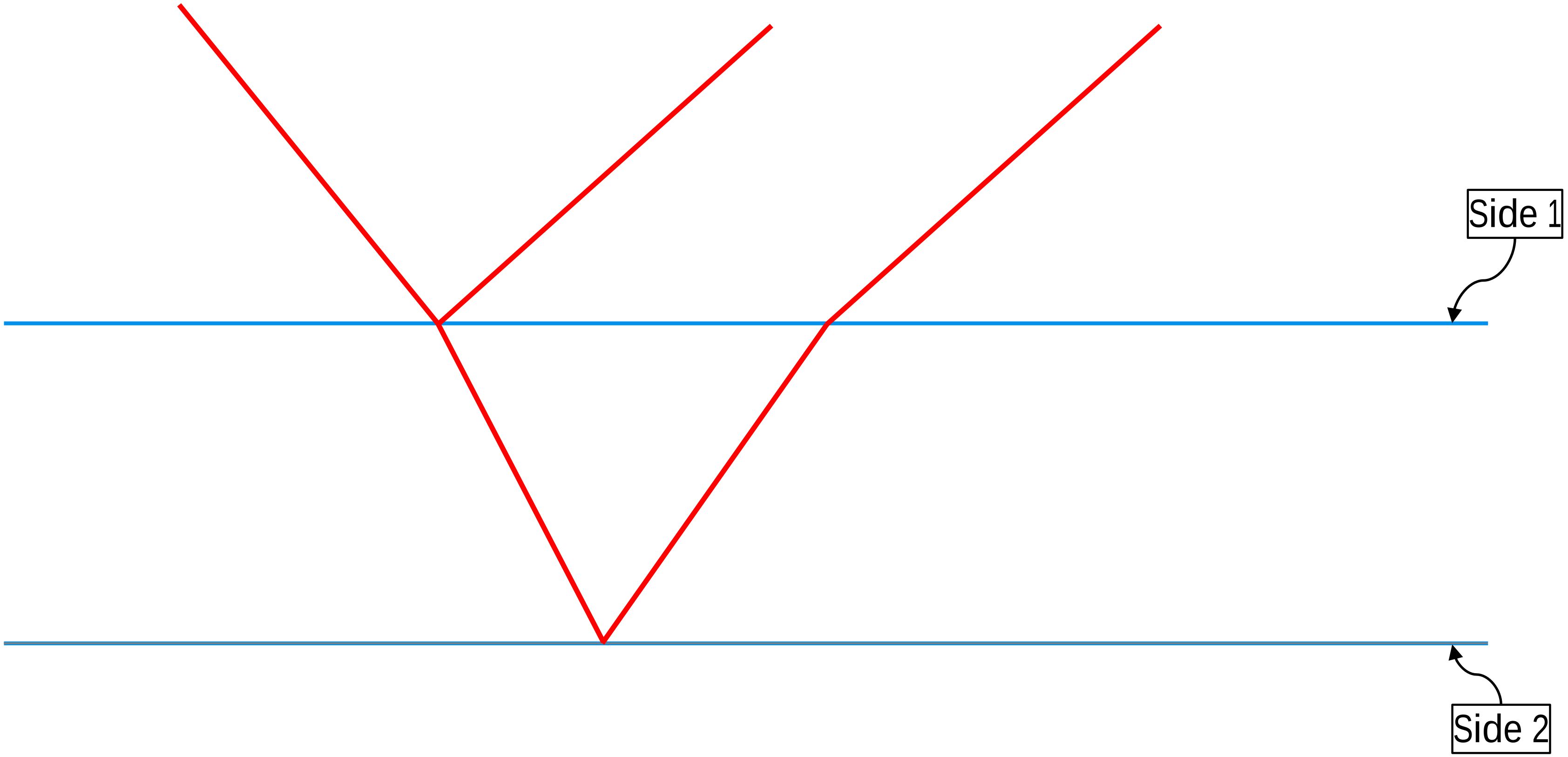
Fit values:
Depth $(10.02 \pm 0.721)\ mm$

Comparing predicted vs. measured data

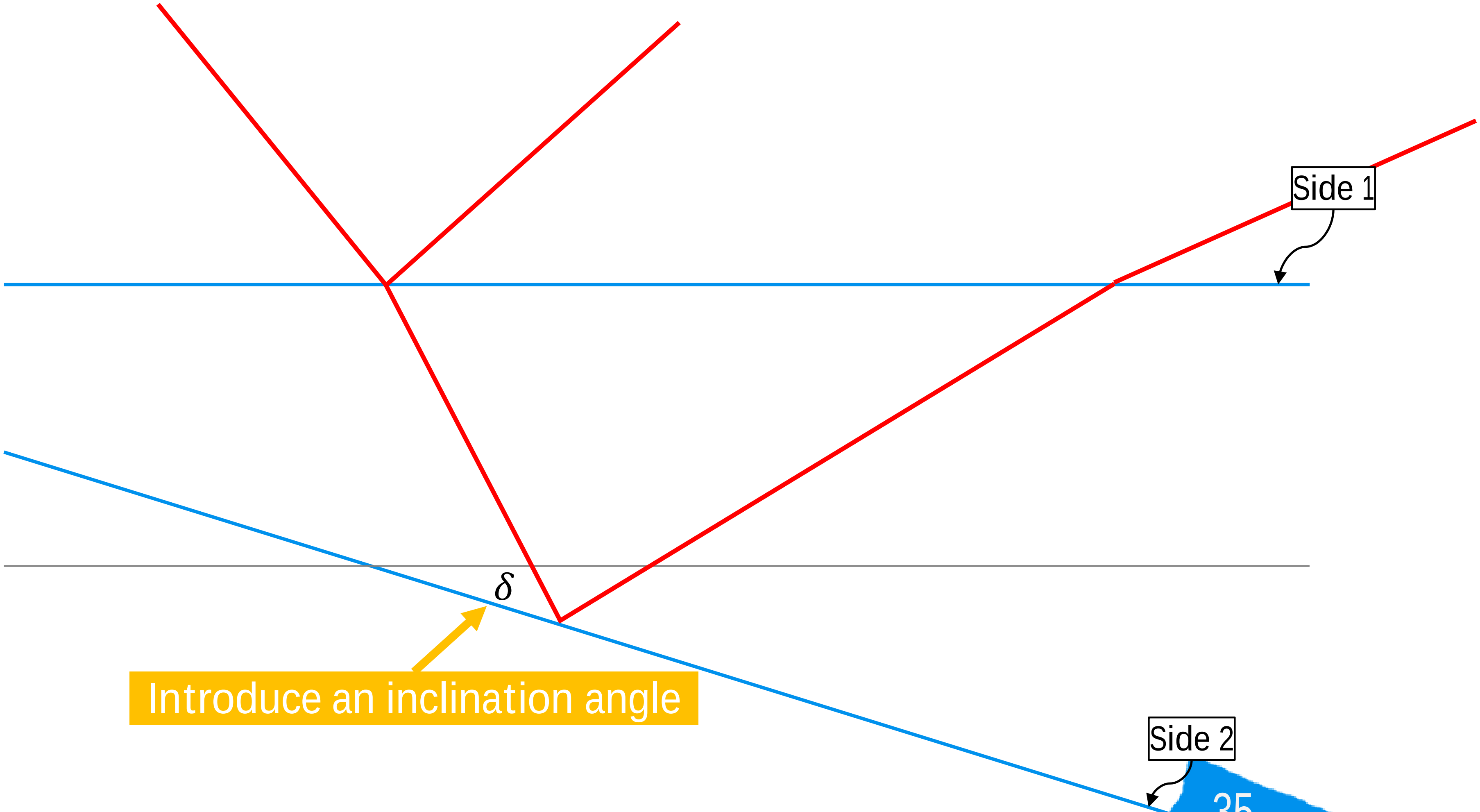


In search for an explanation

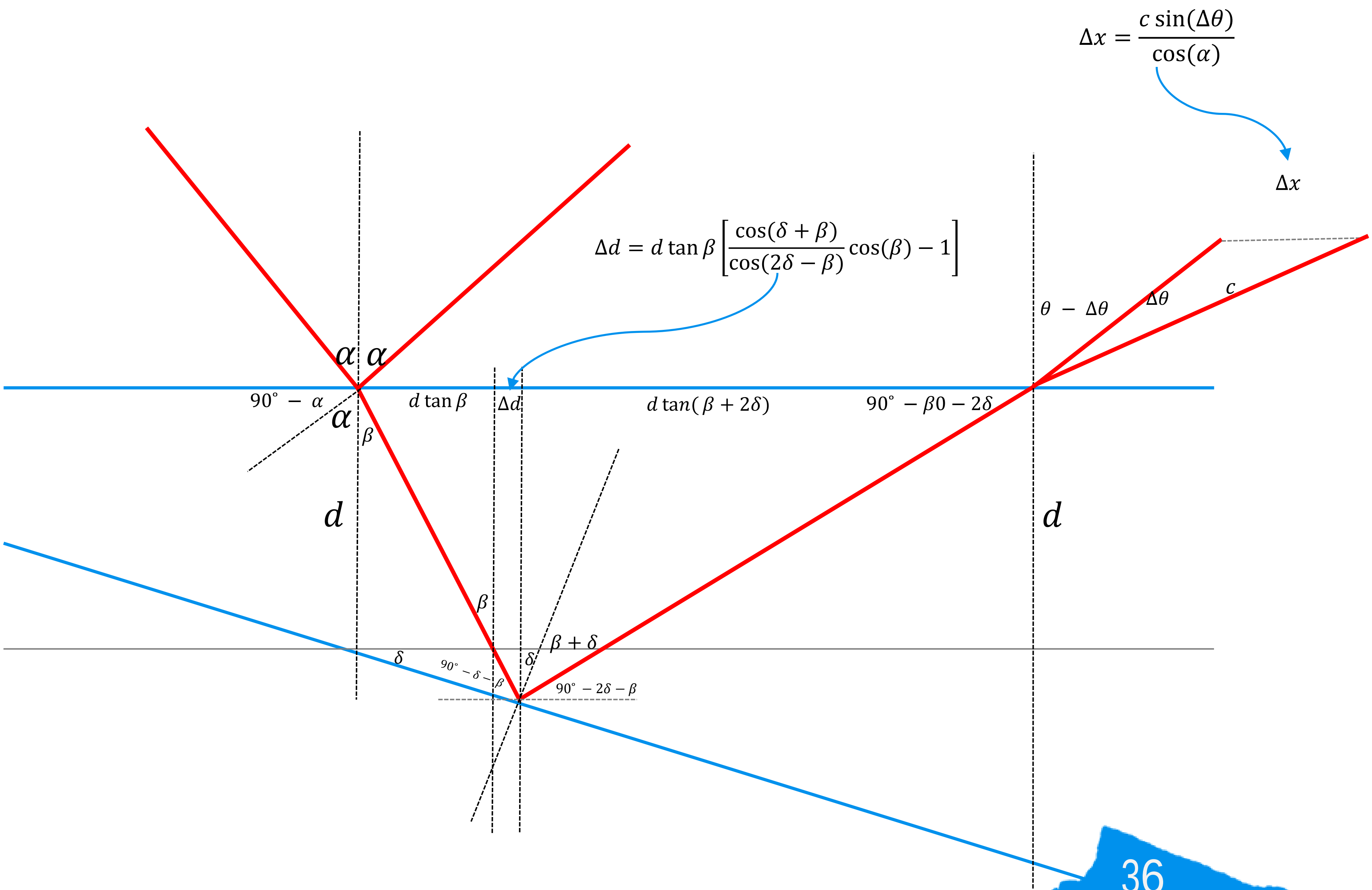
What if the sides are inclined?



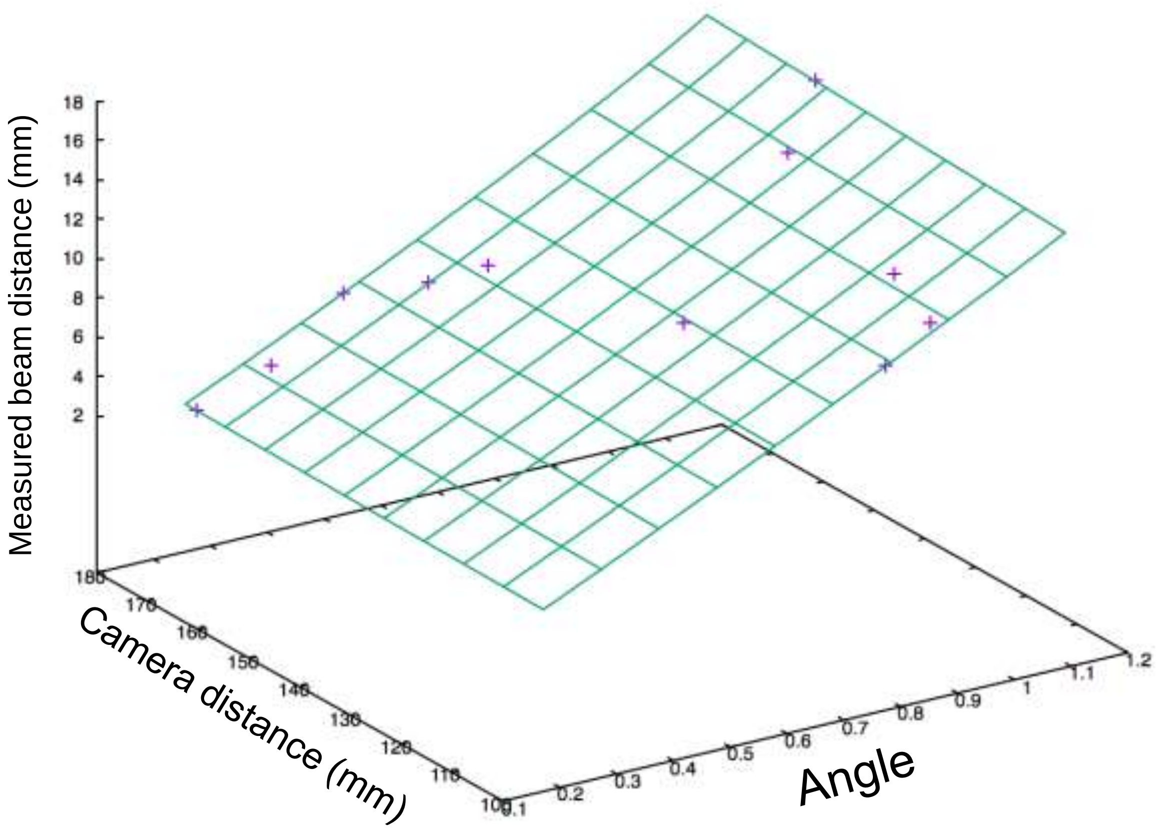
What if the sides are inclined?



Inclination of the sides of sheet of glass



Reflections: 3D plot

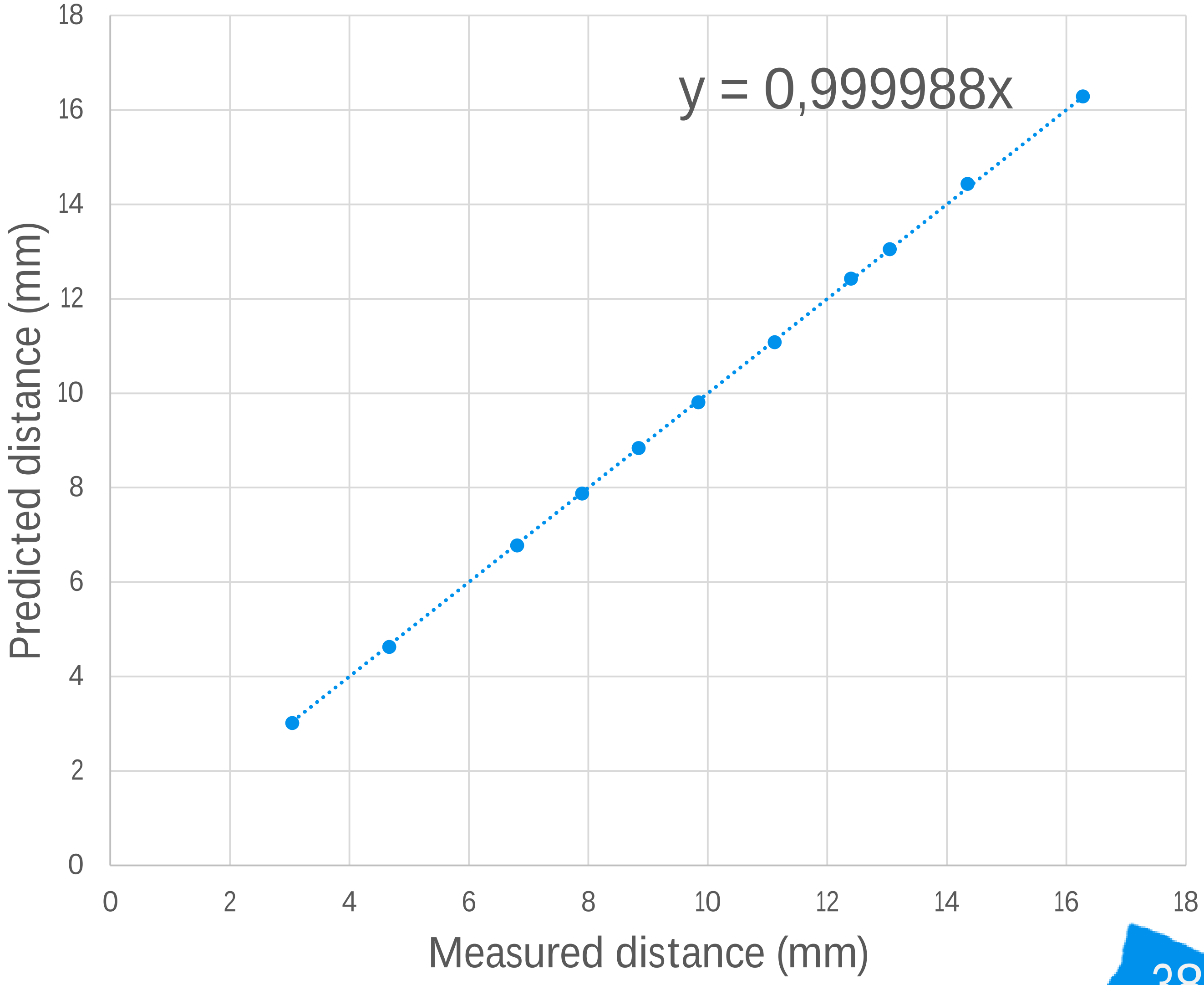


Input values:
Index of refraction 1.502

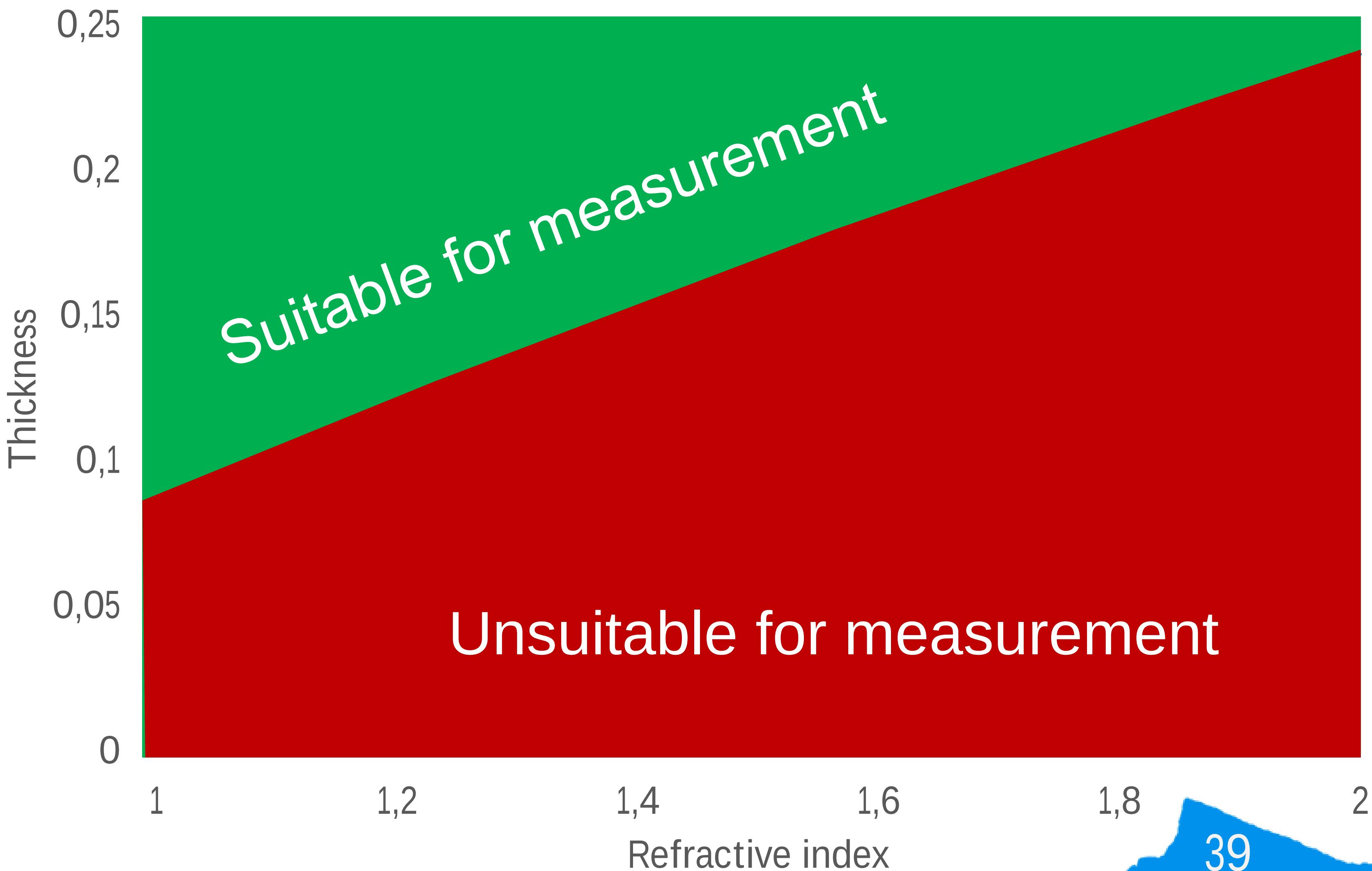
Fit Values:
$\delta = 0.055 \pm 0.002^{\circ}$
$d = 9.78 \pm 0.005$mm

Micrometer:
$d = 9.77 \pm 0.005$mm

Measurement vs. theory correlation

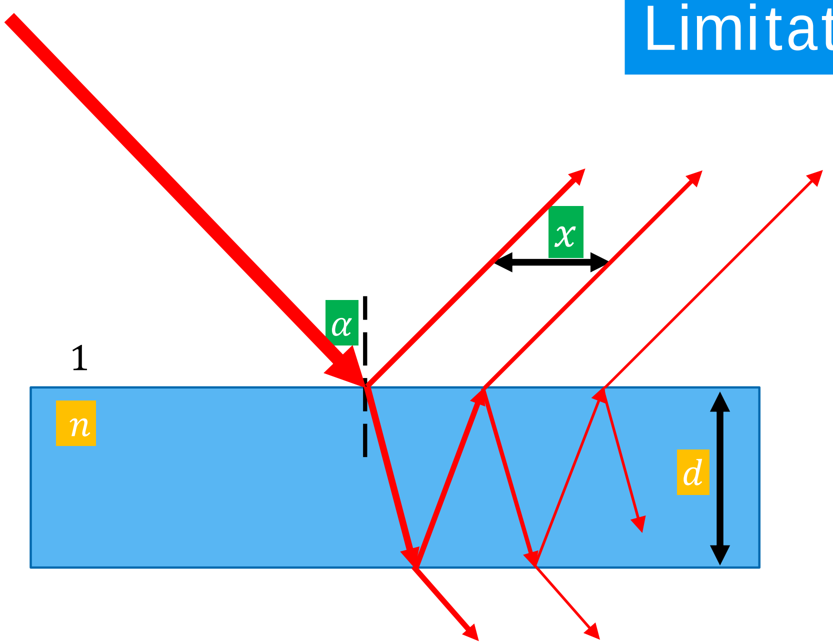


Limitations of reflections method: phase diagram



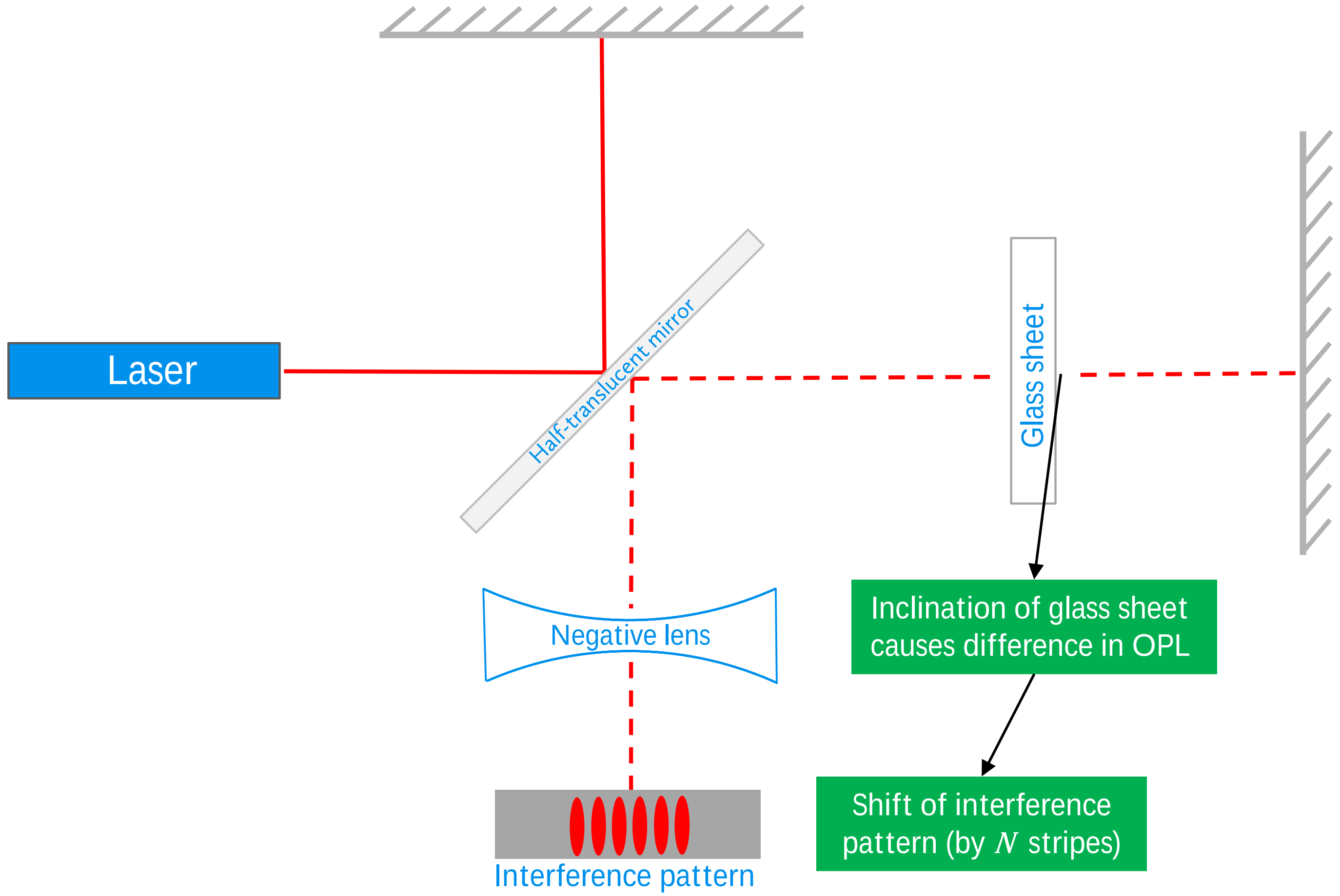
Internal reflections: conclusion

Usage:	Thickness Inclination angle
Error:	± 0.005mm ± 0.002˚
Limitation (n = 1.5):	0.16mm

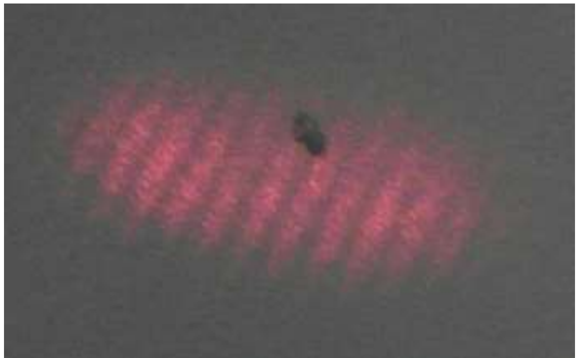


Measuring thickness using Michaelson interferometer

Michelson interferometer



Michelson interferometer: measurement output



Observed pattern

Shift of interference pattern by N stripes

Sheet width

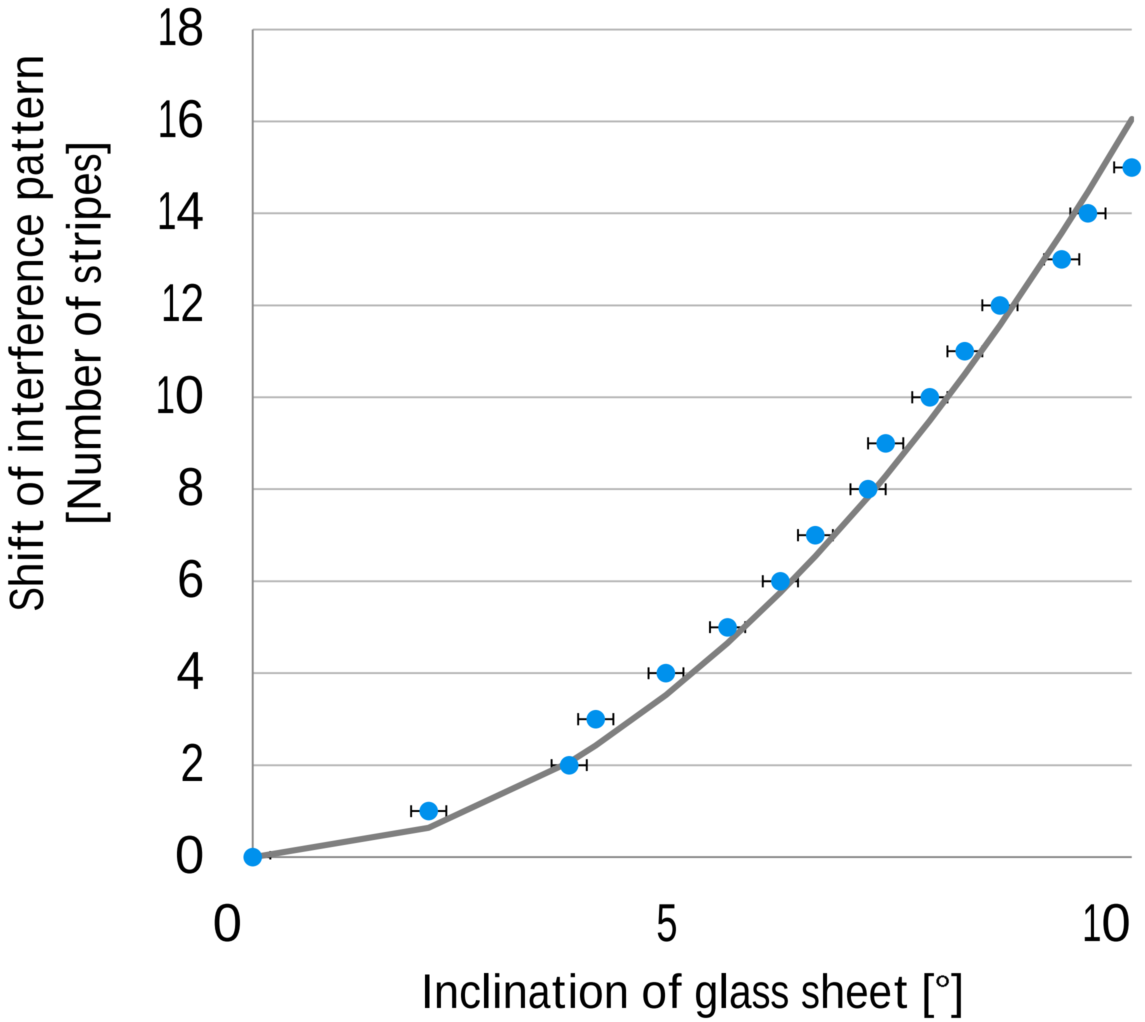
$$N = \frac{2d(1-\cos(\alpha))(n-1)}{n\lambda - \lambda(1-\cos(\alpha))}$$

Refractive index

n or d can be obtained by regression

[Deepak N. Iyer , Lehigh University, A Michelson interferometric technique for measuring refractive index of sodium zinc tellurite glasses]

Michelson interferometer: measuring thickness by regression



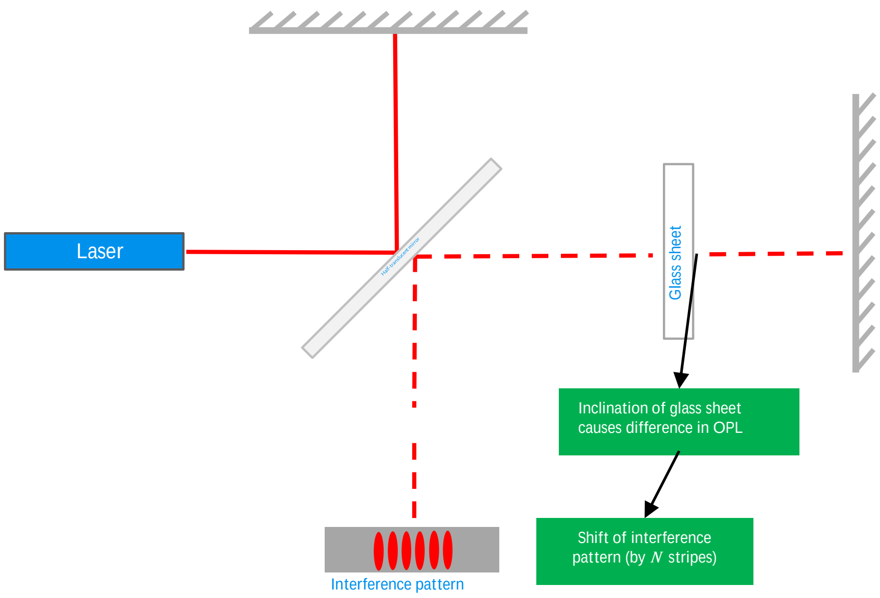
Input values:
Index of refraction 1.48

Fit Values:
$\boldsymbol{d = 1.03 \pm 0.043}$ mm

Micrometer:
$\boldsymbol{d = 1.05 \pm 0.005}$mm

Michelson interferometer: conclusion

Usage:	Thickness
Error:	± 0.043mm
Limitation (n = 1.5):	(0.3 – 2)mm

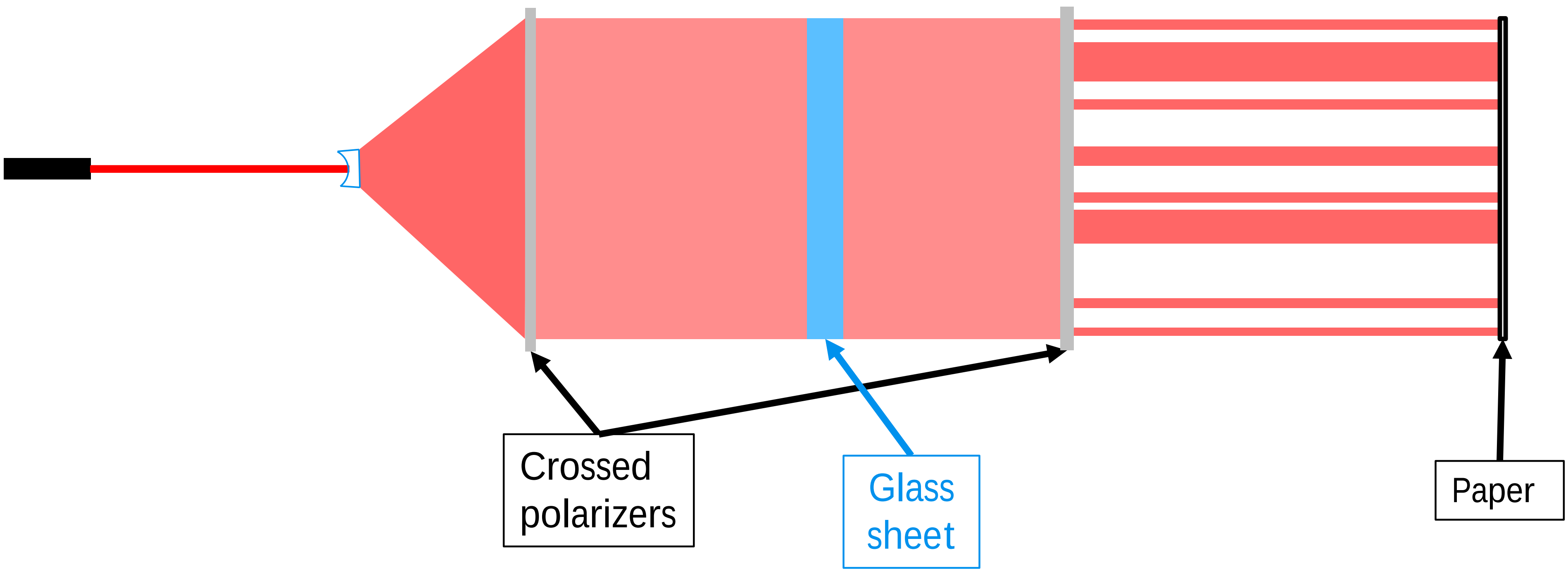


Back to the definition of the problem…

Invent and construct an optical device that uses a laser pointer and allows contactless determination of thickness, refractive index, and other properties of a glass sheet. ✓

Observing internal tension

Observing internal tension



Observing internal tension: plastic protractor

No object inserted

No change of polarization

Object with an internal tension

Change of polarization

Observing internal tension: plastic protractor

Observing internal tension: sheet of glass

No object

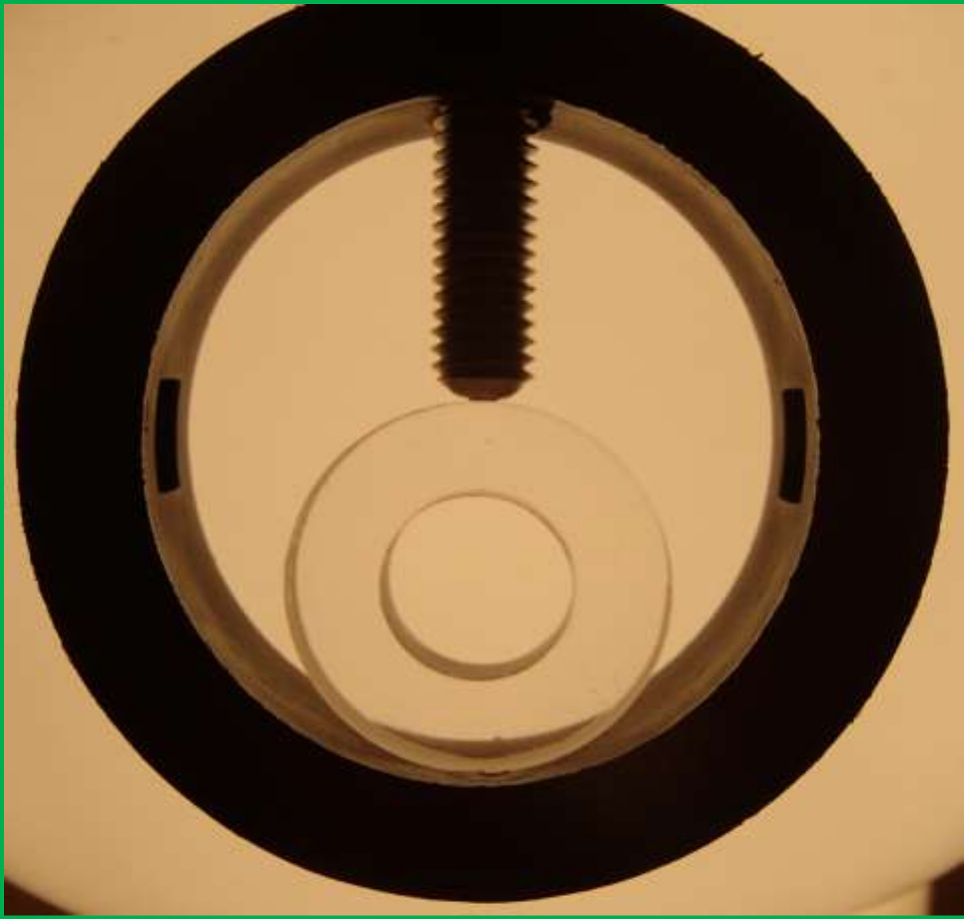
Glass sheet

Extremely low internal tension

Applying external force to create internal tension

Relaxed

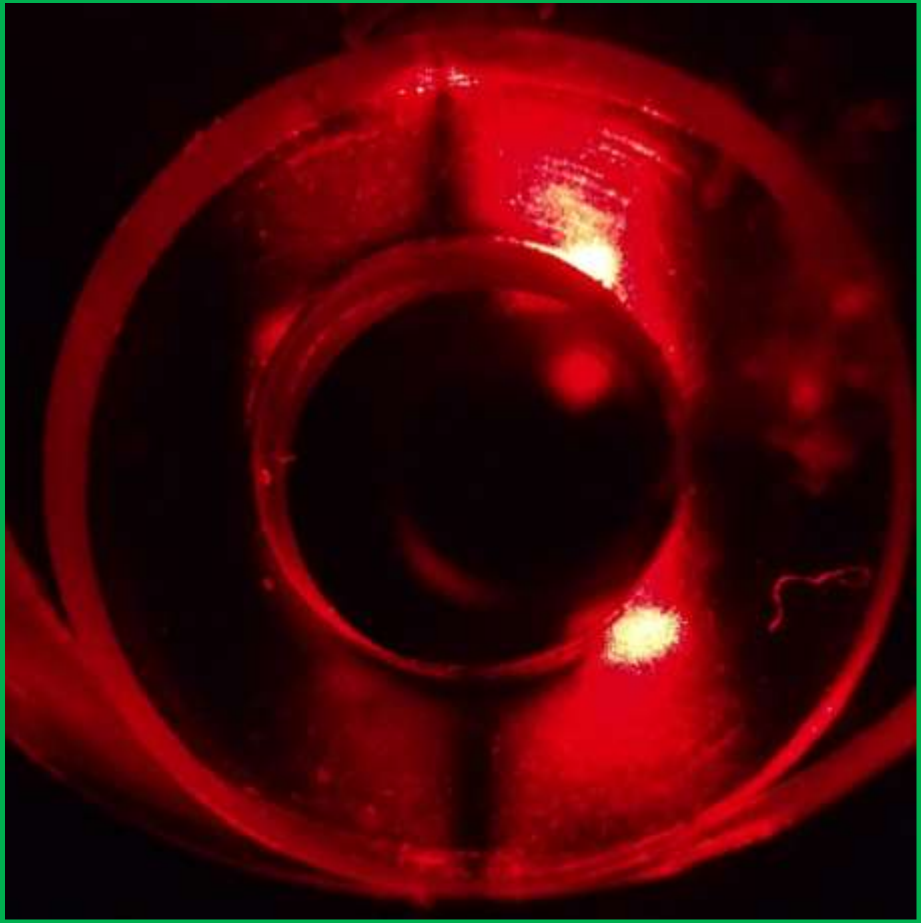
Stressed



Applying external force to create internal tension

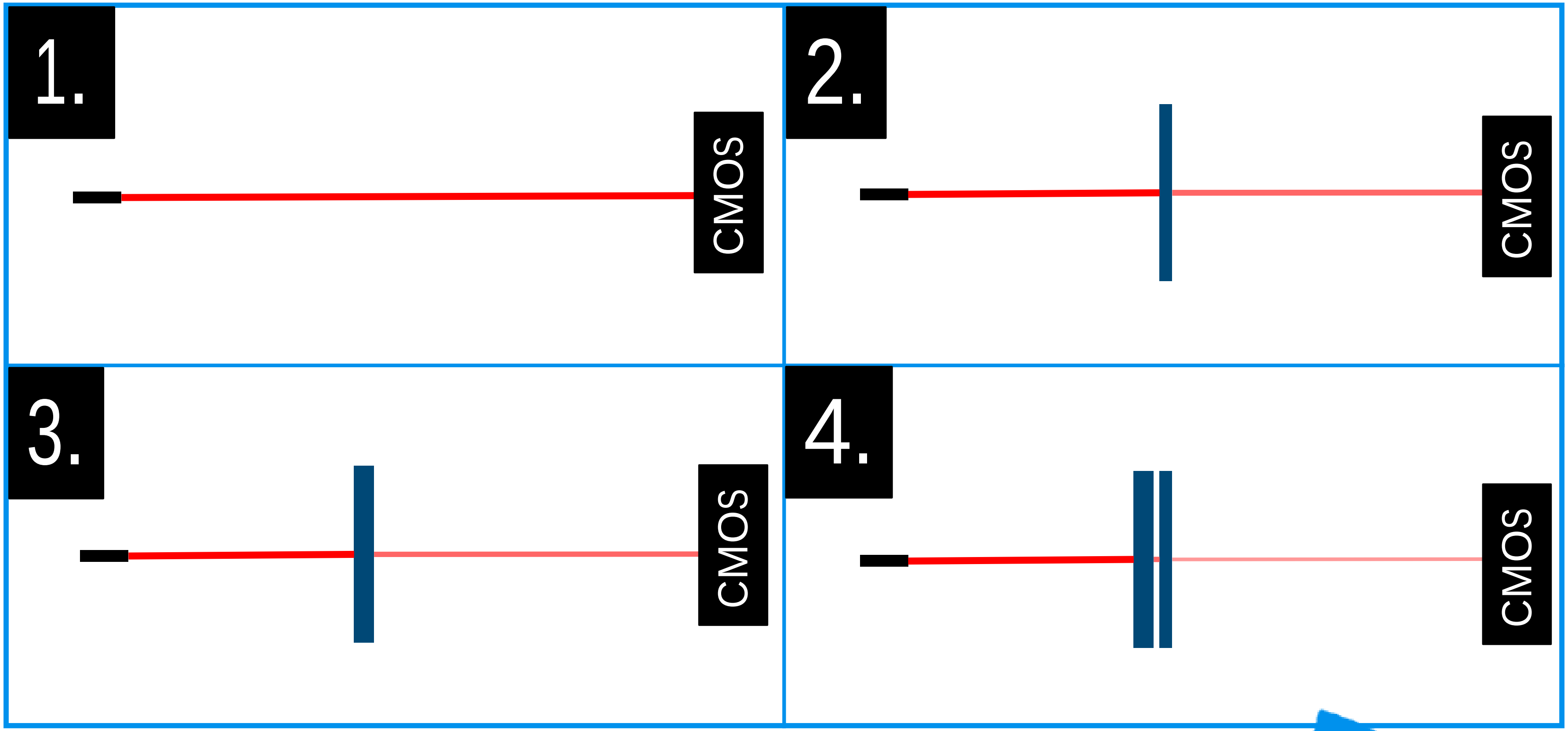
Relaxed

Stressed

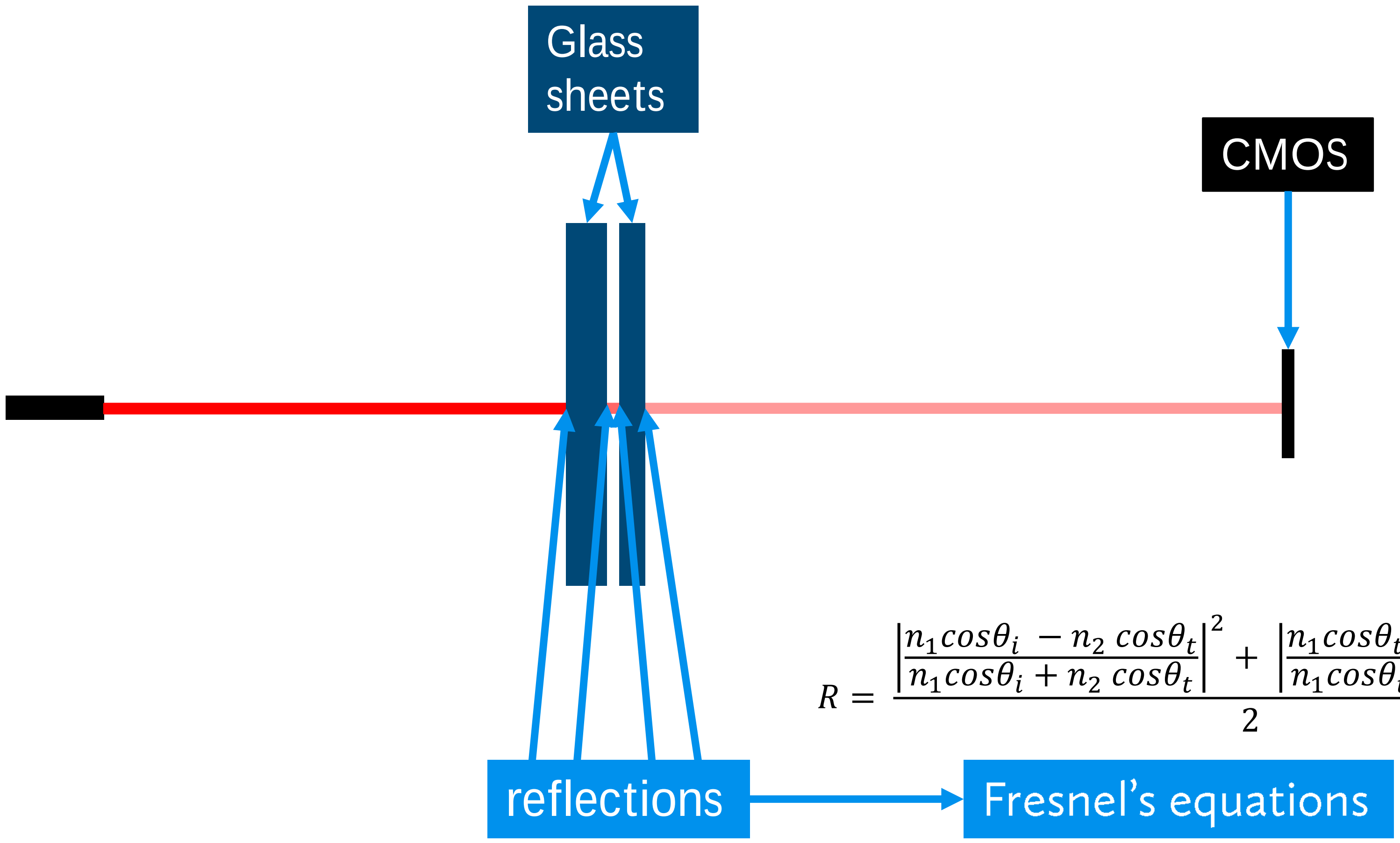


Measuring absorption coefficient

Measuring the absorption coefficient

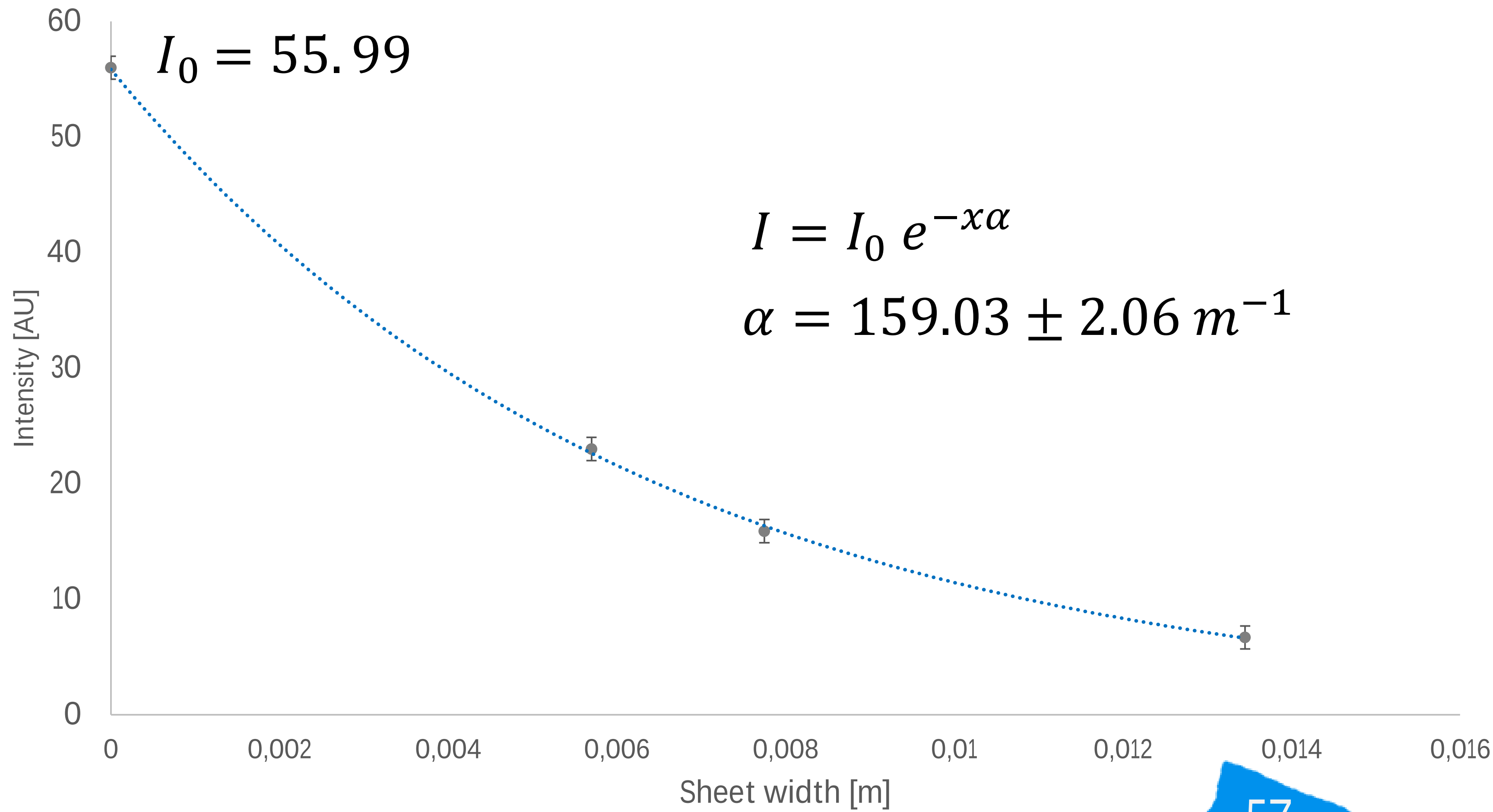


Measuring the absorption coefficient



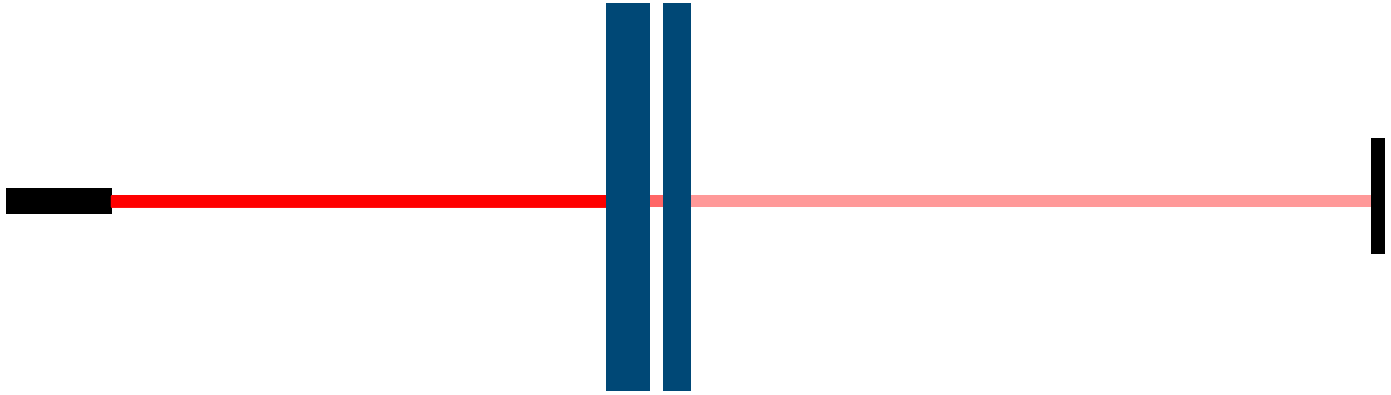
$$R = \frac{\left|\frac{n_1 cos\theta_i - n_2 cos\theta_t}{n_1 cos\theta_i + n_2 cos\theta_t}\right|^2 + \left|\frac{n_1 cos\theta_t - n_2 cos\theta_i}{n_1 cos\theta_t + n_2 cos\theta_i}\right|^2}{2}$$

Measuring the absorption coefficient



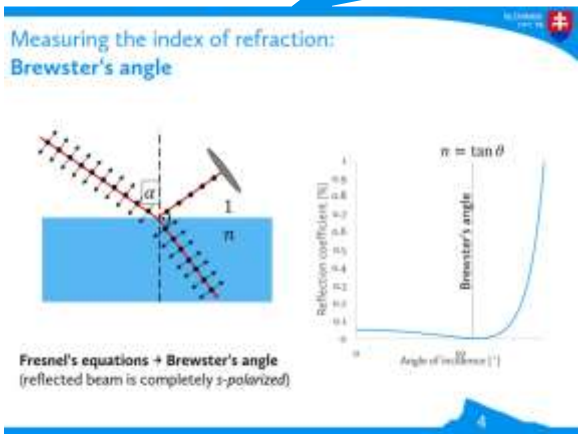
Absorption: conclusion

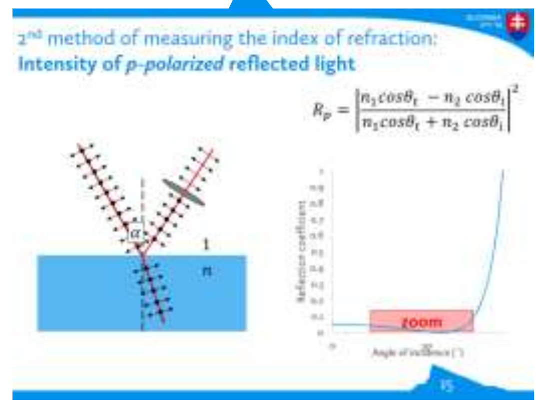
Usage:	Absorption
Error:	$\pm\ 2m^{-1}$
Limitation:	$\alpha d \gg -\ln\left(1-\frac{\Delta I}{I_0}\right)$
Approximated limitation:	$\alpha d \gg 0.02$

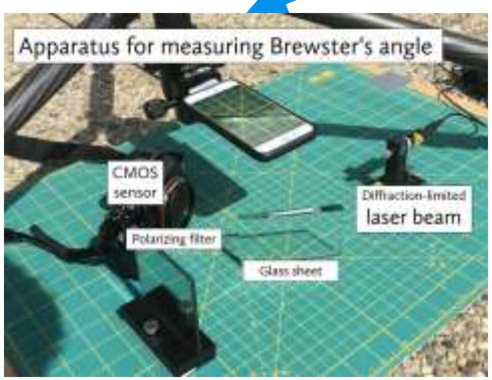


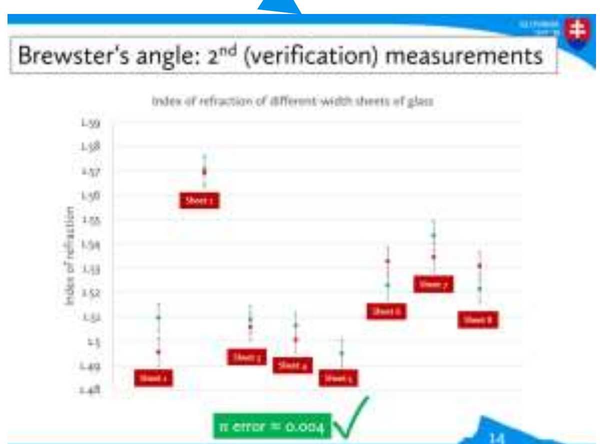
Contactless calliper

Refractive index

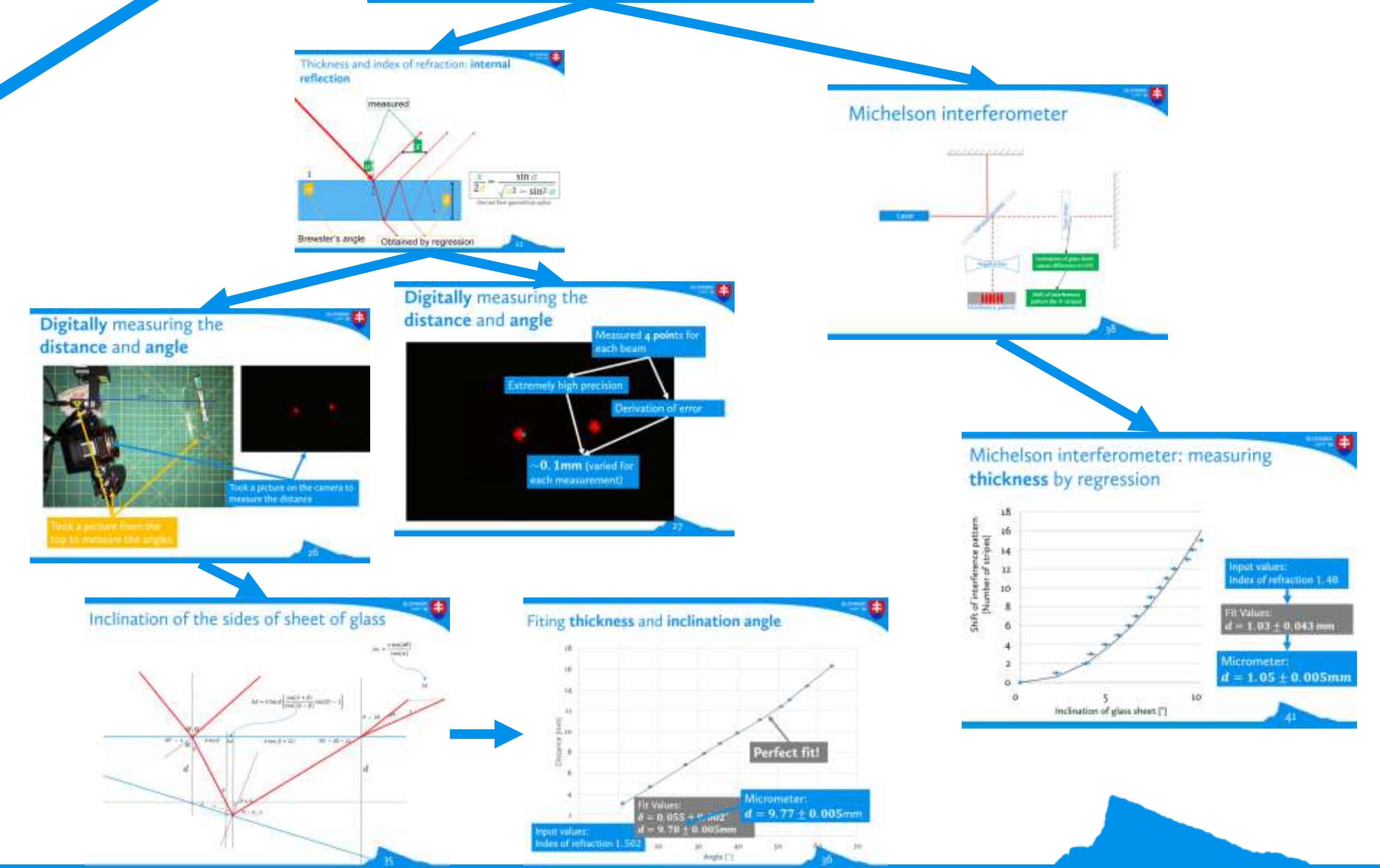






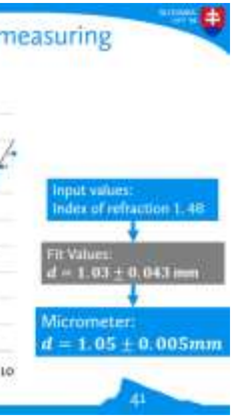


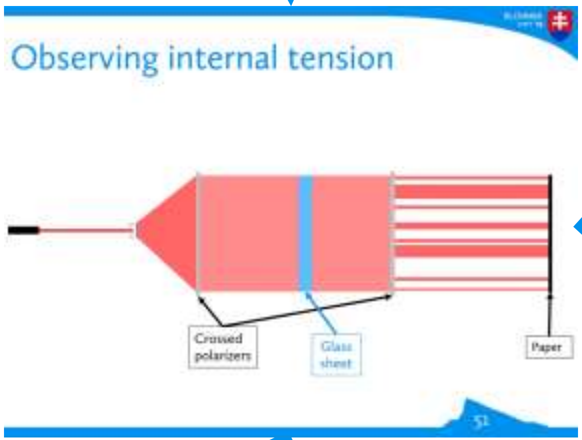
Thickness

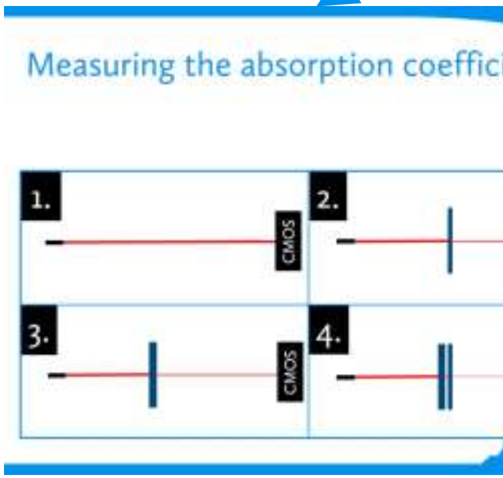

Thickness and index of refraction: internal reflection
measured
Brewster's angle
Obtained by regression
Michelson interferometer
Laser
Digitally measuring the distance and angle
Took a picture on the camera to measure the distance
Took a picture from the top to measure the angles
Measured 4 points for each beam
Extremely high precision
Derivation of error
~0. 1mm (varied for each measurement)
Michelson interferometer: measuring thickness by regression
Shift of interference pattern [Number of stripes]
Inclination of glass sheet [°]
Input values:
Index of refraction 1.48
Fit Values:
d = 1.03 ± 0.043 mm
Micrometer:
d = 1.05 ± 0.005mm
Inclination of the sides of sheet of glass
Fiting thickness and inclination angle
Perfect fit!
Micrometer:
d = 9.77 ± 0.005mm
Input values:
Index of refraction 1.502
Angle [°]

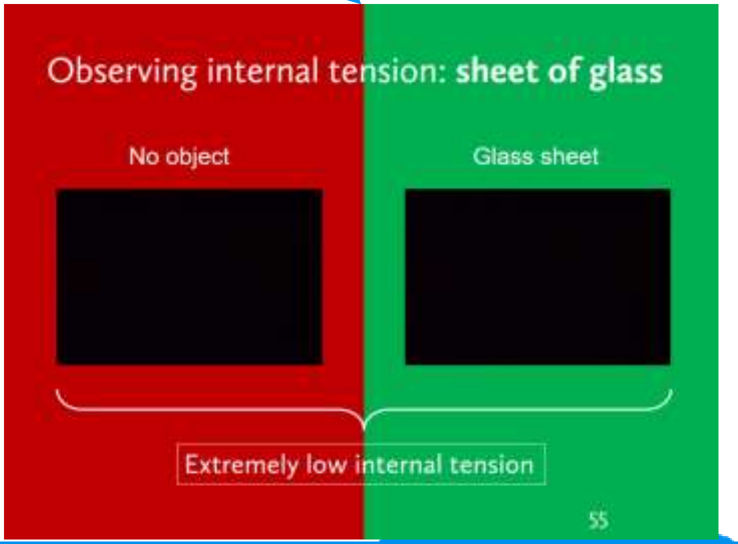
Absor

Internal tension



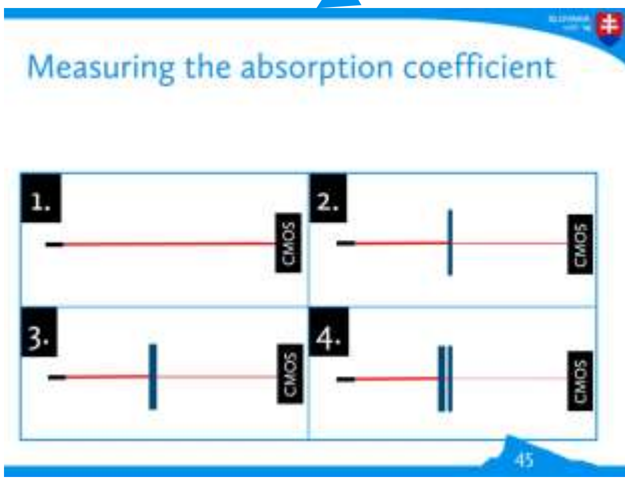


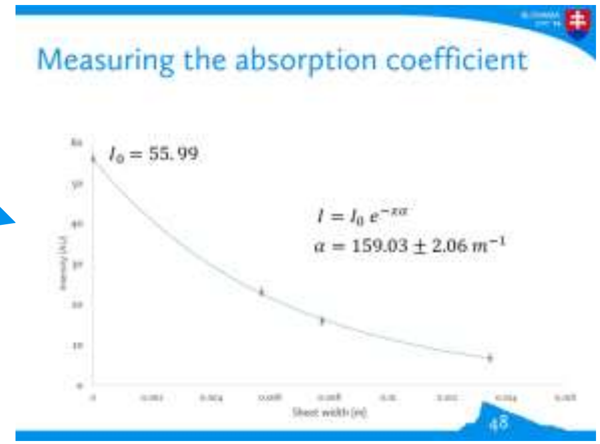


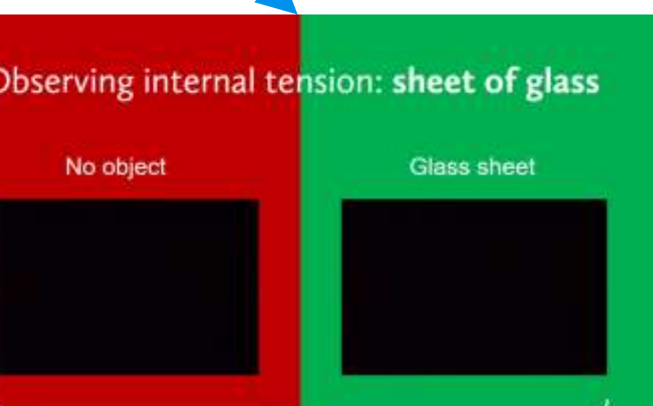


Absorbtion

on







Output

Quantity	Uncertainty	Range	Method
Index of refraction	± 0.008	$d \gtrsim 0.16$mm	Brewster angle
Thickness	± 0.005mm	$d \gtrsim 0.16$mm	Internal reflections
Inclination angle	± 0.002°	$d \gtrsim 0.16$mm	Internal reflections
Absorption	± 2m^{-1}	$\alpha d \gg 0.02$	Absorption

Appendix

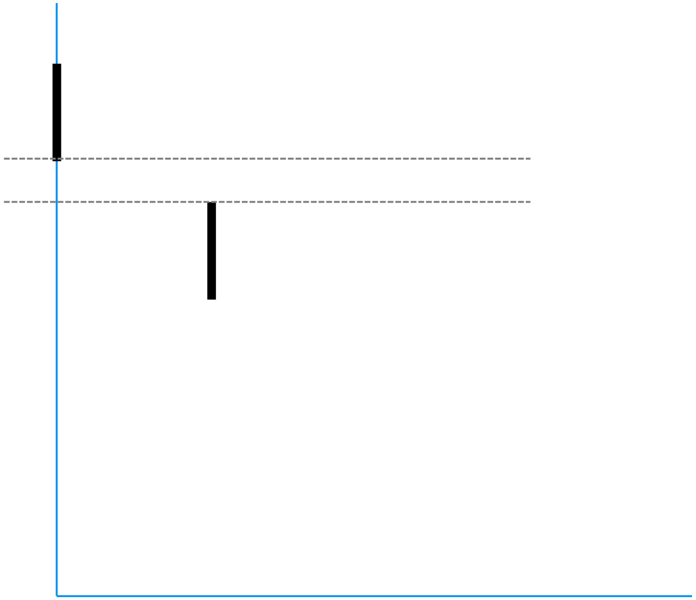
Deriving minimal αd

$$I_0 - I_0 e^{-\alpha d} \gg \Delta I$$

$$1 - e^{-\alpha d} \gg \frac{\Delta I}{I_0}$$

$$e^{-\alpha d} \gg 1 - \frac{\Delta I}{I_0}$$

$$\alpha d \gg -\ln\left(1 - \frac{\Delta I}{I_0}\right)$$



Complex index of refraction

$$\hat{n} = n + i\frac{\alpha}{k}$$

Absorption coefficient

Wavenumber

Refraction

Absorption

$$n = 1.5025 \pm 0.0013 + i(1{,}573 \pm 20) \times 10^{6}$$

Speed of light

$$v = \frac{c}{n}$$

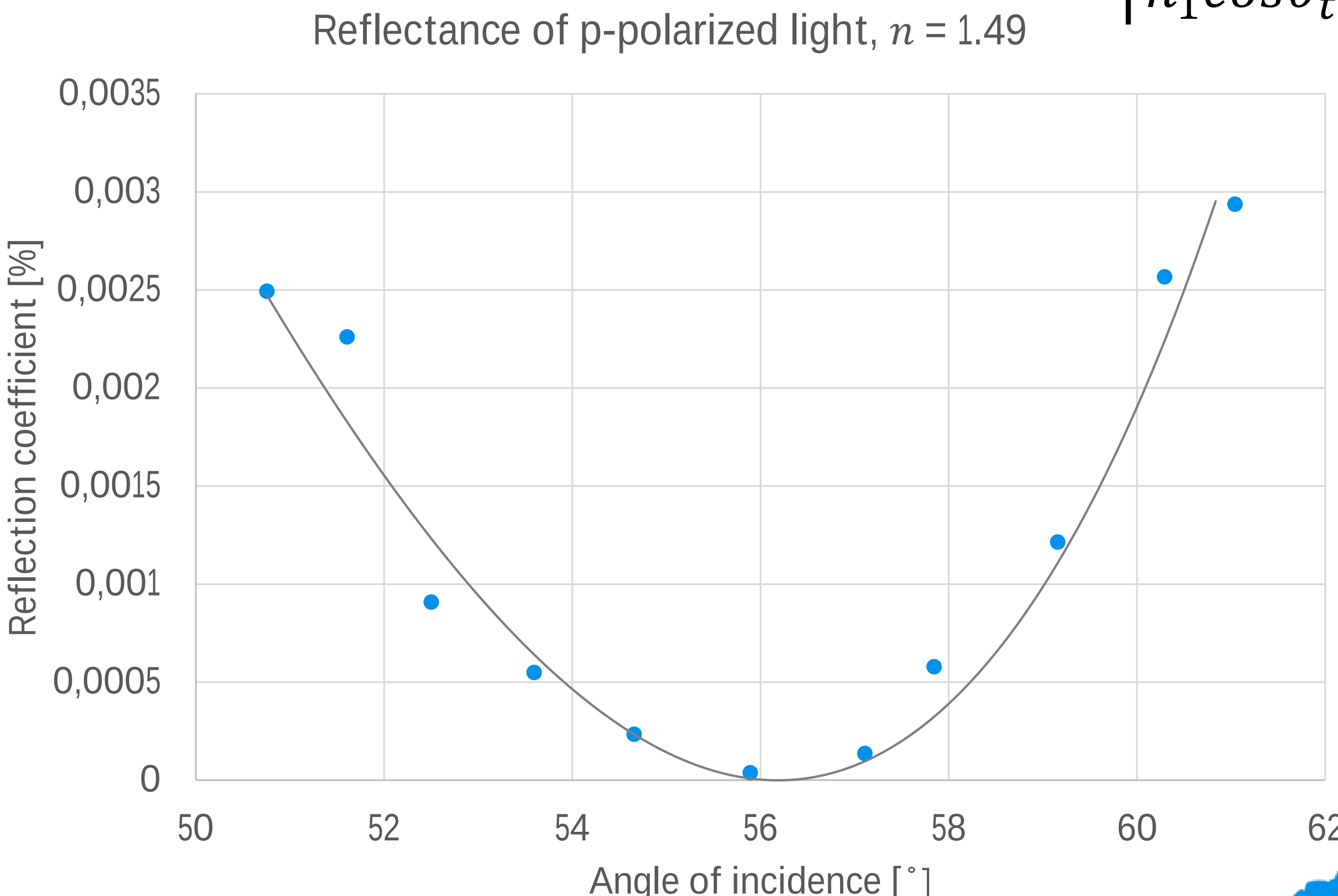
Speed of light in vacuum

Speed of light in the glass

Index of refraction

2nd method of measuring the index of refraction: Intensity of *p-polarized* reflected light

$$R_p = \left| \frac{n_1 cos\theta_t - n_2 \, cos\theta_i}{n_1 cos\theta_t + n_2 \, cos\theta_i} \right|^2$$

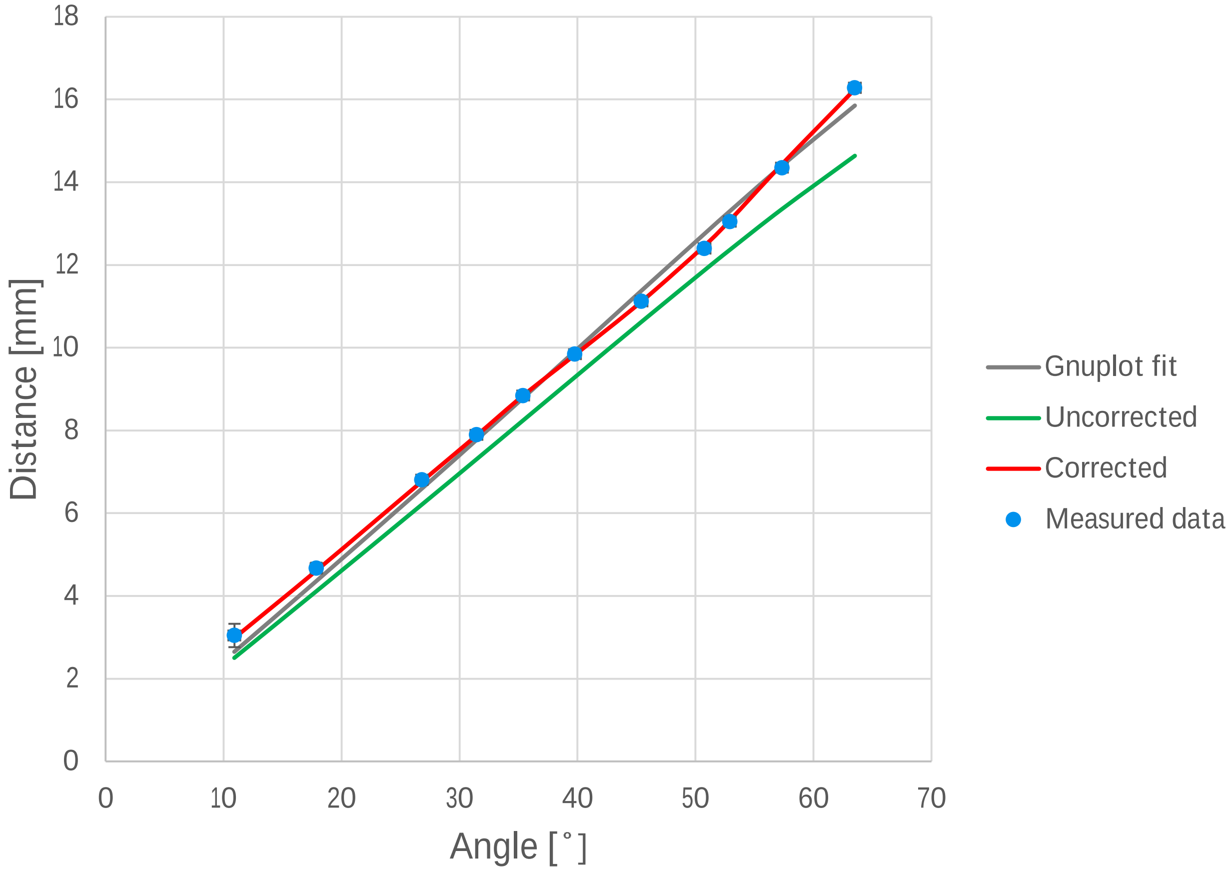


Fresnel: 56.18˚

Brewster: 56.47˚

Observing internal tension

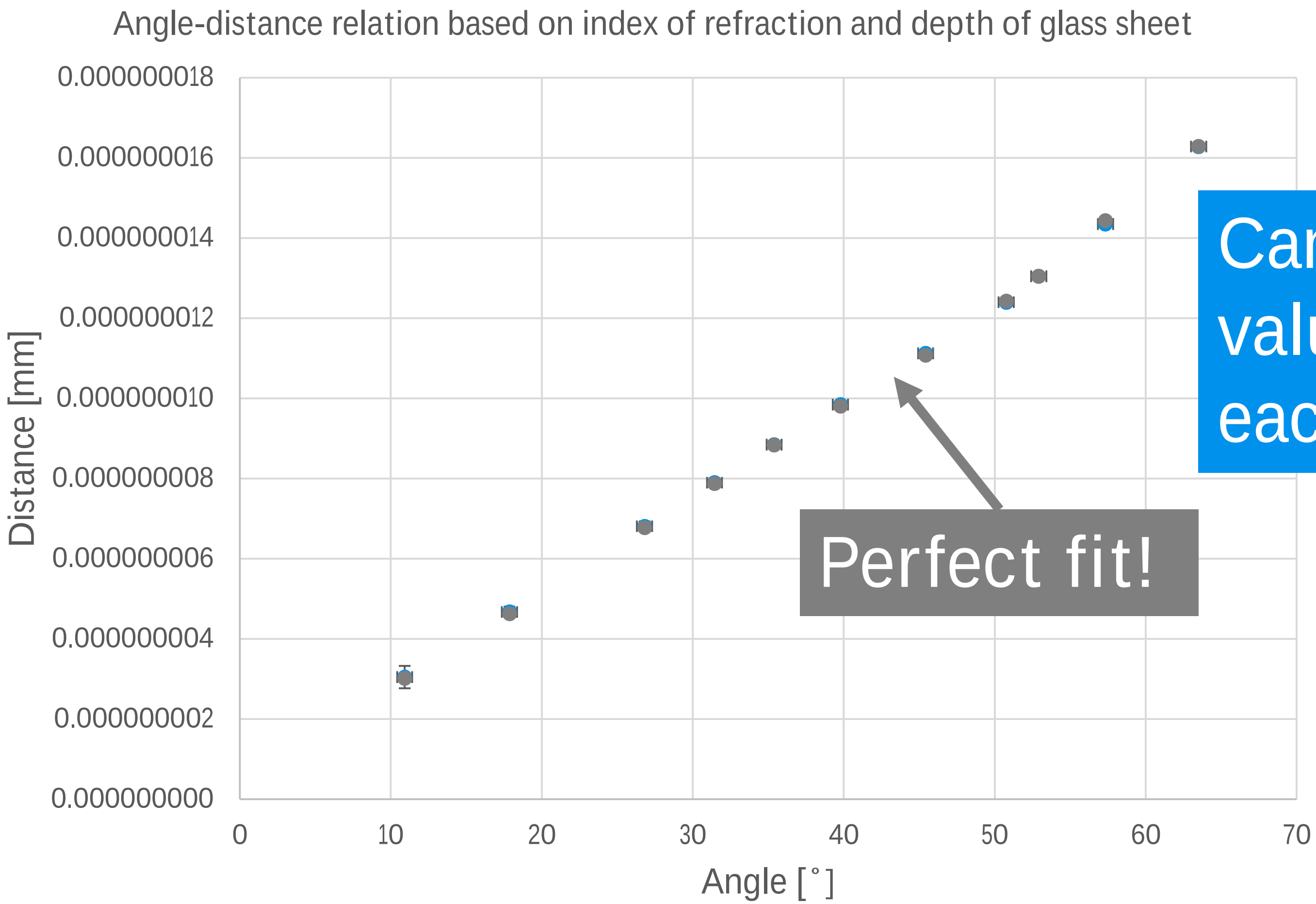
All data



Fit values:
Depth $(\mathbf{10.02} \pm \mathbf{0.721})\ \boldsymbol{mm}$
Index of **refraction** $(\mathbf{1.44} \pm \mathbf{0.073})$

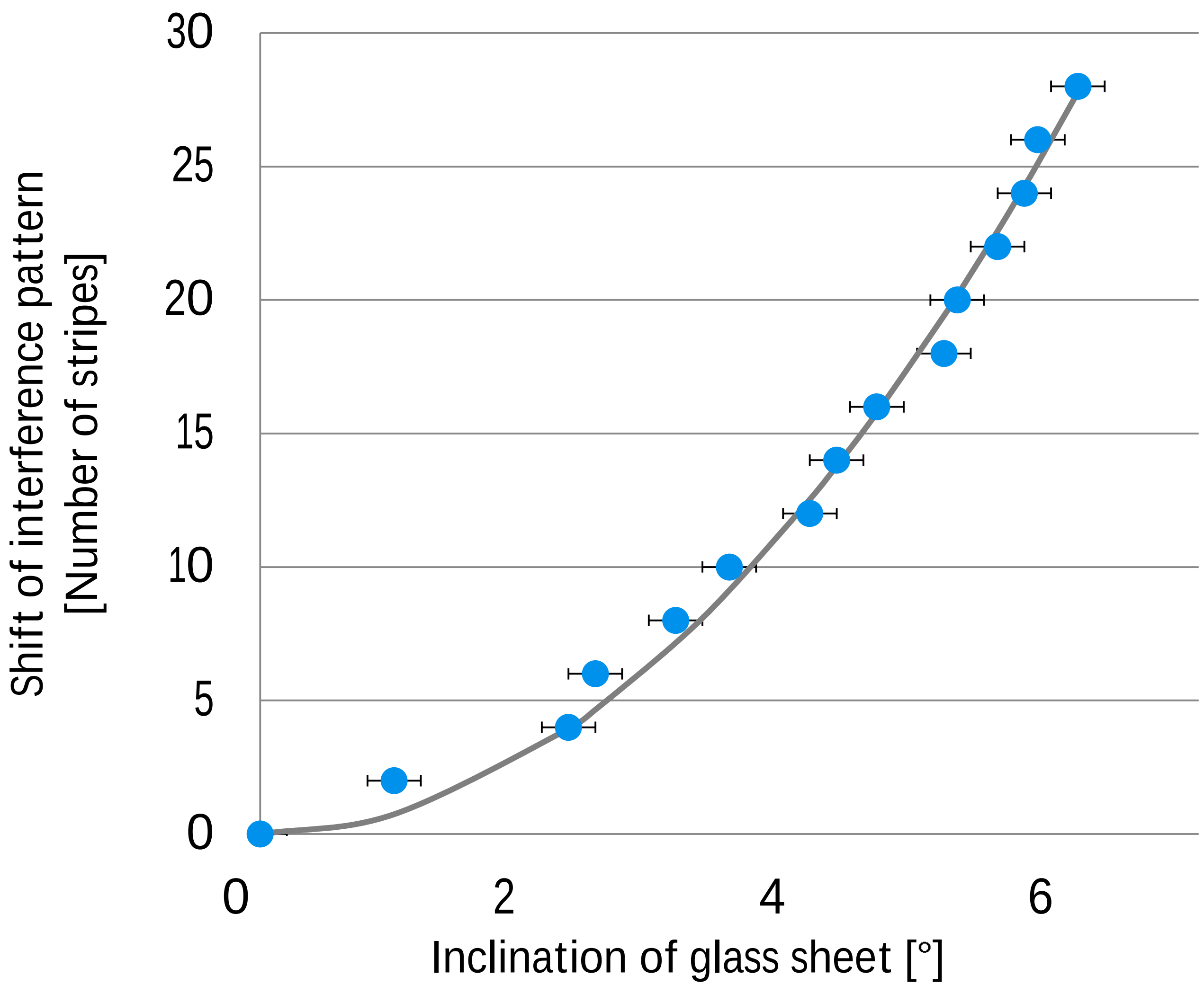
Actual Values:
$\boldsymbol{n} = \mathbf{1.502}$
$\boldsymbol{d} = \mathbf{9.77}$mm

Fiting thickness and inclination angle



Camera distance values unique for each point

Michelson interferometer: measuring refractive index by regression



Input values:
$d = 4.88$mm

Fit Values:
$n = 1.46$

Brewster's angle:
$n = 1.48$