BRAZIL

IYPT 2018

Problem 3

Dancing Coin

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Take a strongly cooled bottle and put a coin on its neck. Over time you will hear a noise and see movements of the coin. Explain this phenomenon and investigate how the relevant parameters affect the dance.

CONTENTS

Place a coin in the mouth of a very cold bottle.

Heat sources.

Sounds and motion:

Figure 1: Coin dancing.

Relevant principles

Conduction

$$\frac{dQ}{dt} = \frac{kA\Delta T}{L}$$

$$\frac{dQ}{dt} = c_c A \Delta T$$

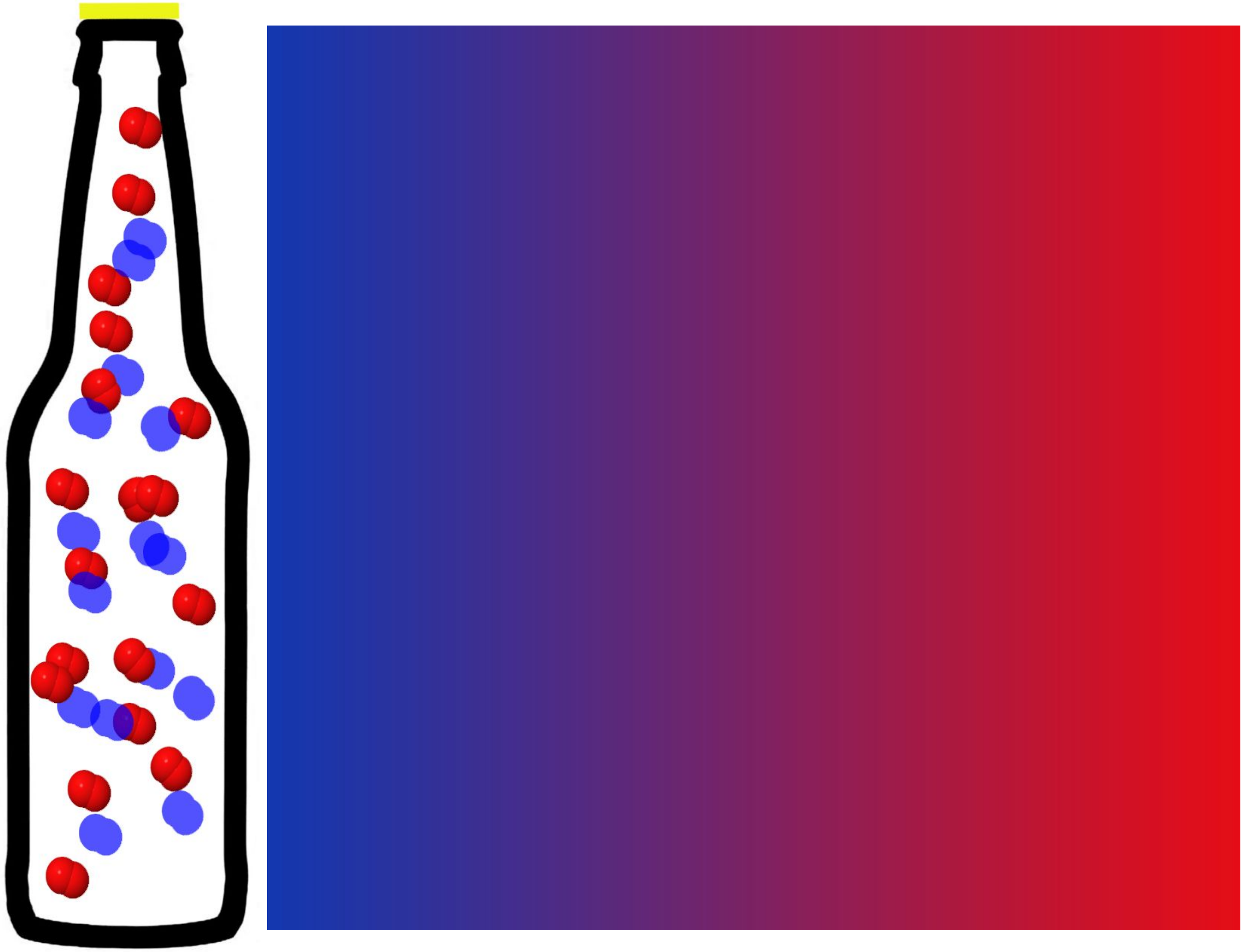


Figure 2: Scheme of heat conduction in the system.

Relevant principles

Convection

$$\frac{dQ}{dt} \approx (c_e + c_i)A\Delta T$$

$c_e \rightarrow$ **External Convection**

$c_i \rightarrow$ **Internal Convection**

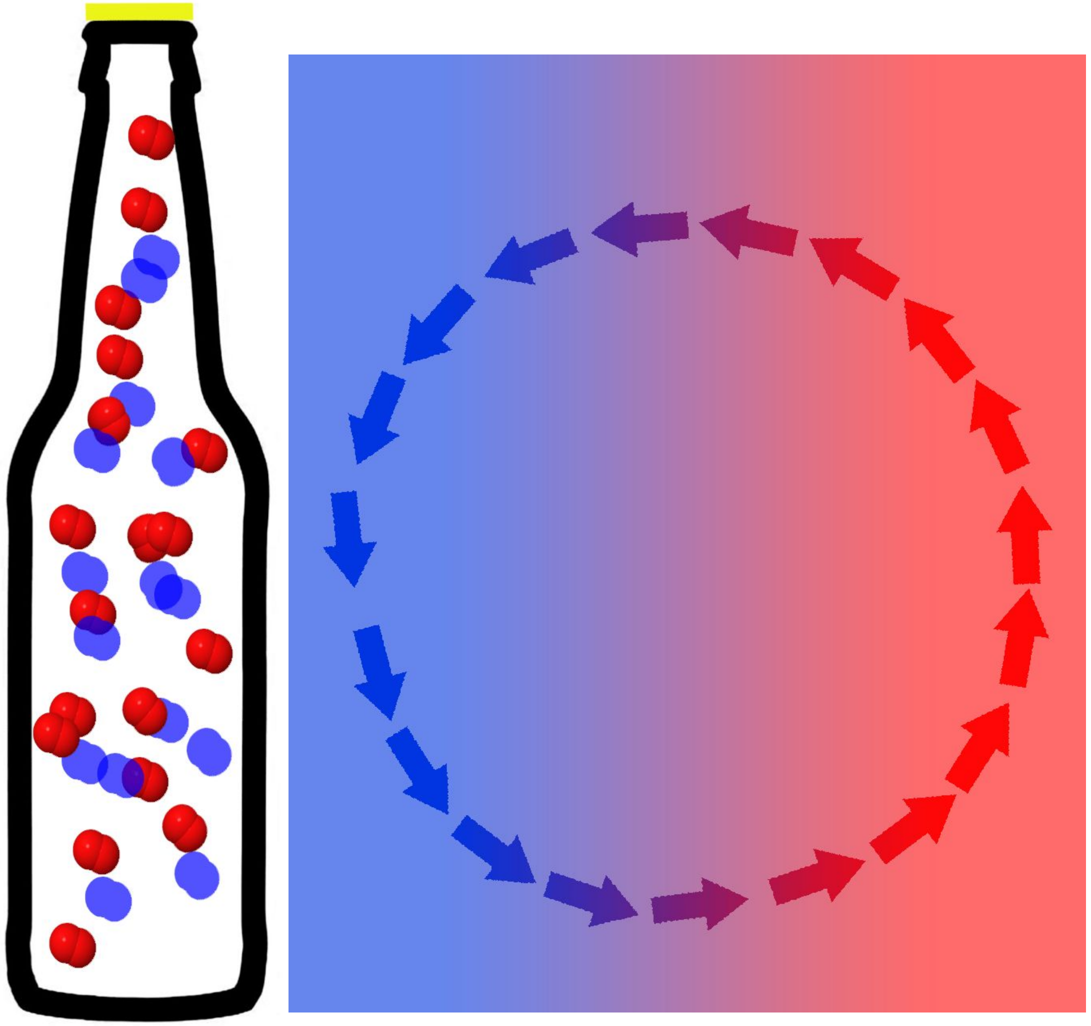


Figure 3: Scheme of heat convection in the system.

Relevant principles

Radiation

Perfect black body:

$$\frac{dQ}{dt} = \sigma A T^4$$

Real case model:

$$\frac{dQ}{dt} \approx c_r A \Delta T$$

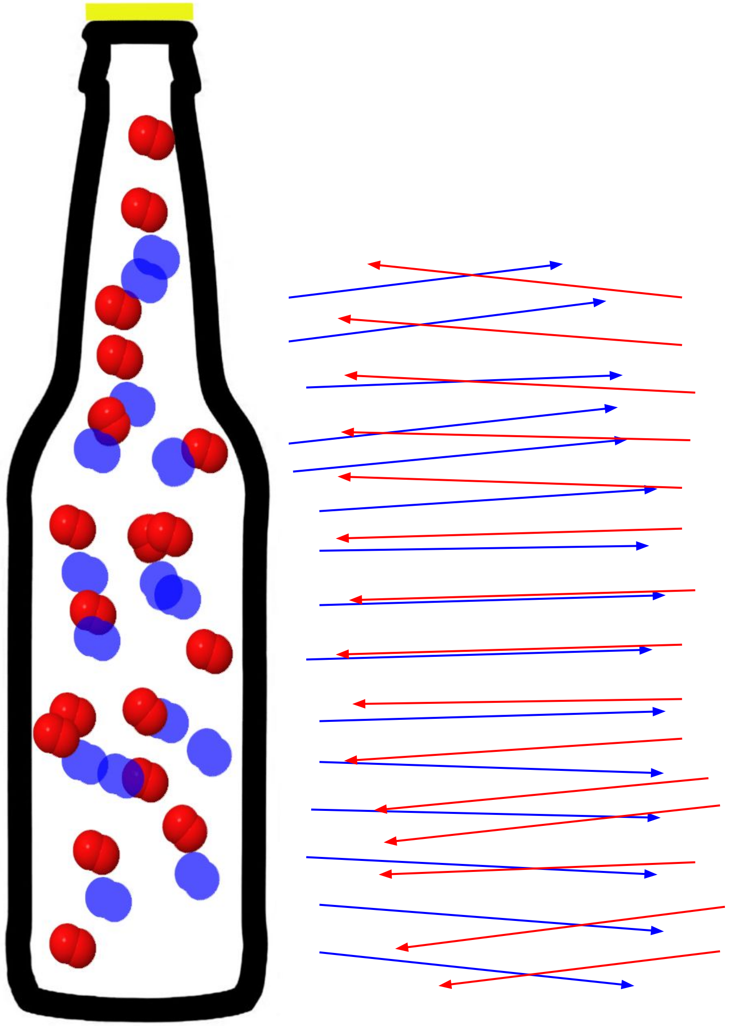


Figure 4: Scheme of heat radiation on the system.

Relevant principles

$$k = \frac{1}{\frac{1}{c_e + c_i} + \frac{L}{c_c} + \frac{1}{c_r}}$$

Convection: $c_e + c_i$

Conduction: c_c

Radiation: c_r

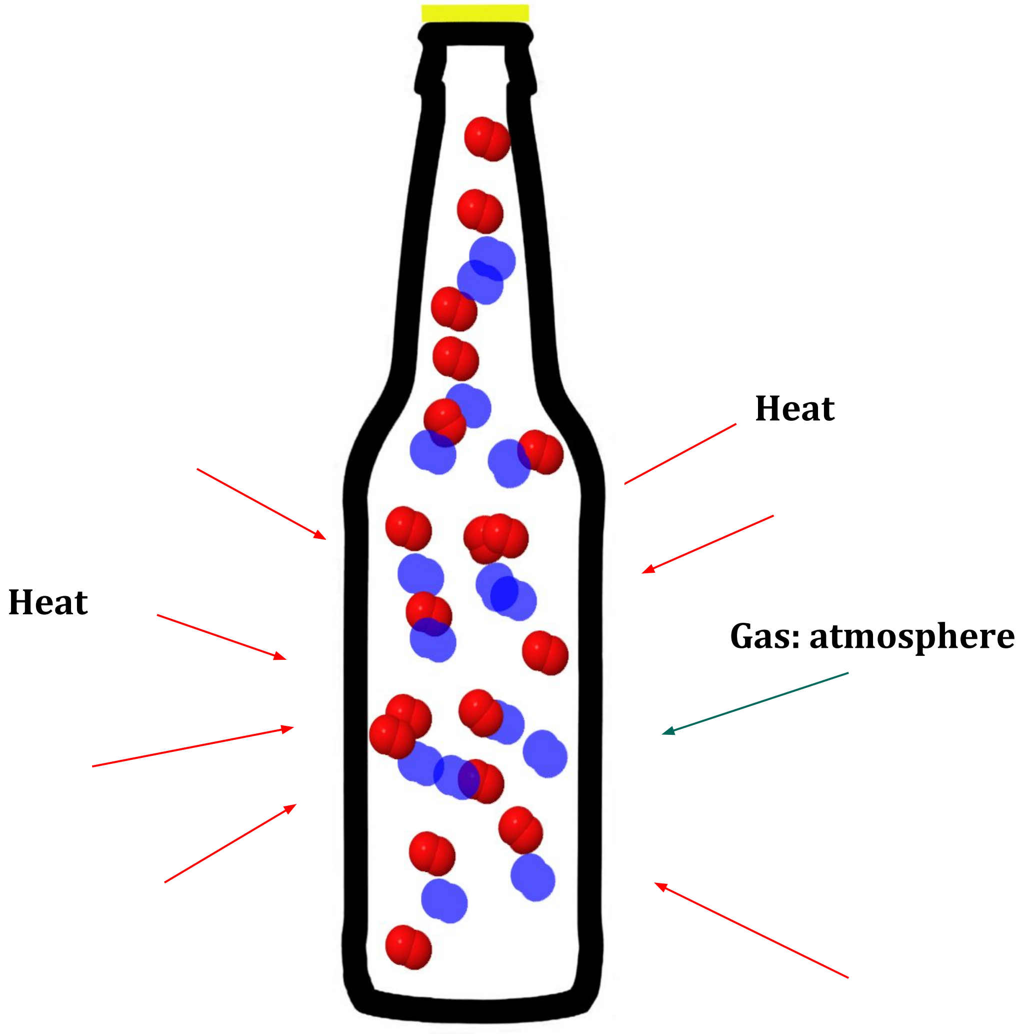
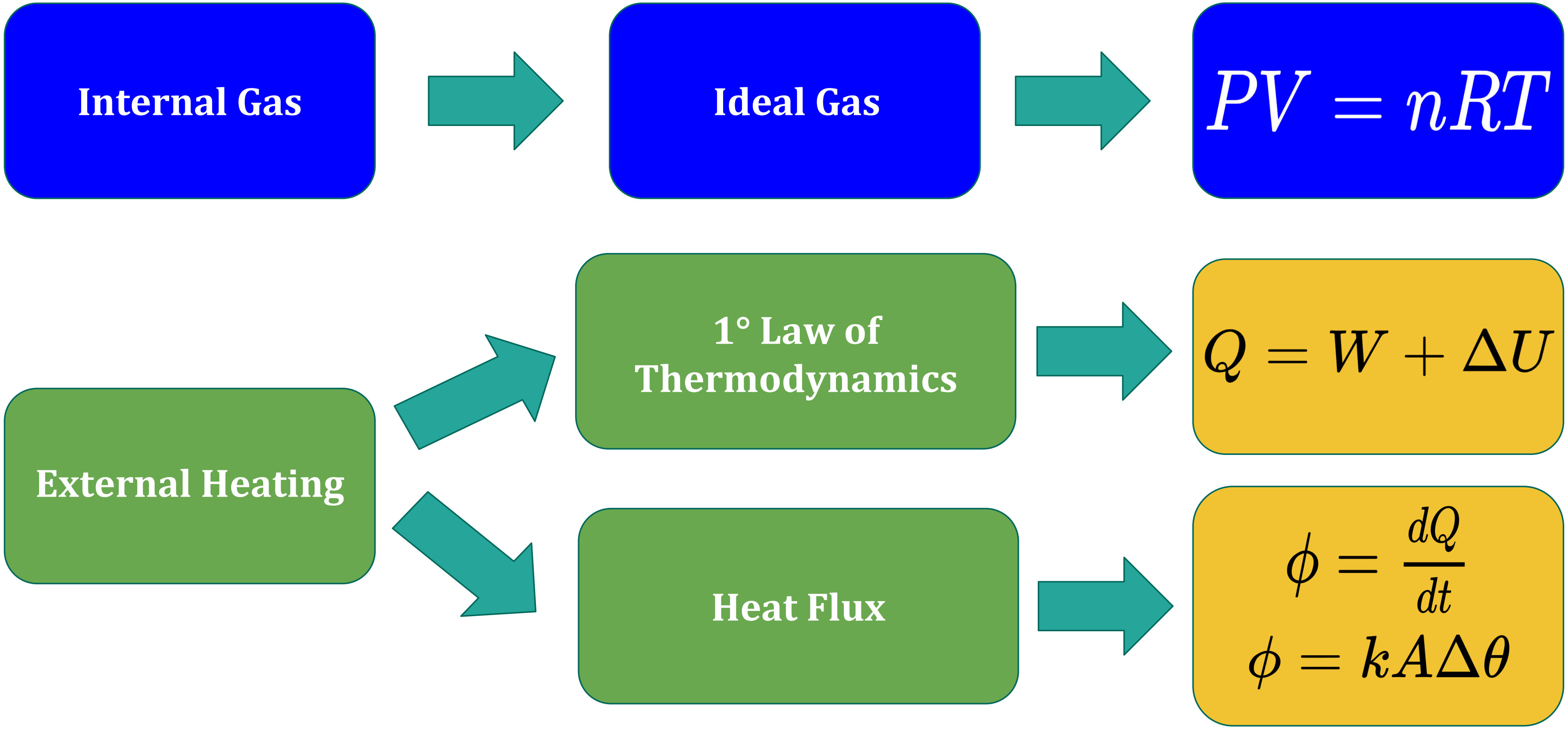


Figure 5: Scheme of heat transfer on the system.

Theoretical model and analysis



Theoretical model and analysis

$$dQ = dW + dU \longrightarrow dU = (nC_v + C_g)d\theta = \phi dt$$

$$\phi dt = kA\Delta\theta dt$$

$$\Delta\theta = \theta_a - \theta(t)$$

$$\longrightarrow \frac{d\theta}{\Delta\theta} = \left(\frac{kA}{nC_v + C_g}\right)dt$$

$$b = \frac{kA}{nC_v + C_g}$$

$$\theta_g(t) = \theta_a - (\theta_a - \theta_0) \cdot e^{-bt}$$

$$P(t) = \frac{nR[\theta_a - (\theta_a - \theta_0)e^{-bt}]}{V}$$

nCv = **Thermal Cap. of gas.**

C_g = **Thermal Cap. of the bottle.**

$\theta(t)$ = **Bottle's Temperature as a function of time.**

θ_a = **Room Temperature.**

θ_0 = **Bottle's Initial Temperature**

Theoretical model and analysis

$$\Delta\theta_i = \frac{V\Delta P}{nR} \longrightarrow \Delta\theta_i = \Delta\theta_n \forall\, n$$

$$\Delta\theta_i = (\theta_i - \theta_0) - \sum_{n=1}^{i-1} \Delta\theta_i \longrightarrow \Delta\theta_i = (\theta_i - \theta_0) - \frac{(i-1)V\Delta P}{nR}$$

$$\theta_i = \theta_a - (\theta_a - \theta_{i-1})e^{-bt}$$

$$\Delta t(i) = \frac{ln(1+\frac{\Delta\theta_i}{\Delta\theta_f - i\Delta\theta_i})}{b}$$

$$\Delta\theta_f = \theta_a - \theta_0$$

$$\Delta\theta_i = \frac{V\Delta P}{nR}$$

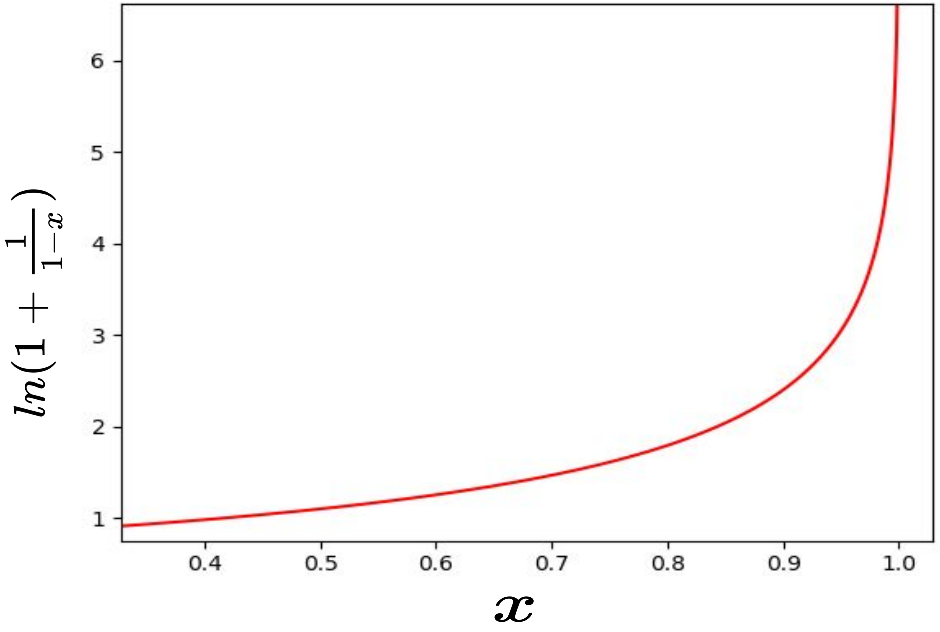
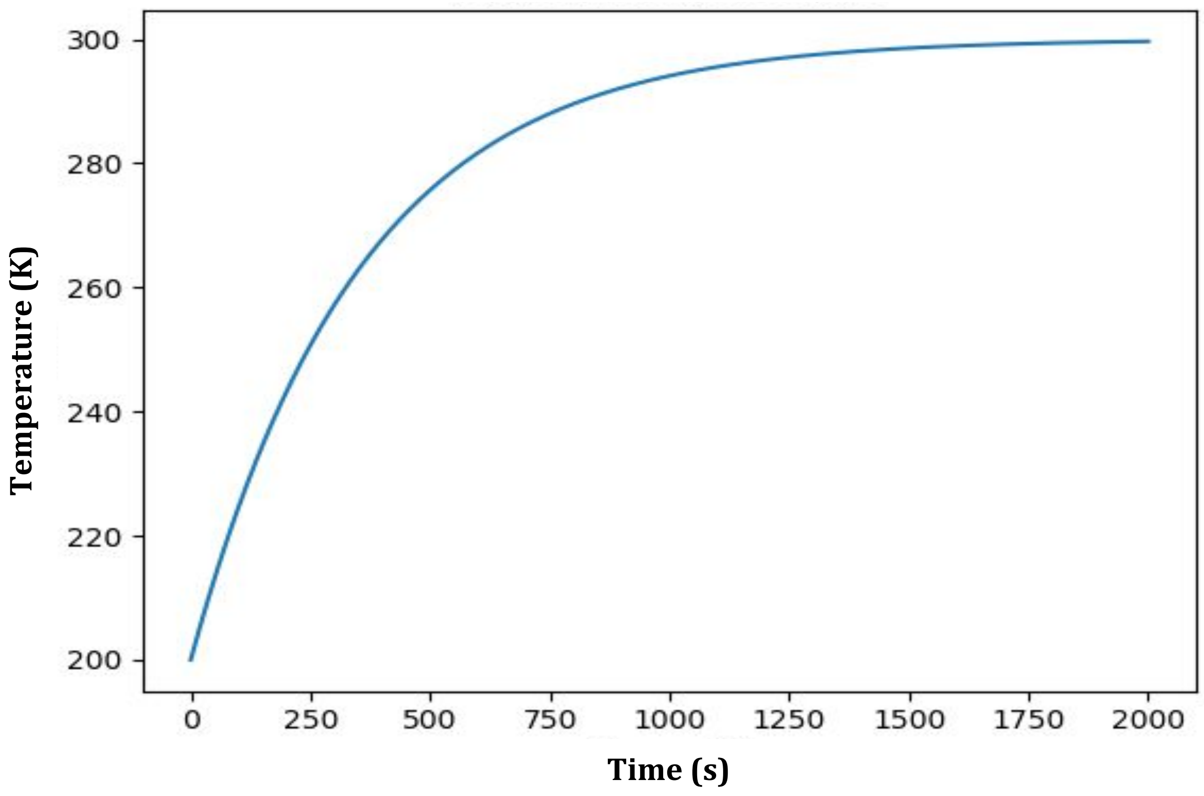


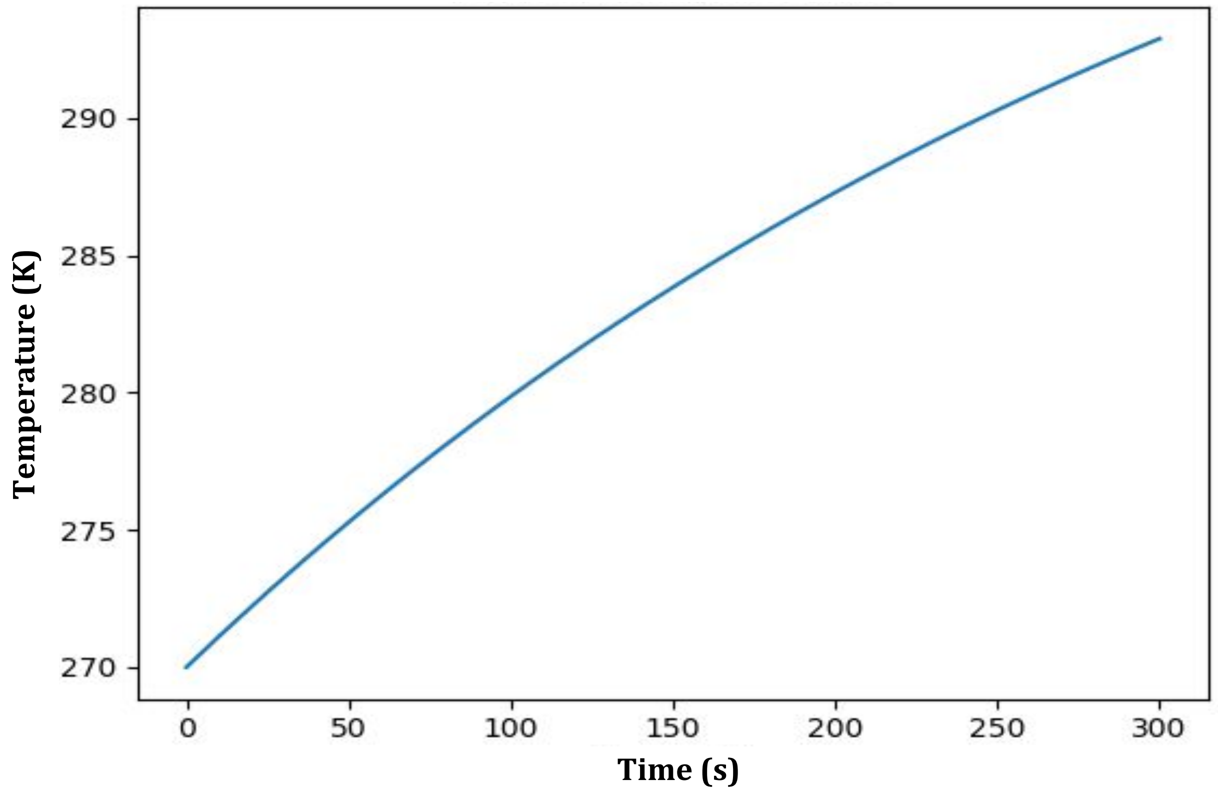
Figure 6: Example plot.

Theoretical model and analysis

Analytical temperature as a function of time.

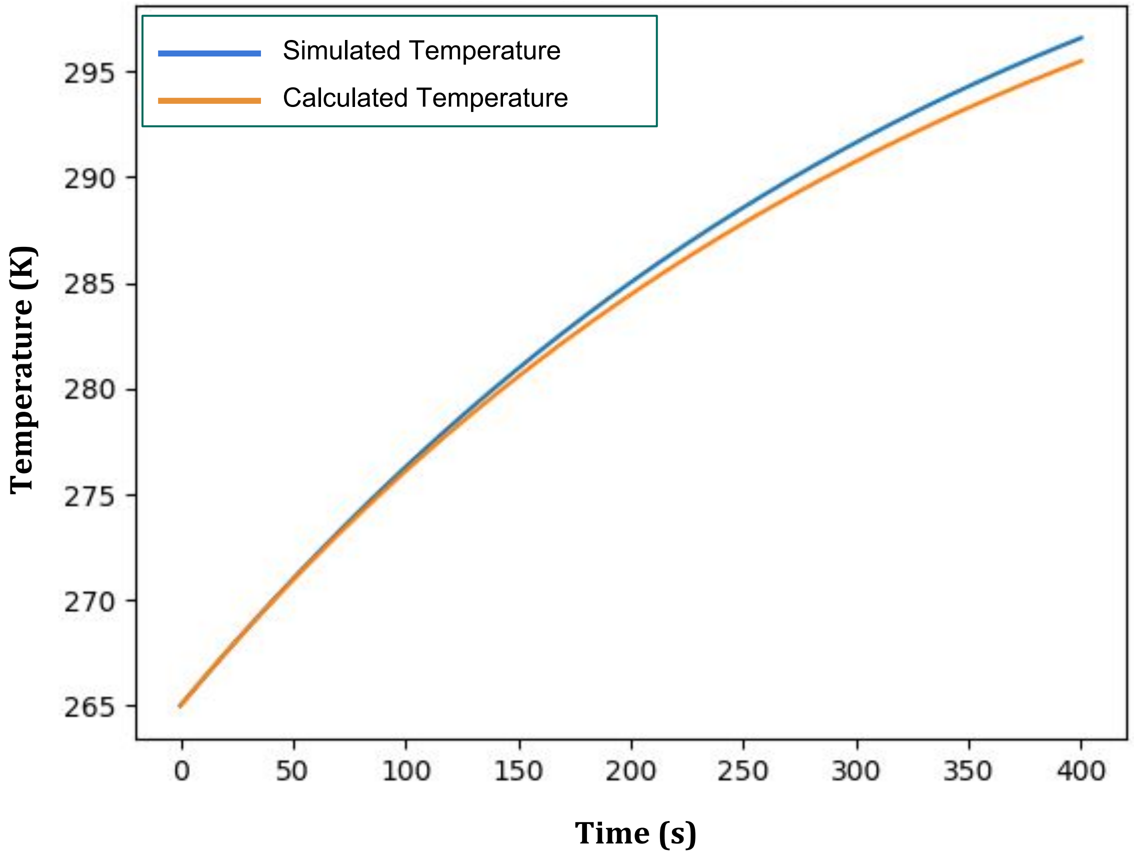


Analytical temperature as a function of time during smaller time intervals.

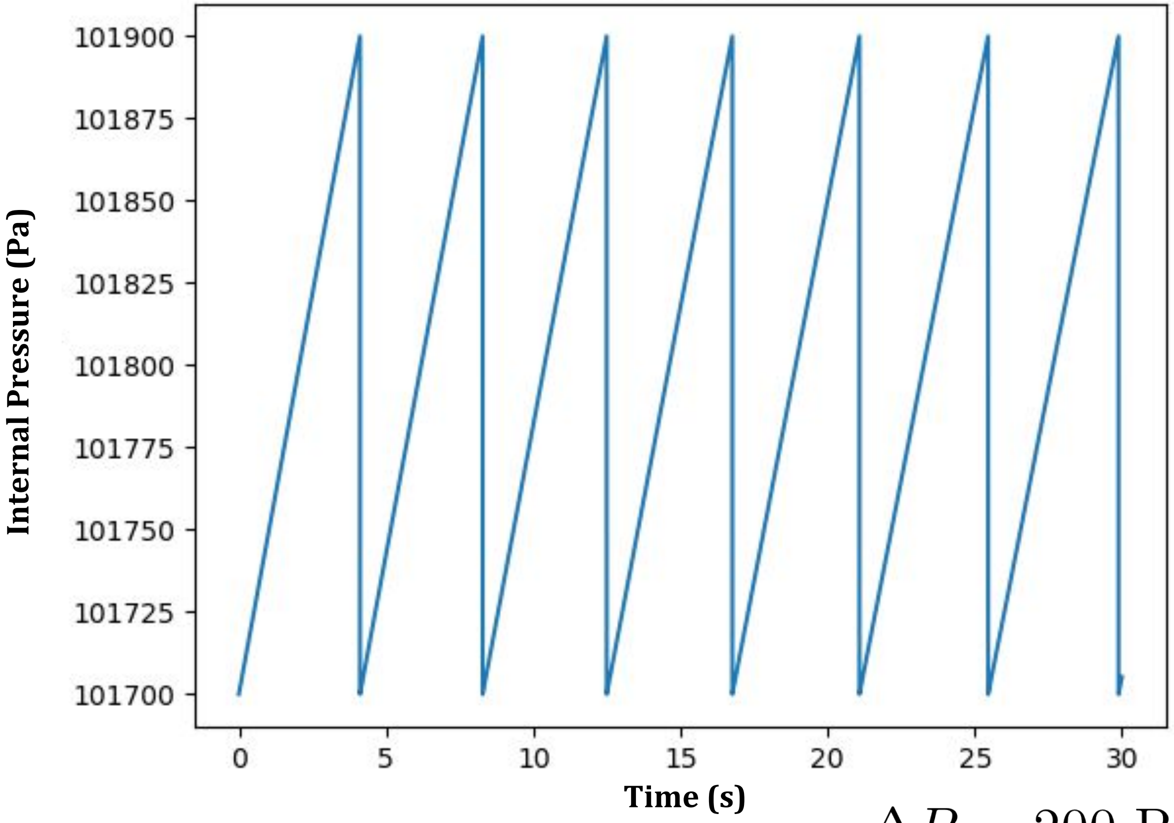


Theoretical model and analysis: Simulations

Simulation temperature *versus* calculated temperature



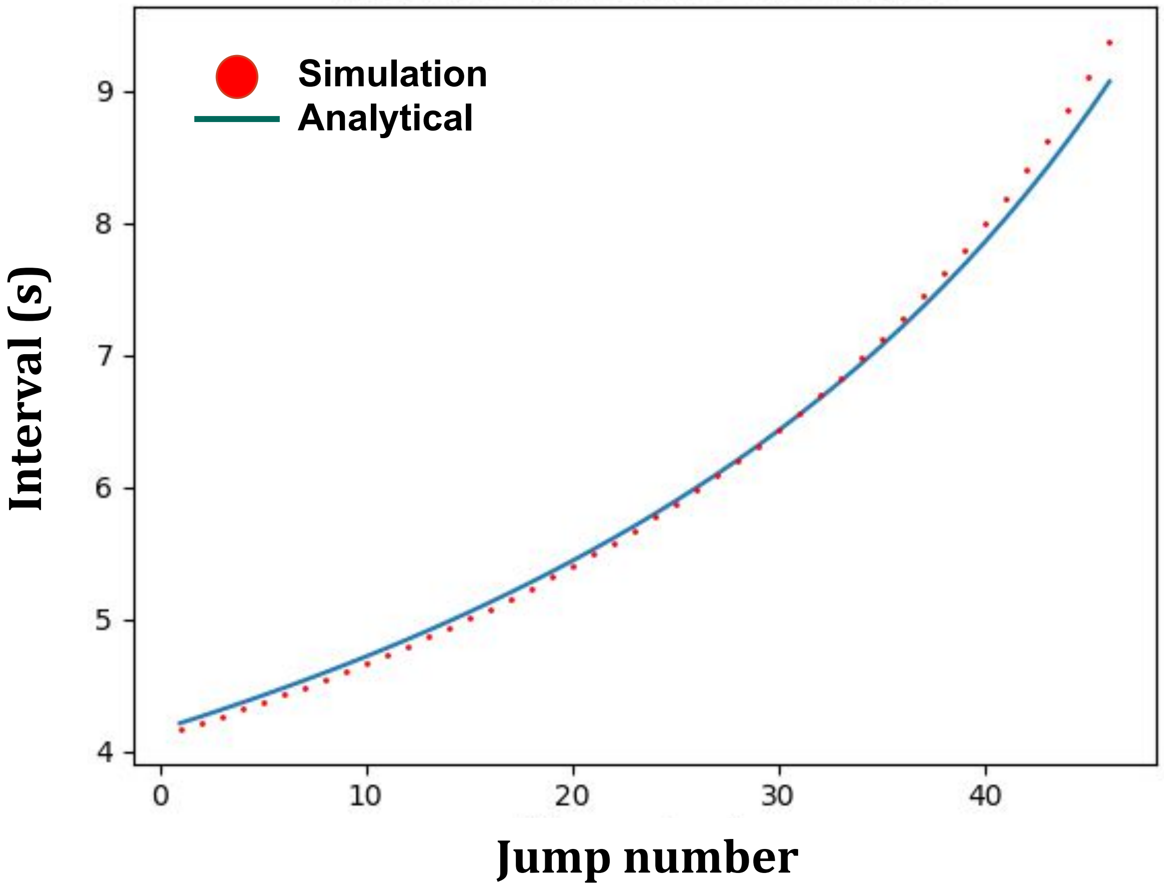
Simulation of pressure *versus* time



$$\Delta P = 200 \text{ Pa}$$

Theoretical model and analysis: Simulations

Intervals between consecutive jumps as simulated and as calculated.



$$\Delta t(i) = \frac{ln(1+\frac{\Delta\theta_i}{\Delta\theta_f - i\Delta\theta_i})}{b}$$

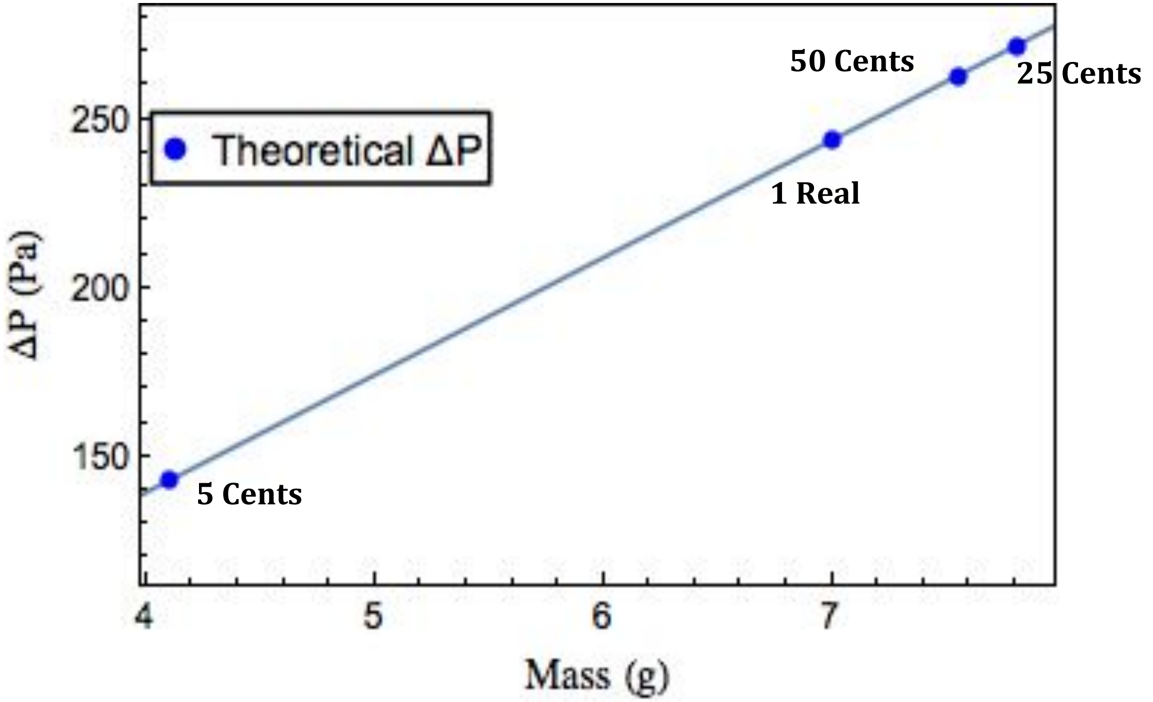
$$b = 2.83 \times 10^{-3}\ s^{-1}$$

Theoretical model and analysis

Pressure needed to raise the coin (theoretical):

$$(P - P_a)A = mg \quad \Rightarrow \quad \Delta P = \frac{mg}{A}$$

Estimated ΔP for each coin.



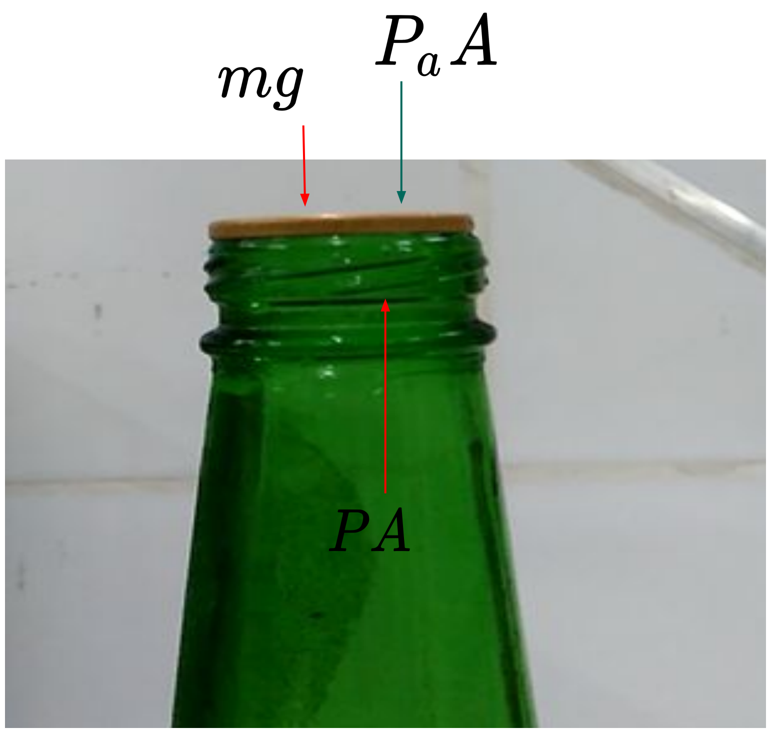


Figure 7: Forces acting on the coin.

Relevant Parameters

- **Heat Sources** → $\Delta\theta$ → **Medium**, **Temperature**
- **The coin and the opening** → ΔP → **Area**, **Mass**
- **Bottle** → **Coefficient *b*** → **Shape**, **Material**

Experimental set-up: The materials

1. Various Bottles;

2. Modern BRL coins of 5 Cents, 25 Cents, 50 Cents and 1 Real;

3. Barometric pressure and temperature module BMP180[1] (Error: P ± 1 Pa and T ± 0,1 K)

4. Arduino UNO;

5. Computer and Python 3 for data-logging;

6. A good freezer.

Figure 8: Bottle similar to what was used.

Figure 9: Modern BRL coins used.

Figure 10: BMP 1801 sensor.

Experimental set-up: The assembly

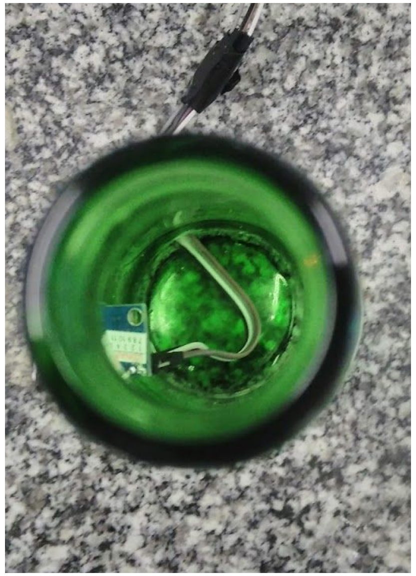


Figure 11: Sensor inside the bottle.

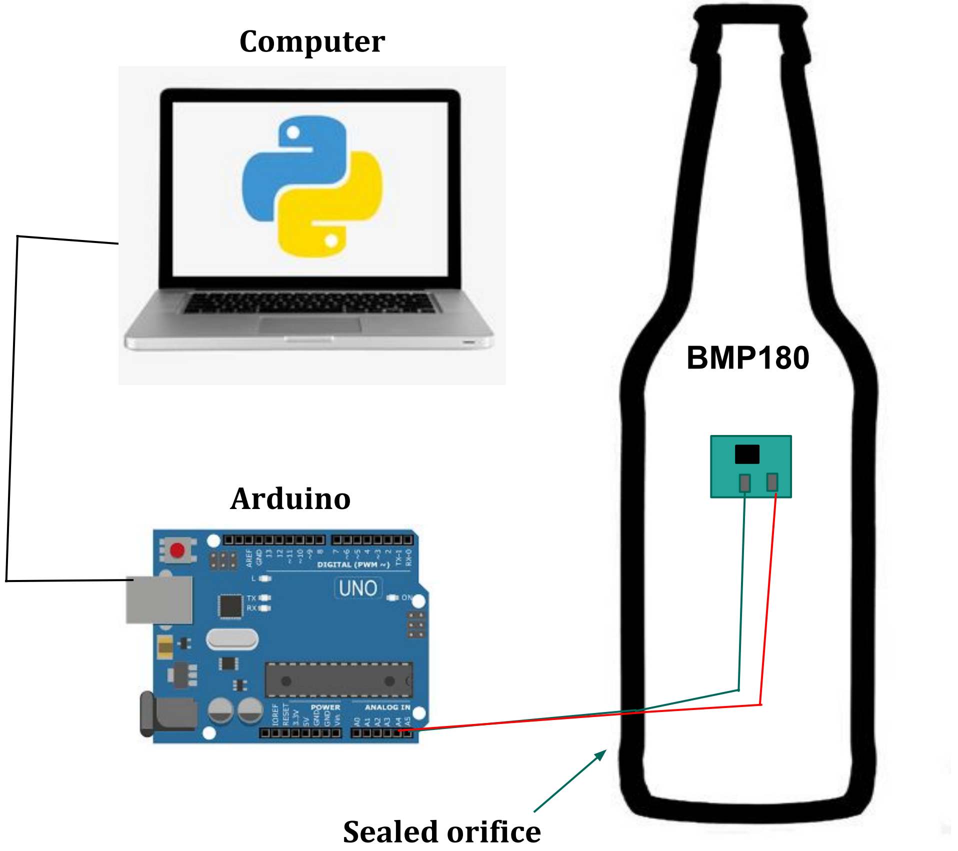


Figure 12: Scheme of experimental set-up..

Figure 13: Part of the experimental assembly.

Experimental set-up: The bottles

From left to right:

1 - Small Bottle (Glass)

2 - Medium Bottle (Glass)

3 - Plastic Bottle (Plastic)

4 - Reference Bottle (Glass)

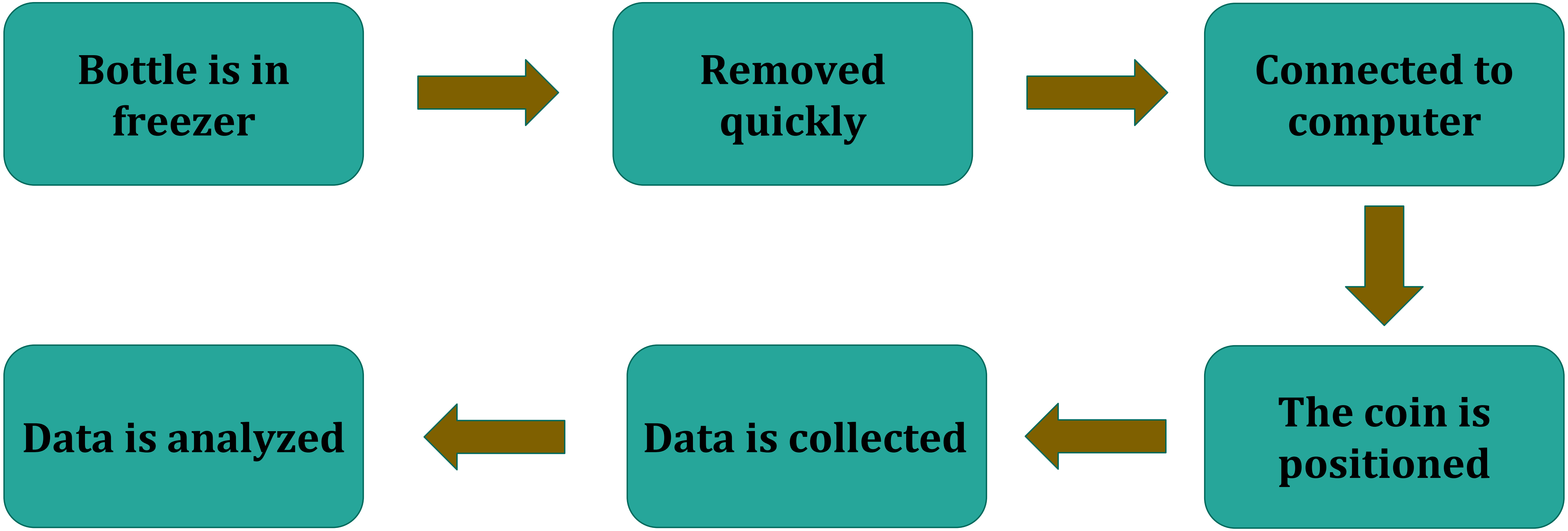
5 - Big Bottle (Glass)

6 - Small Cylindrical Bottle (Glass)

7 - Big Cylindrical Bottle (Glass)

Figure 14: The bottles used in experiments.

Experimental set-up: The procedure



Parameter variation: Medium

ΔP ≈ 260 Pa

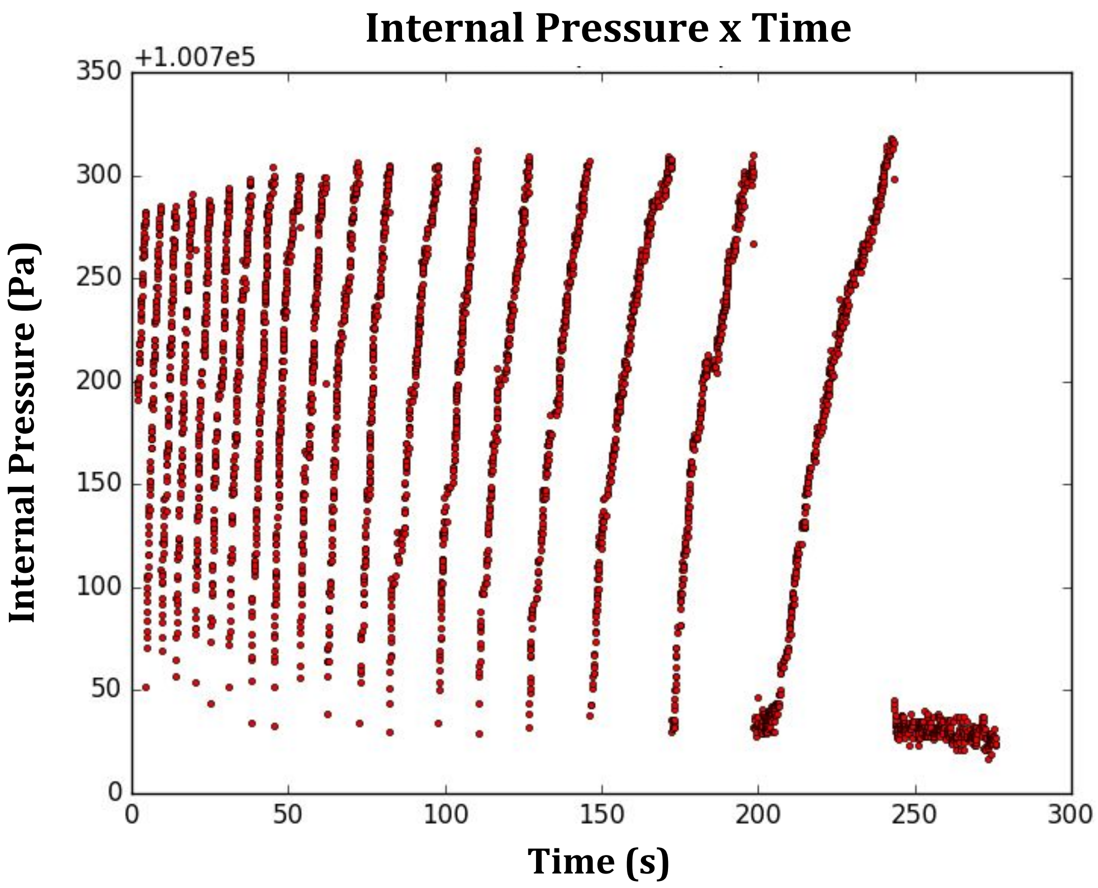


Figure 15: Typical plot from an experiment with no manual contact. The cyclical nature of the internal pressure is noticeable.

Medium
Temperature
Area
Mass
Shape
Material

Parameter variation: Temperature

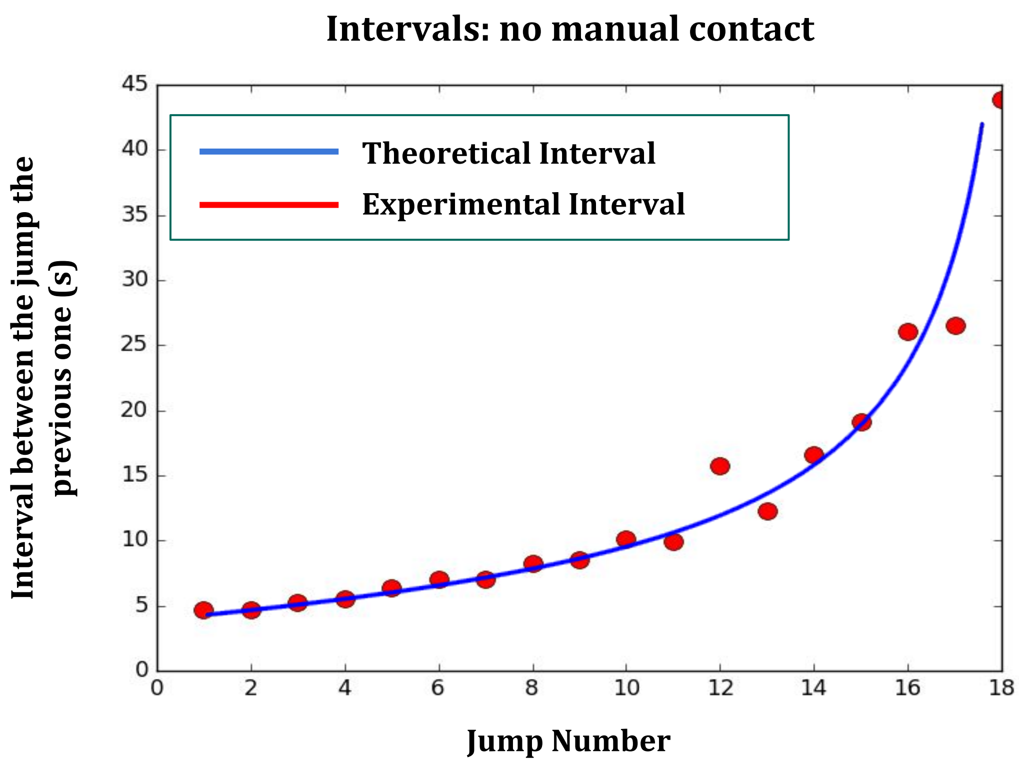


Figure 17: Plot of the interval between the *n*-th jump and the previous one. No manual contact.

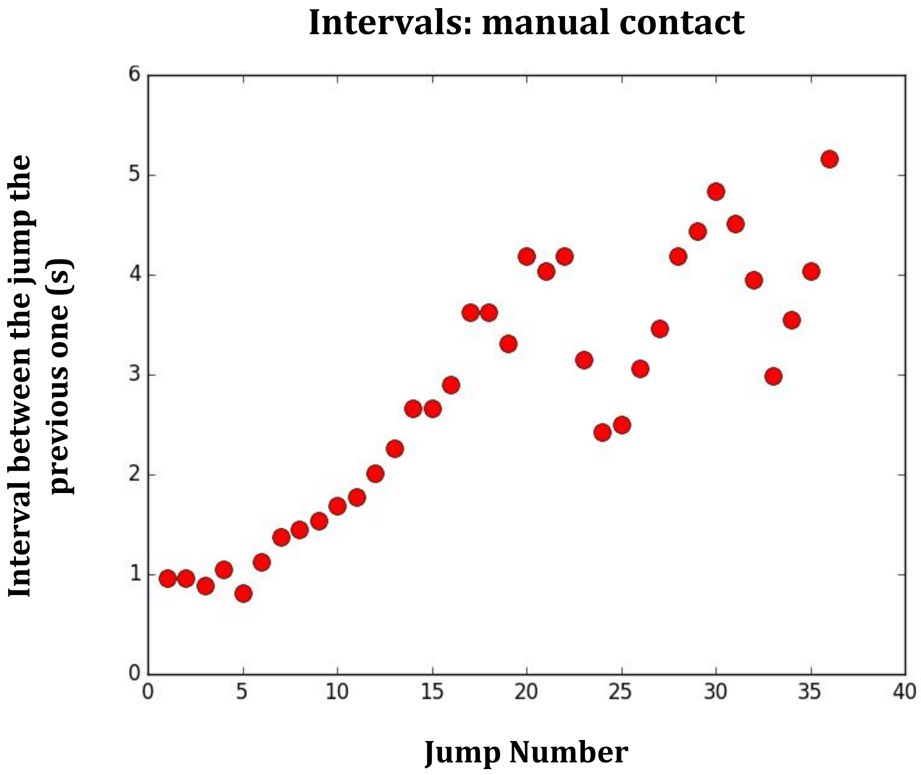
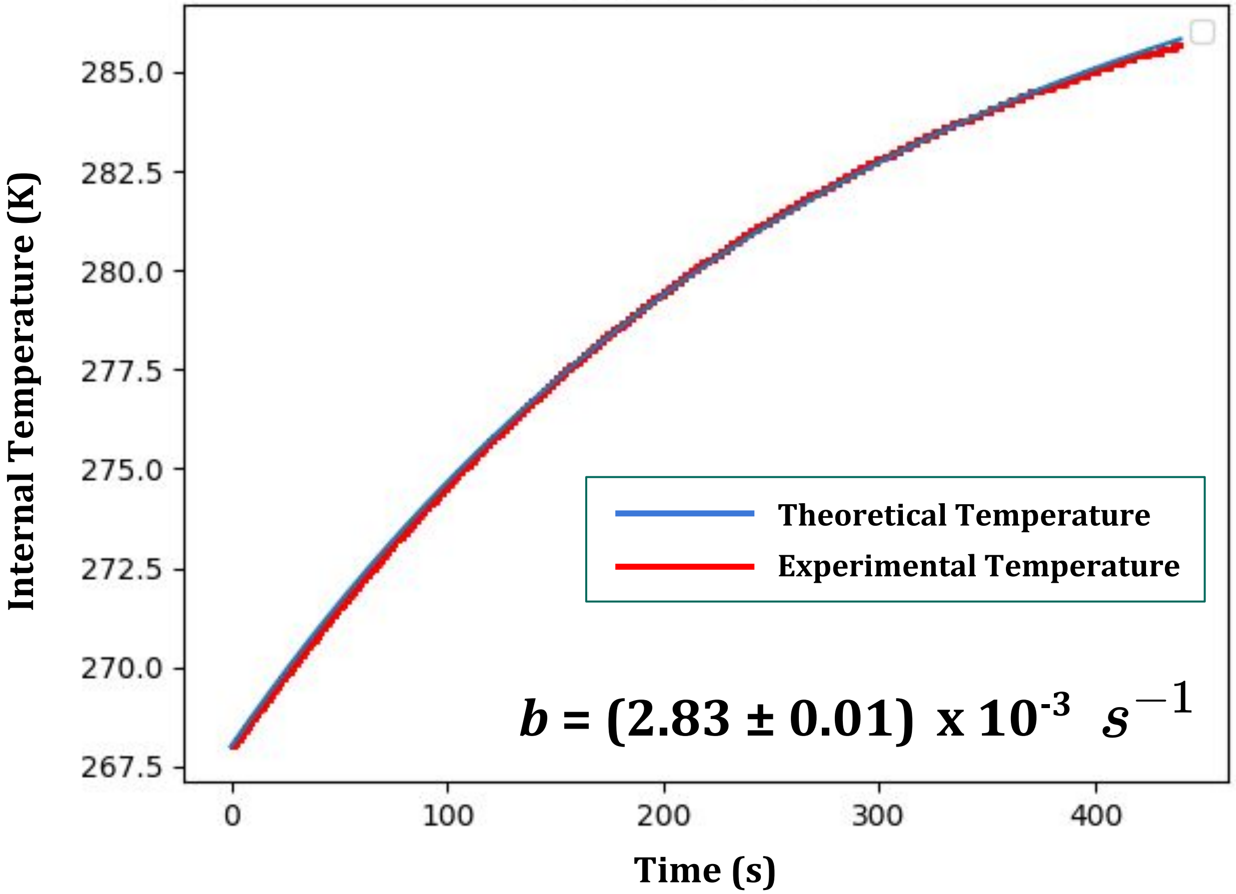


Figure 18: Plot of the interval between the *n*-th jump and the previous one. With manual contact.

Medium
Temperature
Area
Mass
Shape
Material

Parameter variation: Temperature

Temperature as a function of time.



Medium

Temperature

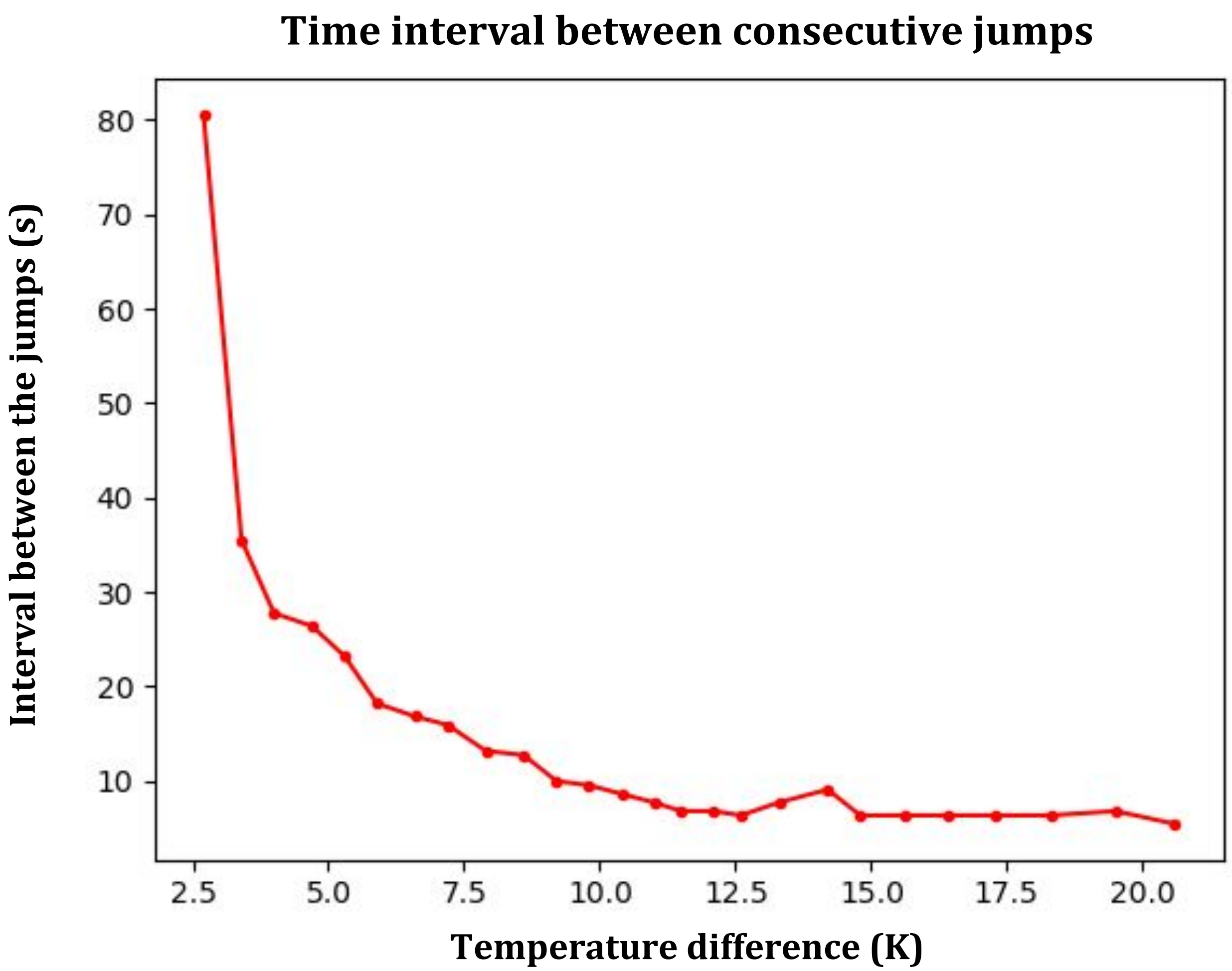
Area

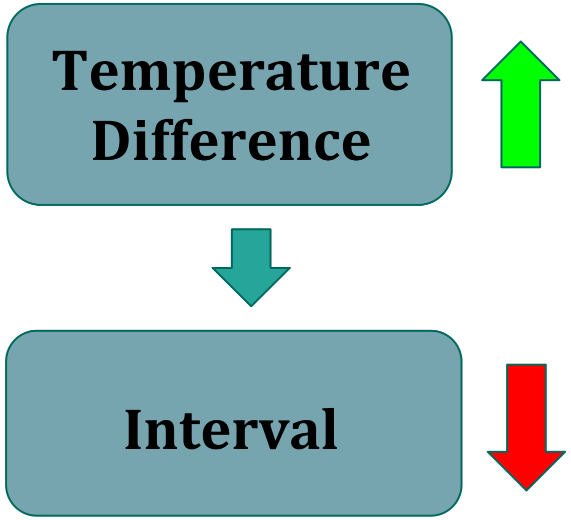
Mass

Shape

Material

Parameter variation: Temperature





Medium

Temperature

Area

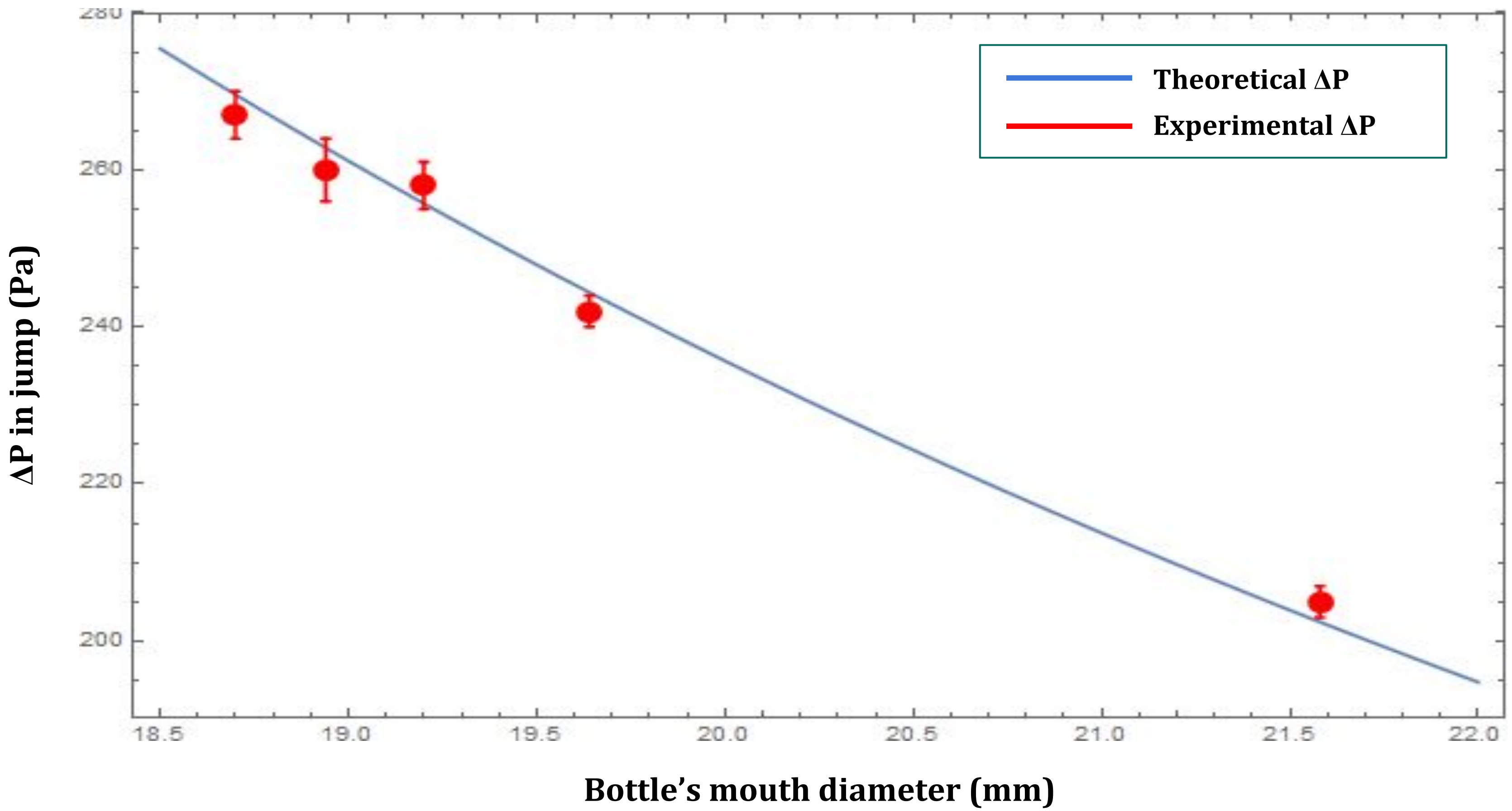
Mass

Shape

Material

Parameter variation: Area

ΔP as a function of the bottle's mouth diameter.



Medium

Temperature

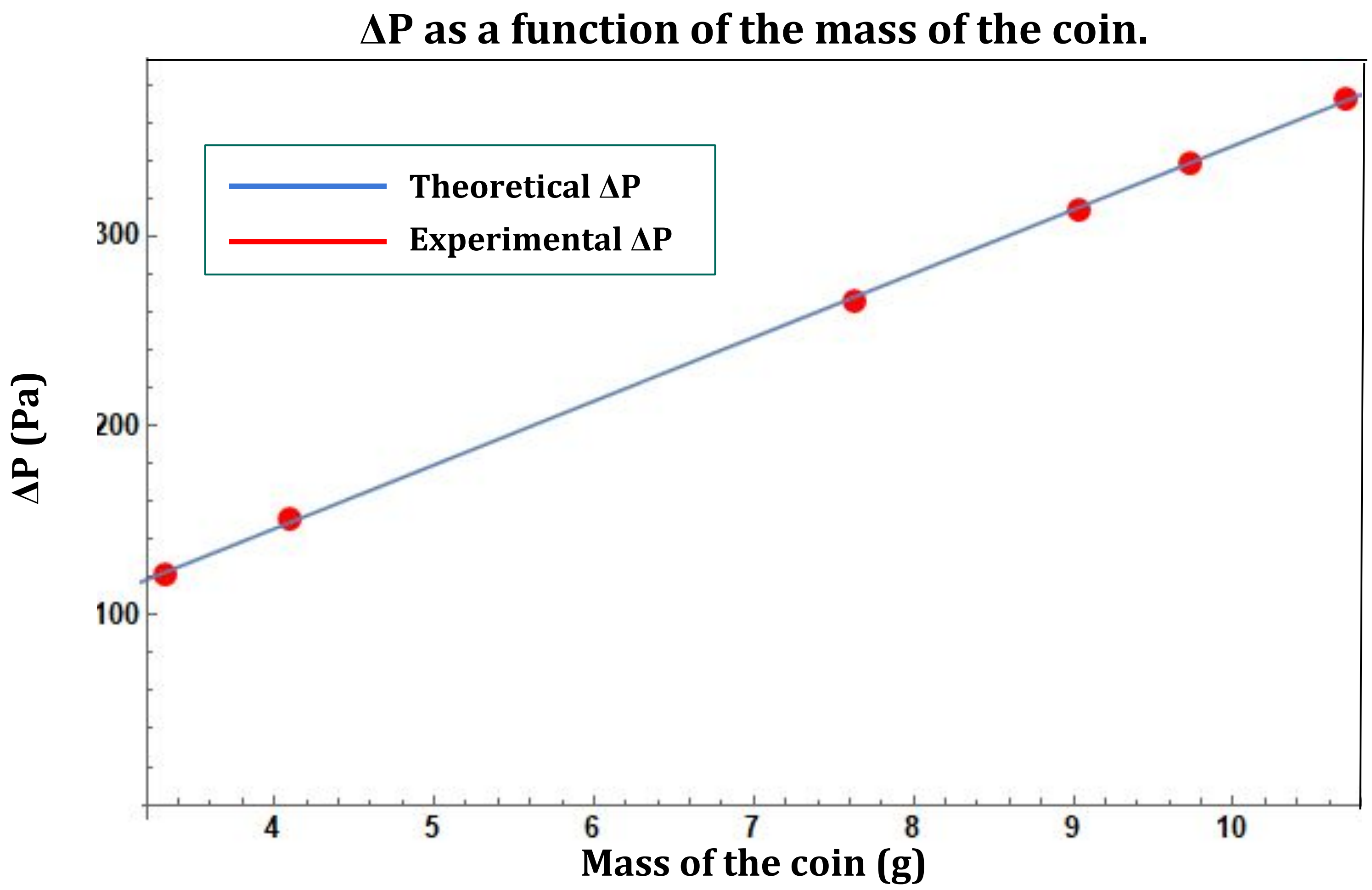
Area

Mass

Shape

Material

Parameter variation: Mass



Medium

Temperature

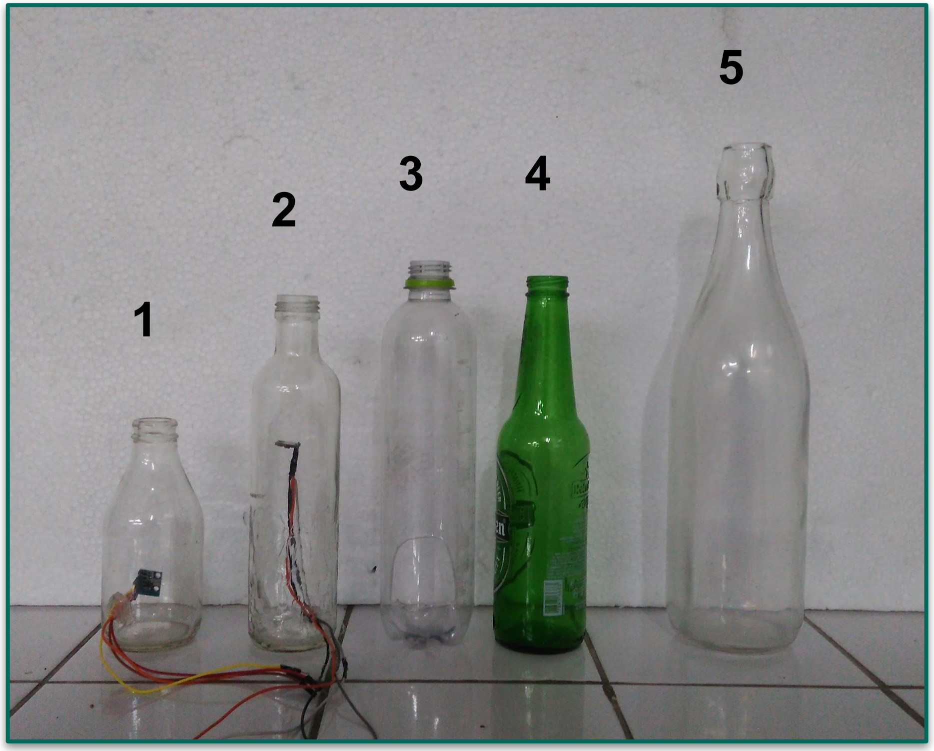
Area

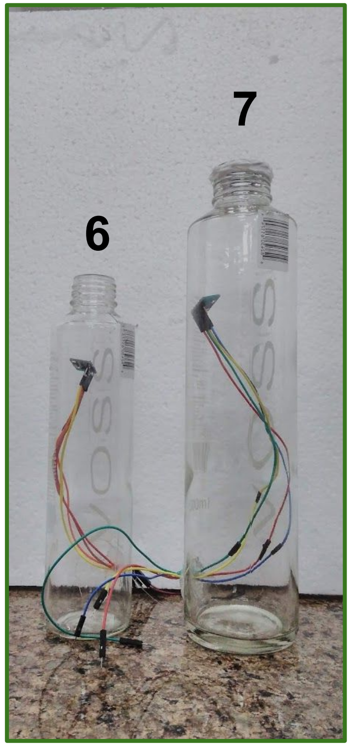
Mass

Shape

Material

Parameter variation: Shape and material





Bottle	Experimental *b* (s^{-1} x 10^{-3})
1	4.50 ± 0.02
2	3.70 ± 0.02
3	10.3 ± 0.4
4	2.83 ± 0.01
5	2.63 ± 0.01
6	3.42 ± 0.01
7	2.77 ± 0.07

Table 1 - Coefficient *b* measured for each bottle.

- Medium
- Temperature
- Area
- Mass
- **Shape**
- **Material**

Sound

Pressure rises → The coin lifts → Sound in a tube → The coin falls

$76\text{ms} \leq \Delta T \leq 153\text{ms}$

$f \approx (1845 \pm 2)\text{Hz}$

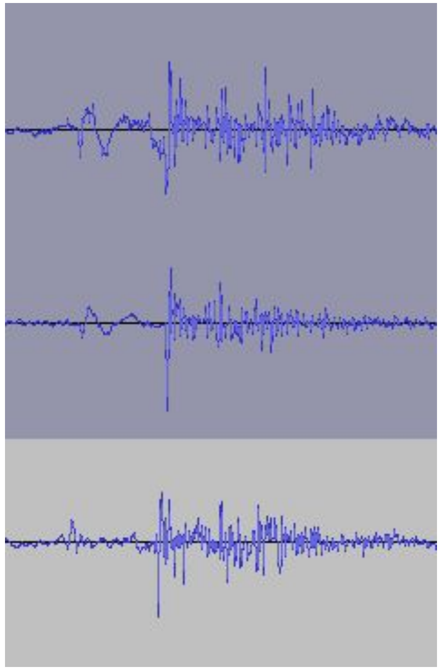
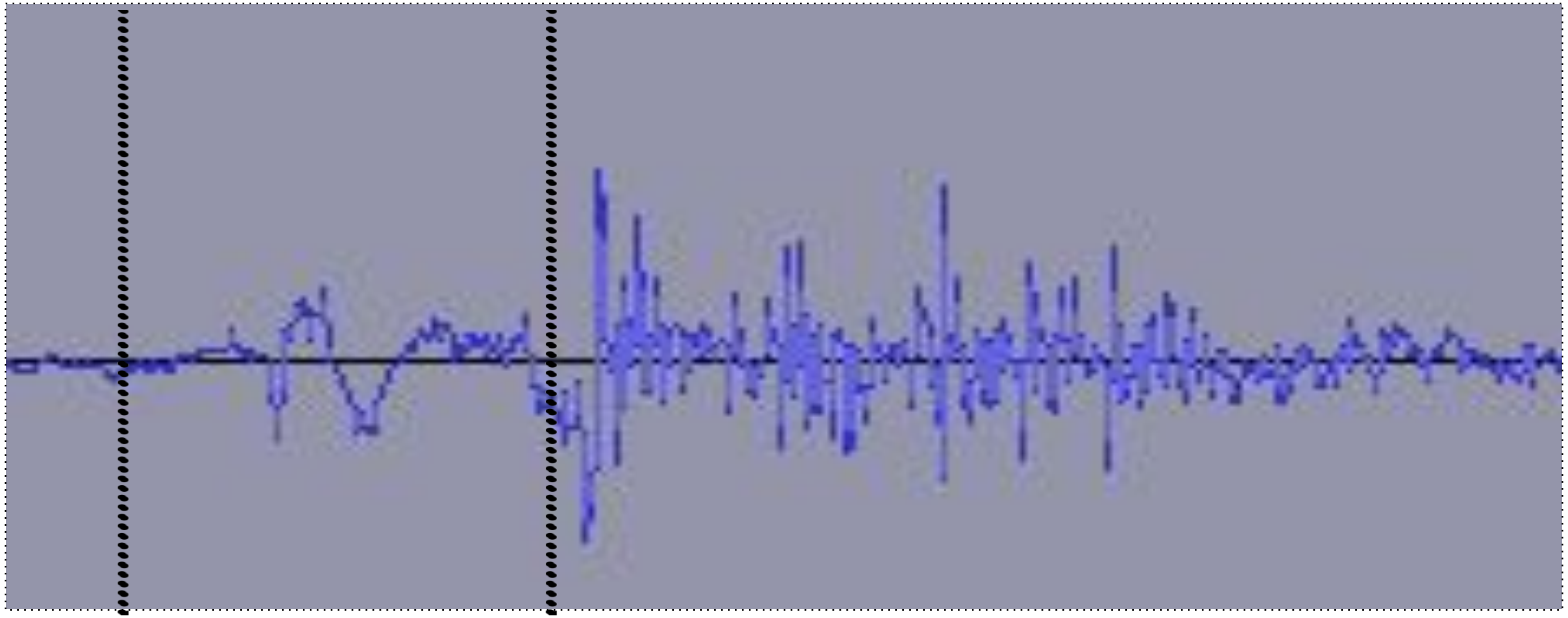


Figure 22: Graphical visualization of the jump sound.

Sound



Lift

Fall

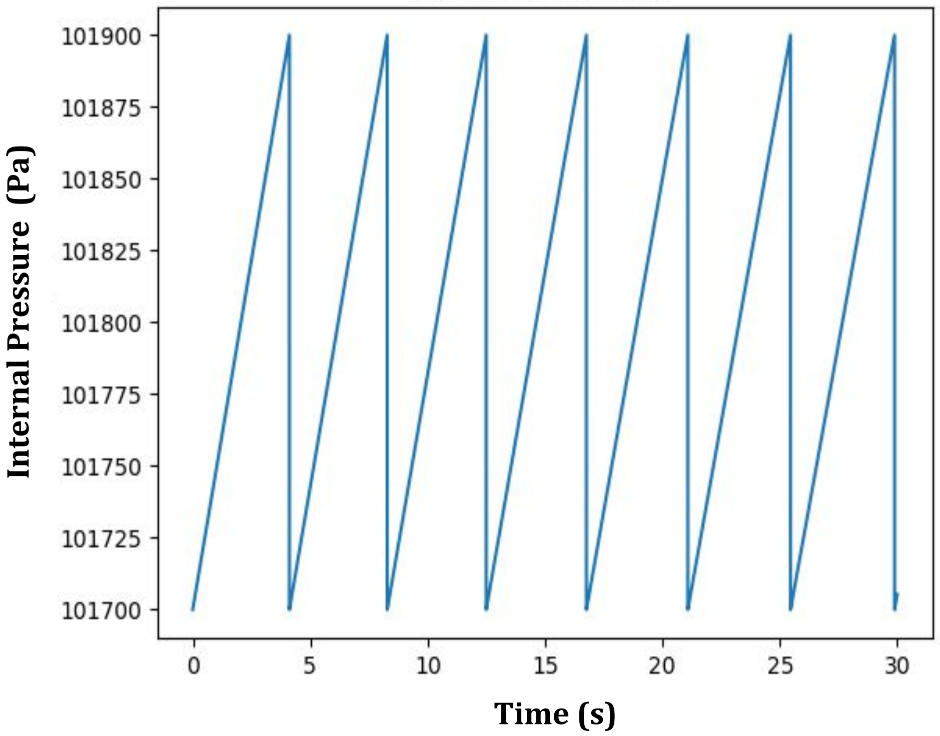
$$f_{low} = (188 \pm 4)Hz$$

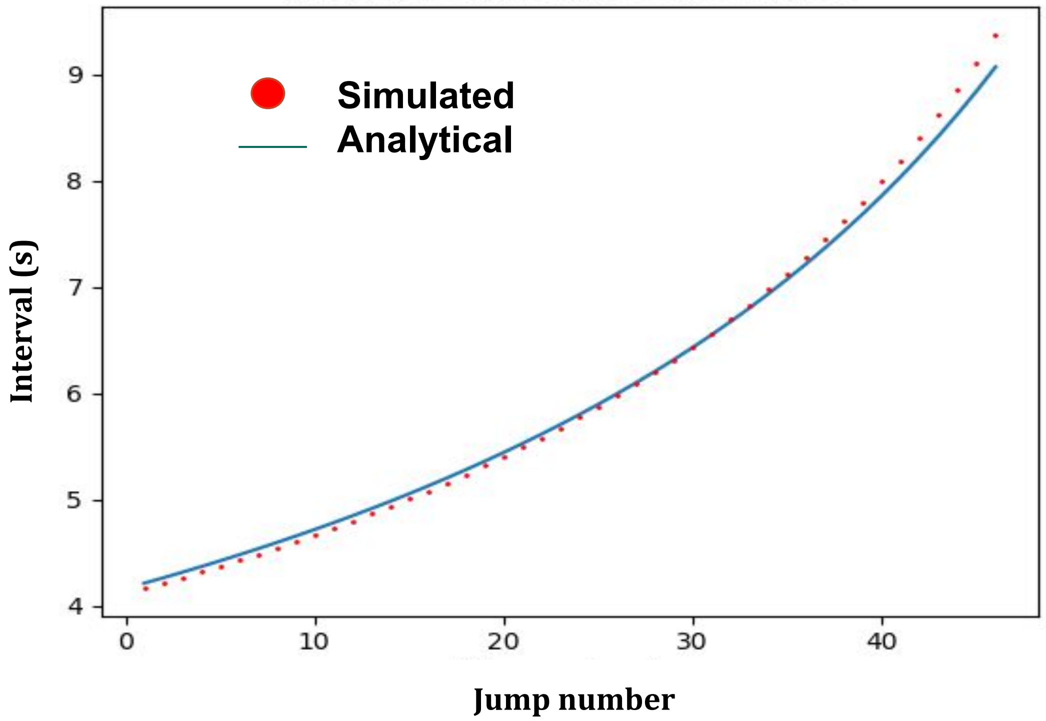
$$f_{high} = (2387 \pm 7)Hz$$

Summary: Theory

$$b = \frac{kA}{nC_v + C_g}$$

$$\Delta t(i) = \frac{ln\left(1 + \frac{\Delta\theta_i}{\Delta\theta_f - i\Delta\theta_i}\right)}{b}$$





Medium

Temperature

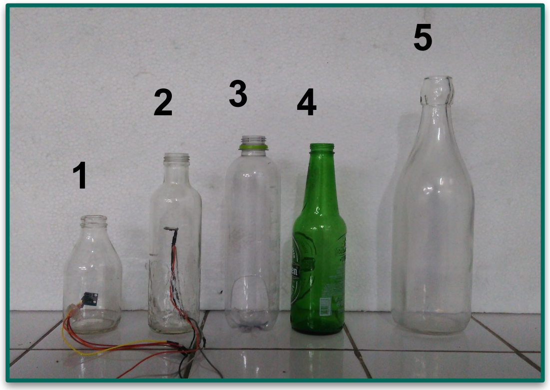
Area

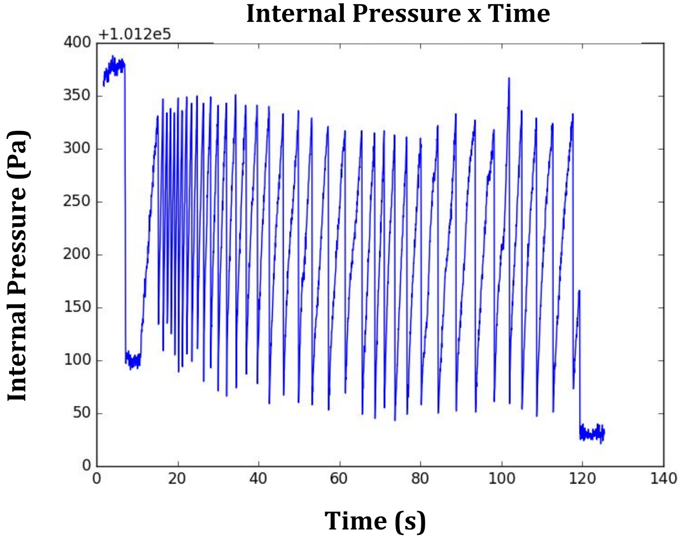
Mass

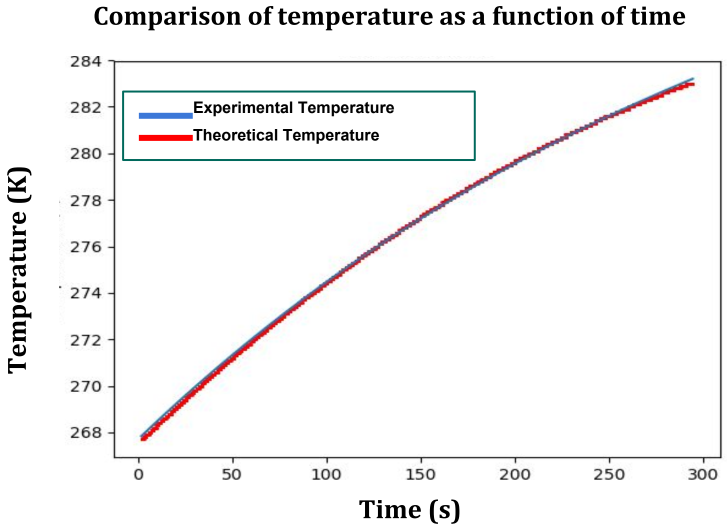
Shape

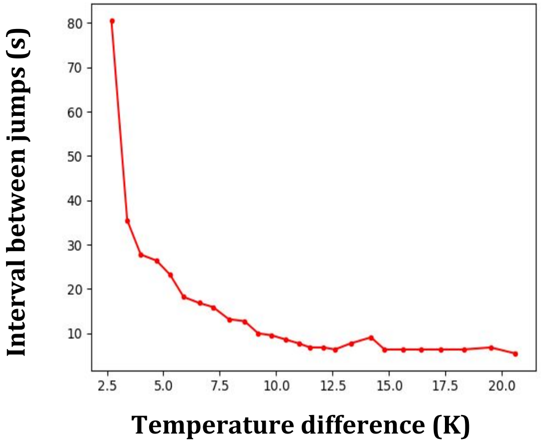
Material

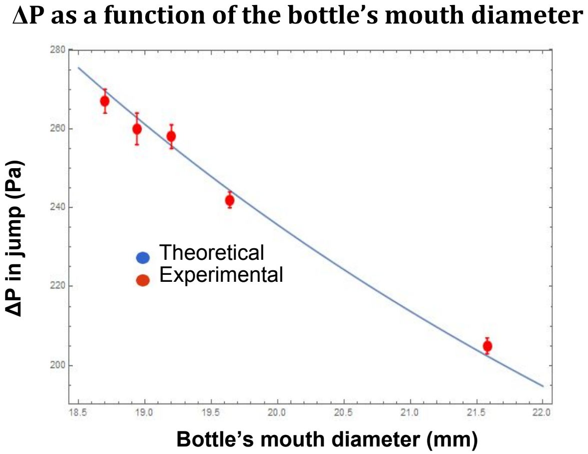
Summary: Experiments

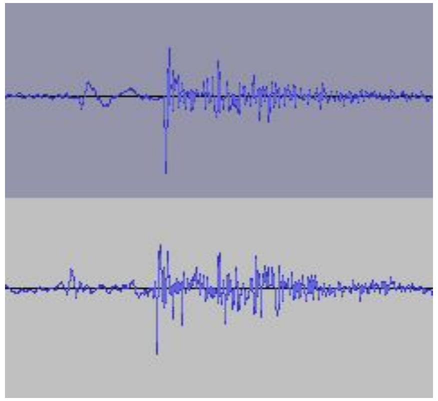












Sound

Bibliography

[1] BOSCH, (5 de Abril de 2013), BMP180 Data sheet,

<https://cdn-shop.adafruit.com/datasheets/BST-BMP180-DS000-09.pdf>, Acessed in: 29/01/2018.

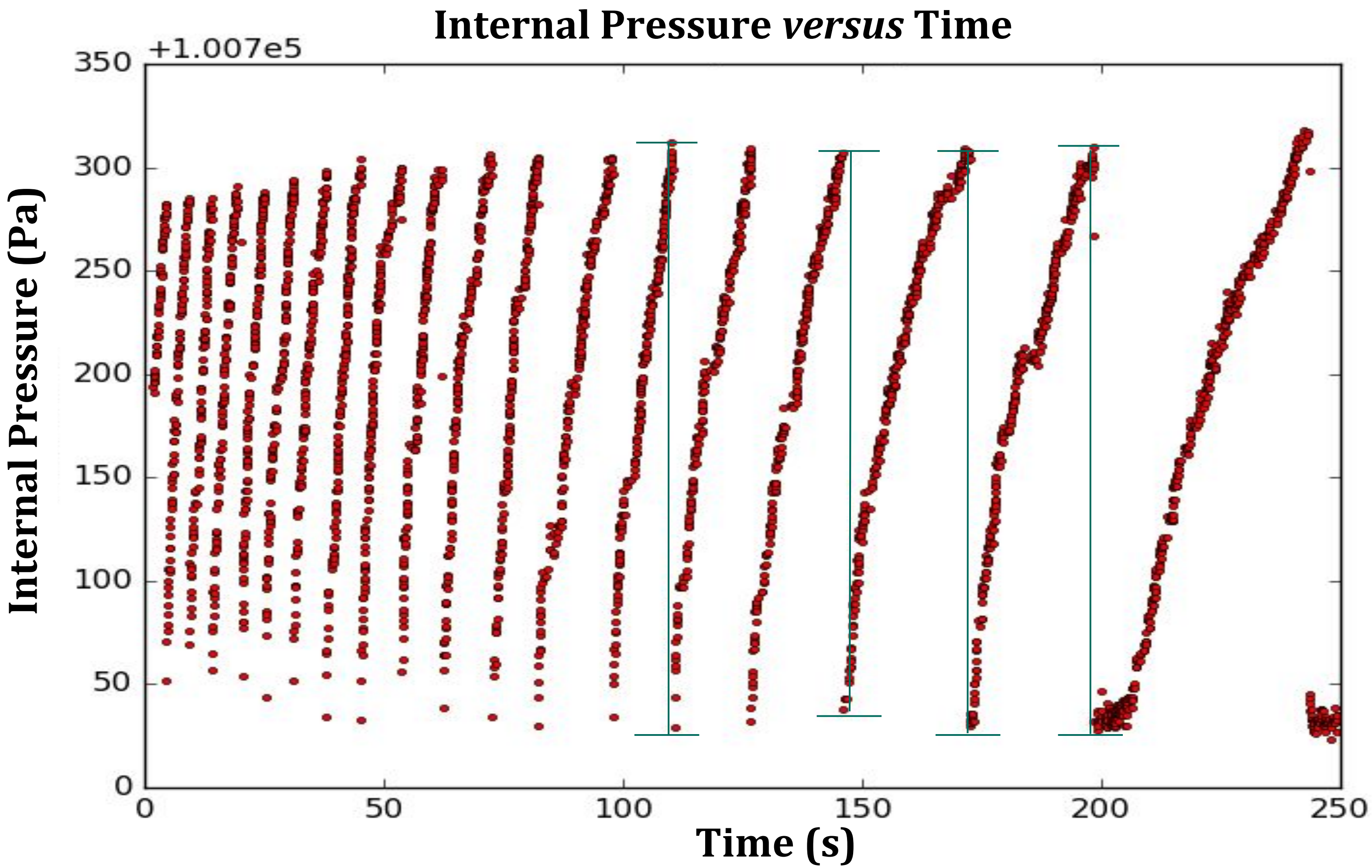
[2] KREITH, F. et al. Princípios de Transferência de Calor. 1 ed. São Paulo:Cengage Learning, 2003.

[3] H.Moysés Nussenzveig, Curso de Física Básica, vol 2, Editora Edgard Blücher, LTDA (1999)

[4] Bailey, R. and Elban, W. (2018). Thermal Performance of Aluminum and Glass Beer Bottles.

Thank you!

Appendix 1: Measuring the ΔP



Appendix 2: Geometry of the bottle

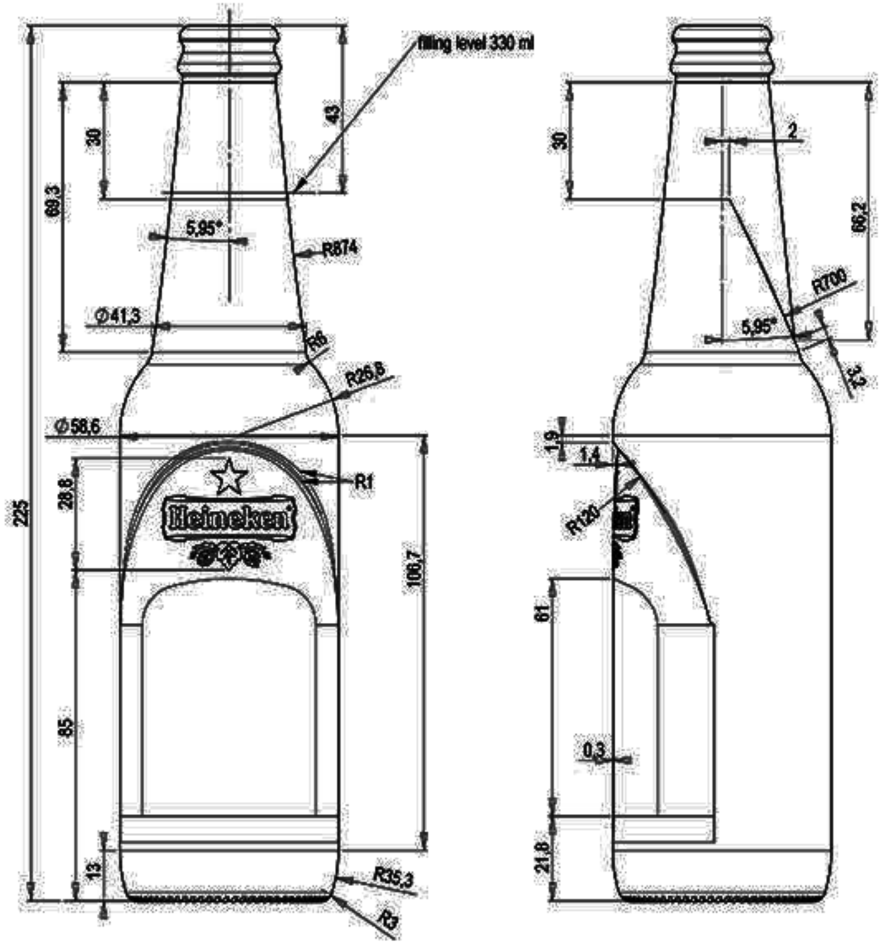


Table - Bottle dimensions.

Area (m²)	0,0306
Mouth diameter (mm)	19,30
=Height (mm)	225
Volume (L)	0,330
Average thickness (mm)	3,54

Thickness $\approx$ Constant

Appendix 3: Van der Waals vs Clapeyron

$$P_w = \frac{nRT}{V-nb} + n\left(\frac{a}{V}\right)^2$$

$$\Delta P_w = \frac{nRT_1}{V-nb} + n\left(\frac{a}{V}\right)^2$$

$$-\left(\frac{nRT_2}{V-nb} + n\left(\frac{a}{V}\right)^2\right)$$

$$\Delta P_w = \frac{nR(T_1-T_2)}{V-nb}$$

$$P_c = \frac{nRT}{V}$$

$$\Delta P_c = \frac{nRT_1}{V} - \frac{nRT_2}{V}$$

$$\Delta Pc = \frac{nR(T_1-T_2)}{V}$$

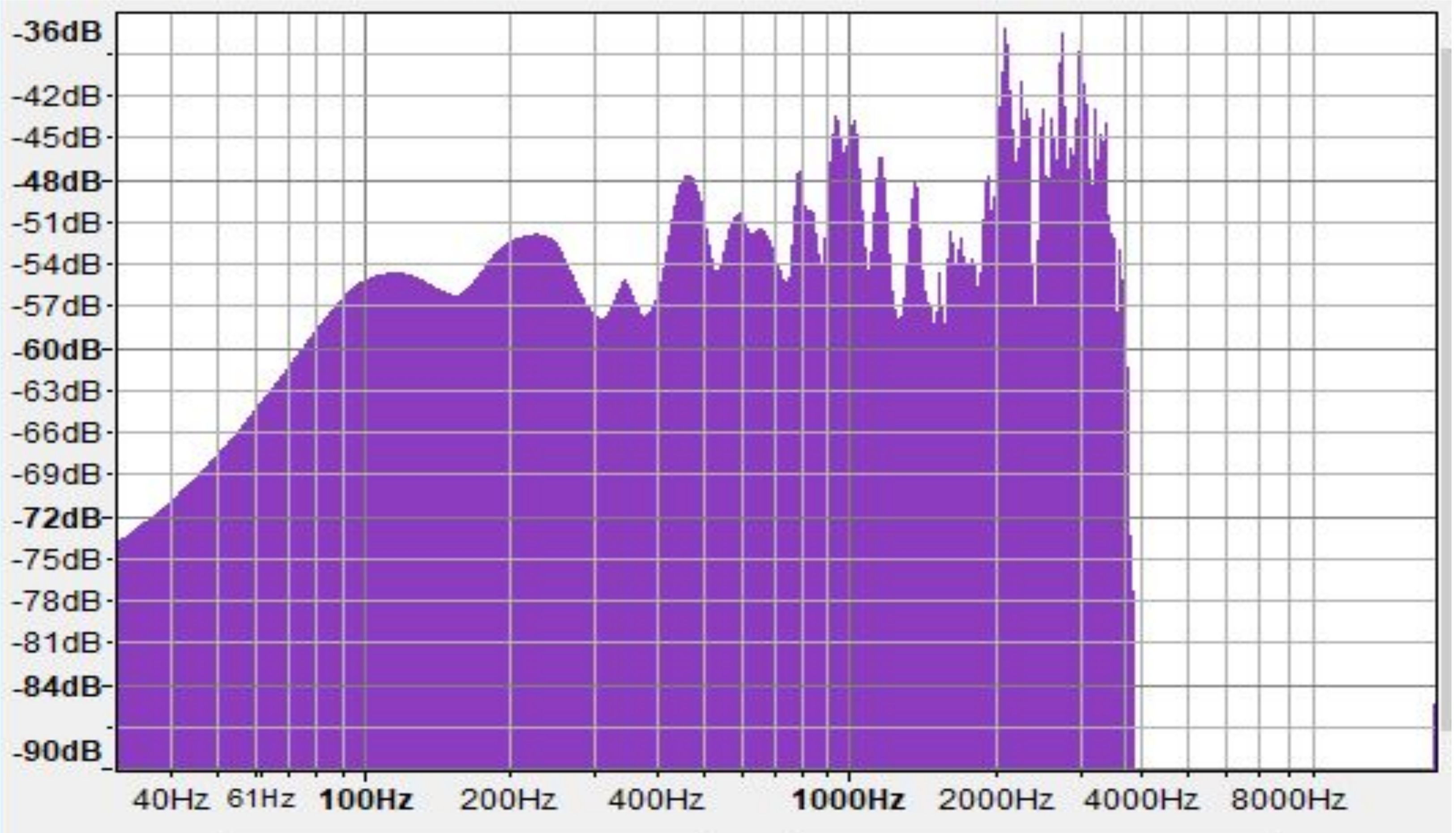
$$n \approx 0.0147\, mol$$

$$b \approx 0.0387\, \frac{L}{mol}$$

$$V \approx 0.33L$$

$$\frac{V-nb}{V} \approx 0.998$$

Appendix 4: Sound frequency analysis



Sound spectrum given through fourier transform using Audacity

Appendix 5: Experimental errors