BRAZIL

IYPT 2018

Problem 1

Invent Yourself

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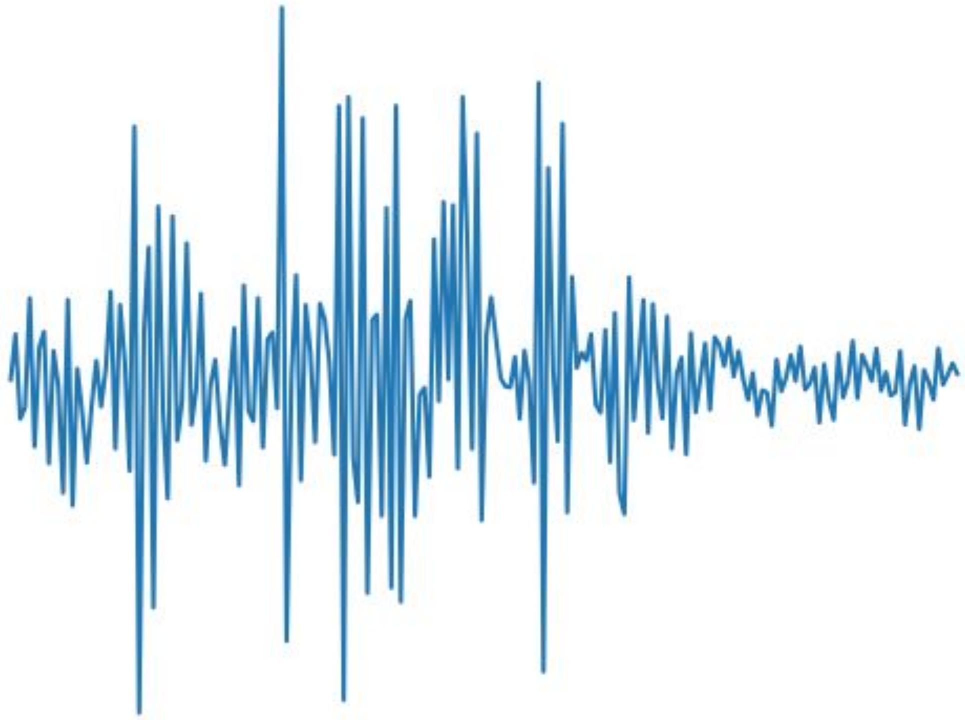
Construct a simple seismograph that amplifies a local disturbance by mechanical, optical or electrical methods. Determine the typical response curve of your device and investigate the parameters of the damping constant. What is the maximum amplification that you can achieve?



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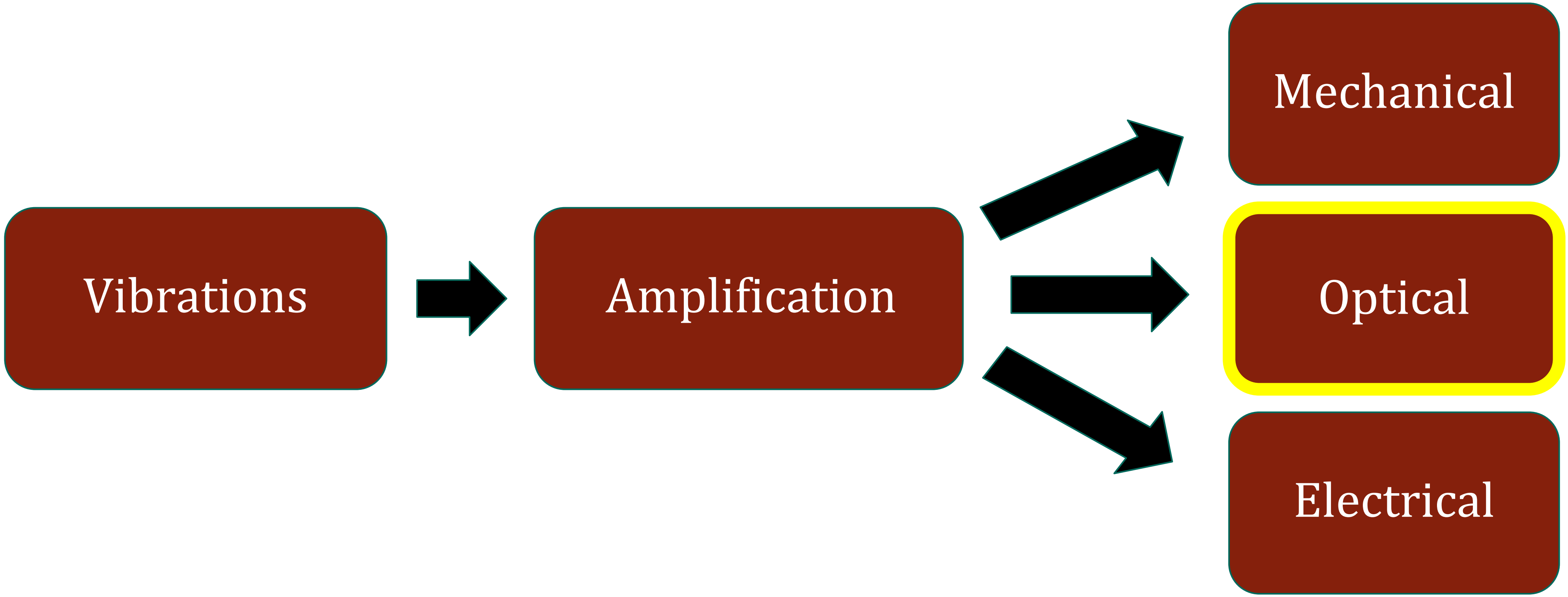

Vibrations
Amplification
Mechanical
Optical
Electrical

The Seismograph

Amplification → Optical

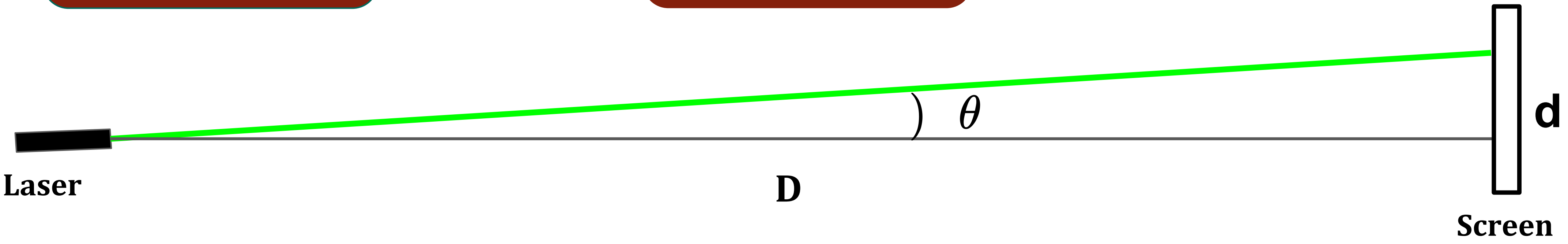


Figure 1: Laser scheme.

$$d = D \cdot tan(\theta) \approx D \cdot \theta$$

↑ D → ↑ d

The Seismograph

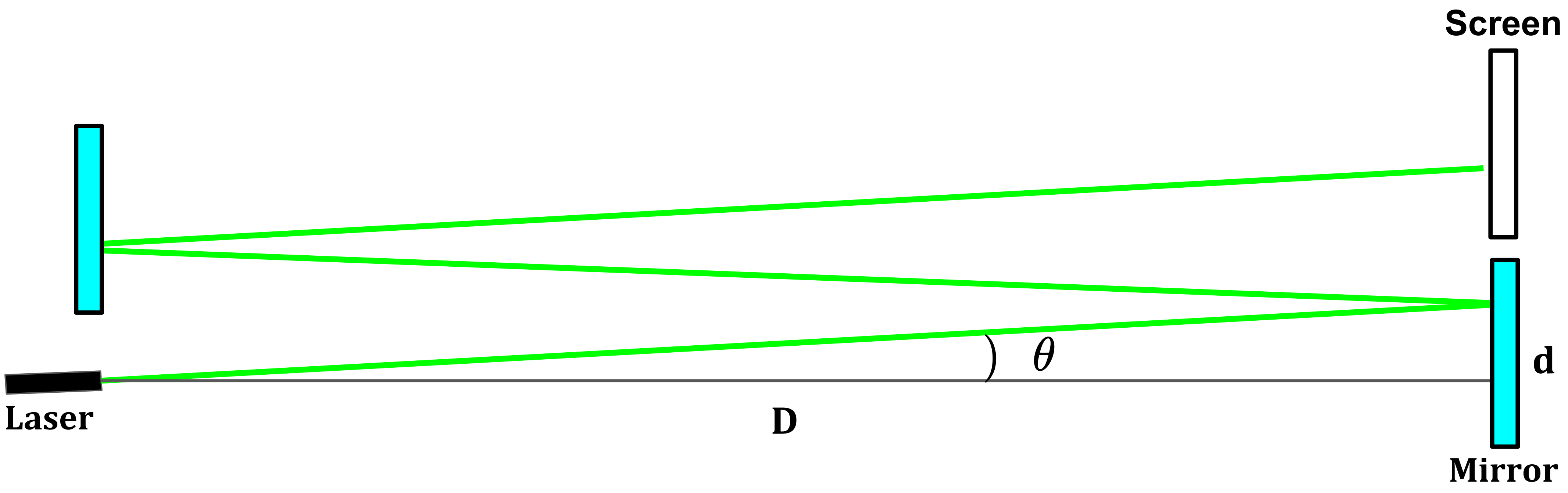


Figure 2: Scheme of an optical system with multiple mirrors.

The Seismograph

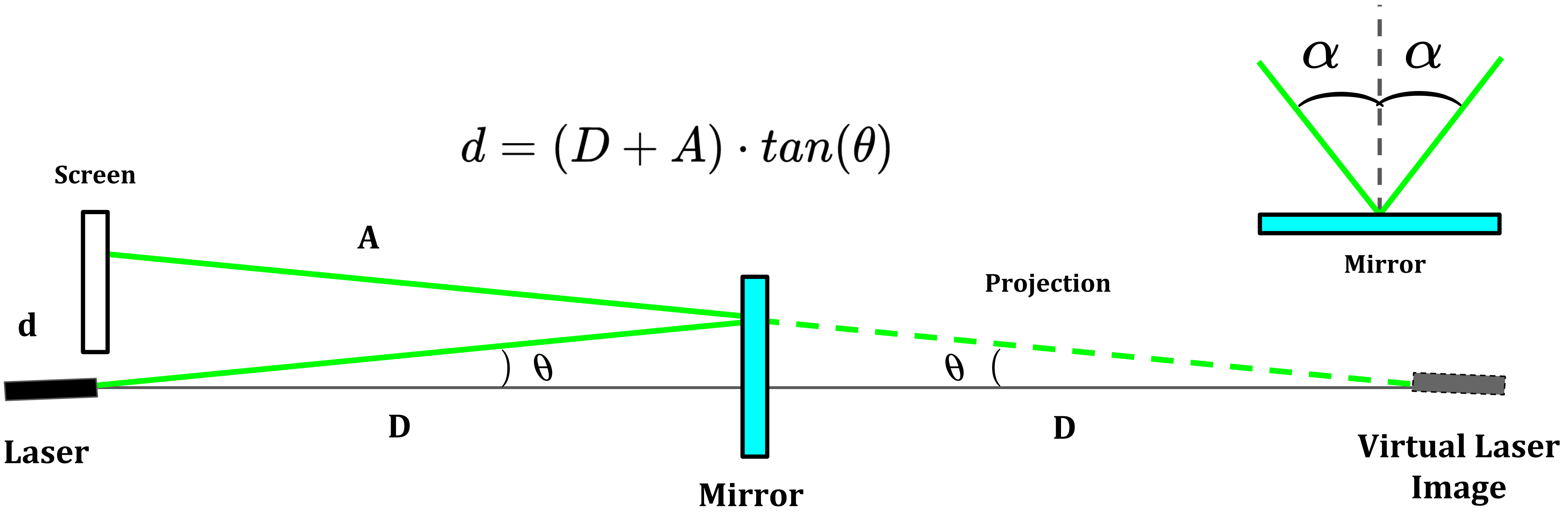
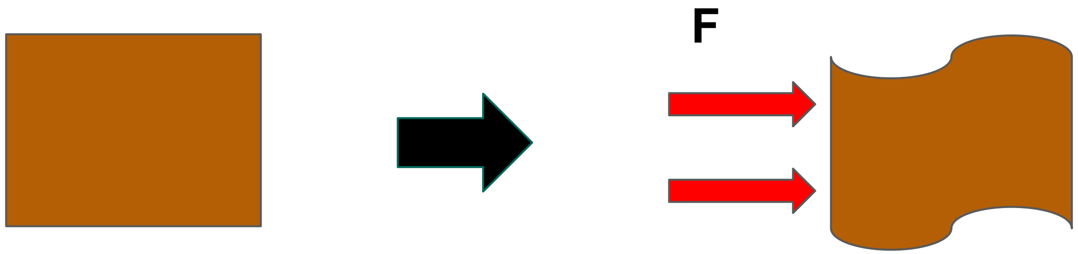


Figure 3: Scheme of the optics of the problem.

Theoretical Model



Young’s Modulus

$$F = -\left(\frac{EA}{L_0}\right)x = -kx$$

$$F_{dissipative} = -cv$$

$$\boxed{ma = -kx - cv}$$

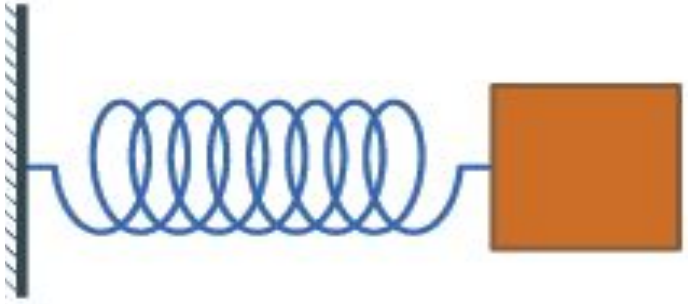


Figure 4: Damped mass-spring system.

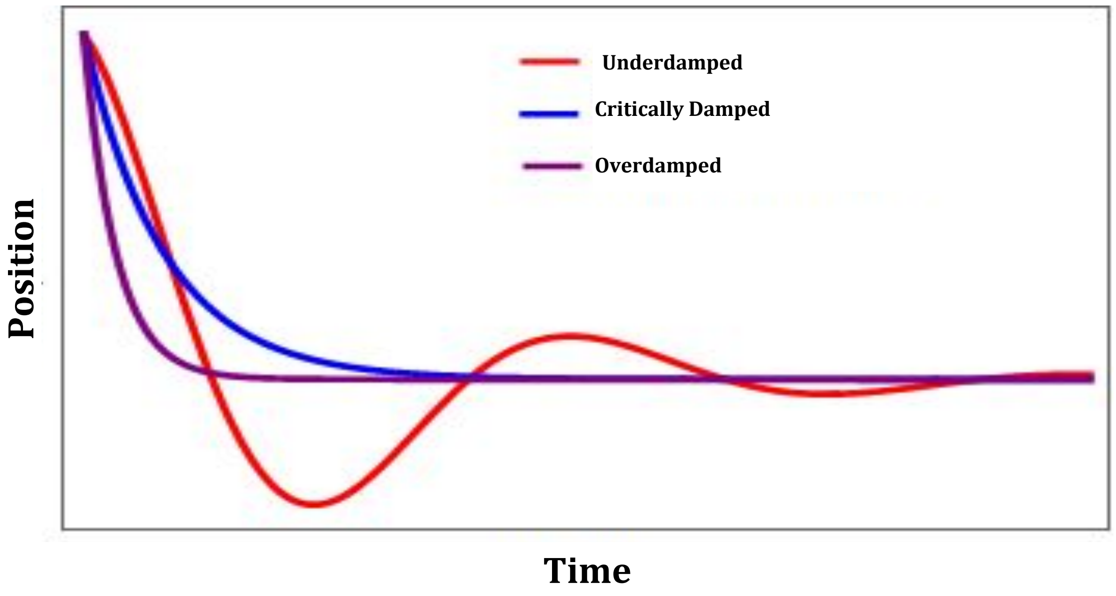
Theoretical Model: $F_{dissipative} = -cv$

$$ma = -kx - cv \Rightarrow m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

$$x(t) = Ae^{\lambda t} \Rightarrow mA\lambda^2 e^{\lambda t} + cA\lambda e^{\lambda t} + kAe^{\lambda t} = 0$$

$$m\lambda^2 + c\lambda + k = 0 \Rightarrow \lambda = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

Theoretical Model



$c^2 > 4mk$ ➡ Overdamped

$c^2 = 4mk$ ➡ Critically damped

$c^2 < 4mk$ ➡ Underdamped ➡ $\lambda \in \mathbb{C}$

$$\gamma = \frac{c}{2m} \quad ; \quad \omega = \frac{\sqrt{c^2 - 4mk}}{2m} \quad \Rightarrow \quad x(t) = Ae^{-\gamma t} cos(\omega t + \varphi_0)$$

Theoretical Model

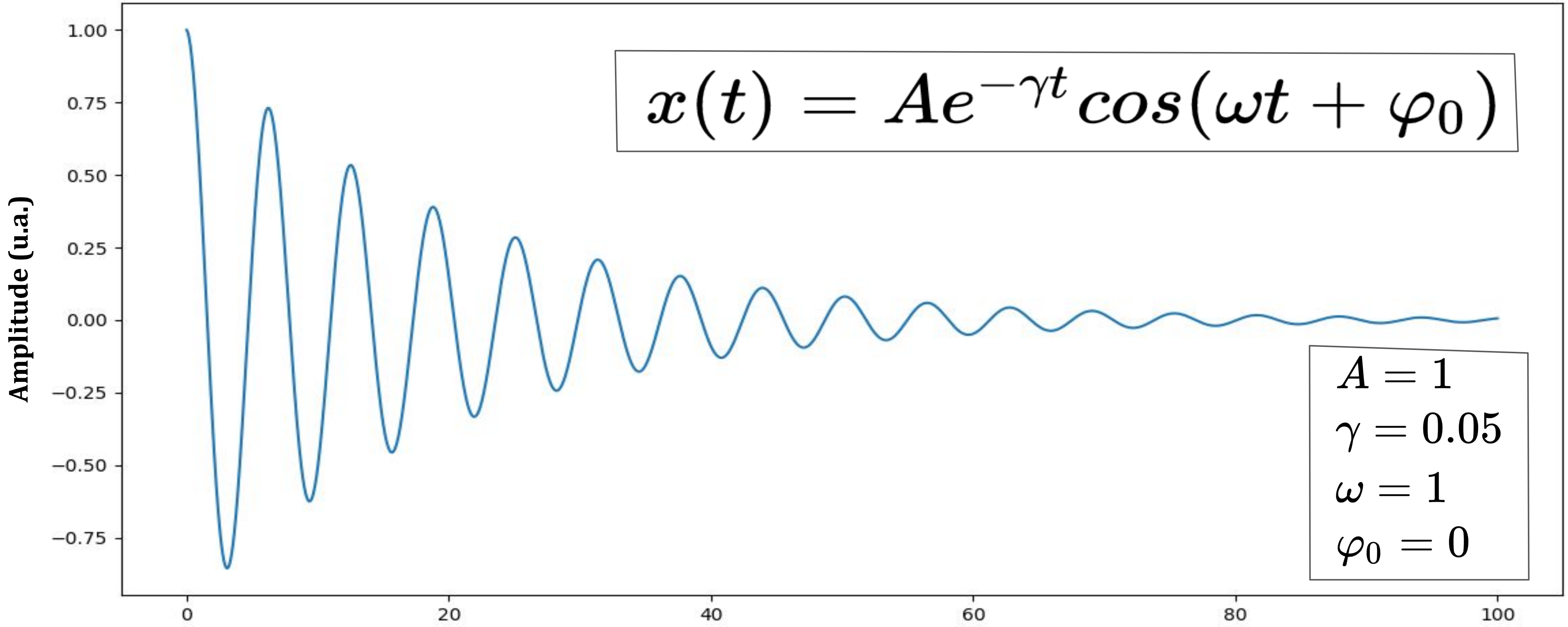


Figure 5: Function x(t) using the indicated parameters.

Theoretical Model

$$x_1(t) = A_1 e^{-\gamma_1 t} \cos(\omega_1 t + \varphi_1) \qquad x_2(t) = A_2 e^{-\gamma_2 t} \cos(\omega_2 t + \varphi_2)$$

$$x_r(t) = x_1(t) + x_2(t)$$

$$x_r(t) = A_1 e^{\gamma_1 t} \cos(\omega_1 t + \varphi_1) + A_2 e^{-\gamma_2 t} \cos(\omega_2 t + \varphi_2)$$

General case: $x_r(t) = \sum_{i=1}^{n} x_i(t)$

Theoretical Model

Damping

Sensibility

Response

Experimental Materials

1 - Well polished mirrors held by stable supports;

2 - Powerful green laser beam;

3 - Stable supports;

4 - High frame rate camera (240 fps);

5 - White screen;

6 - Computer for data analysis.

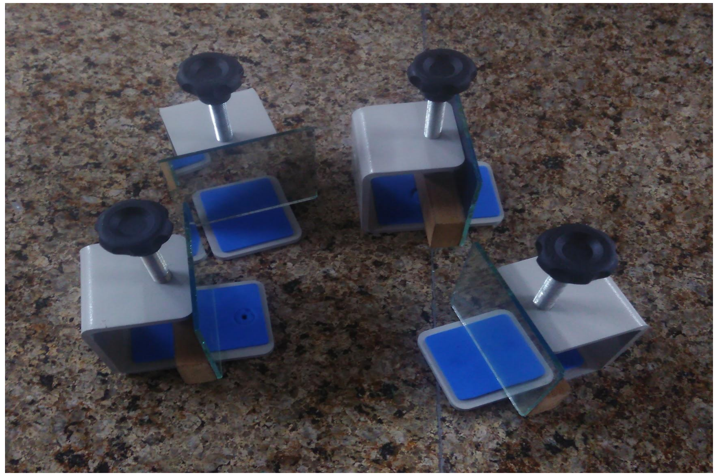
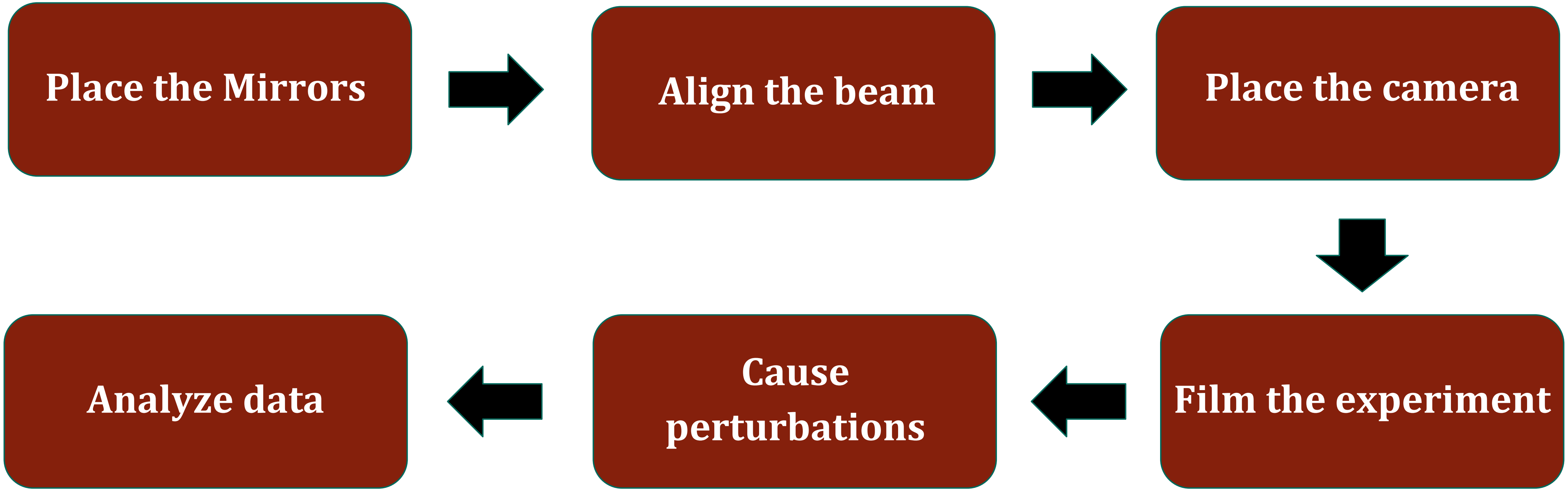


Figure 6: Mirrors held by stable supports.

Figure 7: Seismograph.

Experimental Procedure



Experimental Procedure

$$d = \sum_{i=1}^{n} Di \cdot tan(\theta)$$

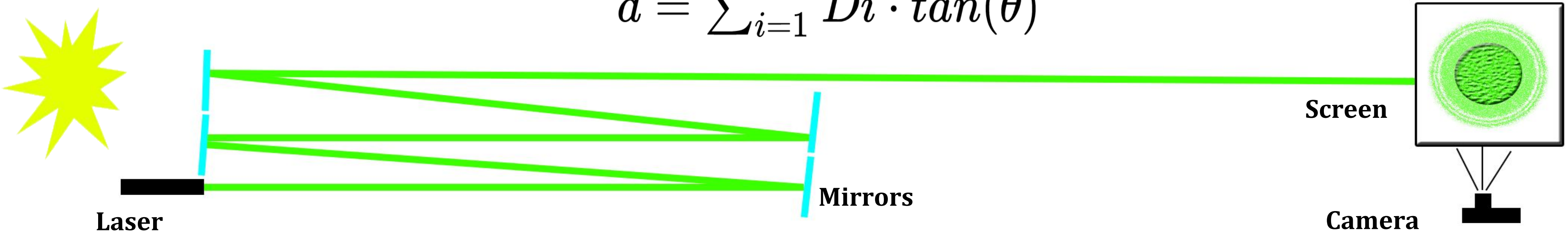


Figure 8: Scheme for experimental set-up.

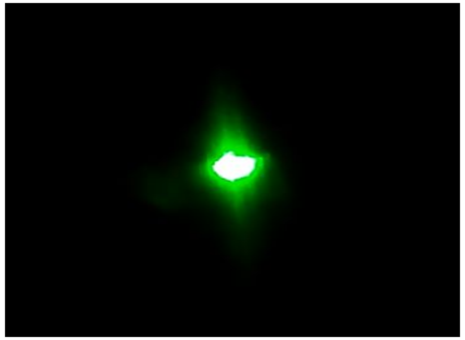


Figure 9: Laser projection over the screen.

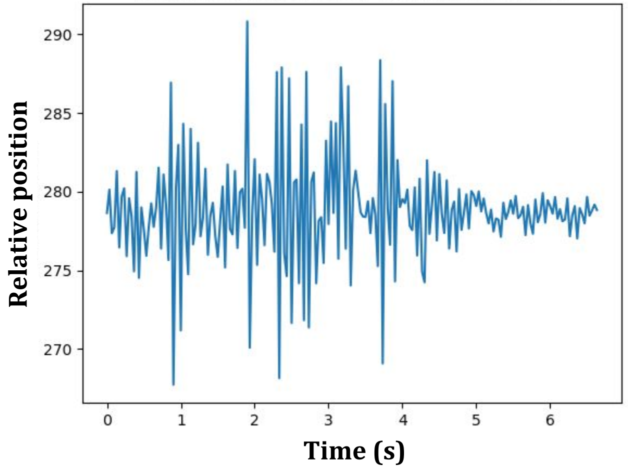
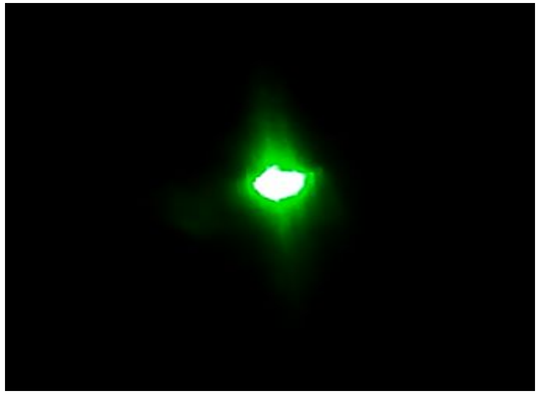
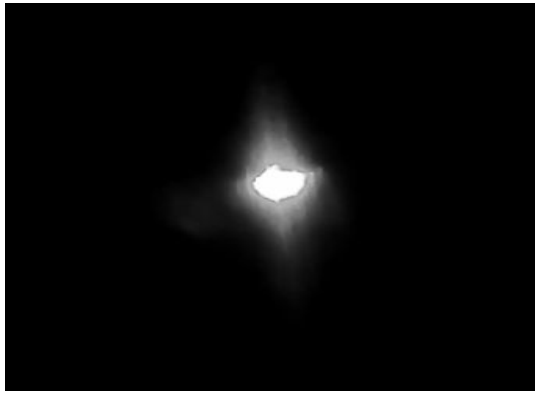


Figure 10: Typical signal.

Experimental Procedure: Data Analysis



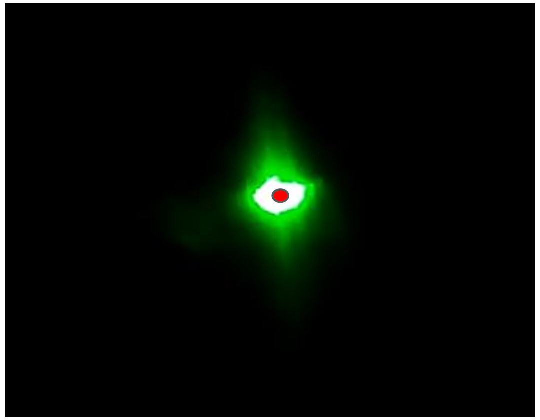
Luminosity scale



$$A = \begin{bmatrix} 0 & 10 & 26 & \dots & 0 \\ 10 & 22 & 23 & \dots & 5 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 250 & 250 & 254 & \dots & 220 \end{bmatrix}$$

$$X_{cv} = \frac{\sum_{i=1}^{n} \sum_{k=1}^{m} x_{i,k} \cdot i}{\sum_{i=1}^{n} \sum_{k=1}^{m} x_{i,k}}$$

$$Y_{cv} = \frac{\sum_{i=1}^{n} \sum_{k=1}^{m} x_{i,k} \cdot k}{\sum_{i=1}^{n} \sum_{k=1}^{m} x_{i,k}}$$



Experiment 1: Random vibrations

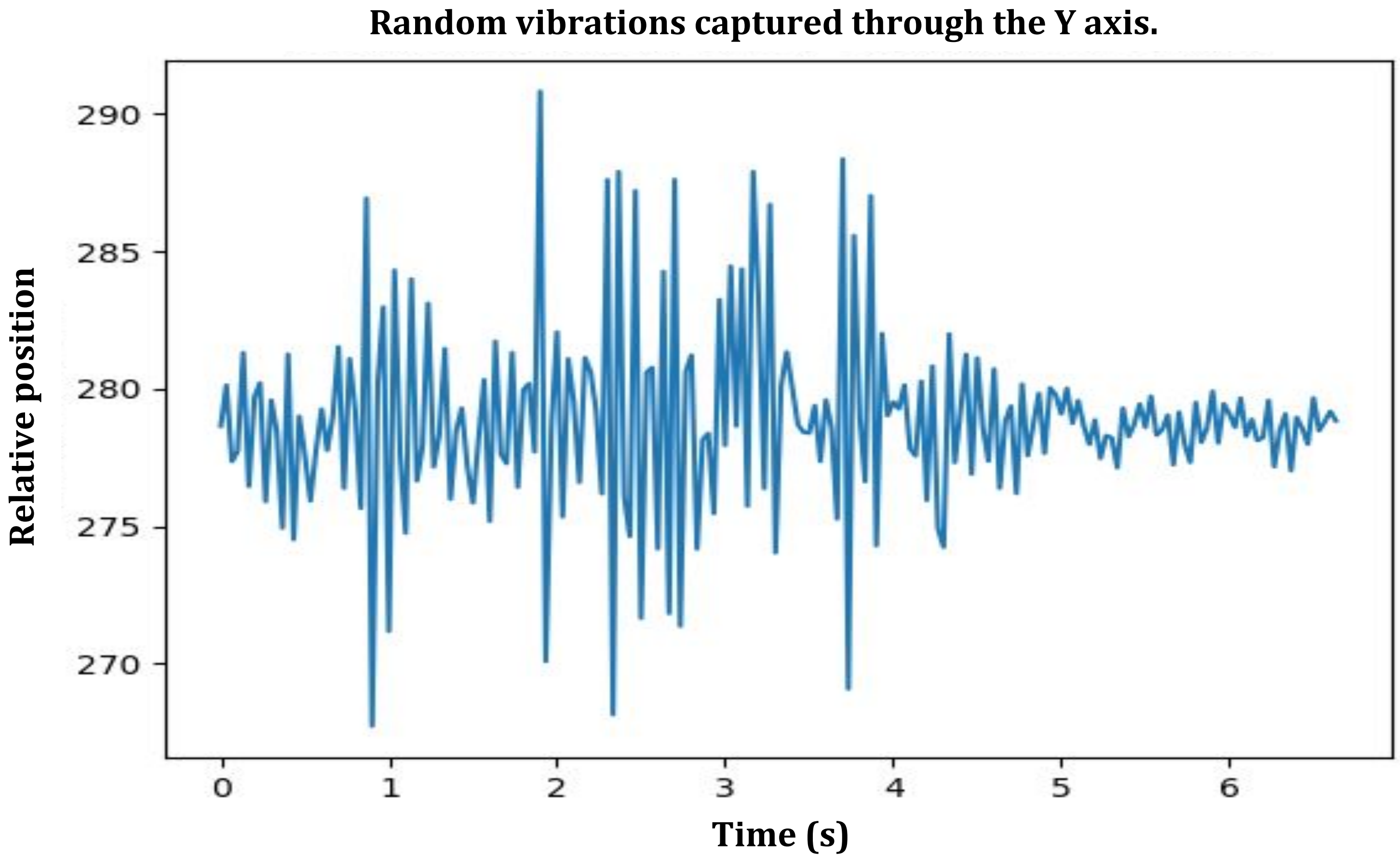


Figure 11: Seismograph response to random perturbations.

Experiment 2: Controlled vibrations

Controlled Vibration → **Measuring parameters**

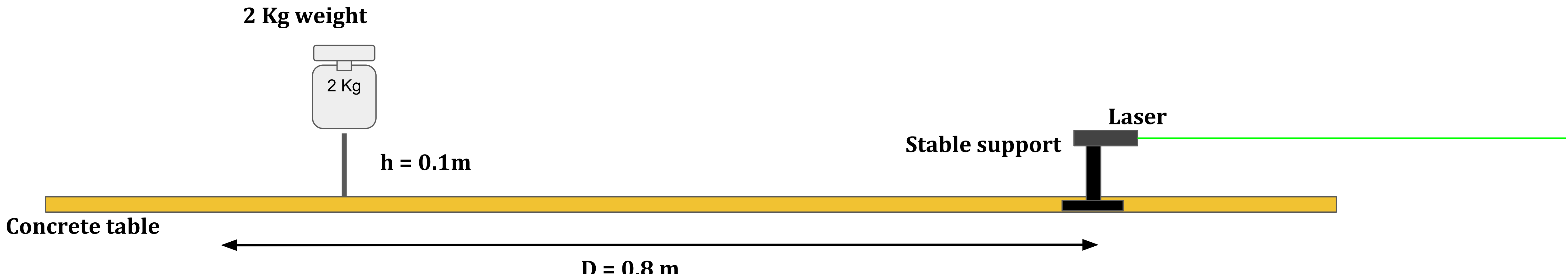


Figure 12: Experimental scheme.

Experiment 2: Controlled vibrations

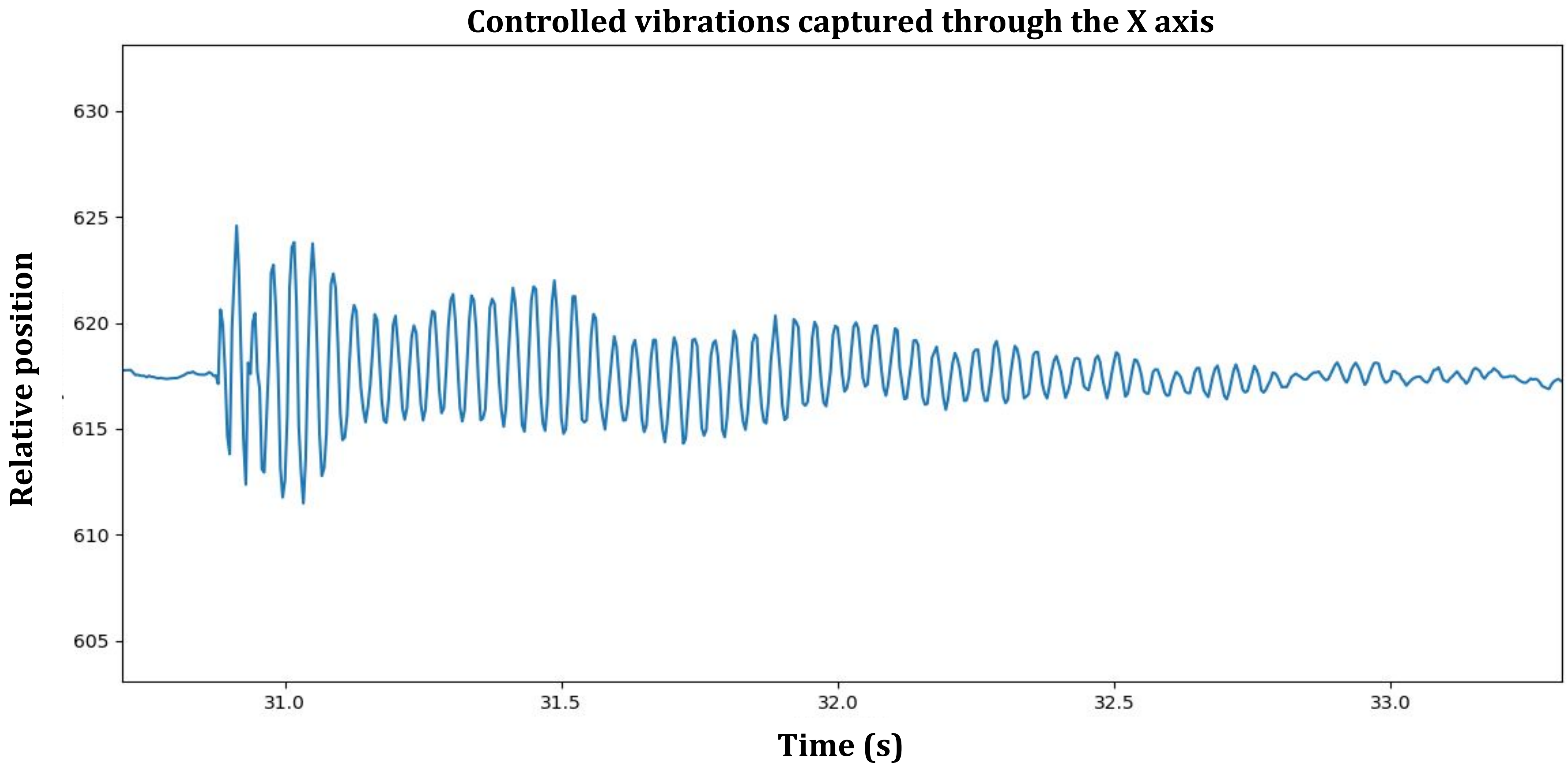
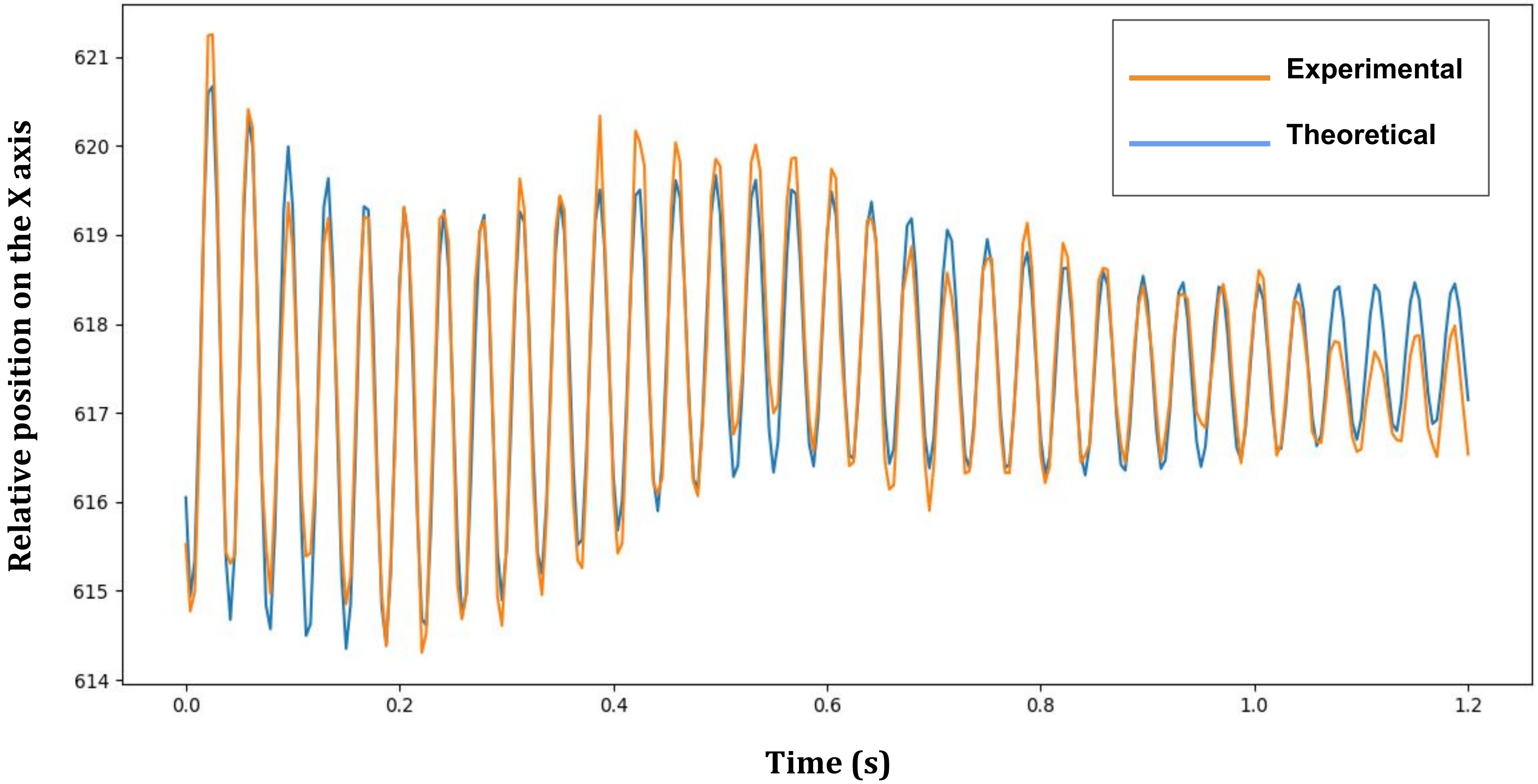


Figure 13: Response curve to a controlled vibration.

Experiment 2: Controlled vibrations



P wave:

$$A = 3.10$$
$$f = 27.51\,Hz$$
$$\gamma = 1.15Hz$$

S wave:

$$A = 1.10$$
$$f = 1.39\,Hz$$
$$\gamma = 1.92Hz$$

Experiment 3: Controlled distances

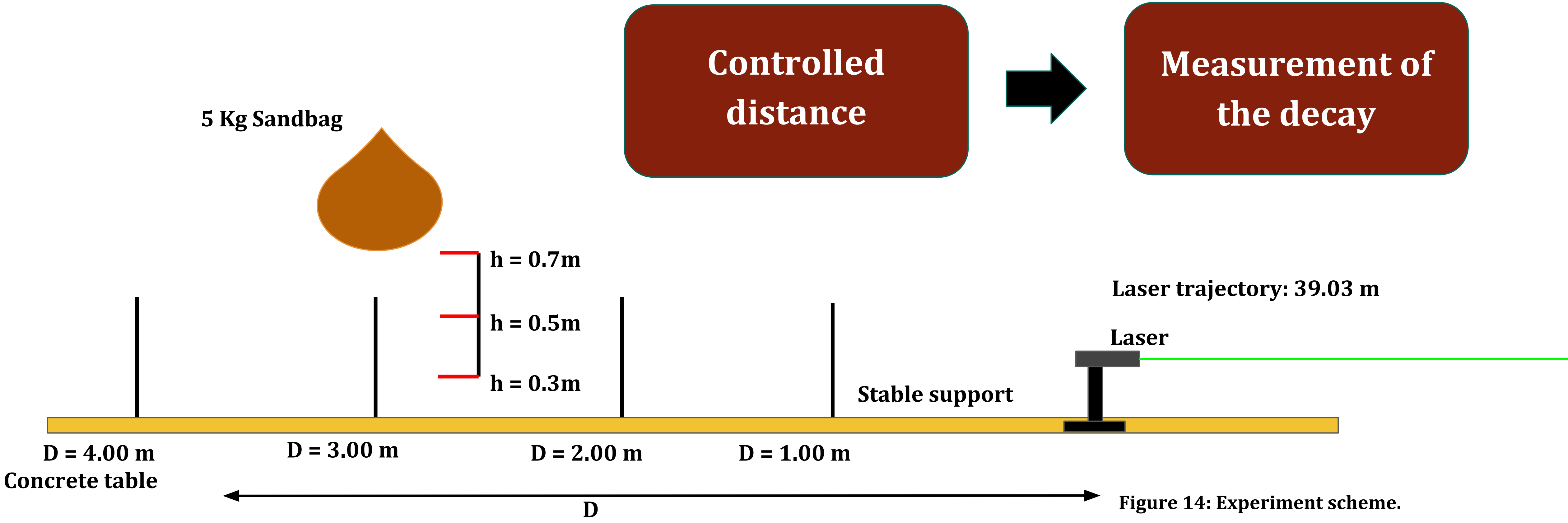


Figure 14: Experiment scheme.

Experiment 3: Controlled distances

Specific vibrations

$$A(x) = A_0 e^{-bx}$$

$$< b > \approx (1.08 \pm 0.02)\, m^{-1}$$

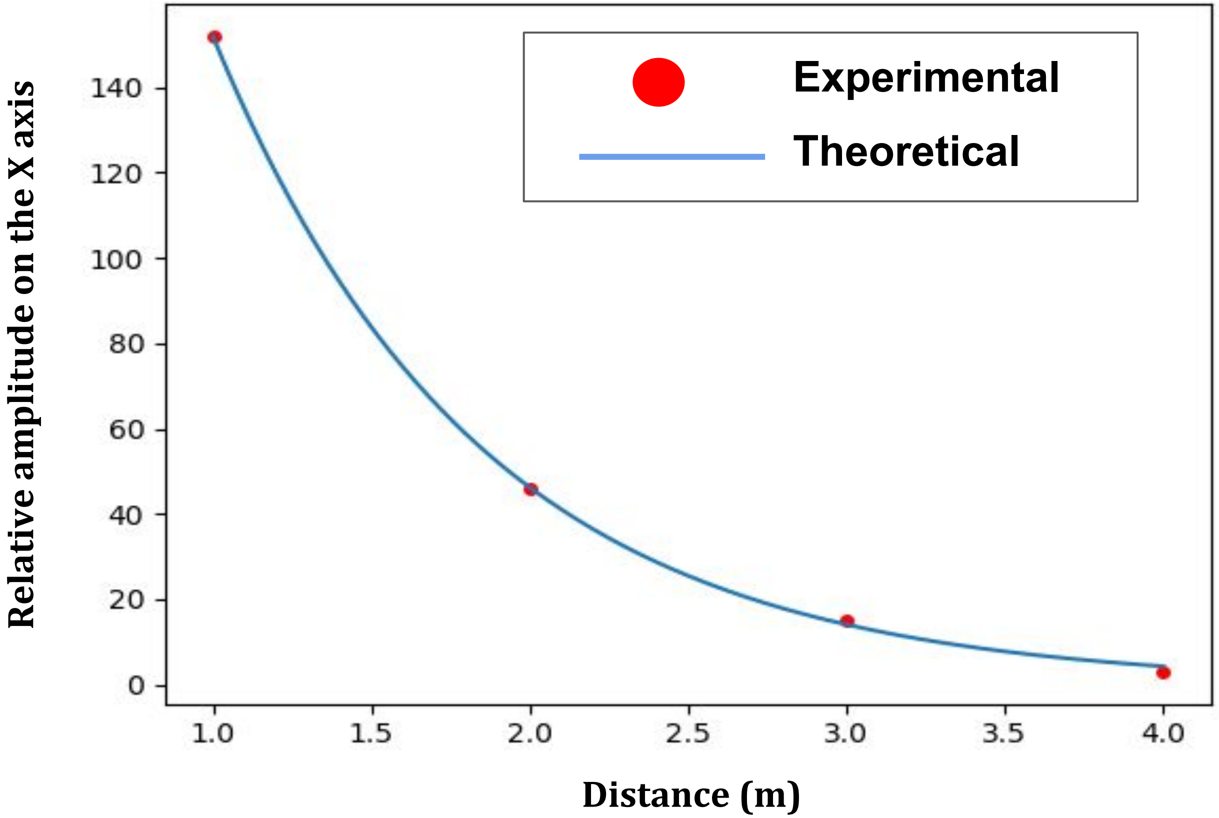
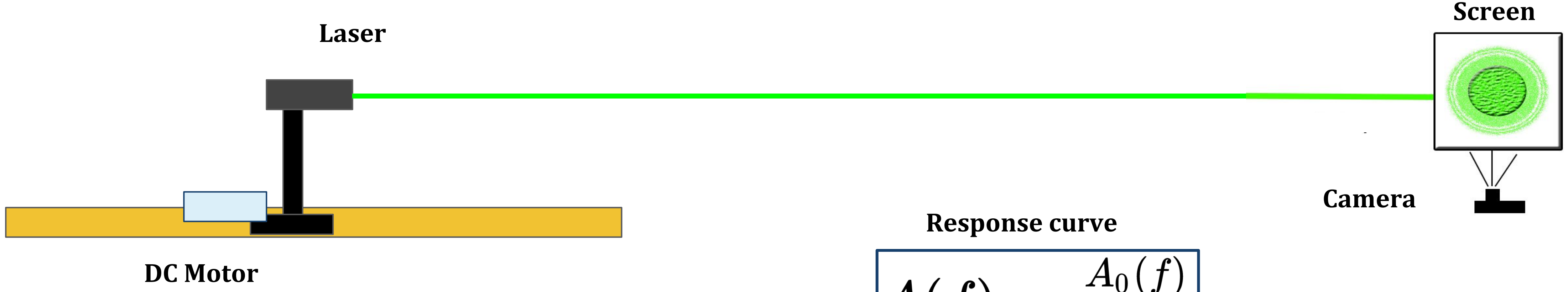


Figure 15: Computational fitting for the experimental plot from 50cm launches.

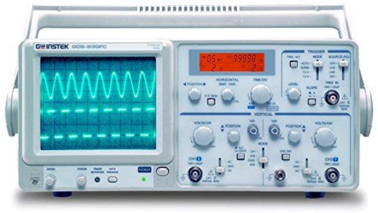
Experiment 4: Response curve



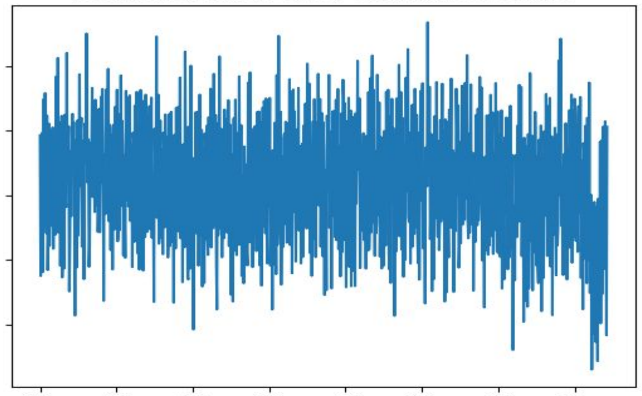
Response curve

$$A(f) = \frac{A_0(f)}{Ai(f)}$$

Frequency Generator

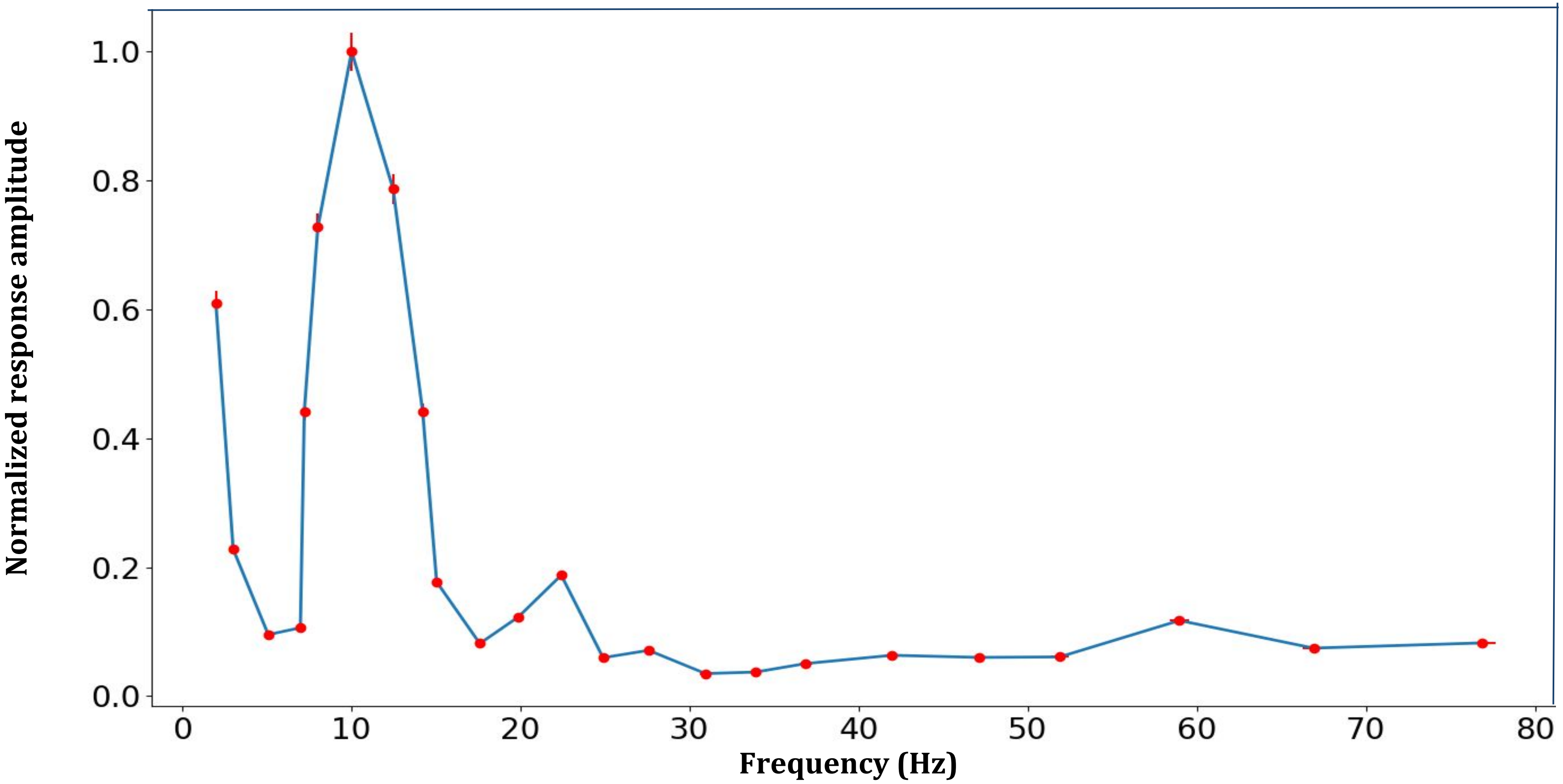


Oscilloscope



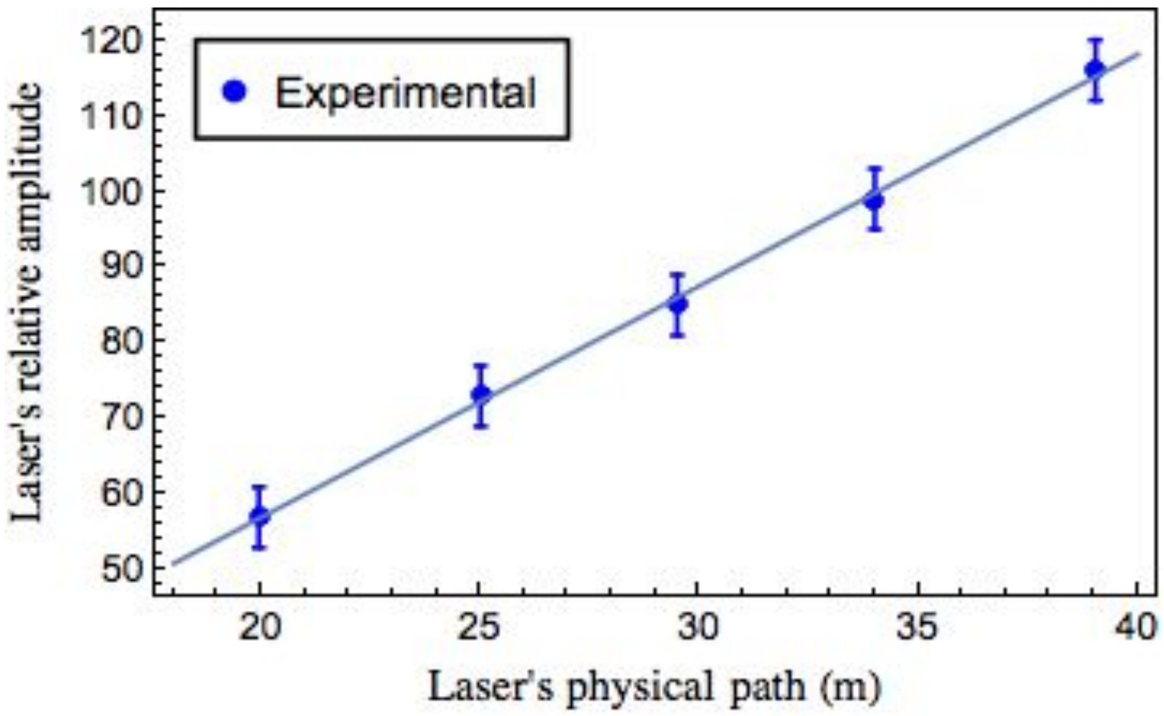
→ **DFT**

Experiment 4: Response curve



Experiment 4: Controlled vibrations

Relative amplitude as a function of the laser's physical path.



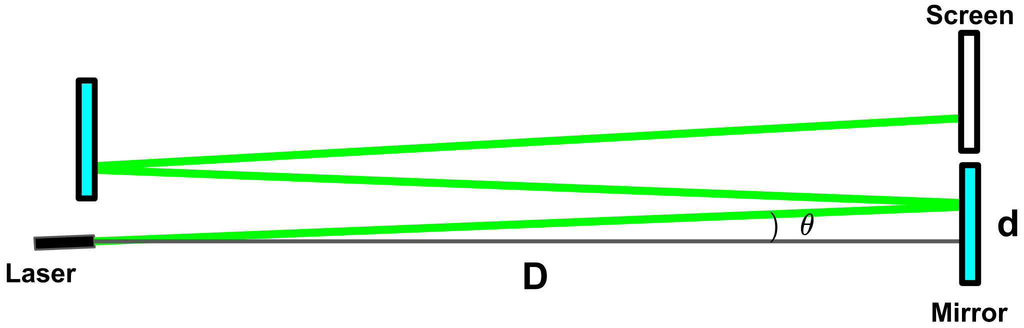
$$A_{literature} \approx 5\,\mu m$$ [5]

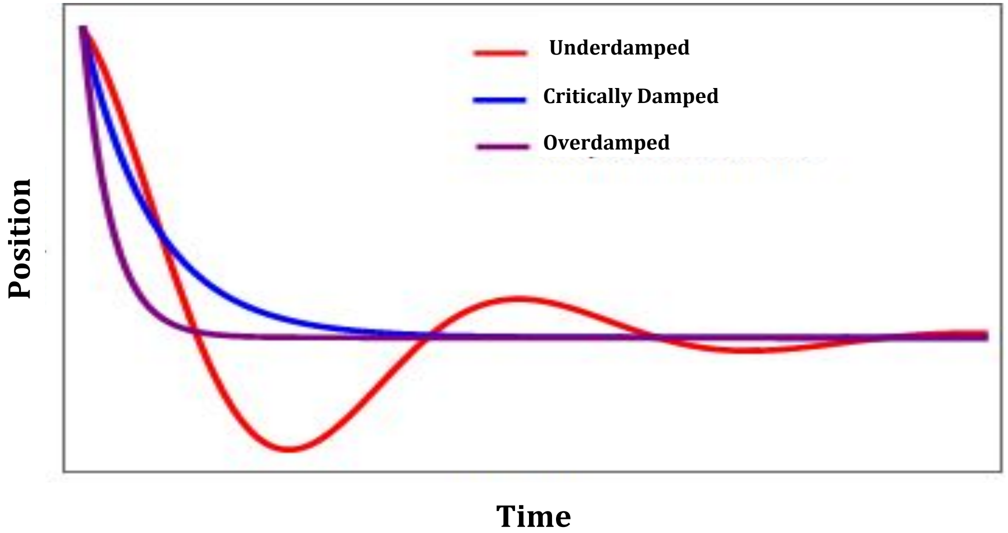
$$A_{amplified} \approx 4\,cm$$

$$\phi = \frac{A_{amplified}}{A_{literature}}$$

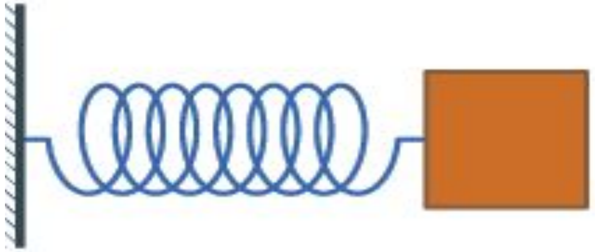
$$\phi \approx \frac{4\,cm}{5\mu m} \approx 10^4$$

Summary: Theory



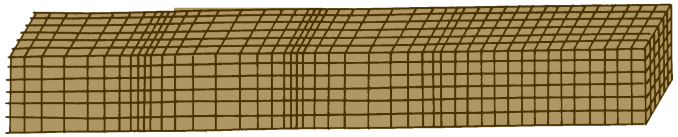


$$ma = -kx - cv$$

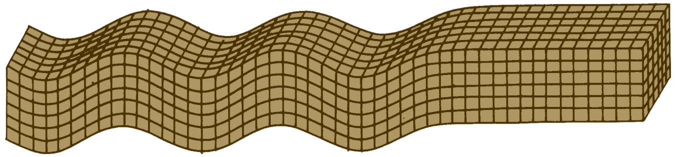


$$x(t) = Ae^{-\gamma t}cos(\omega t + \varphi_0)$$

P wave

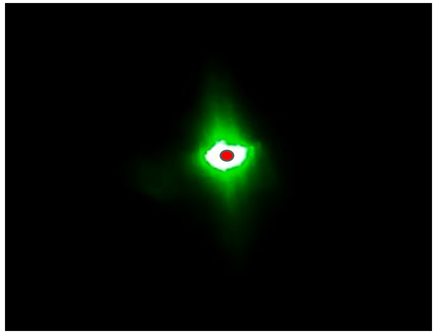


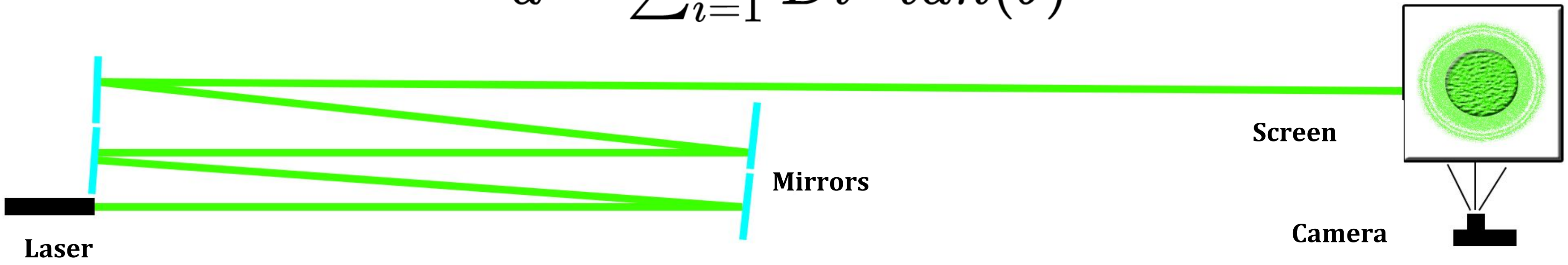
S wave

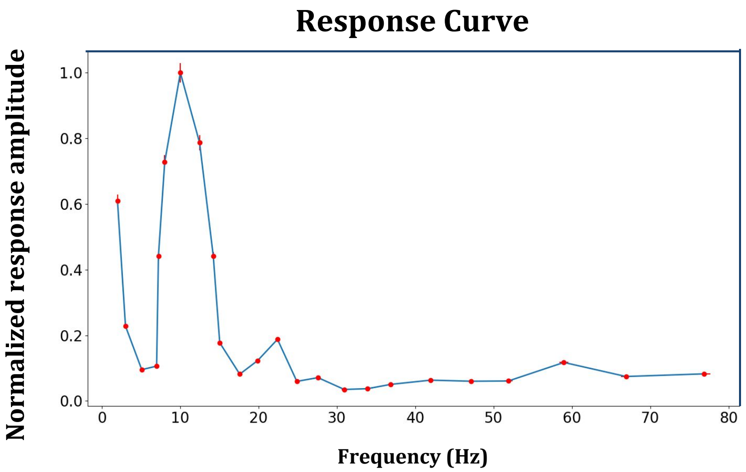


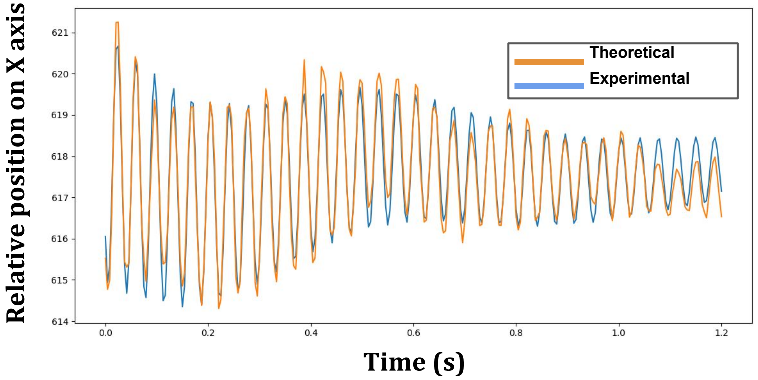
Summary: Experiment

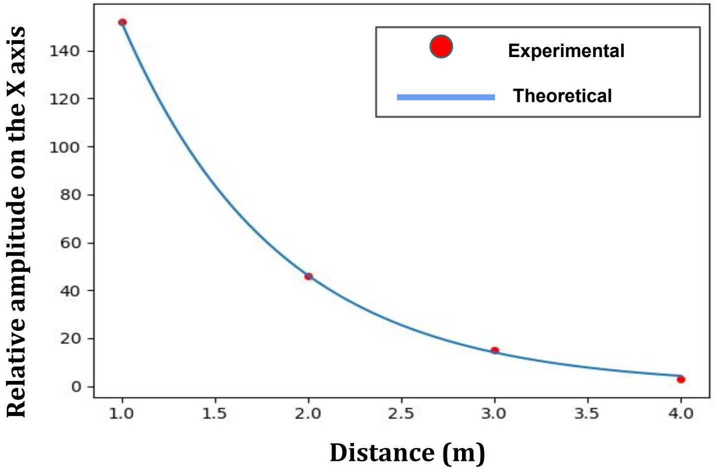
$$d = \sum_{i=1}^{n} Di \cdot tan(\theta)$$











Bibliography

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Figure: Oleg Alexandrov, 2007

Thank you!

Appendix: Experiment 3 - Controlled distances

$$A(x) = A_0 e^{-bx} \Rightarrow I \propto A^2 \Rightarrow E \propto A_0^2$$

$$E = 5.0\,kg \cdot 9.8 \cdot 0.5 = 14.7\,J$$

$$E = k \cdot 500^2 \Rightarrow k = 5.9 \cdot 10^{-5}$$

$$E_{min} = 5.9 \cdot 10^{-5}\,J$$

$$x \approx 5.8\,m$$