PROBLEM 9: BOTTLE TONE

IYNT 2018

Team Switzerland

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THE PROBLEM

Bottle tone

Take an empty bottle and blow air across its mouth to produce a sound. Now fill the bottle with some water and study how the sound changes.

DEMONSTRATION

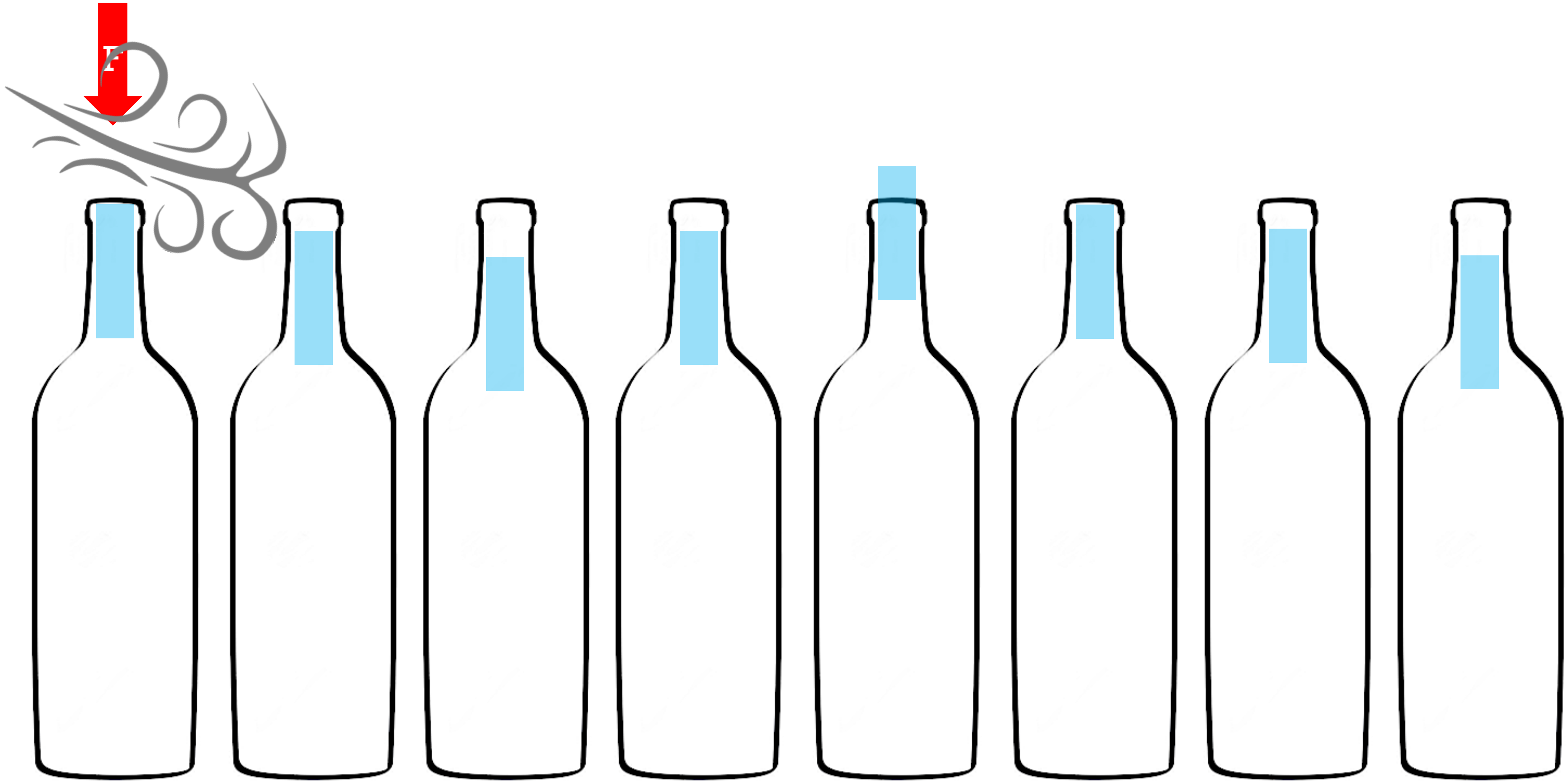
OPEN/CLOSED

$$f = \frac{v}{4L} \longrightarrow f = 504\text{Hz}$$

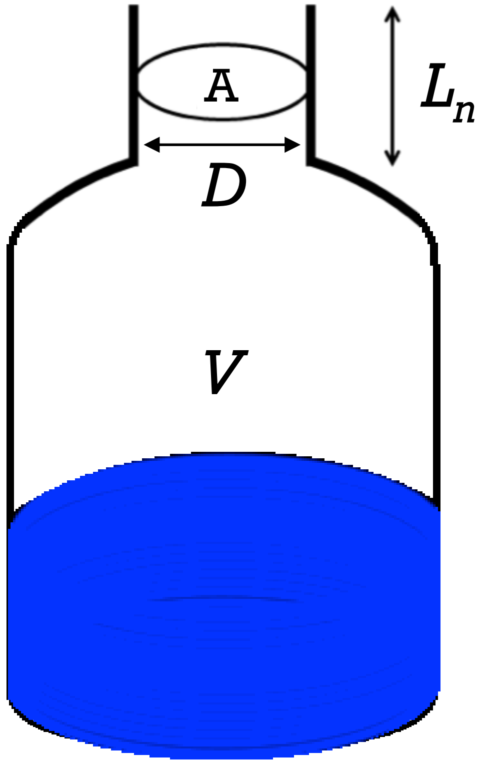
Actual frequency = 236Hz

16.5 cm

THEORY: HOW THE SOUND IS CREATED



THEORY: HELMHOLTZ RESONANCE



$$f = \frac{v}{2\pi}\sqrt{\frac{A}{V \cdot L_{eq}}}$$

$$L_{eq} = L_n + 0.3D$$

Sound parameters

f : frequency of the resonance [Hz]

v : propagation speed of sound [m/s]

Bottle parameters

L_{eq} : equivalent length of neck [m]

L_n : length of neck [m]

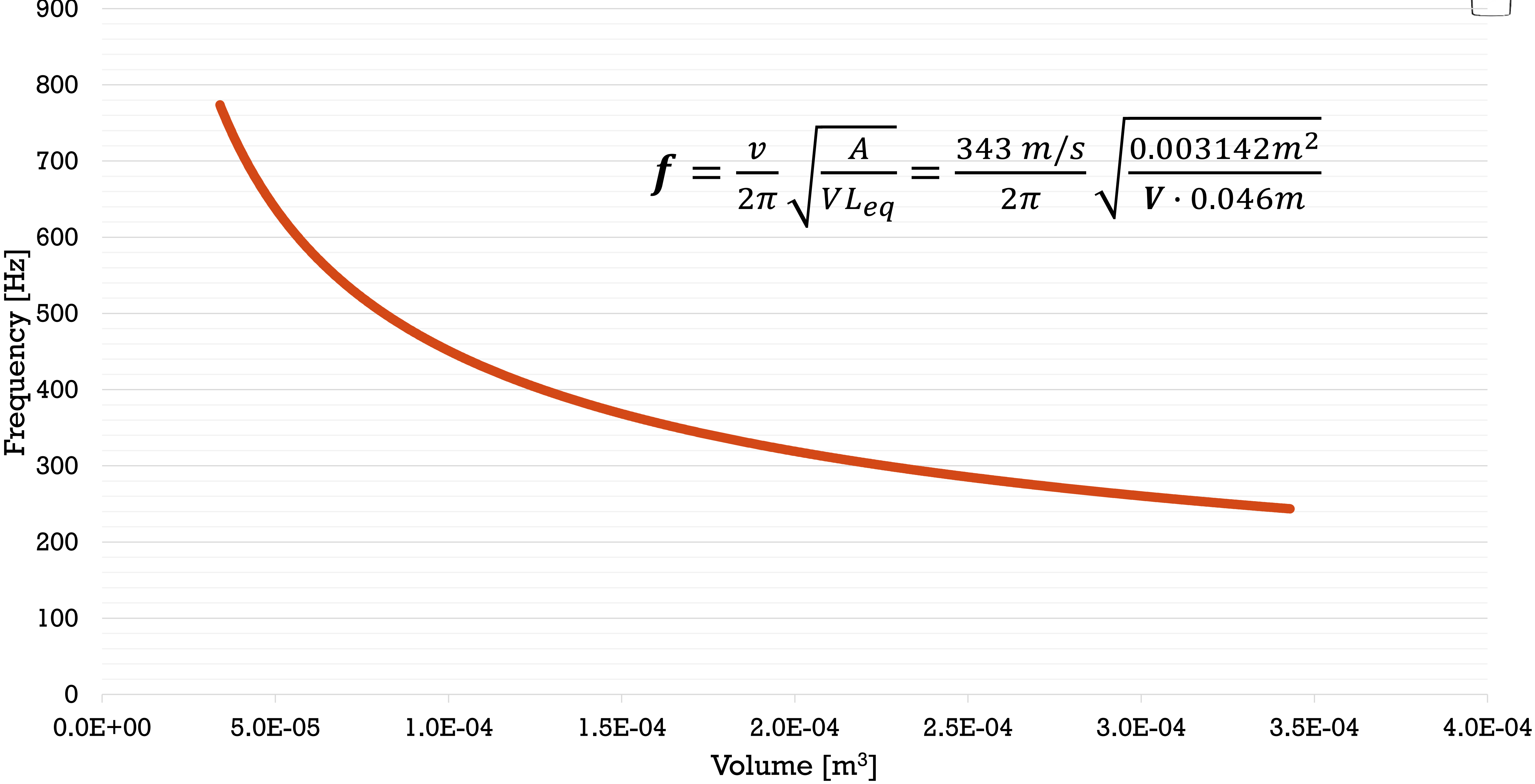
D : diameter of neck [m]

A : cross-sectional area of neck [m^2]

V : static volume of cavity [m^3]

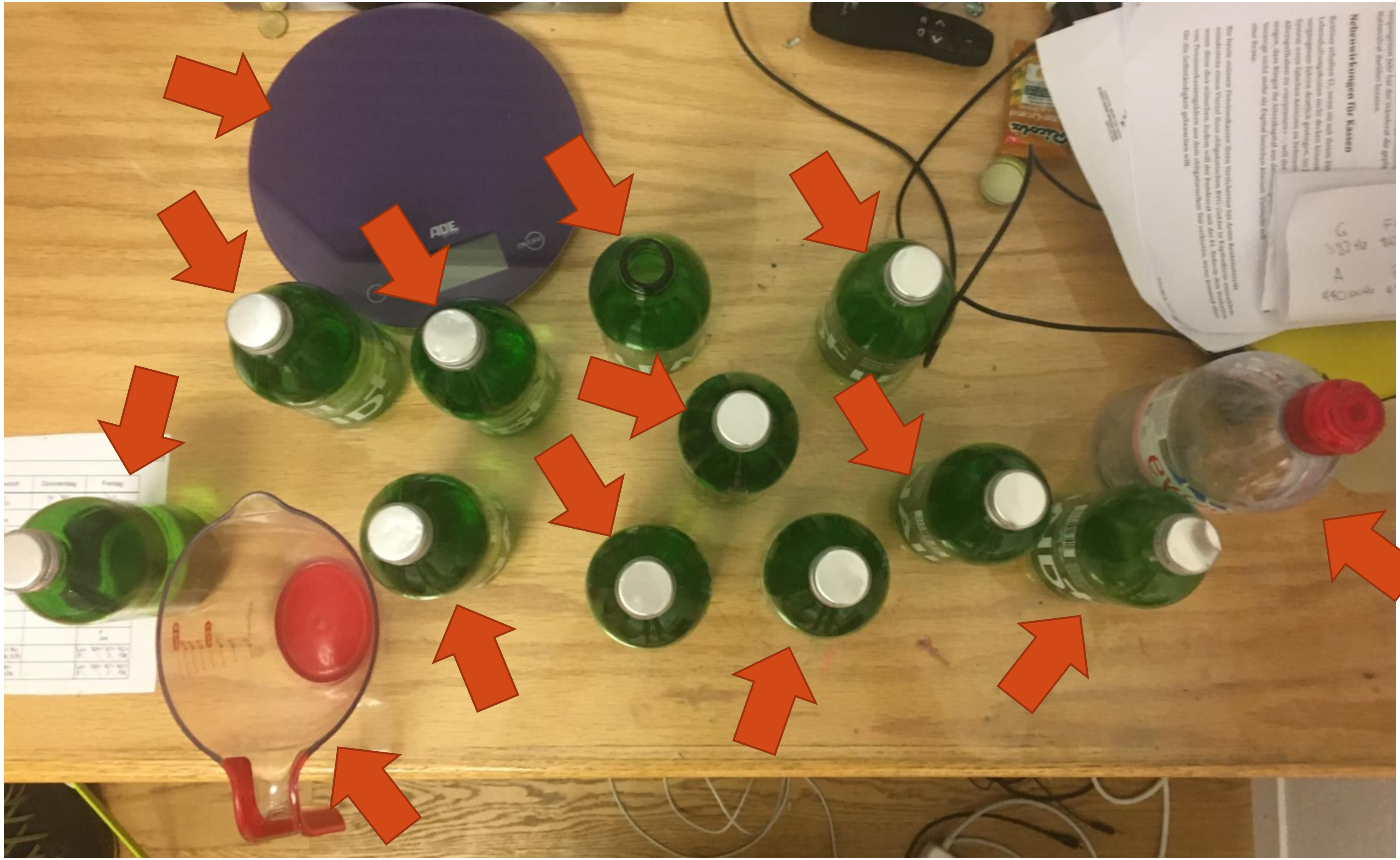
Theory suggests: frequency and volume vary inversely

Model of Frequency



$$\boldsymbol{f} = \frac{v}{2\pi}\sqrt{\frac{A}{VL_{eq}}} = \frac{343\ m/s}{2\pi}\sqrt{\frac{0.003142m^2}{\boldsymbol{V}\cdot 0.046m}}$$

EXPERIMENTS

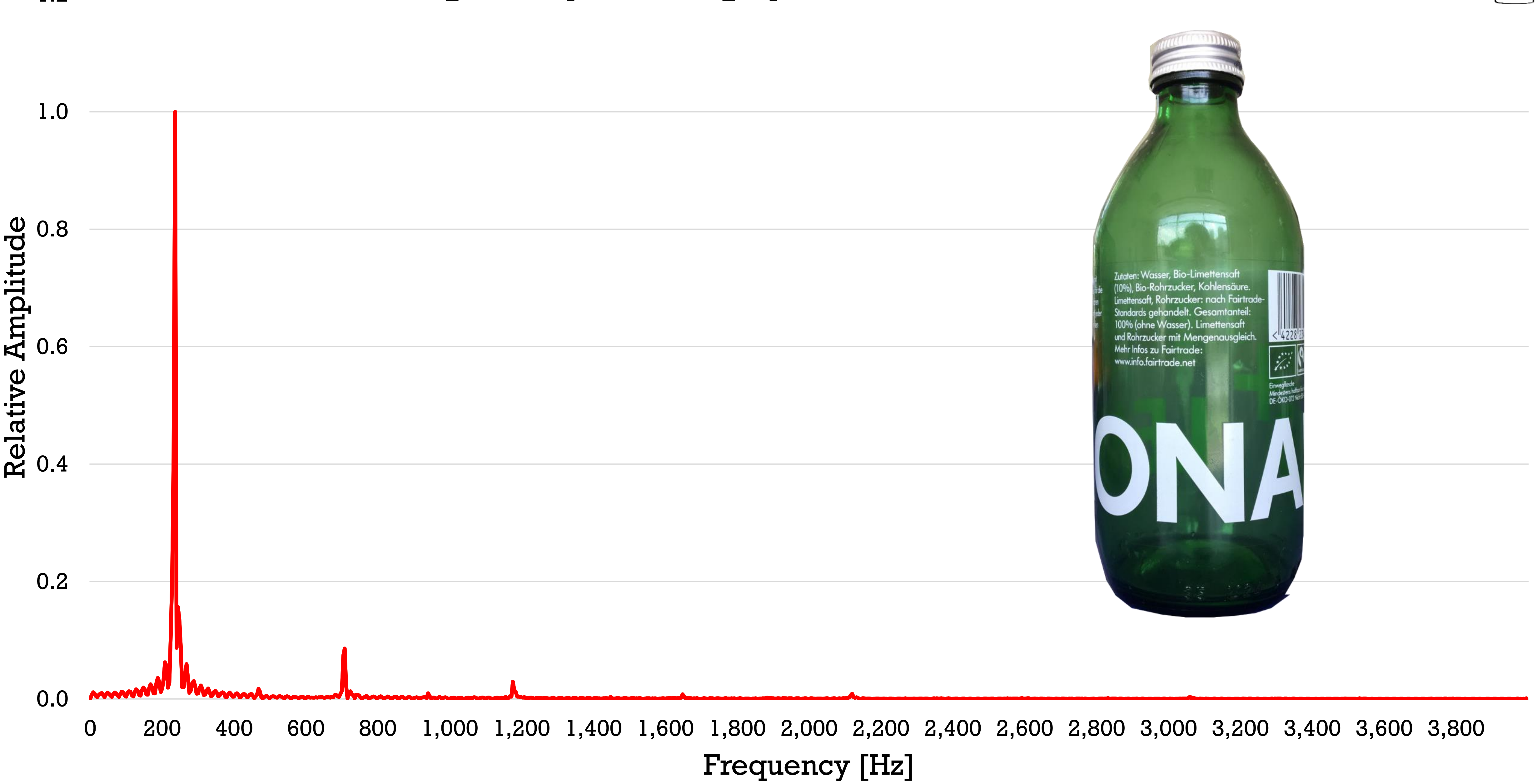


Set up

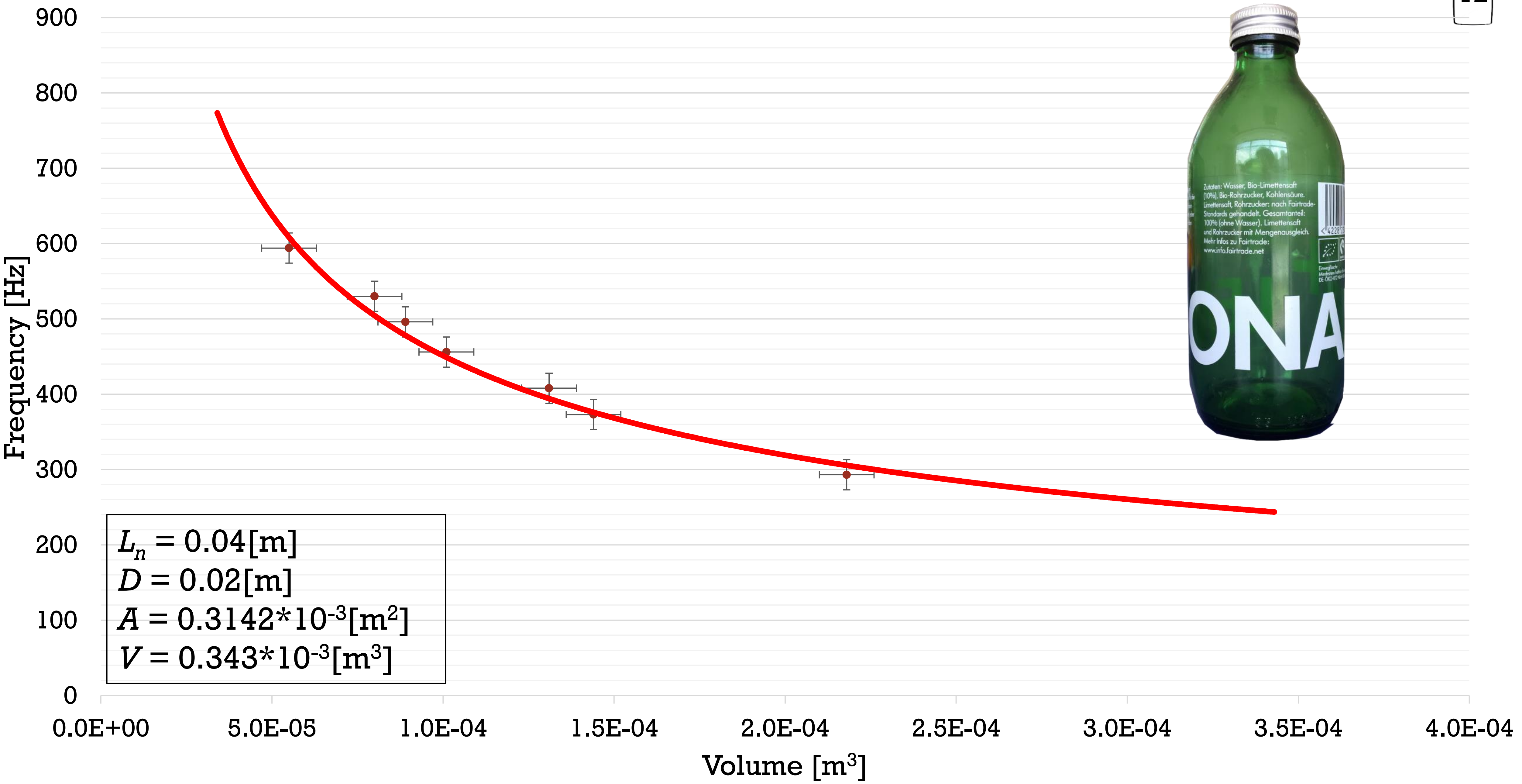
Glass bottles
Scale
Plastic bottle
Measuring cup

THE BOTTLES

Frequency of Empty Bottle with Volume 343 cm^3



Comparing Experiments to the Theory



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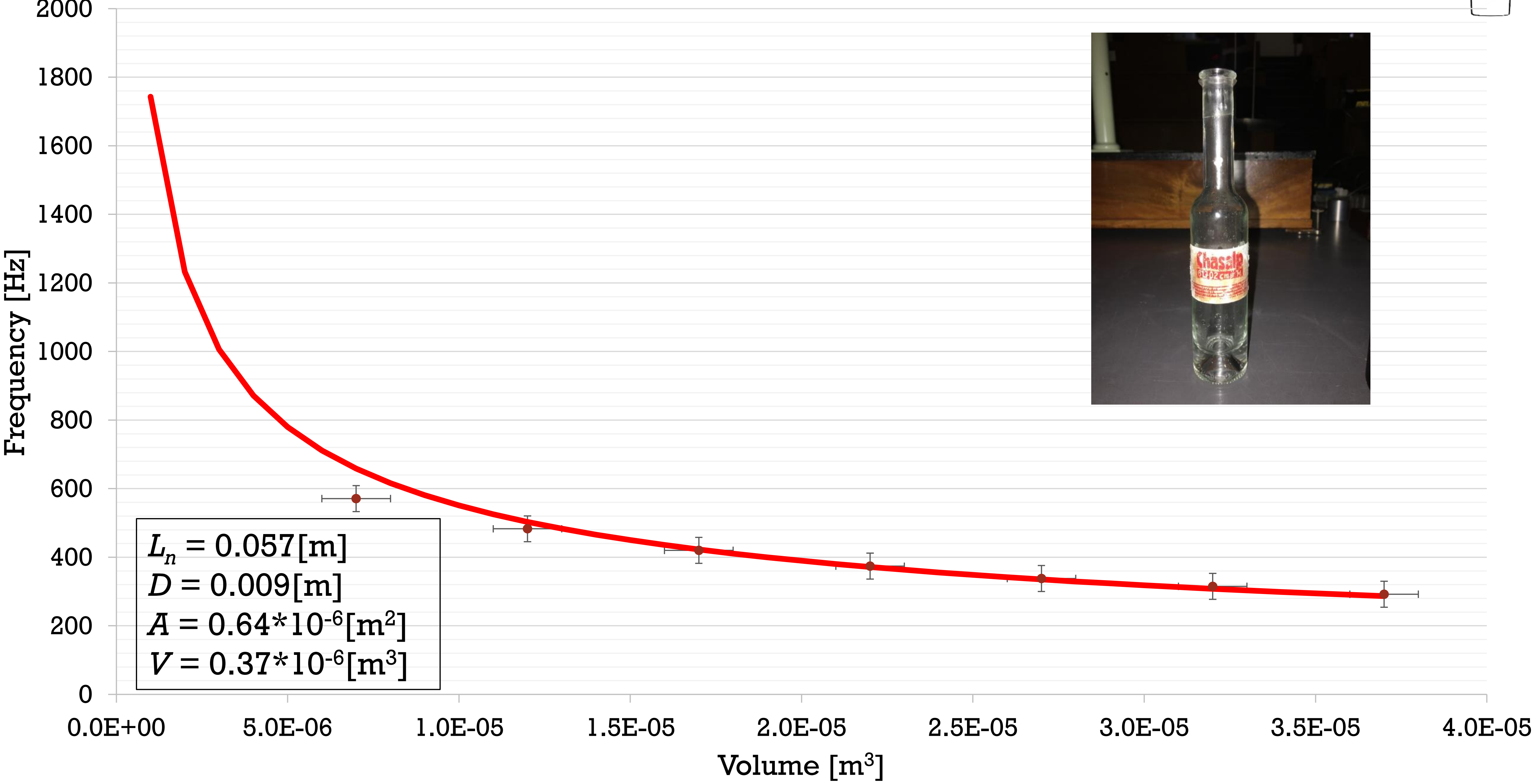

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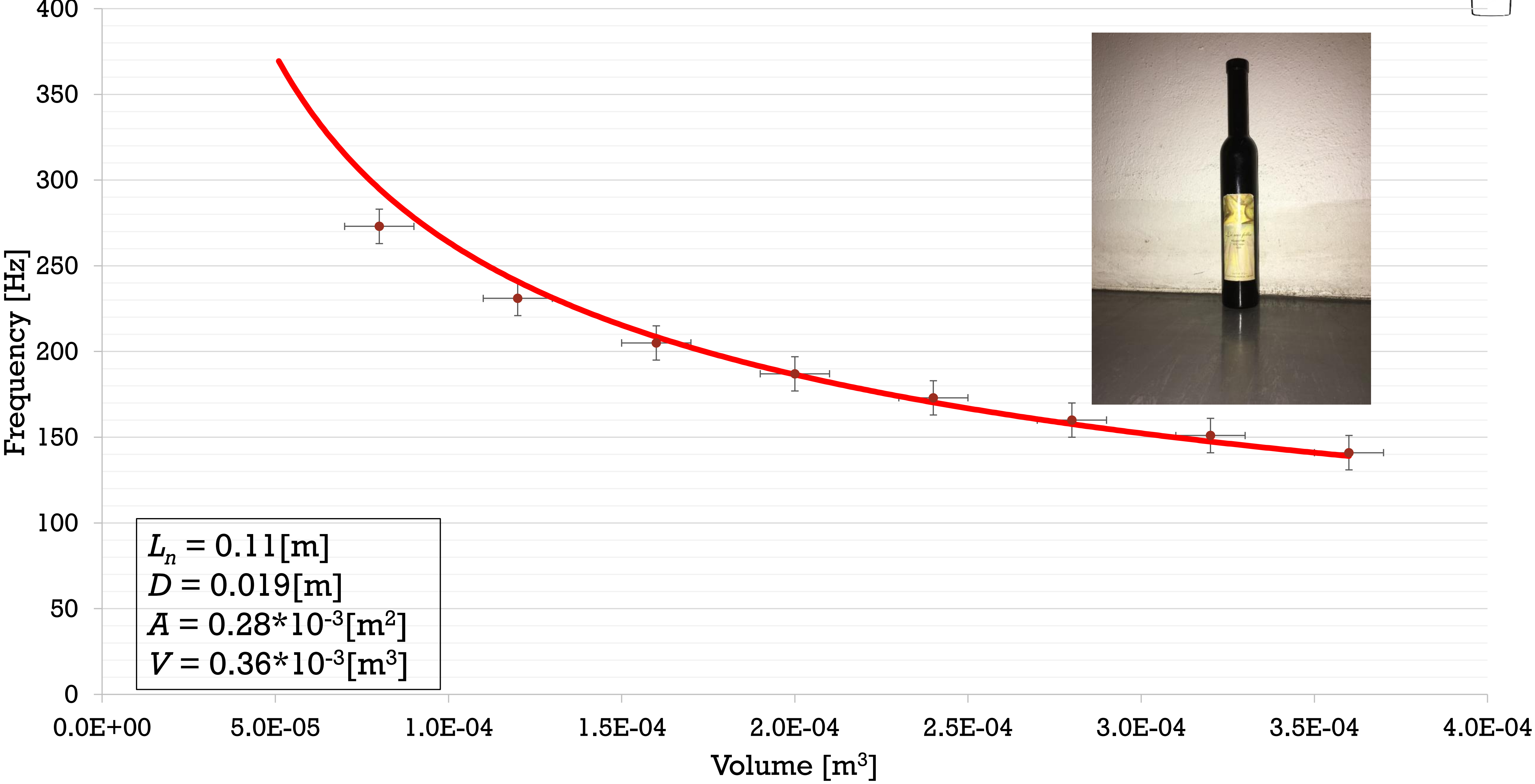
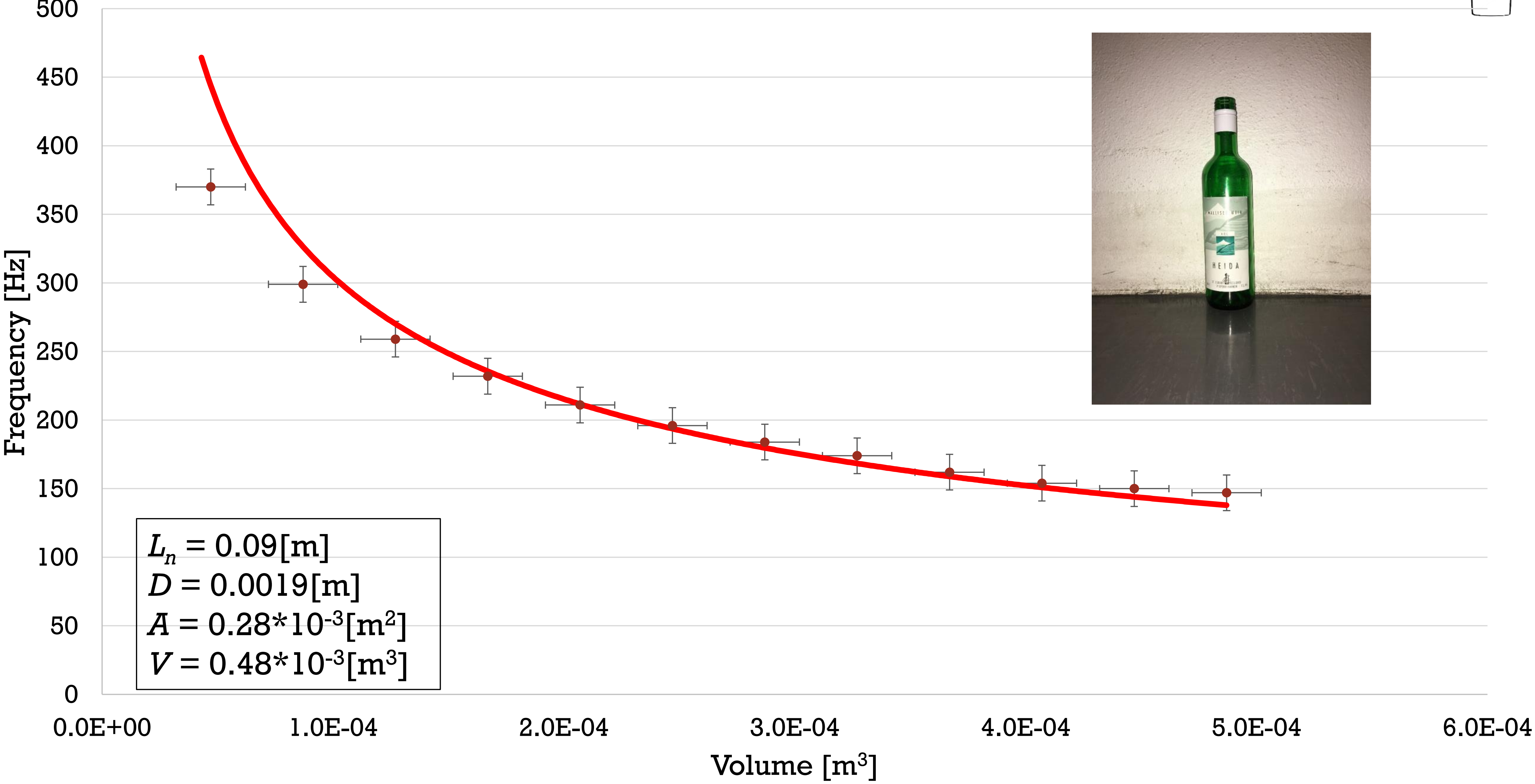
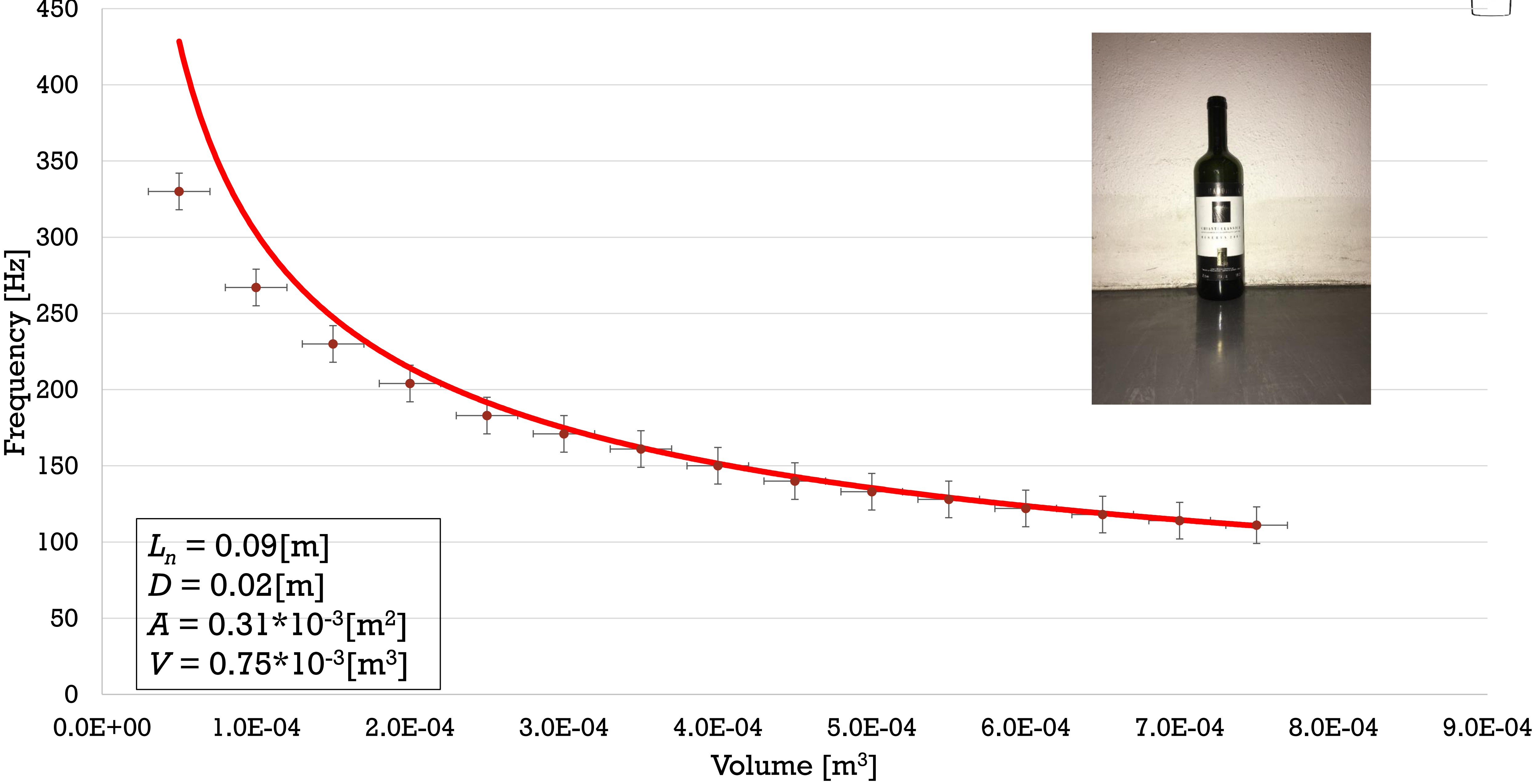


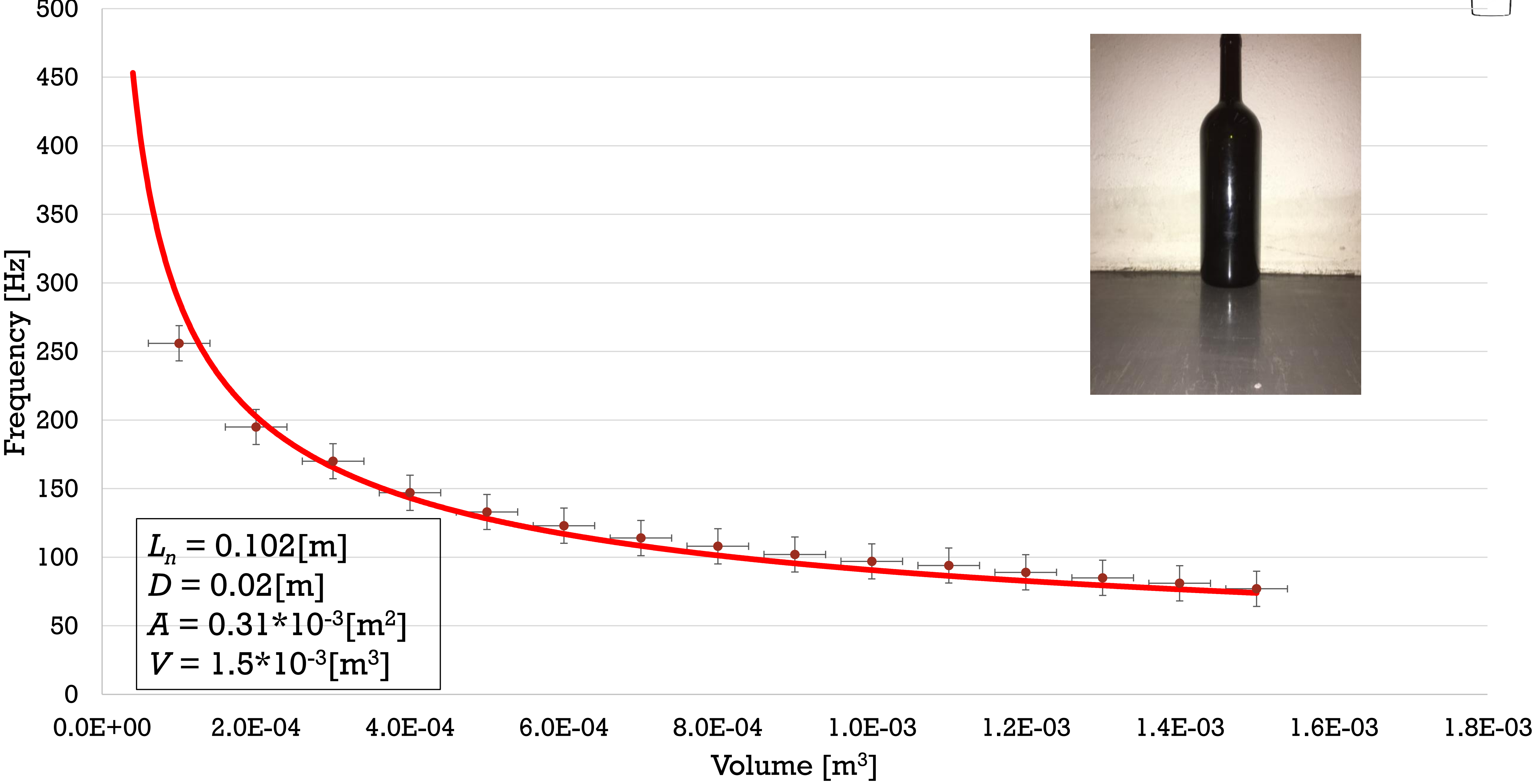
Comparing Experiments to the Theory



Comparing Experiments to the Theory



Comparing Experiments to the Theory



WHAT DOES THIS MEAN?

Helmholtz resonation is valid

Volume comparable neck volume ⟶ Edge case (assumptions break down)

EXPERIMENT WITH "MY HEART WILL GO ON"

low D F# G A

B C high D

$$f \sim \frac{1}{\sqrt{V}} \quad \Rightarrow \quad V \sim \frac{1}{f^2}$$

TILT OF THE BOTTLE

CONCLUSION

Notes with bottles and water:

$$f(V) = \frac{contant}{\sqrt{V}}$$

Same bottle:

- the shape of cavity does not influence the frequency

THANK YOU FOR LISTENING

SOURCES

- Special thanks to Marc Bitterli and Ella Blakely
- Akira Hirose and Karl E. Lonngren, Fundamentals of Wave Phenomena, 2nd Ed., (SciTech Publishing, 2010). ISBN: 978-1-891121-92-0
- Why you hear what you hear: An experimental approach to Sound, Music, and Psychoacoustic, ISBN 9780691148585, Princeton University Press, 2012
- The Use of Helmholtz Resonance for Measuring the Volume of Liquids and Solids, E.S. Webster, C.E. Davies, Journal of Engineering and Advanced Technology, New Zealand, 2010
- http://hyperphysics.phy-astr.gsu.edu/hbase/shm.html
- https://hyperphysics.phy-astr.gsu.edu/hbase/Waves/cavity.html
- https://newt.phys.unsw.edu.au/jw/Helmholtz.html
- https://pages.mtu.edu/~suits/notefreqs.html
- www.youtube.com/channel/UCp96ZKaODsGQwWYCR5T3DeA

EXPERIMENT WITH SPECIFIC NOTES

$$V = \frac{S \cdot v^2}{4\pi^2 \cdot L_{eq} \cdot f^2}$$

Musical note	low D	F#	G	A	B	C	high D
Frequency in Hz (theory)	293.66	369.99	392.00	440.00	493.88	523.25	587.33
Volume in cm^3 (model)	236	148	132	105	83	74	59
Frequency in Hz (experiment)	293	373	408	456	496	530	594
Frequency error	0.2%	0.8%	4.1%	3.6%	0.4%	1.3%	1.1%

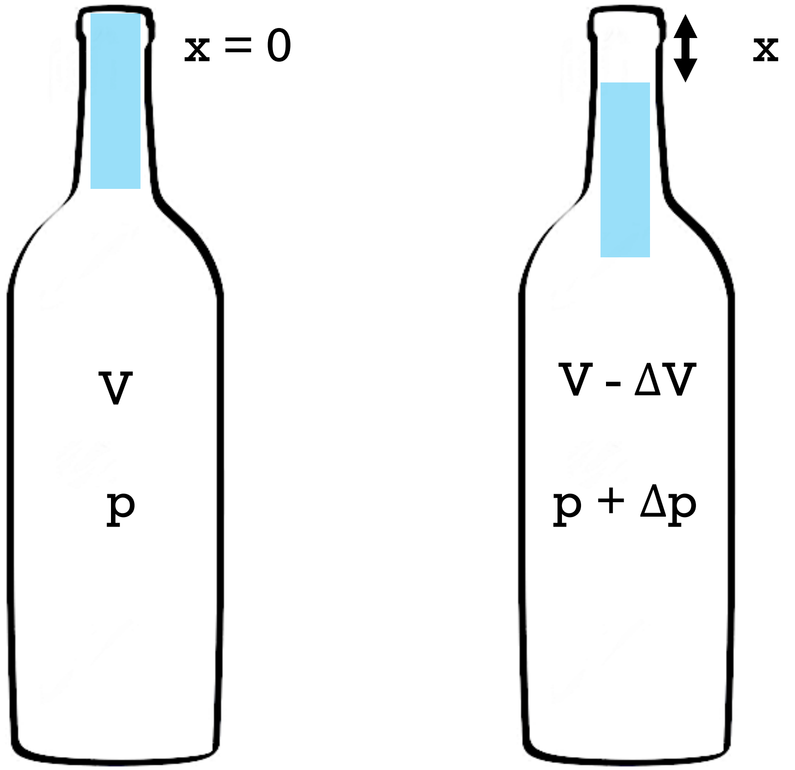
SPEED OF SOUND IN FLUIDS

$$v = \sqrt{\frac{\gamma R T}{M}} = \sqrt{\frac{\gamma p}{\rho}}$$

γ : Adiabatic index
R : Universal gas constant
T : Temperature
M: Molecular mass

v : Propagation speed of sound
p : Pressure
ρ: Density

HELMHOLTZ DERIVATION



$$a = \frac{F}{m} = \frac{\Delta p\ A}{\rho\ A\ L_{eq}}$$

$$= -\frac{\gamma\ p\ A\ x}{\rho\ V\ L_{eq}} = -\frac{v^2\ A}{V\ L_{eq}}\ x$$

$$x(t) = B\sin(\omega t + \varphi)$$

$$\omega = 2\pi f = v\sqrt{\frac{A}{V\ L_{eq}}}$$

DIMENSIONS OF THE BOTTLE

$L_n = 0.04[m]$
$D = 0.02[m]$
$A = 0.3142*10^{-3}[m^2]$
$V = 0.343*10^{-3}[m^3]$

$L_n = 0.11[m]$
$D = 0.019[m]$
$A = 0.28*10^{-3}[m^2]$
$V = 0.36*10^{-3}[m^3]$

$L_n = 0.09[m]$
$D = 0.02[m]$
$A = 0.31*10^{-3}[m^2]$
$V = 0.75*10^{-3}[m^3]$

$L_n = 0.057[m]$
$D = 0.009[m]$
$A = 0.64*10^{-6}[m^2]$
$V = 0.37*10^{-6}[m^3]$

$L_n = 0.09[m]$
$D = 0.0019[m]$
$A = 0.28*10^{-3}[m^2]$
$V = 0.48*10^{-3}[m^3]$

$L_n = 0.102[m]$
$D = 0.02[m]$
$A = 0.31*10^{-3}[m^2]$
$V = 1.5*10^{-3}[m^3]$