PROBLEM 8: FAIR COIN

IYNT 2018

Team Switzerland

PROBLEM

8. Fair coin

In many cases, disputes are resolved with a coin toss. It is presumed that this procedure gives equal chances of winning to both sides. Investigate how the chances depend on the tossing mechanism and the coin properties.

WHAT IS A COIN TOSS?

1. Coin gets thrown into the air

2. Coin rotates around its diameter (edge-over-edge)

3. Coin lands

Coin landing on its edge: negligible probability (angular momentum)

WHAT IS A FAIR COIN?

A sequence of independent Bernoulli trials with probability of ½ of success on each trial is metaphorically called a fair coin.

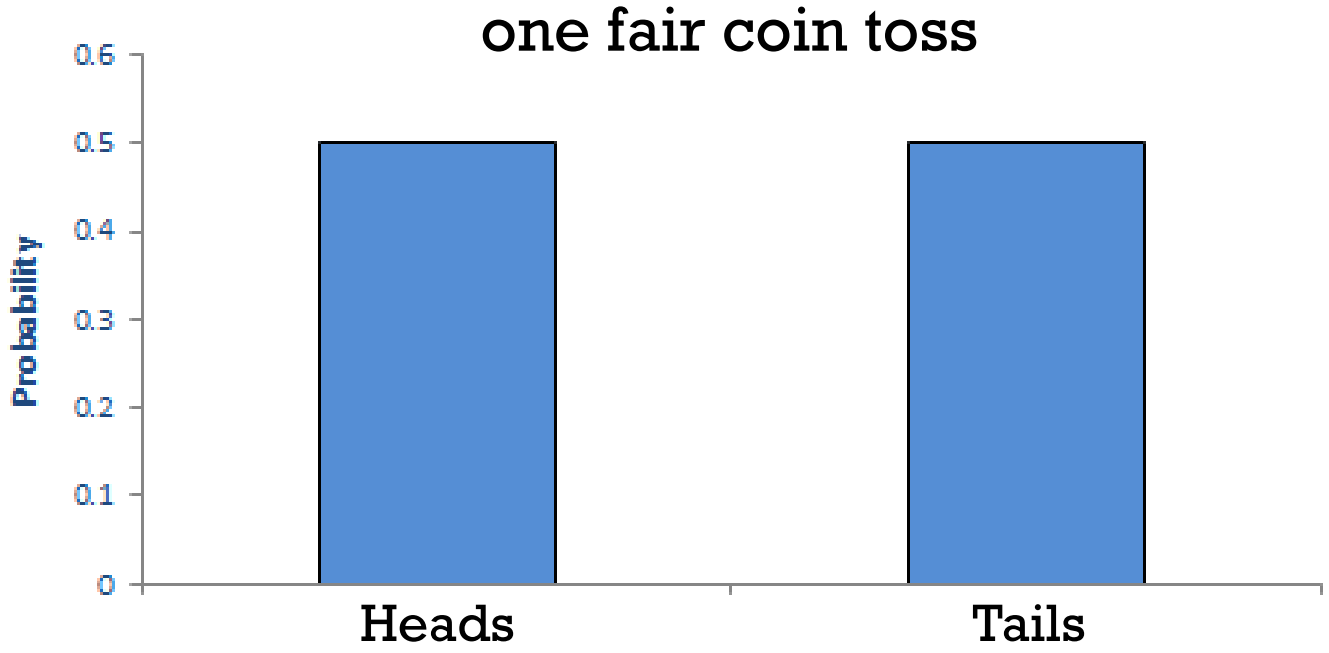
$$\frac{1}{2}$$

$$\frac{1}{2}$$

COIN PROPERTIES

Material:	rigid structure
	hard material
Mass:	moderate total mass
Uniformity:	density
	center of mass = geometric center
	symmetrical about center of mass

THEORY: EQUAL CHANCE

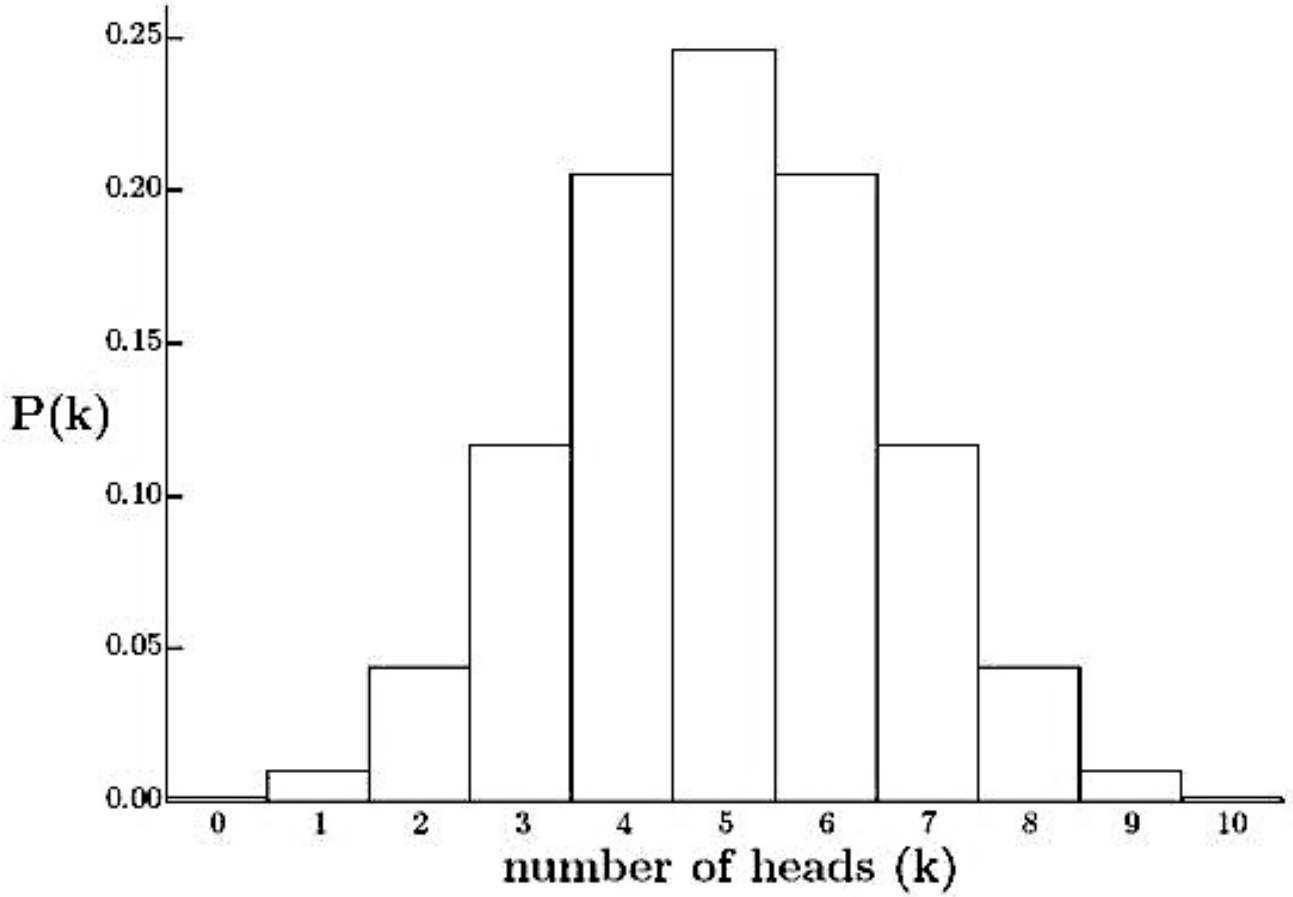


two outcomes → binomial

outcome unpredictable → random variable

same probability of outcomes → **uniform probability distribution**

THEORY: MULTIPLE COIN TOSSES



Binomial probability distribution

THEORY: LAW OF LARGE NUMBERS

Empirical probability of success

= relative frequency

$$= \frac{\text{number of tails}}{\text{number of tosses}}$$

Theoretical probability of success

= p

$$= \frac{1}{2}$$

Law of large numbers:

As the number of experiments (coin tosses) increases,
the empirical probability will converge on the theoretical probability.

HYPOTHESIS

Human inaccuracy is responsible for the equal chance of any side in a coin toss.

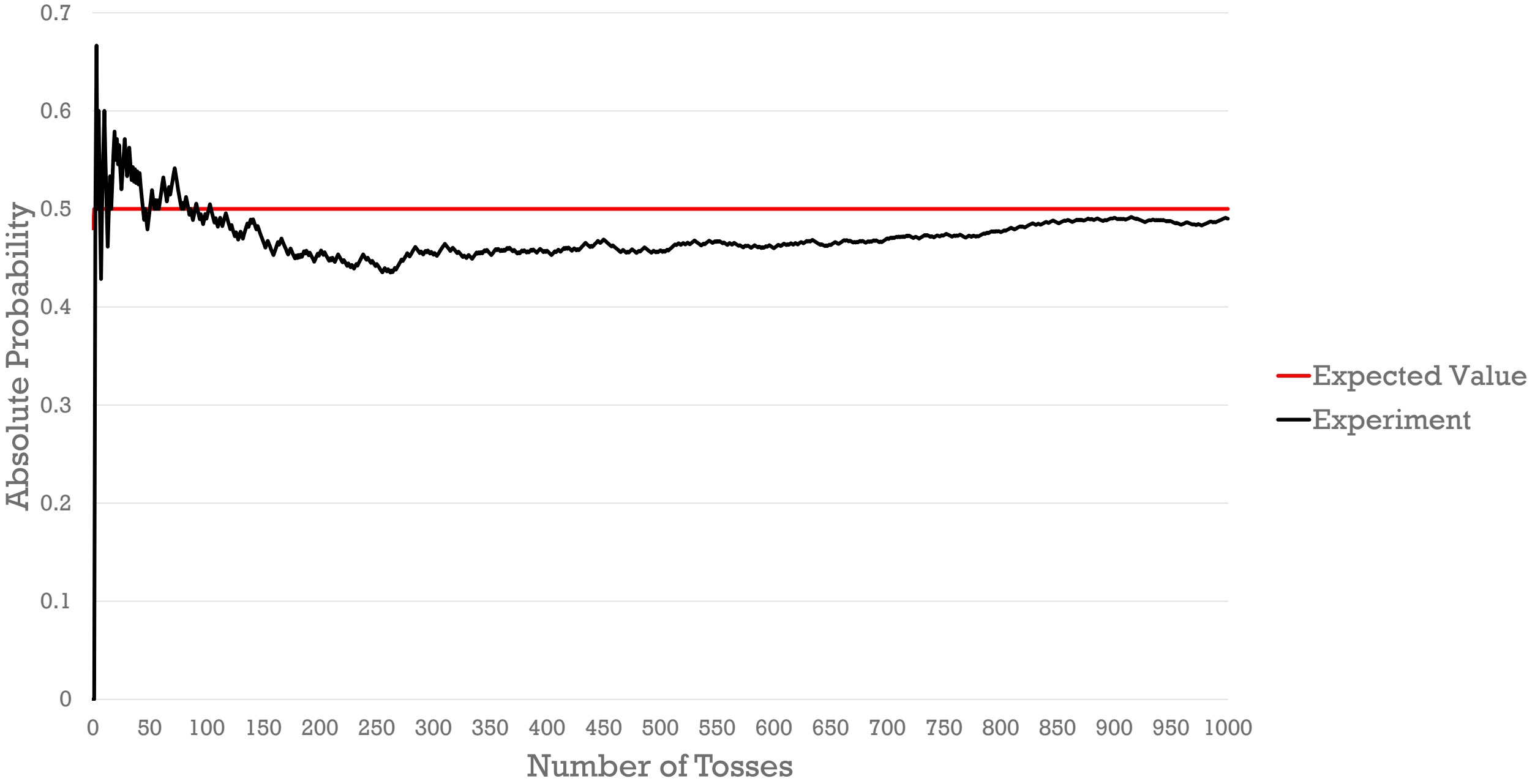
EXPERIMENT 1: TOSSING BY HAND

Tossed a two Swiss Franc coin 1000 times by hand.

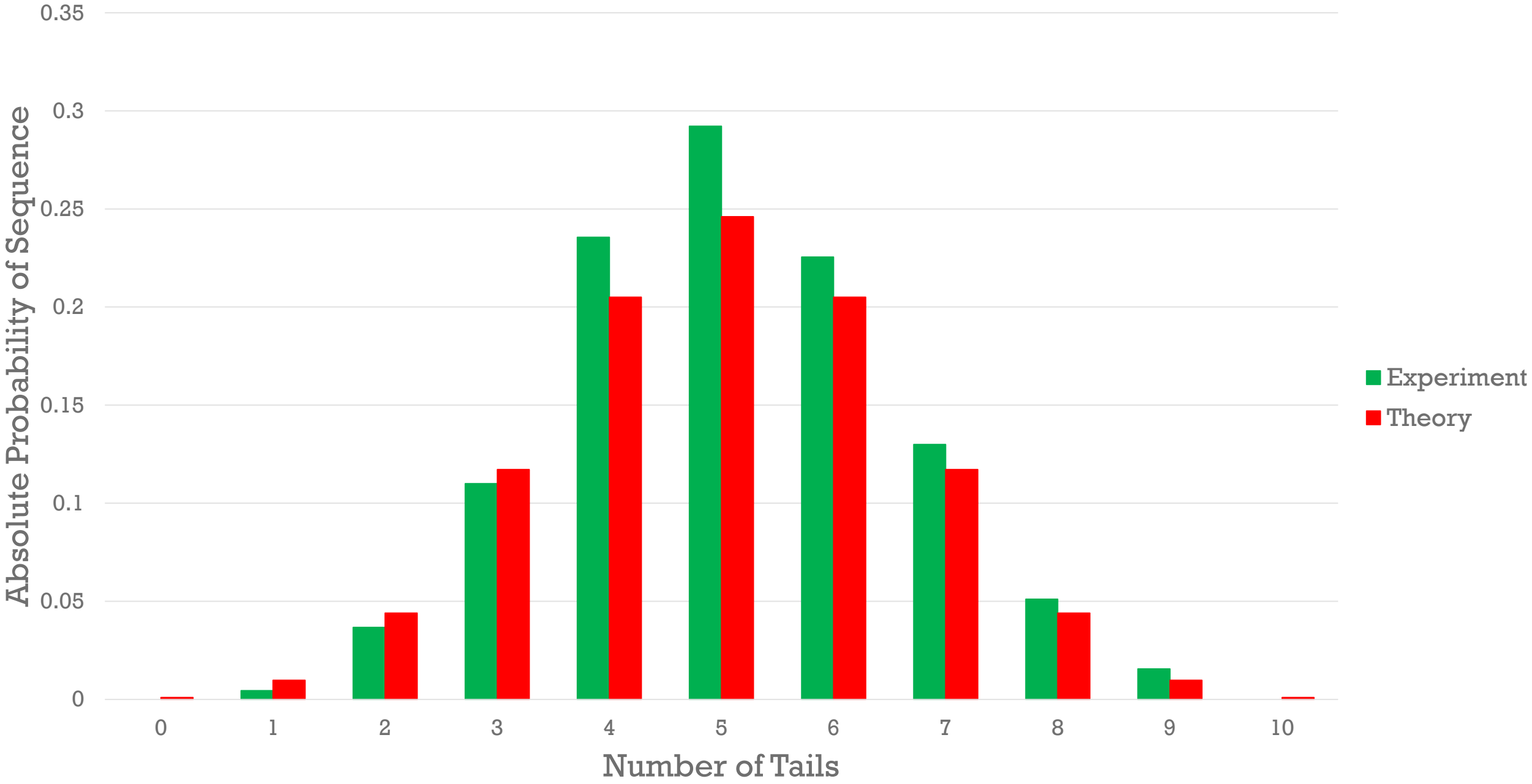
Heads
510

Tails
490

Law of Large Numbers (Hand)



Binomial Distribution (Hand)



ANALYSIS OF TOSSING BY HAND

Empirical probability converges on expected probability

law of large numbers ✓

Empirical probability distribution of sequences of 10 tosses converges on binomial distribution

fair coin ✓

EXPERIMENT 2: REMOVING THE INACCURACY

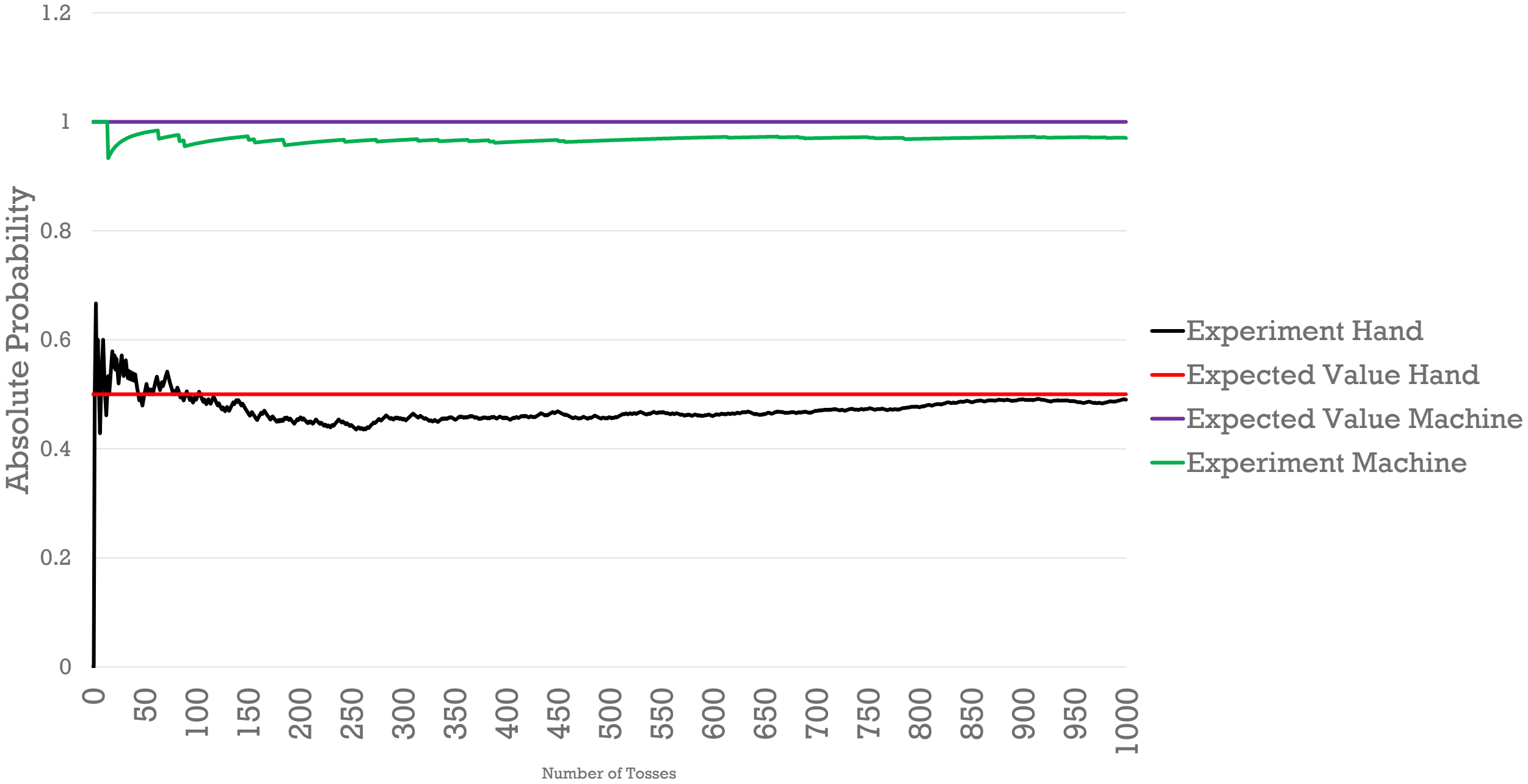
EXPERIMENT 2: TOSSING WITH MACHINE

Tossed a two Swiss Franc coin 1000 times with a machine

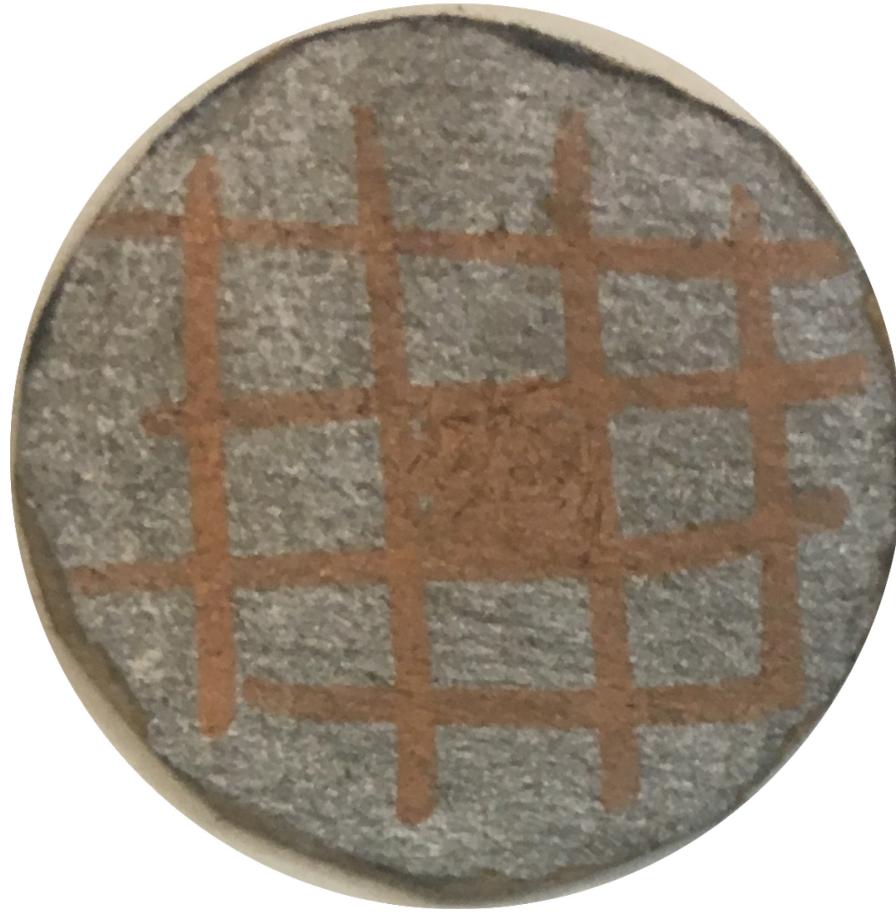
Heads
30

Tails
970

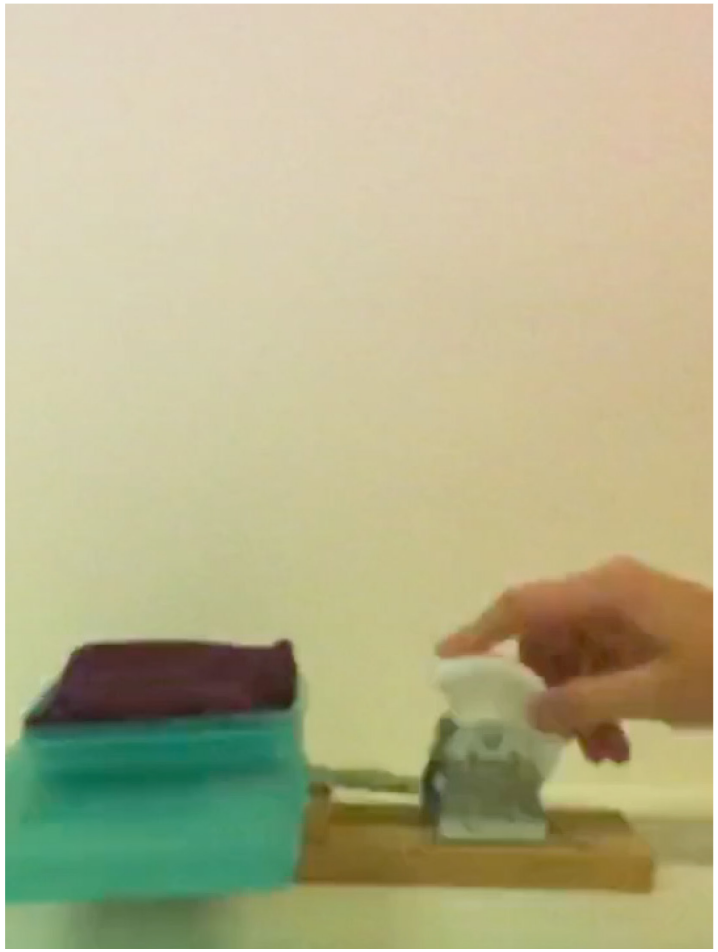
Law of Large Numbers (Hand & Machine)



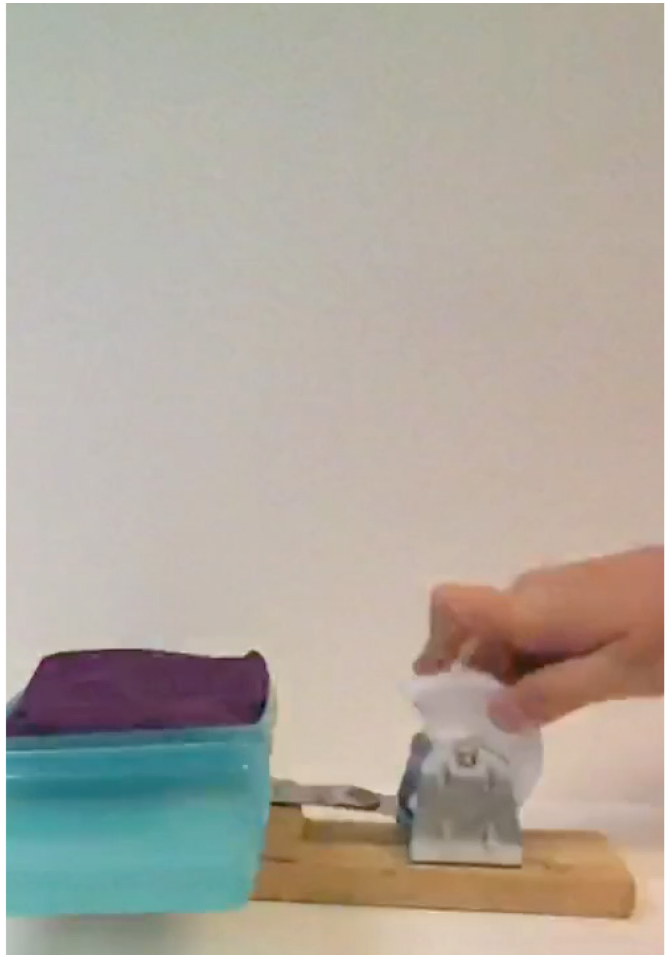
PROPERTIES OF THE COIN



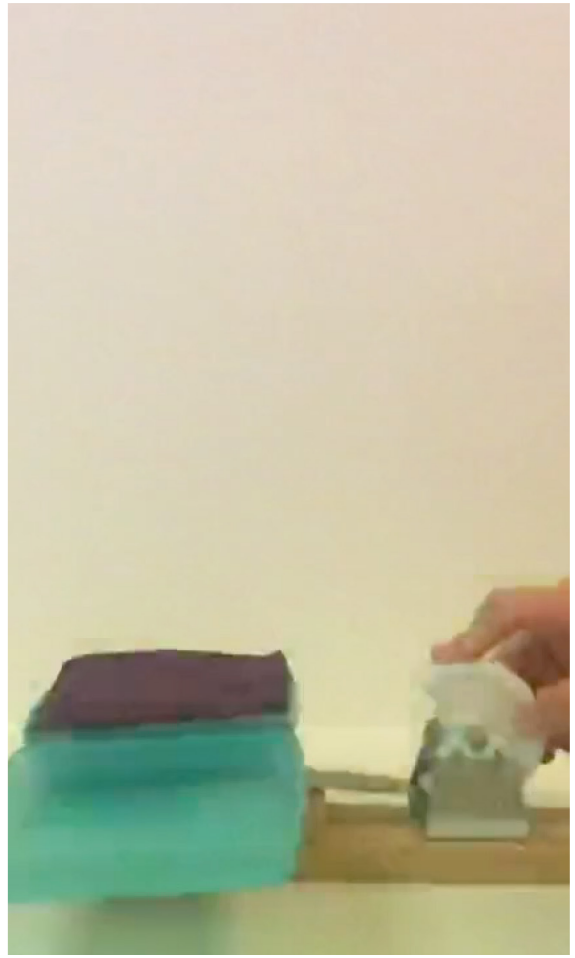
COMPARISON OF PROPERTIES



Normal



Hole



Cardboard

CONCLUSION

The apparent random aspect of the coin toss is due to inconsistency of the human tossing the coin, not because it is an inherently random procedure.

(classical mechanics)

The material has an effect, as air resistance and inertia play an important role.

SOURCES

- https://www.khanacademy.org/math/statistics-probability/probability-library
- http://www.statisticshowto.com/probability-and-statistics/binomial-theorem/binomial-distribution-formula/
- http://www.stat.yale.edu/Courses/1997-98/101/binom.htm
- http://www.btwaters.com/probab/flip/coinmainD.html
- https://econ.ucsb.edu/~doug/240a/Coin%20Flip.htm

THANK YOU FOR LISTENING