

The 6th International Young Naturalists' Tournament

Problem № 1
«Buffon's needle»



Team «12FM»
Diana Ignatovich

chnmk@mail.ru

The task

Draw a series of parallel equally spaced lines on a horizontal surface. Pick a bunch of sticks (e.g. matches or needles) slightly shorter or longer than the separation between the lines, and randomly drop them on the surface. It is claimed that the number of times the sticks cross the lines allows estimating the constant π to a high precision. What accuracy can you achieve?

Hypothesis

If you draw parallel lines on paper and drop matches on it, accuracy π will depend on the distances between the lines.

Aim of the study

Get the most accurate possible value of π using the Buffon method.

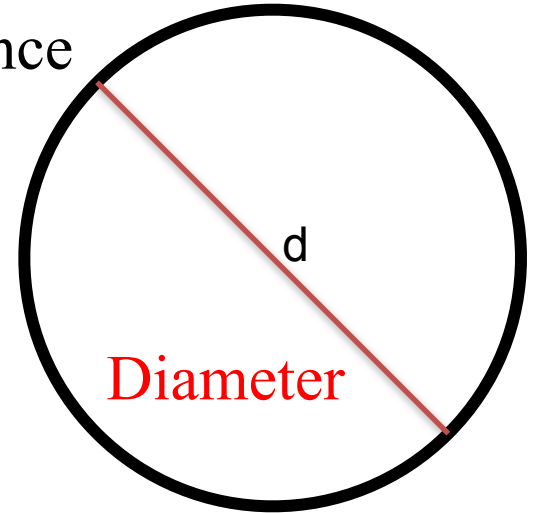
Objectives

1. Study the literature;
2. Study possible cases of intersection of the needle and the straight line;
3. Use the Buffon method to calculate the number π ;
4. Compare the experimental data with the theoretical data.

Theory

$$\pi = \left(\frac{16}{9}\right)^2 = 3,160\dots$$

Circumference



π — irrational number: that is, it cannot be expressed as a fraction $\mathbf{a/b}$, where \mathbf{a} is an integer and \mathbf{b} is a non-zero integer.

π — transcendental number: that is, it cannot be the root of any polynomial with integer coefficients.



Theory

MN, M_1N_1 – parallel lines;

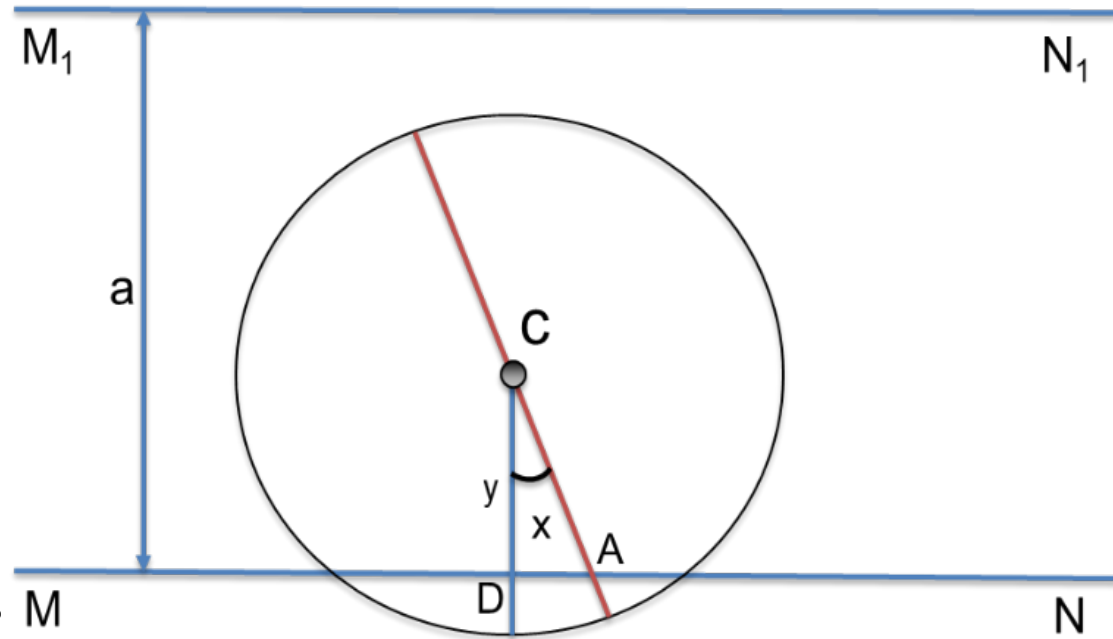
a – distances between lines;

C – the center of the needle;

CD (y) – the perpendicular from the center of the needle to the line;

A – the end of the needle;

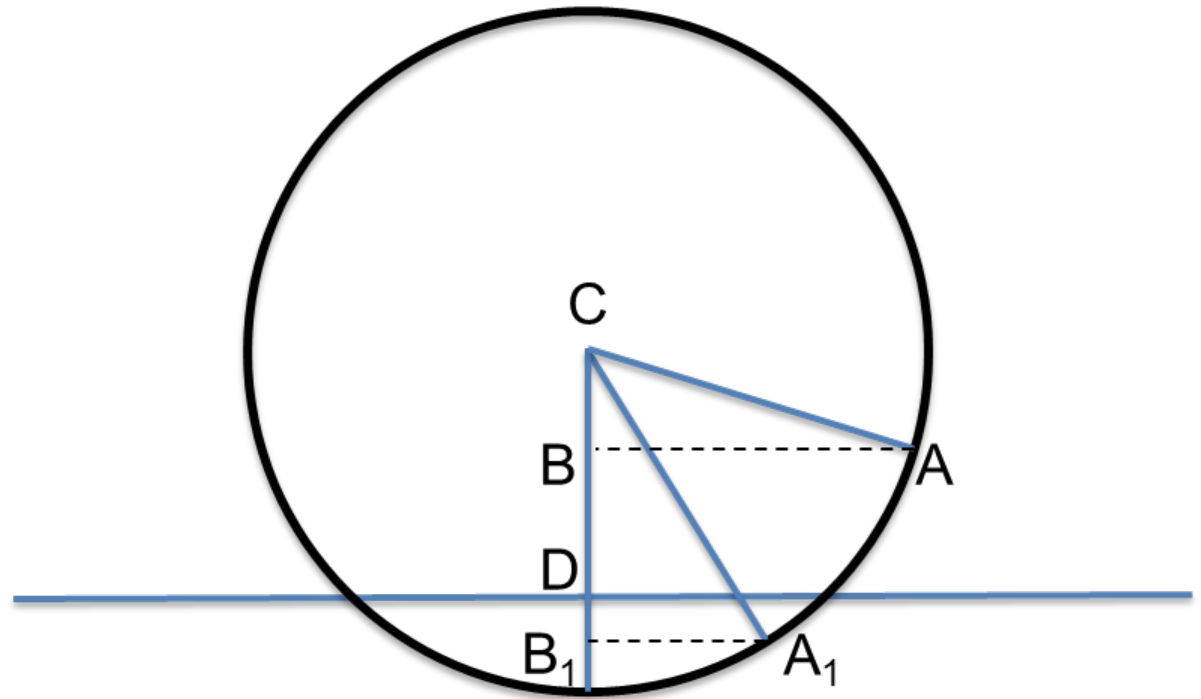
x – the angle between the perpendicular and the needle;



$$0 < x < \frac{\pi}{2}, \quad 0 < y < \frac{a}{2}$$

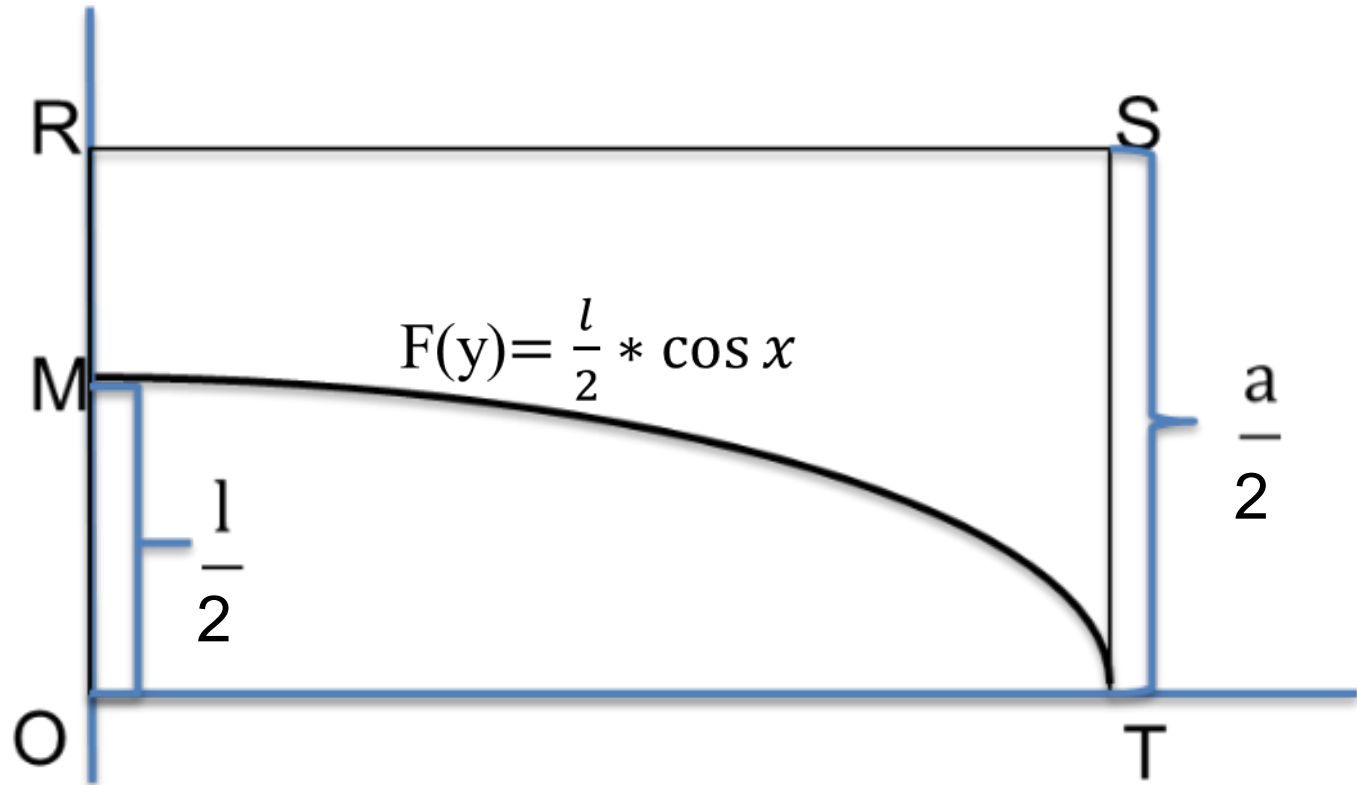
Theory

$$CB = \frac{l}{2} \cos \alpha$$



$$0 < x < \frac{\pi}{2}, \quad 0 < y < \frac{a}{2}$$

$$OT = \frac{\pi}{2}, \quad OR = \frac{a}{2}$$

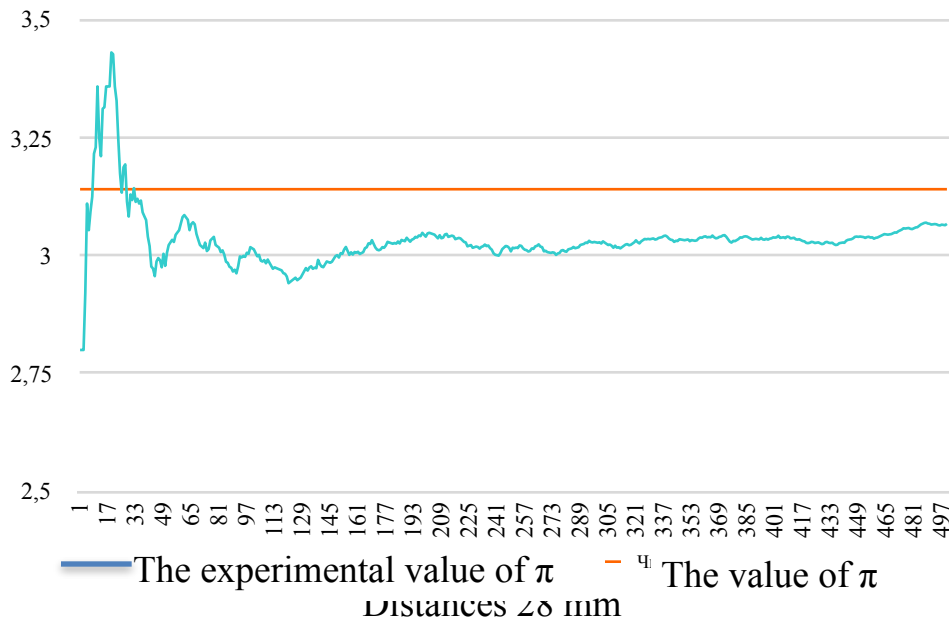
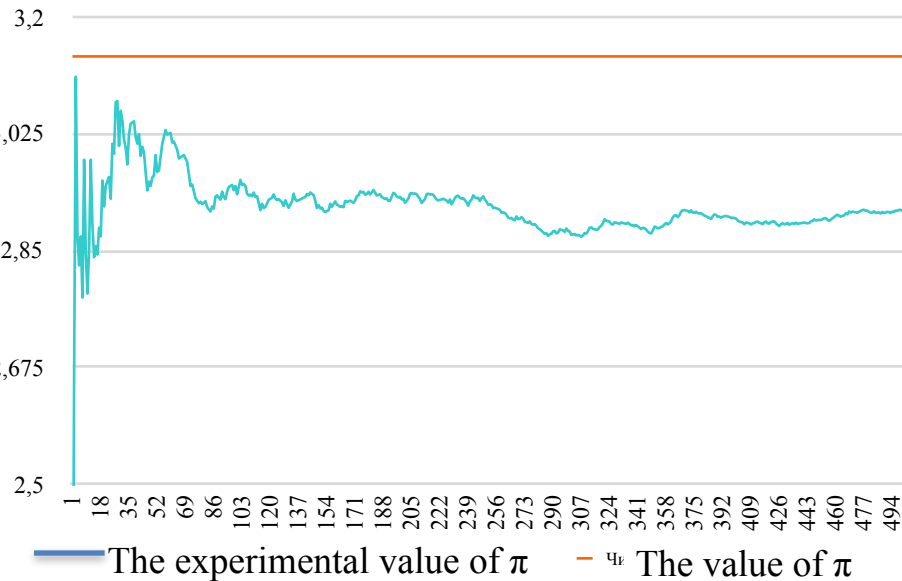


$$p = \frac{S_{OMT}}{S_{ORST}} = \frac{s}{S} = \frac{l}{2} \div \frac{\pi a}{4} = \frac{2l}{\pi a}$$

Experiment 1

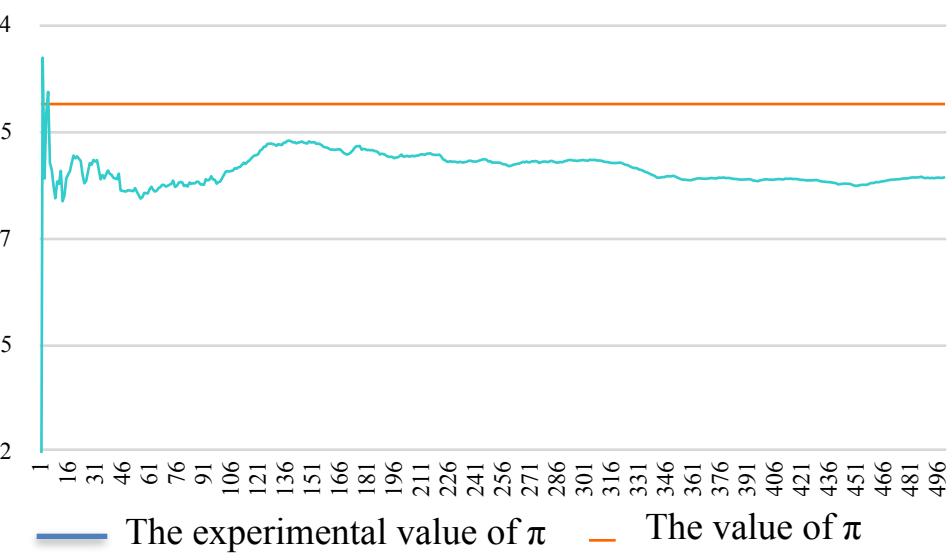
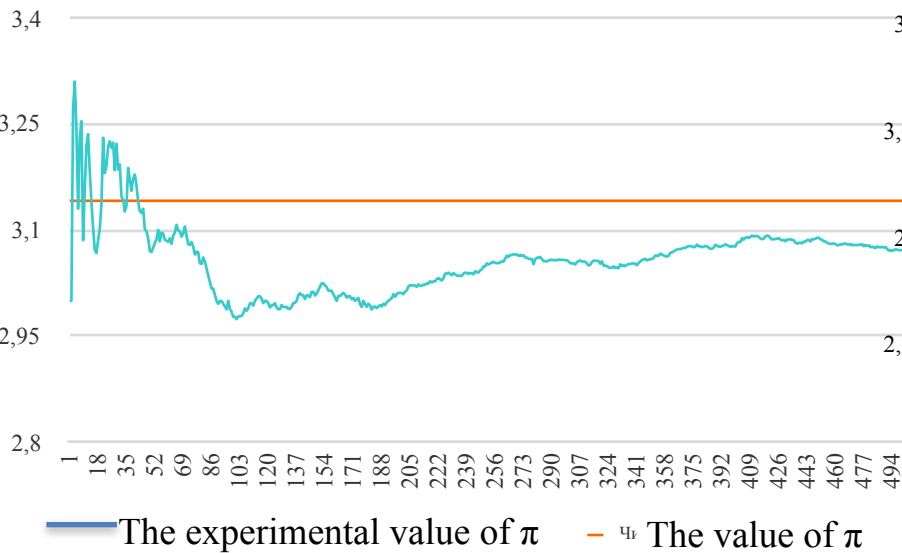
Distances 45 mm

Distances 50 mm



Distances 35 mm

Distances 28 mm



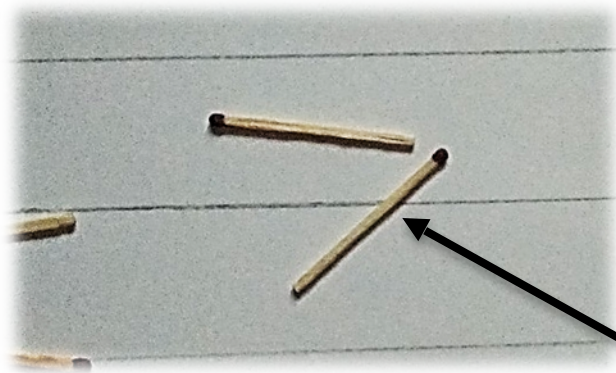
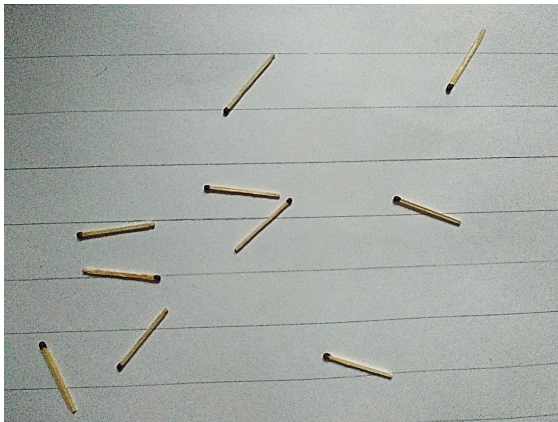
Experiment 1

$$\pi = \frac{2l}{pa}$$

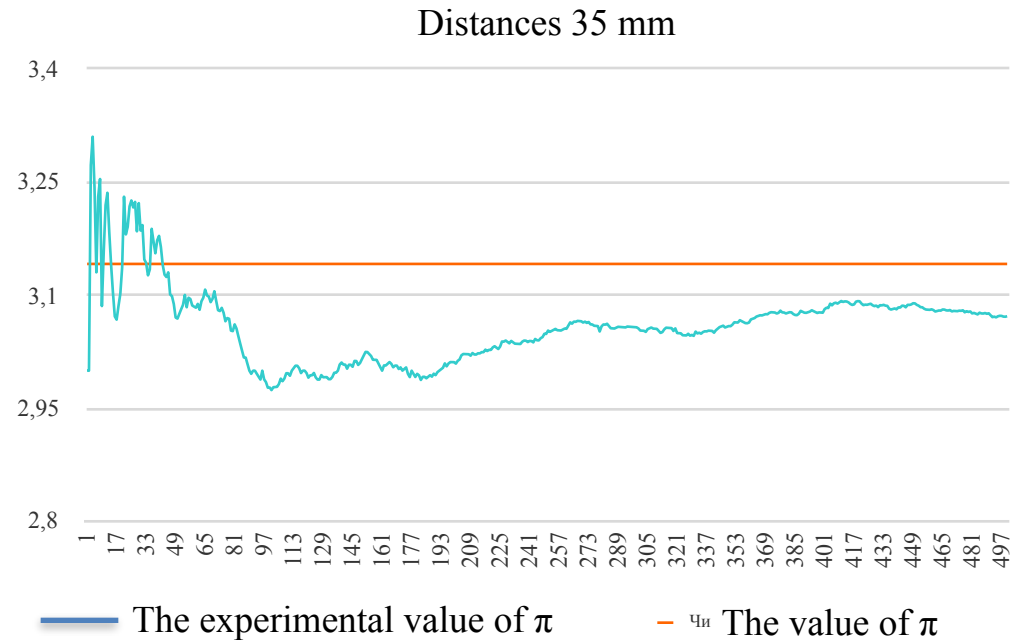
$$p = \frac{m}{n}$$

m – intersections

n – number of the thrown matches



intersections

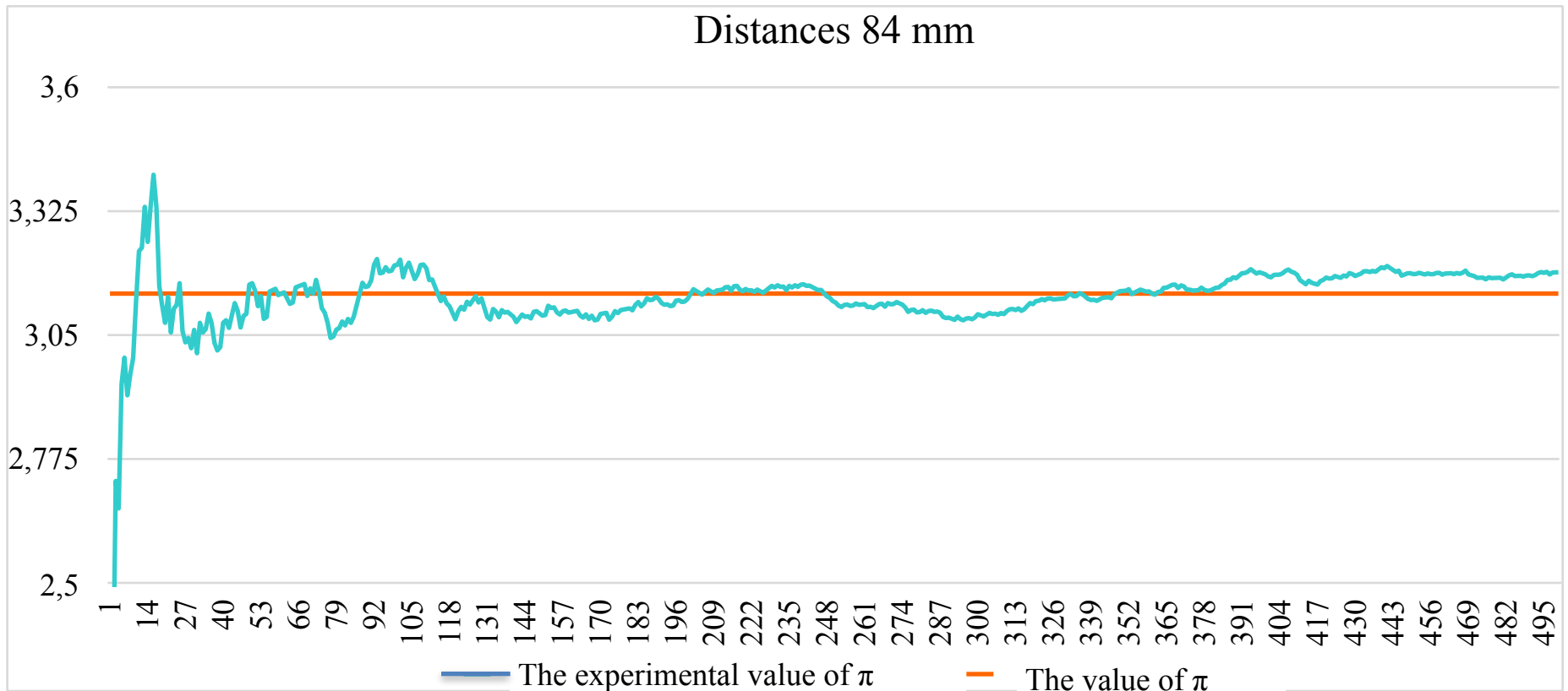


Experiment 2

Purpose: to determine the most accurate value of π .

Equipment: matches (length - 42 mm); a sheet of paper lined with the parallel straight lines at a distance of 84 mm.

Distances 84 mm

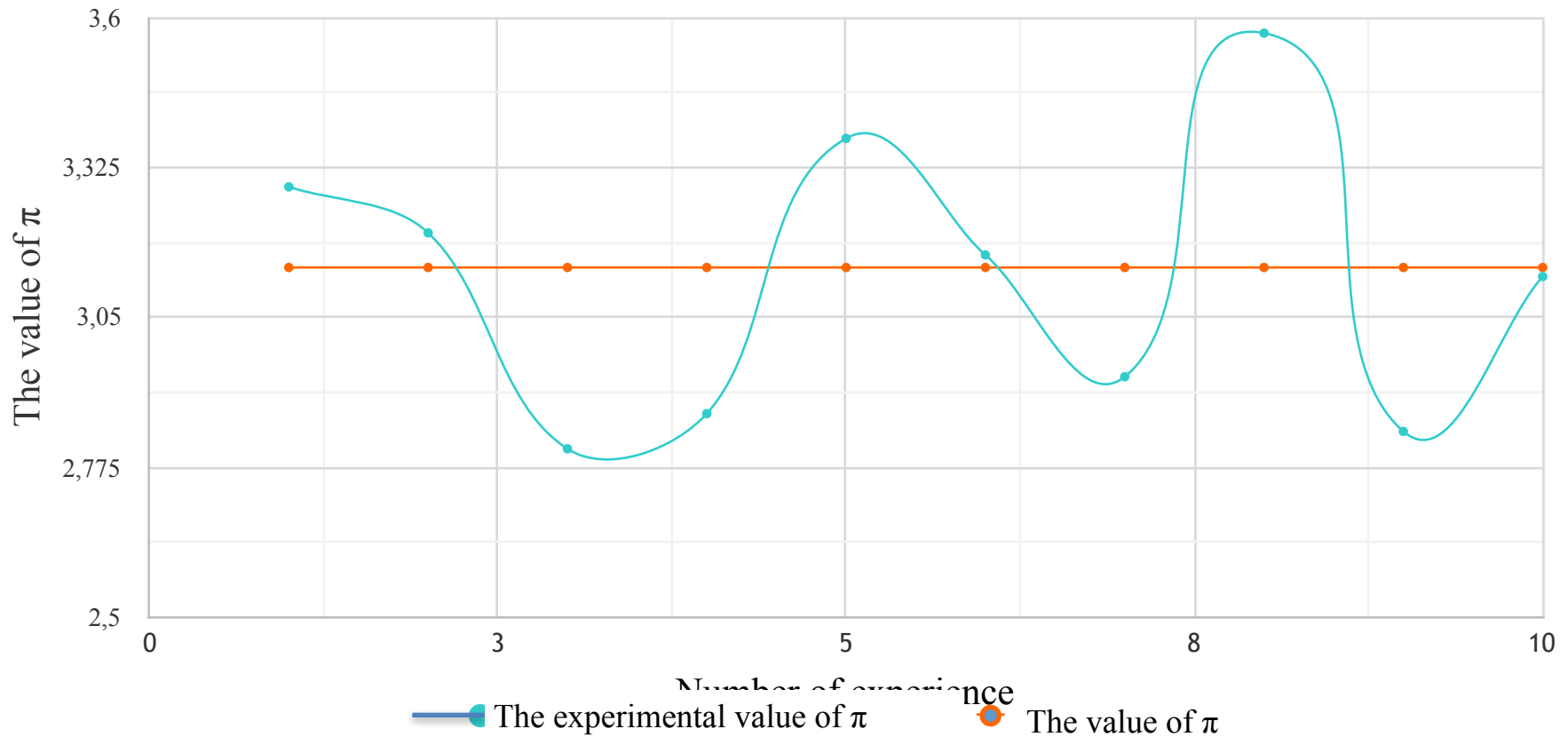


Conclusion: the experimental value of the number π : 3,14159292.

Experiment 3

Purpose: to determine the most accurate value of π .

Equipment: 250 matches (length - 42 mm); a sheet of paper lined with the parallel straight lines at a distance of 84 mm.



Conclusion: the possibility to get exact value of the π is slight. The error of the closest result is 0.03.

Experiment 4

Purpose: to calculate the value of π by using the ratio of the circumference to diameter.

Equipment: tennis ball, glass, mug, glass jar, box of tennis balls.

<i>Object</i>	<i>Circumference, cm</i>	<i>Diameter, cm</i>	<i>The ratio of the circumference to its diameter</i>
A tennis ball	20	6,4	3,125
A glass	17,5	5,5	3,18181818
A mug	26,7	8,5	3,14117647
A glass jar	19,0	6,0	3,16666667
A box of tennis balls	23,7	7,5	3,16

Conclusion: To calculate the accuracy of the number π by using this method is small: in only one case out of 5, the found constant contains the correct number in the hundredth digit.

Conclusion

- Even if we have a slight difference between the length of the "needle" and the distance between the lines, we do not get even a close value of the number π . But if the distance between the lines is twice larger than the length of the needle, then the possibility of obtaining the most accurate value of the number π increases several times.
- The experimental value of the number π : 3,14159292.

References

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