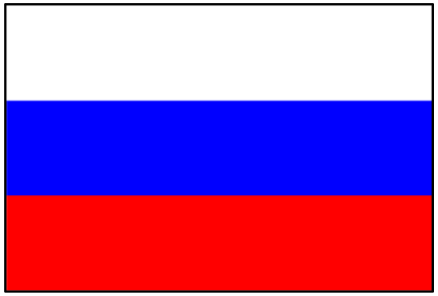


Problem №1:
“Buffon’s needle”

Reporter:
Ekaterina Rosnovskaya,
Team “12 kids of science”,
Russia, Voronezh

Tbilisi, 2018

The problem:

Draw a series of parallel equally spaced lines on a horizontal surface. Pick a bunch of sticks (e.g. matches or needles) slightly shorter or longer than the separation between the lines, and randomly drop them on the surface. It is claimed that the number of times the sticks cross the lines allows estimating the constant π to a high precision. What accuracy can you achieve?

Plan of the work:

1) To derive the formula calculating the constant π from the times of the intersections of the needles with lines;
2) To design and conduct the experiments according to method that is mentioned in the task;
3) To calculate the number π from the collected data
4) To compare experimental results with constant π

Location of the needle in relation to the line is defined by

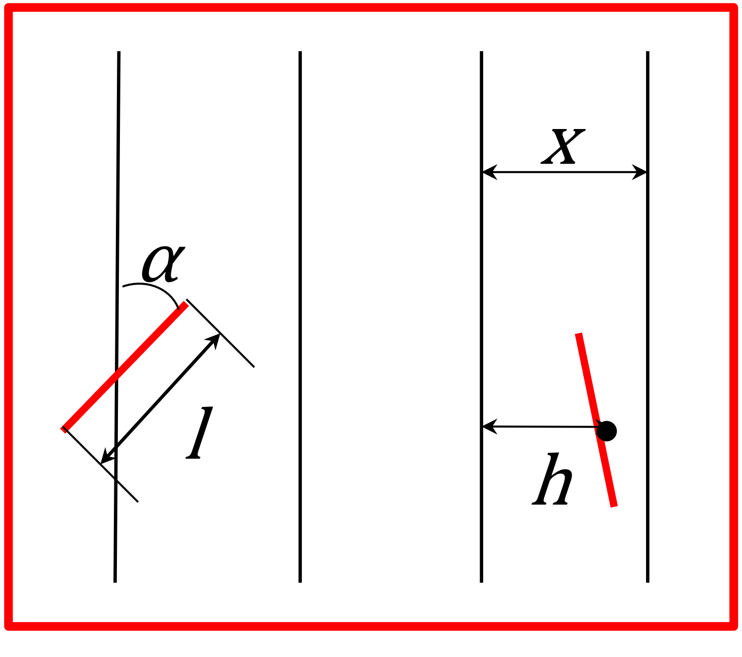
Angle α between the needle and the line

Distance h from the middle of the needle to the line

l – length of the needle,
x – distance between the lines

$$\mathbf{0 \le h \le \frac{x}{2}}$$

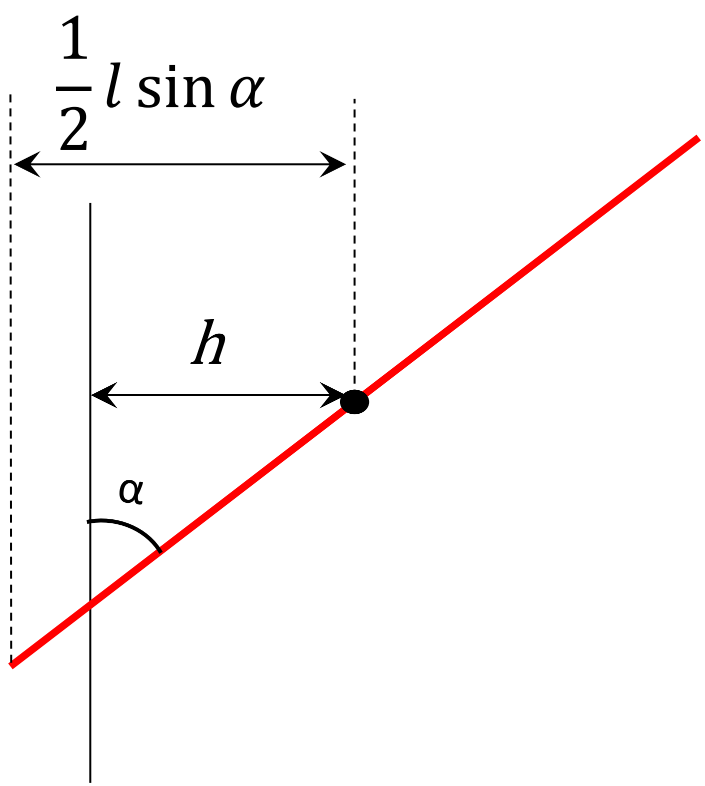
$$\mathbf{0 \le \alpha \le \frac{\pi}{2}}$$



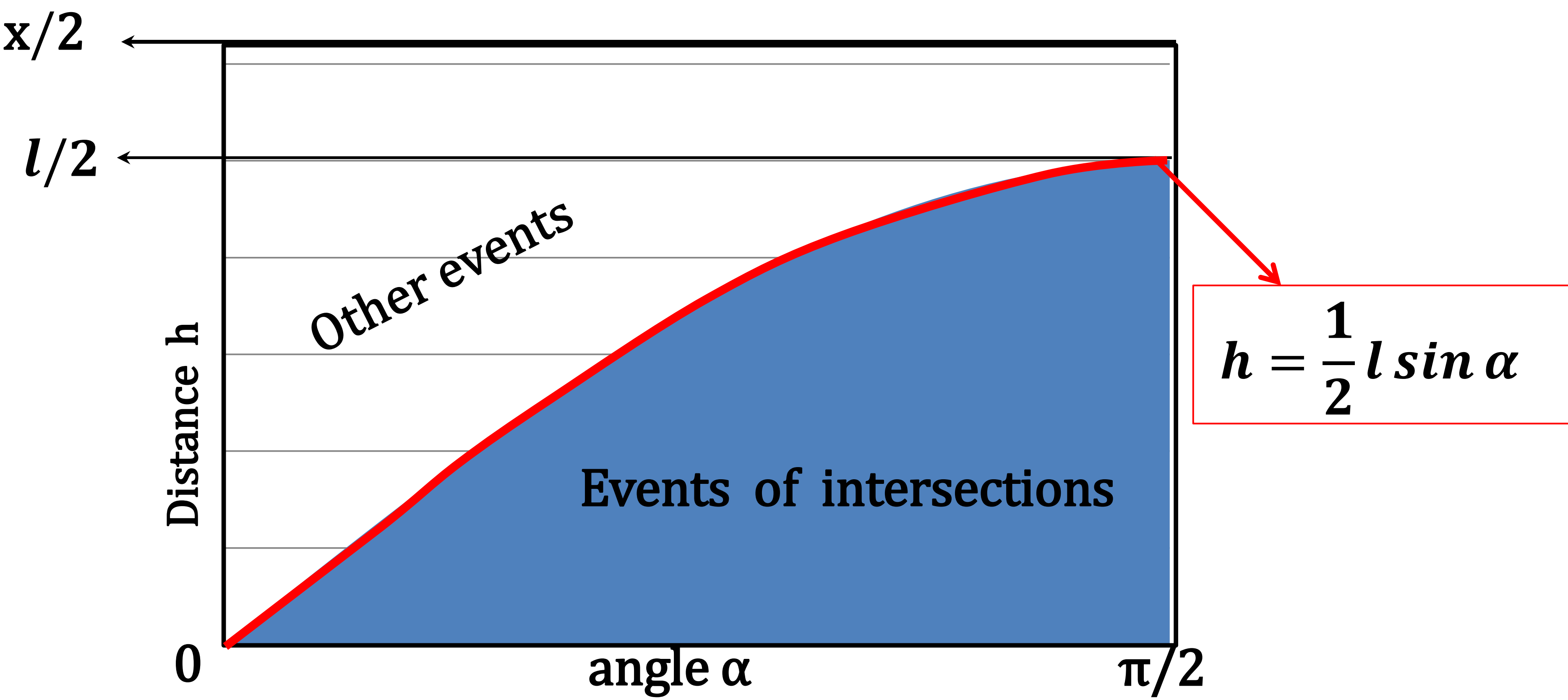
Event of intersection

Condition of intersection:

$$\boldsymbol{h \le \frac{1}{2} l \sin \alpha}$$



Probability of intersection:



$$p = \frac{Area\ under\ the\ sinusoid}{Rectanle's\ area} = \frac{2l}{\pi x}$$

Conducting of experiment:

1) We drew the parallel lines on a horizontal surface. Distances between the lines were equal. Then we dropped matches on that surface.
2) After that we counted up the number of intersections
3) Using the formula:

$$p = \frac{2l}{\pi x}; \quad \pi = \frac{2l}{px},$$

where $p = \frac{n\ of\ intersections}{n\ of\ all\ drops},$

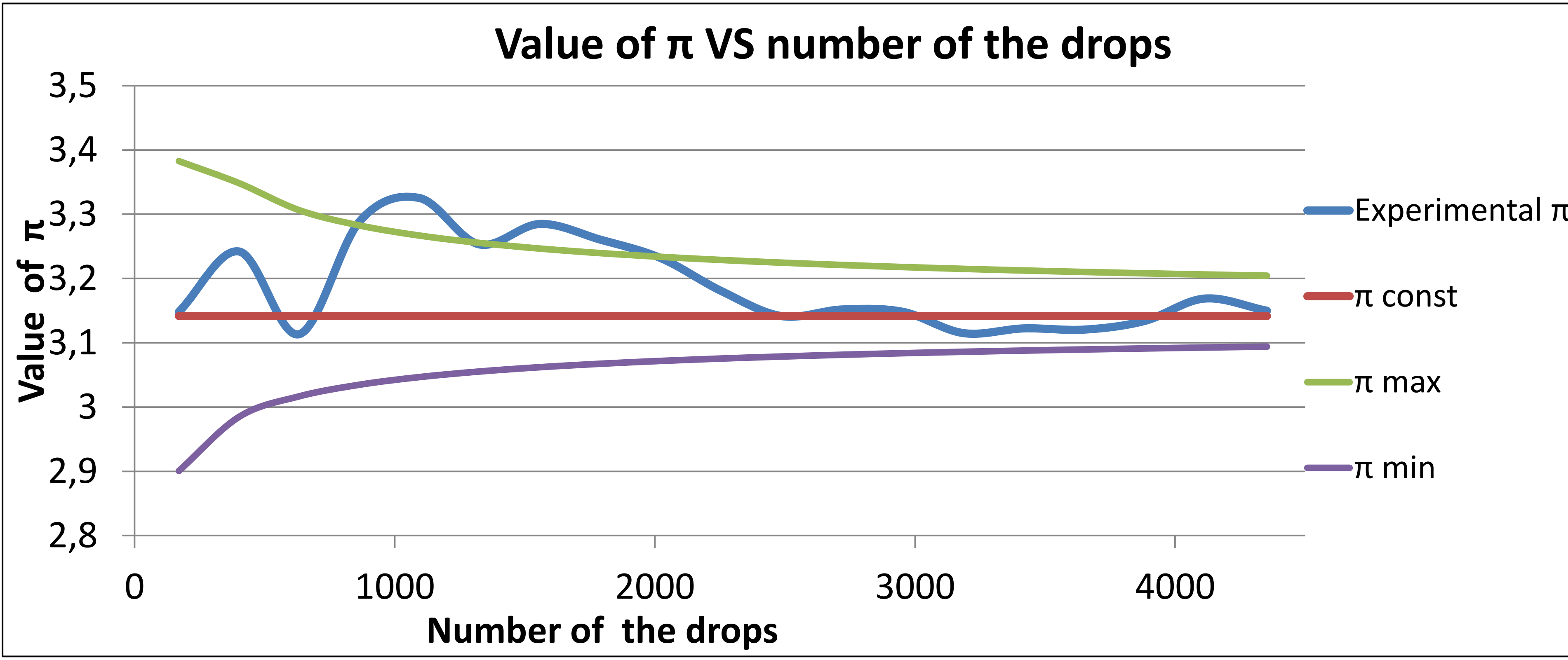
We calculated experimental π:

$$\boxed{\pi = \frac{2l \times n\ of\ all\ drops}{x \times n\ of\ intersections}}$$

Length of the match is shorter than distance between the lines

Number of drops: 4353

Length of the match is shorter than distance between the lines



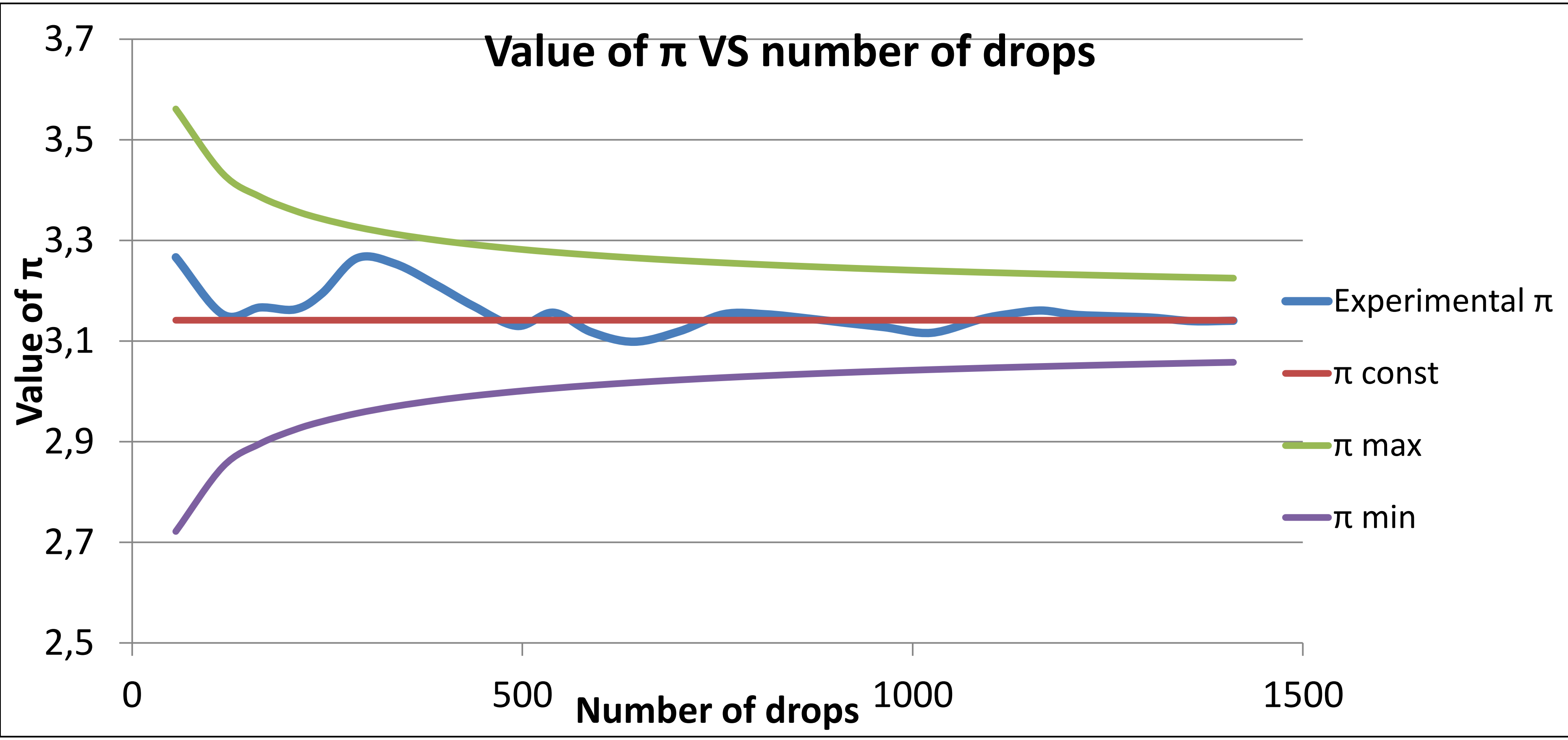
Conditions of experiment:
Distance between the lines – 8,4 cm
Length of the match– 4,2 см
Final value of π – **3,14978292**

Constant π = **3,14159265358979**
Difference between constant π and experimental π:
0,00819

Length of the match is longer than distance between the lines

Number of drops: 1411

Length of the match is longer than distance between the lines



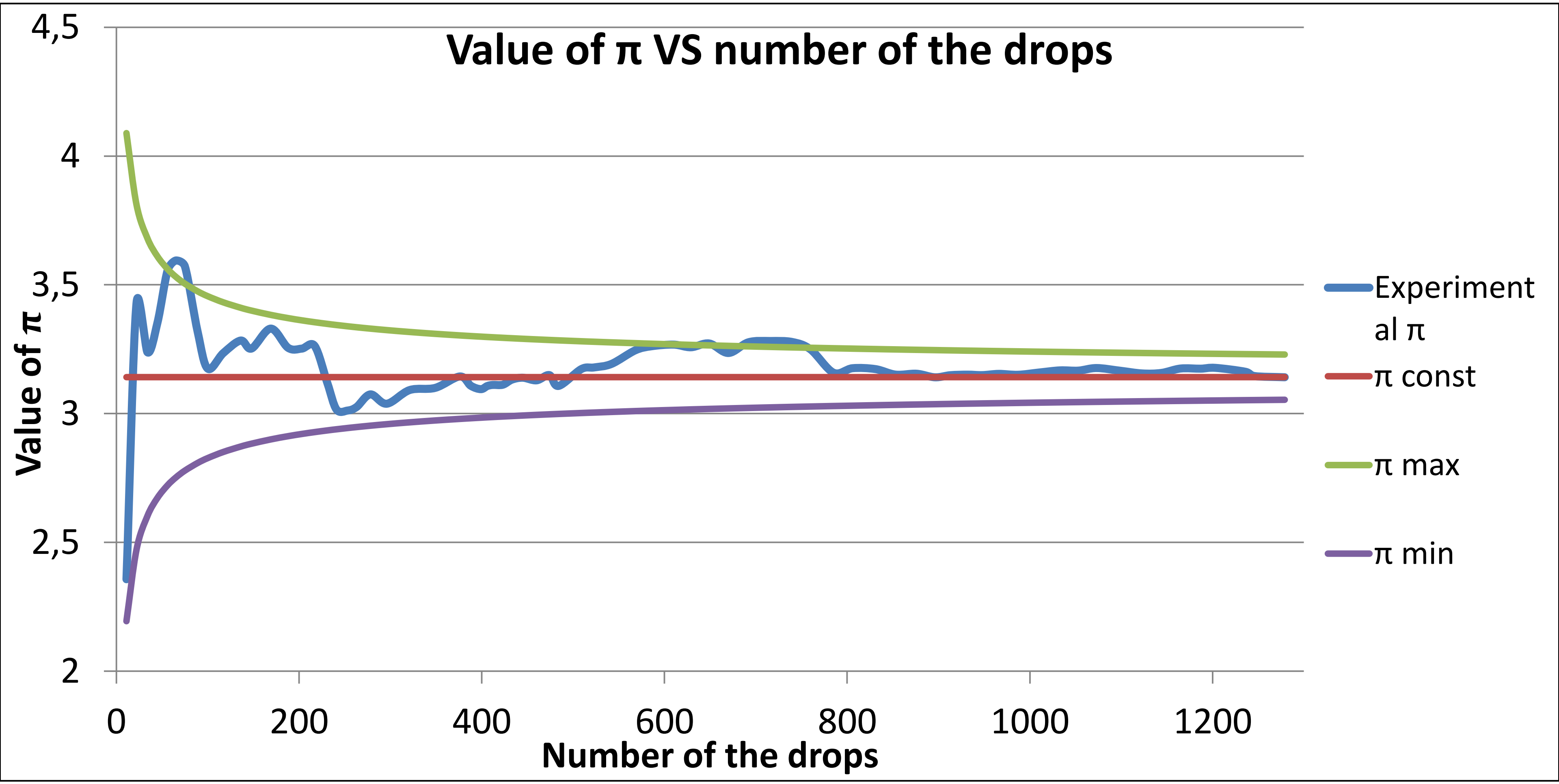
Conditions of experiment:
Distance between the lines– 3 см
Length of the match– 4,2 см
Final value of π – **3,1405405405**

Constant π = **3,14159265358979**
Difference between constant π and experimental π:
0,001052

Spinning surface

Number of the drops: 1279

Spinning surface



Conditions of experiment:
Distance between the lines – 4,9 см
Length of the match – 4,2 см
Final value of π– 3,141219812

Constant π = **3,14159265358979**
Difference between constant π and experimental π:
0,00037284

Conclusion

1. Two factors determining the location of the needle are:
   - **Distance h** from the center of the needle to the line
   - **Angle α** between the needle and the line
2. Condition of intersection: $\boldsymbol{h \leq \frac{1}{2} l \sin \alpha}$
3. We can calculate π from numbers of drops and intersections using the following formula:

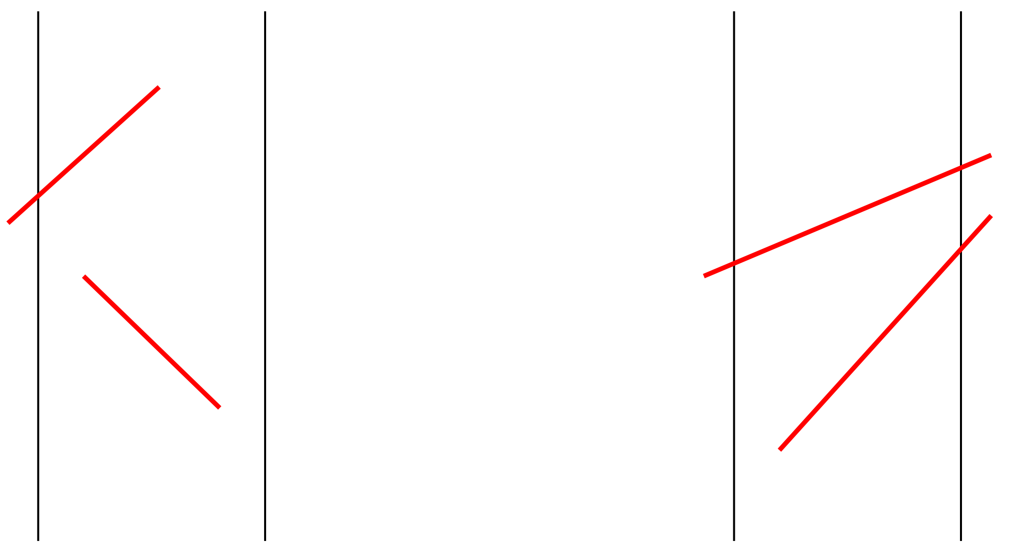
$$\pi = \frac{2l \times n\, of\ all\ drops}{x \times n\, of\ intersections}$$

4. Value of experimental π becomes closer to constant π when number of drops increases. The least difference between experimental π and constant π equals **0,00037284**

Thank you for attention!

How result depends on the characteristics of the needle

When length of the needle increases, but distance between the lines stays constant, number of the intersections increases too.



We would obtain the bigger number of the intersections after equal number of the drops.

$$p = \frac{Events\ of\ intersections}{All\ events}$$

All events of the drops:

Rectangle's area =

$$\frac{\boldsymbol{x}}{\mathbf{2}} \times \frac{\boldsymbol{\pi}}{\mathbf{2}} = \frac{\boldsymbol{x\pi}}{\mathbf{4}}$$

Events of intersections:

Area under the sinusoid

$$p = \frac{4\int_0^{\pi/2} \frac{l}{2} \sin\alpha\, d\alpha}{x\pi} = \frac{2l\int_0^{\pi/2} \sin\alpha\ d\alpha}{x\pi} = \frac{-2l\left(\cos\alpha \Big|_0^{\pi/2}\right)}{\pi x} =$$

$$= \frac{\mathbf{2}\boldsymbol{l}}{\boldsymbol{\pi x}}$$

Where *p* – probability of intersection;

l – length of the needle,

x – distance between the lines