Team of Brazil

Problem 07
Drawing pins

reporter:

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Problem 07
Drawing pins

A drawing pin (thumbtack) floating on the surface of water near another floating object is subject to an attractive force. Investigate and explain the phenomenon. Is it possible to achieve a repulsive force by a similar mechanism?

Contents

Theoretical introduction

- Surface tension
- Viscosity
- Capillarity
- Horizontal Forces
  - *Interaction energy*
- Attraction and repulsion
- Dimensionless parameters
- Time of approach

Experiments

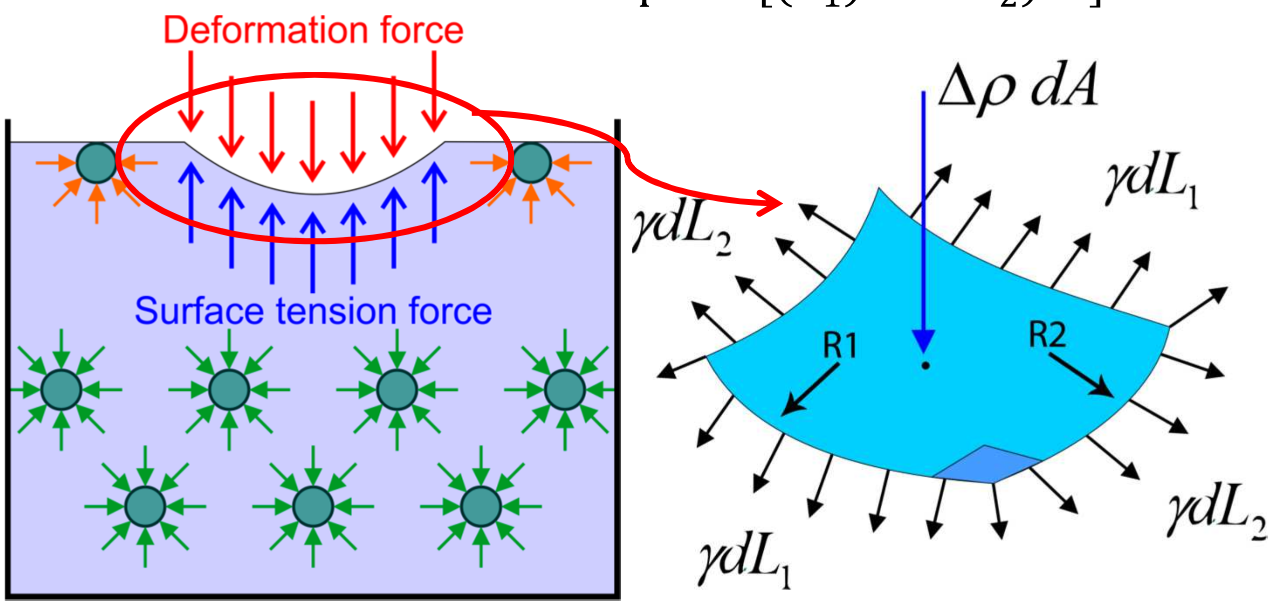
- Tests
- Comparison between theory and experiments

Conclusion

Surface tension

Definition:	*Is a property of fluids caused by intermolecular forces, and tends to minimize the surface area to minimize his energy.*

$$\Delta p = \Upsilon[(R_1)^{-1} + R_2)^{-1}]$$

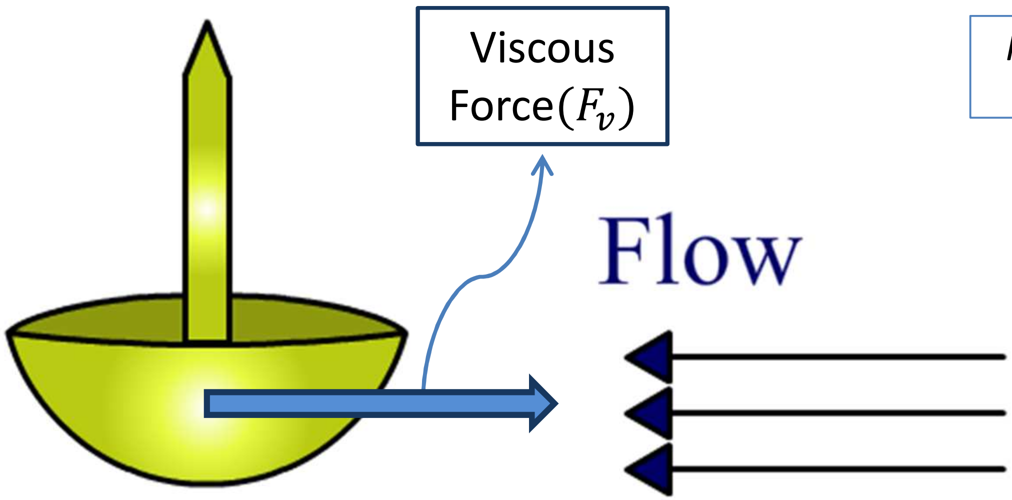


Surface tension table

Liquid	Temperature °C	Surface tension, γ
Acetic acid	20	27.6
Acetic acid (40.1%) + Water	30	40.68
Acetone	20	23.7
Ethanol	20	22.27
Glycerol	20	63
Isopropanol	20	21.7
Mercury	15	487
Methanol	20	22.6
n-Octane	20	21.8
Water	25	71.97

Drag force

We have a drag force caused by the viscosity of the fluid.



$$F_v = 6\pi R \mu \alpha \frac{dL}{dt}$$

- R: *Radius of the shape*
- μ: *Absolut viscosity (Pa.s)*
- α: *Adjustment parameter*
- $\frac{dL}{dt}$: *Velocity*

Capillarity

The interaction between contacting surfaces of a liquid and a solid that distorts the liquid surface from a planar shape.

Considering water as the fluid, we can classify the particles in:

Hydrophilic particle

Hydrophobic particle

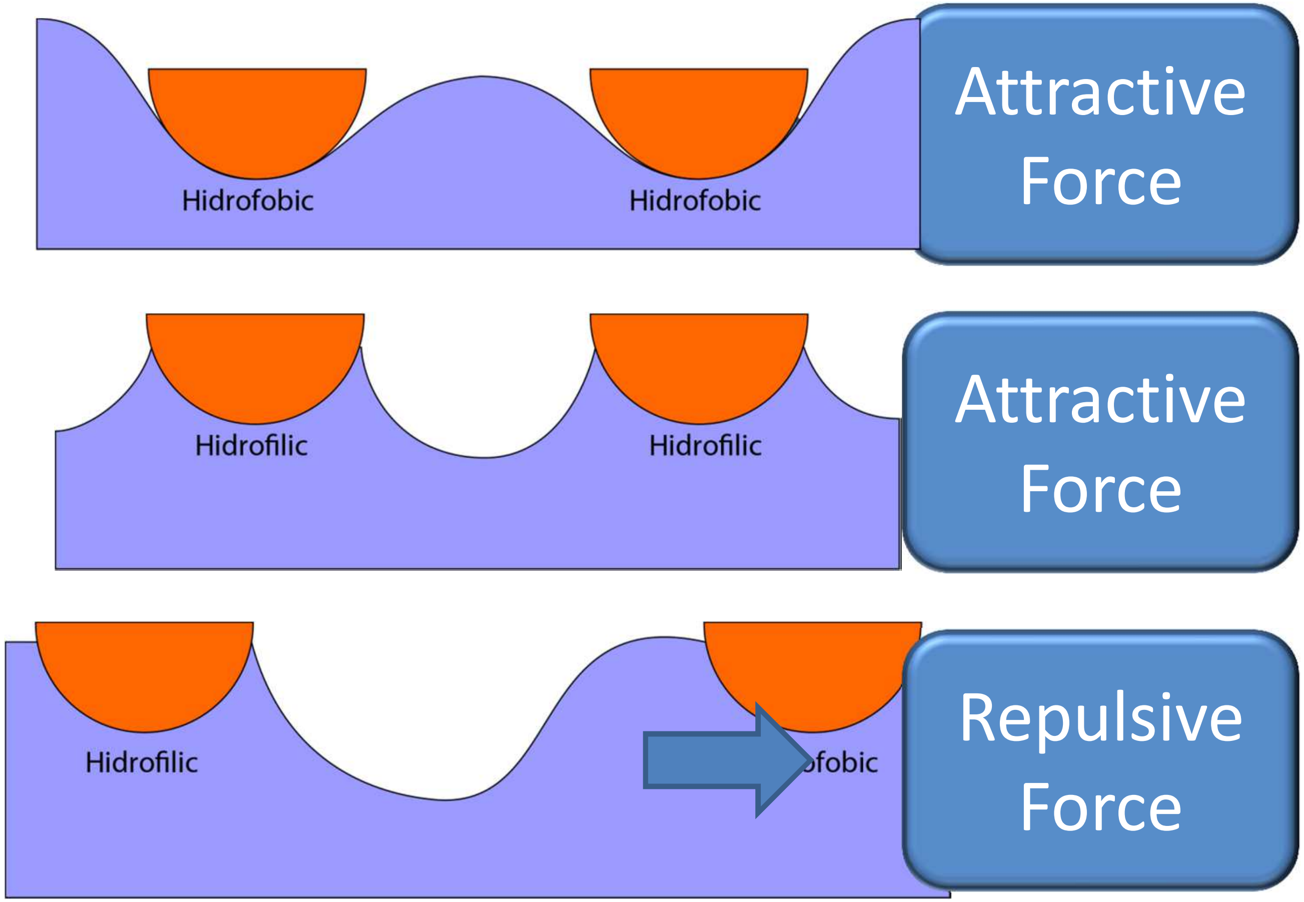
Hydrophilic: attract the w

Hydrophobic: repels the

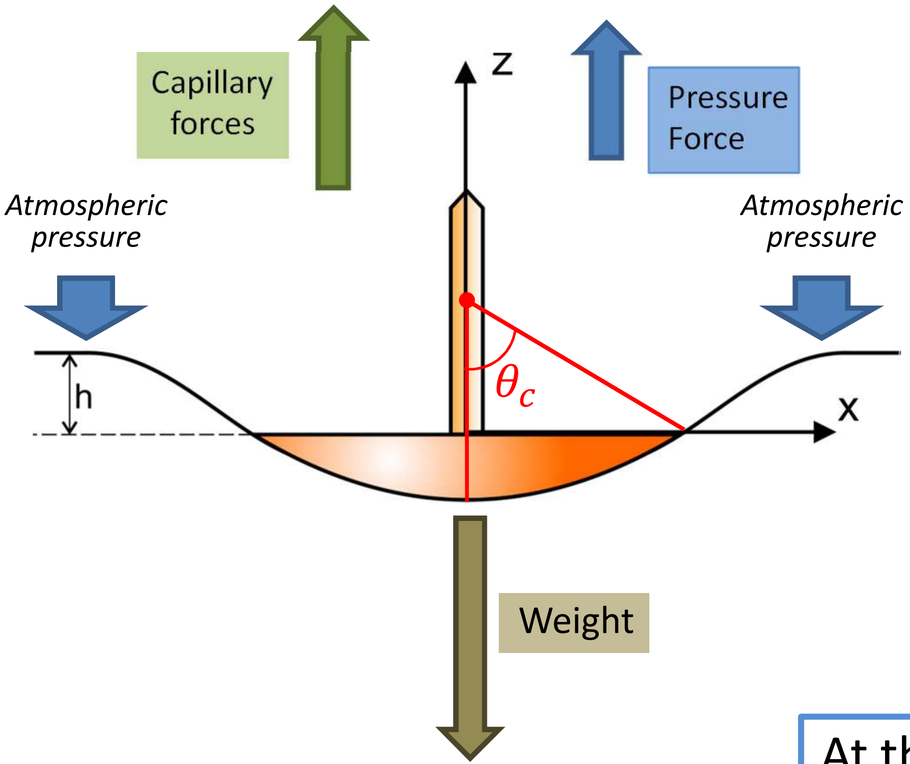
Hydrophilic particle

Hydrophobic particle

Meniscus – kinds of forces



Vertical forces



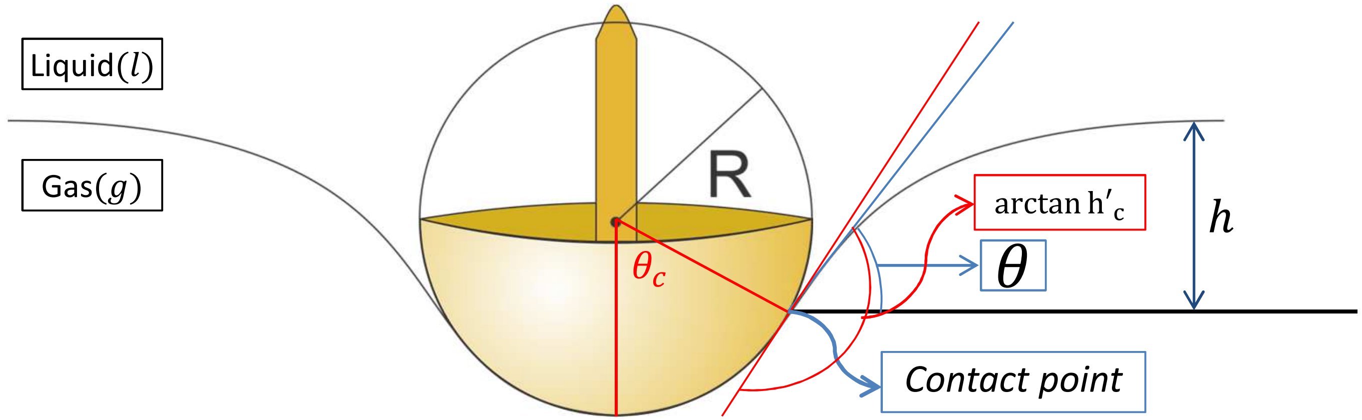
$$\begin{cases} F_c = 2\pi(R\sin\theta_c)\gamma\sin[\theta_c - (\pi - \alpha)] \\ F_p = \int_0^{\theta_c} p\cos\theta(2\pi R\sin\theta)R\,d\theta \end{cases}$$

$$\boxed{F_p = \rho_l g V_w - \Delta\rho h_2 A}$$

$$\begin{cases} V_w = \pi R^3\left(\frac{2}{3} - \cos\theta_c + \frac{1}{3}\cos^3\theta_c\right) \\ A = 2\pi R^2(1 - \cos\theta_c) \end{cases}$$

At the equilibrium $\Rightarrow F_c + F_p = Mg$

Horizontal forces



The interaction energy is given by the Nicolson's superposition approximation:

Bound number

$$\begin{cases} E_i = -2\pi\gamma B_o^2 \zeta_1 \zeta_2 \left[K_0\left(\frac{L}{L_c}\right)\right] \\ F_r(L) = \frac{dE}{dL} \end{cases}$$

Bessel function

$h \sin\theta$

Capillary length: $L_c = \sqrt{\frac{\gamma}{\Delta\rho g}}$

$$F_r(L) = -2\pi\gamma R \sqrt{B}(\beta)^2 \left[K_1\left(\frac{L}{L_c}\right)\right]$$

Dimensionless parameters

Bound number → $B_o \cong \frac{R^2}{L_c{}^2}$

Radius of the semi sphere

$D = \frac{\rho_s}{\rho_l}$

$$\zeta_k = \frac{2D - 1}{3} - \frac{1}{2} cos\theta_c + \frac{1}{6}\cos^3\theta_c \cong \frac{h \sin\theta_c}{B_o}$$

$k \in \{1,2\}$

Contact angle

Attraction and repulsion

Attraction cases → $F_r < 0$

- $sin\,\theta_1 < 0$ + $sin\,\theta_2 < 0$

Or

- $sin\,\theta_1 > 0$ + $sin\,\theta_2 > 0$

Attraction and repulsion

Repulsion cases → $F_r > 0$

$\sin\theta_1 < 0$ + $\sin\theta_2 > 0$

Or

$\sin\theta_1 > 0$ + $\sin\theta_2 < 0$

Space function for small L

For small values of L, the Bessel function can reduced to:

$$K_1(L \ll 1) \approx \frac{1}{L} \Rightarrow F_r(L) = \frac{-2\pi\gamma B_o^{\frac{5}{2}}\zeta_1\zeta_2 L_c}{L}$$

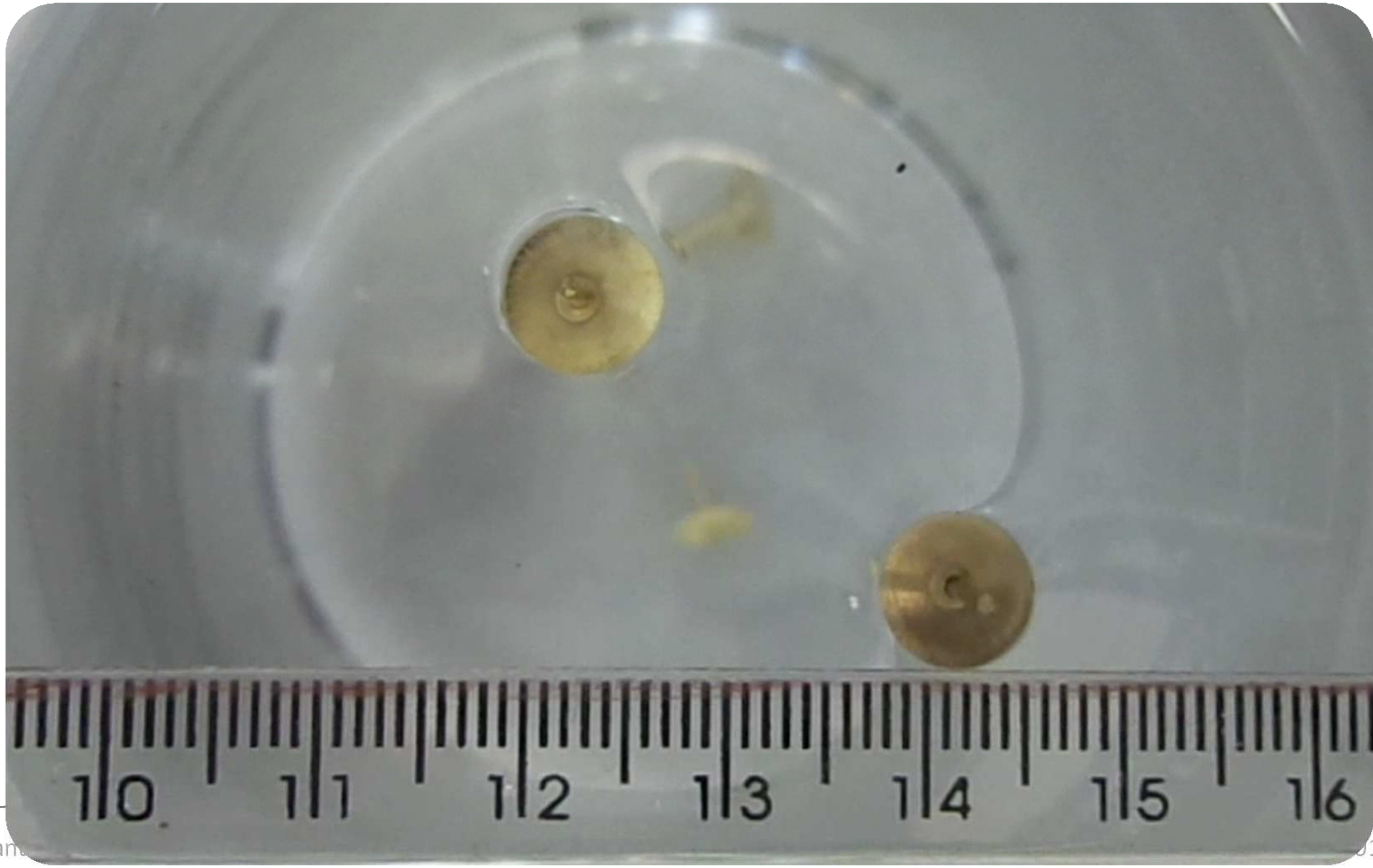
Using Torricelli's equation: $\int_0^v v\,dv = \int_0^L a\,dL$

We get:

$$L(t) = \sqrt{L(0)^2 - \frac{2\gamma L_c B_o^{\frac{5}{2}}\zeta_1\zeta_2}{3\mu a m}t}$$

Experiment 1: Two pins in water - attraction

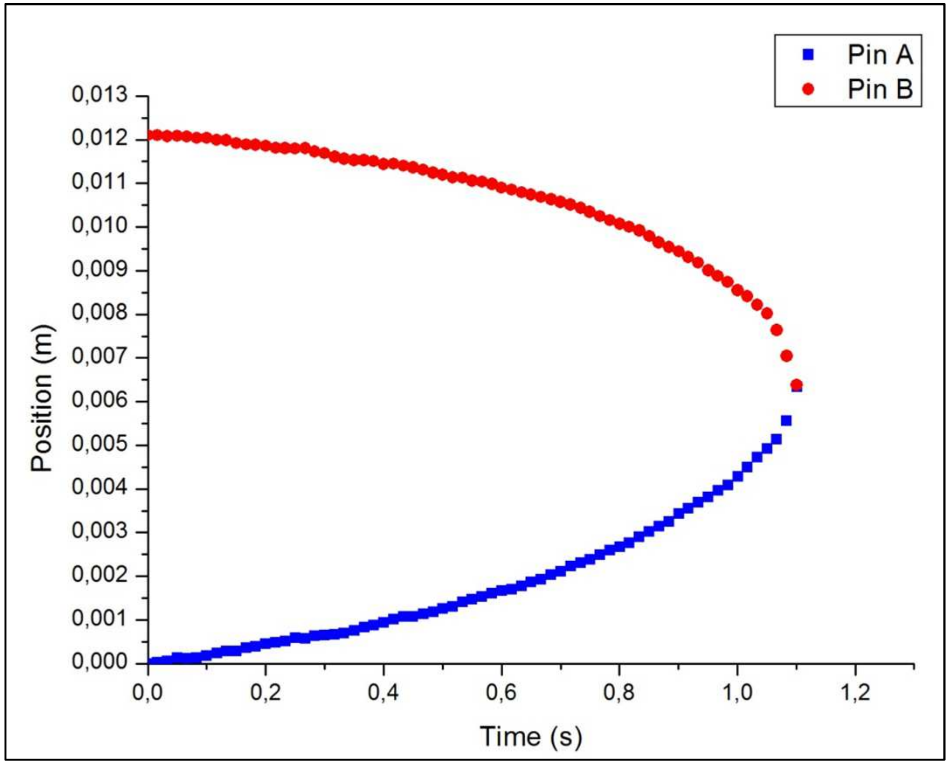
1
2
3
4
5



Experiment 1: Two pins in water - attraction

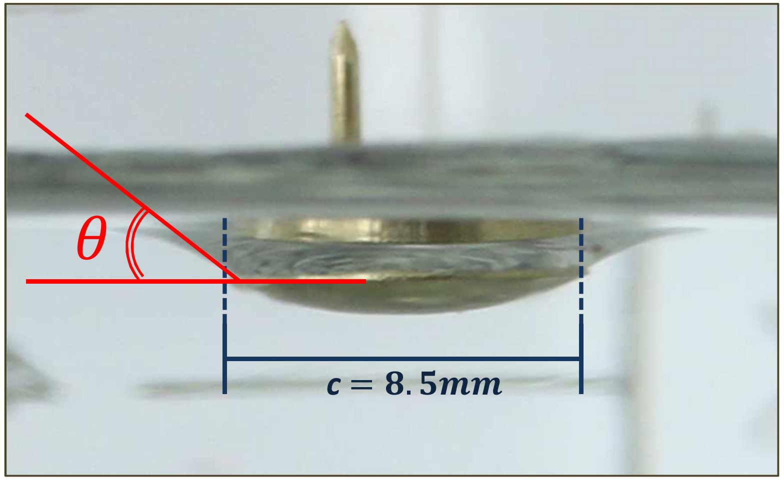
1
2
3
4
5

Position in function of time



Experiment 1: Two pins in water

1
2
3
4
5



$\boldsymbol{m} = 0.0003kg$
$\boldsymbol{\gamma_{water}} = 0.0728\ {}^{N}/_{m}$
$\boldsymbol{\theta} = 38°$

Temperature: 20℃

Radius: *24.62 mm*

h: *0.00103 m*

Source of errors

Rule and conversion: $\pm\ 0.5mm$

$$\beta = h \sin\theta$$

$$\beta = 0.00616$$

$$B_o = \frac{R^2}{{L_c}^2}$$

$$B_o \cong 74$$

Experiment 2: Water with surfactant

1
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4
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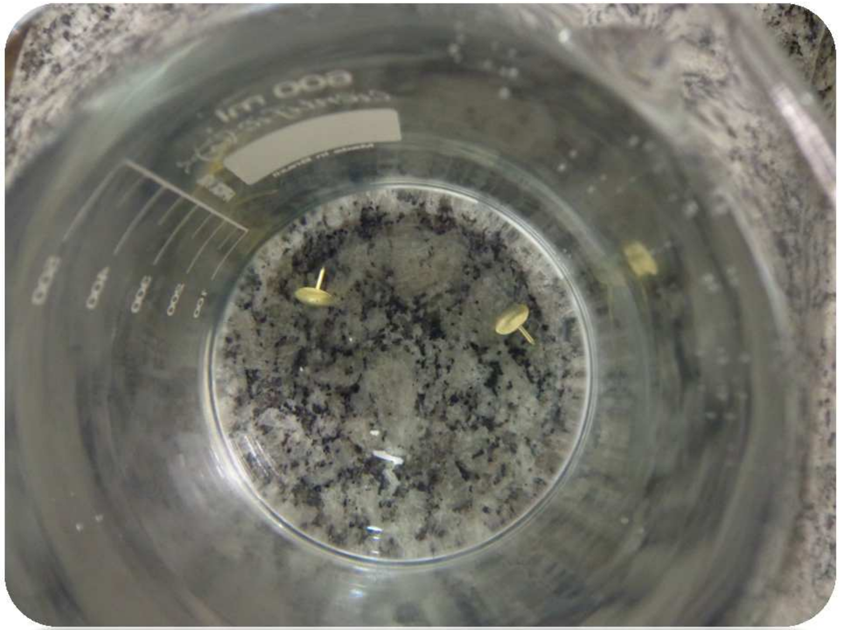

The phenomena does not occur, because the surfactant break the surface tension, and the force became 0, according to the theory.

$$F_{\gamma}(L) = \frac{-2\pi\gamma B_o^{\frac{5}{2}}\zeta^2 L_c}{L}$$

$$\gamma \to 0$$

$$F \to 0$$

Experiment 3: distance vs. time of approach

1
2
3
4
5

Distance (cm)	Time of approach(s)
1.09	0.82
1.95	9.81
1.36	2.17
1.54	2.95
1.37	1.33
0.76	0.92

There's no pattern.

Source of errors:

- Measure of the distance and conversion

- **Imprecisions:**
- **Time:** $\pm 0.05s$
- **Conversion and measurement:** $\pm 0.5mm$

Experiment 4:Repulsion

The phenomenon

1
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5

Experiment 4:Repulsion

1
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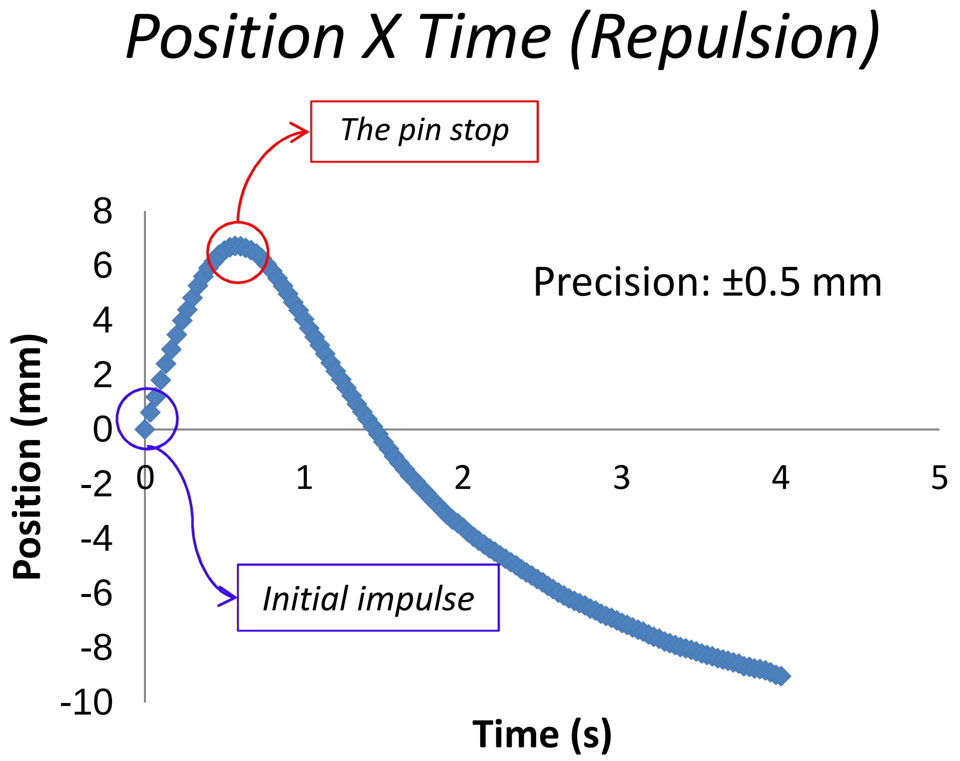
3
4
5



Experiment 5: Two pins in glycerin

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4
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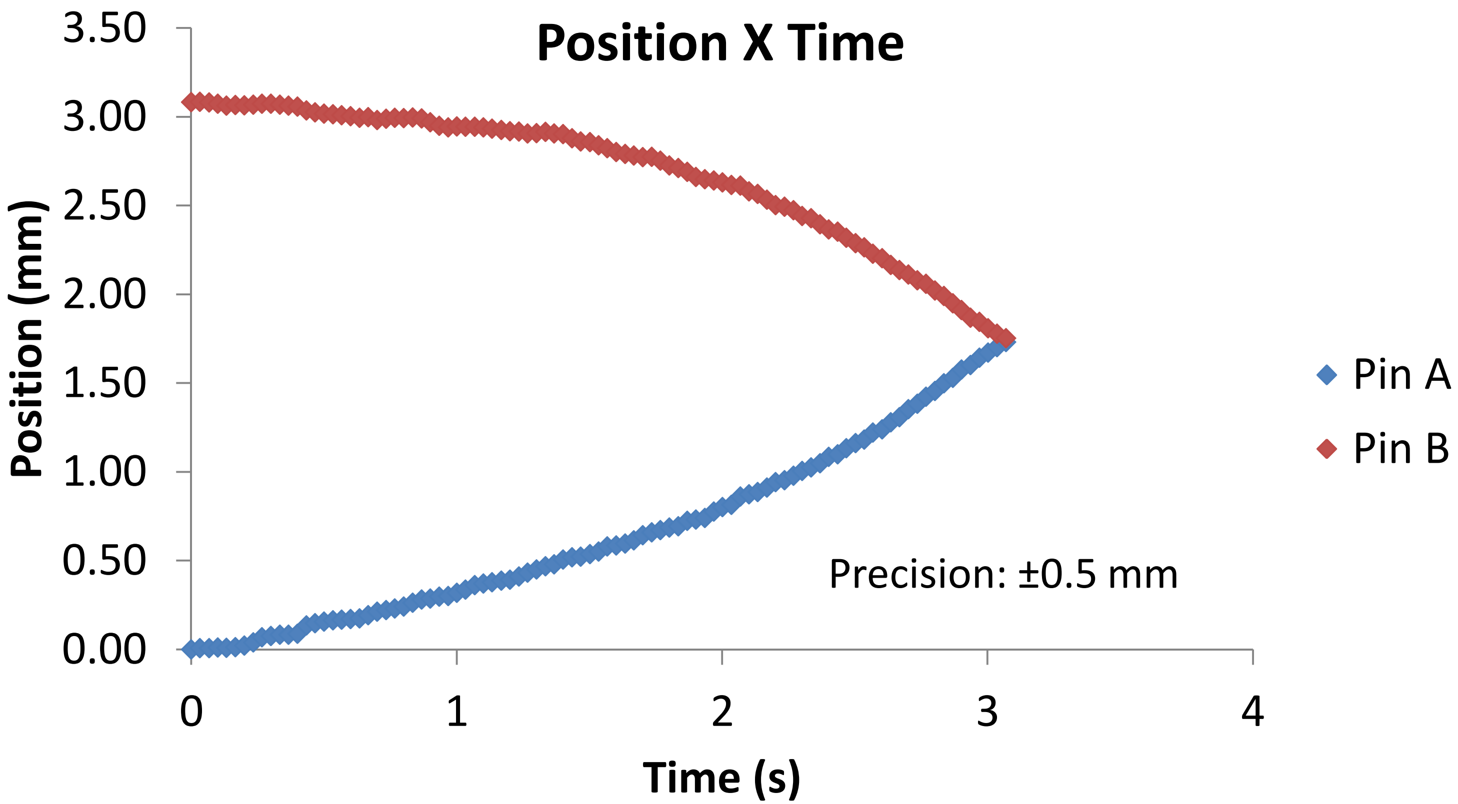

Conclusion

- The movement
- Repulsion
- Viscosity
- Time of approach

References

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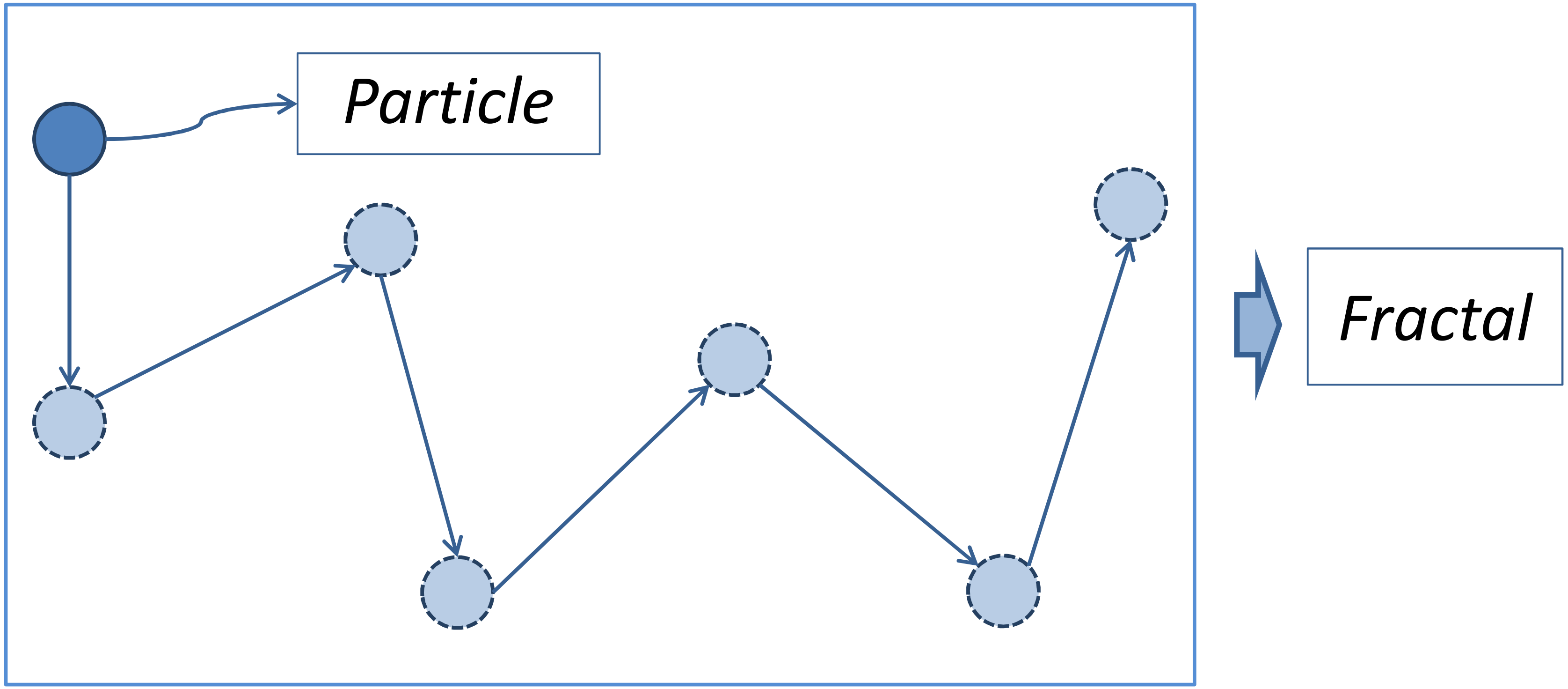
Thank you!

Appendix A

- Brownian motion



Bessel function

- There's no analytical solution for the equation in no asymptotic cases.

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - (x - \boxed{\alpha})$$

Order

Software used to data collect and analysis

- Tracker Physics Video analysis V 4.62
- Origin Pro 7.0
- Corel Draw X5