Team of Brazil

Problem 09
Magnet and coin

reporter:

Bárbara Cruvinel Santiago

Problem 09
Magnet and coin

Place a coin vertically on a magnet. Incline the coin relative to the magnet and then release it. The coin may fall down onto the magnet or revert to its vertical position. Study and explain the coin's motion.

Contents

Introduction
- Threshold angle
- Rotational motion
- Oscillations

Experiments
- Experimental data
- Theory validation

Conclusion
- Analysis
- Comparison

Magnetization of the coin

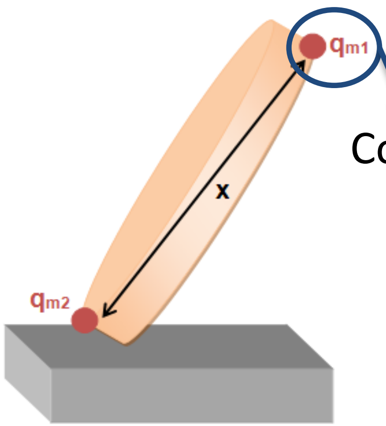
- Magnetization is the main cause of the phenomenon.

↓

- Difficult to predict mathematically for ferromagnetic material.

↓

- Equivalent to “magnetic charges” for simplification.

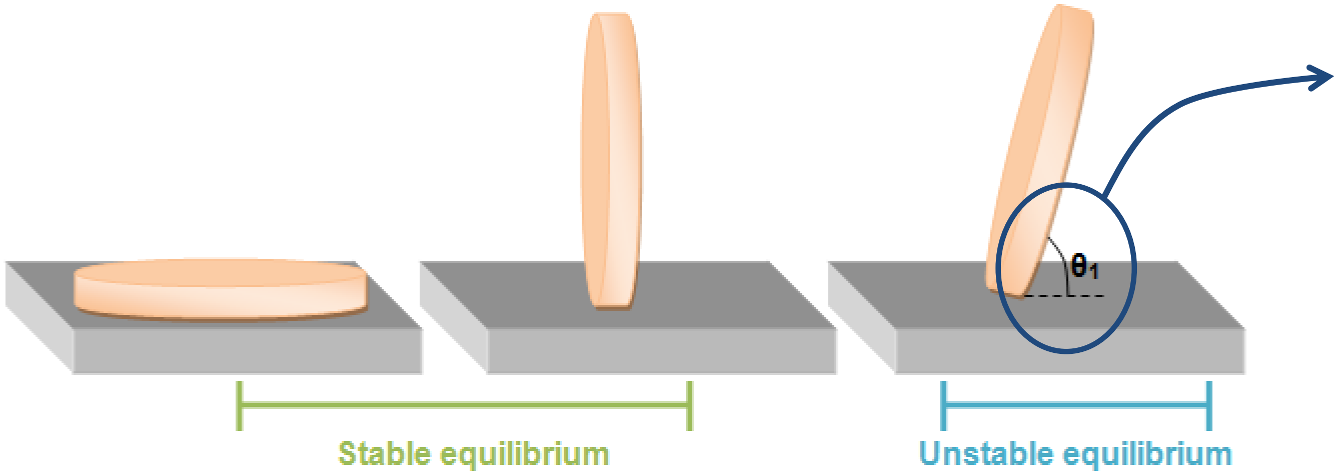


Considered constant as just one position is taken.

$$p = q_m \cdot x$$

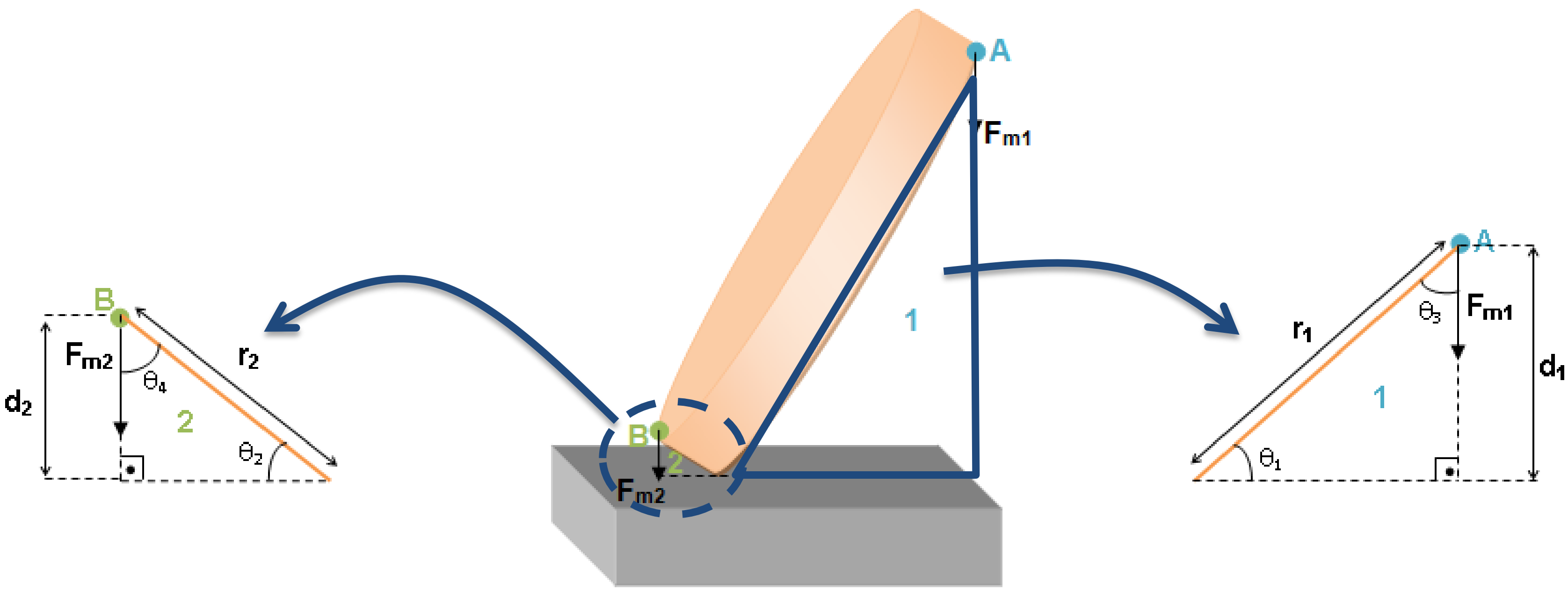
Acts as kind of an average for the magnetic momentum

Threshold angle

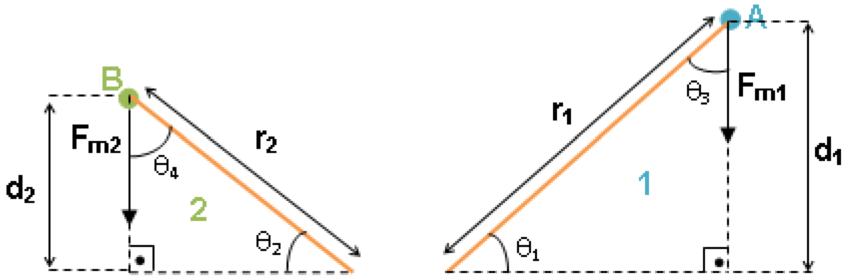


Threshold angle is defined

Threshold angle



Threshold angle



$$F_m = \frac{\mu \cdot q_{m1} \cdot q_{m2}}{4\pi d^2} = \frac{c}{d^2}$$

The regions taken must be small enough to be considered **single points**, as "point magnetic charges".

$$d_1 = r_1 \cdot sin\theta_1$$

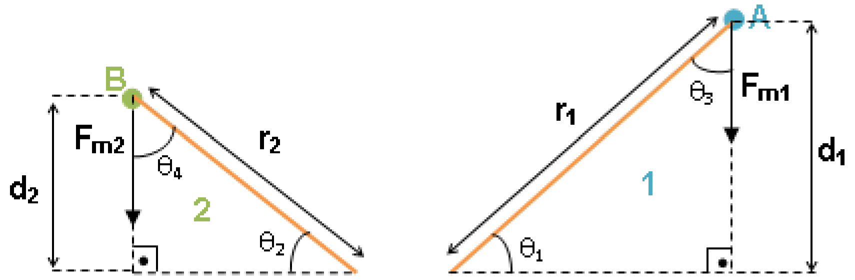
$$d_2 = r_2 \cdot sin\theta_2$$

$$\theta_1 + \theta_2 = 90°$$

$$sin\theta_1 = cos\theta_2$$

$$sin\theta_2 = cos\theta_1$$

Threshold angle



- In the equilibrium:

$$\tau_1 = \tau_2$$

$$F_{m1} \cdot r_1 \cdot sin\theta_3 = F_{m2} \cdot r_2 \cdot sin\theta_4$$

$$\frac{c}{(r_1 \cdot sin\theta_1)^2} \cdot r_1 \cdot cos\theta_1 = \frac{c}{(r_2 \cdot sin\theta_2)^2} \cdot r_2 \cdot cos\theta_2$$

Threshold angle:

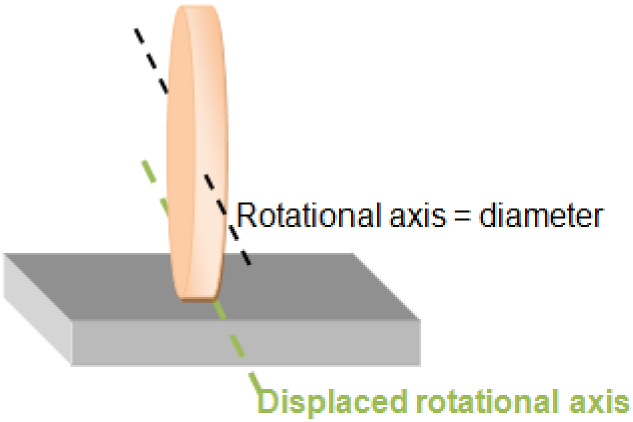
$$\tan^3\theta_1 = \frac{r_2}{r_1}$$

Threshold angle

$\tan^3\theta_1 < r_2/r_1$	• The coin falls over the magnet.
$\tan^3\theta_1 = r_2/r_1$	• The coin is in instable equilibrium.
$\tan^3\theta_1 > r_2/r_1$	• The coin comes back to the original position.

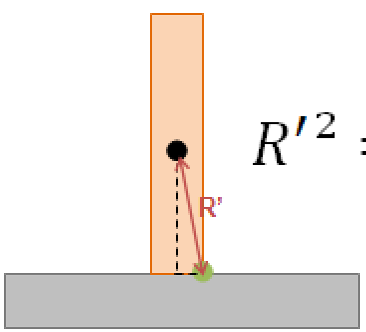
- Magnetic properties:
  - Condition for effect occurrence.
  - Have no influence in the threshold angle determination.
- Weight: can be neglected as it is going to be showed.

Rotational motion



$$I = \frac{mR^2}{4} + \frac{mL^2}{12} + mR'^2$$

$$I = \frac{m}{4}\left(\frac{r_1}{2}\right)^2 + \frac{m}{12}\left(\frac{r_2}{2}\right)^2 + m\left(\frac{r_1^2 + r_2^2}{4}\right)$$



$$R'^2 = \left(\frac{r_1}{2}\right)^2 + \left(\frac{r_2}{2}\right)^2$$

Moment of inertia:

$$\boldsymbol{I = \frac{m(15r_1^2 + 13r_2^2)}{48}}$$

Rotational motion

$$\tau = I \cdot \alpha$$

- Angular acceleration:

$$\boldsymbol{\alpha = \frac{48 \cdot c \cdot cos\theta_1}{m \cdot r_1(15r_1^2 + 13r_2^2) \cdot sin^2\theta_1}}$$

→ Dependent on the inclination angle and on the magnetic properties.

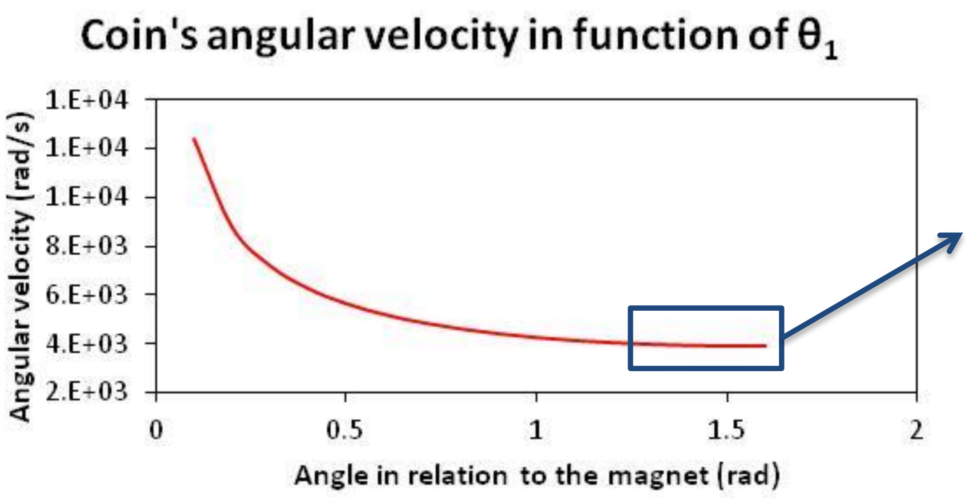
$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta_1} \cdot \frac{d\theta_1}{dt} = \frac{d\omega}{d\theta_1} \cdot \omega$$

$$\int_{-\theta}^{\frac{\pi}{2}} \alpha \cdot d\theta_1 = \int_0^{\omega} \omega \cdot d\omega$$

$$\frac{\omega^2}{2} = \int_{-\theta}^{\frac{\pi}{2}} \alpha \cdot d\theta_1$$

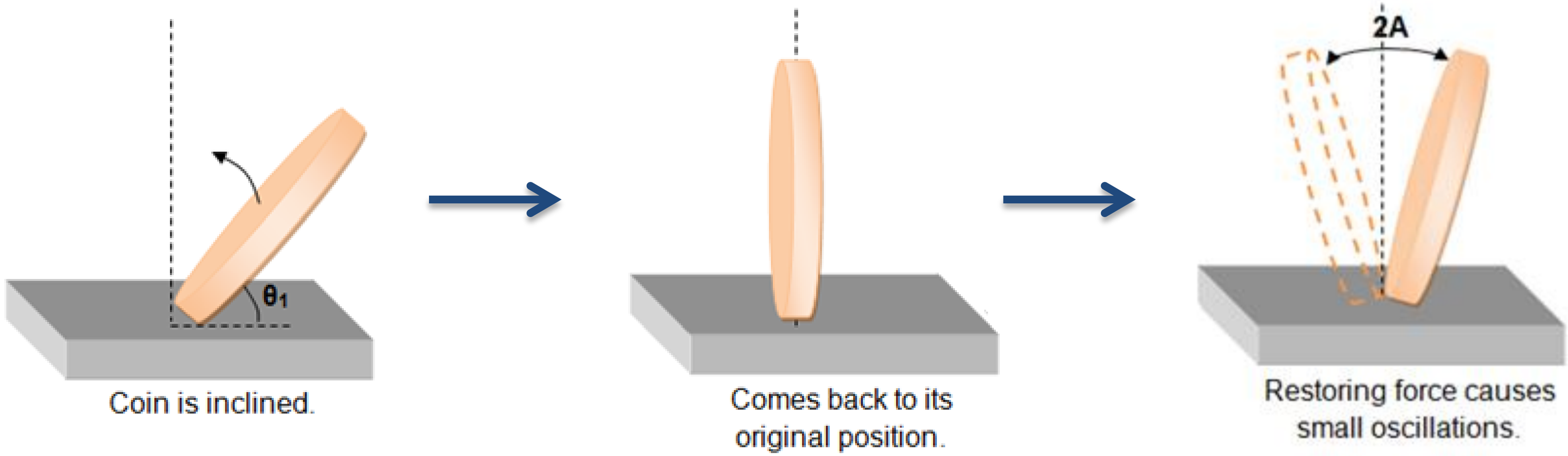
Rotational motion

$$\omega = \sqrt{-2 \cdot \frac{48 \cdot c}{m \cdot r_1 (15 r_1^2 + 13 r_2^2) \cdot sin\theta_1}}$$



Angular velocity approximately constant near the normal line to the magnet.

Oscillations



Angular velocity approached to constant + Amplitude that decreases with time + A < 15° = **Under-damped Harmonic Oscillation**

Oscillations

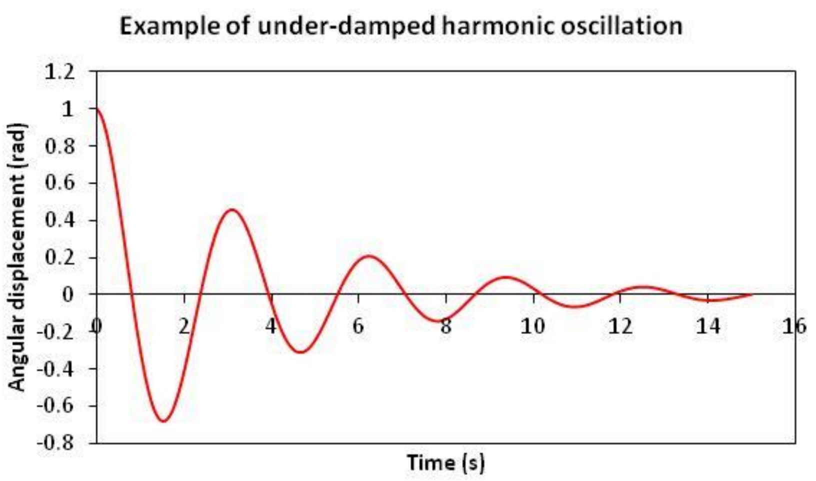
- Dissipative force of type –bv.

$$m\frac{d^2x}{dt} + b\frac{dx}{dt} + kx = 0$$

$$\omega_0^2 = \frac{k}{m} \qquad \gamma = \frac{b}{m}$$

$$\omega = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2}$$

$$x = Ae^{-(\gamma/2)t} \cdot \cos(\omega t + \varphi)$$



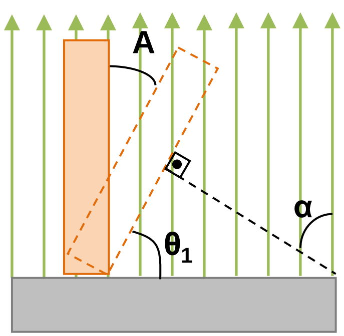
However, magnetic force does not vary linearly and so does the damping. Hence, the period varies. So, that's just a first approximation!

Eddy currents: source of dissipative force

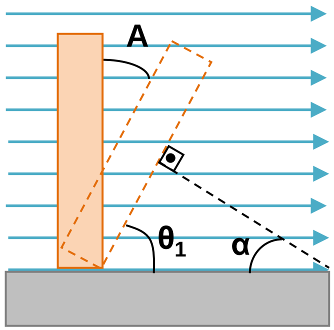
Variation of magnetic flux → **Induction of electrical currents** → **Dissipative force generated**

$$\Phi = BS \cdot \cos\alpha$$

A≈5°



1st case: on the poles



2nd case: on the sides

Case of our experiments

1st case:

$$\theta_1 = \alpha$$

$$\Phi = BS \cdot cos\theta_1$$

$$\Delta\Phi_1 = BS \cdot \Delta cos\theta_1$$

$$\boldsymbol{\Delta\Phi_1 = 0.087 \cdot BS}$$

2nd case:

$$\theta_1 = 90^\circ - \alpha$$

$$\Phi = BS \cdot sin\theta_1$$

$$\Delta\Phi_2 = BS \cdot \Delta sin\theta_1$$

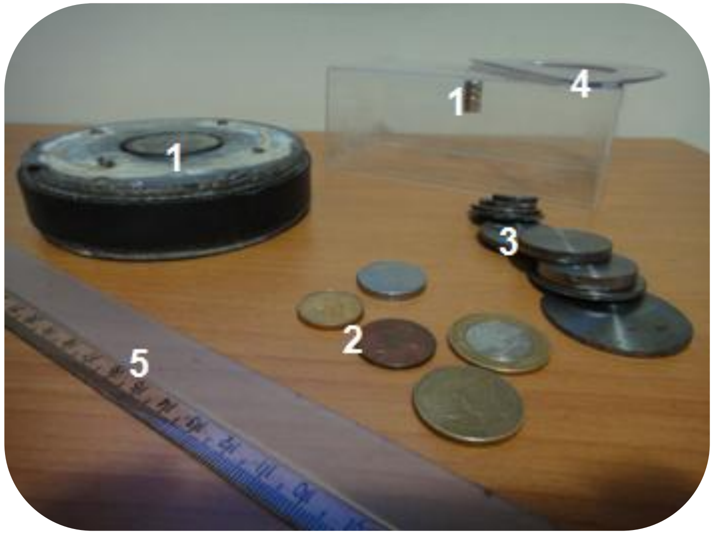
$$\boldsymbol{\Delta\Phi_2 = -0.004 \cdot BS}$$

In our case, the eddy currents have their maximum influence in relation to the position on the magnet.

Material

1. Magnets
2. Real coins
3. Steel coins
4. Protractor (±0,5°)
5. Acrylic ruler

- Iron filings
- White paper
- Dynamometer (±0.05N)
- Support for the dynamometer
- Camera



Experimental description

- **Experiment 1:** show that the weight is negligible in comparison to the magnetic force.
- **Experiment 2:** threshold angle analysis.
- **Experiment 3:** magnet analysis.
- **Experiment 4:** oscillation analysis.

Experiment 1: weight analysis

- **Weight**
- Threshold angle
- Magnet
- Oscillations

	Force (N)
1st	7.9
2nd	7.7
3rd	7.6
4th	7.8
5th	7.6
Average	**7.72**
Standard Deviation	0.13

Experiment 1: weight analysis

Weight
Threshold angle
Magnet
Oscillations

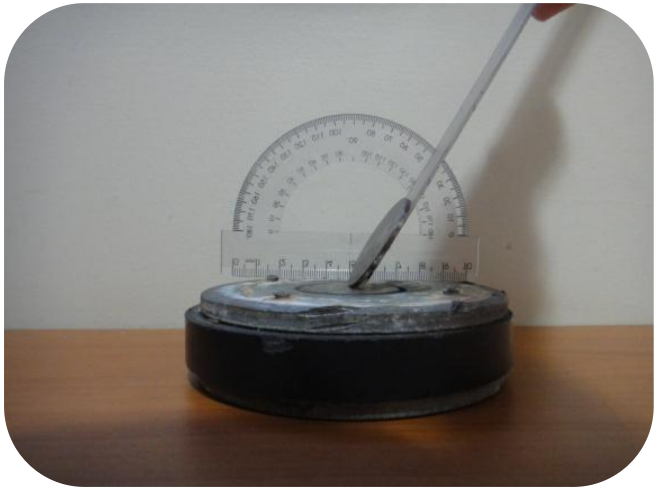
Total force (N)		7.72
Weight (N)		0.04
Magnetic force (N)		7.68
The weight represents	**0.52%**	**of the magnetic force**

The weight can actually be neglected.

Experiment 2: threshold angle analysis

- Weight
- Threshold angle
- Magnet
- Oscillations

- 8 coins are used.
- They are inclined 1 by 1 degree.
- This is repeated 10 times for each coin.
- The points are averages.



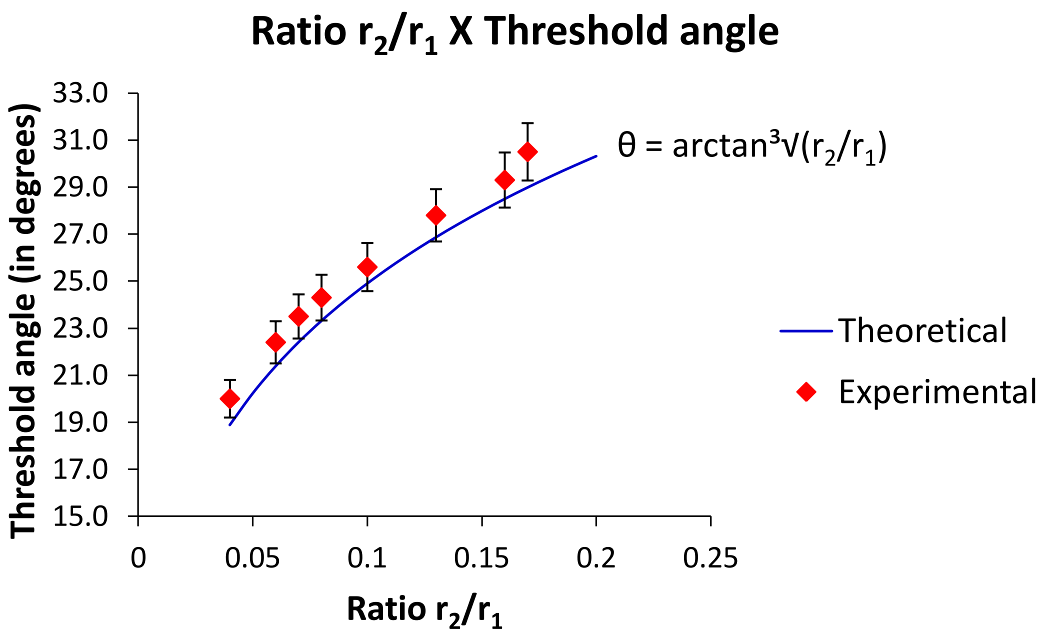
Experiment 2: threshold angle analysis

Weight

Threshold angle

Magnet

Oscillations



Average deviation: 4%

Precision of the protractor: ±0,5°

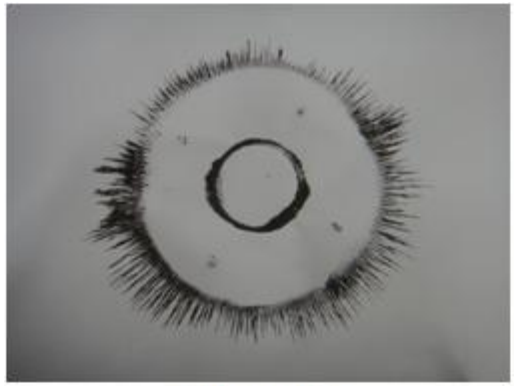
Experiment 3: different magnets

Weight

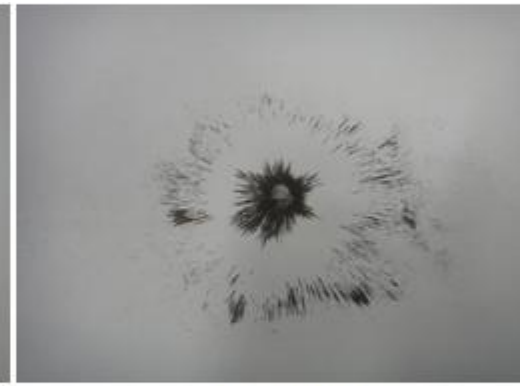
Threshold angle

Magnet

Oscillations



Magnet 1



Magnet 2

Magnet 1

Magnet 2

Experiment 3: different magnets

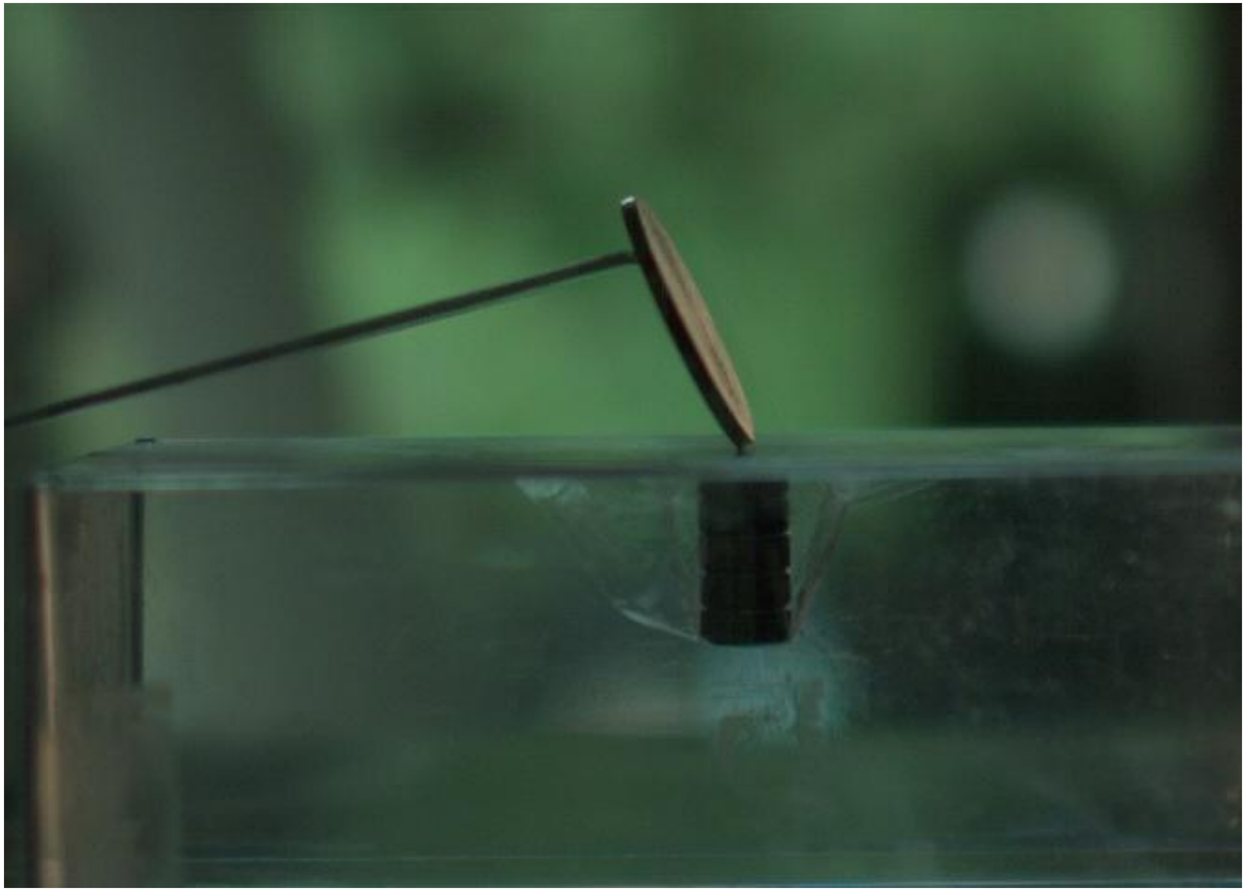
- Weight
- Threshold angle
- Magnet
- Oscillations

Threshold angle in degrees		
	Magnet 1	Magnet 2
1st	26	30
2nd	25	26
3rd	24	27
4th	27	26
5th	26	28
6th	25	27
7th	25	25
8th	25	24
9th	26	26
10th	27	24
Average	**25.6**	**26.3**
Standard deviation	**1.0**	**1.8**

The angles are the same considering the standard deviation.

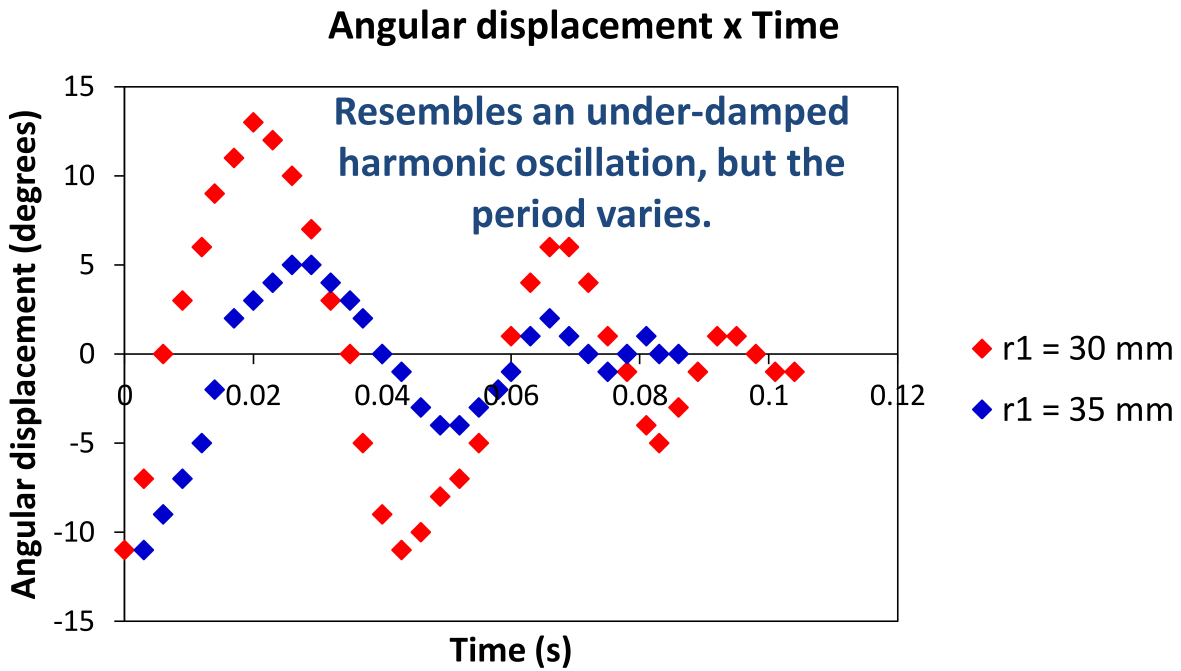
Experiment 4: oscillation analysis

- Weight
- Threshold angle
- Magnet
- Oscillations



Experiment 4: oscillation analysis

- Weight
- Threshold angle
- Magnet
- Oscillations



Coin with larger area → Greater damping → Evidence of eddy currents

Conclusion

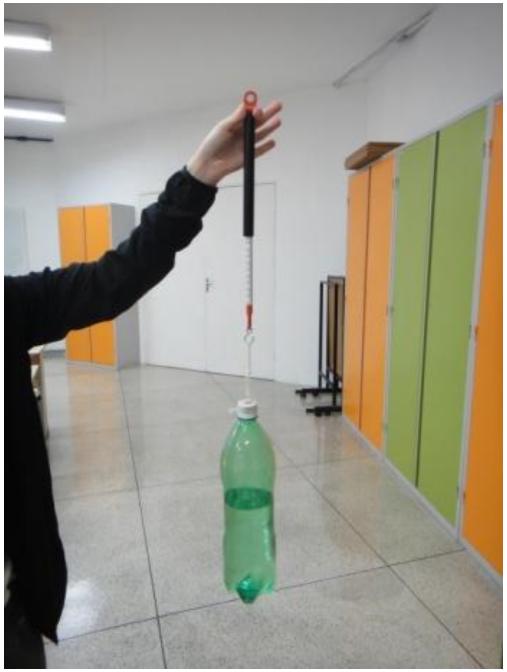
- Magnetic properties are important for the motion itself and are the initial condition for the effect occurrence.
- Threshold angle is dependent only on the coins dimensions (exp. 2/3).
- The fall or reverting motion is due to the torque of the magnetic force.
- Oscillations resembles an under-damped harmonic oscillations but the period varies (exp. 4).
- Evidences of dissipation by eddy currents (exp. 4).

References

- 1. http://www.bcb.gov.br/?MOEDAFAM2
- 2. http://www.infomet.com.br/acos-e-ligas-conteudo-ler.php?cod_tema=9&cod_secao=10&cod_assunto=81&cod_conteudo=123
- 3. http://www.usmint.gov/mint_programs/circulatingCoins/index.cfm?action=CircPenny
- 4. http://geophysics.ou.edu/solid_earth/notes/mag_basic/mag_basic.html
- 5. PRÄSS, Prof. Alberto Ricardo. Oscilações: Movimento Harmônico Simples – MHS. Física.net

Thank you!

Dynamometer calibration



- The dynamometer was calibrated with PET bottle full of water, with exact 1 kg, measured with a scale of precision of 0.1 g. The maximum mark of the dynamometer is 10N.

Real coin's table

Technical characteristics of the coin				
Value (R$)	**Diameter (mm)**	**Thickness (mm)**	**Mass (g)**	**Material**
0.01	17	1.65	2.43	Steel covered with copper
0.05	22	1.65	4.10	Steel covered with copper
0.10	20	2.23	4.80	Steel covered with bronze
0.25	25	2.25	7.55	Steel covered with bronze
0.50	23	2.85	6.80	Stainless steel
1.00	27	1.95	7.00	Stainless steel (core) a steel covered with bronze (ring)

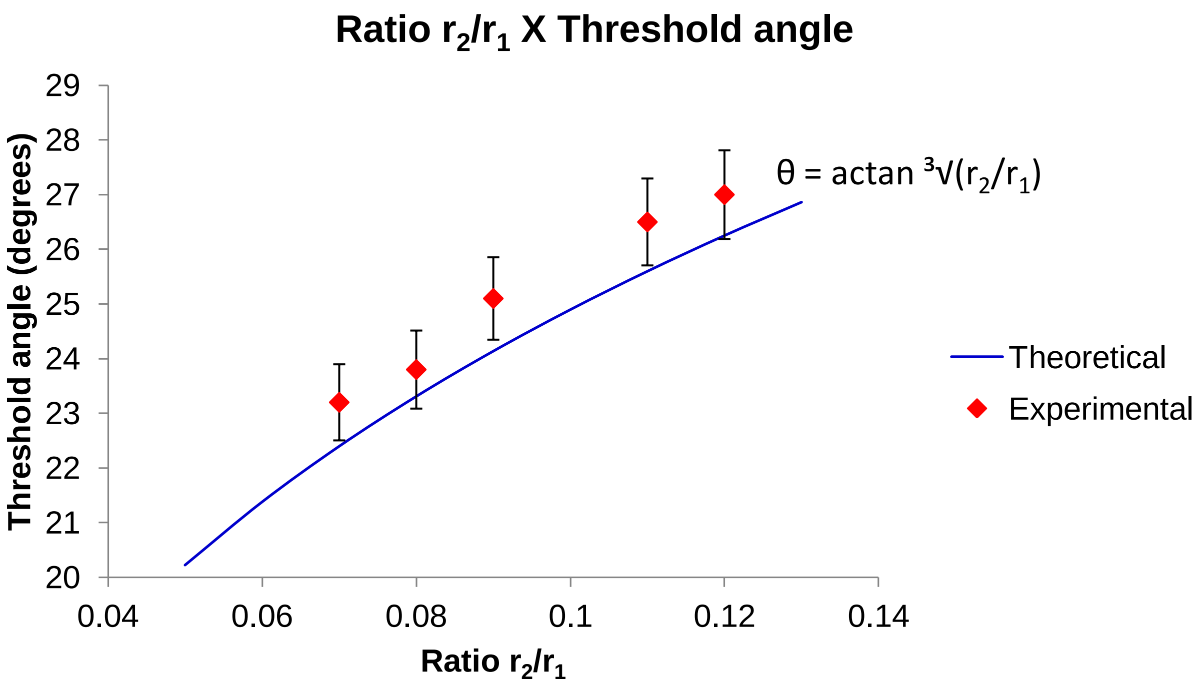
Table for experiment 2

Threshold angle (in degrees)								
Coins	**0.16**	**0.08**	**0.04**	**0.07**	**0.1**	**0.13**	**0.17**	**0.06**
1	33	28	21	25	26	25	28	23
2	31	25	22	24	25	25	32	25
3	30	25	23	22	24	24	34	21
4	29	24	20	23	27	30	31	23
5	28	23	19	24	26	29	29	22
6	28	24	18	24	25	28	33	21
7	29	23	19	23	25	33	30	23
8	28	24	19	23	25	31	29	22
9	28	24	20	23	26	27	29	21
10	29	23	19	24	27	26	30	23
Average	29.3	24.3	20	23.5	25.6	27.8	30.5	22.4
Theory	28.5	23.3	18.9	22.4	24.9	26.9	29.0	21.4
Standard deviation	1.64	1.49	1.56	0.85	0.97	2.94	1.96	1.26
Percentage error	3%	4%	6%	5%	3%	3%	5%	5%
Average Percentage Error								4%

Experiment 2 for real coins - table

Threshold angle (in degrees)					
Coins	**R$ 1.00**	**R$ 0.50**	**R$ 0.25**	**R$ 0.10**	**R$ 0.05**
1	25	26	25	25	25
2	26	27	26	26	24
3	23	25	25	29	23
4	22	27	24	25	23
5	24	28	26	26	25
6	22	29	24	27	24
7	24	27	25	27	23
8	23	28	27	25	25
9	22	27	24	27	23
10	21	26	25	28	23
Average	23.2	27	25.1	26.5	23.8
Theory	22.6	26.5	24.1	25.7	22.9
Standard deviation	1.55	1.15	0.99	1.35	0.92
Percentage error	3%	2%	4%	3%	4%
Average Perc. Error					3%

Experiment 2 for real coins - graphic

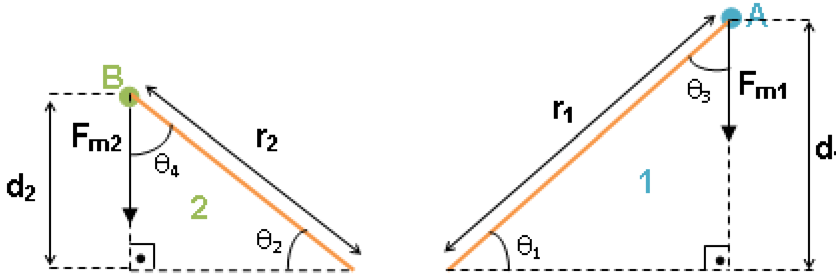


Percentage error = 3%

Threshold angle: Ampère model consideration

Ampère Model:

$$F_m = \frac{3\mu m_1 m_2}{2\pi d^4} = \frac{k_a}{d^4}$$



$$\tau_1 = \tau_2$$

$$F_{m1} \cdot r_1 \cdot sin\theta_3 = F_{m2} \cdot r_2 \cdot sin\theta_4$$

$$F_{m1} \cdot r_1 \cdot cos\theta_1 = F_{m2} \cdot r_2 \cdot cos\theta_2$$

$$\theta_1 + \theta_2 = 90°$$

$$d_1 = r_1 \cdot sin\theta_1$$

$$sin\theta_1 = cos\theta_2$$

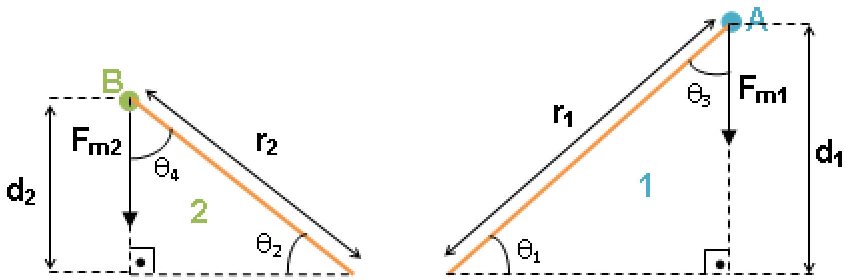
$$d_2 = r_2 \cdot sin\theta_2$$

$$sin\theta_2 = cos\theta_1$$

Threshold angle: Ampère model consideration

Ampère Model:

$$F_m = \frac{3\mu m_1 m_2}{2\pi d^4} = \frac{k_a}{d^4}$$



$$\frac{k_a}{(r_1 \cdot sin\theta_1)^4} \cdot r_1 \cdot cos\theta_1 = \frac{k_a}{(r_2 \cdot sin\theta_2)^4} \cdot r_2 \cdot cos\theta_2$$

$$\frac{cos^3\theta_1}{sin^3\theta_1} = \frac{r_1}{r_2}$$

$$tan^5\theta_1 = \left(\frac{r_2}{r_1}\right)^3$$

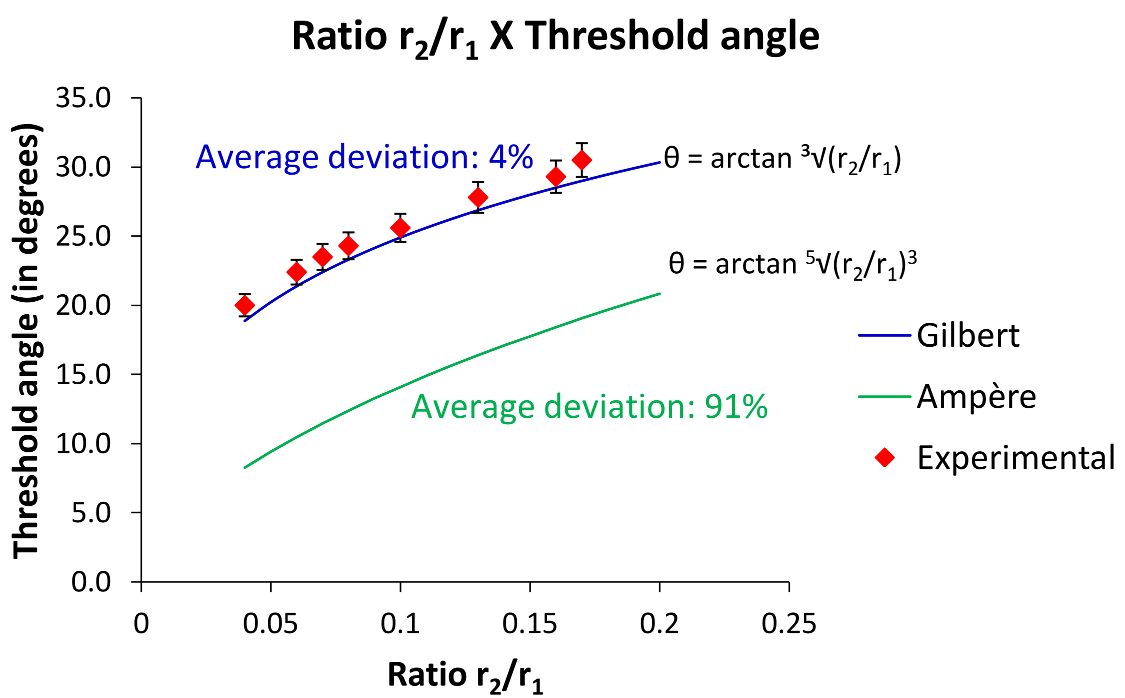
Experiment 2: threshold angle analysis

Weight

Threshold angle

Magnet

Oscillations



Precision of the protractor: ±0,5°

Coin's material

- Used coins: steel (iron + carbon).

↓

- Ferromagnetic: existence condition for the phenomenon.
  - Iron, cobalt, nickel and gadolinium;
  - Magnetic domains are oriented in the direction of the external field.
- Certain coins cannot be tested, like dollar coins (zinc+ copper).

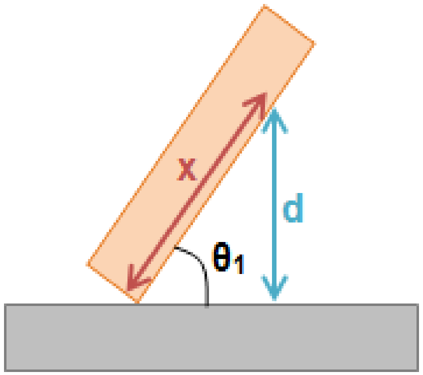
Magnetic properties

$$p = q_m \cdot d$$

$$M = \frac{p}{V}$$

$$B = \frac{\mu}{2} M$$

$$q_m = 2\frac{V}{\mu d} \cdot B$$

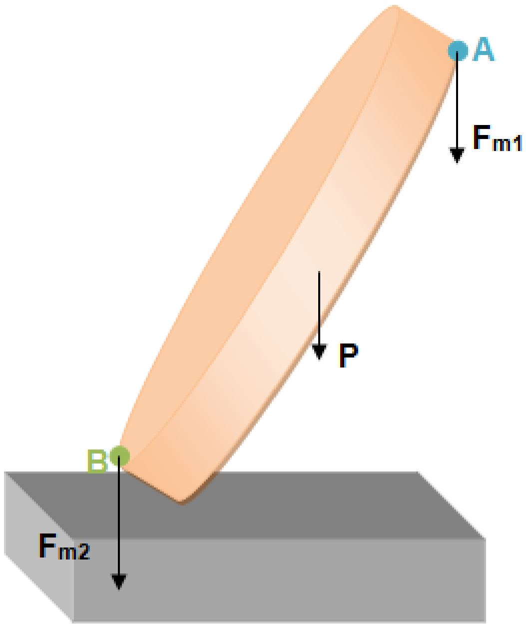


$$q_m = \int\int\int\int 2\frac{V}{\mu \cdot x \cdot sin\theta_1} \cdot B \cdot dB \cdot dV \cdot dx \cdot d\theta_1$$

$$d = x \cdot sin\theta_1$$

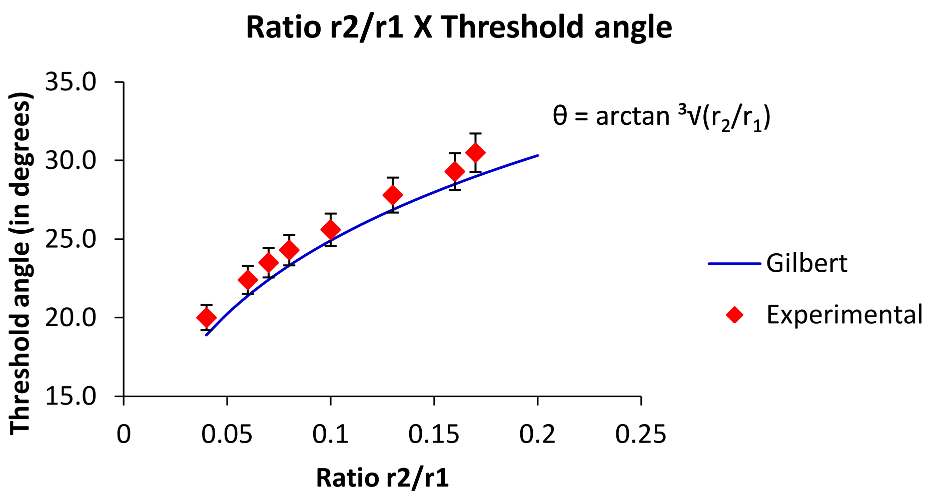
$$q_m = \frac{B^2V^2 \cdot \log(x) \cdot \left[\log\left(\sin\left(\frac{\theta_1}{2}\right)\right) - \log\left(\cos\left(\frac{\theta_1}{2}\right)\right)\right]}{2\mu}$$

Contribution of the weight for the torque



$$F_{m2} > F_{m1} > W$$

Correction factor for systematic error.



$$\tan^3\theta_1 = \frac{r_2}{r_1}$$

Without disregarding the weight, the relation without **c** wouldn't be gotten.