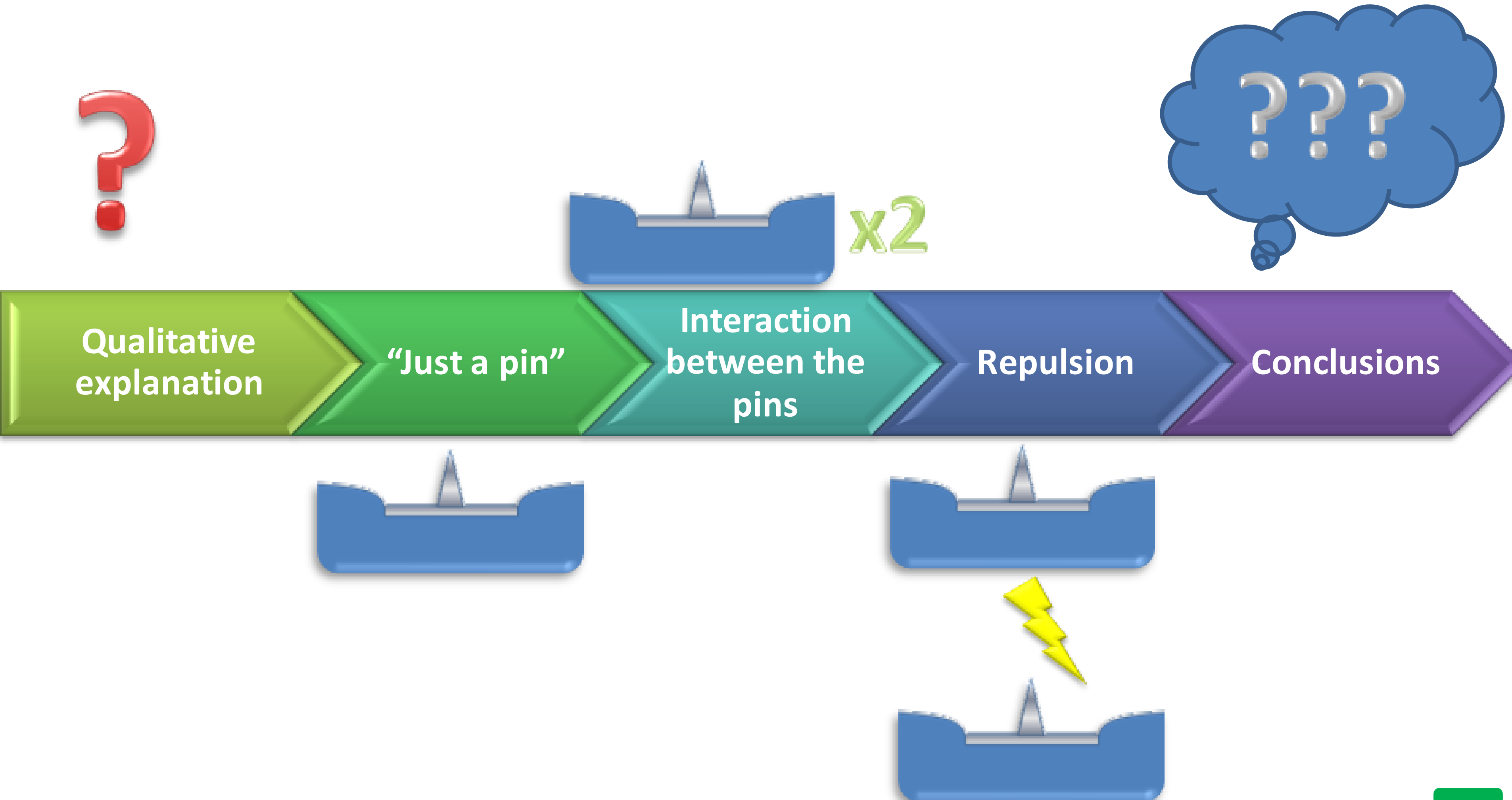
Problem #7
“Drawing pins”

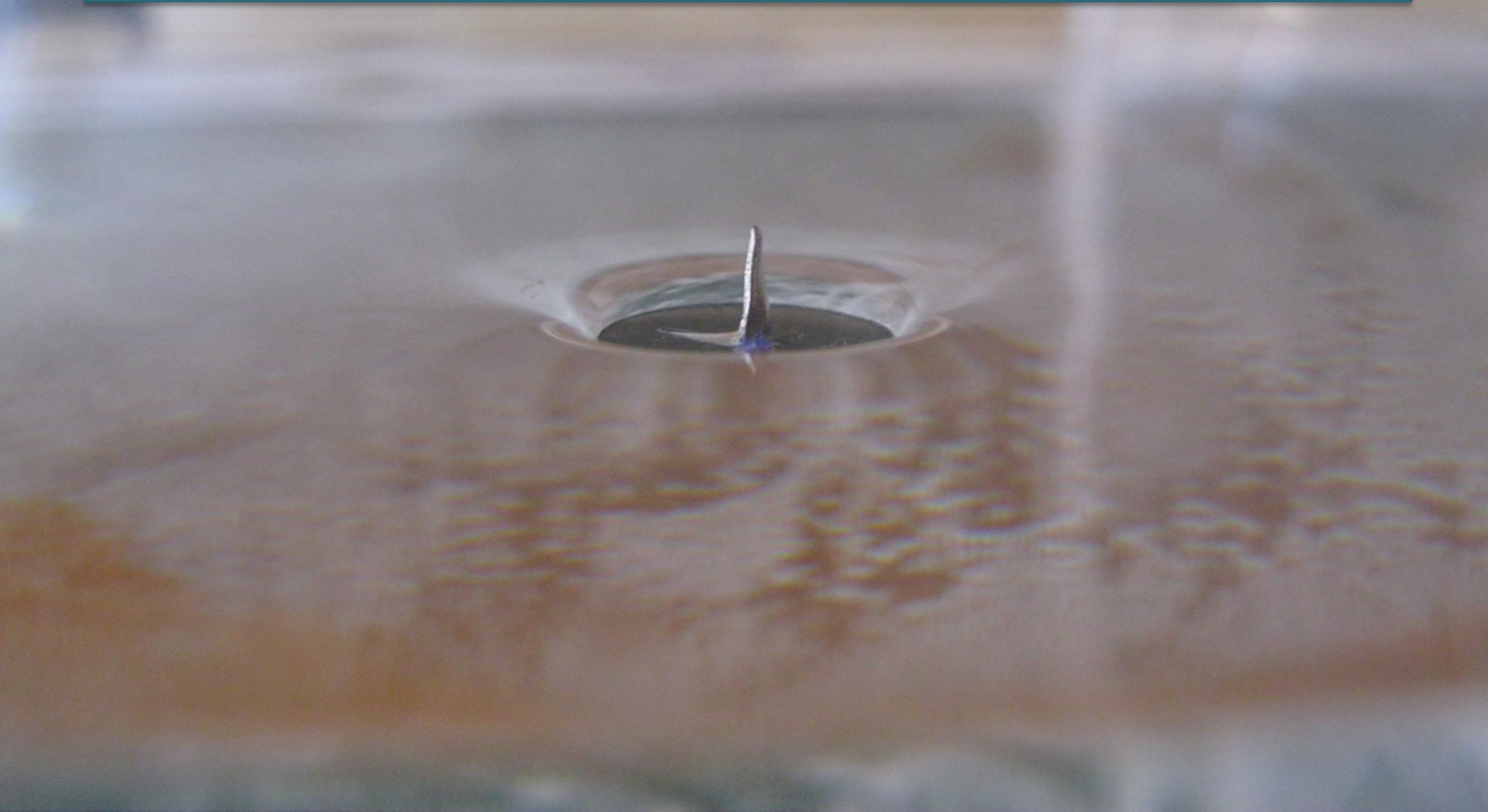
The problem

A drawing pin (thumbtack) floating on the surface of water near another floating object is subject to an attractive force. Investigate and explain the phenomenon. Is it possible to achieve a repulsive force by a similar mechanism?

The work plan



Qualitative explanation

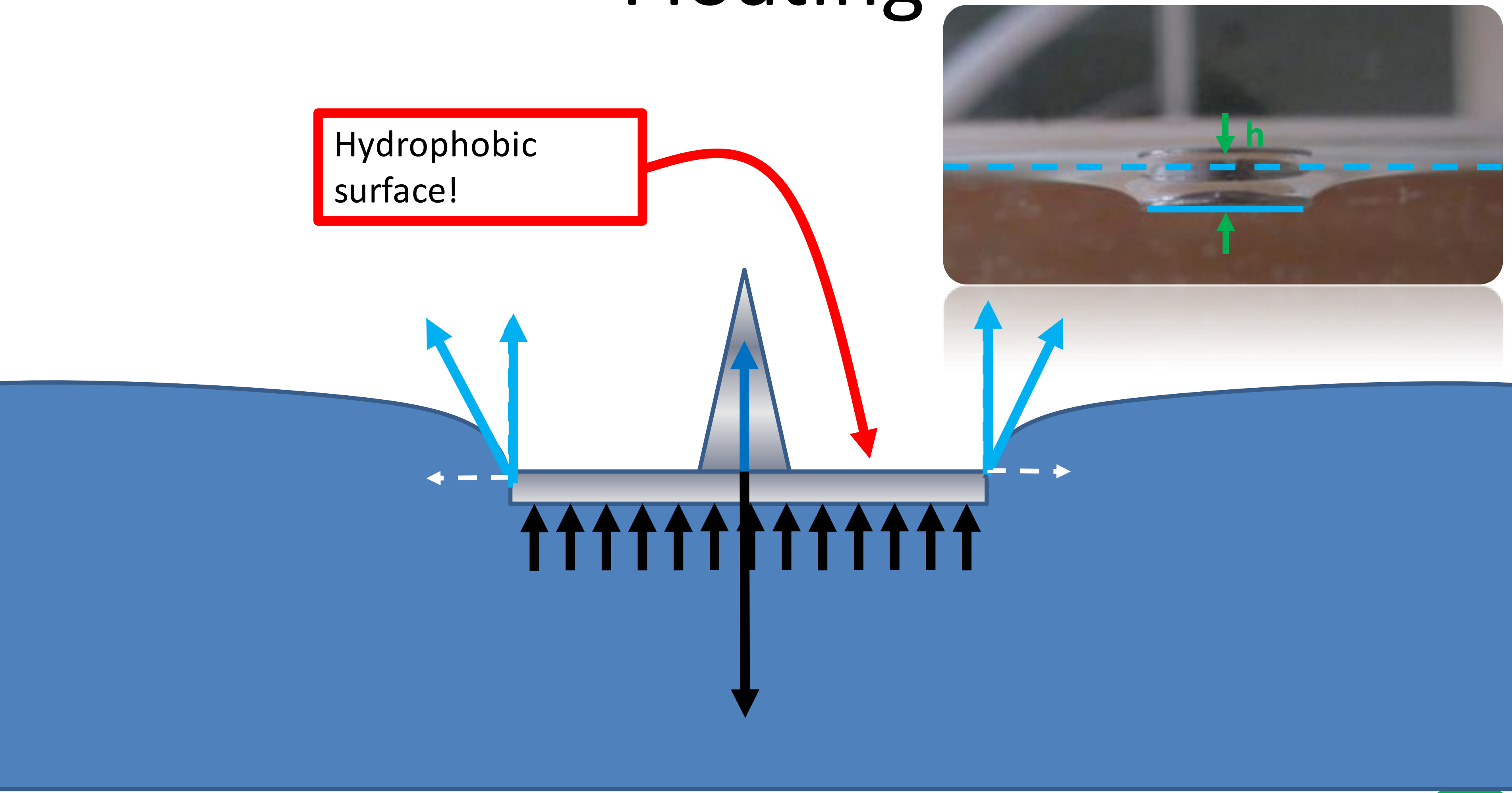


What to explain?

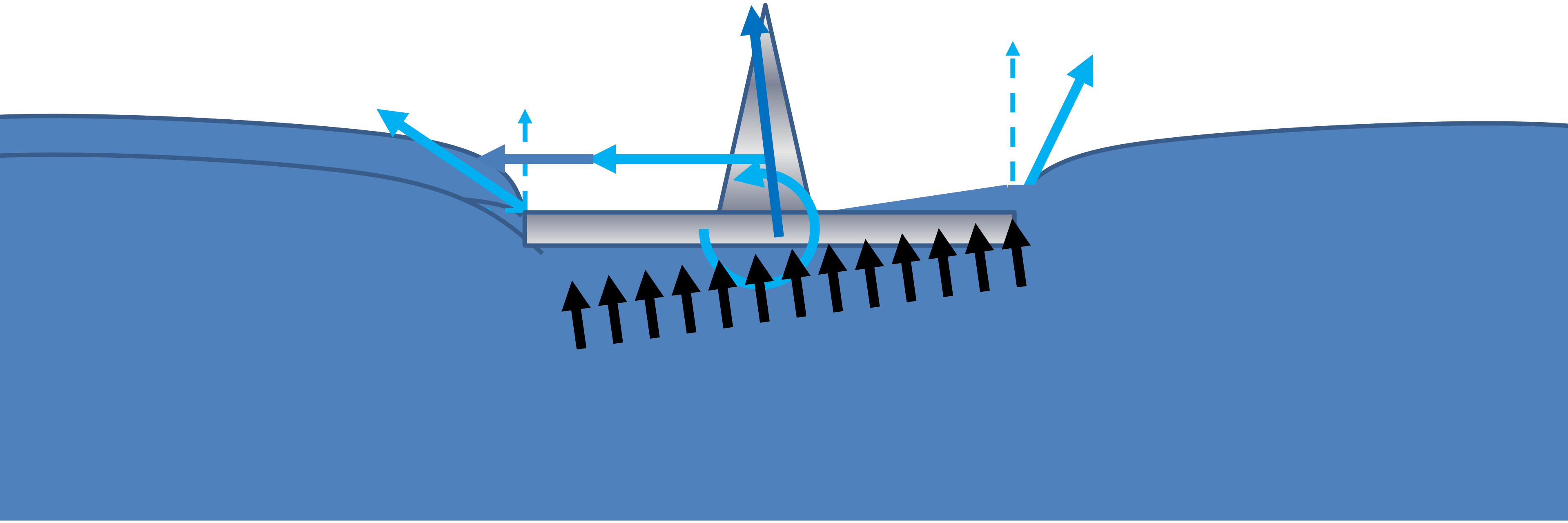
The pin is *floating*

There is a *force*…

Floating



The force




“Just a pin”

The balance of the **pin**

2

Surf

$$z(r) \approx h\left(1 - e^{-\alpha(r-R)}\right)$$

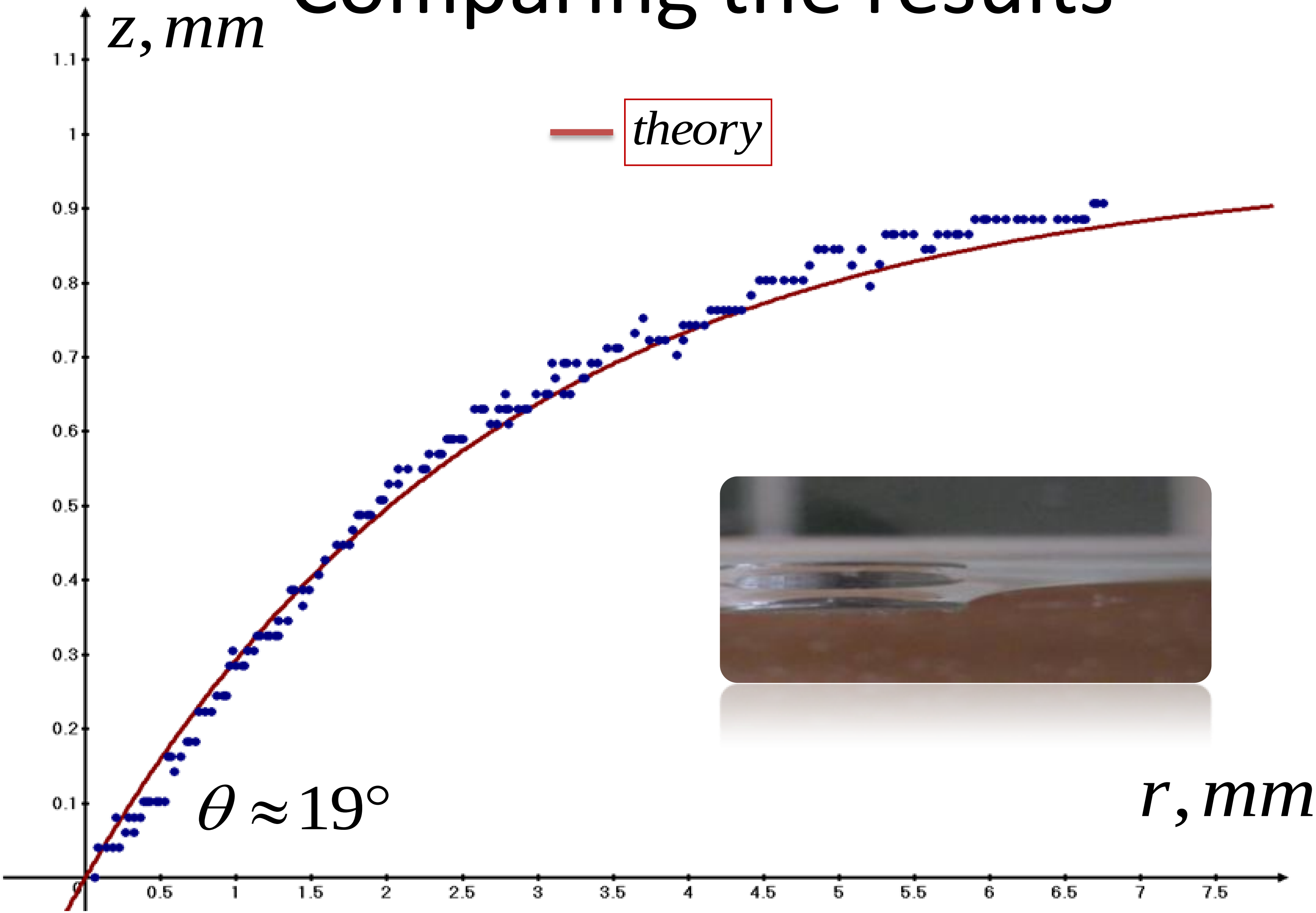
$$\alpha = \sqrt{\frac{\rho g}{\sigma}}$$

s

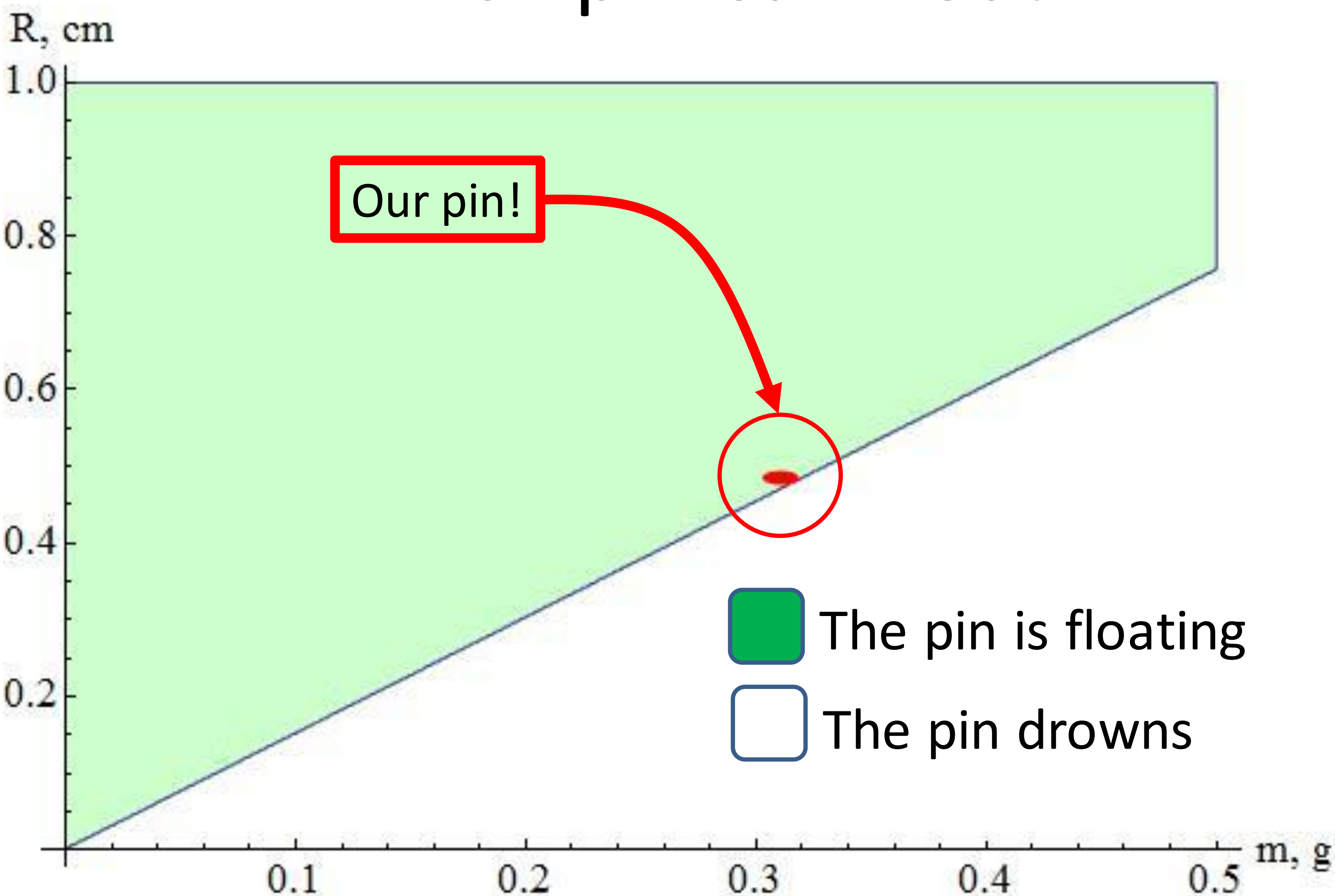
r))r

ressure

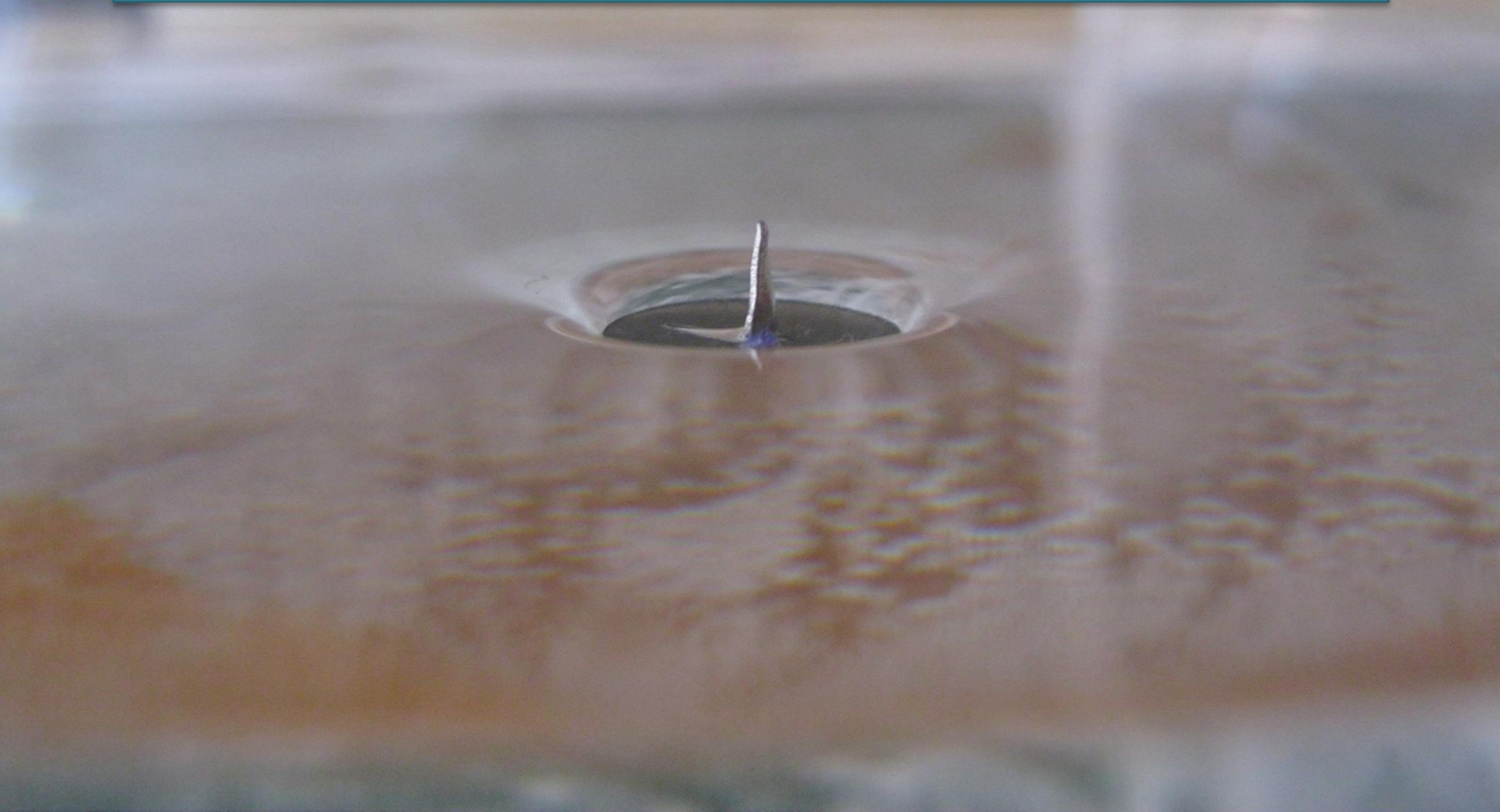
Comparing the results



Which pin can float?

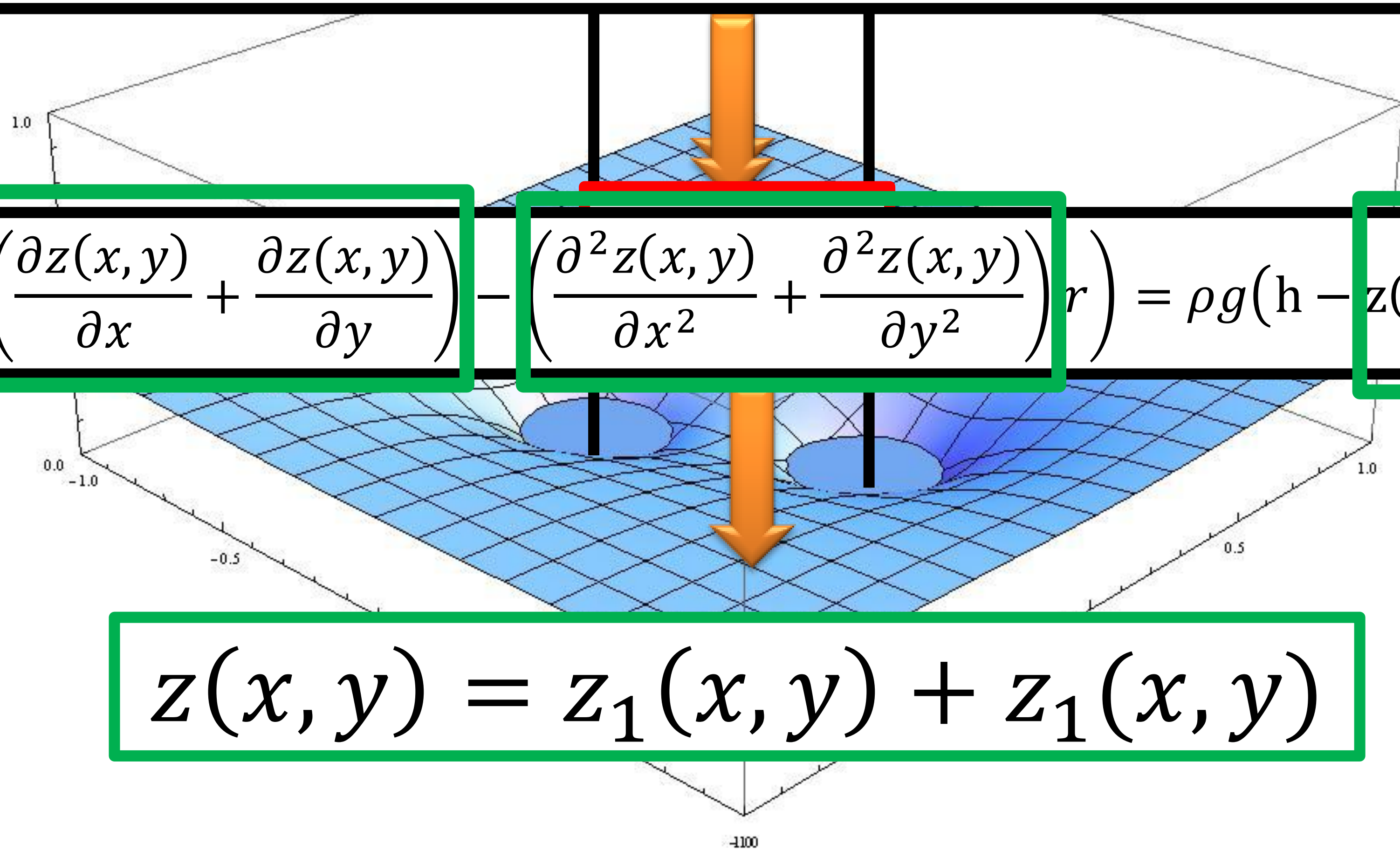


Interaction of the pins



Asymmetric case

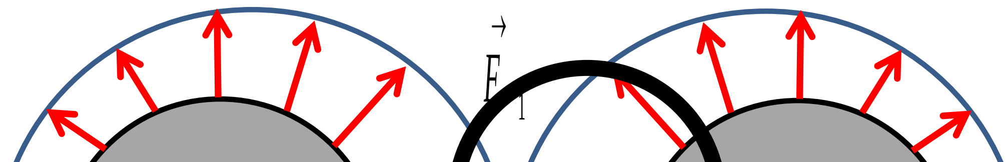
$$\sigma\left(-z'(r) - z''(r)r\right) = \rho g\left(\mathrm{h} - \mathrm{z}(\mathrm{r})\right)\mathrm{r}$$



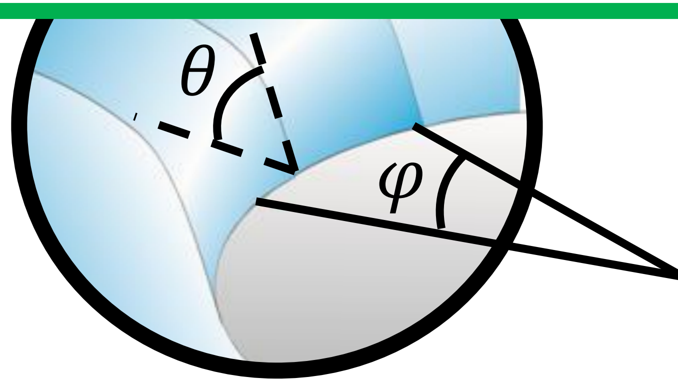
$$\sigma\left(-\left(\frac{\partial z(x,y)}{\partial x} + \frac{\partial z(x,y)}{\partial y}\right) - \left(\frac{\partial^2 z(x,y)}{\partial x^2} + \frac{\partial^2 z(x,y)}{\partial y^2}\right) r\right) = \rho g\left(\mathrm{h} - \mathrm{z}(\mathrm{x},\mathrm{y})\right)\mathrm{r}$$

$$z(x,y) = z_1(x,y) + z_1(x,y)$$

The forces. Force of surface tension



$$F_1 = 2\sigma R\int_0^{\pi}\cos\varphi\,\cos(\theta(\varphi))\,d\varphi$$

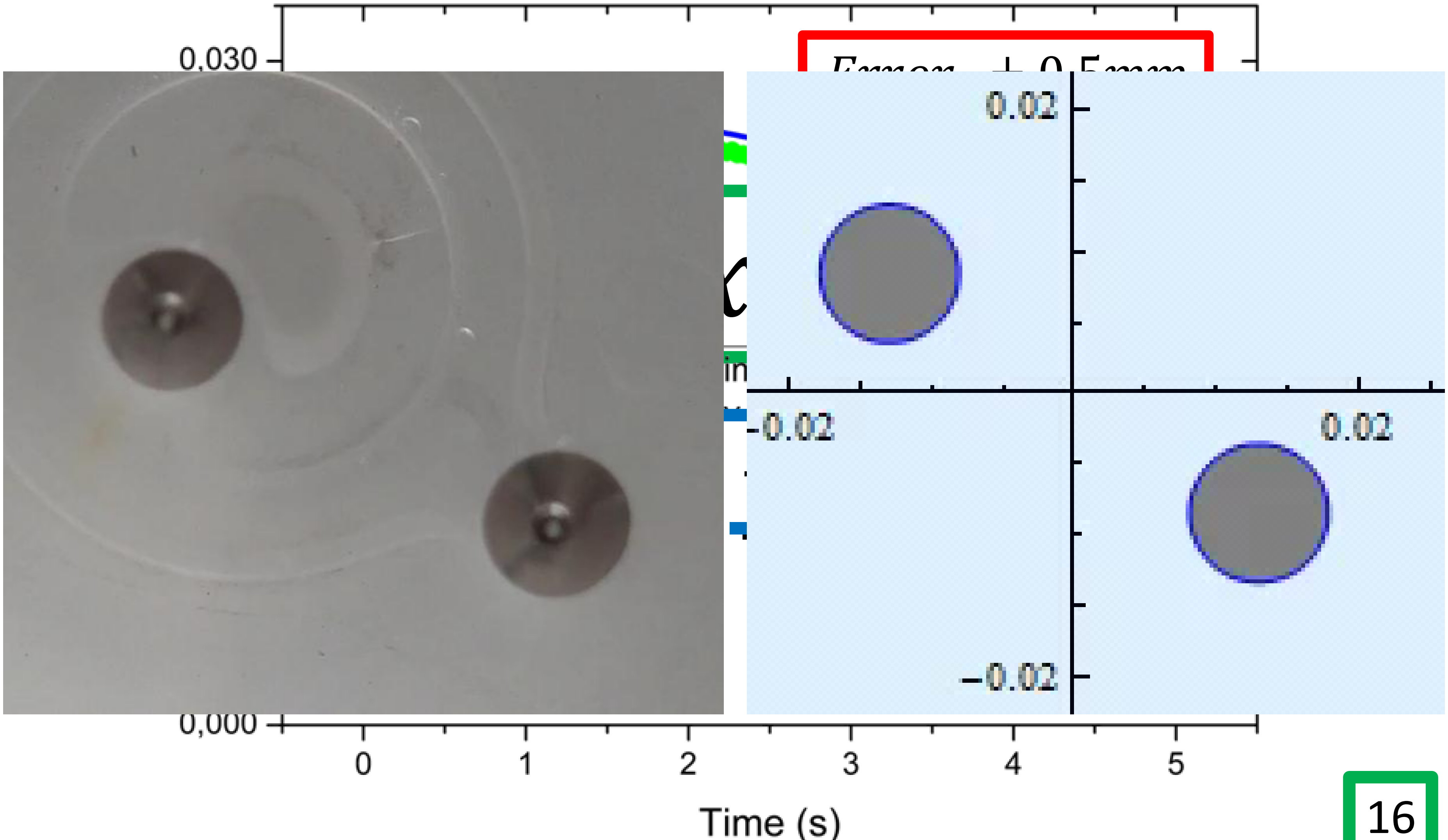


The forces. Force of hydrostatic pressure

$$F_p = \rho g h_1 \pi R^2 \sin\gamma$$

$$\sin\gamma = \frac{z_0(r,\varphi)\big|_{r=R,\varphi=0} - z_0(r,\varphi)\big|_{r=R,\varphi=\pi}}{2R}$$

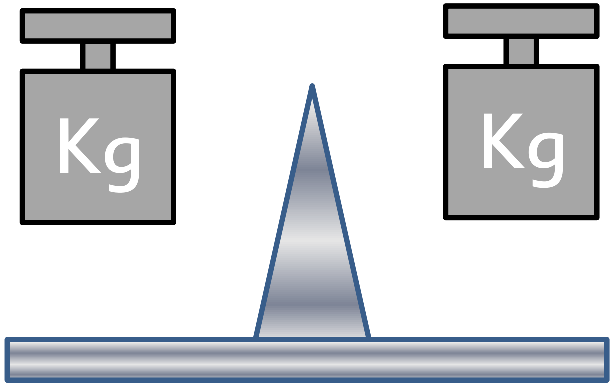
The law of motion



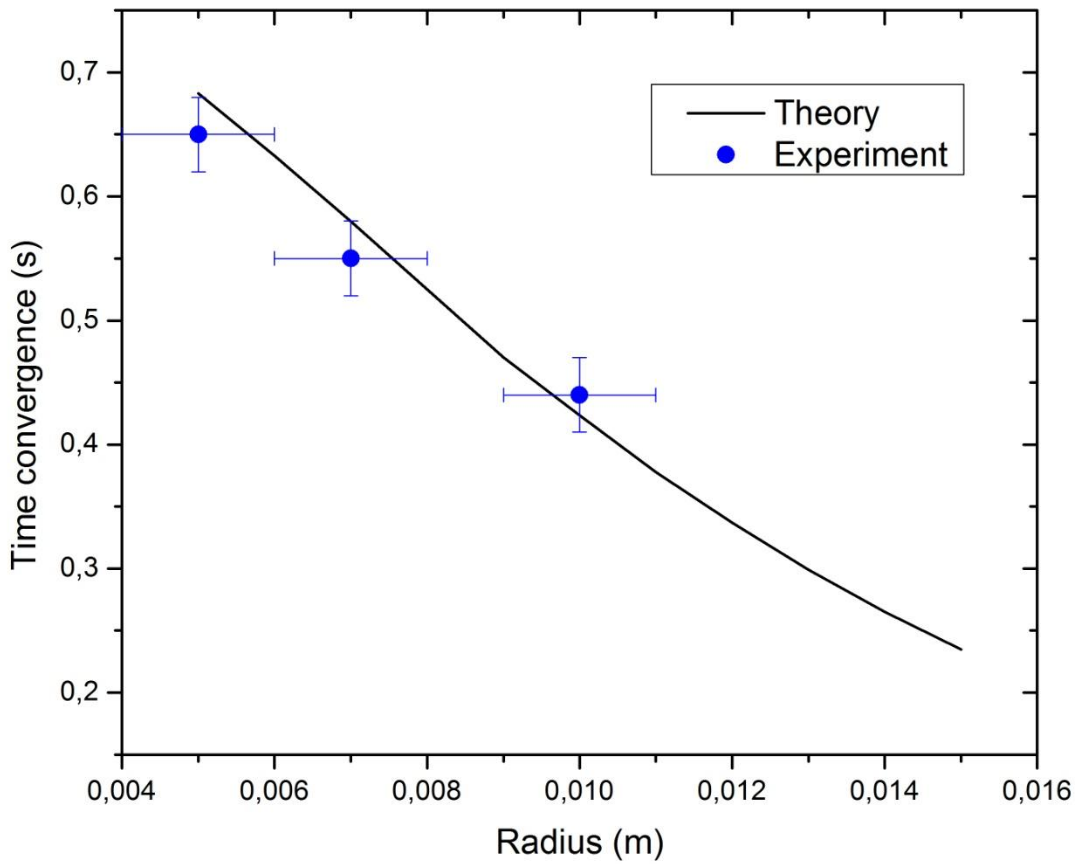
Different parameters of the pins

Adjustable parameters:

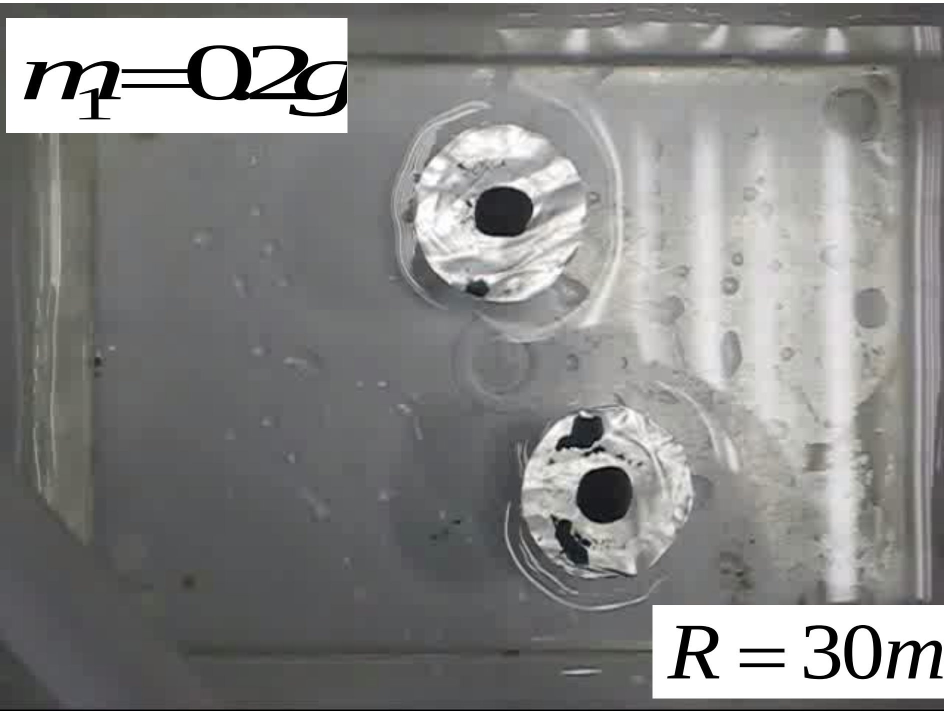
- Radius
- Mass



The radius of the pins



The mass of the pins



$R = 30mm = const$

$Experiment: t_1 = 27s$

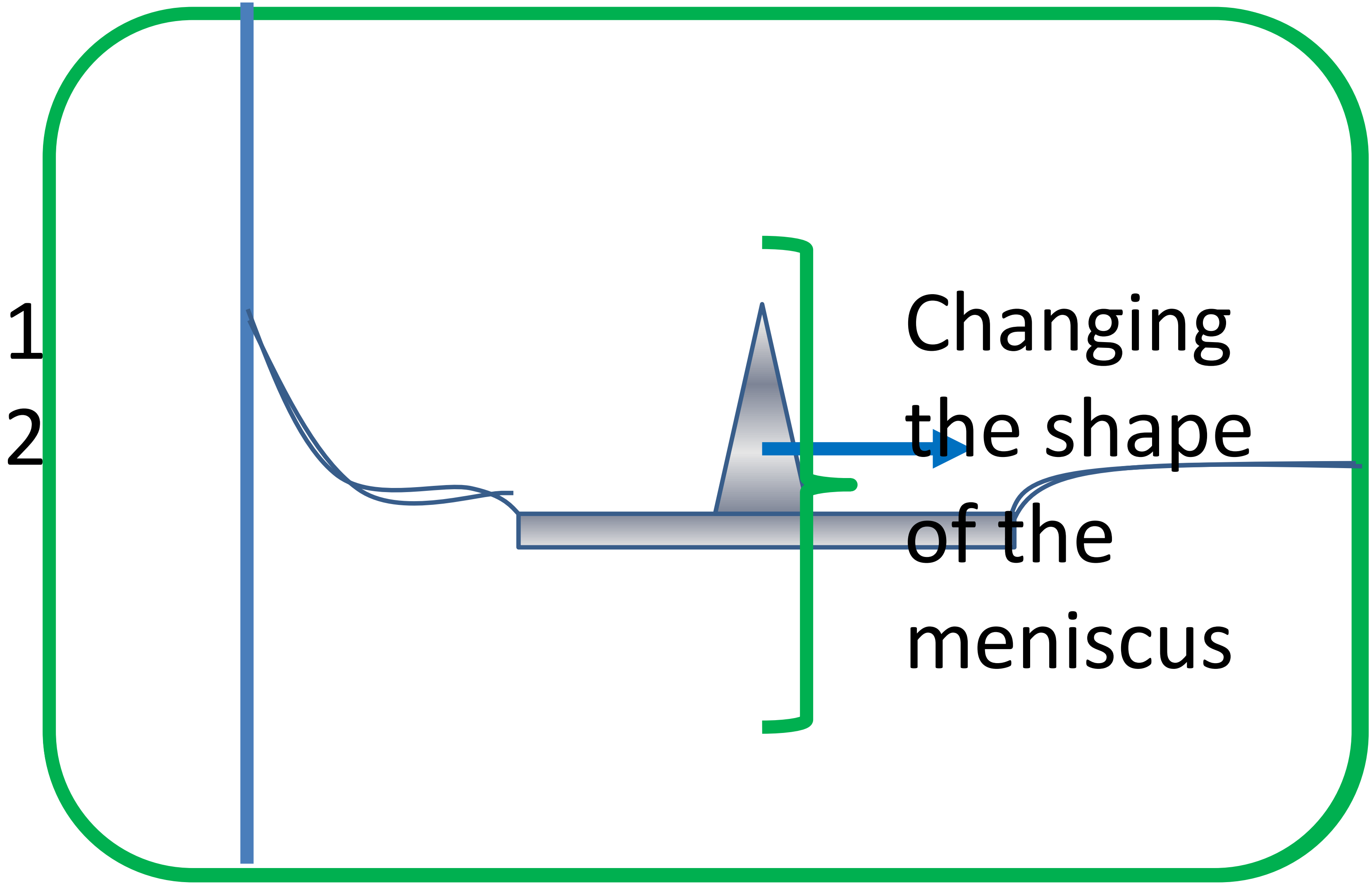
$Theory: t_1 = 26{,}3s$

$Experiment: t_2 = 6s$

$Theory: t_2 = 5{,}7s$


The repulsion

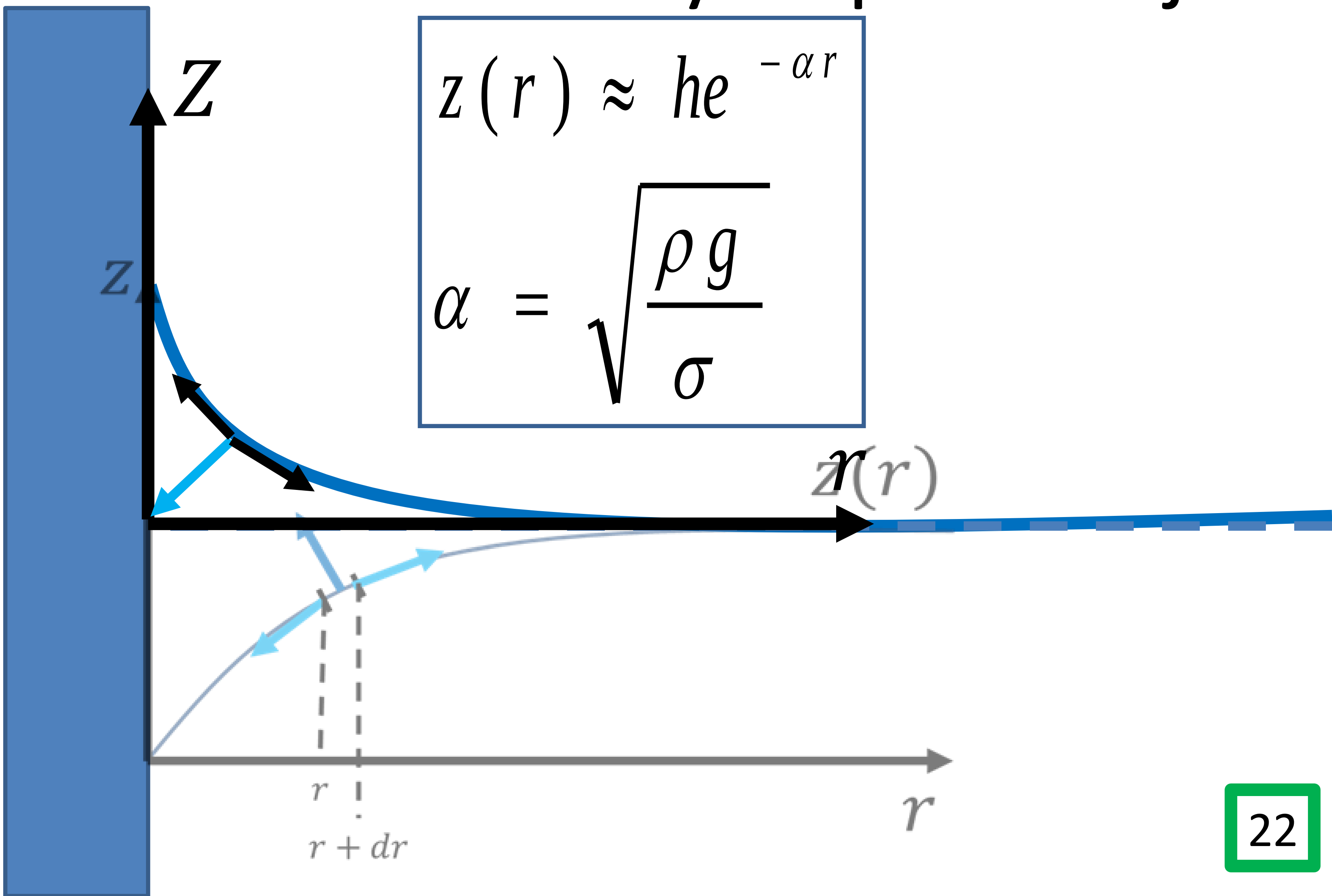
Ways of getting the repulsive force



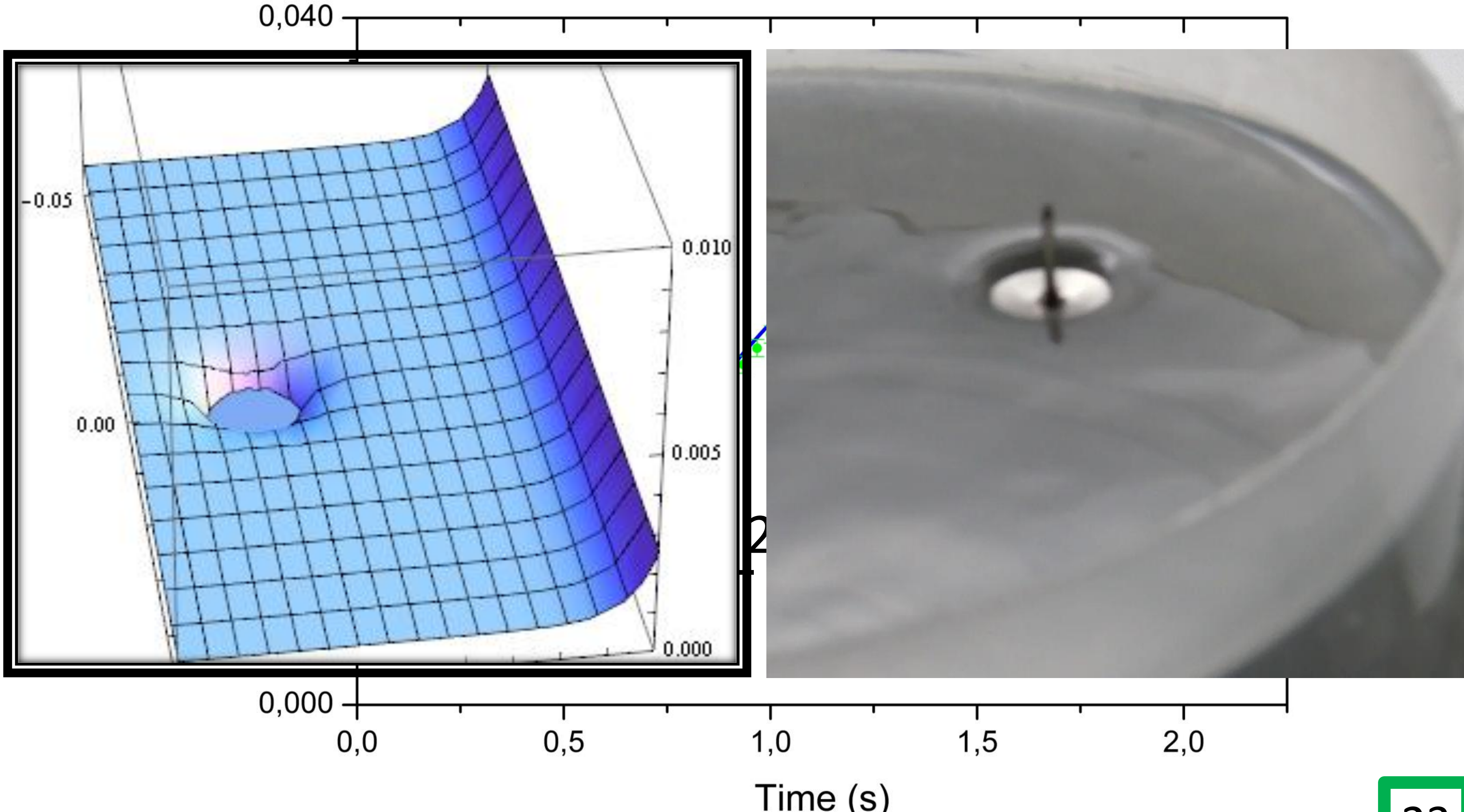
The interaction with hydrophilic object

$$z(r) \approx he^{-\alpha r}$$

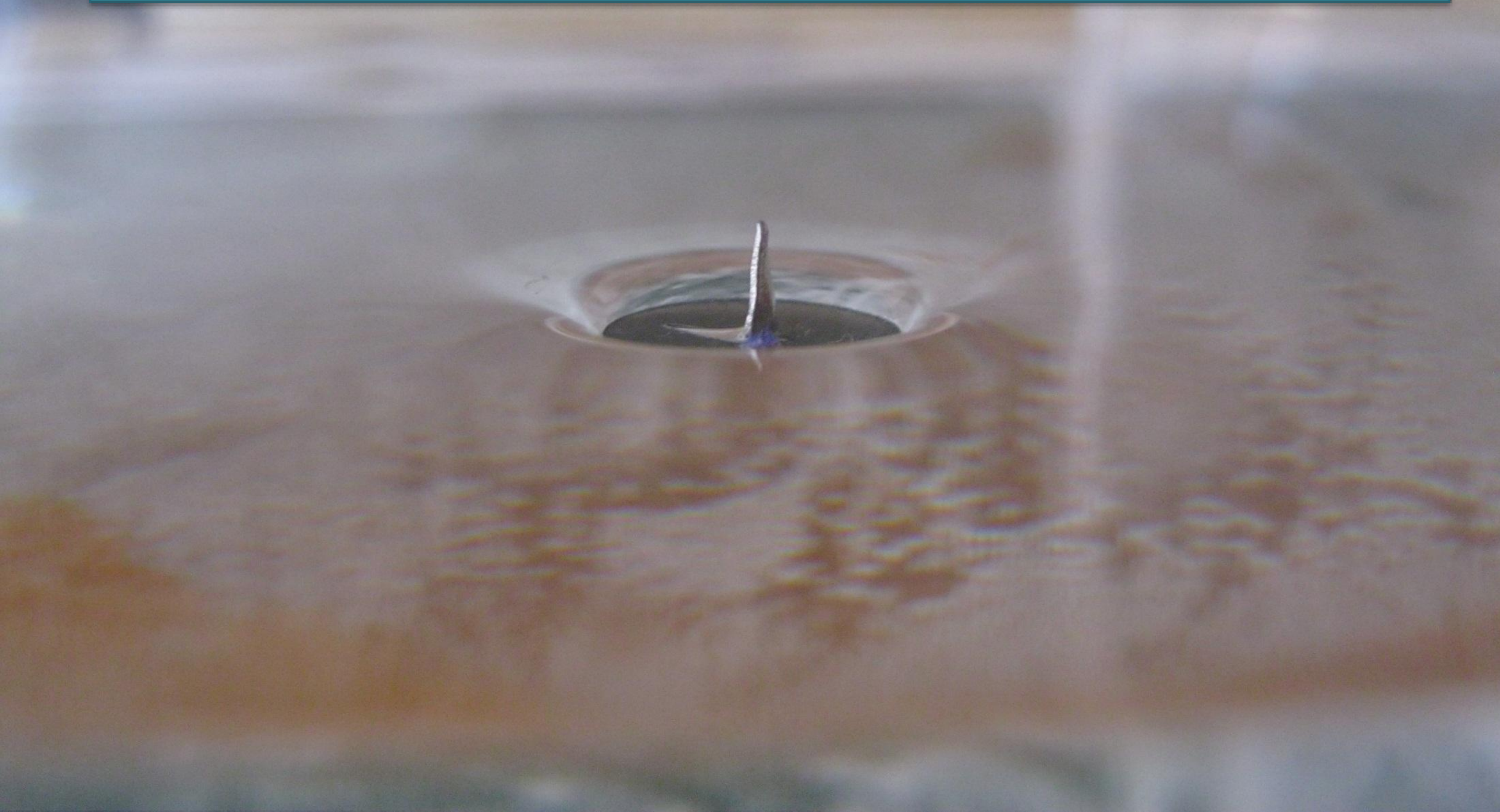
$$\alpha = \sqrt{\frac{\rho g}{\sigma}}$$



The law of motion



Results and conclusions



Final list of relevant parameters

- **The parameters of the pin**
  - Mass
  - Radius
  - Shape
- **The parameters of the liquid**
  - Density
  - Viscosity
  - Surface tension

Results of our work

Our work

- Investigate the phenom
- Explain the pheno

The problem

- Build a theore del
  - Invent to describe the meni
  - a quantitative description e phenomena
  - Get all the properties of the interaction

Can predict the process of interaction!

Conclusions

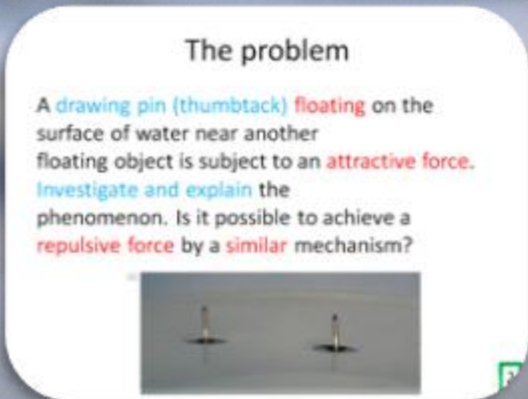
1. The pin is floating due to surface tension, forming the determinate meniscus

2. The shape of the meniscus is determined by the parameters of the system and can be approximated as an exponent

3. The interaction of the pins is caused by changing in the shape of the meniscus and can be explained by two forces – force of hydrostatic pressure and force of surface tension


Thank you for your attention!

Review


The problem
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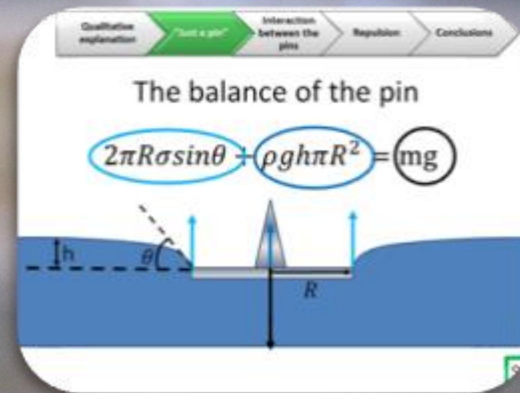
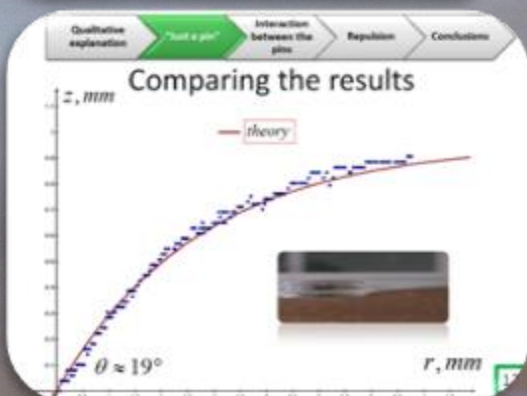
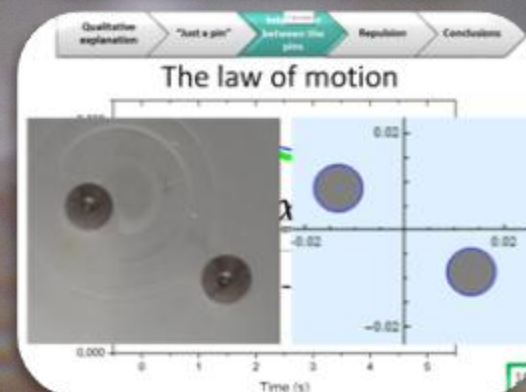
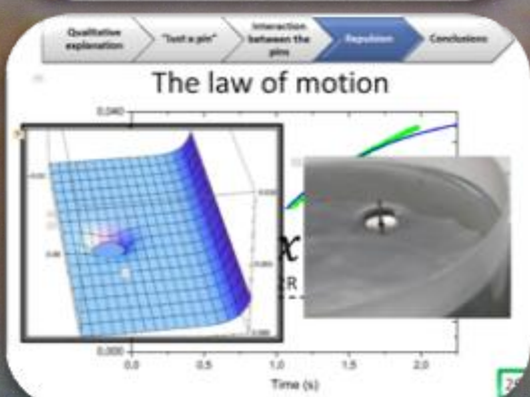
Qualitative explanation


The balance of the pin
$2\pi R\sigma sin\theta + \rho gh\pi R^2 = mg$
h
θ
R

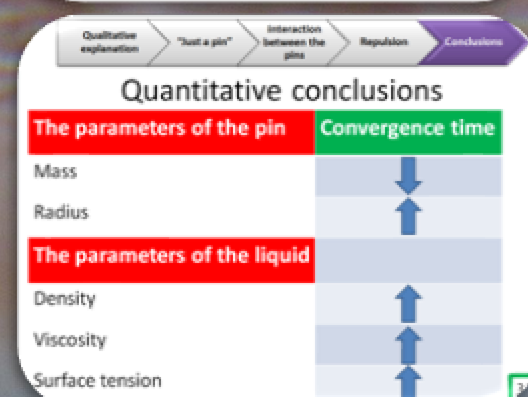

Comparing the results
z, mm
theory
θ ≈ 19°
r, mm


Interaction of the pins


The law of motion
Time (s)


The law of motion
Time (s)

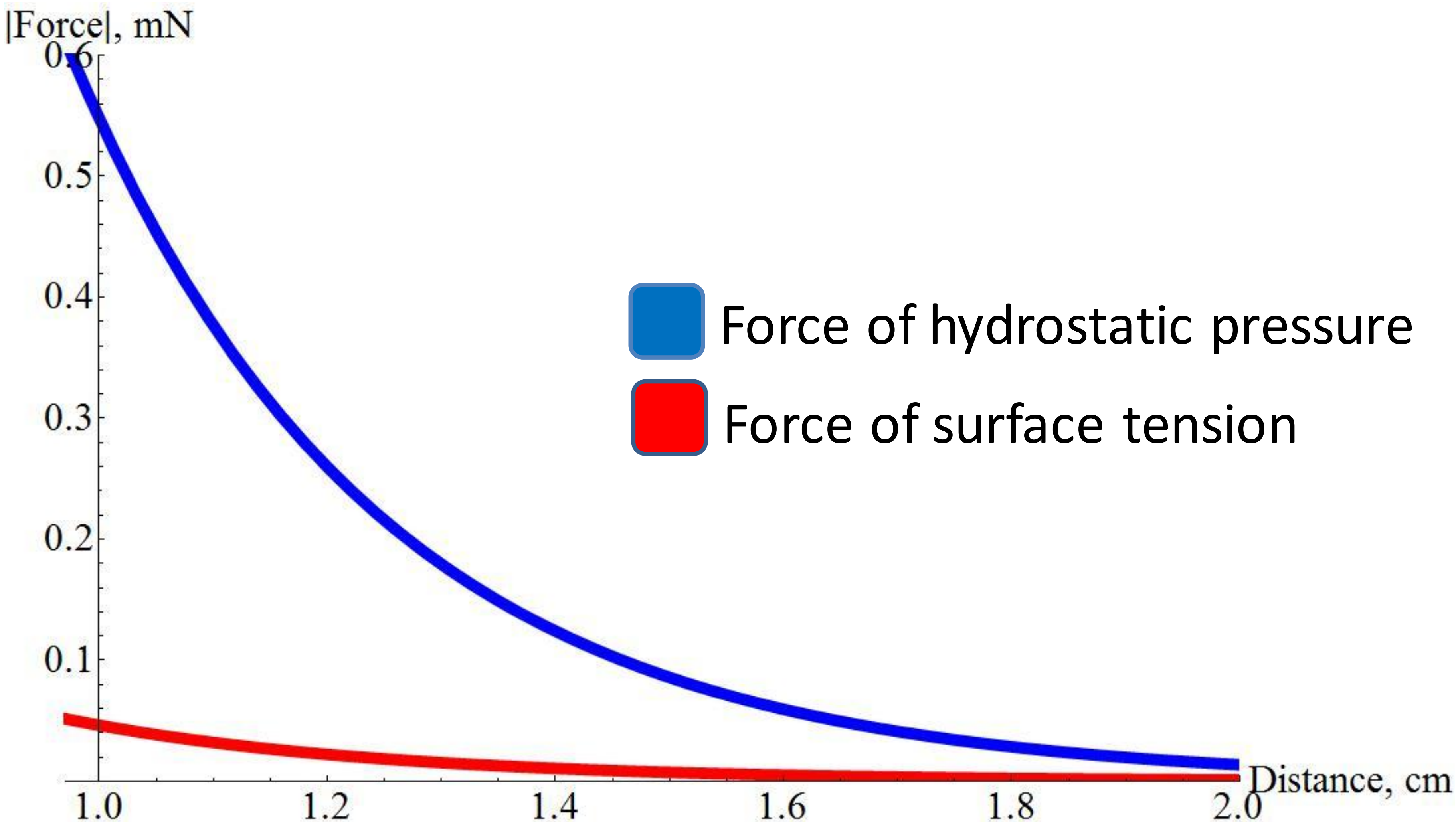

Results and conclusions


Quantitative conclusions
The parameters of the pin
Convergence time
Mass
Radius
The parameters of the liquid
Density
Viscosity
Surface tension

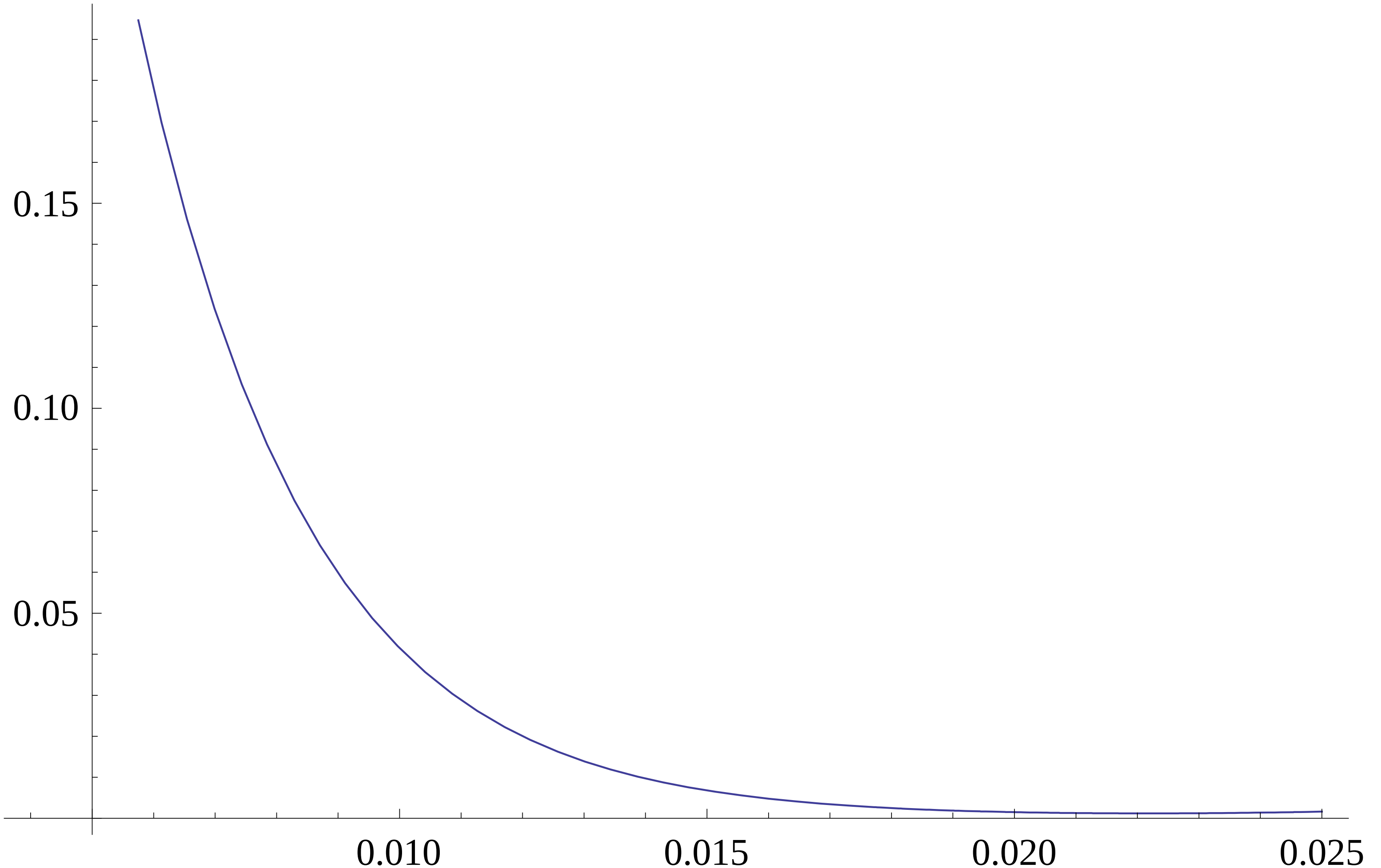
Additional slides

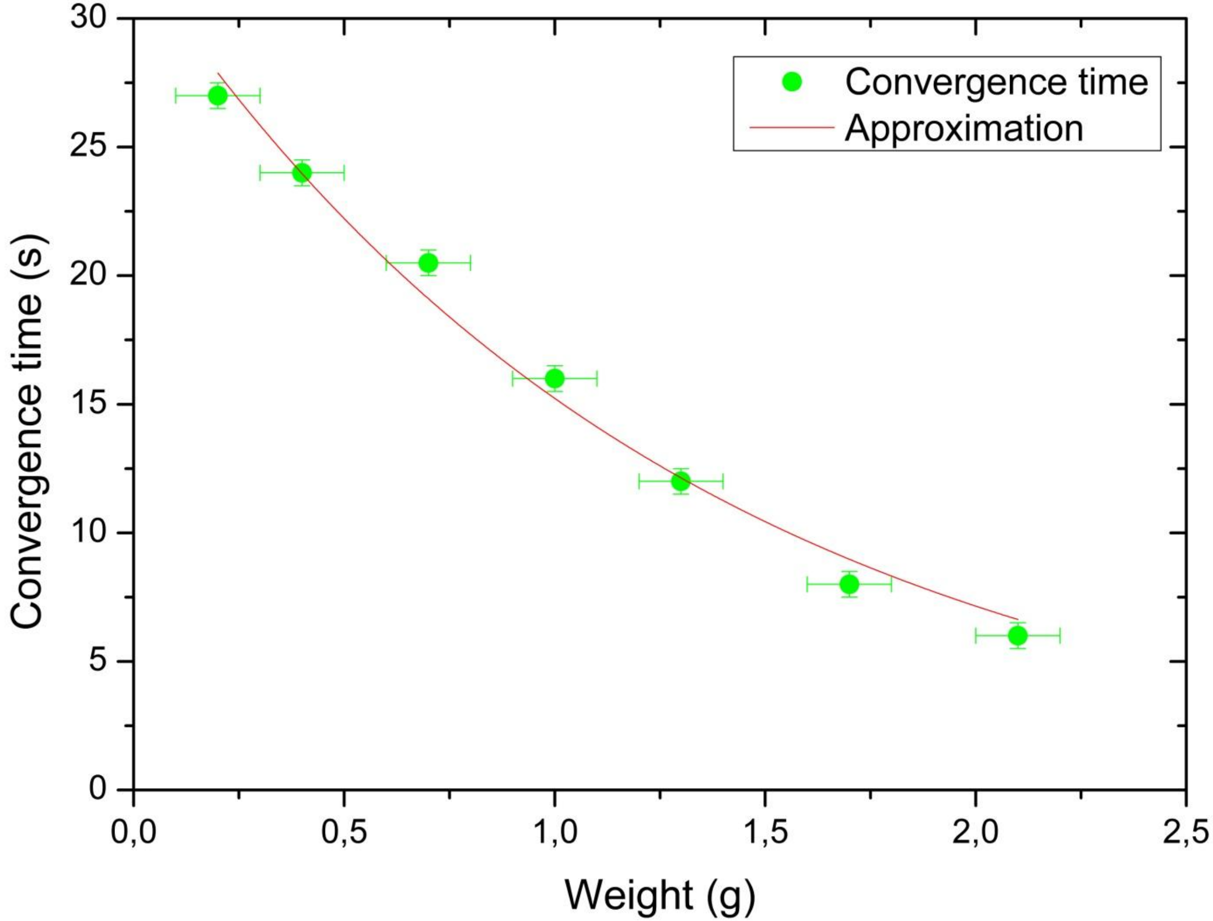
- Comparison of forces
- Full theory
- Influence of parameters

Comparing the influence of forces



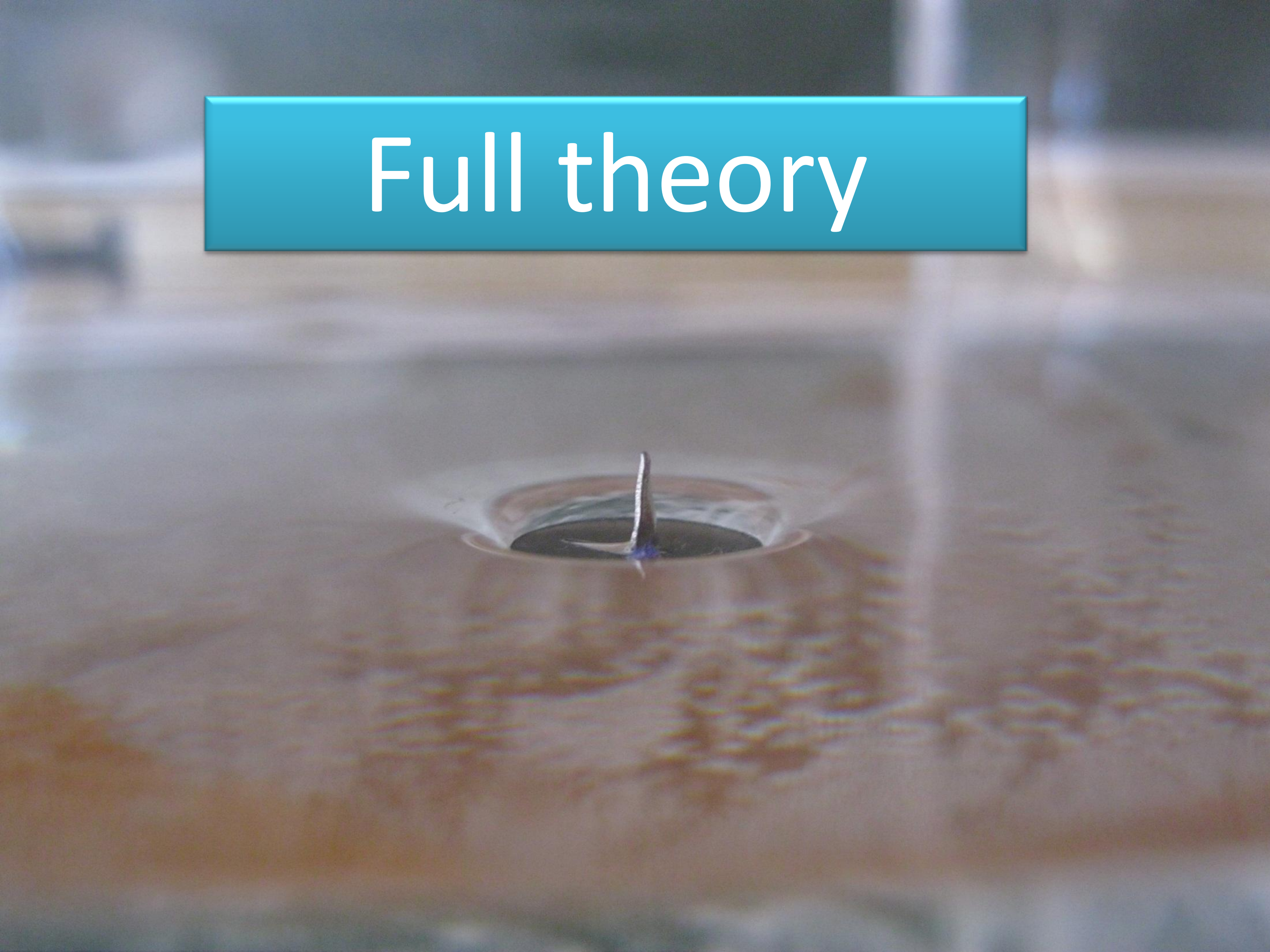
Angle of contact vs. radius





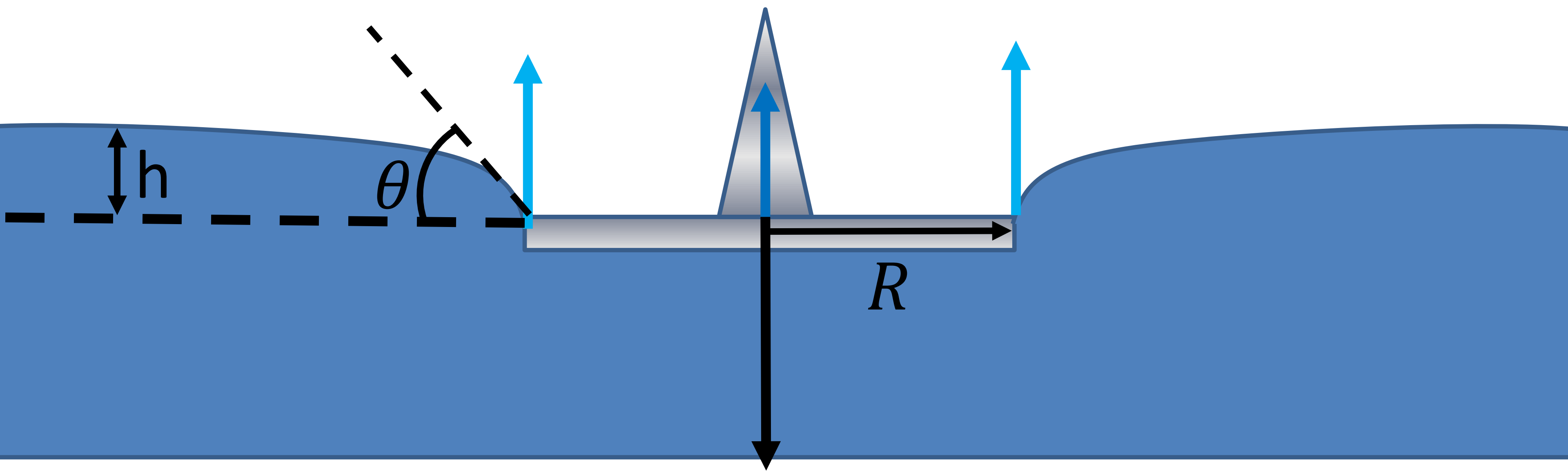
Convergence time
Approximation
Convergence time (s)
Weight (g)
0
5
10
15
20
25
30
0,0
0,5
1,0
1,5
2,0
2,5

Full theory



The balance of the pin

$$2\pi R\sigma \sin\theta + \rho g h \pi R^2 = mg$$



The balance of the meniscus

$$z(r) \approx h\left(1 - e^{-\alpha(r-R)}\right)$$

$$\alpha = \sqrt{\frac{\rho g}{\sigma}}$$

The integral balance of the meniscus

$$z(r) \approx h\left(1 - e^{-\alpha(r-R)}\right)$$

$$\alpha = \sqrt{\frac{\rho g}{\sigma}}$$

$h(m, R, \sigma \ldots)$ $\theta(m, R, \sigma \ldots)$

Full list of relevant parameters

- **The parameters of the pin**
  - Mass
  - Radius
  - Shape
- **The parameters of the liquid**
  - Density
  - Viscosity
  - Surface tension
  - ~~Temperature~~

Influence of parameters

The parameters of the pin		Convergence time
Mass	↑	↓
Radius	↑	↑
The parameters of the liquid		
Density	↑	↑
Viscosity	↑	↑
Surface tension	↑	↑