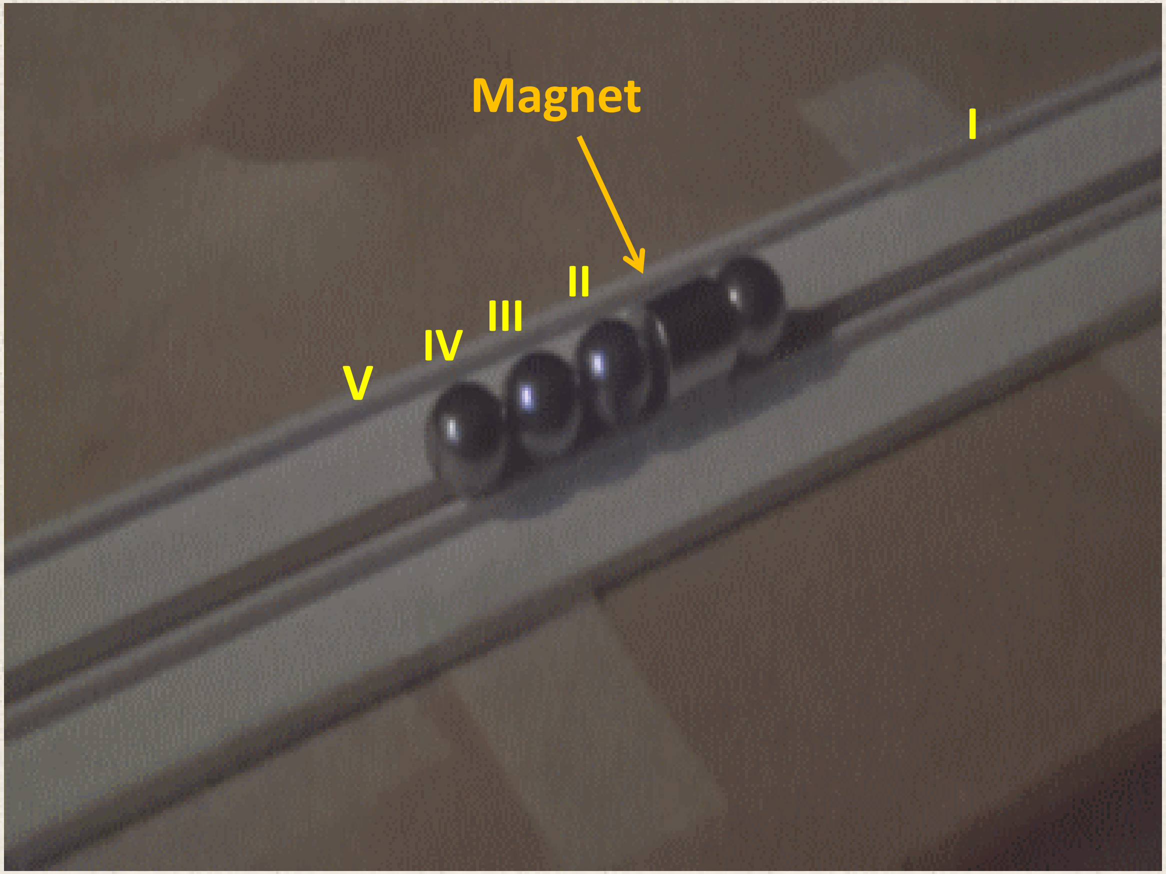
1. Gaussian Cannon



Alexander Barnaveli

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A sequence of identical steel balls includes a strong magnet and lies in a nonmagnetic channel. Another steel ball is rolled towards them and collides with the end ball. The ball at the opposite end of the sequence is ejected at a surprisingly high velocity. Optimize the magnet's position for the greatest effect.

Presentation Plan

1. Experiment

2. Gaussian Cannon action principles

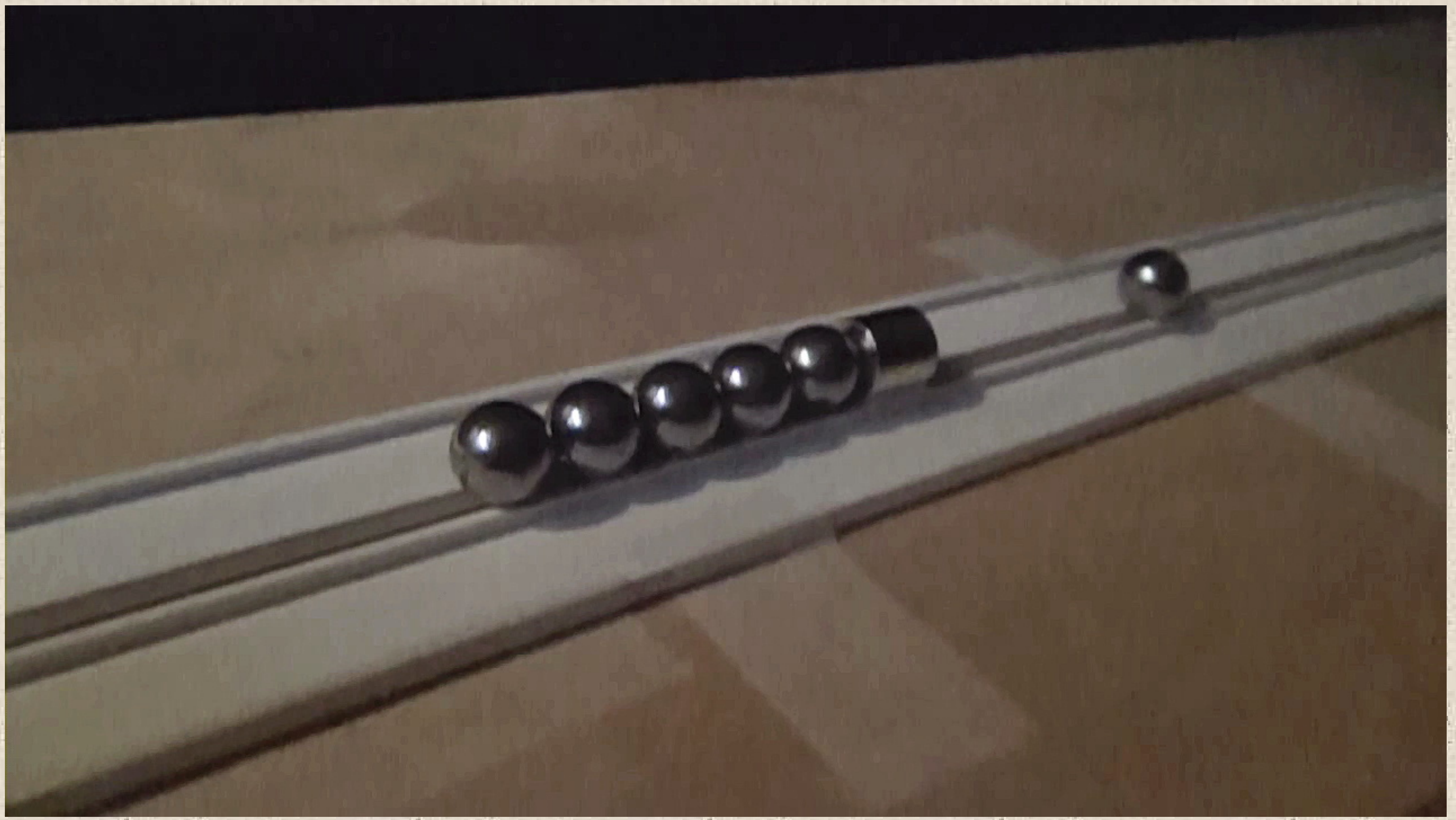
3. Interaction of magnet and ball

4. “Energy source”

5. Gaussian Cannon optimization

6. Multi-stage Cannon

Experiment



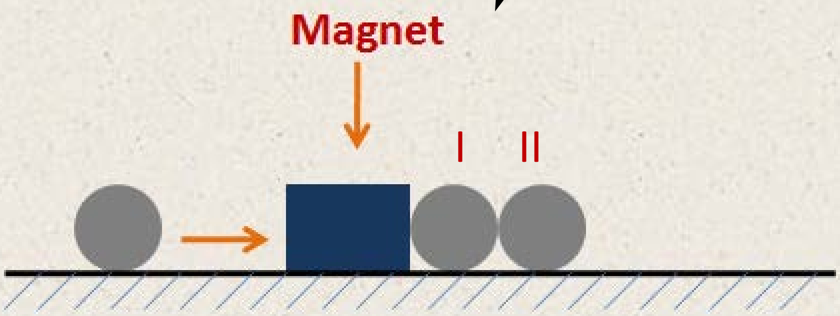
Slow motion shot

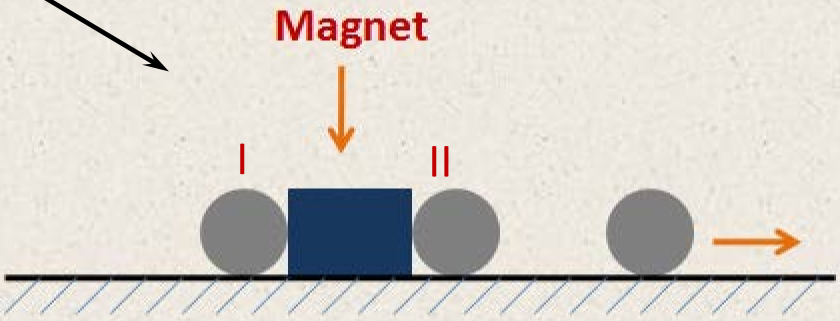
Video

Gaussian Cannon action principles

- ✓ Momentum conservation law
- ✓ Potential (magnetic) energy difference of initial and final conditions

Initial and final conditions

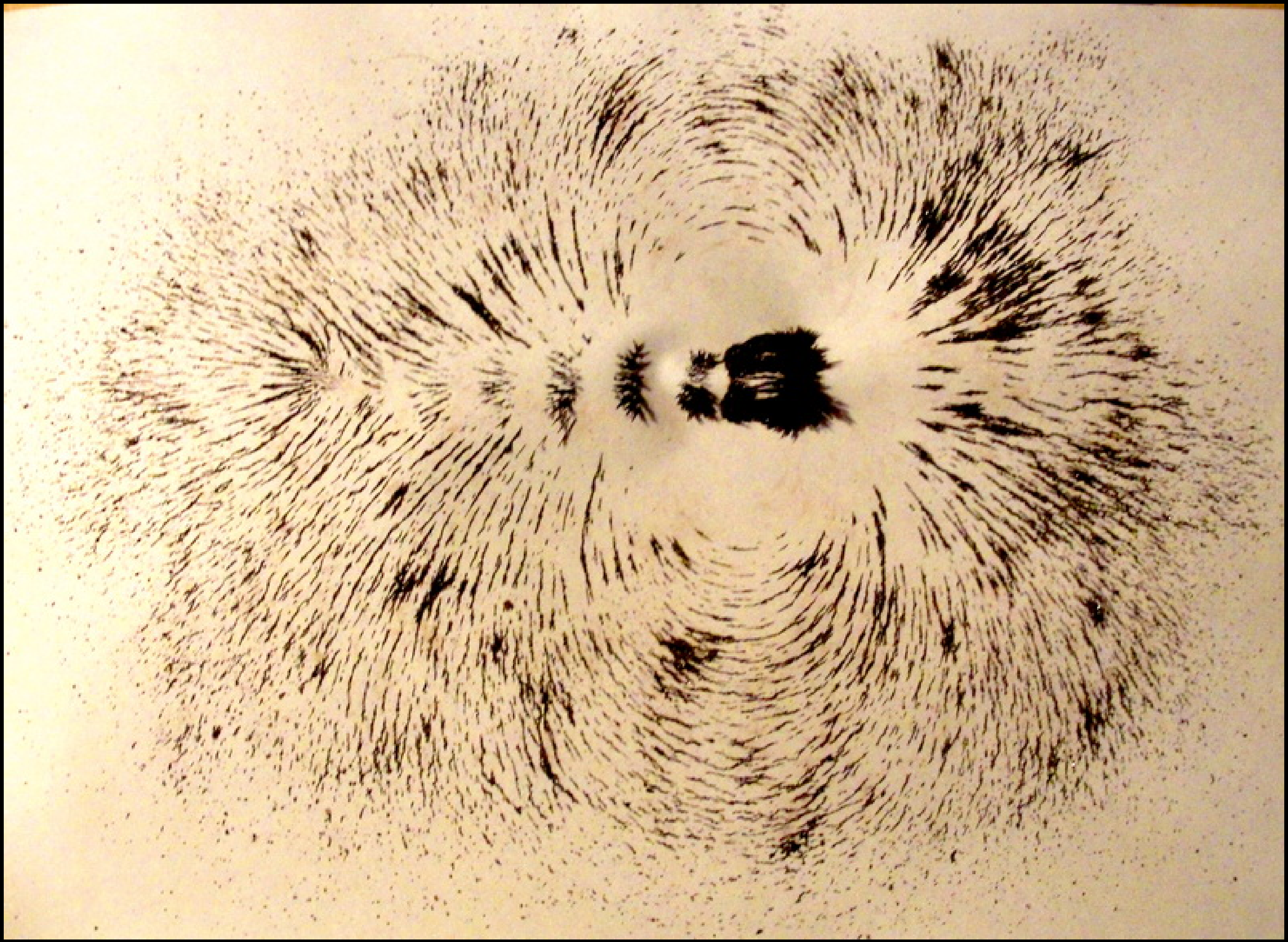




- ✓ Potential (magnetic) energy: $$U = U_{01} + U_{02} + U_{12} \quad (1)$$

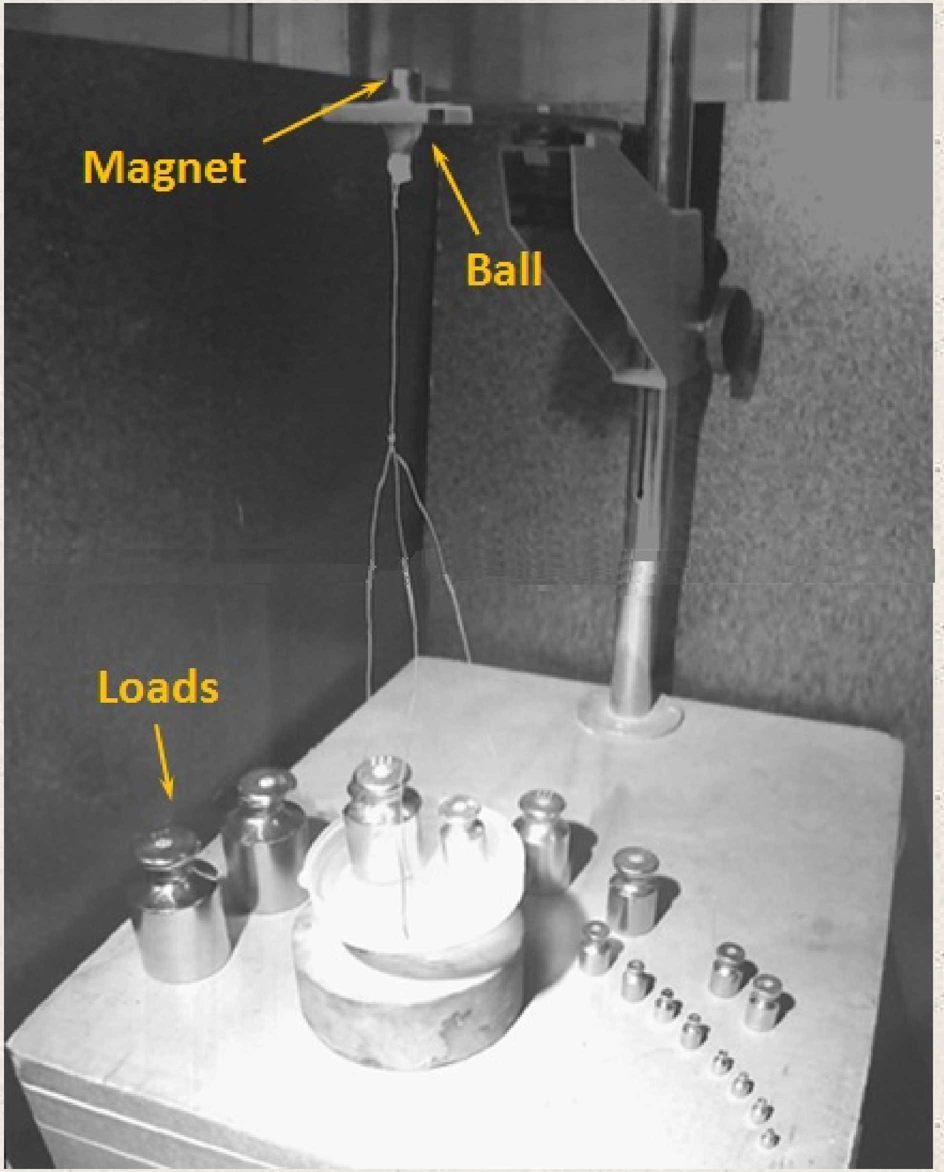
- ✓ Ball velocity: $$U_{initial} - U_{final} = \frac{mv_{final}^2}{2} \quad (2)$$

Magnet and Ball – Magnetic Dipoles

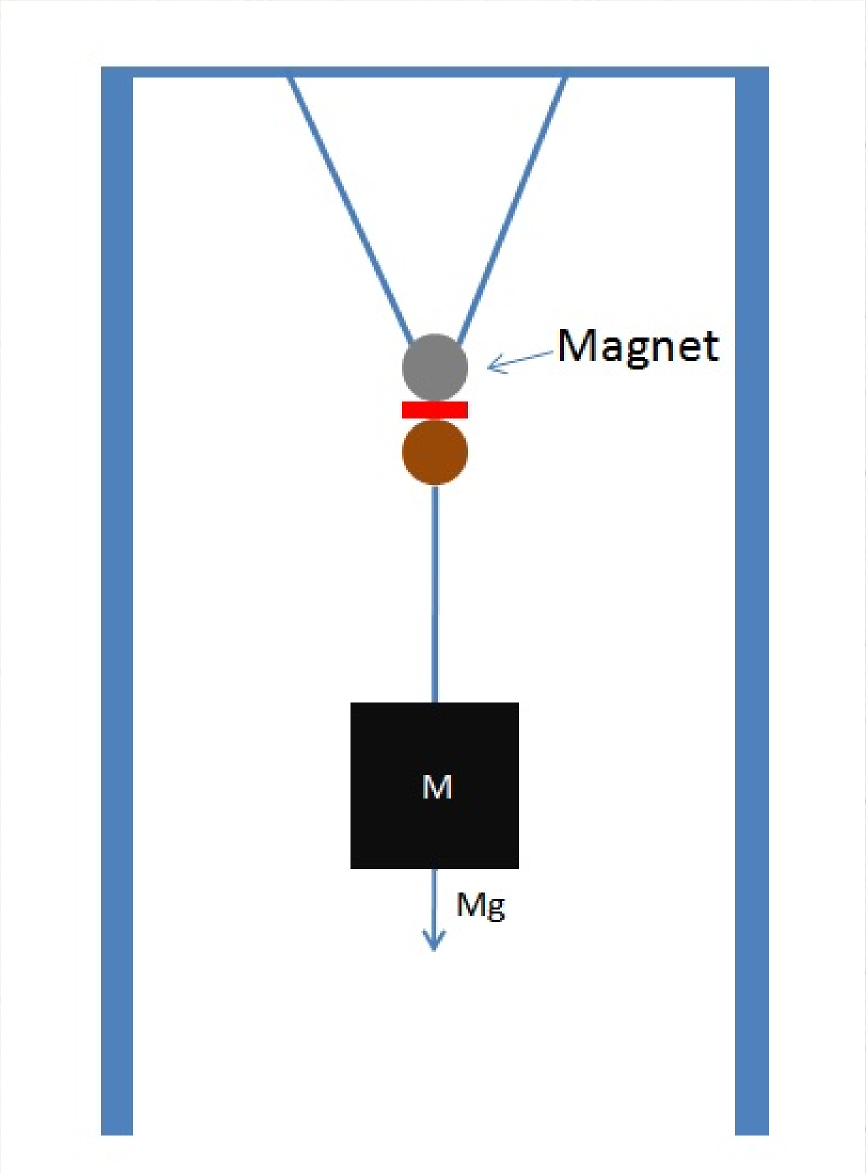


Magnetic field picture

Force measurement

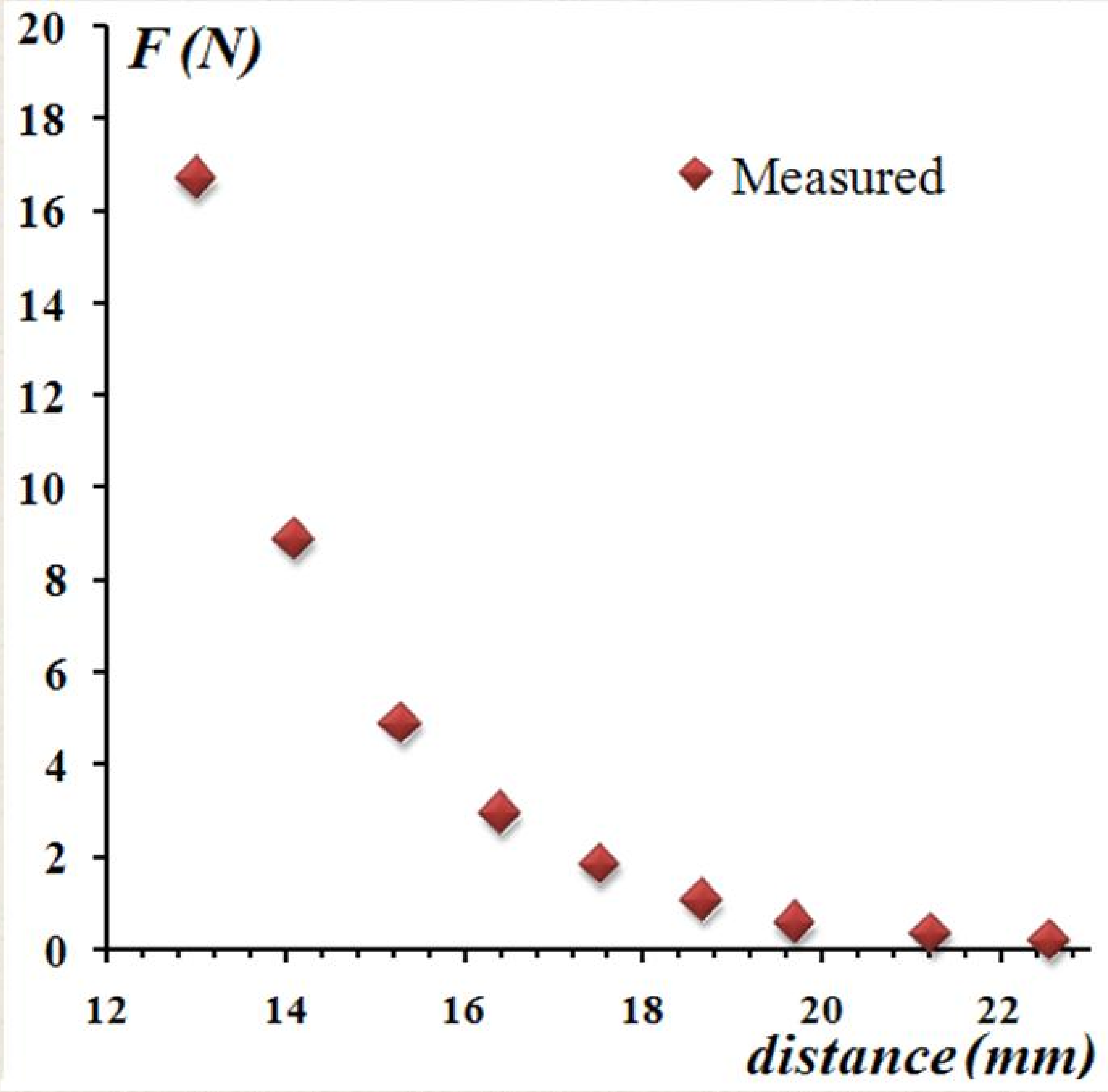


Our installation



Schematic representation

Force measurement



Acting Force

Interaction force of magnetic dipoles:

$$F\,(z, \mathfrak{M}_1, \mathfrak{M}_2) = -\frac{6\mathfrak{M}_1\mathfrak{M}_2}{z^4}$$

(z – Distance between centers,
Magnet dipole moment: $\vec{\mathfrak{M}} = \mathfrak{M}\hat{z}$)

$$F \sim z^{-7}$$

Magnetic dipole creates magnetic field in vacuum equal to [1]:

$$B = \frac{2\mathfrak{M}}{z^3}$$

Due to magnetic field, magnetic dipoles are induced in balls [1]:

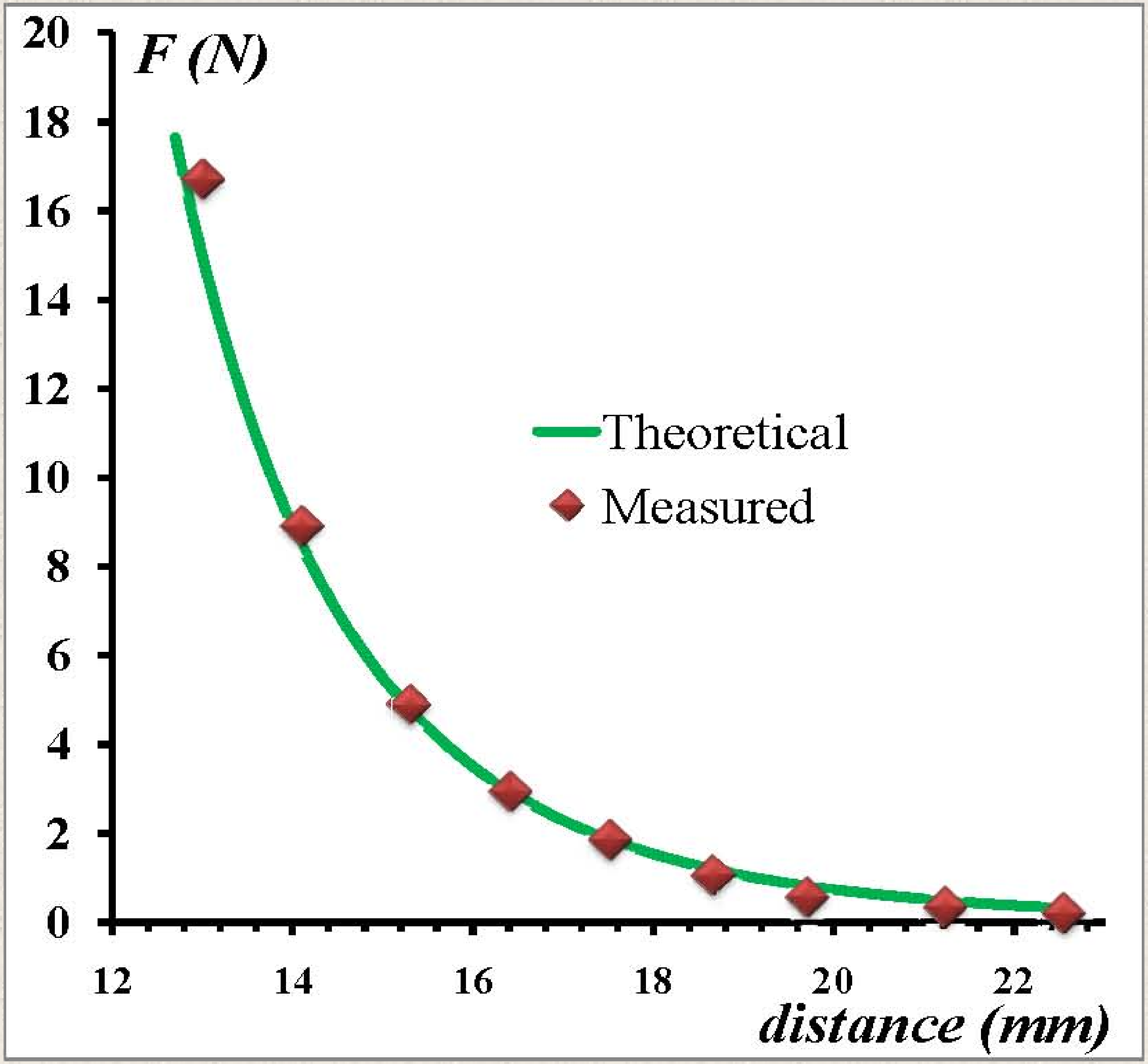
$$\mathfrak{M}_{ind} = \frac{\mu - 1}{\mu + 2} a^3 B \approx a^3 B \approx \frac{2a^3\mathfrak{M}}{z^3}$$

μ- magnetic permeability of steel

Thus the interaction force of magnet and ball is:

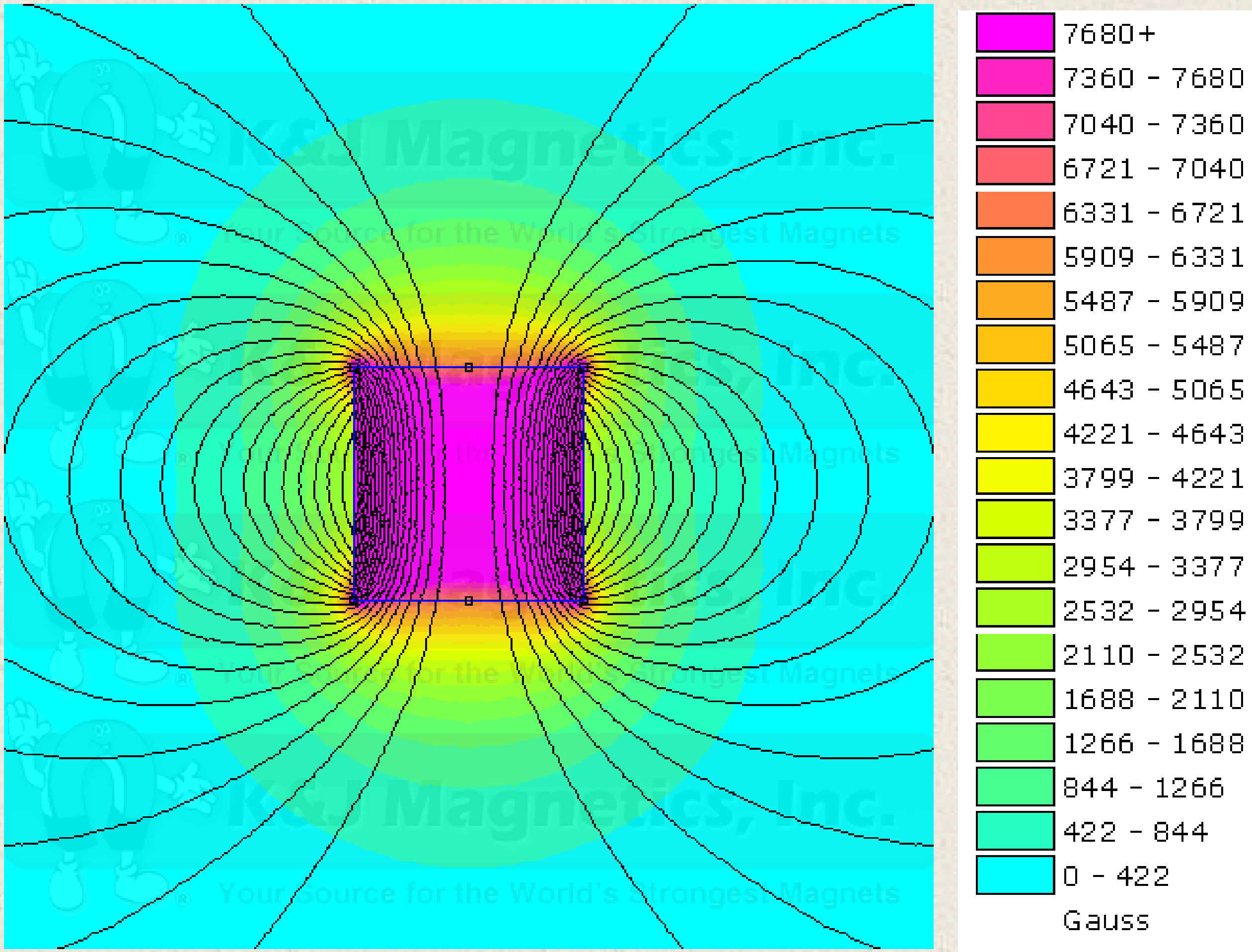
$$F\,(z, \mathfrak{M}) \approx -\frac{a^3\mathfrak{M}^2}{z^7}$$

Acting Force



Experimental Magnets

Magnetic Field Visualization
Single Magnet in Free Space
Grade = N52
Diameter = 0.5in
Thickness = 0.5in

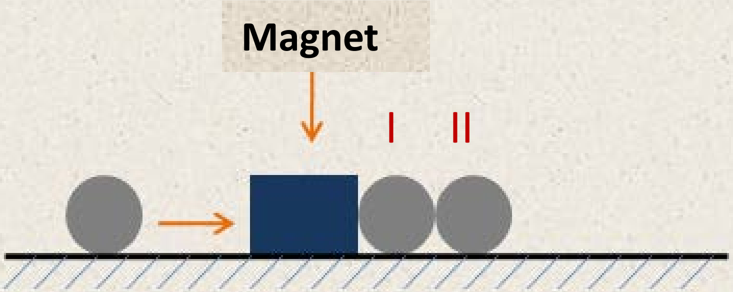


Magnet length: $2a$

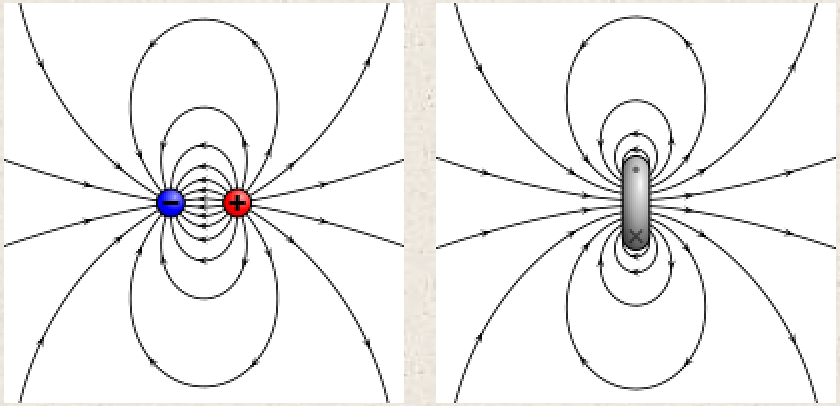
Dipole moment of magnet: $\vec{\mathfrak{M}} = \mathfrak{M}\hat{z}$

$$B_{surface} = \frac{2\mathfrak{M}}{a^3} \approx 7600 \text{ Gauss}$$

Energy – Initial Condition



- Steel ball radius: ***a=0,64 cm***
- Mass: ***m=8 g***
- Magnetic permeability of steel: ***μ = 700 » 1***
- Magnet length: ***2a***
- Dipole moment of magnet: $\vec{\mathfrak{M}} = \mathfrak{M}\hat{z}$



- Magnetic dipole creates magnetic field in vacuum equal to [1]:

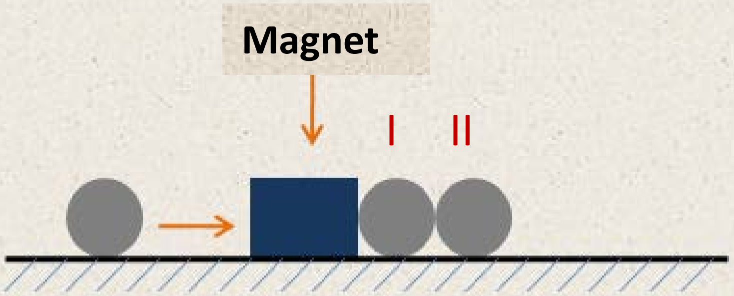
$$B = \frac{2\mathfrak{M}}{z^3} \qquad (3)$$

- Due to magnetic field, magnetic dipoles are induced in balls [1]:

$$\boxed{\mathfrak{M}_{ind}} = \frac{\mu - 1}{\mu + 2} a^3 B \boxed{\approx a^3 B} \qquad (4)$$

- Dipole energy in magnetic field: $\boxed{U = \mathfrak{M}B}$ **(5)**

Energy – Initial Condition



- Center of the 1st ball: $z=2a$
- Center of the 2nd ball: $z=4a$
- *Magnetic moment induced in ith ball* : $\mathfrak{M}_i$

- Magnetic moments in centers of 1st and 2nd balls are:

$$\mathfrak{M}_1 \approx a^3 B_1 \quad ; \qquad \mathfrak{M}_2 \approx a^3 B_2 \qquad (6)$$

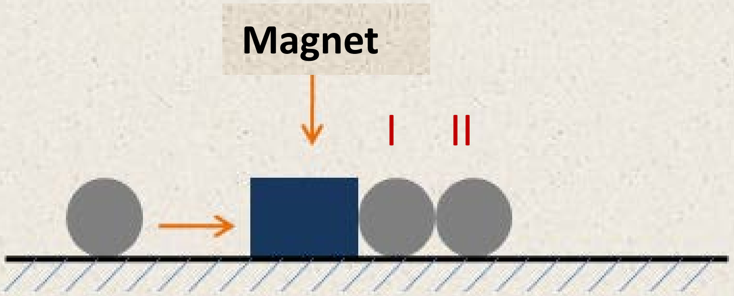
- Magnetic field in centers of 1st and 2nd balls :

$$B_1 = \frac{2\mathfrak{M}}{(2a)^3} + \frac{2\mathfrak{M}_2}{(2a)^3} = \frac{\mathfrak{M} + \mathfrak{M}_2}{4a^3} \qquad B_2 = \frac{2\mathfrak{M}}{(4a)^3} + \frac{2\mathfrak{M}_1}{(2a)^3} = \frac{\mathfrak{M} + 8\mathfrak{M}_1}{32a^3} \qquad (7)$$

- B_i - Magnetic field in the center of ith ball

Energy – Initial Condition

$$U = \mathfrak{M}B \qquad (5)$$



- Using (6) and (7) - magnetic moments induced in balls are:

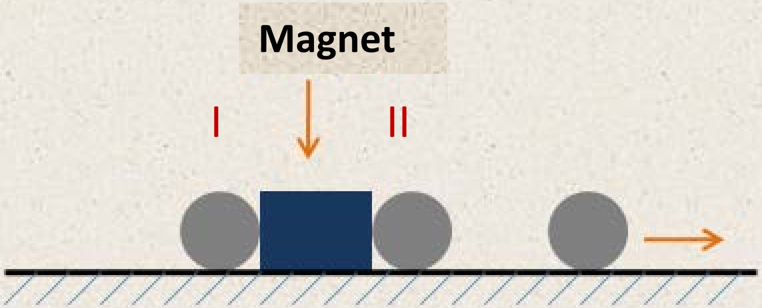
$$\mathfrak{M}_1 = \frac{11}{40}\mathfrak{M} \qquad \mathfrak{M}_2 = \frac{1}{10}\mathfrak{M} \qquad (8)$$

$$B_1 = \frac{2\mathfrak{M}}{(2a)^3} + \frac{2\mathfrak{M}_2}{(2a)^3} = \frac{\mathfrak{M} + \mathfrak{M}_2}{4a^3} \qquad B_2 = \frac{2\mathfrak{M}}{(4a)^3} + \frac{2\mathfrak{M}_1}{(2a)^3} = \frac{\mathfrak{M} + 8\mathfrak{M}_1}{32a^3} \qquad (7)$$

- Using (5),(7),(8) - Magnetic energy of initial condition is:

$$U_{Initial} = U_{01} + U_{02} + U_{12} = -\frac{63}{800}\frac{\mathfrak{M}^2}{a^3} \approx -0.079\frac{\mathfrak{M}^2}{a^3}$$

Energy – Final Condition



Magnetic field in centers of 1st and 2nd balls:

$$B_1 = \frac{2\mathfrak{M}}{(2a)^3} + \frac{2\mathfrak{M}_2}{(4a)^3} = \frac{8\mathfrak{M} + \mathfrak{M}_2}{32a^3} = B_2$$

Induced magnetic moments in balls: $\mathfrak{M}_1 = \mathfrak{M}_2 = \frac{8}{31}\mathfrak{M}$

Using (5), (10) and (11) potential (magnetic) energy of final condition is:

$$U_{Final} = U_{01} + U_{02} + U_{12} = -\frac{126}{961}\frac{\mathfrak{M}^2}{a^3} \approx -0.131\frac{\mathfrak{M}^2}{a^3}$$

Theoretical estimation of velocity

Potential (magnetic) energy difference of initial and final conditions:

$$\left|U_{initial} - U_{final}\right| \approx 0.052\frac{\mathfrak{M}^2}{a^3}$$

The velocity is calculated using kinetic energy:

$$\frac{mv_{final}^2}{2} = \left|U_{initial} - U_{final}\right|$$

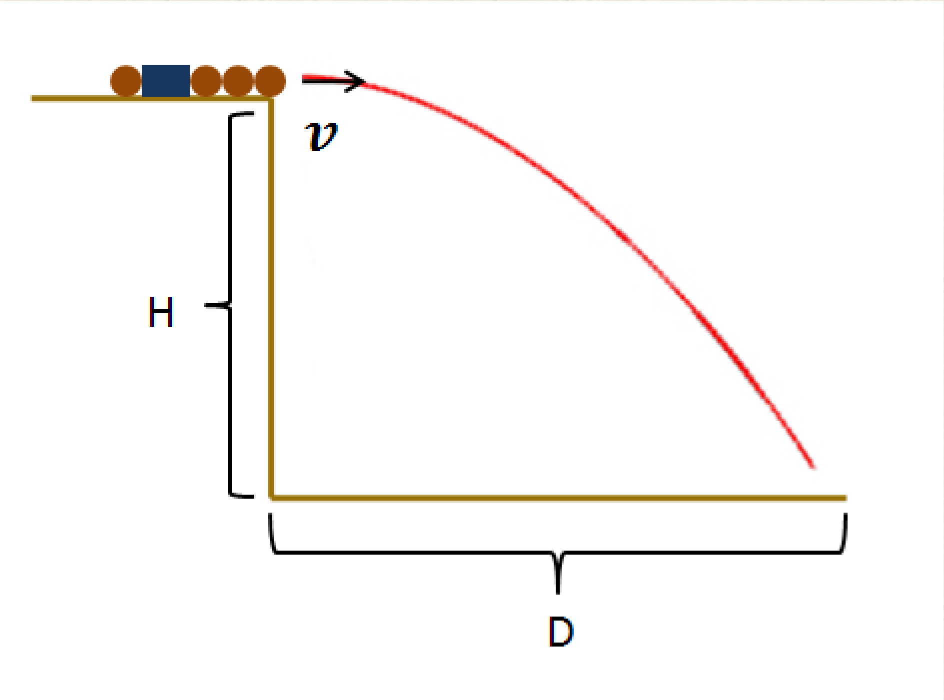
In our experiments:

$$v_{final}^2 = \frac{0.052}{2m} \cdot B_{surface}^2 a^3 \approx 49\,000\left(\frac{cm}{sec}\right)^2$$

For single-stage 1+2 balls case:

$$v_{final} \approx 2.2\,\frac{m}{sec}$$

Experimental calculation of velocity



$$\frac{gt^2}{2} = H \longrightarrow t = \sqrt{\frac{2H}{g}}$$

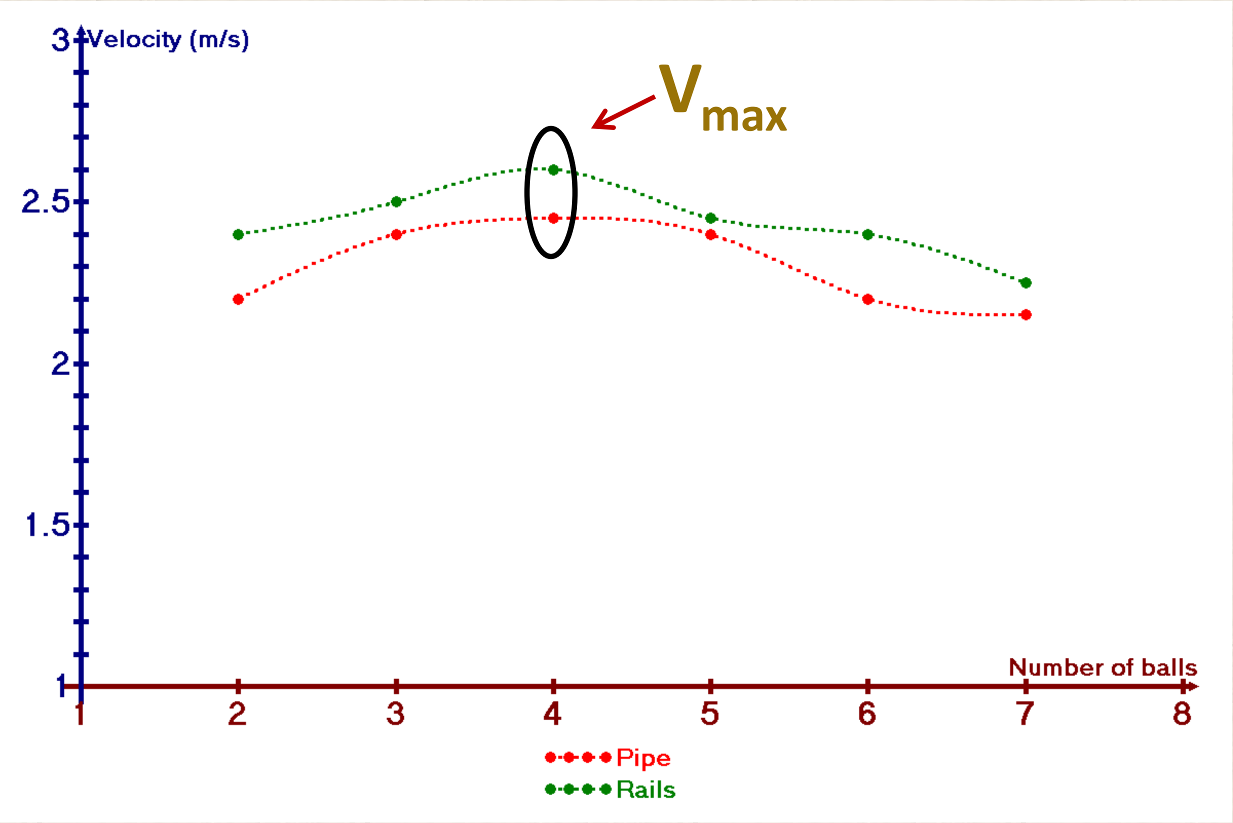
$$v = \frac{D}{t} \longrightarrow v = D\sqrt{\frac{g}{2H}}$$

$$v_{exper} \approx 2.2 \div 2.5 \ \frac{m}{sec}$$

Different number of balls for 1 section

Video (3-7)

Different number of balls for 1 section



➢ **Maximal velocity is achieved with 1+4 balls**

Reasons:

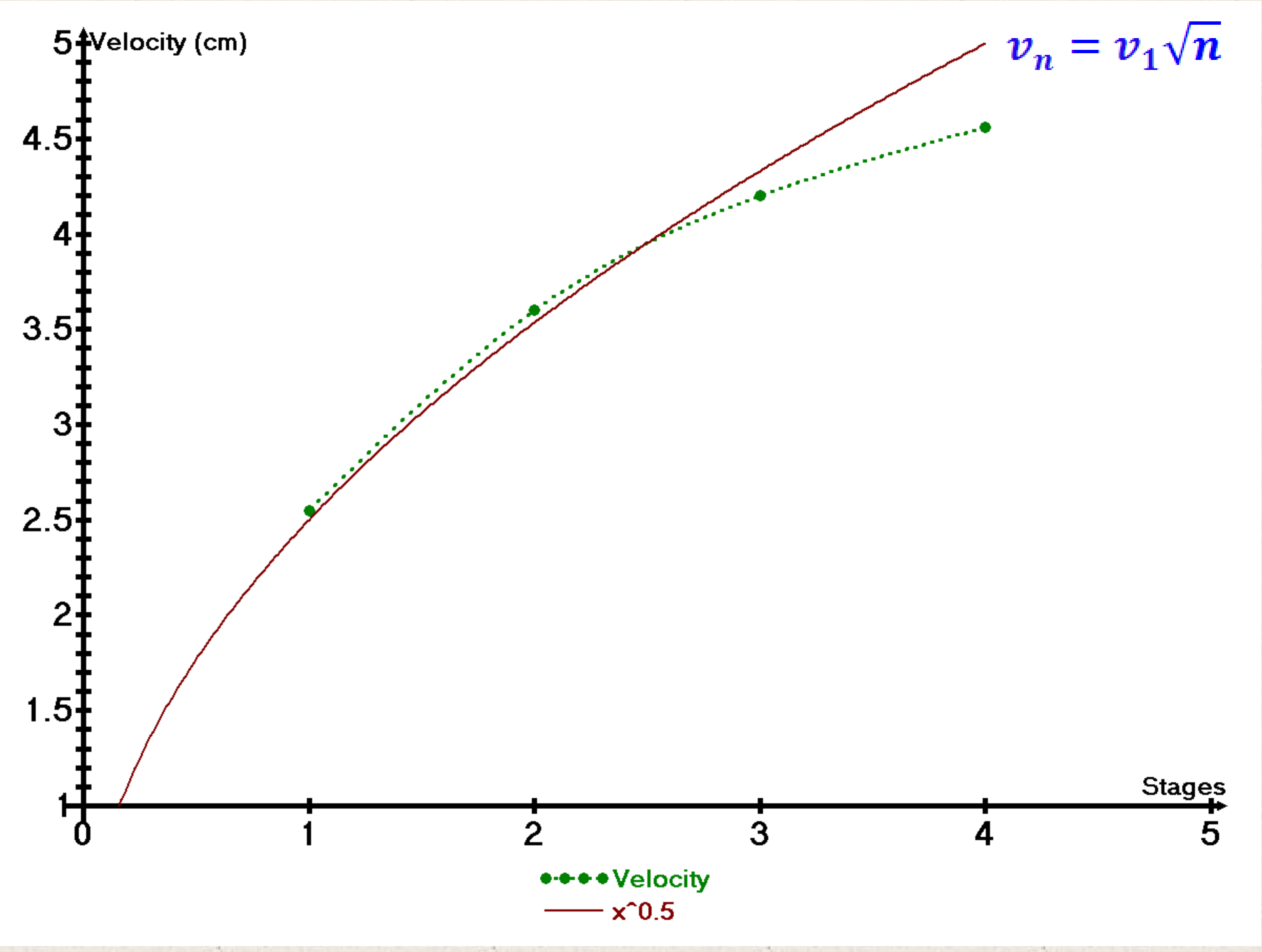
- **Energy loss (heat, rotation)**
- **Alignment of balls** → **Comparison of Pipe and Rails**

Multi-sectional installation

Slow motion shot

Shot

Velocity for "n" sections



Important Points of construction

- Fixing the magnet to avoid it’s backward movement
- Alignment of balls strictly along one line
- In case of multi-stage cannon alignment of magnet poles
- Using same-size balls and magnet
- Consideration of using non-magnetic balls behind the magnet
- Consideration of different masses of the last ball

Conclusion

- We constructed the Gaussian cannon
- Energy source of cannon - difference of the initial and the final potential energies
- Interaction of magnet and ball = interaction of magnetic dipoles
- Optimal configuration of cannon:
  One ball before magnet, other balls (2 or more) - behind the magnet
- We calculated the ball velocity for 1+2 ball variant of cannon. Theoretical calculations coincide with experimental results.
- In one-stage cannon maximal velocity is achieved for 1+4 ball configuration.
- We considered some features of multi-stage cannon
- **The very important points of the construction are:**
  - Fixing magnet to avoid it's backward movement
  - Alignment of balls strictly along one line
  - Using same-size balls and magnet

Thank you for attention!

References:

1. Kirk T. McDonald. **A Magnetic Linear Accelerator.** *Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544.* (March 3, 2003)

2. Christian Ucke, Hans Joachim Schlichting. **Die Magnetkanone.** Phys. Unserer Zeit. 3/2009 (40). P. 152

3. http://en.wikipedia.org/wiki/Force_between_magnets

4. http://en.wikipedia.org/wiki/Dipole ; http://en.wikipedia.org/wiki/Magnetic_dipole