Rising bubble

A vertical tube is filled with a viscous fluid. On the bottom of the tube there is a large air bubble. Study the bubble rising from the bottom to the surface.

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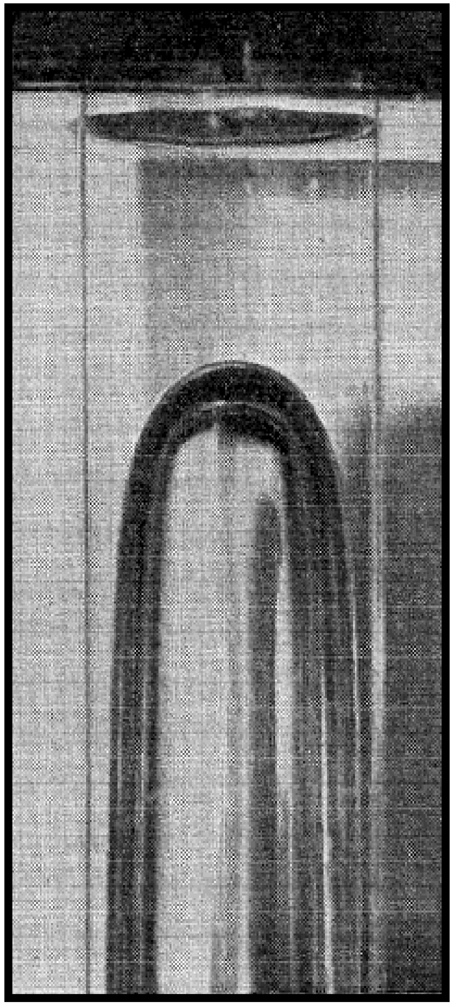
Taylor bubbles

Large bubble is a long bubble.

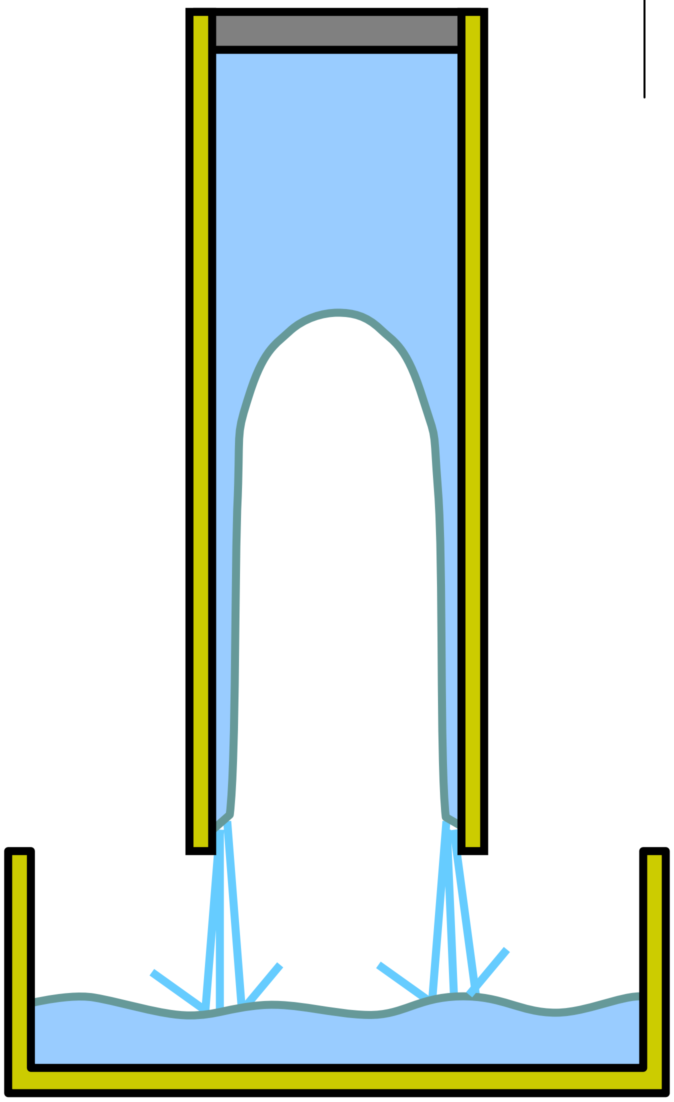
Rising of long air bubbles in vertical circular tubes (not capillaries) was studied by Taylor & Davies (1950). Tubes were filled with water, so the rising regime was nonviscous with Re >> 1.



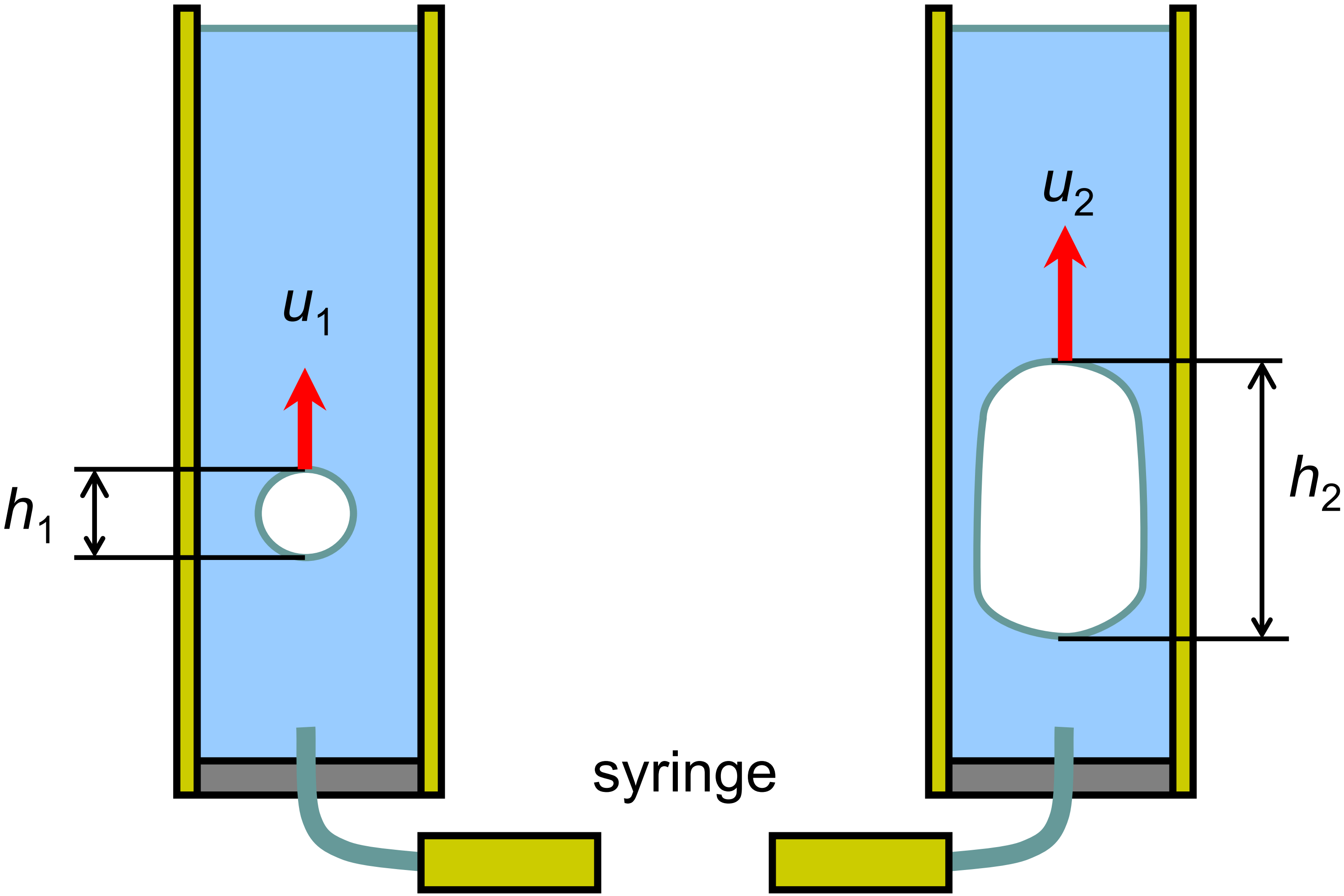
Taylor & Davies 1950

Main property of the long bubble

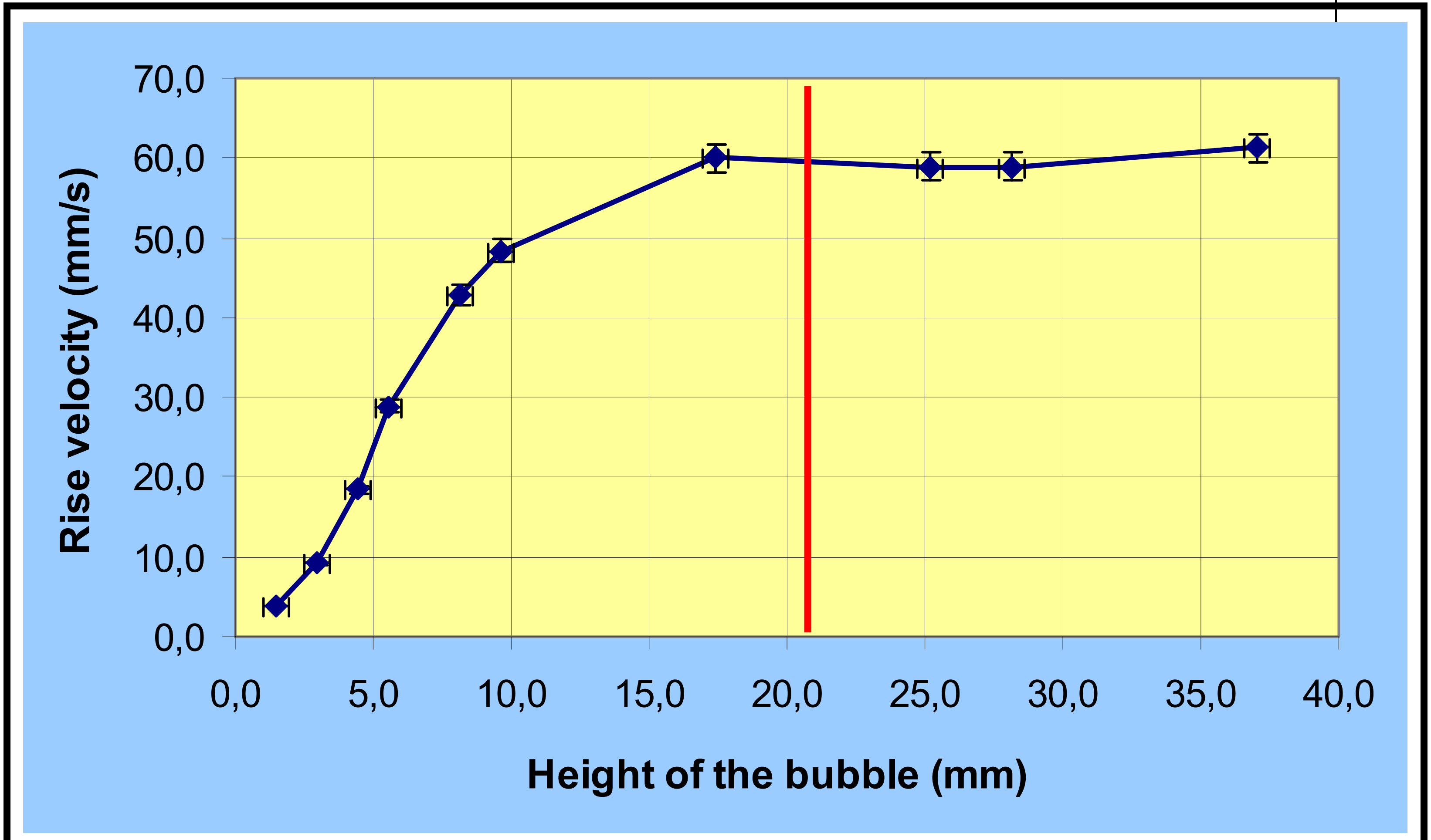
Rise velocity of long bubble does **not depend on its length**. This velocity will be the same even if the bottom of the tube is opened to the air.



What happens when the volume of the bubble increases?



Experimental results



Diameter of the tube = 21 mm; liquid = glycerin.

Rise velocity in nonviscous regime (Re >> 1)

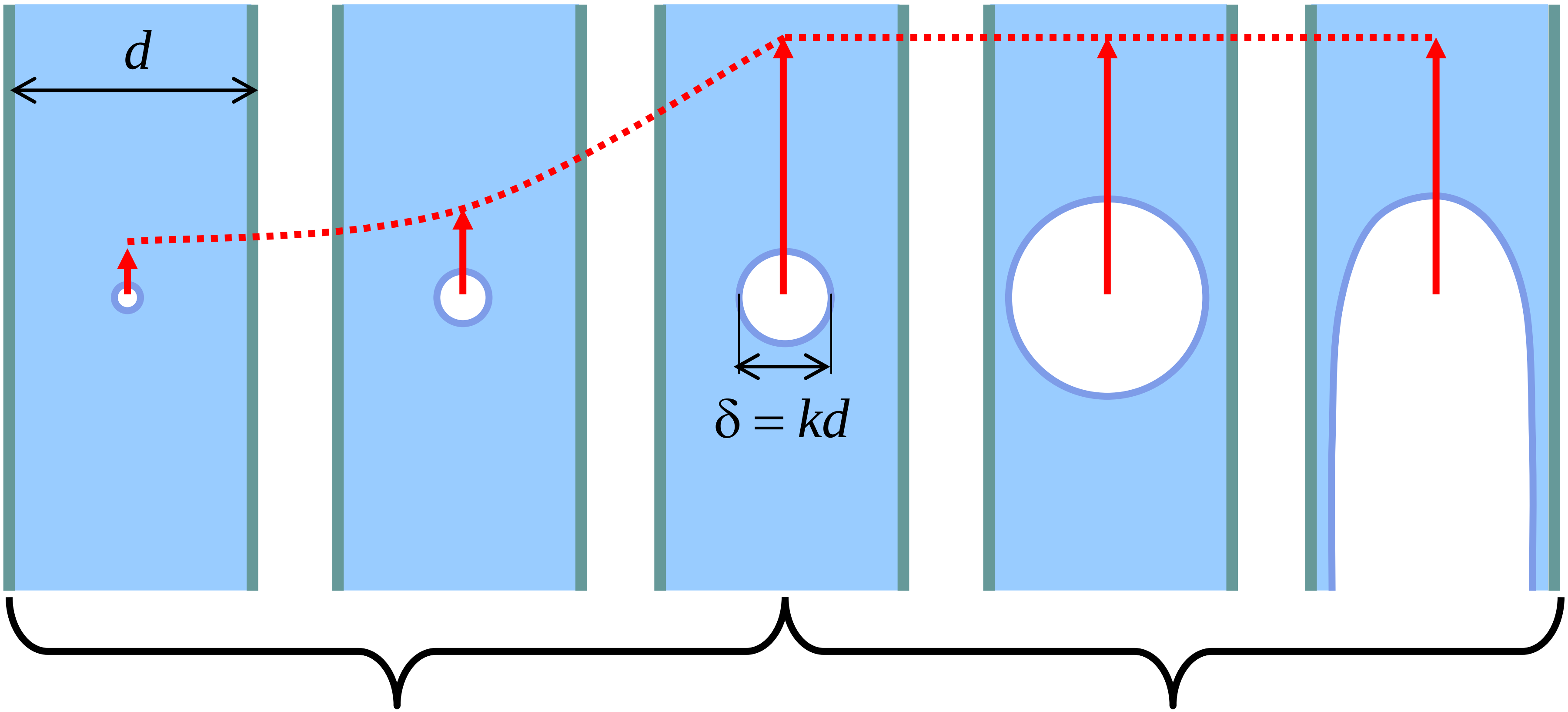
- If viscosity of water is negligible, then

$$u \sim \sqrt{gd}$$

- Taylor & Davies found theoretically and experimentally that

$$u = 0.33\sqrt{gd}$$

A model for rise velocity in viscous regime (Re << 1)



Stokes formula $u = \dfrac{g\delta^2}{12\nu}$

Velocity is const $u = k^2 \cdot \dfrac{gd^2}{12\nu}$

Liquids we used

T = 27° C	Viscosity ν, 10^{-6} m^2/s
Water	1
Sunflower oil	65 ± 3
Glycerin	1030 ± 50
Shampoo	2900 ± 150

Viscosity was measured with a capillary viscometer.

Viscometer

$$\nu = \frac{g(H+L)}{L} \cdot \frac{d^4}{32D^2} \cdot \frac{t}{h_1 - h_2}$$

ν — kinematic viscosity;

h_1, h_2 — liquid level at the beginning and end of measurement;

$H = (h_1 + h_2)/2$;

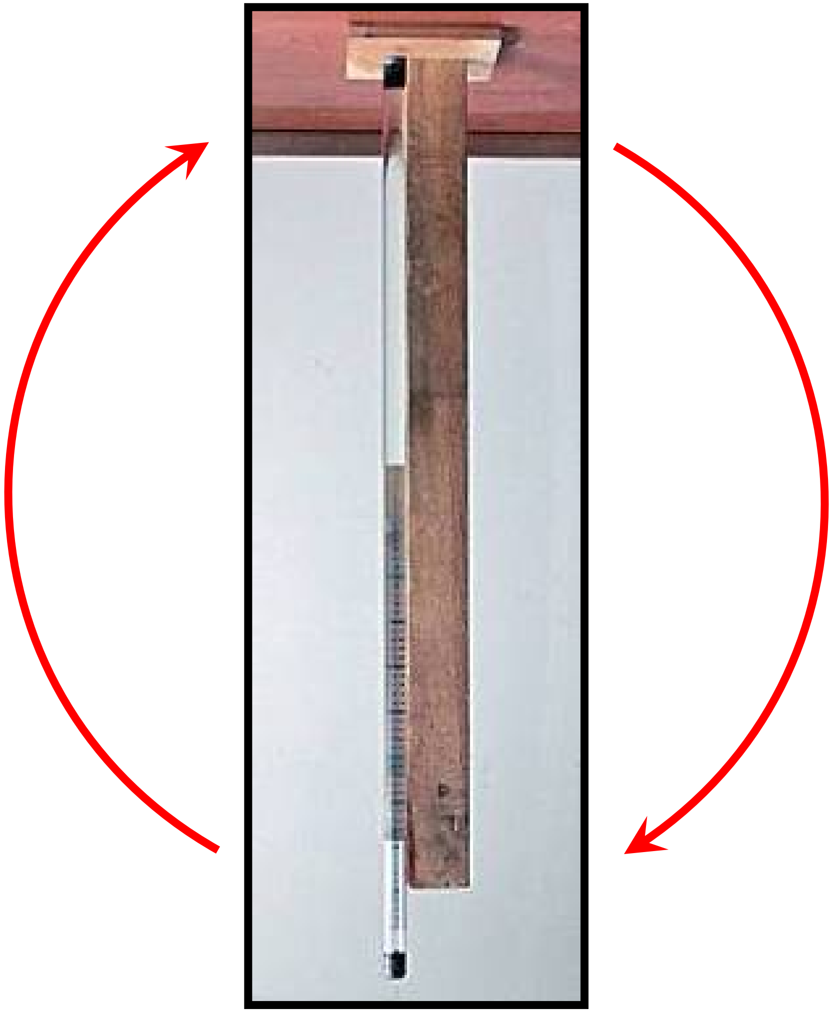
t — time of measurement;

L — length of the tube;

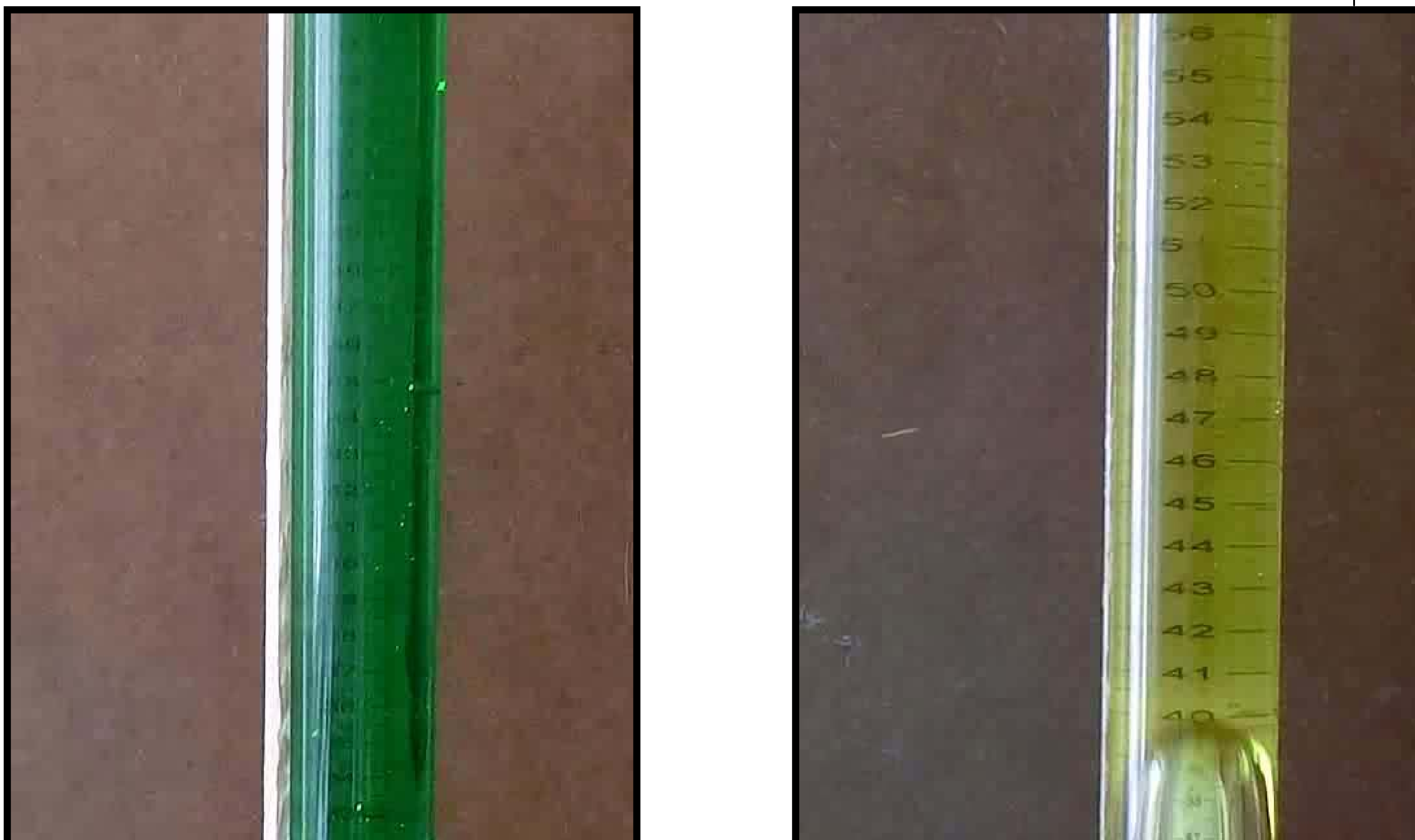
d — diameter of the tube;

D — diameter of the vessel.

Scheme of the experiment

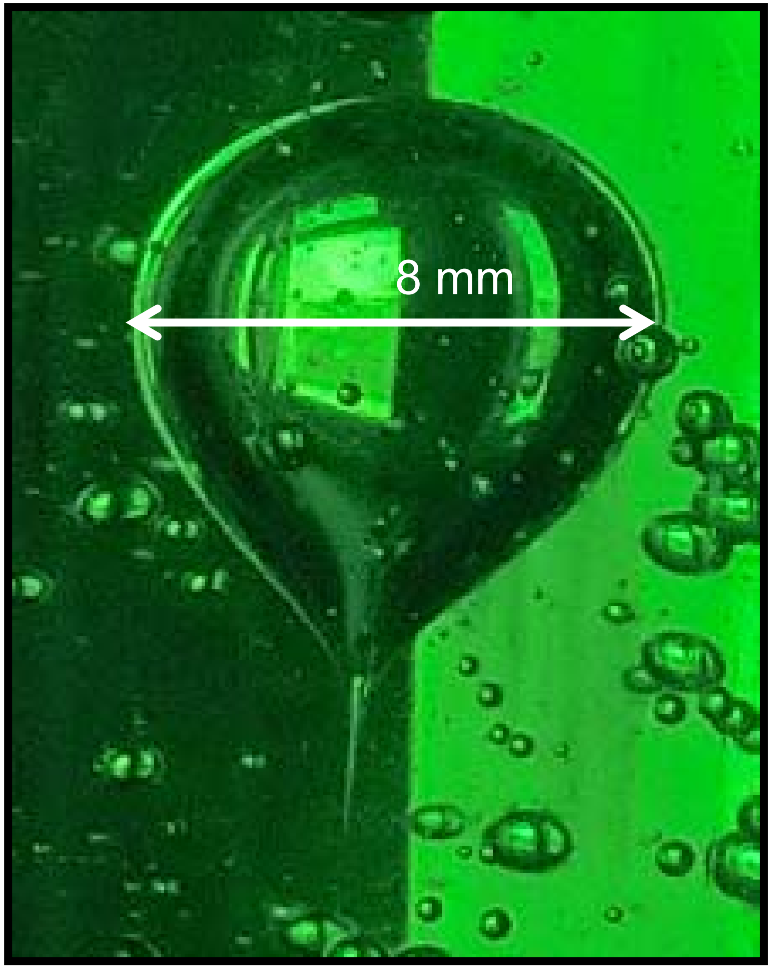


Shape of the bubbles



Diameter = 35 mm. Shampoo = 13.6 cm/s, sunflower oil = 20.8 cm/s

Shampoo is a non-Newtonian fluid

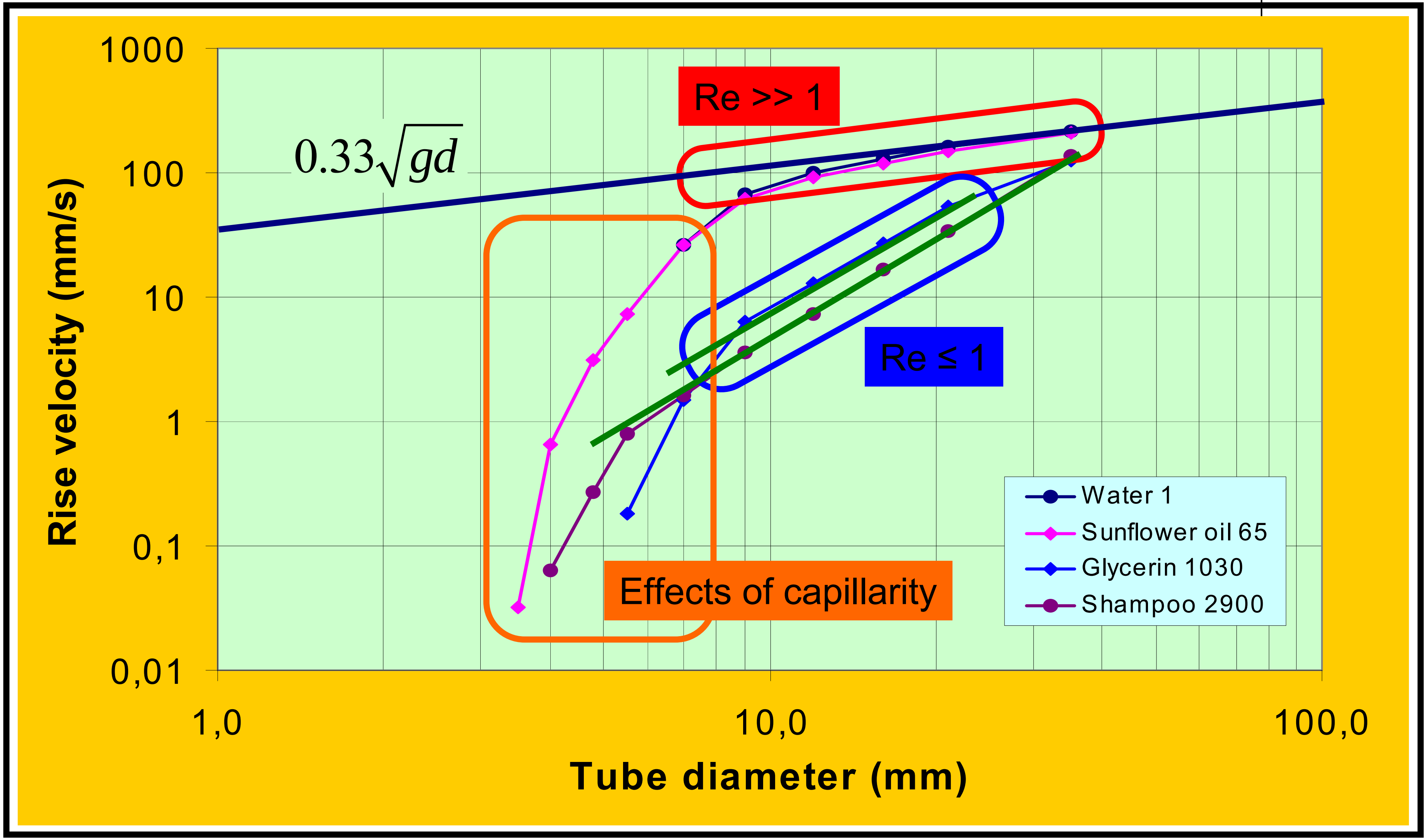


- In a Newtonian fluid the viscous drag force (shear stress) is proportional to the velocity gradient (strain rate).
- In a non-Newtonian fluid this relation does not hold.

Experimental results with Reynolds numbers

Diameter of the tube (mm)	7,0	9,0	12,0	16,0	21,0	35,0
Velocity in water (mm/s)	26	68	101	131	163	217
Re in water	200	600	1000	2000	3000	7000
Velocity in sunflower oil (mm/s)	26	62	91	120	150	208
Re in sunflower oil	2,5	7	15	25	50	100
Velocity in glycerin (mm/s)	1,5	6,3	13	27	54	124
Re in glycerin	0,001	0,05	0,2	0,4	1	4
Velocity in shampoo (mm/s)	1,5	3,7	7,3	16	34	136
Re in shampoo	0,004	0,01	0,03	0,09	0,25	1,7

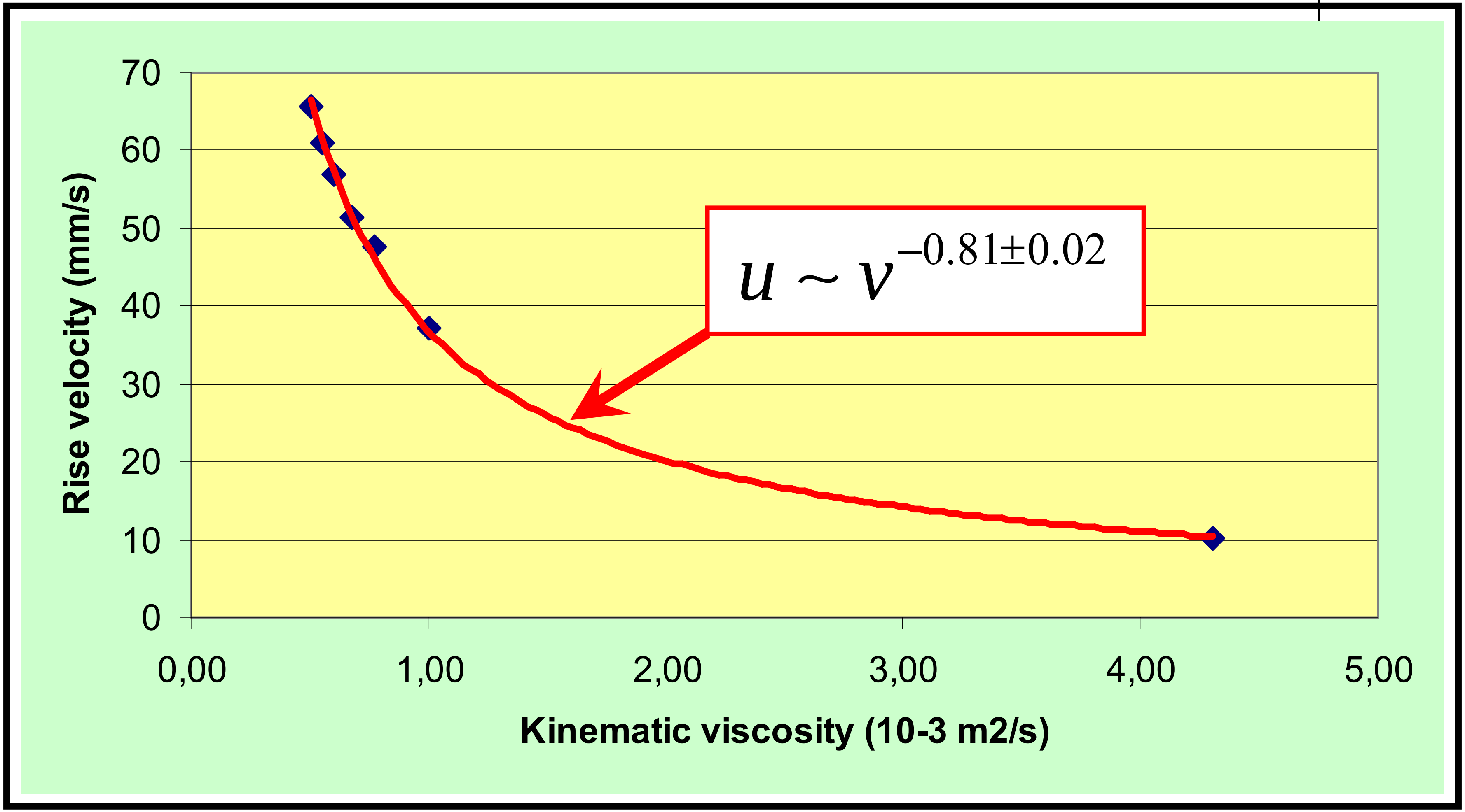
Rise velocity vs. the tube diameter



Determination of the exponent

- Theoretical prediction: $$u \sim \frac{d^2}{\nu}$$
- Suppose $u \sim d^{\alpha}$
- $(u_1 : u_2) = (d_1 : d_2)^{\alpha}$
- $\alpha = (\log u_1 - \log u_2) : (\log d_1 - \log d_2)$
- Glycerin **$\alpha \approx 2.5 \pm 0.1$**
- Shampoo **$\alpha \approx 2.7 \pm 0.1$**

Rise velocity vs. kinematic viscosity



Diameter of the tube = 16 mm; liquid = glycerin.

Empirical formula for the rise velocity in glycerin

- If surface tension is negligible (d ≥ 9 mm),
- and rising regime is viscous (d ≤ 25 mm),
- then the rise velocity of Taylor bubbles in glycerin depends on the tube diameter and on the kinematic viscosity such a way:

$$u \sim \frac{d^{2.5}}{\nu^{0.8}}$$

Instead of **2**

Instead of **1**

Summary

- Historical background
- Main property of long bubbles
- Nonviscous and viscous regimes
- Model for a viscous regime
- Experiment with various liquids
- Determination of exponents
- Comparison of experiment with theory

Bibliography

- Batchelor G.K. (1967) *An introduction to fluid mechanics*.
- Taylor G., Davies R.M. (1950) "The mechanics of large bubbles rising through extended liquids and through liquids in tubes". *Proc. Royal Soc. A*, **200**, 375-390.

Coefficient *k* for glycerin

Tube diameter (mm)	9,0	12,0	16,0	21,0
Measured velocity of Tailor bubble v_T (mm/s)	7,5	18	33	62
Calculated velocity of Stokes bubble v_S (mm/s)	98	174	310	534
$k = \sqrt{v_T / v_S}$	0,28	0,32	0,33	0,34

$$u \approx 0,01 \cdot \frac{gd^2}{\nu}$$