# String of beads

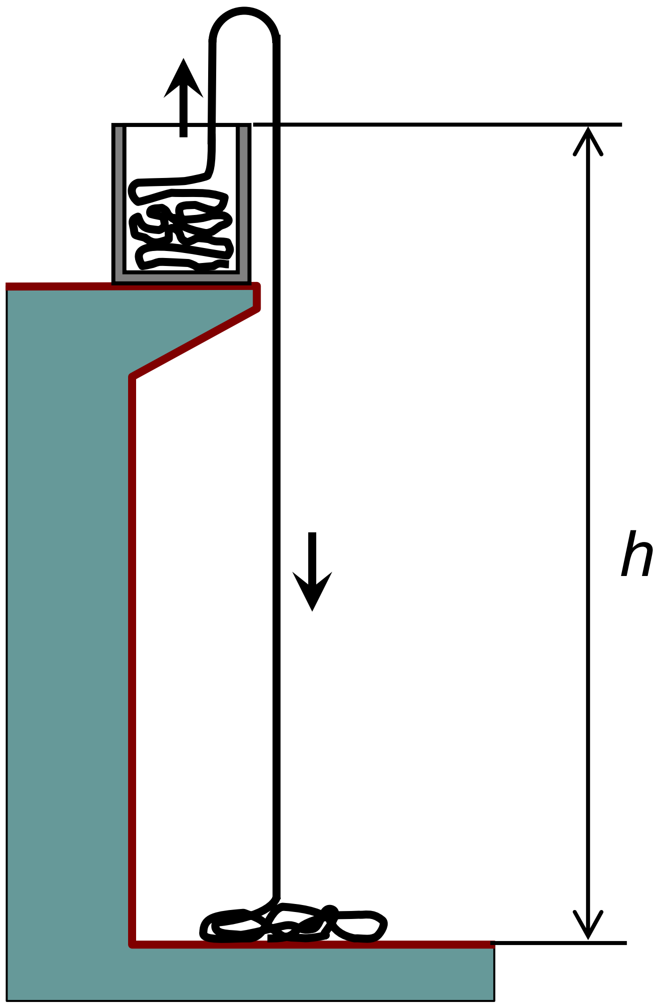
A long string of beads is released from a beaker by pulling a sufficiently long part of the chain over the edge of the beaker. Due to gravity the speed of the string increases. At a certain moment the string no longer touches the edge of the beaker. Investigate and explain the phenomenon.

Vyacheslav Matyunin, Russia

# Beads for the fountain

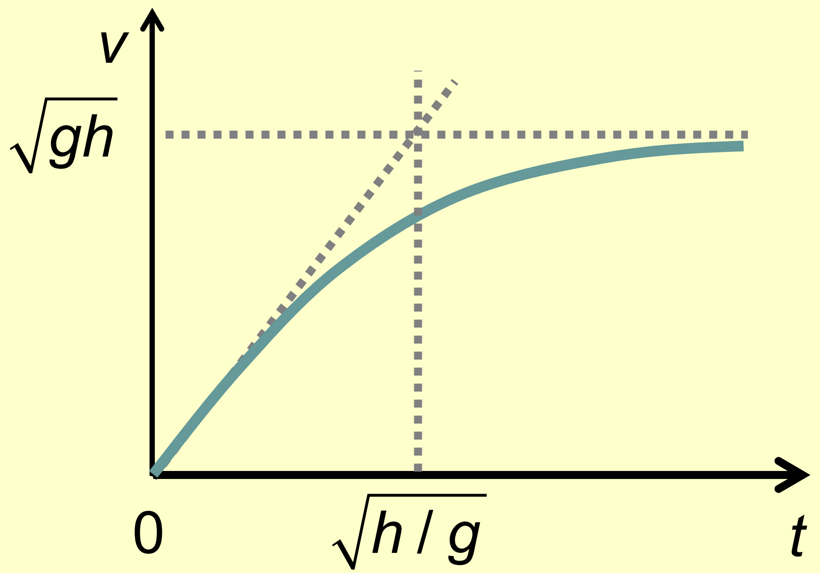
The fountain is created only with beads, which are separated from each other.

# Scheme of the experiment



# Velocity of beads vs. time

$$v = \sqrt{gh} \cdot \mathrm{th}\left(\frac{t}{\sqrt{h/g}}\right)$$



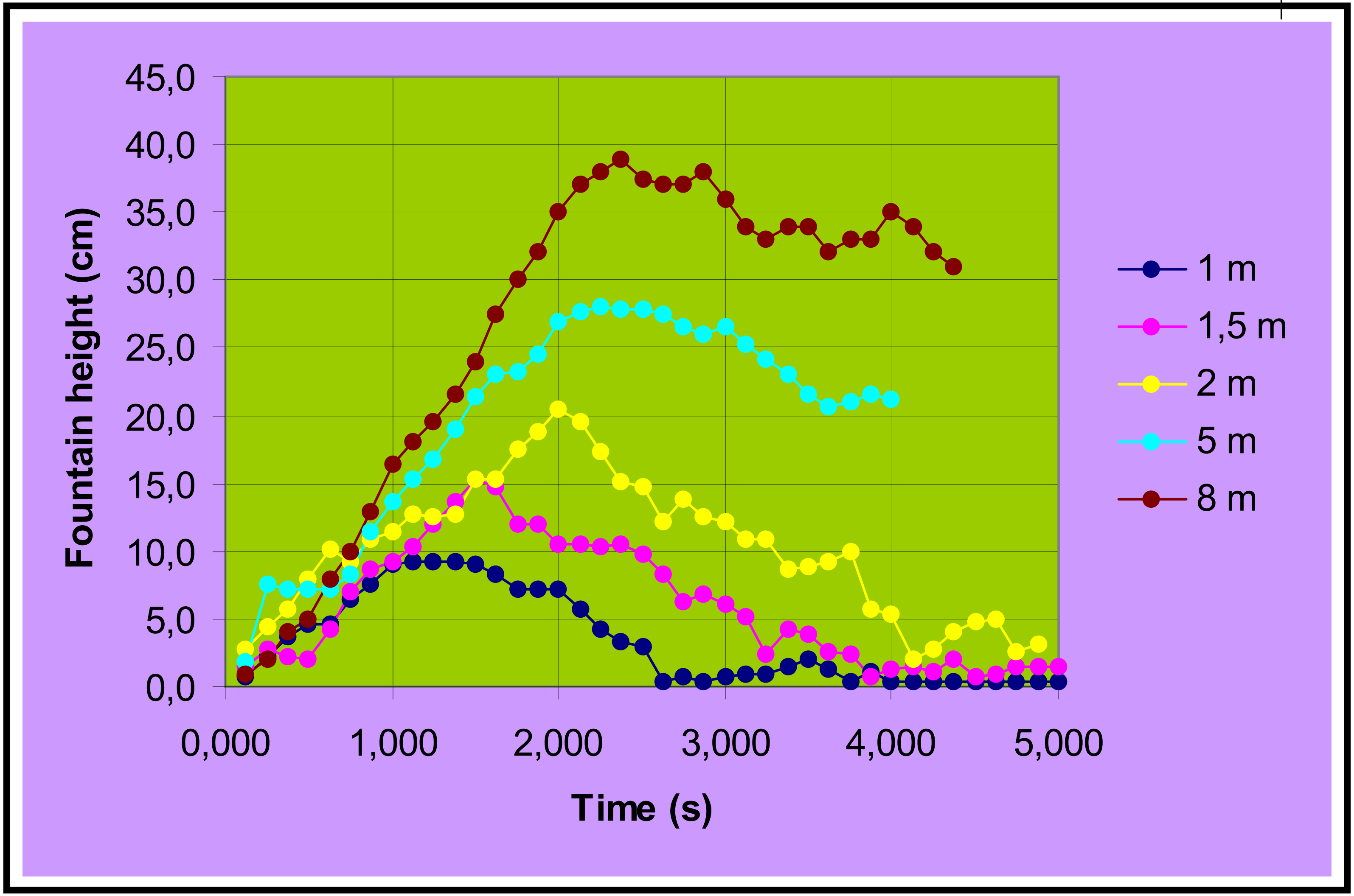
# Video 240 fps

35 meters of beads fall from the height 8 m.

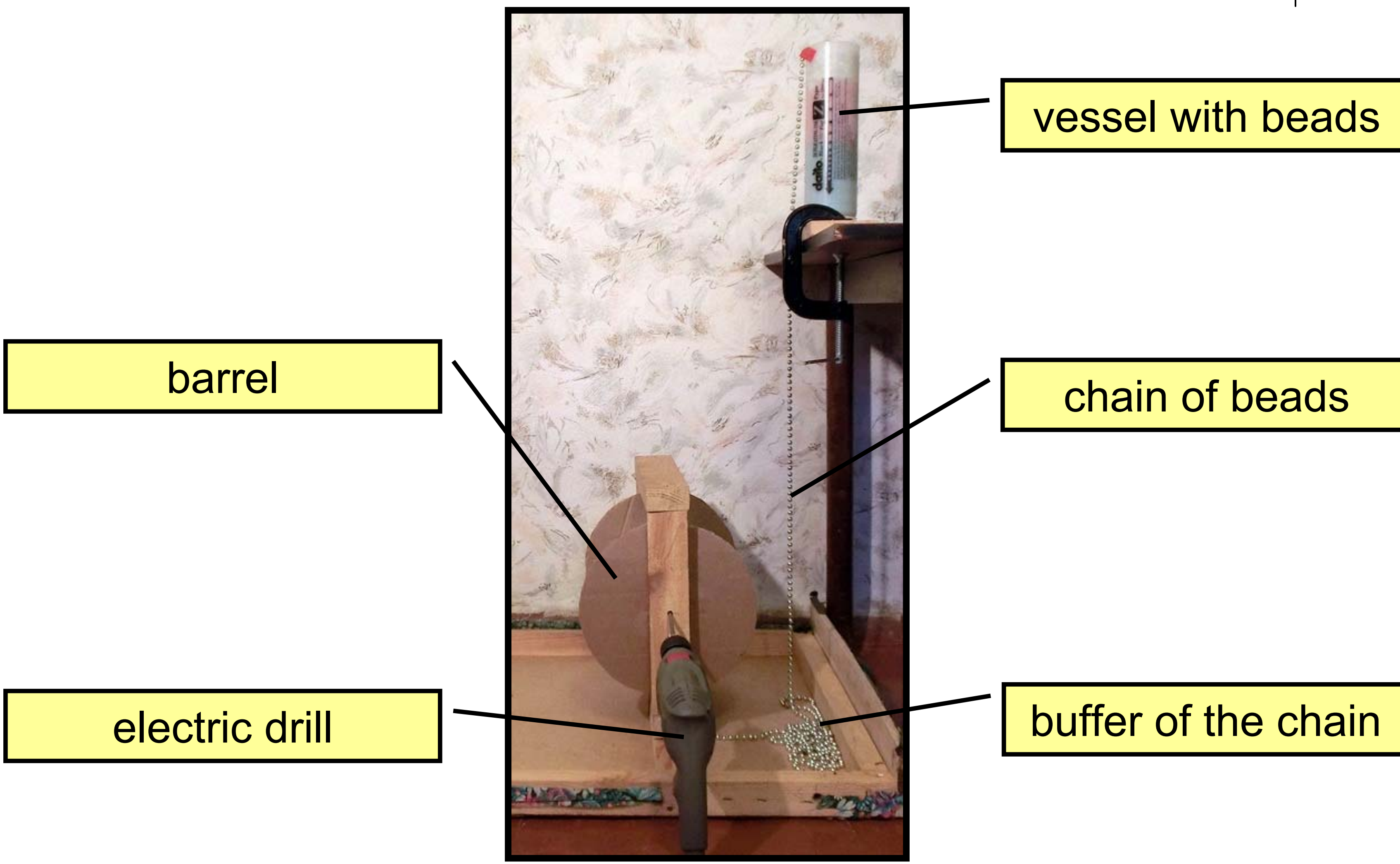
# What happens when we change the diameter of the vessel

$D$ = 8 cm $D$ = 12 cm $D$ = 20 cm

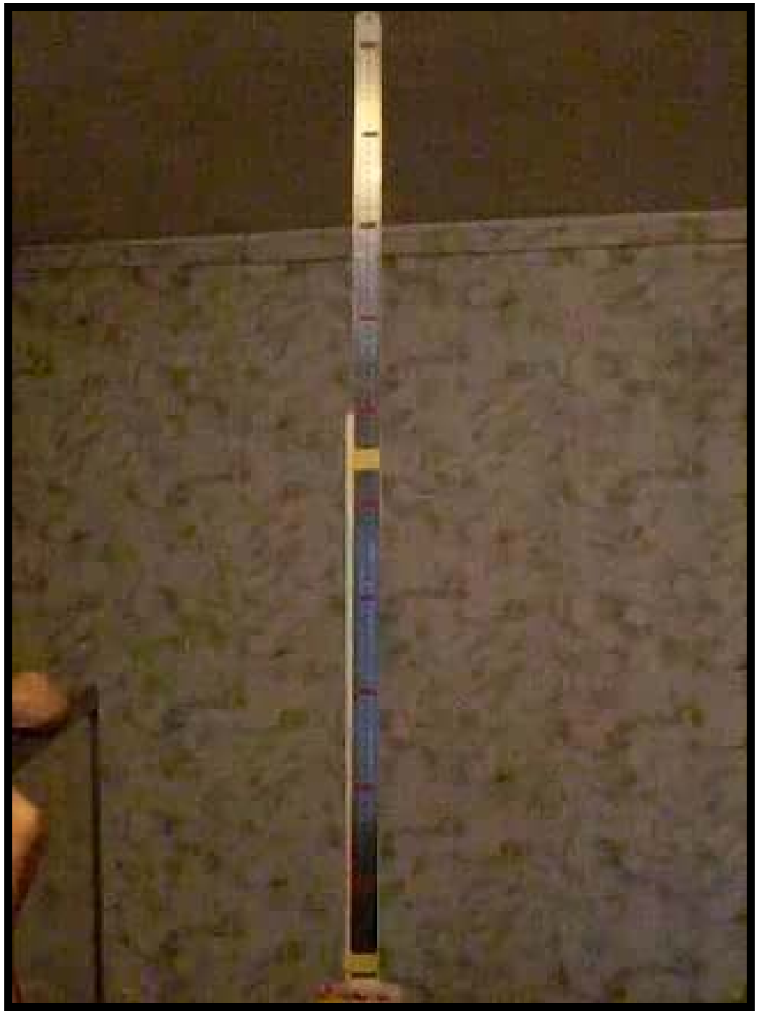
# Results of the experiment



# Experiment with an electric drill

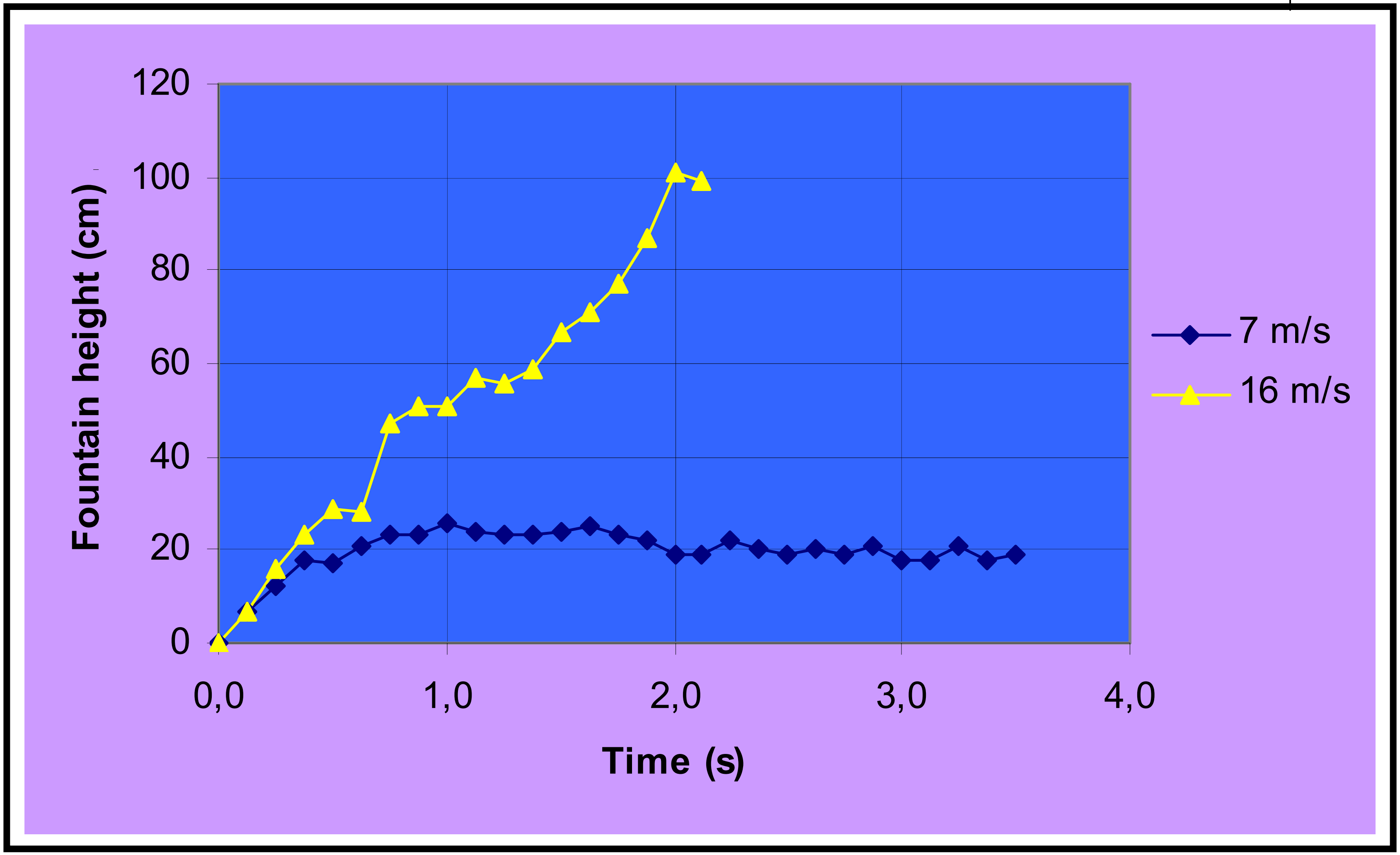


# Video 240 fps: the record height and the accident

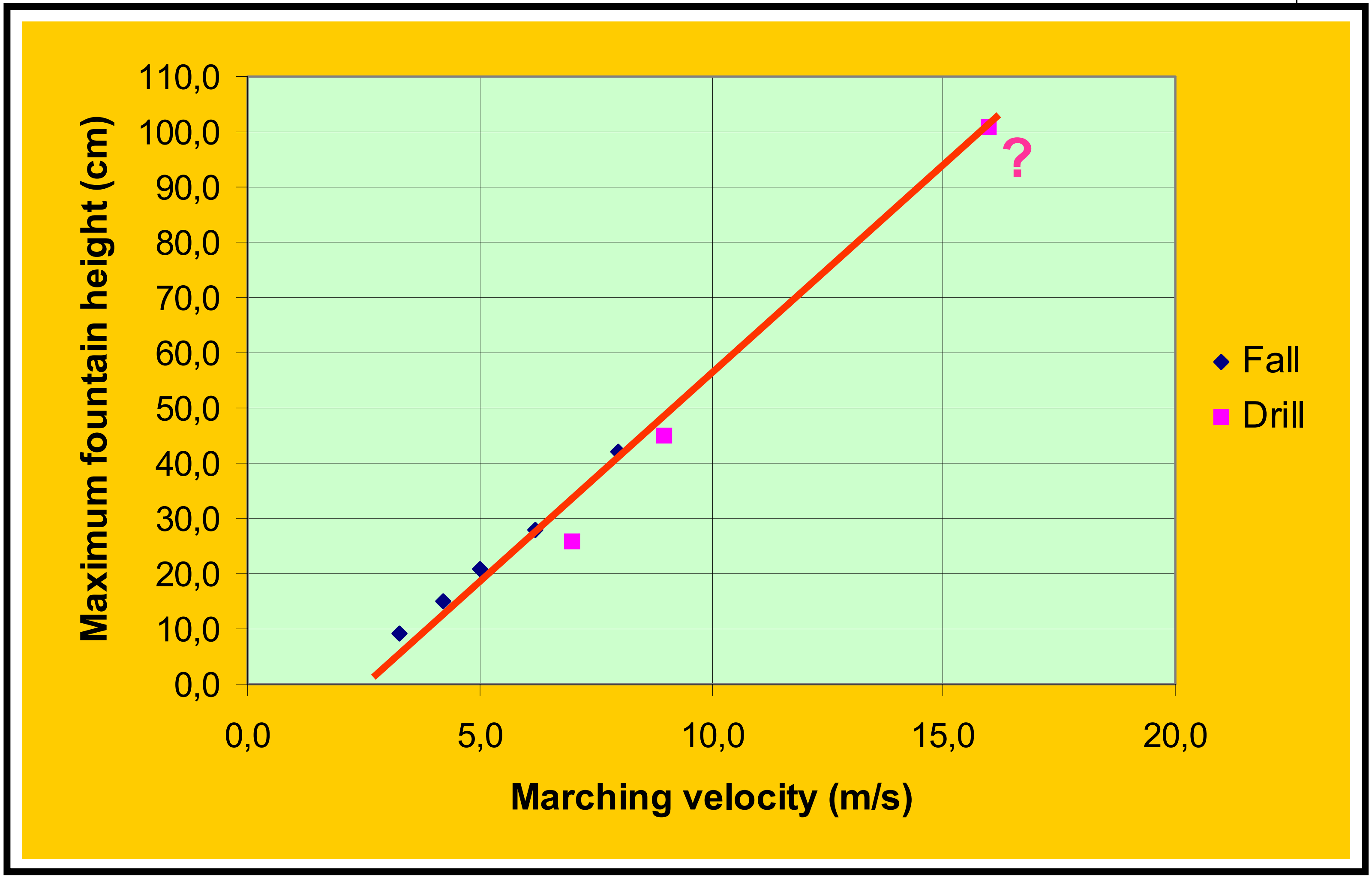


45 meters of beads flow at the velocity 16 m/s.

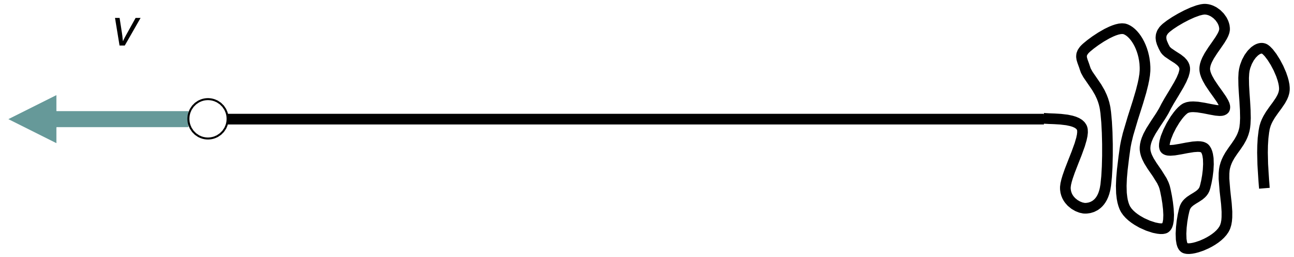
# Results of the experiment with an electric drill



# Maximum fountain height vs. marching velocity



# The problem of the rope

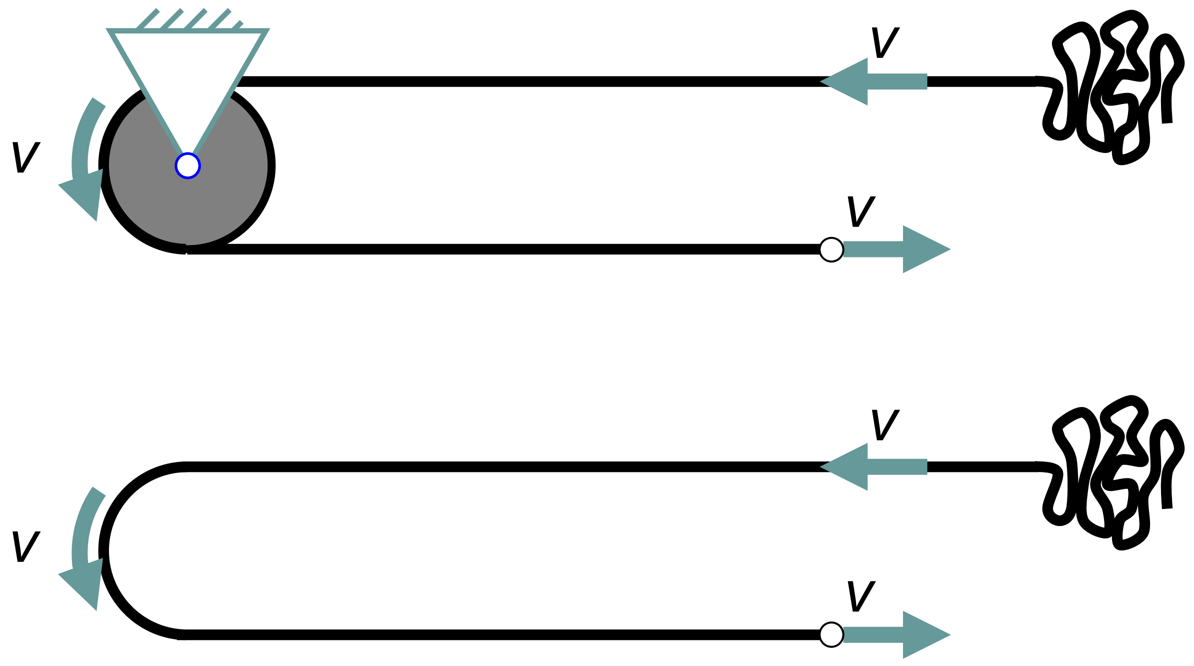



$$F = \frac{dp}{dt} = \frac{dm}{dt} \cdot v = \rho v^2 \qquad N = Fv = \rho v^3$$

$$\frac{d}{dt}\left(\frac{mv^2}{2}\right) = \frac{dm}{dt} \cdot \frac{v^2}{2} = \frac{\rho v^3}{2}$$

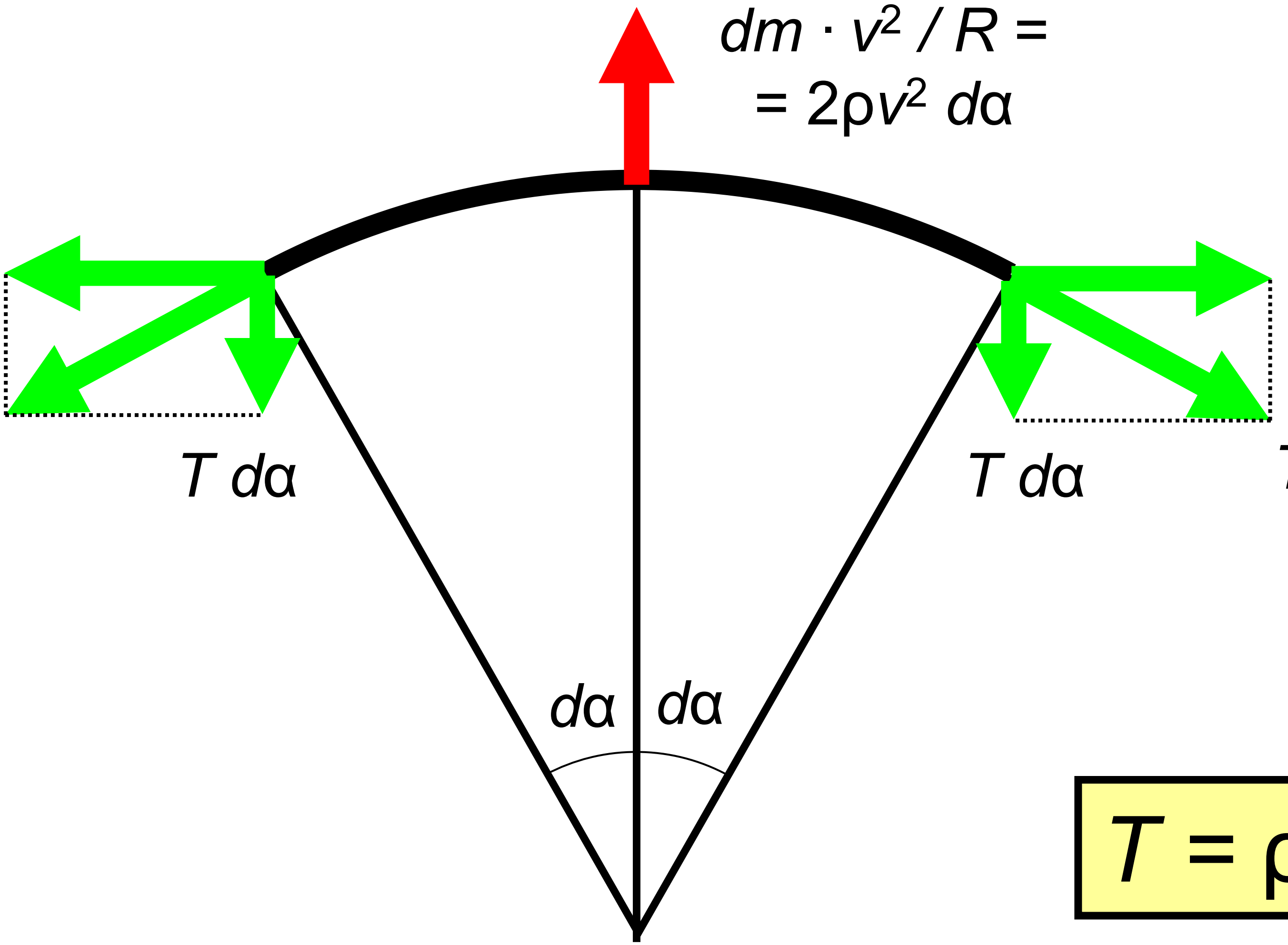
**Half the power is converted into heat**
**(or go through other channels)**

# The problem of the rope (continued)




The rope is accelerated to constant speed, and then the block disappears.
Whether the band of the rope stand still?

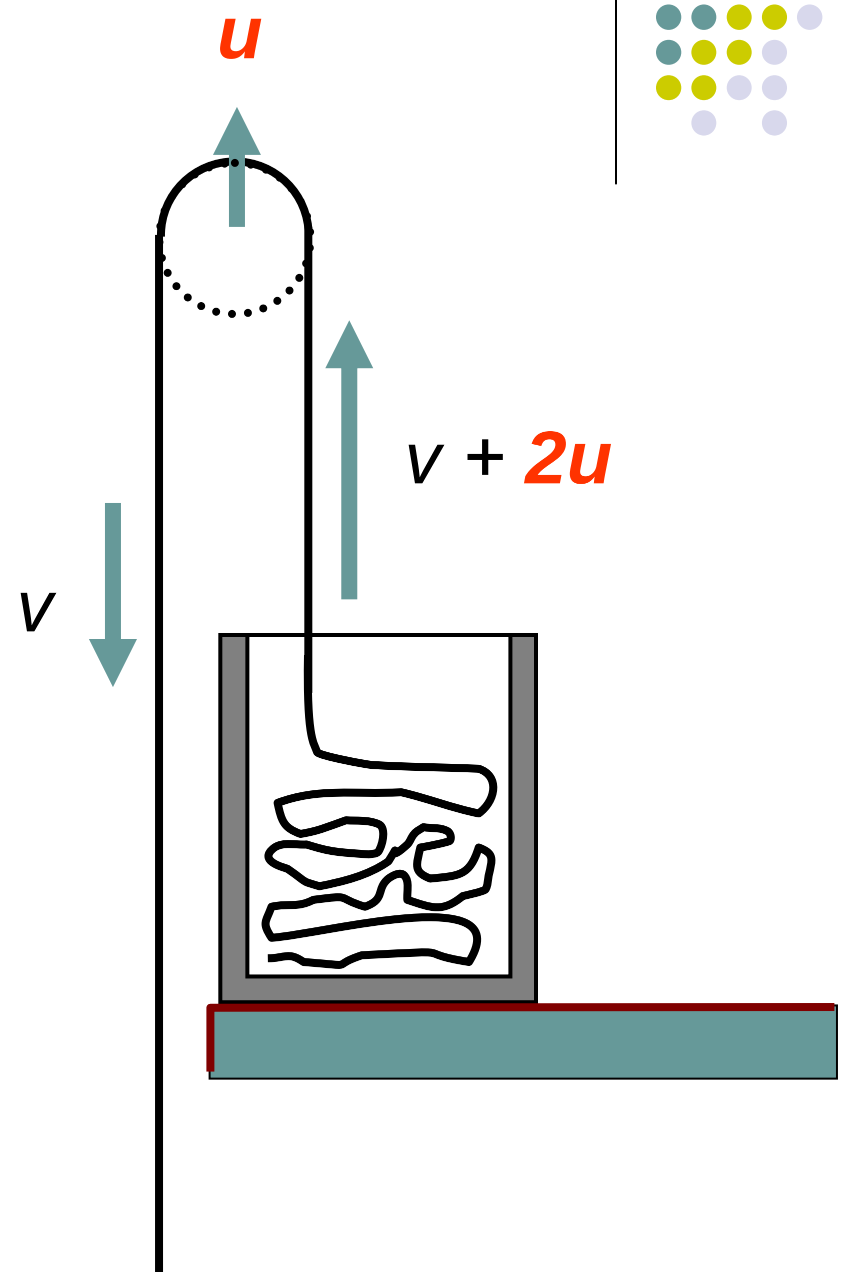
# The balance of the arc of rope




$$T = \rho v^2$$

# Kinematics of the growing fountain

- If the fountain grows with the velocity *u*, then the ascending rope has an additional velocity **2*u***.
- And if the ascending rope has an additional velocity **2*u***, then the fountain grows with the velocity *u*.
- What is the cause? What is the effect?

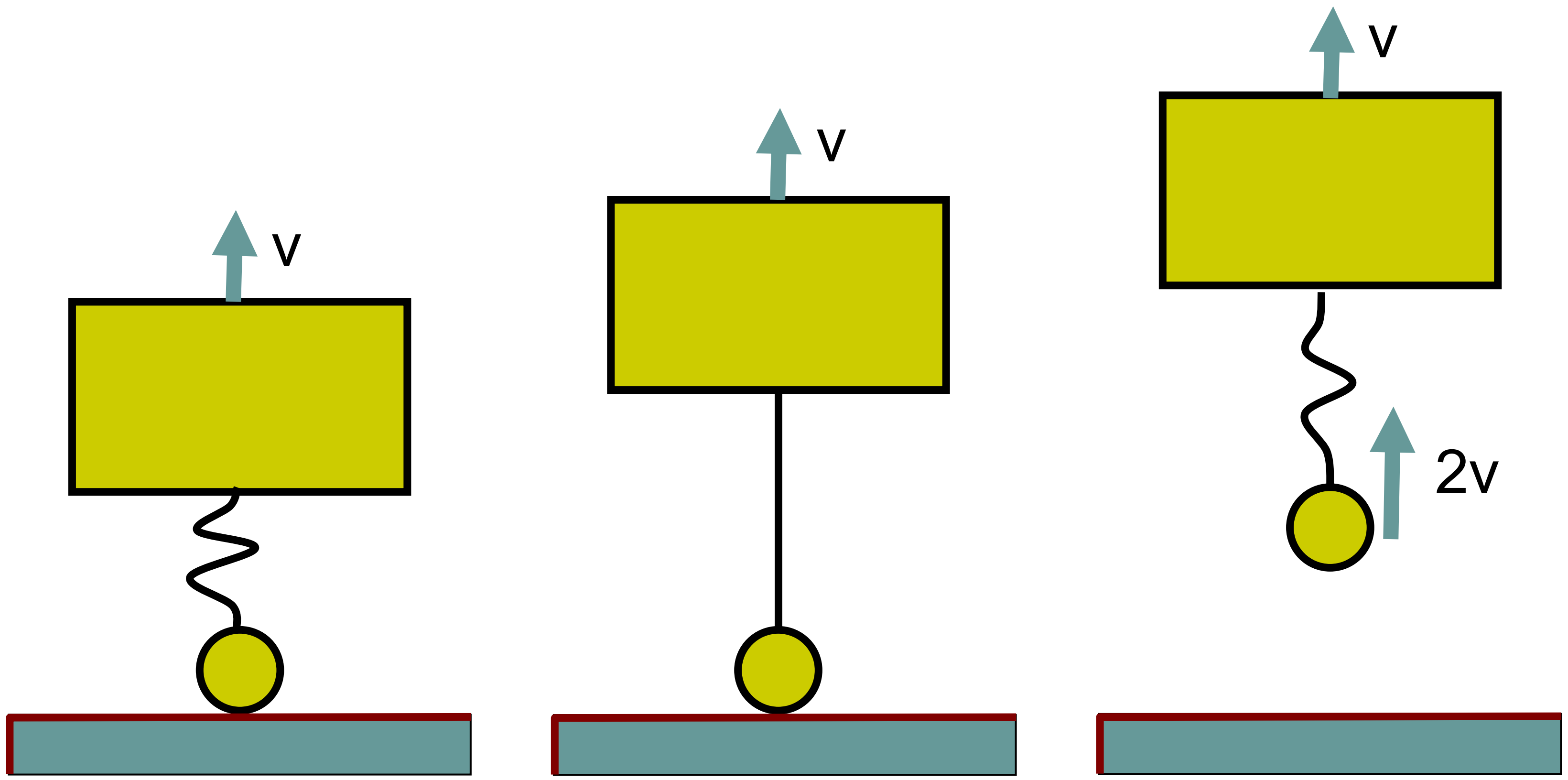


# A model for the fountain growth

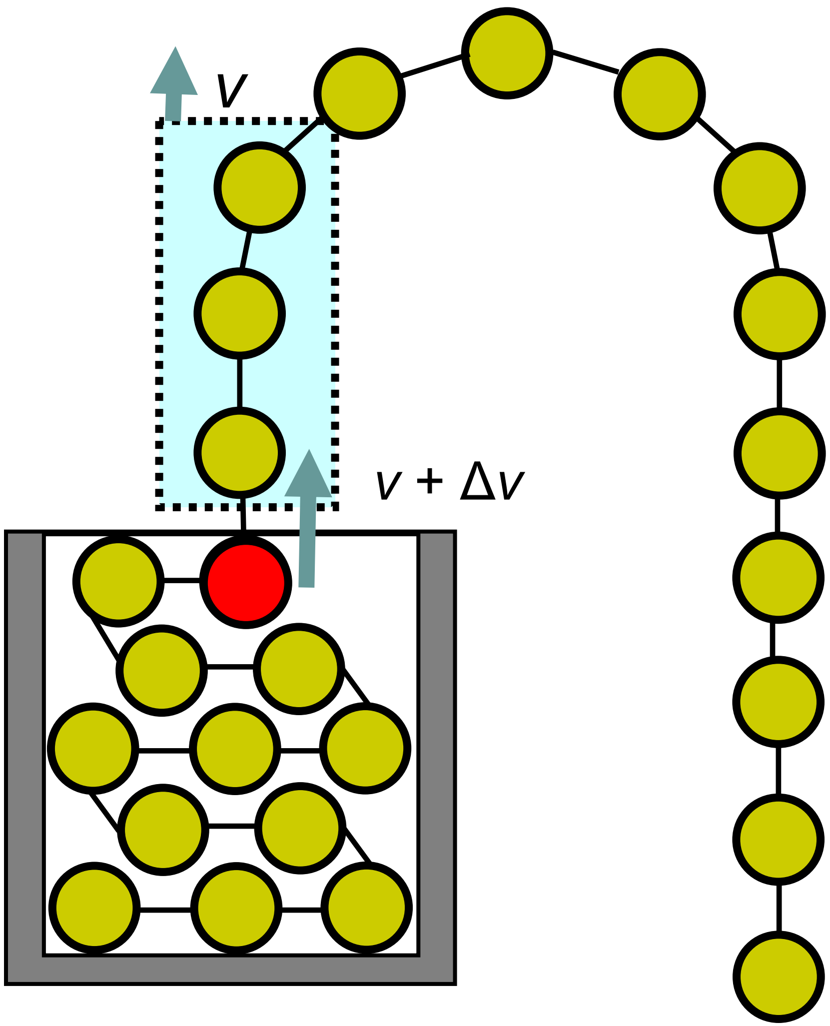
1st stage:
The bead is at rest

2nd stage:
The thread is elastically stretched

3rd stage:
The bead moves



# Why the fountain grows?



# Questions
# we have no answer

- What determines the growth rate of the fountain?
- What determines the maximum height of the fountain?

# Transverse waves

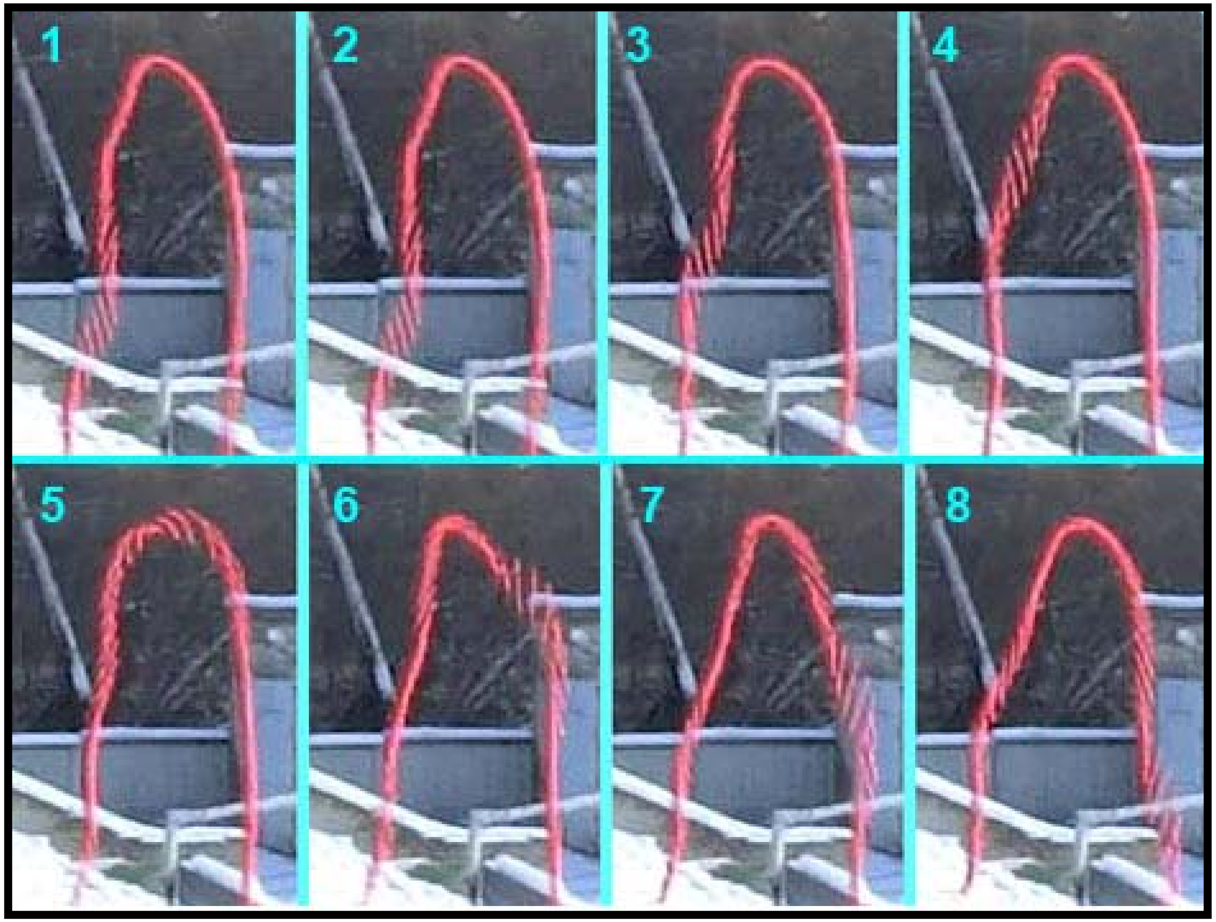
Propagation velocity
of transverse waves in the chain of beads
is equal to the velocity of the chain:

$T = \rho v^2$

$$c = \sqrt{\frac{T}{\rho}} = v$$

The codirectional wave has a velocity of 2*v*.
The opposite wave stands still.

# Transverse waves




Time between frames = 1/240 s.
Velocity of beads = 7 m/s. Velocity of the wave = 14 m/s.

# Summary

- Beads to create a fountain
- Experiment with falling beads
- Experiment with a drill (cool!)
- Balance of a moving rope
- Kinematics of the growing fountain
- Fountain growth is caused by elasticity of the thread (hypothesis)
- Transverse waves

# Bibliography


- Calkin M. G., March R. H. (1989) “The dynamics of a falling chain”. *American Journal of Physics*, **57**, 154–158
- Гельфгат И. (1993) “Сколько веревочке ни виться”. *Квант*, № 1, 55–56.

# Acceleration of the beads

When the beads move *ds* down, the work of gravity is:

$$dA = \rho gh \cdot ds = \rho gh \cdot vdt$$

The increment of the kinetic energy of the beads:

$$dE_1 = \rho h \cdot d\left(v^2 / 2\right) = \rho hv \cdot dv$$

Energy expended in accelerating new beads:

$$dE_2 = \rho v^3 dt$$

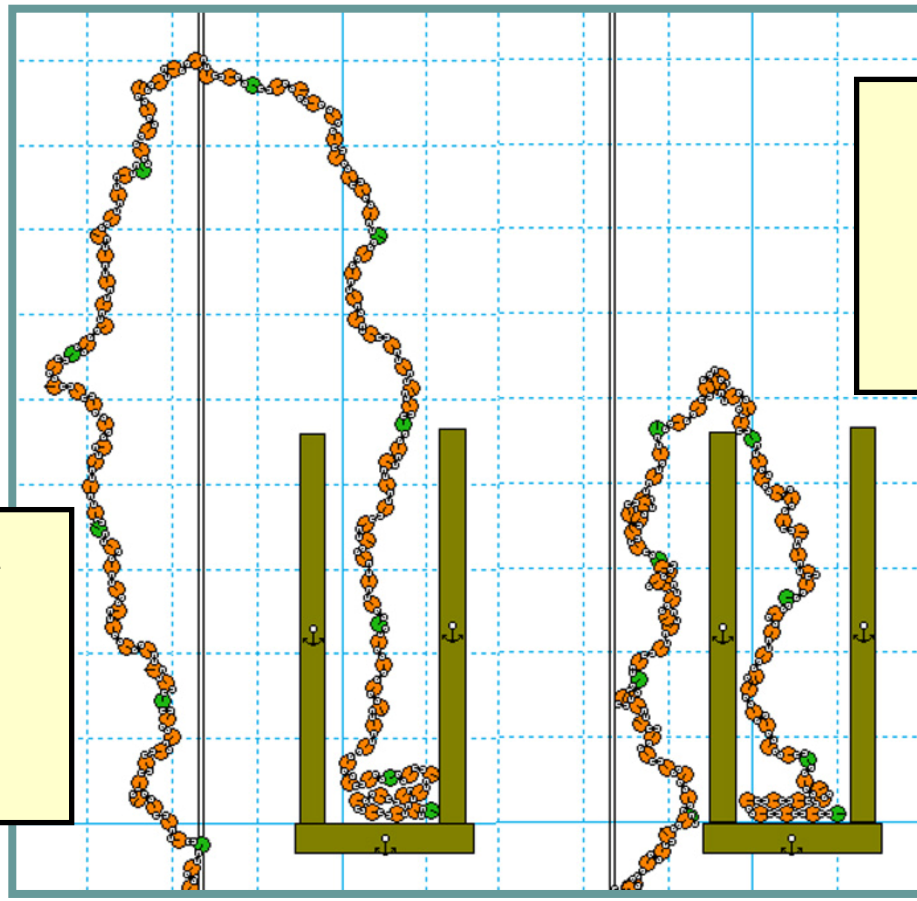
The balance of energy:

$$gh \cdot dt = h \cdot dv + v^2 dt$$

The solution:

$$v(t) = \sqrt{gh} \cdot \mathrm{th}\left(\frac{t}{\sqrt{h / g}}\right)$$

# Computer simulation



Beads are not transparent to each other: **the fountain arises**

Beads are transparent to each other: **the fountain not arises**

“Interactive physics” program. String of beads is drawn down at constant velocity. Gravity is turned off.

# Centrifugal acceleration

$v \sim 5$ m/s     $R \sim 0.05$ m

$a = v^2/R \sim 500$ m/s$^2$ **= 50*g***