Team of Brazil

Problem 02
Elastic Space

reporter:

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Problem 2
Elastic Space

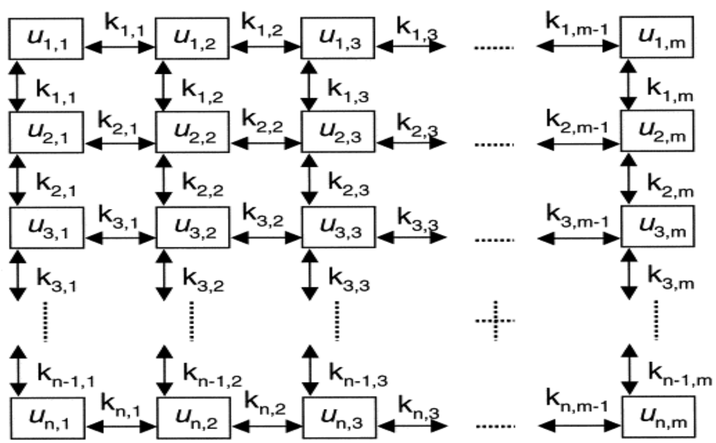
The dynamics and apparent interactions of massive balls rolling on a stretched horizontal membrane are often used to illustrate gravitation. Investigate the system further. Is it possible to define and measure the apparent "gravitational constant" in such a "world"?

Contents

Elastic Membrane - Deformation

- Vertical displacement of the membrane:
  - Continuous isotropic membrane
  - Discrete system $x = i * a$ $y = j * a.$
  - Using Hooke's Law:

$$F_{i,j} = m\frac{d^2u_{i,j}(t)}{dt^2} = -k(u_{i,j}(t) - u_{i,j+1}(t)) - k(u_{i,j}(t) - u_{i,j-1}(t)) - k(u_{i,j}(t) - u_{i+1,j}(t)) - k(u_{i,j}(t) - u_{i-1,j}(t))$$

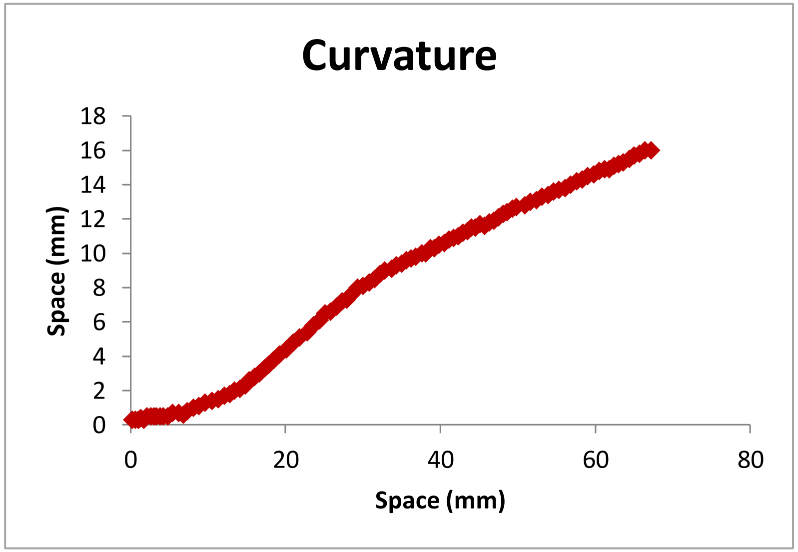


Elastic Membrane - Deformation

$$m\frac{\partial^2 u}{\partial t^2} = ka \cdot (a \cdot \frac{\partial^2 u}{\partial x^2}) + ka \cdot (a \cdot \frac{\partial^2 u}{\partial y^2})$$

$$\frac{m}{a^2} \equiv \sigma \rightarrow$$ **Massic density (constant)**

$$\sigma\frac{\partial^2 u}{\partial t^2} = k\nabla^2 u \quad \rightarrow \quad \nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$$



Elastic Membrane - Potential

- Membrane obeys wave equation:

$$\frac{1}{c^2}\frac{d^2u}{dt^2} - \nabla^2 u = 0$$

- Stationary form:

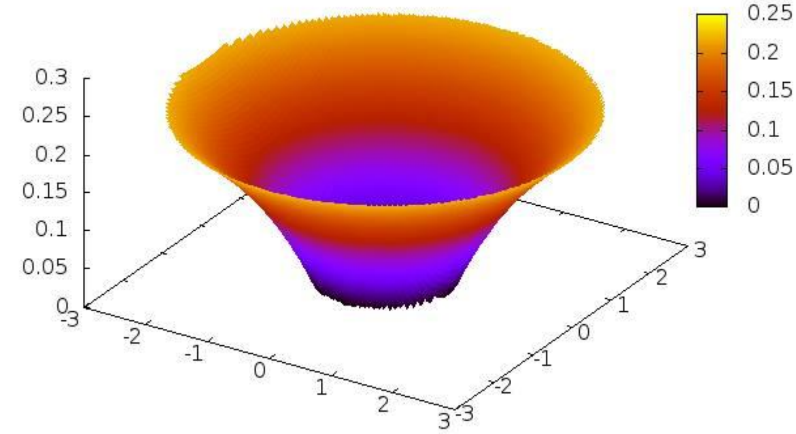
$$\nabla^2 u = 0$$

Vertical displacement ↑

Distance from the center to any other position (radial coordinates) ↑

$$u = C_1 . \log r + c_2$$

Adjustment constant



Elastic Membrane - Potential

$$U_M \sim U_g = mgu$$

- Ball is constrained to ride on the membrane.

$$F_r = -\frac{dU_g}{dr} = mg\frac{d}{dr}[\log r]$$

Mass of the particle

Force due to membrane's potential

Earth's Gravity

$$F_r = \frac{mg}{r}$$

As the force does not vary with $1/r^2$, our system is not gravitational.
We find, then, an analogue G.

Elastic Membrane - C_1 (and analog G) definition

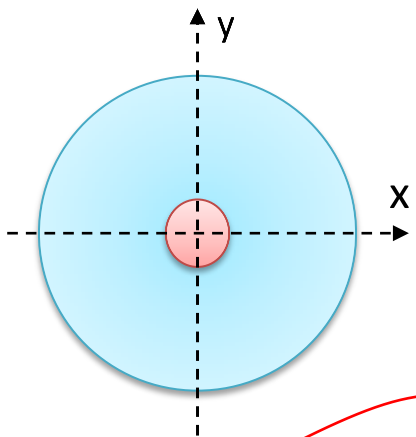
$$C_1 = \frac{L}{\log\left(\frac{R}{r}\right)}$$

$$L \sim \frac{Mg}{k}$$

$$F_r = \frac{g}{k.\log\left(\frac{R}{r}\right)} \frac{Mm}{r}$$

Analog G

Elastic Membrane – Motion Equation

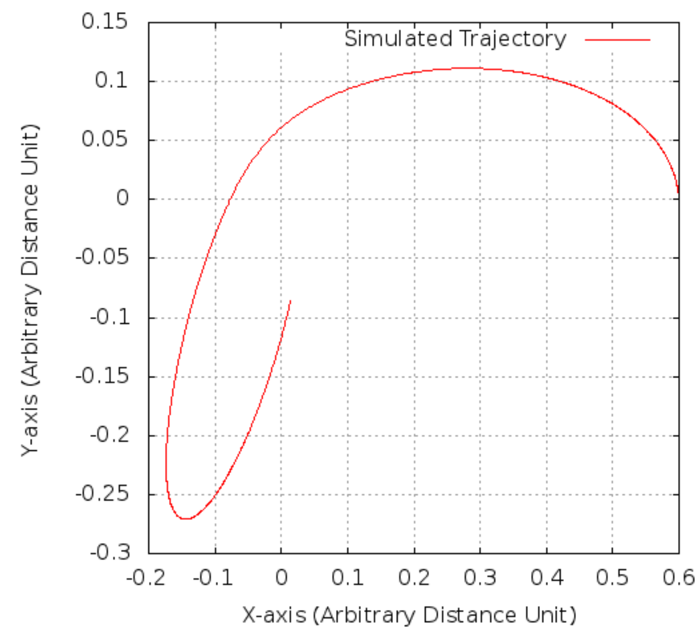


- Gravity
- Masses
- Radii
- Elastic constants

$$x'' = -\frac{ax}{(x^2+y^2)} - bx'$$

$$y'' = -\frac{ay}{(x^2+y^2)} - by'$$

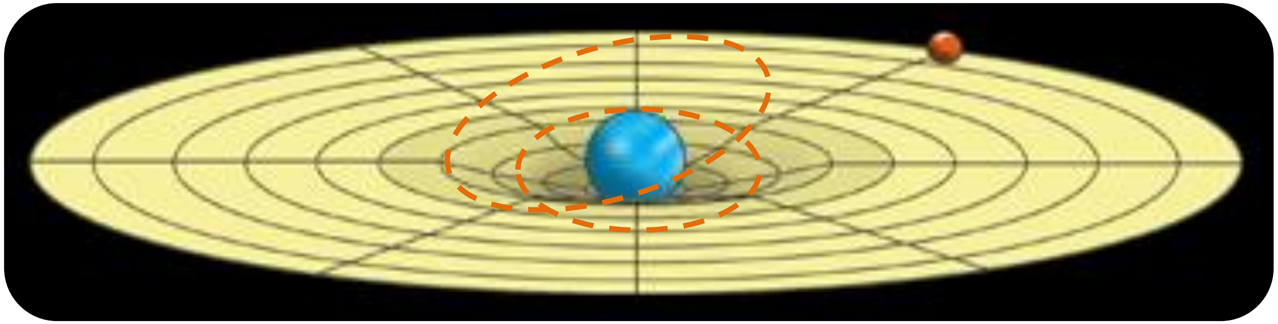
Damping constant (dissipative forces)



The validity of this trajectory will be shown qualitatively in experiment.

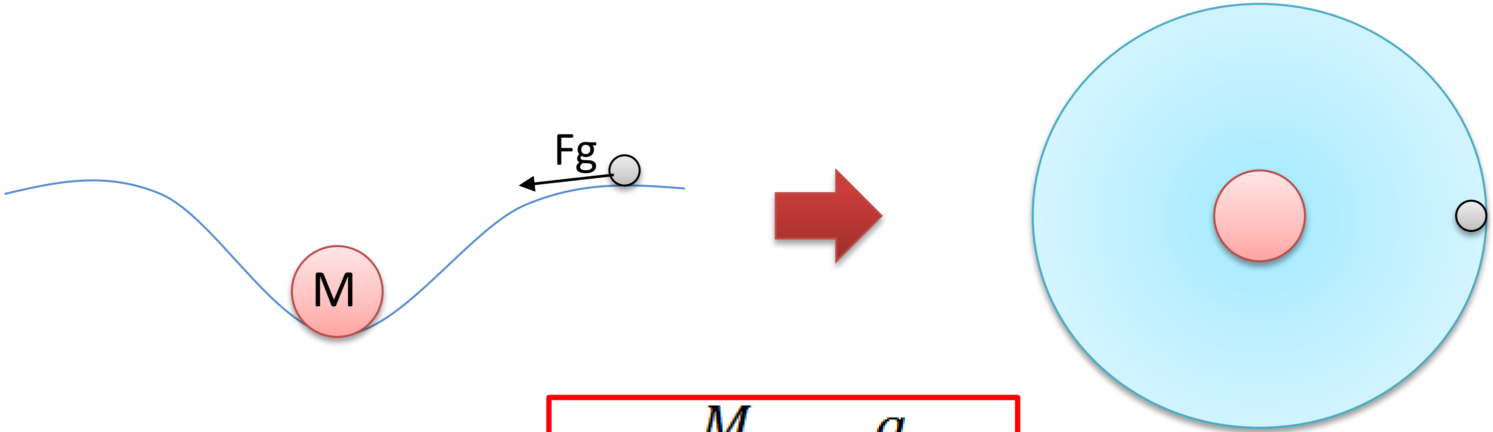
Elastic Membrane – Motion Equation

- Rolling ball's behavior:
  - Close to the great mass ("singularity")
  - Far from singularity
  - The closer to singularity, the greater deviation from parabola



Approach Movement

- Disregarding vertical motion (one-dimensional movement), the smaller ball's acceleration towards the other is equivalent to the gravitational pull



$$a_g = \frac{M}{r} \frac{g}{k \log\left(\frac{R}{r}\right)}$$

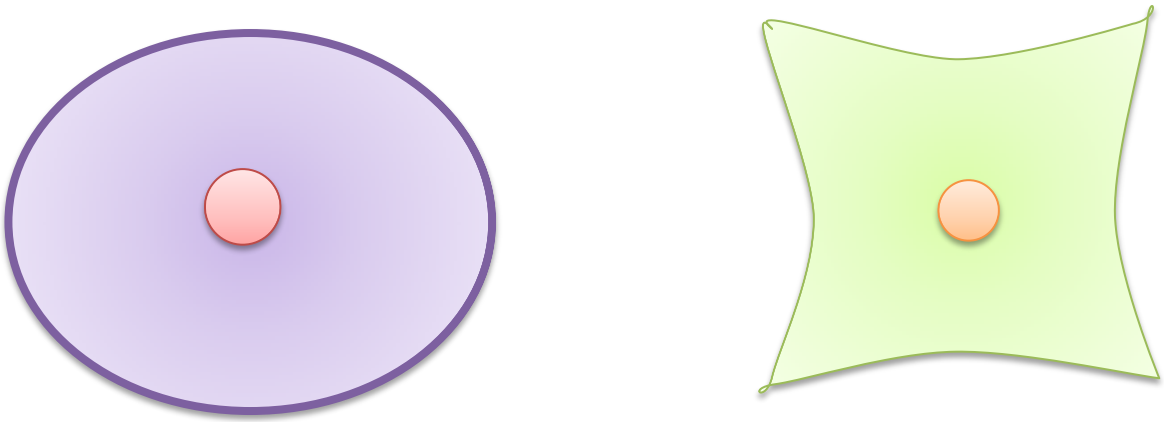
Elastic Membranes

Different rolling masses → Great masses compared to the central one → Both deform the sheet → Constant dependent on depth of deformation → Deformations add themselves meniscus-like → Greater than real gravitational constant

Model applicability finds limitation

Gravitational Field's Dimension

- Space-time: continuous
- Membrane: limited



Limit= Borders

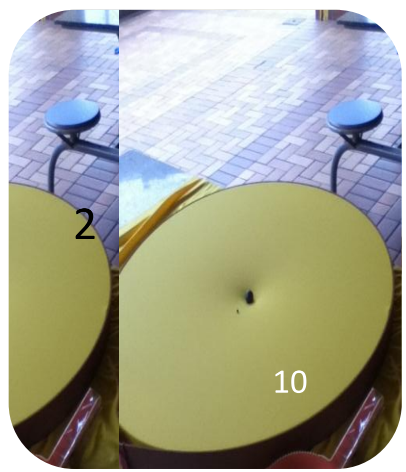
Relativity and Membranes

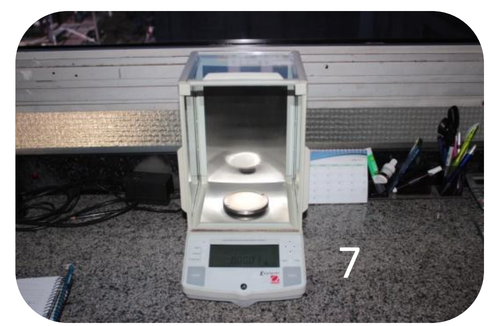
- Basic concepts: space-time and gravity wells
- Fundamental differences:
  - Time
  - Friction
  - Kinetic energy
  - Gravitational potential
- Our model: Earth's gravity causes deformation (great ball's weight) (Rubber-sheet model)
- It's not a classical gravitation model, but it's possible to make an analogy.

Material

1. Strips of rubber balloon and lycra
2. Half of rubber balloon and lycra (sheet)
3. Baste
4. Styrofoam balls with lead inside
5. Metal spheres, marbles and beads
6. Two metal brackets
7. Scale (±0.0001 g - measured)
8. Sinkers
9. Measuring tape (±0.05 cm)
10. Plastic shovel with wooden block (velocity control)

- Adhesive tape
- Camera





Experimental Description

- **Experiment 1:** Validation of theoretical trajectory.
- **Experiment 2:** Determination of rubbers' spring constant.
- **Experiment 3:** Same balls interacting, variation of rubber-sheet and dimensions.
- **Experiment 4:** Same membrane and center ball (great mass), different approaching ball.

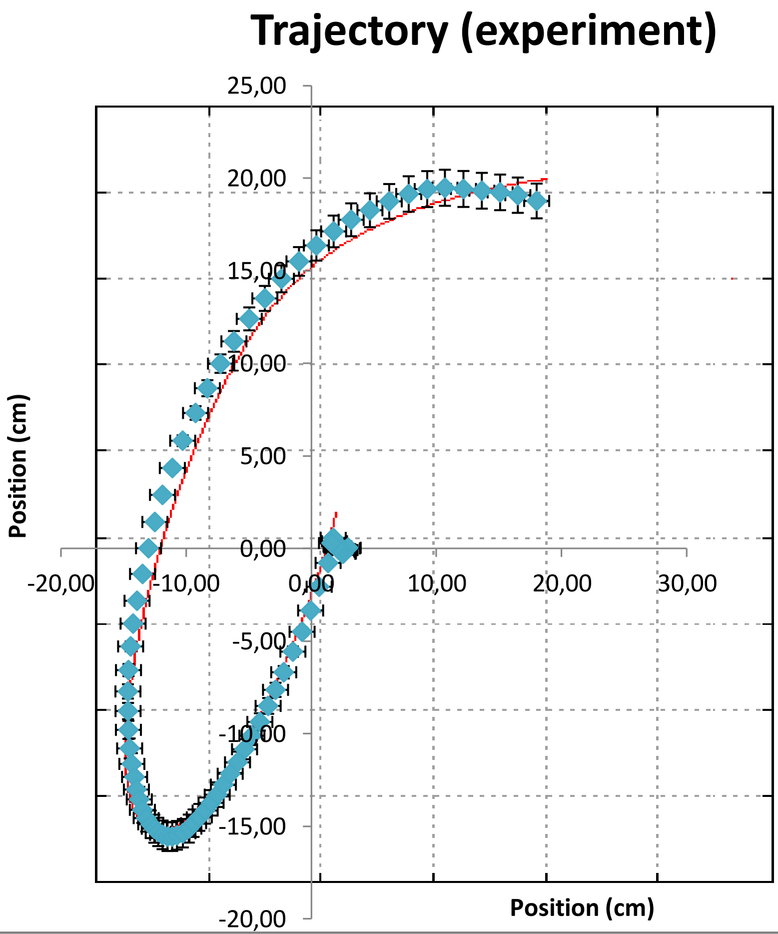
Experiment 1: Trajectory

- Trajectory
- Spring Constant
- Dimensions
- Different ball

- Using the shovel, we set the same initial velocity for all the repetitions.

Experiment 1: Trajectory

- Trajectory
- Spring Constant
- Dimensions
- Different ball



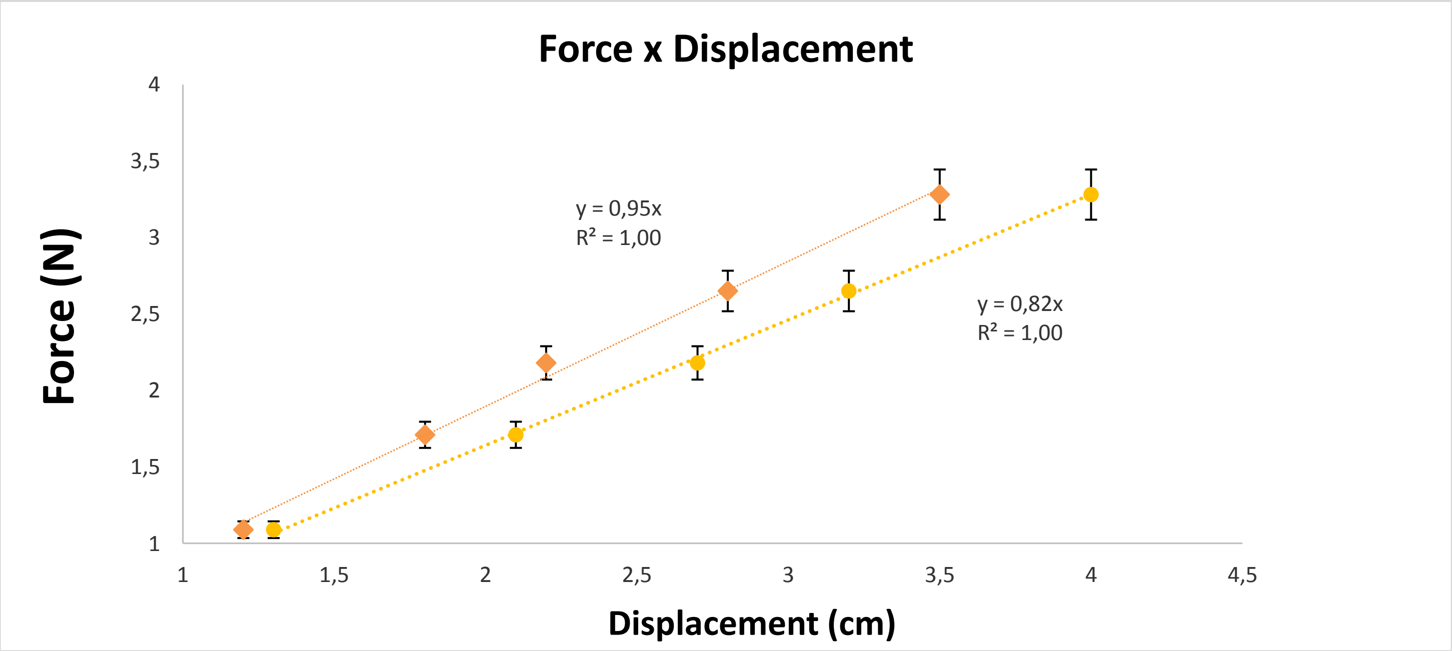
The obtained trajectory fits our model, thus, validating the proposed equations.

Experiment 2: Spring Constant

- Experimental Setup:

- 5 sinkers, added one at a time.

Experiment 2: Spring Constant



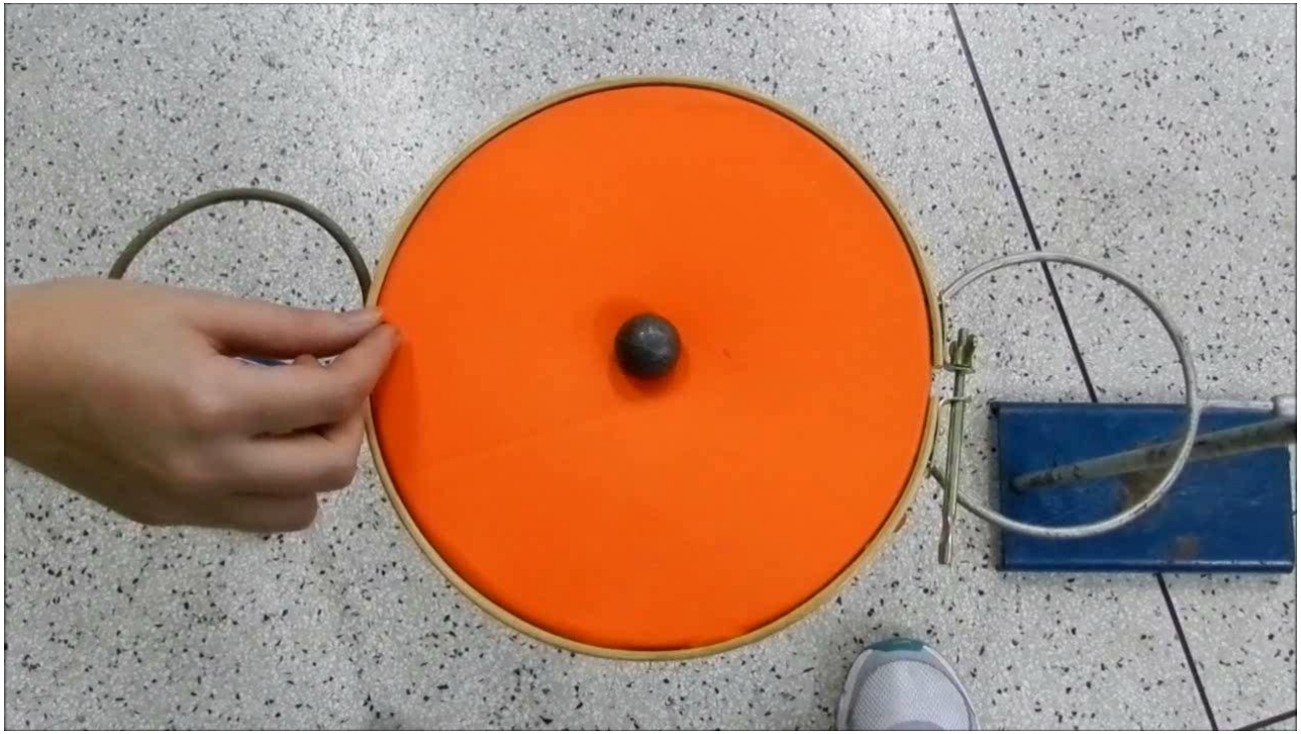
- Yellow: k=0.82 N/cm
- Orange: k=0.95 N/cm

The yellow sheet will be the most deformed: higher gravitational constant.

Experiment 3: Different rubber-sheets

- Trajectory
- Spring Constant
- Dimensions
- Different ball

- Experimental procedure:



- Five repetitions for every ball/sheet combination

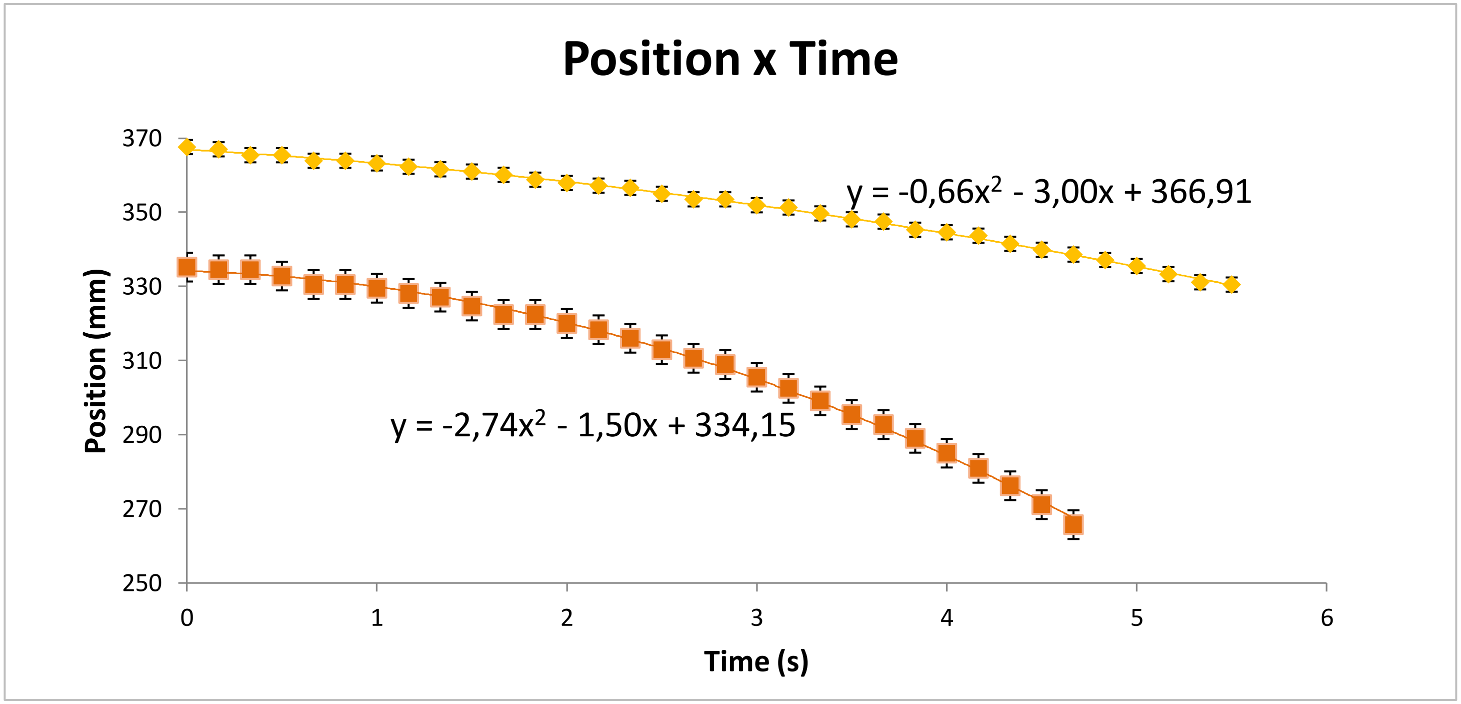
Experiment 3: Different rubber-sheets

Trajectory

Spring Constant

Dimensions

Different ball



- G_{aO}: $6,3.10^{-12}$ $m^3/kg.s^2$
- G_{aY}: $1,18.10^{-11}$ $m^3/kg.s^2$
- M = 77,3 g

Experiment 3: Different rubber-sheets

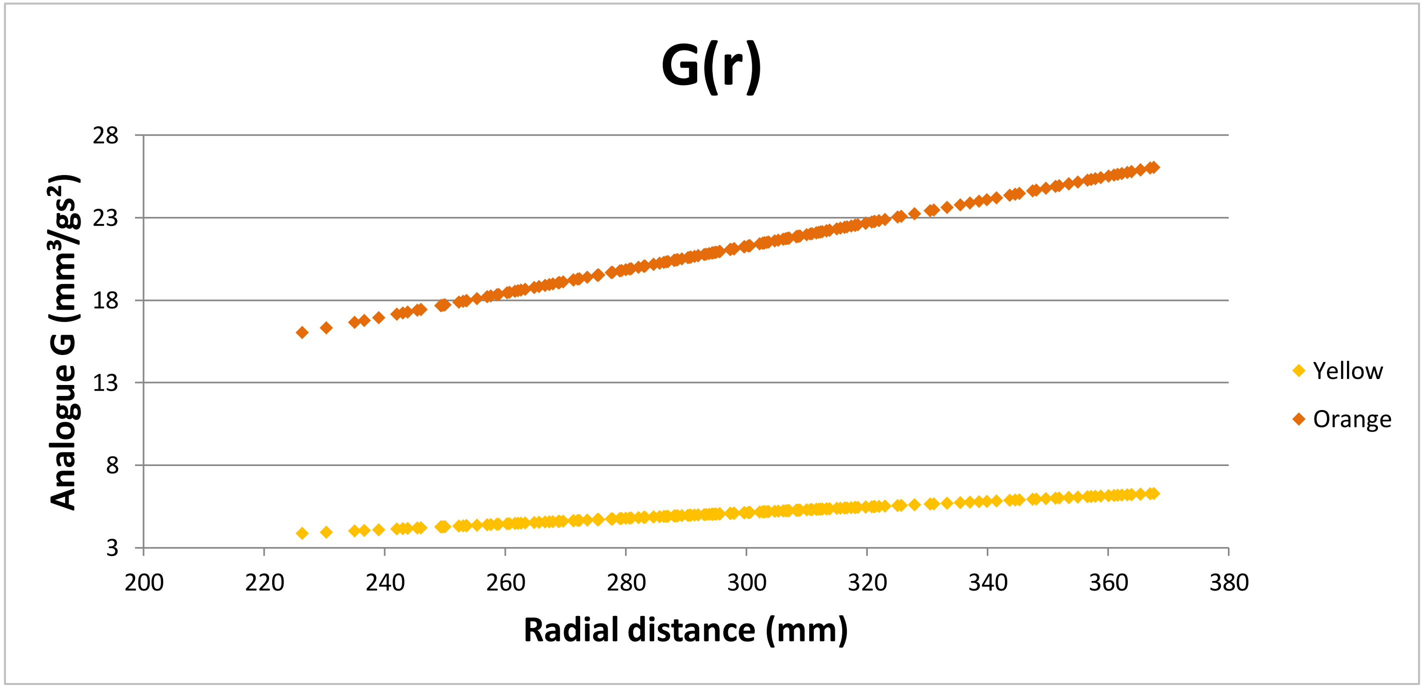
Gravitational Constant		
Theoretical	G_y	G_o
6,67E-11	6.29E-12	1.17E-11
Deviation	**90.56**	**82.45**

- Trajectory
- Spring Constant
- Dimensions
- Different ball

$m \approx 6.9.10^{-6}\%M$ + **$6,67.10^{-11}$ $m^3kg^{-1}s^{-2}$** = **Analogue gravitational constant**

Experiment 3:

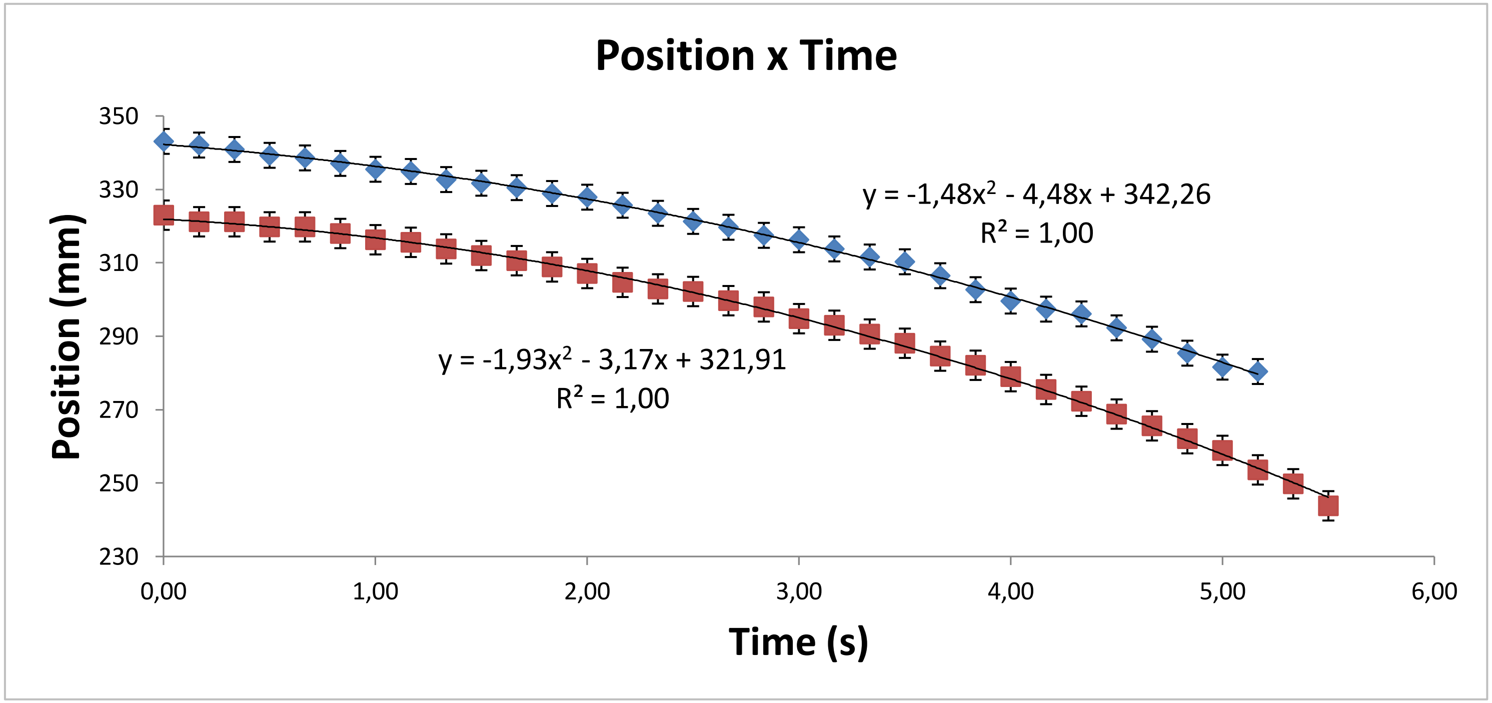
- Trajectory
- Spring Constant
- Dimensions
- Different ball



The "gravitational constant" is dependent on the distance from the center, showing that, again, our system is not gravitational.

Experiment 4: Different rolling ball

- Trajectory
- Spring Constant
- Dimensions
- Different ball



- Blue: marble
- Red: bead

Conclusion

- The proposed system is not a gravitational system;
- It is possible to determine analogue analogue gravitational constant of the system (exp. 3);
- This G_a is dependent on geometric and elastic characteristics of the system;
- G_a not equal to G, as the system's not gravitational;
- The model's approach is limited to very different masses (exp.4). Otherwise, the model is not applicable.

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Thank you!