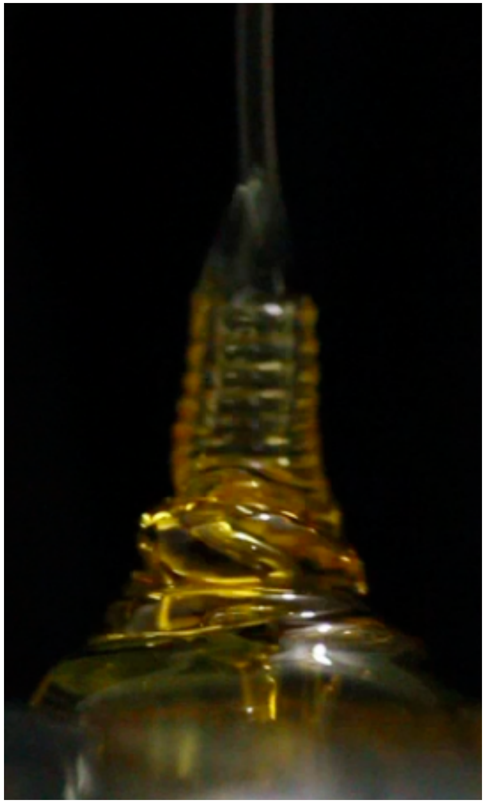
Team Brazil

Problem 13
Honey coils

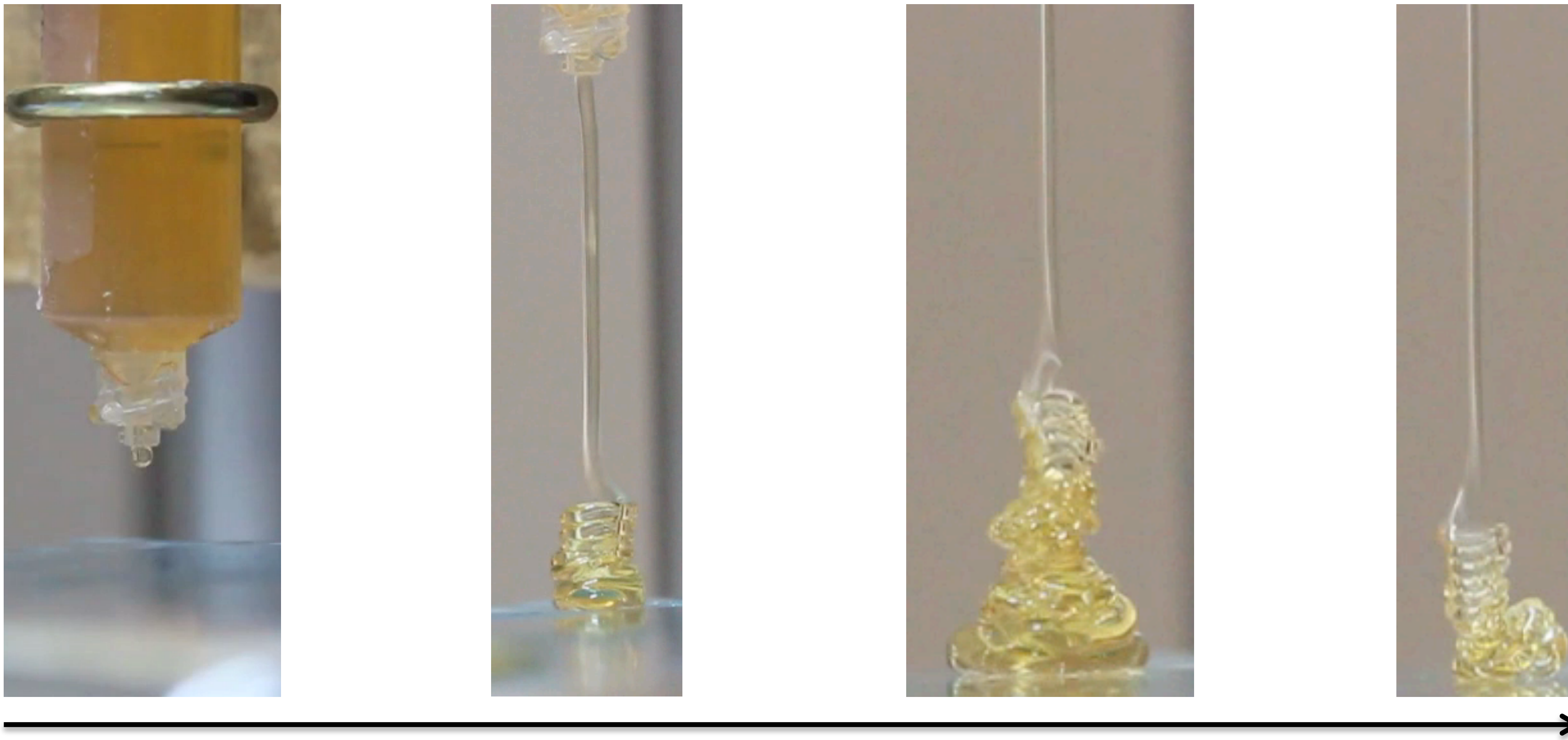
Liara Guinsberg



Problem 13

A thin, downward flow of viscous liquid, such as honey, often turns itself into circular coils. Study and explain this phenomenon.

Video



Increasing height

Introduction

Theoretical formulation

- Surface tension
  - Plateau-Rayleigh instability
- Viscosity
- Newtonian fluids
- Fluid conditions
  - Initial conditions
- Coiling conditions
- Torsions and Tensions
- Regimes

Experiments

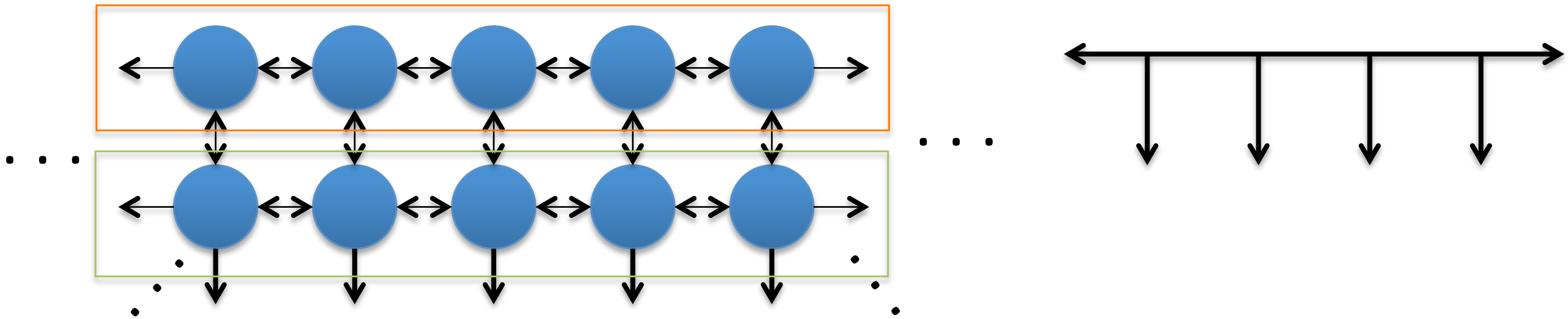
- Honey
- Cane molass
- Oils
- Shampoo

Comparison between the theory and the experiments

- Graph comparison
- Regimes and conditions

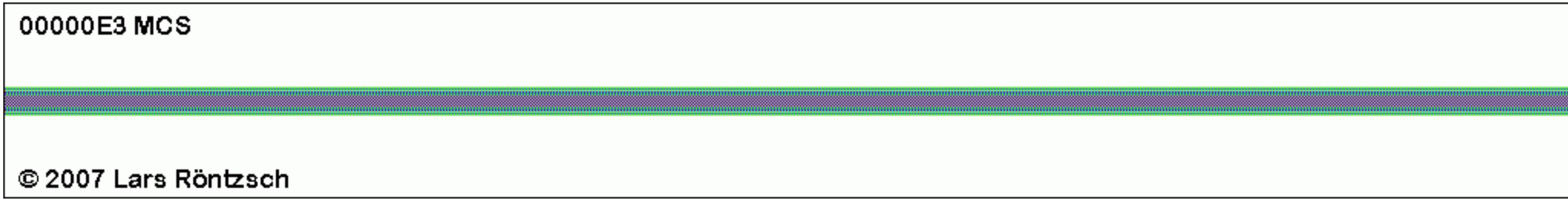
Surface tension

- The interior molecules have as many neighbors as they can.
- For the liquid to minimize its energy state, the number of higher energy boundary molecules must be minimized.
- The minimized quantity of boundary molecules: minimized surface area.



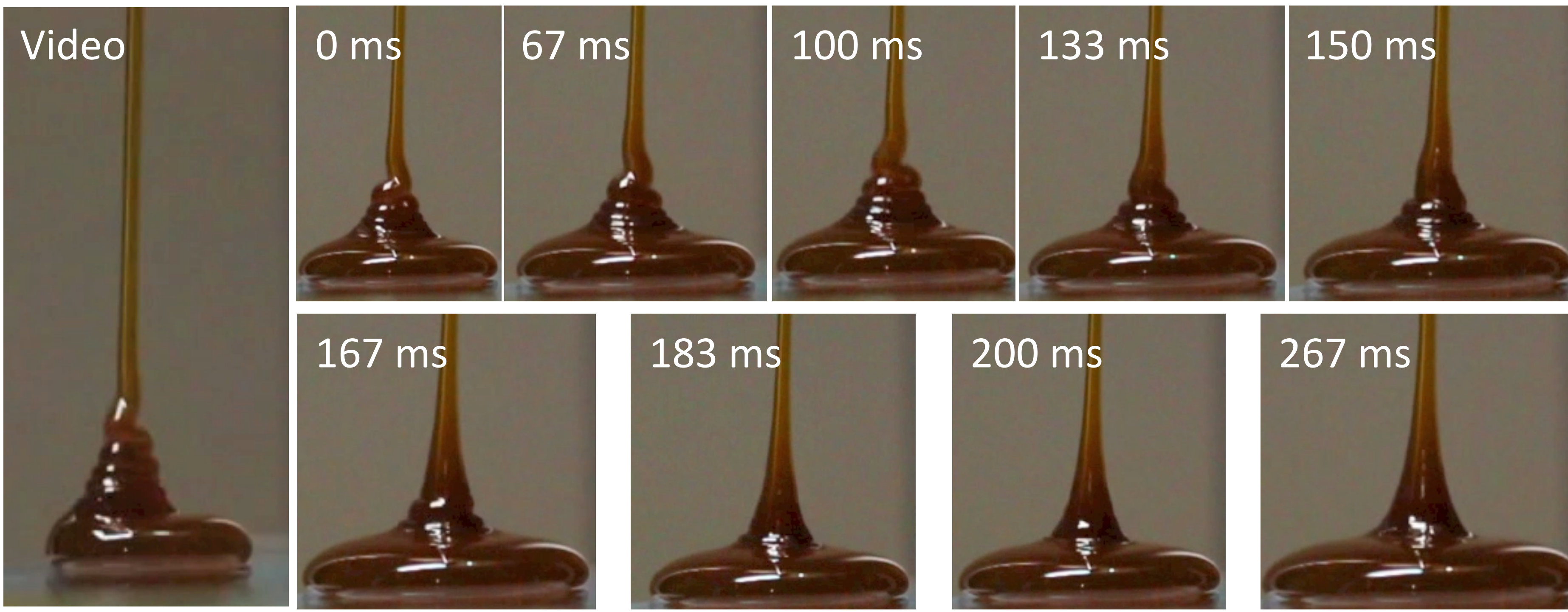
Plateau-Rayleigh instability

- The surface tension causes some oscillations in the jet, sometimes breaking it into droplets, to minimize surface area.



Instabilities

- Surface tension:

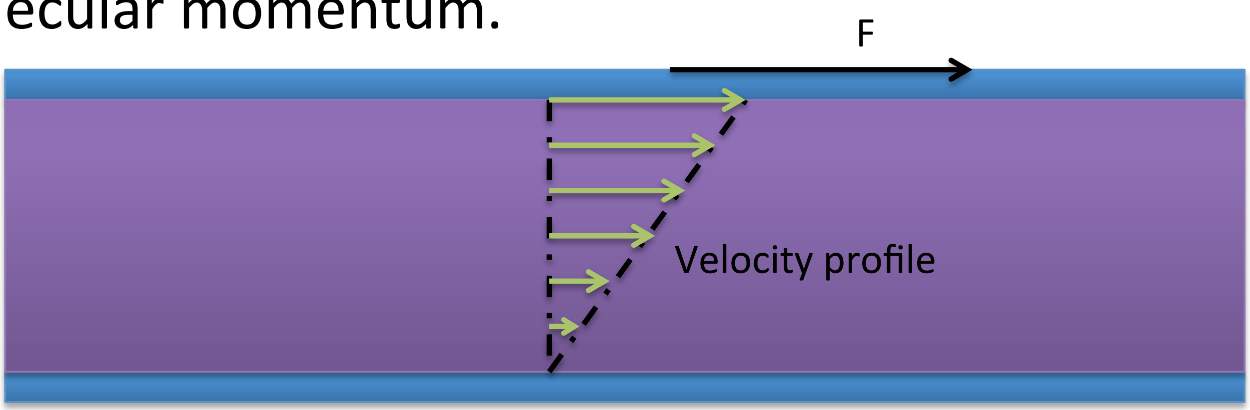


Viscosity

- It's the fluid property by which a fluid offers resistance to shear stresses.

$$\tau = \mu \frac{du}{dy}$$

- The physical origins are the intermolecular forces and transfer of molecular momentum.

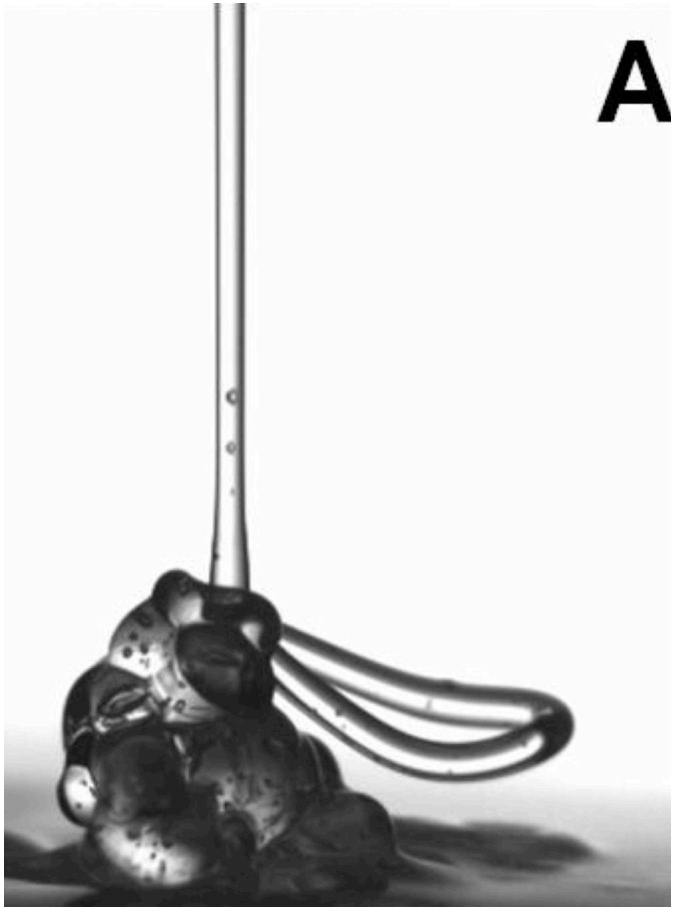


Newtonian and non-Newtonian fluids

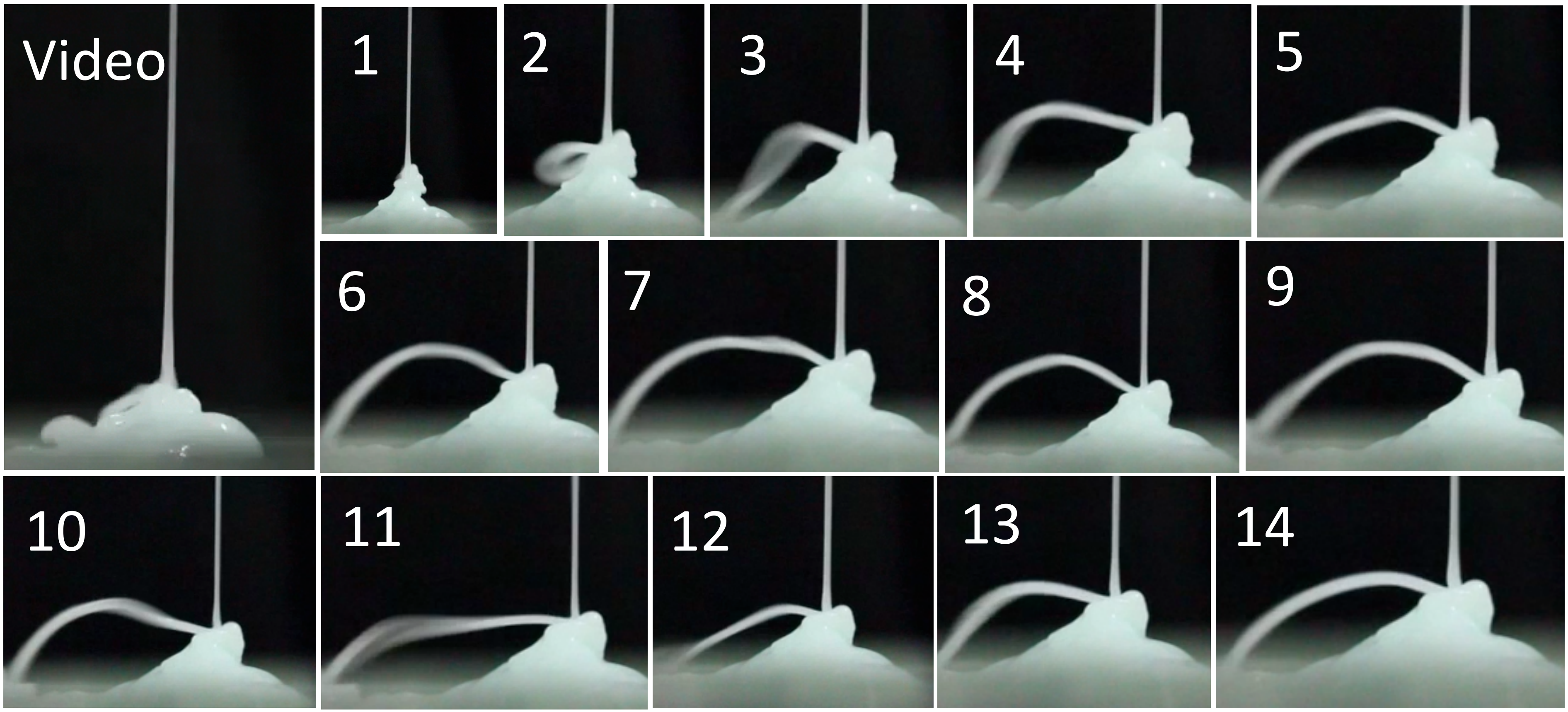
- A fluid can be considered Newtonian when the viscous stress is proportional to the deformation rate with time, in every point of the fluid.
- A fluid is non-Newtonian has variable viscosity with applied shear stress

Shear thinning

- Happens on non Newtonian fluids
- The viscosity decreases with the increase of the shear stress.
- Causes phenomena like Kaye effect
- We can study it briefly with shampoo, glycerin and many others.

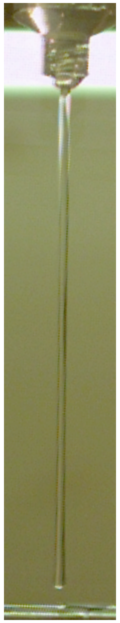


Kaye effect



Initial conditions

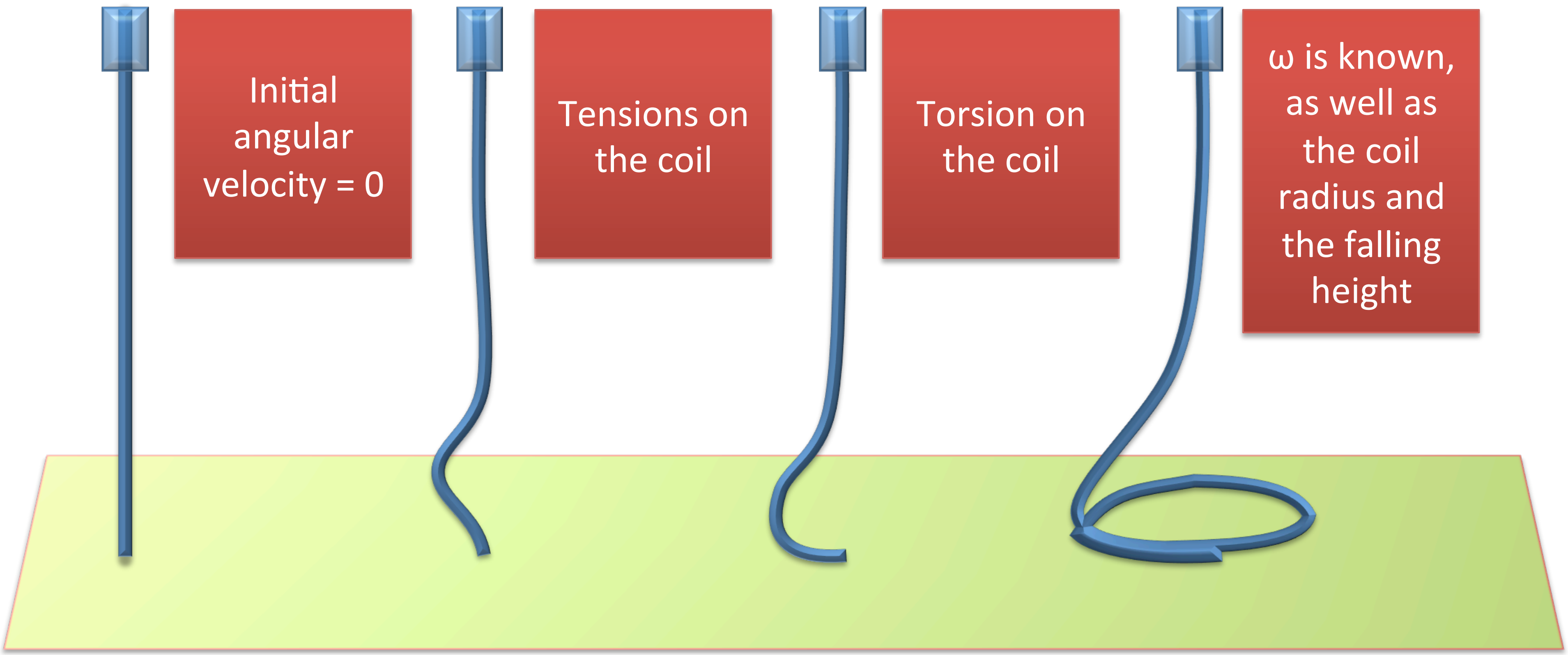
- First, we suppose the fluid stream is completely vertical before touching the solid surface:

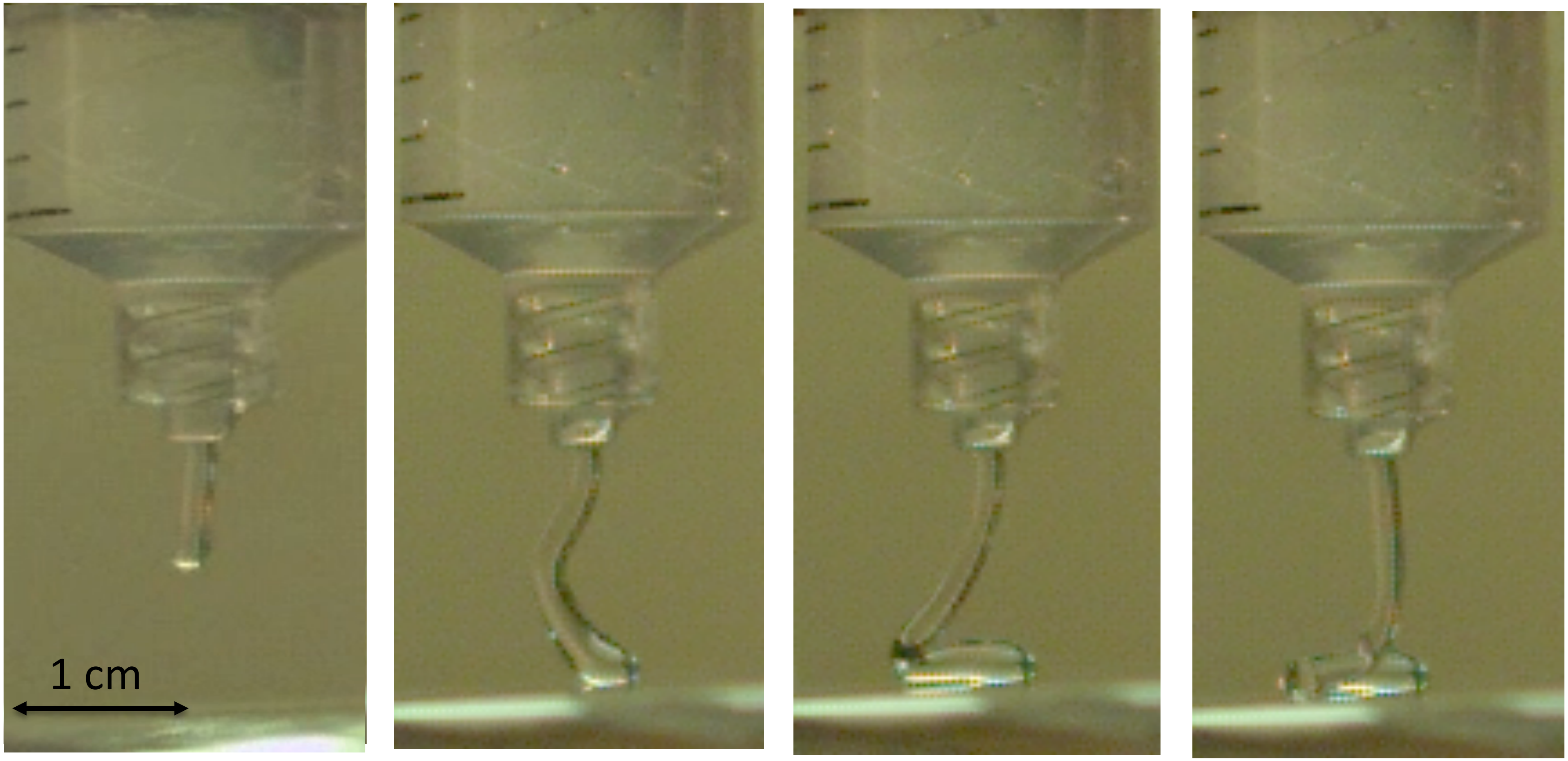


That implies:

- No coiling formation before the fluid touches the surface

Boundary conditions



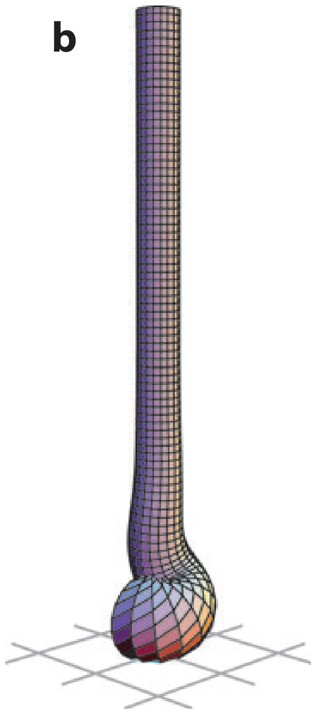

1 cm

Torsions and tensions along the fluid rope

- Why does the jet changes its format after its collision with a rigid surface?
  - The jet has a velocity when touching the rigid surface
  - It has to slow down to zero, so there's a force directed upwards, that goes along the fluid stream and changes its form
  - There's a torsion caused by this tension
  - Thus, for the smaller energy, we can have the coiling phenomena.

Coiling conditions

- Minimal viscosity for the coil to happen
- Maximum height, because of the Plateau-Rayleigh instability
- Surface tension relevance, for visible coils and minimum height.

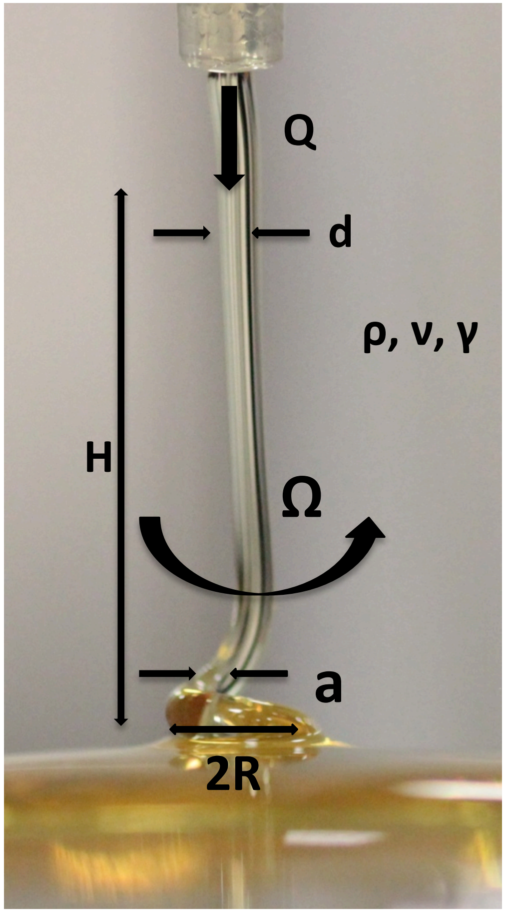


Liquid Rope Coiling
Neil M. Ribe, Mehdi Habibi, and Daniel Bonn

Regimes

- We can see a relation of the coiling frequency with the height of fall
- As we increase the height, some buckling instabilities appear, and we define 4 regimes.

Theoretical analysis



H- Height
R- Coil radius
Ω- Rotational frequency
ρ- Density
ν- Viscosity
γ- Surface tension
a- Fillet radius at the contact point
d- Injection diameter
Q- Volumetric rate of fluid insertion

Definitions

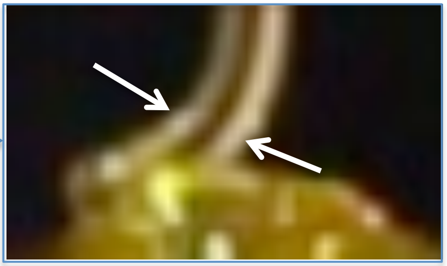
$$U_1 = R\Omega$$

→ Used in formulations

$$\delta \sim \left(\frac{\nu Q}{g}\right)^{1/4}$$

→ Defines geometrical properties in the bending region

a_1 →

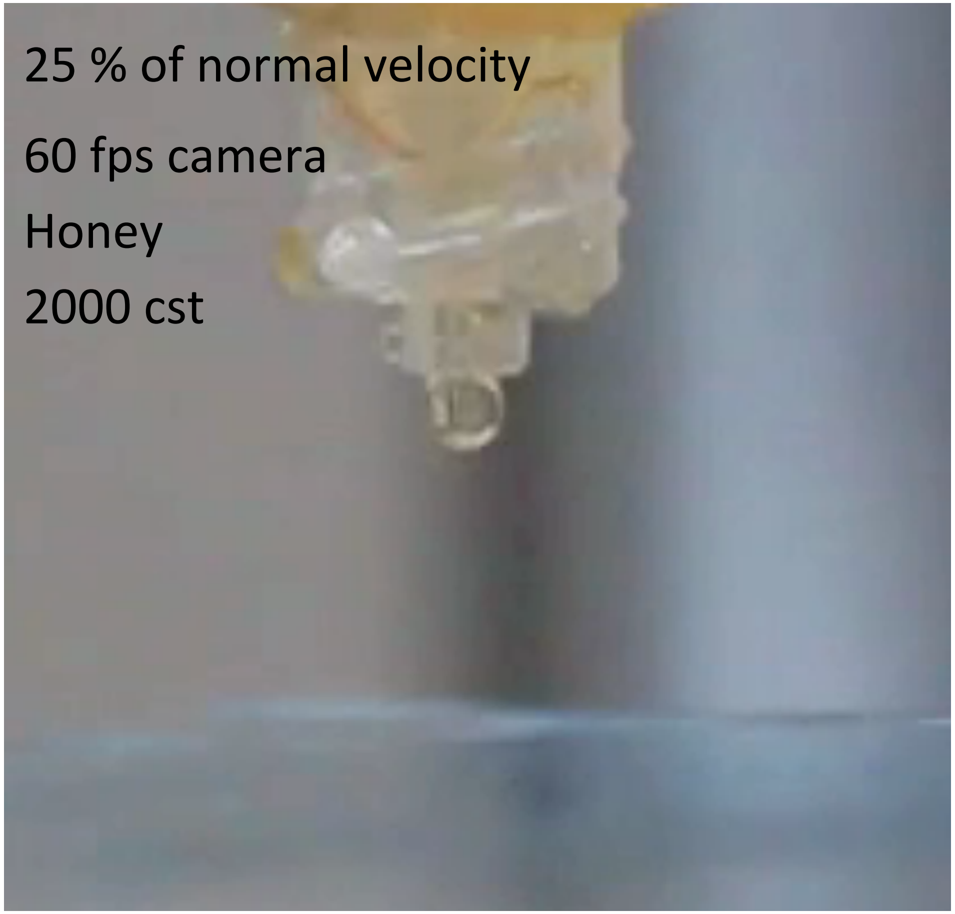


Theoretical analysis

- Viscous regime:
  - Smaller heights
  - Gravitational effect negligible
  - Inertial effect negligible

$$R \sim H$$

$$\Omega \sim \frac{Q}{H a_1^2}$$


25 % of normal velocity
60 fps camera
Honey
2000 cst


1.5 % of normal velocity
2000 fps camera
Silicone oil
5000 cst

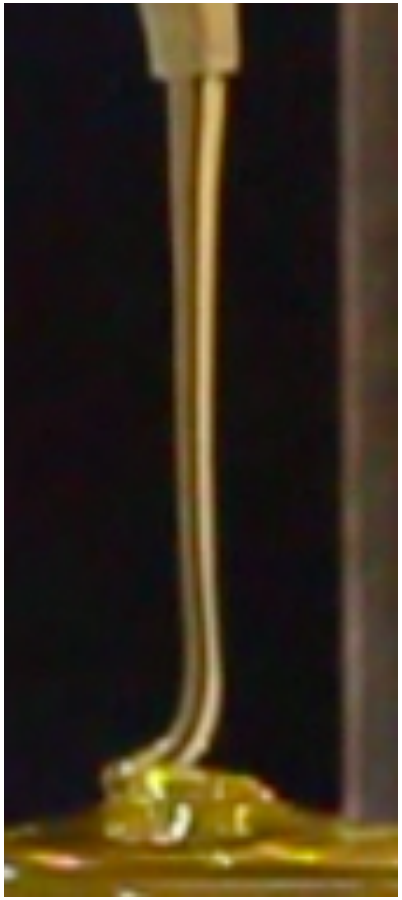
Theoretical analysis

- Gravitational regime:
  - Gravitational effects are the most relevant ones

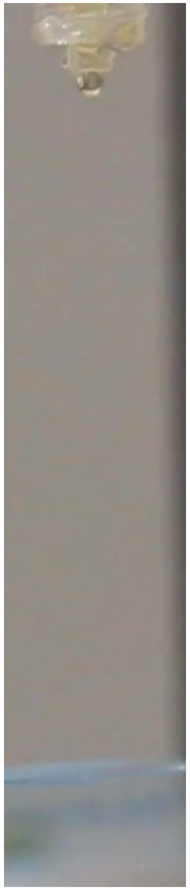
$$\delta \sim \left(\frac{\nu Q}{g}\right)^{1/4}$$

$$R \sim \delta \left(\ln \frac{H}{\delta}\right)^{1/2}$$

$$\Omega \sim \frac{U_1}{\delta}\left(\ln \frac{H}{\delta}\right)^{-\frac{1}{2}}$$



Gravitational regime movies

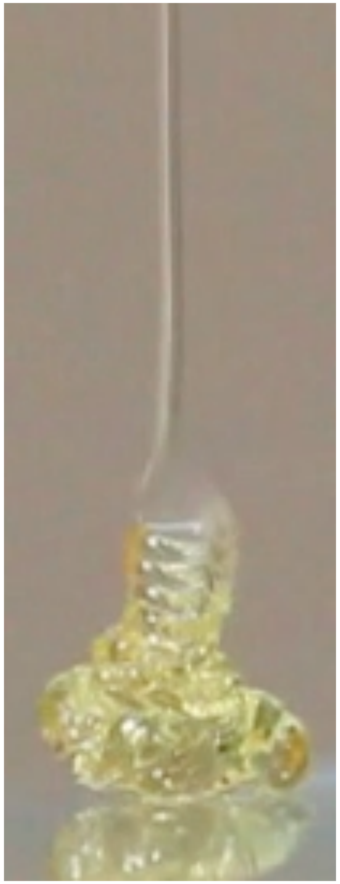


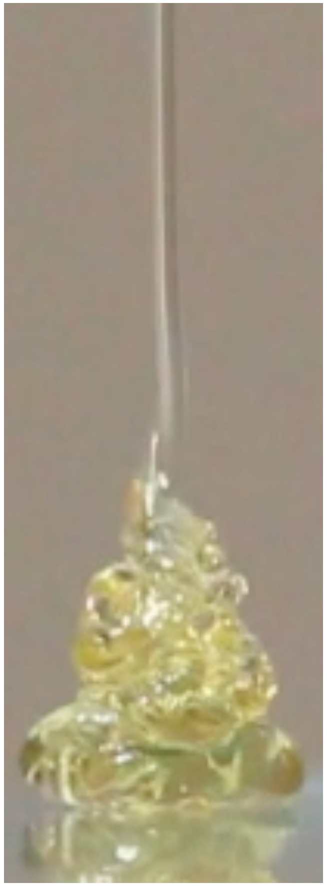
Honey, 2000 cst
60 fps
Fall height: 5 cm
Flow rate:
25% of original velocity

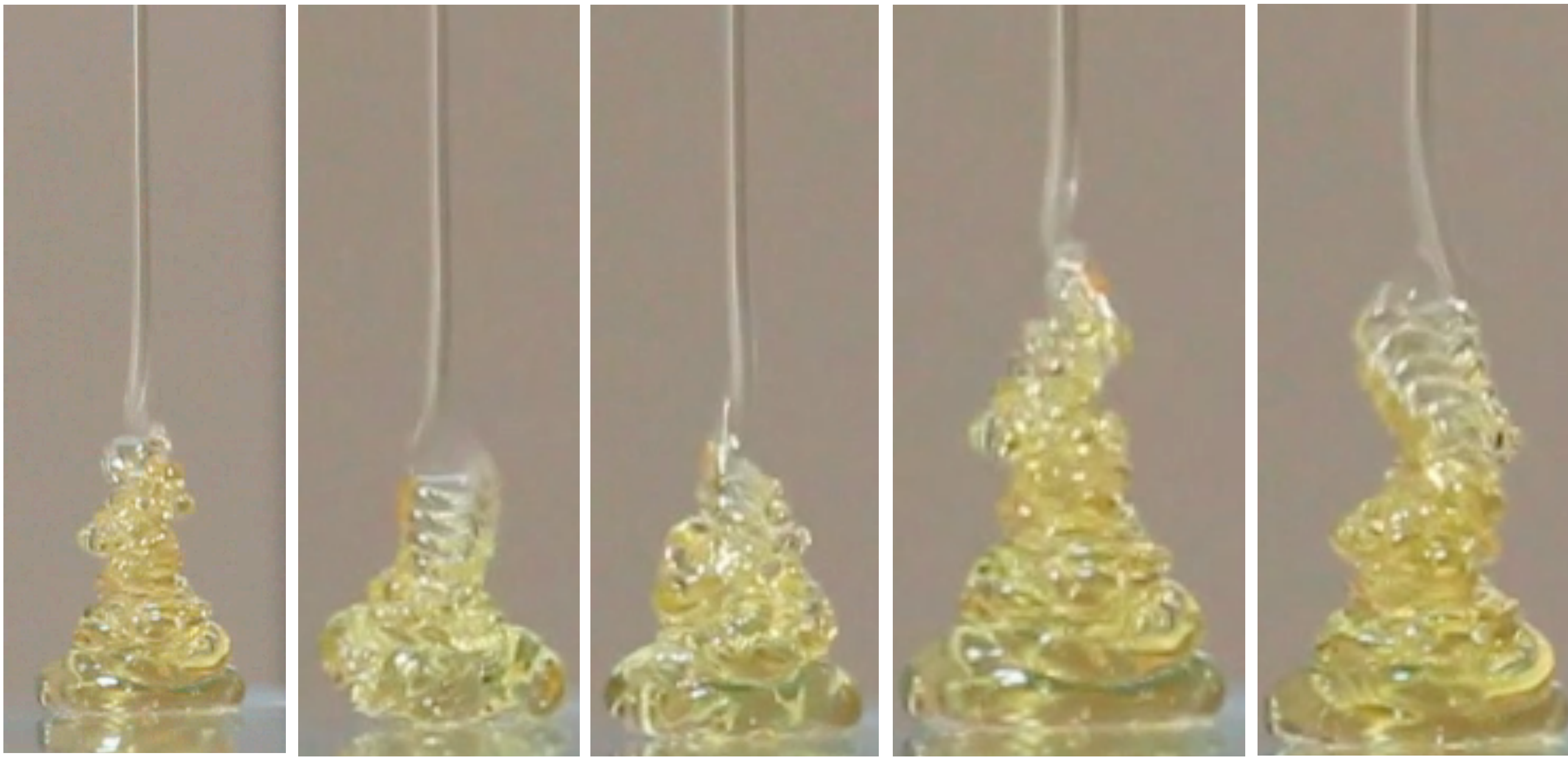
Silicone oil, 5000 cst
2000 fps
Fall height: 5 cm
Flow rate:
1.5% of original velocity

Theoretical analysis

- Inertial-Gravitational regime
  - Both gravitational and inertial forces are considerable
  - There're resonant frequencies







Theoretical analysis

- Inertial regime:
  - Inertial effects are more important than the gravitational and viscous

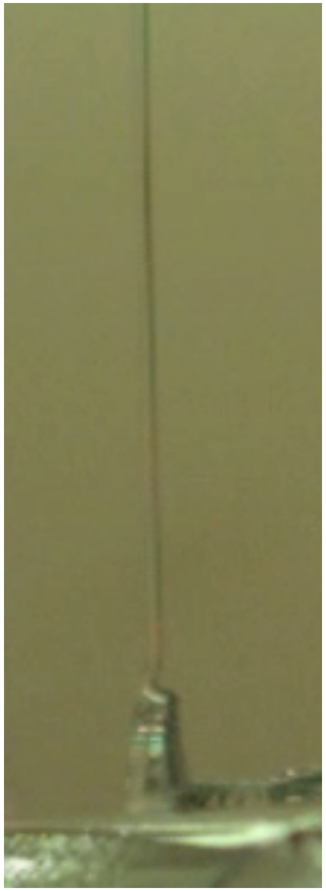
$$R \sim \left(\frac{\nu a_1^4}{Q}\right)^{1/3}$$

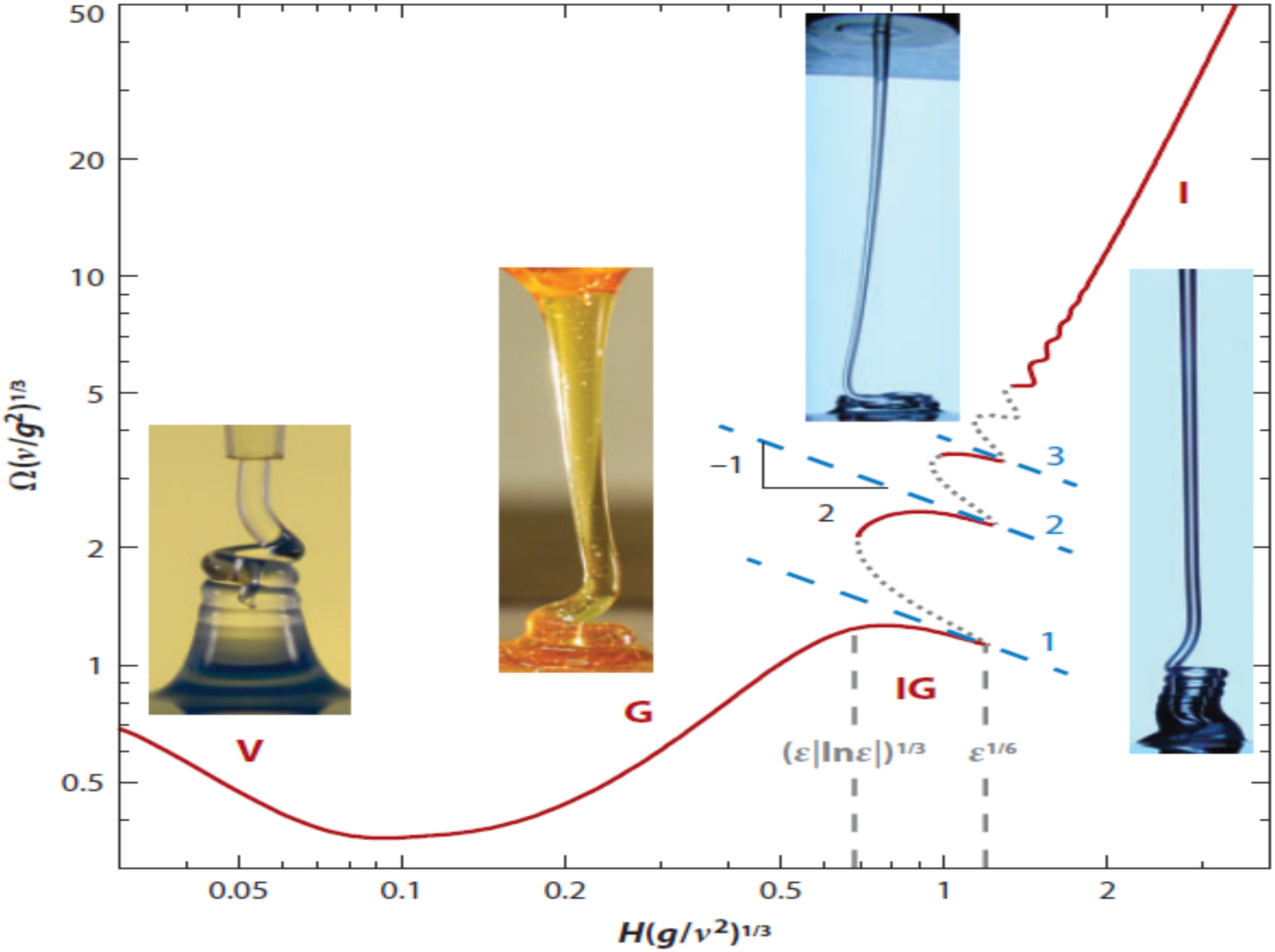
$$a_1 \sim (\nu Q / g H^2)^{1/2}$$

$$a_1 \sim (Q^2 / g H)^{1/4}$$

$$\Omega \sim \left(\frac{Q^4}{\nu a_1^{10}}\right)^{\frac{1}{3}}$$

Inertial regime movies





Liquid Rope Coiling, Neil M. Ribe,1 Mehdi Habibi,2 and Daniel Bonn3, Annu. Rev. Fluid Mech. 2012.

Instabilities

- From the equations: $R \sim \left(\frac{\nu a_1^4}{Q}\right)^{1/3}$ $a_1 \sim (\nu Q/gH^2)^{1/2}$
- We see, from the substitution of R, we have:

$$R \sim \left(\frac{\nu^3 Q}{gH^2}\right)^{\frac{1}{3}} \propto \nu \qquad a_1 \sim \left(\frac{\nu Q}{gH^2}\right)^{\frac{1}{2}} \propto \sqrt{\nu}$$

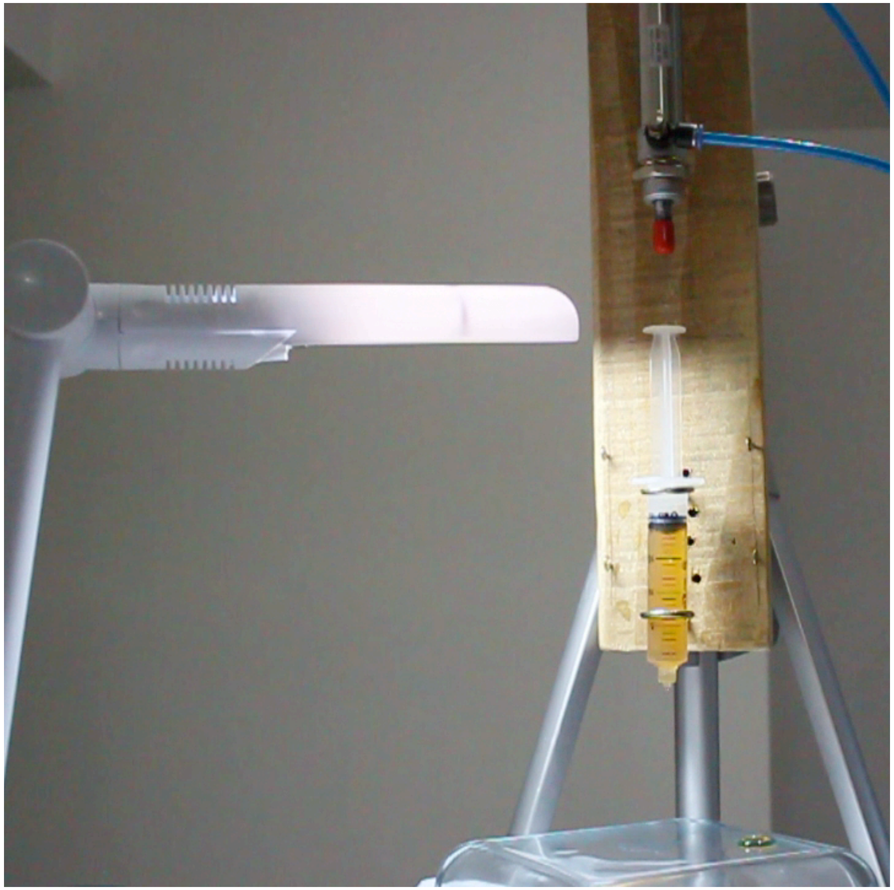
- There's a moment when the radius R is smaller than the filet radius a. The coil formation is ceased.

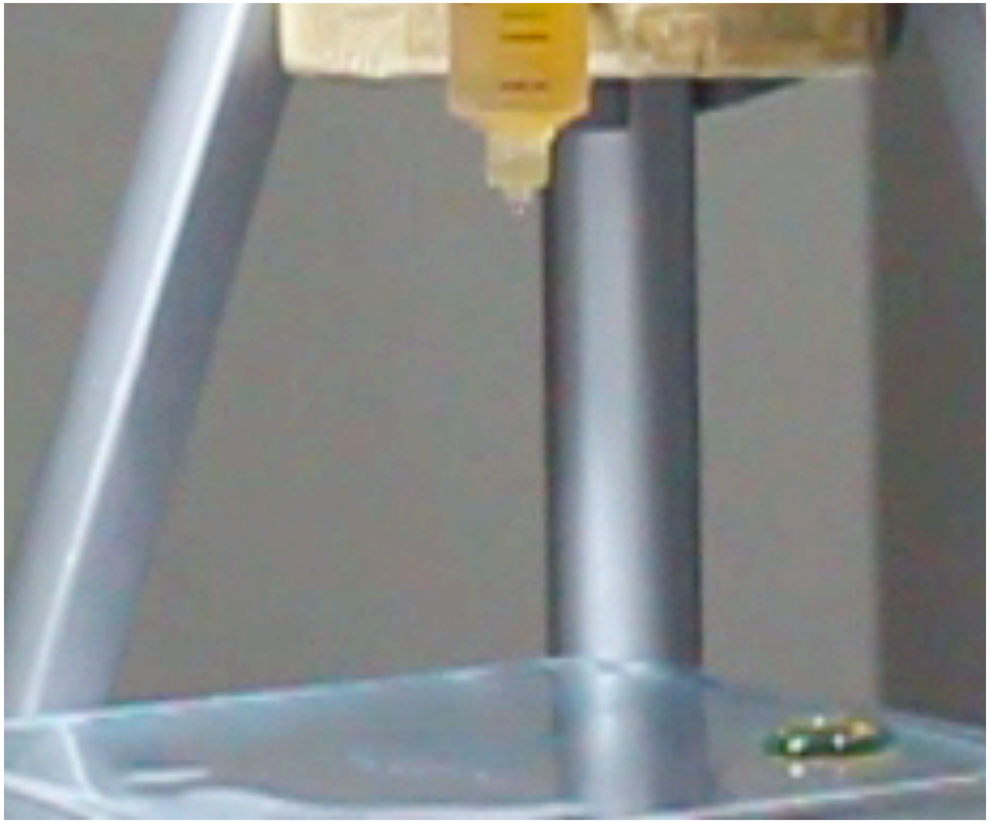
Materials used

- Honey
- Corn syrup
- Silicone oils
- Shampoo
- Glycerin

Experimental setup

Experimental procedure

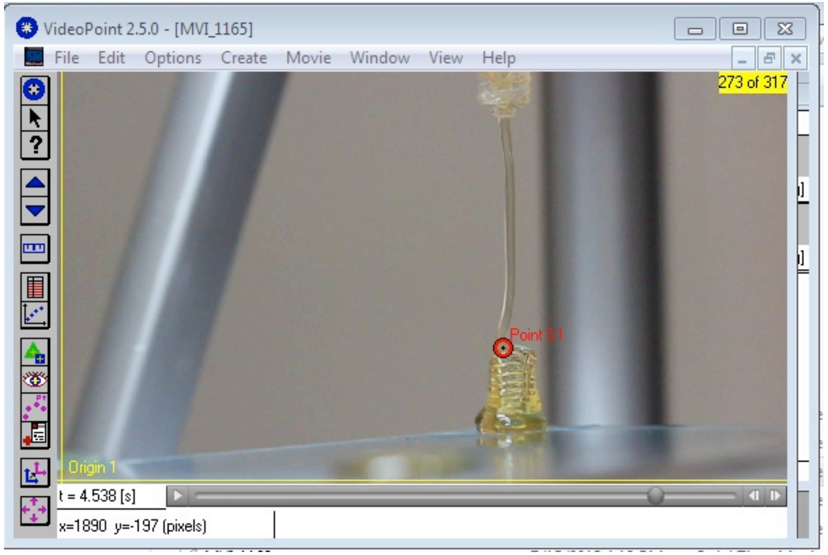




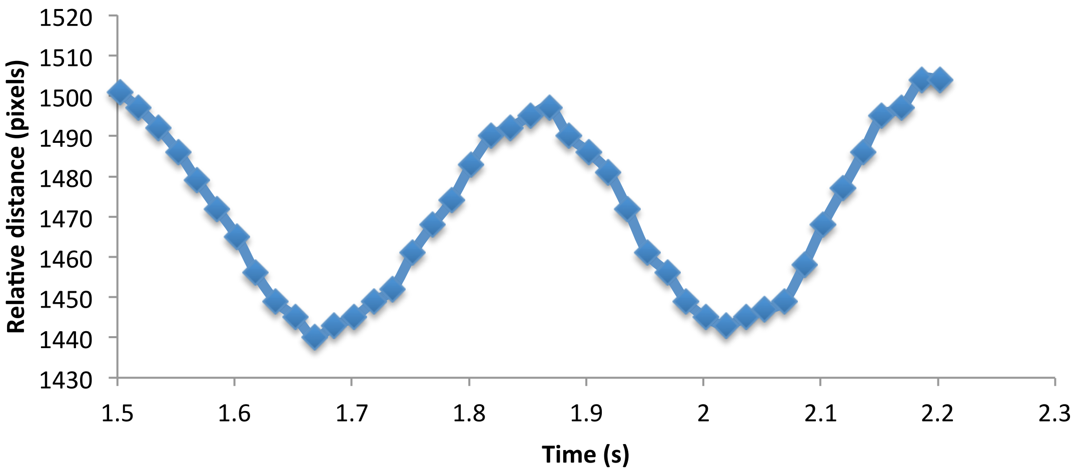
Material used	Viscosity
Corn syrup	1800 cst
Honey	2000 cst
Silicone oil	5000 cst
Silicone oil	500 cst

Obtaining data

- We used a high speed camera (2000 fps)
- Height of the fluid measured by using a fixed ruler and Video Point®
- Viscosities from the fabricant of each fluid



Obtaining data



Viscous regime – Honey, 2000 cst

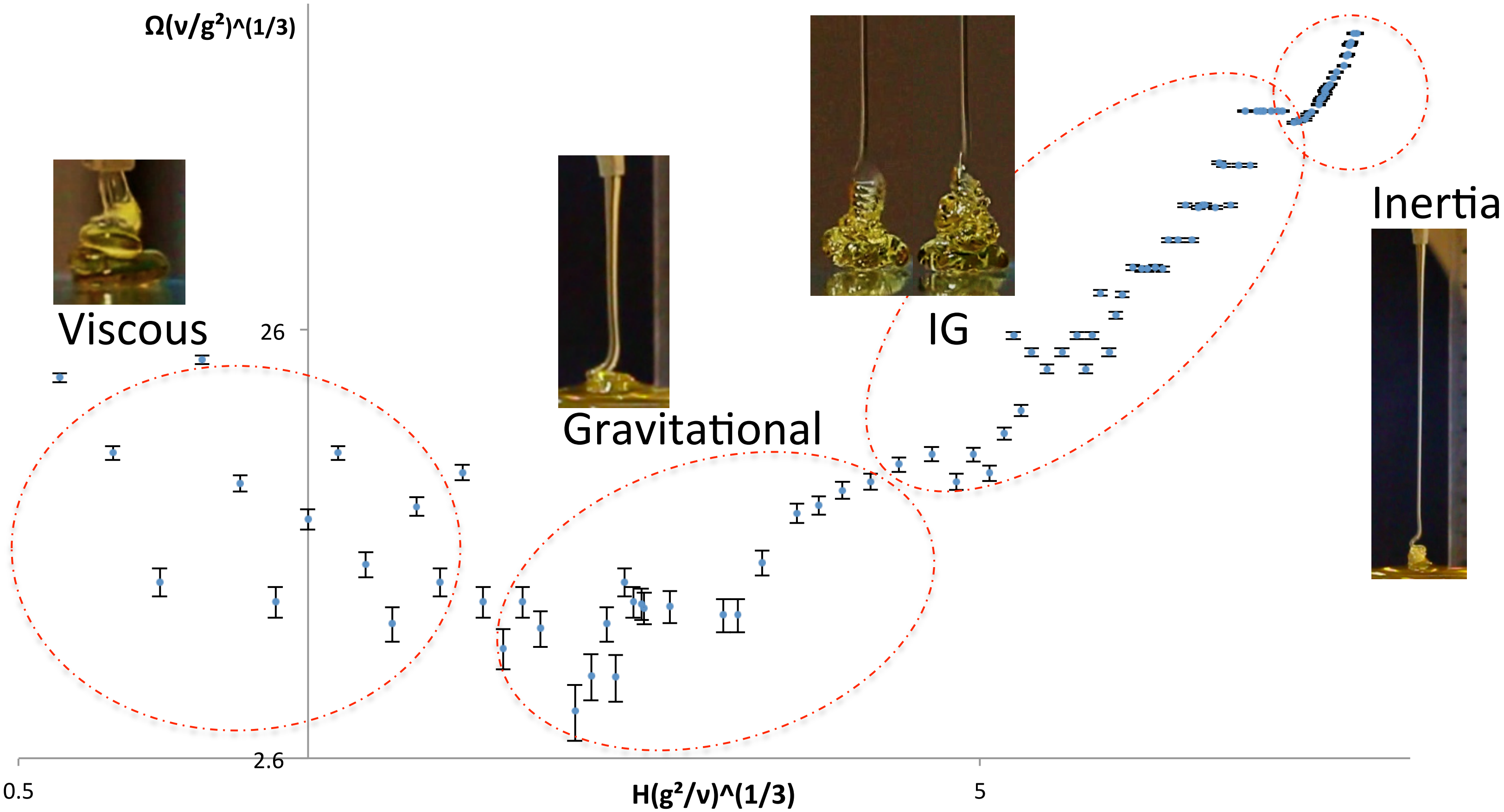
Data analysis
Viscous regime
Gravitational regime
Inertial gravitational regime
Inertial regime
Instabilities

Fall height	Angular frequency	Theoretical prediction	Relative error
1.0 cm	25 Hz	27 Hz	7.4%
4.0 cm	45 Hz	39.0 Hz	15.3%
8.0 cm	140 Hz	148 Hz	5.1%

Viscous regime | Gravitational regime | Inertial regime

Honey - Graph

- Data analysis
- Viscous regime
- Gravitational regime
- Inertial gravitational regime
- Inertial regime
- Instabilities



Viscous regime – Silicone oil, 5000 cst

Fall height	Angular frequency	Theoretical prediction	Relative error
0.5 cm	121.2 Hz	111.2 Hz	8.2%
3.2 cm	154.8 Hz	156.4 Hz	0.9%
13.6 cm	689.2 Hz	662.8 Hz	3.8%

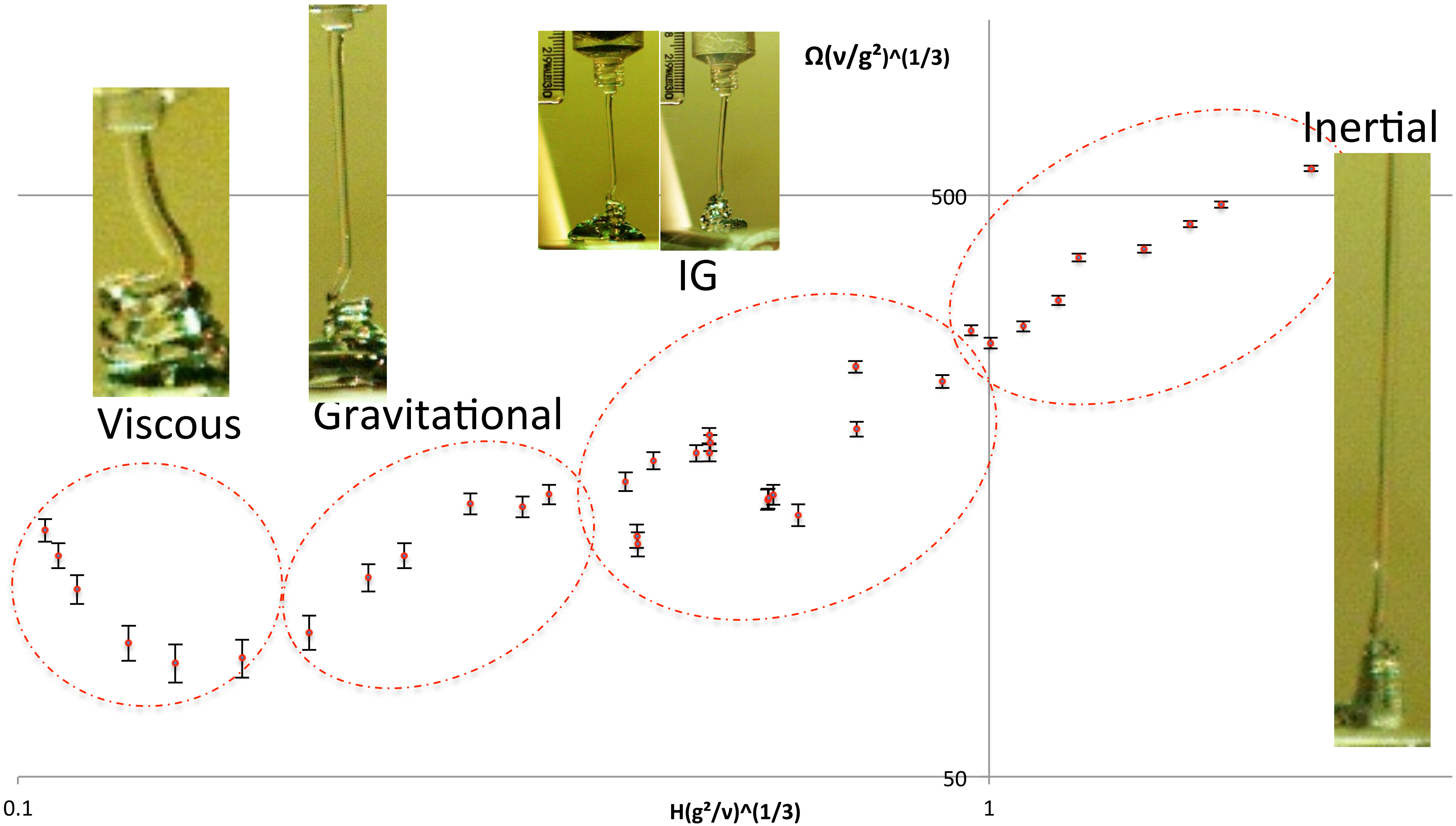
Viscous regime

Gravitational regime

Inertial regime

Silicone oil graph – 5000 cst

- Data analysis
- Viscous regime
- Gravitational regime
- Inertial gravitational regime
- Inertial regime
- Instabilities



Viscosity variation

- For the viscosity variation, we used silicone oils with the same surface tension, density and fall height.

Viscosity	Surface tension	Density
500 cst	21.2 dynes/cm	0.970 g/cm^3
1000 cst	21.2 dynes/cm	0.970 g/cm^3
5000 cst	21.4 dynes/cm	0.975 g/cm^3
60000 cst	21.5 dynes/cm	0.976 g/cm^3
63775 cP	--	1.03 g/cm^3

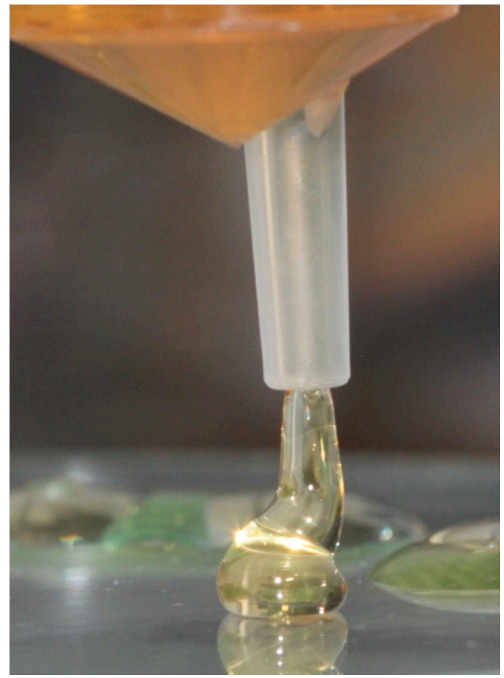
Comparative videos

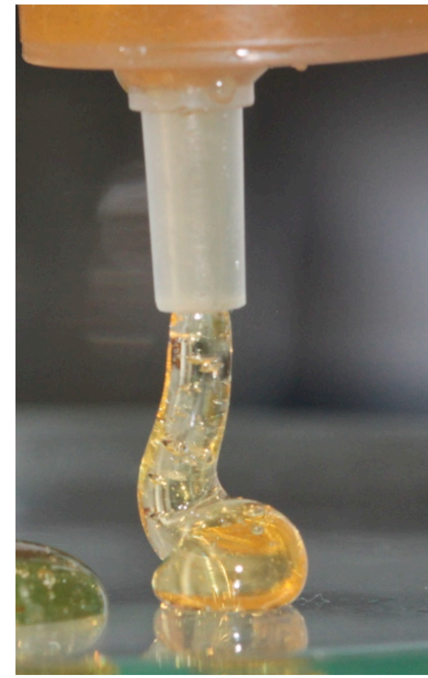
Comparing theoretical and experimental results

- From our instability analysis:
  - Surface tension reduces the visibility of the phenomena
- Experiment to see its effects:
  - Observe the smaller heights for the phenomena still be seen:
    - Honey (control)
    - Honey with salt (higher surface tension)
    - Honey with a bit of detergent (smaller surface tension)

Comparing theoretical and experimental results

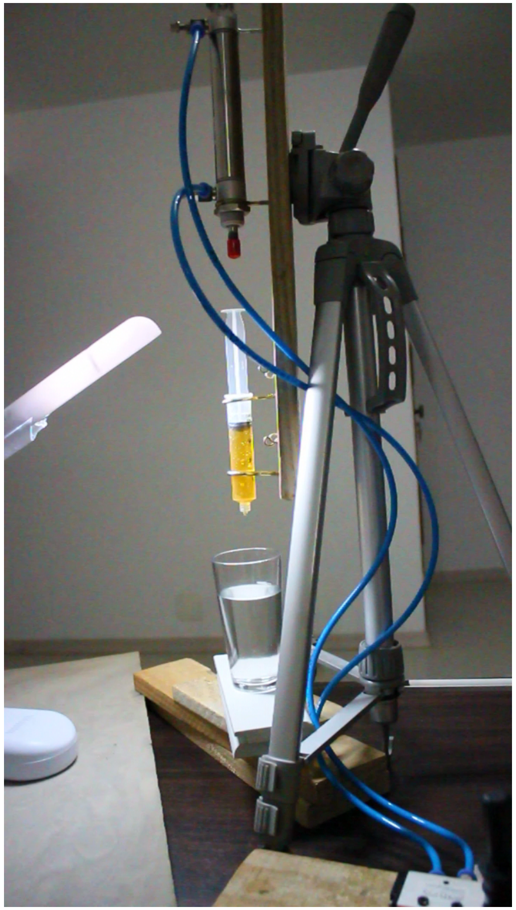
- Data analysis
- Viscous regime
- Gravitational regime
- Inertial gravitational regime
- Inertial regime
- Instabilities





Honey - detergent	Honey - control	Honey - salt
0.6 cm	1.0 cm	1.3 cm

Experimental setup – Honey and water



- Cup with water
- Honey
- Syringe
- Tripod
- Pneumatic piston

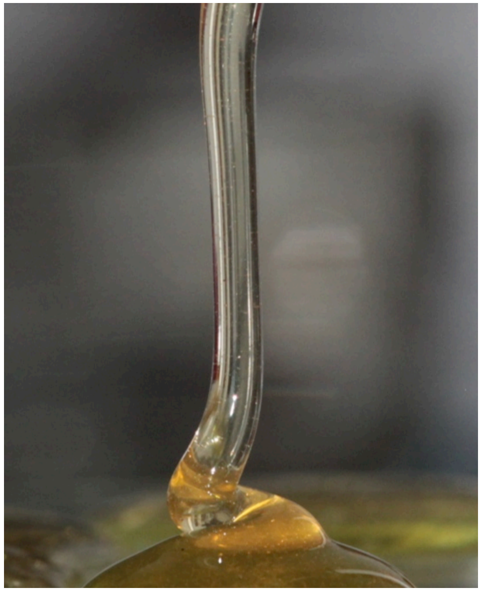
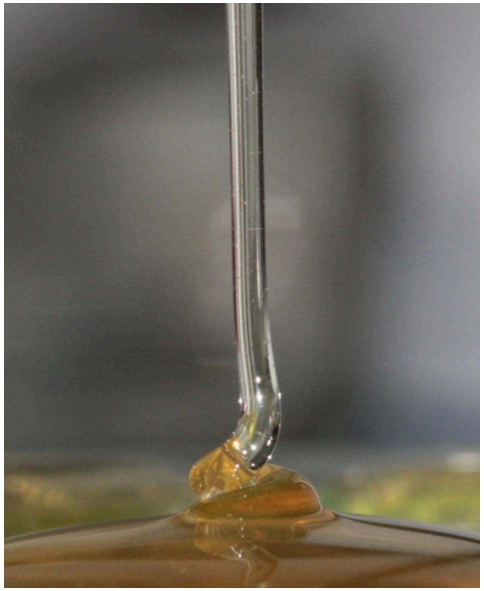
Fluid variation

- We can use another surrounding medium, such as water.
- We get some interesting phenomena:

Comparing theoretical and experimental results

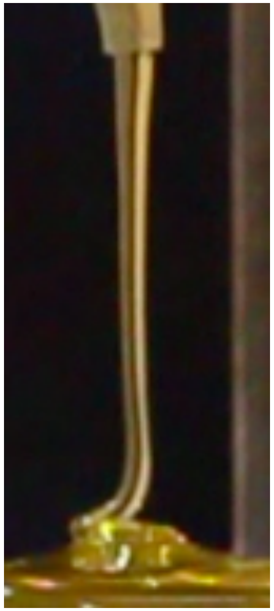
- Data analysis
- Viscous regime
- Gravitational regime
- Inertial gravitational regime
- Inertial regime
- Instabilities

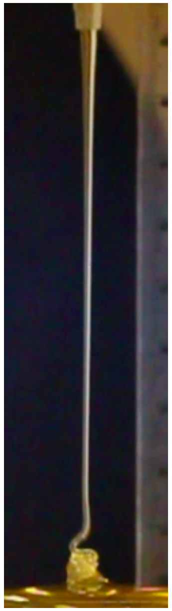
- Viscosity
- It's possible to notice changes in the coil formation by changing its viscosity

Conclusion

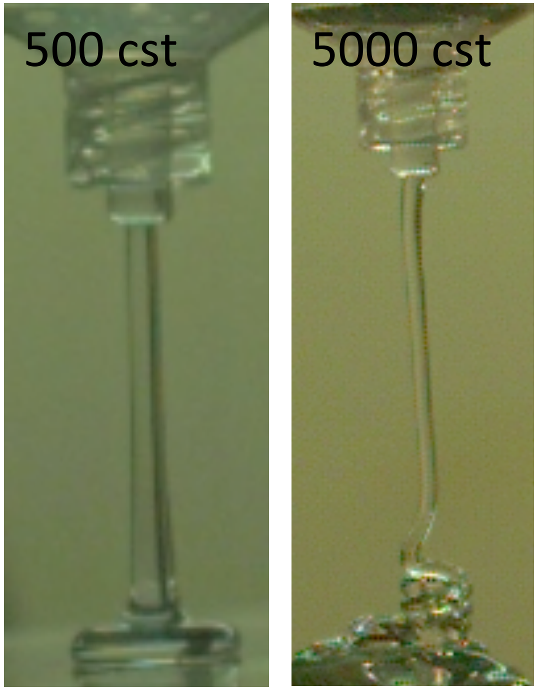
- The phenomena can be divided in 4 phases, depending on the fall height:



Conclusion

- The coiling can appear in many fluids, but the visualization depends on the surface tension and viscosity.



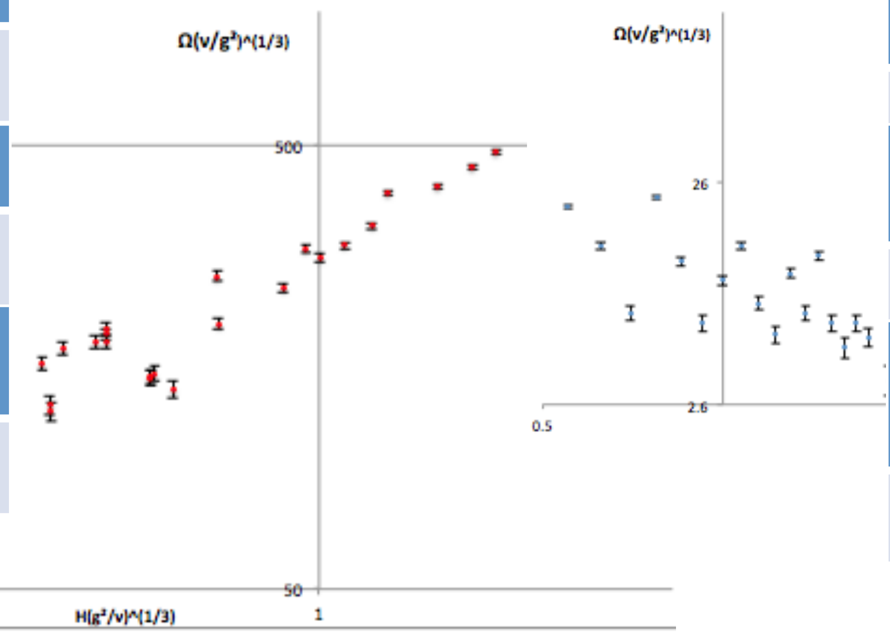
Conclusion

- We can analyze the problem in a quantitative way, depending on height, viscosity and flow rate.

Height	Angular frequency	Flow rate	Theoretical angular frequency	Error
0.5 cm	30.3 Hz	0.49 cm^3/s	27.8 Hz	8.2%

Height	Angular frequency	Flow rate	Theoretical angular frequency	Error
3.20 cm	38.75 Hz	0.49 cm^3/s	39.1 Hz	0.9%

Height	Angular frequency	Flow velocity	Theoretical angular frequency	Error
13.6 cm	172.3 Hz	0.49 cm^3/s	165.7 Hz	3.8%



Fall height	Angular frequency	Flow velocity	Theoretical angular frequency	Error
1.0 cm	25 Hz	25 m/s	27 Hz	7.4 %

Fall height	Flow velocity	Angular frequency	δ parameter	Theoretical angular frequency	Error
4 cm	30 cm/s	45 Hz	0.93 cm	39 Hz	15.3 %

Angular frequency	Fall height	α_1 parameter	Theoretical angular frequency	Error
140 Hz	8.0 cm	0.2	148 Hz	5.1 %

References

- *Multiple coexisting states of liquid rope coiling By N. M. RIBE1, H. E. HUPPERT2, M. A. HALLWORTH2, M. HABIBI3,4 AND DANIEL BONN*
- Coiling of viscous jets By Neil M. Ribe
- Liquid Rope Coiling, by Neil M. Ribe, Mehdi Habibi and Daniel Bonn, Annu. Rev. Fluid Mech. 2012
- Mahadevan L, Ryu WS, Samuel ADT. 1998. Fluid 'rope trick' investigated. *Nature*
- The Bouncing Jet: A Newtonian Liquid Rebounding off a Free Surface, Matthew Thrasher,∗ Sunghwan Jung,† Yee Kwong Pang,‡ Chih-Piao Chuu, and Harry L.
- The meandering instability of a viscous thread, Stephen W. Morris, Jonathan H. P. Dawes, Neil M. Ribe, and John R.
- Bending-Filament Model for the Buckling and Coiling Instability of Viscous Fluid Rope, Shin-ichiro Nagahiro, Yoshinori Hayakawa
- The folding motion of an axisymmetric jet of wormlike-micelles solution, Matthieu Varagnat, Trushant Majmudar, Will Hartt, Gareth H. McKinley

Thank you!

Appendix summary

Theoretical deduction

- Formulations
- Considerations
- Boundary conditions

Experiment variation

- Water and honey
- Honey sewing machine

Data getting

- Graphs and tables
- How to get frequency
- Unity of graphs and physical meaning

Appendix

- We have, in the problem, the parameters:
  - We define the Cartesian coordinates **x**(s, t), radius a(s, t), where s is the measurement of the arc along the axis.
  - In this way, we have:

$$\mathbf{d}_3(s, t) = \mathbf{x}'$$

  - Where the prime denotes a partial derivative in relation to s, and we'll use this notation from now on.
  - The inverters were defined in each point along the axis that follow the section plane of the filet of the fluid.

Appendix

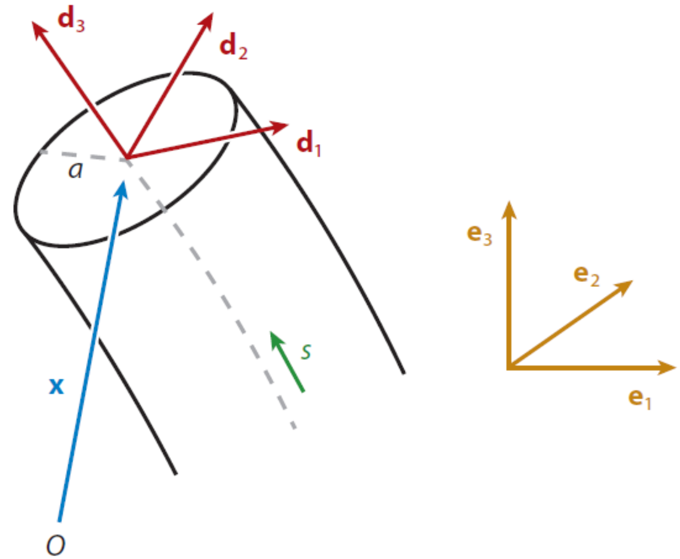
- Other than that, it's necessary to define the inverters, following the fluid's rotation.

$$\mathbf{d}_1(s,t)$$

$$\mathbf{d}_2(s,t) \equiv \mathbf{d}_3 \times \mathbf{d}_1$$

$$\mathbf{d}'_i = \boldsymbol{\kappa} \times \mathbf{d}_i$$

$$\boldsymbol{\kappa} \equiv \kappa_i \mathbf{d}_i$$



Modified image from Ribe (2004) and obtained from Liquid Rope Coiling, Neil M. Ribe,1 Mehdi Habibi,2 and Daniel Bonn3, Annu. Rev. Fluid Mech. 2012

Appendix

- We can introduce a velocity vector of the fluid in the axis: *V≡Vi* ***di***
- Thus, follows the relation of V and x:

$$\frac{D\mathbf{x}}{Dt} = \mathbf{V}$$

- We need then to relate the deformation rate by compression, flexion and rotation. The stretching rate is:

$$\Delta = \mathbf{V}' \cdot \mathbf{d}_3$$

Appendix

- The mass consevation gives us, being ***A = π a²:***

$$\frac{DA}{Dt} = -A\Delta$$

- We can define the rotation rate around the directions ***d1*** and ***d2:***

$$\omega_1 = -\mathbf{V}' \cdot \mathbf{d}_2$$
$$\omega_2 = \mathbf{V}' \cdot \mathbf{d}_1$$

Appendix

- We can define the forces ***N*** and the momentum vector of the flexion and rotation of the fluid, ***M:***

$$\mathbf{N} \equiv N_i \mathbf{d}_i = \int \boldsymbol{\sigma} \cdot \mathbf{d}_3 \, dA$$

$$\mathbf{M} \equiv M_i \mathbf{d}_i = \int \mathbf{y} \times (\boldsymbol{\sigma} \cdot \mathbf{d}_3) dA$$

- Where σ is the stress tensor and the integration is made though a section.

Appendix

- Linear momentum consevation:

$$\rho A \frac{D\mathbf{V}}{Dt} = \mathbf{N}' + \rho A \mathbf{g}$$

- Angular momentum conservation:

$$0 = \mathbf{M}' + \mathbf{d}_3 \times \mathbf{N}.$$

Mathematical analysis

- Imposing the Newtonian fluid condition, we need a relating condition for the dynamic variables with the kinectic, and this analysis gives us: (Ribe et al.2006)

Appendix

$$N_3 = 3\eta A\Delta$$

$$M_1 = 3\eta I\omega' \cdot \mathbf{d}_1$$

$$M_2 = 3\eta I\omega' \cdot \mathbf{d}_2$$

$$M_3 = 2\eta I\omega' \cdot \mathbf{d}_3$$

Where η is the dynamic viscosity, ω is defined as ***ω**=ω’**d*** and *I=(πa^4)/4*

Appendix

- Acceleration term in the global balance of forces:

$$\frac{D\mathbf{V}}{Dt} = U\,(U\mathbf{d}_3)' + 2\Omega U\mathbf{e}_3 \times \mathbf{d}_3 + \Omega^2\mathbf{e}_3 \times (\mathbf{e}_3 \times \mathbf{x})$$

- First term: referential rotating with the fluid
- Second term: Coriolis
- Third term: centrifugal acceleration

Boundary conditions

- Boundary condition:
  - We impose the initial conditions of the problem
  - At first, the fluid doesn't rotate and it's vertical
  - We consider that the fluid rotates in the base with defined angular velocity Ω.
  - Imposing we know the fluid's movement in its base (known anguar velocity, coil radius, the derivatives, the filet orientation and its derivatives)

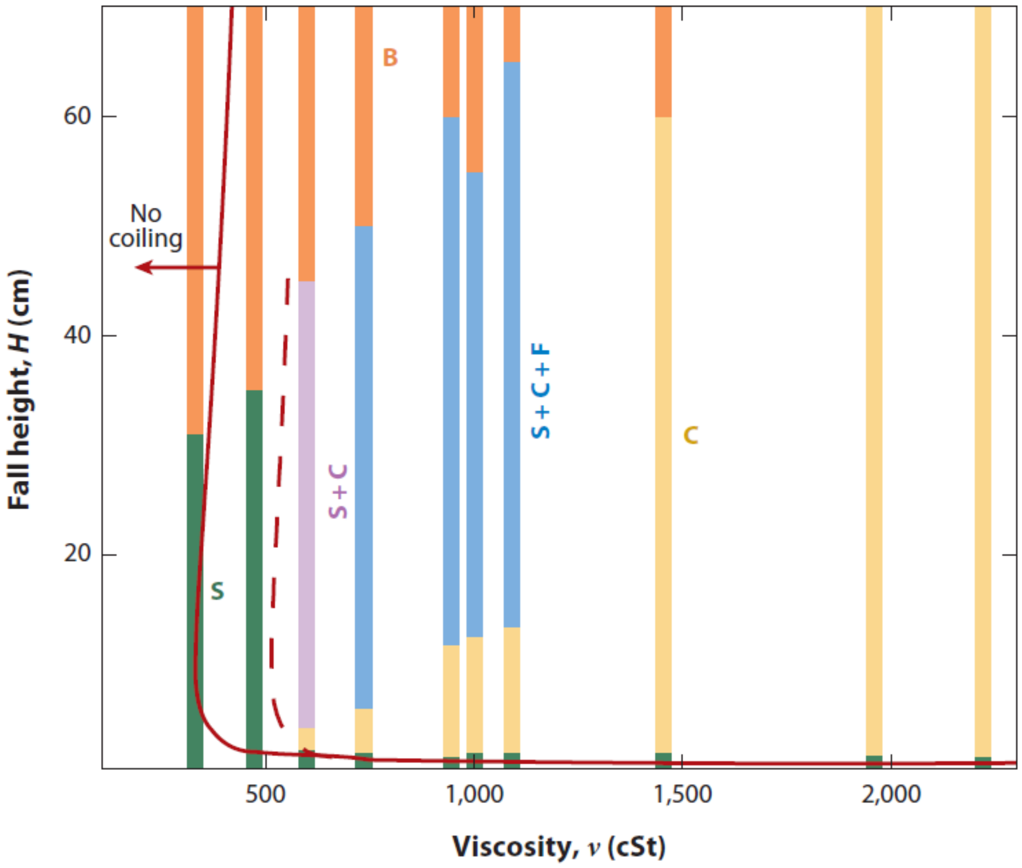
Appendix

- From our previous expressions, we can do a equation system and we can relate the wanted parameters.
- For such procedure, we need a numerical analysis.
- The radius and frequency approximations come from this system of equations.

Appendix

- From the presented formulas, we can obtain, from a numerical solution of the equation system, a graph of frequency x height, already presented.

Appendix



B- Capillary instabilities
S- Stagnation
C- Coil formation
F- Flexion rotation

Appendix

- We can define the forces of each phenomena:
  - Viscous:
    $$F_V \sim \rho \nu a_1^4 U_1 R^{-4}$$
  - Gravitational:
    $$F_G \sim \rho g a_1^2$$
  - Inertial:
    $$F_I \sim \rho a_1^2 U_1^2 R^{-1}$$

Appendix

- Viscous regime:

$$(F_V \gg F_G \approx F_I)$$

- Gravitational regime:

$$(F_G \approx F_V \gg F_I)$$

- Inertial regime:

$$(F_I \approx F_V \gg F_G)$$

Appendix

- Viscous regime:
  - Viscous force:
    - $F \approx 10^{1}$ dyne
  - Gravitational force:
    - $F \approx 10^{-3}$ dyne
  - Inertial force:
    - $F \approx 10^{-3}$ dyne

Appendix

- Gravitational regime:
  - Viscous force:
    - $F \approx 10^{-2}$ dyne
  - Gravitational force:
    - $F \approx 10^{-2}$ dyne
  - Inertial force:
    - $F \approx 10^{-4}$ dyne

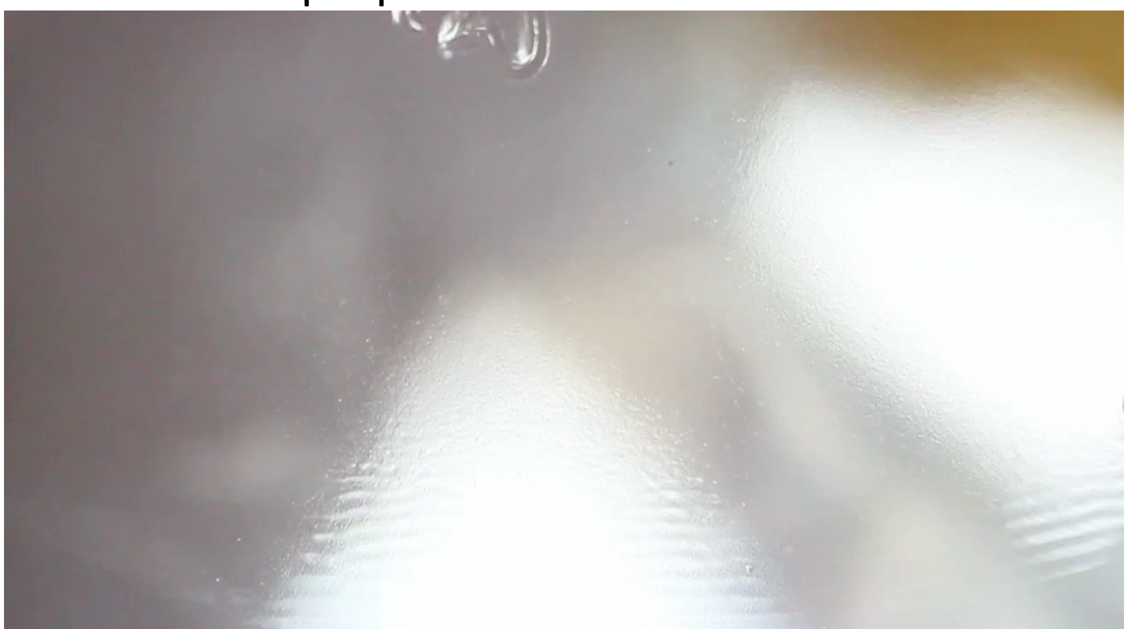
Appendix

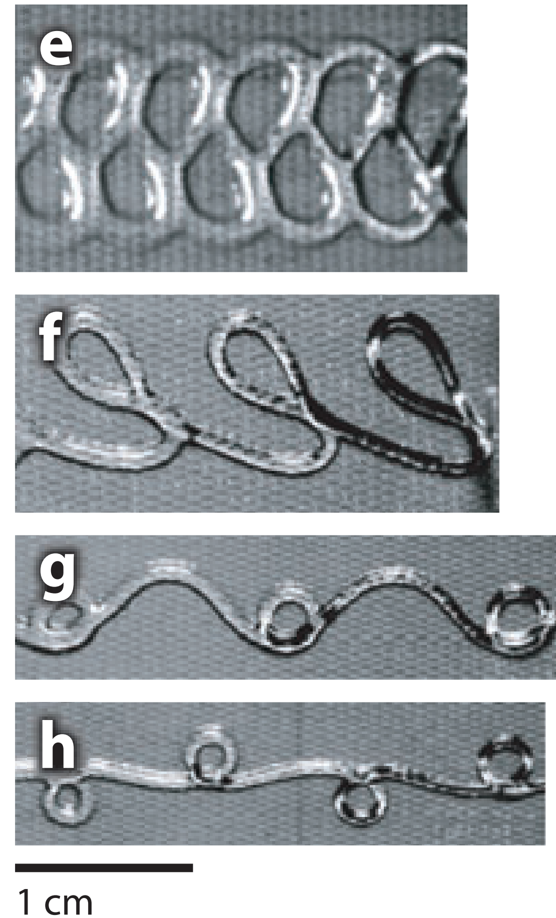
- Inertial regime:
  - Viscous force:
    - $F \approx 10^{-1}$ dyne
  - Gravitational force:
    - $F \approx 10^{-7}$ dyne
  - Inertial force:
    - $F \approx 10^{-1}$ dyne

- 1 dy = 10^{-5}N

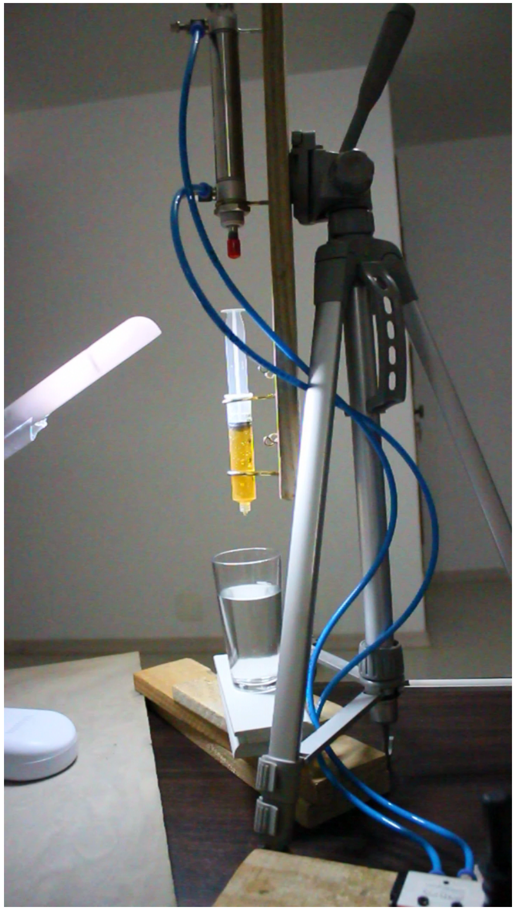
Sewing machine

- A thin thread of viscous fluid falling onto a moving belt generates a surprising variety of patterns depending on the belt speed, fall height, flow rate, and fluid properties





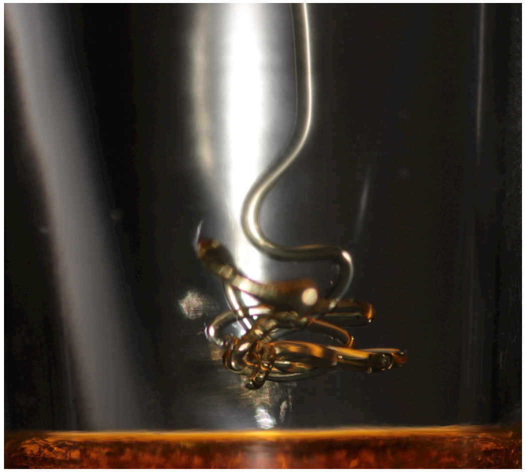
Experimental setup – Honey and water



- Cup with water
- Honey
- Syringe
- Tripod
- Pneumatic piston

Fluid variation

- We can use another surrounding medium, such as water.
- We get some interesting phenomena:

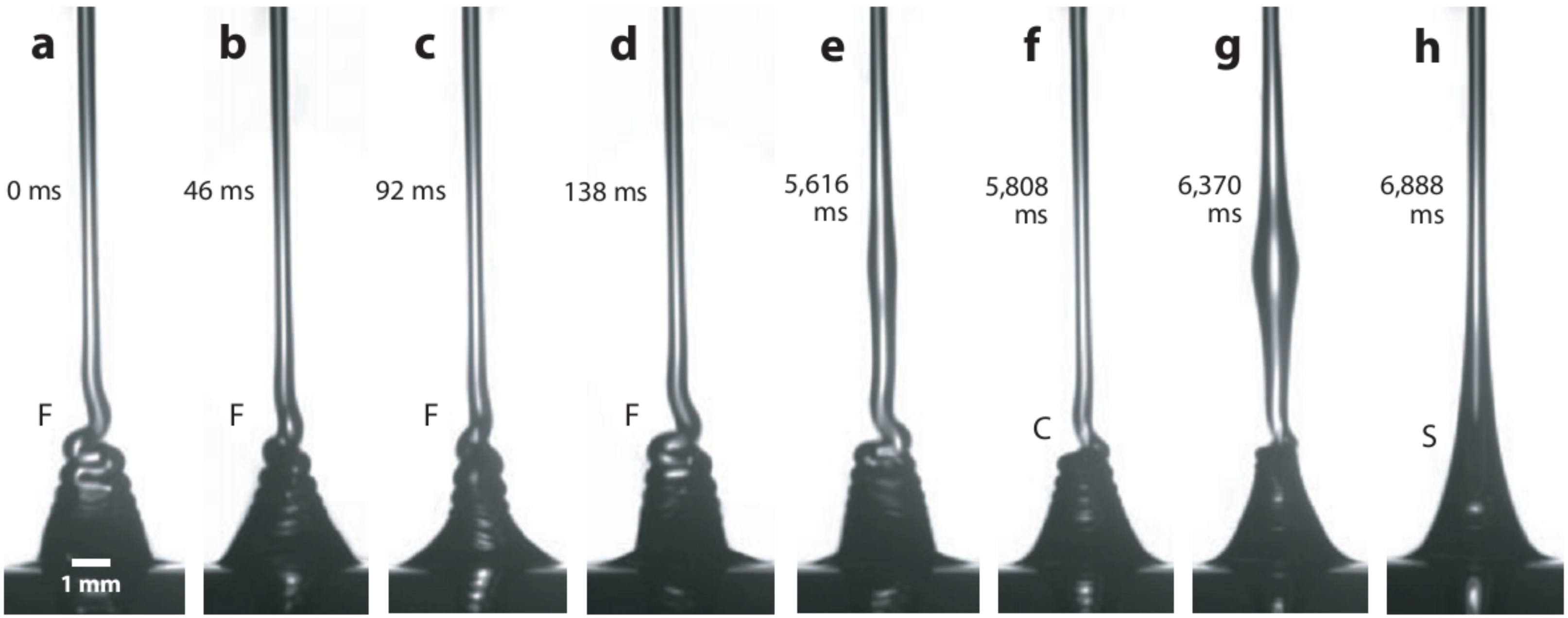


Motion seen from above

Instabilities

[Experimental]

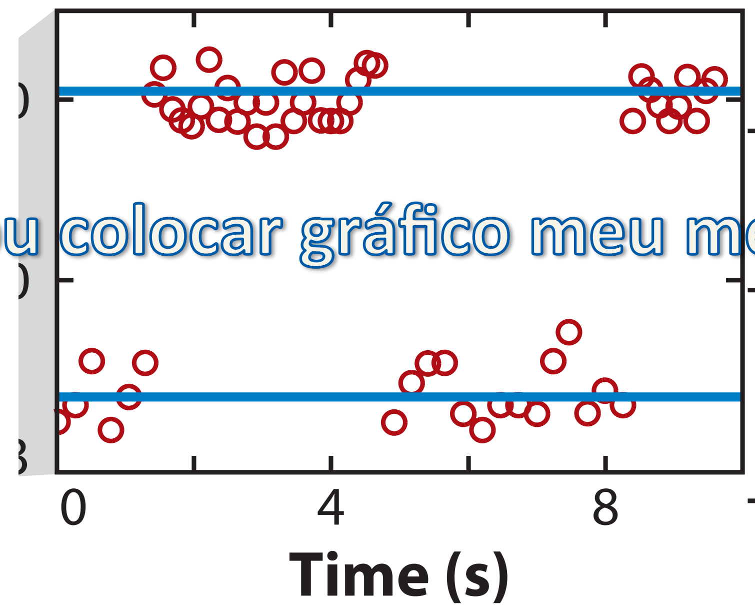
- Surface tension:
- Rayleigh-Plateau instabilities



Liquid Rope Coiling, Neil M. Ribe,1 Mehdi Habibi,2 and Daniel Bonn3, Annu. Rev. Fluid Mech. 2012.

Instabilities study

- In the inertial-gravitational regime, we have resonant forces, causing the phenomena to be time-dependent.
- We studied this resonance:



Vou colocar gráfico meu mesmo ☺

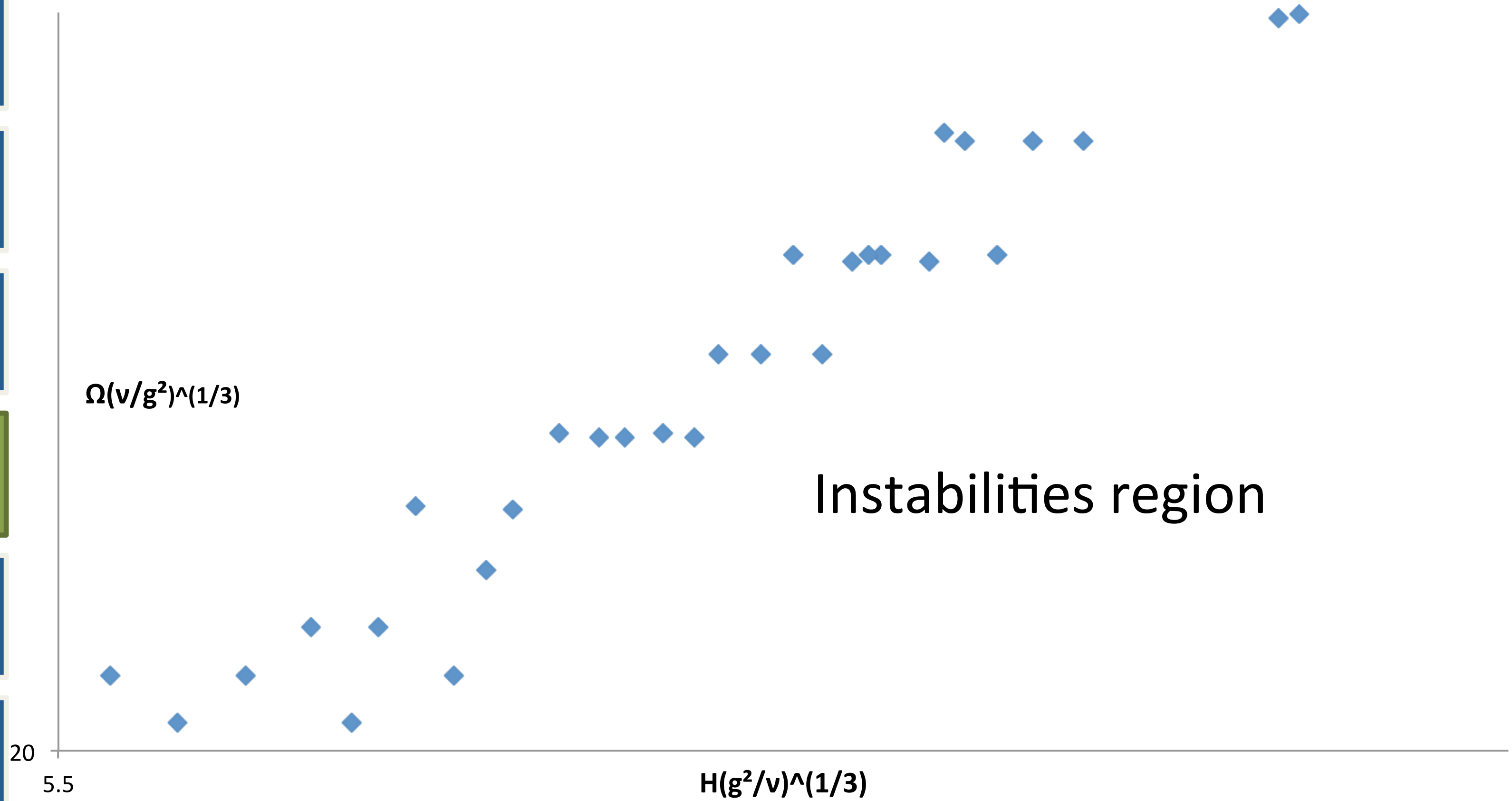
Gravitational regime - Honey, 2000 cst

Fall height	Flow velocity	Angular frequency	δ parameter	Theoretical angular frequency	Error
4 cm	30 cm/s	45 Hz	0.93 cm	39 Hz	15.3 %

Increasing tendency

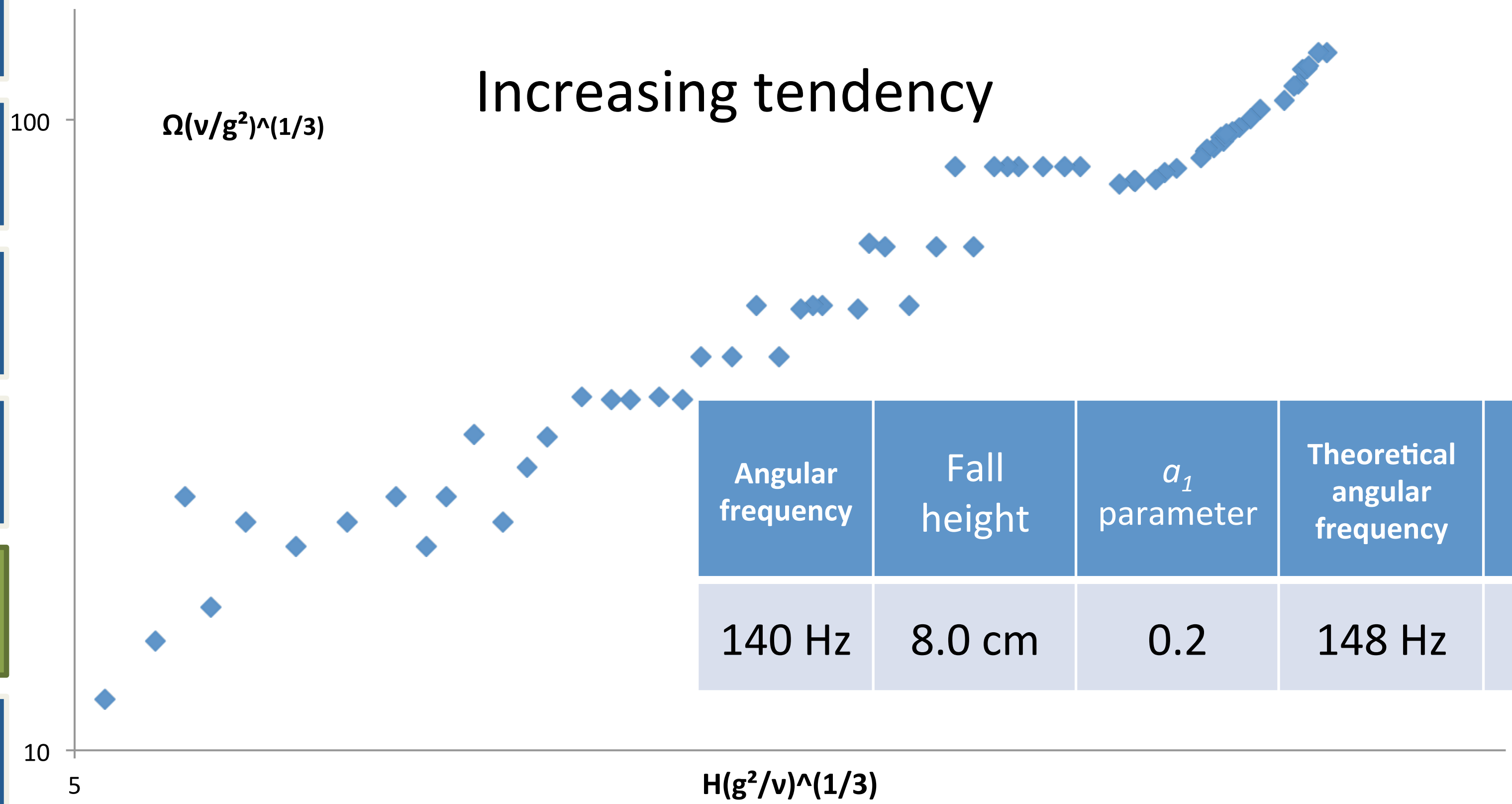
Inertial gravitational regime - Honey, 2000 cst

- Data analysis
- Viscous regime
- Gravitational regime
- **Inertial gravitational regime**
- Inertial regime
- Instabilities



Inertial regime - Honey, 2000 cst

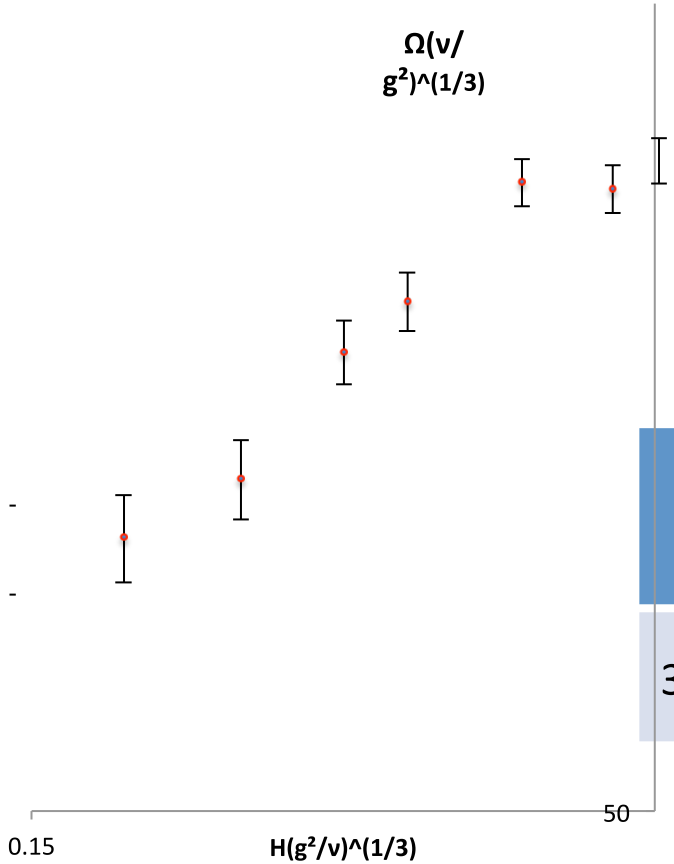
- Data analysis
- Viscous regime
- Gravitational regime
- Inertial gravitational regime
- Inertial regime
- Instabilities



Angular frequency	Fall height	a_1 parameter	Theoretical angular frequency	Error
140 Hz	8.0 cm	0.2	148 Hz	5.1 %

Gravitational regime – Silicone oil, 5000 cst

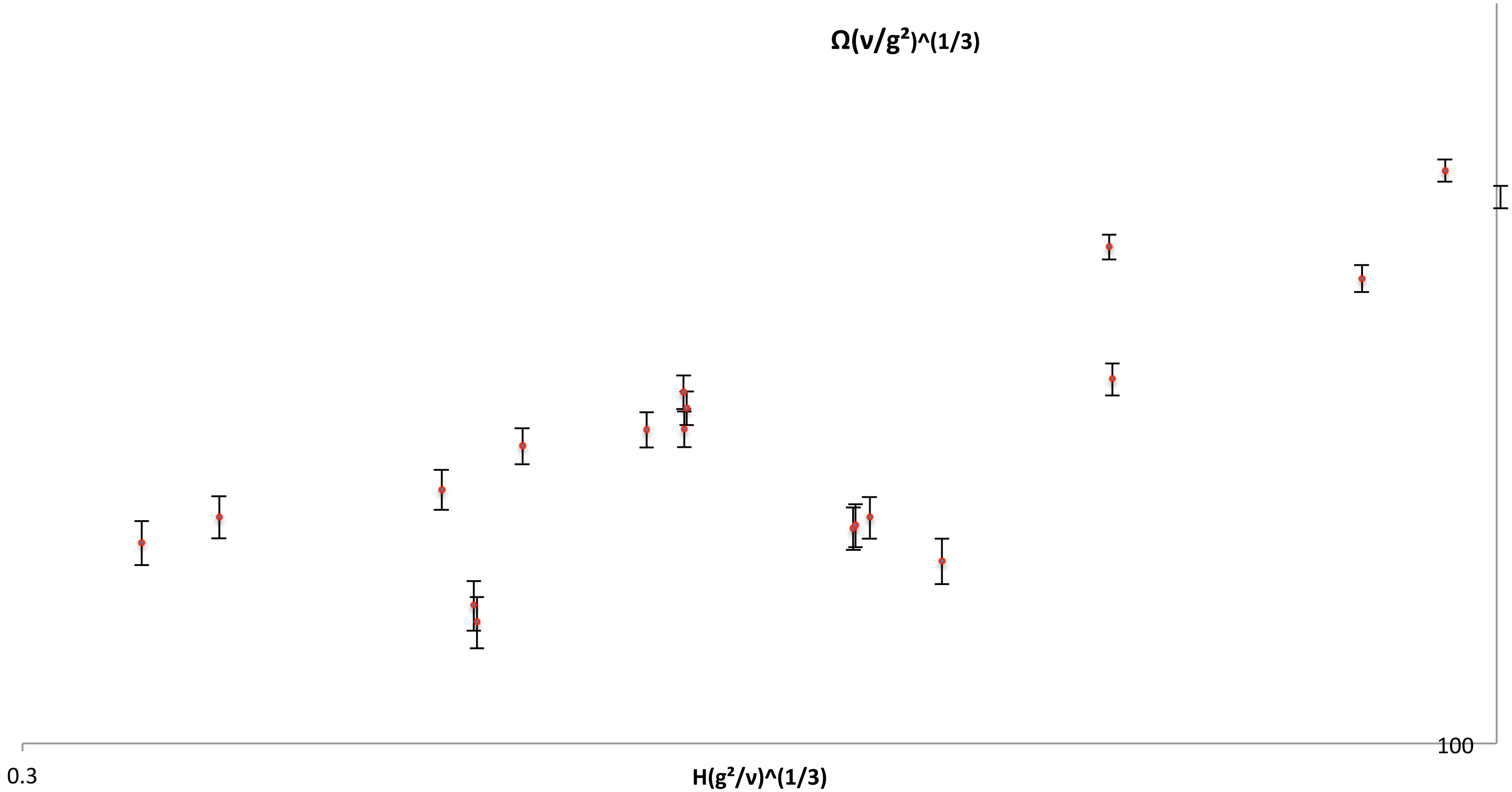
- Data analysis
- Viscous regime
- **Gravitational regime**
- Inertial gravitational regime
- Inertial regime
- Instabilities



Height	Angular frequency	Flow rate	Theoretical angular frequency	Error
3.20 cm	38.75 Hz	0.49 cm^3/s	39.1 Hz	

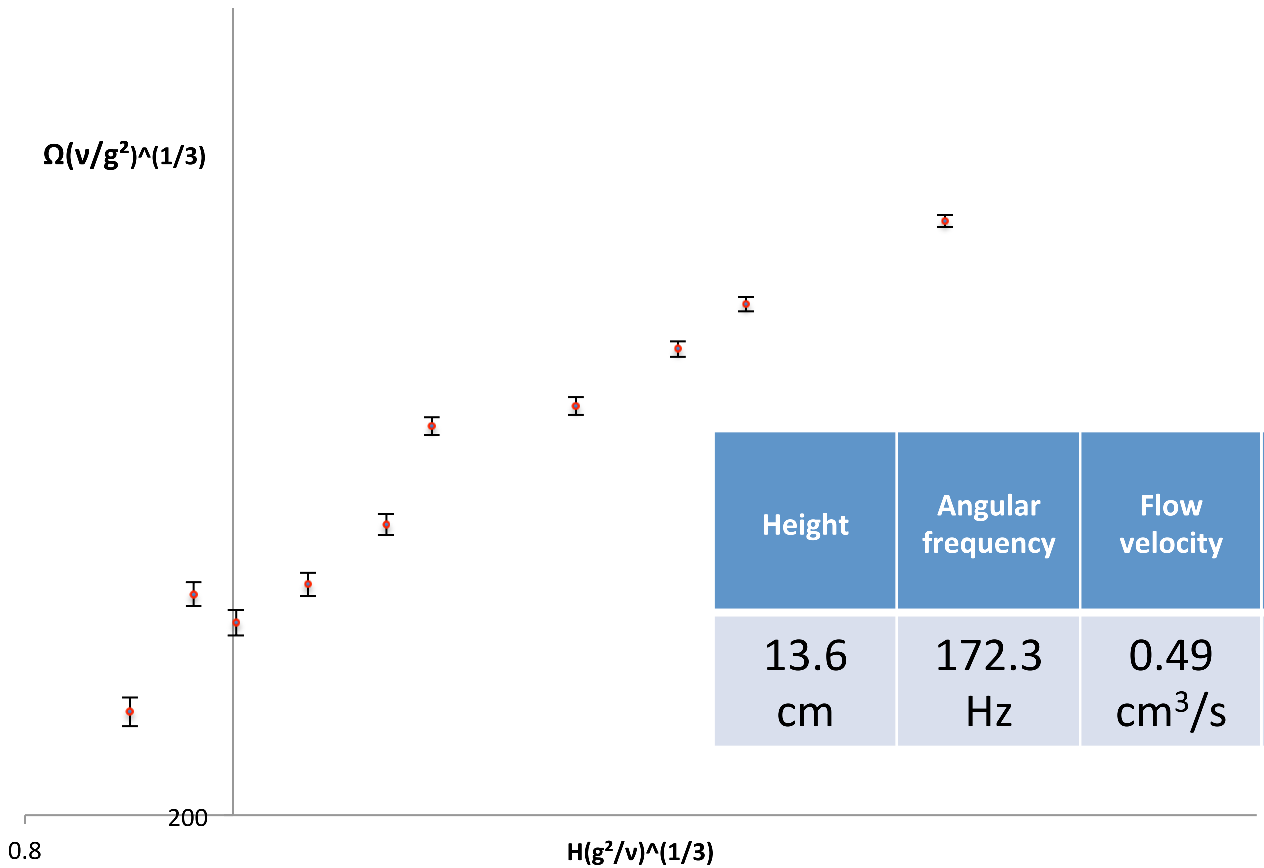
Inertial-gravitational regime – Silicone oil, 5000 cst

- Data analysis
- Viscous regime
- Gravitational regime
- Inertial gravitational regime
- Inertial regime
- Instabilities



Inertial regime – Silicone oil, 5000 cst

- Data analysis
- Viscous regime
- Gravitational regime
- Inertial gravitational regime
- **Inertial regime**
- Instabilities



Height	Angular frequency	Flow velocity	Theoretical angular frequency	Error
13.6 cm	172.3 Hz	0.49 cm^3/s	125.7 Hz	

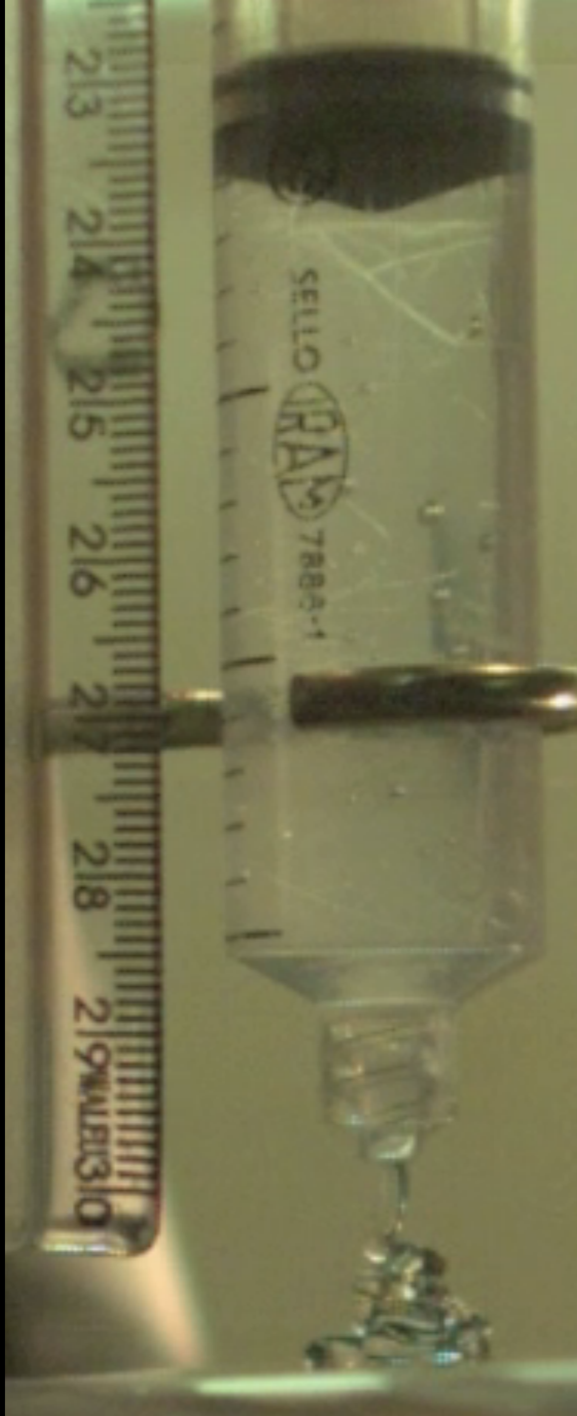
Viscous regime – Honey, 2000 cst

- Data analysis
- Viscous regime
- Gravitational regime
- Inertial gravitational regime
- Inertial regime
- Instabilities

Fall height	Angular frequency	Flow velocity	Theoretical angular frequency	Error
1.0 cm	25 Hz	25 m/s	27 Hz	7.4 %

Fall height	Flow velocity	Angular frequency	δ parameter	Theoretical angular frequency	Error
4 cm	30 cm/s	45 Hz	0.93 cm	39 Hz	15.3 %

Angular frequency	Fall height	a_1 parameter	Theoretical angular frequency	Error
140 Hz	8.0 cm	0.2	148 Hz	5.1 %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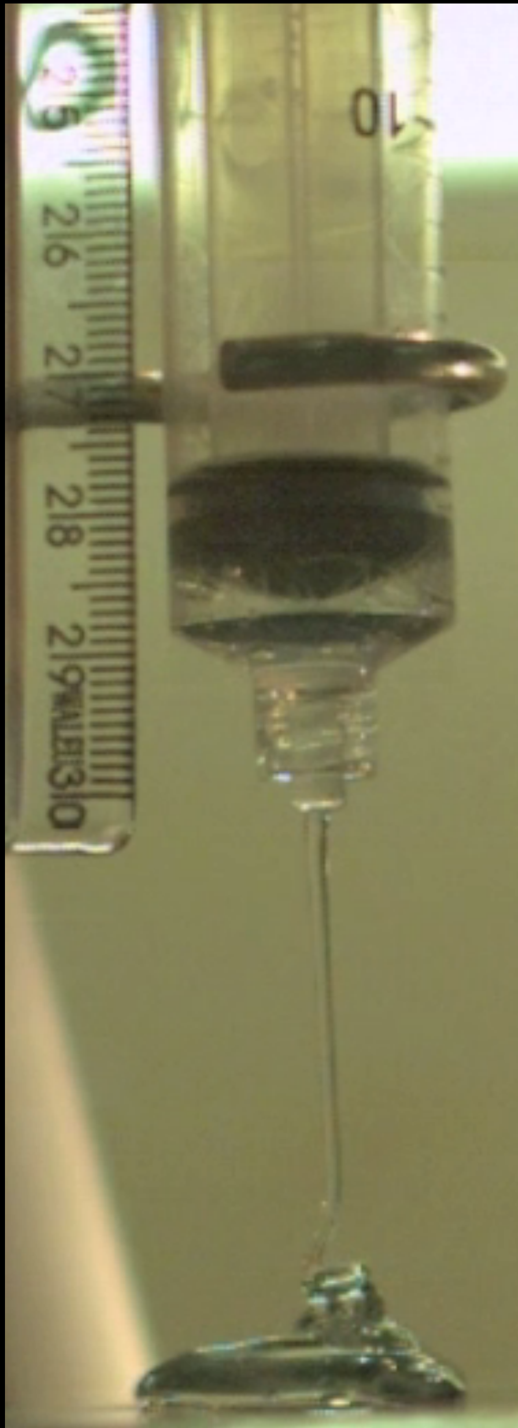
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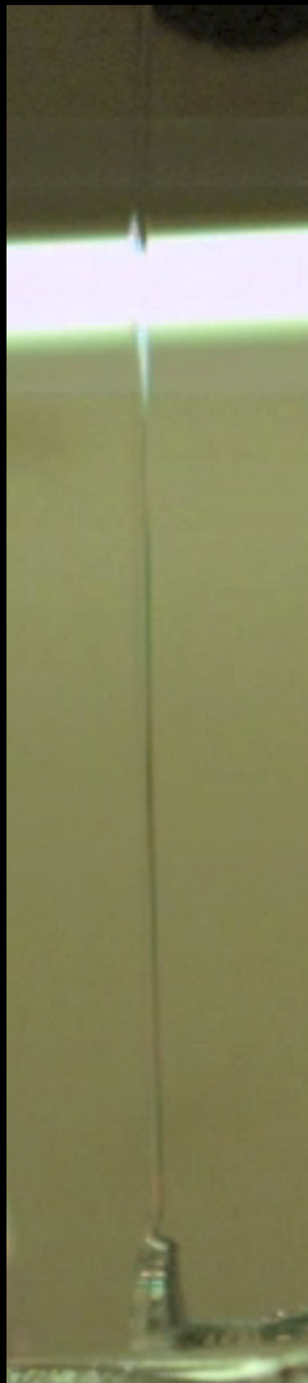
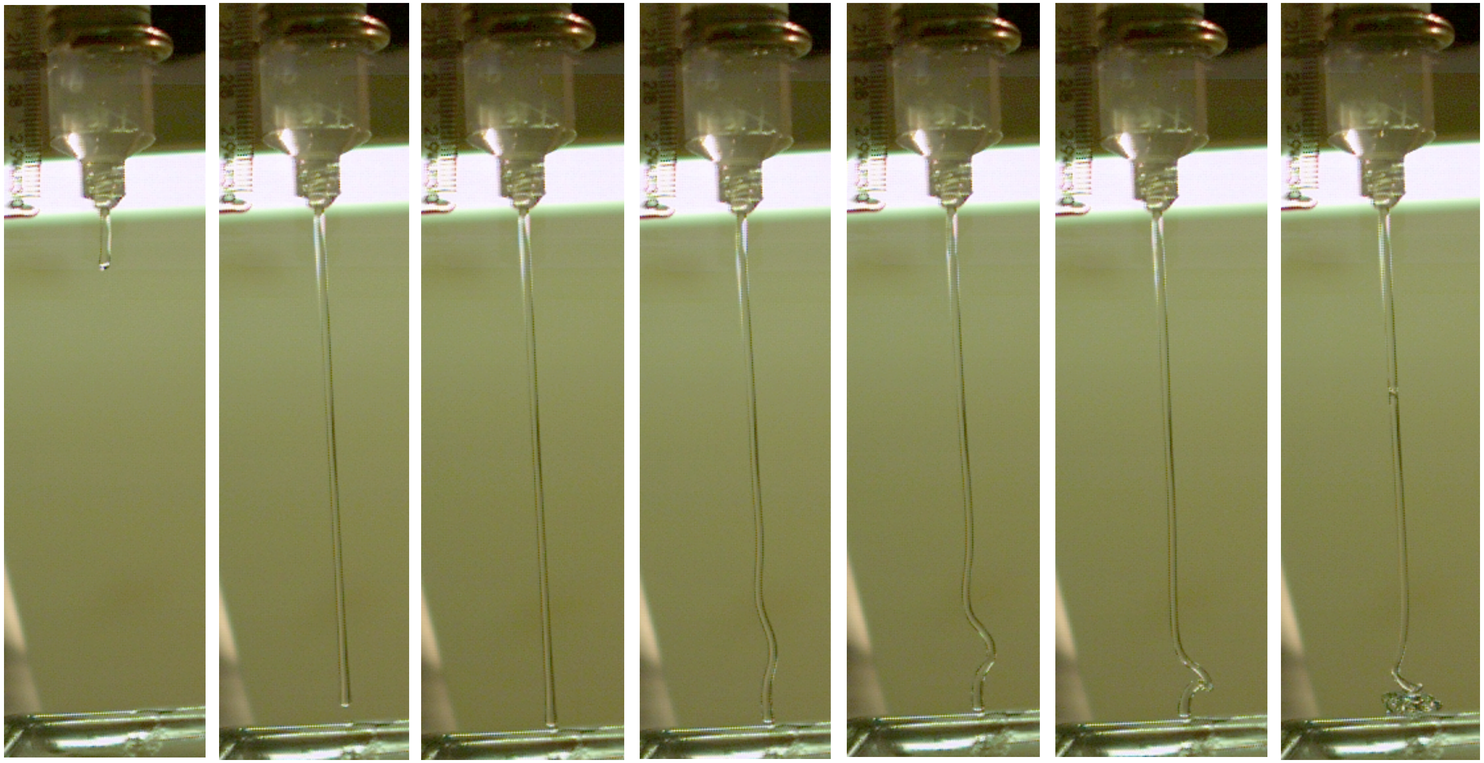



Tensions and torsions: video and images



Navier-Stokes equations

- In a general way, we can apply the Navier-Stokes equation, given by:

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla \mathrm{v}\right) = -\nabla p + \nabla\cdot\mathbf{T} + \mathbf{f}$$

Density (ρ), Velocity ($\mathbf{v}$), Pressure (p), Stress tensor ($\mathbf{T}$), Density of force per unit of mass ($\mathbf{f}$)

- And for non-compressible Newtonian-fluids, we have:

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla \mathbf{v}\right) = -\nabla p + \mu\nabla^2\mathbf{v} + \mathbf{f}$$

How to get the data