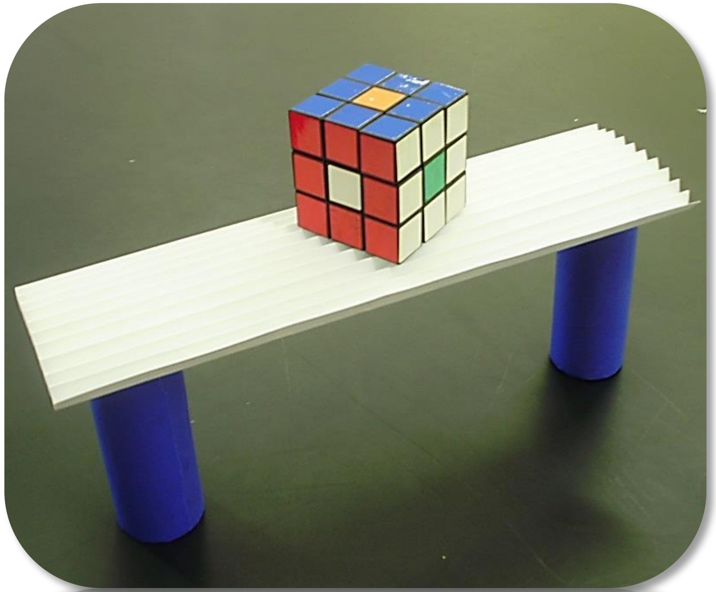
Team of Brazil

Problem 01
Invent Yourself



reporter:

Denise Sacramento Christovam

Problem 1
Invent Yourself

It is more difficult to bend a paper sheet, if it is folded "accordion style" or rolled into a tube. Using a single A4 sheet and a small amount of glue, if required, construct a bridge spanning a gap of 280 mm. Introduce parameters to describe the strength of your bridge, and optimise some or all of them.

Contents

Introduction

- Strength of materials - Paper
- Bridge types
- Force distribution
- Load Distribution

Experiments

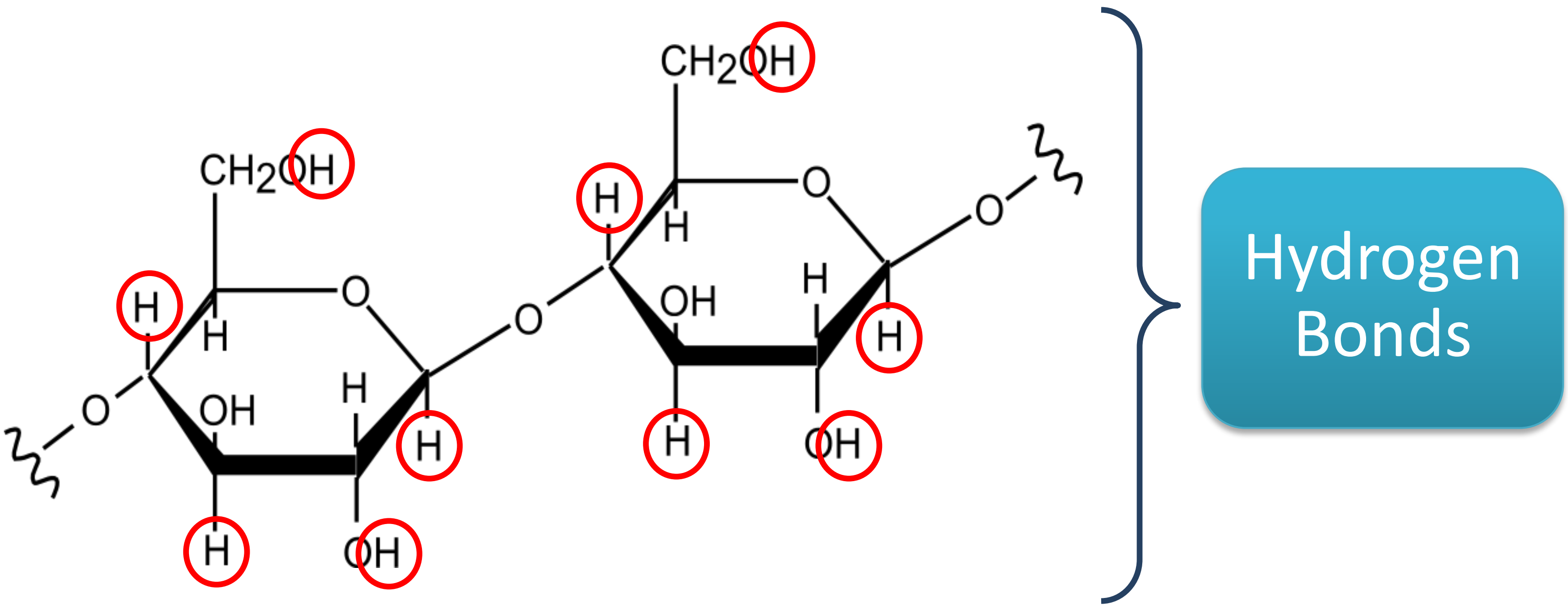
- Load distribution
- Type of Bridge
- Truss Bridge
- Number of folds/”turns”/triangles
- Accessories
- Grammage (paper characteristic)

Conclusion

- Parameters and optimization

Strength of Materials - Paper

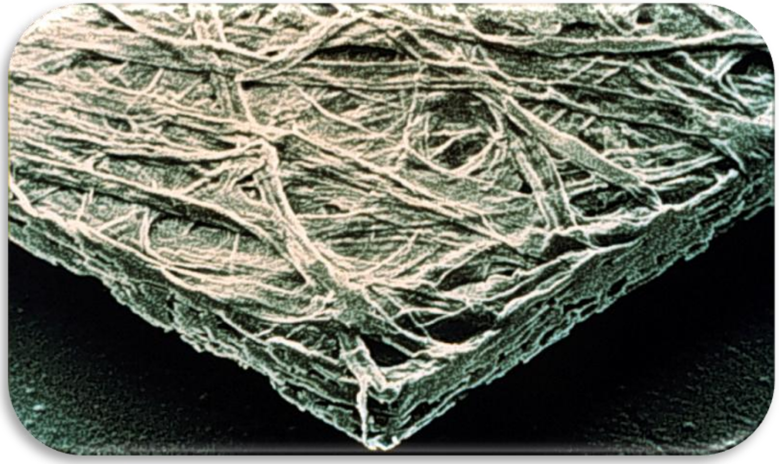
- Intermolecular forces



Cellulose

Strength of Materials - Paper

Linear structure composed of cellulose fibers intertwined.



http://mosmanibphilosophy.wordpress.com/2012/09/05/what-is-contained-in-a-blank-sheet-of-paper/

For calculation purposes, the A4 sheet may be considered isotropic.

Strength of Materials - Paper

Grammage (g/m²)

- Represents superficial density of the paper.

Density

- Mass per unit of volume
- Higher density= more molecules per unit of volume= higher intermolecular interactions.

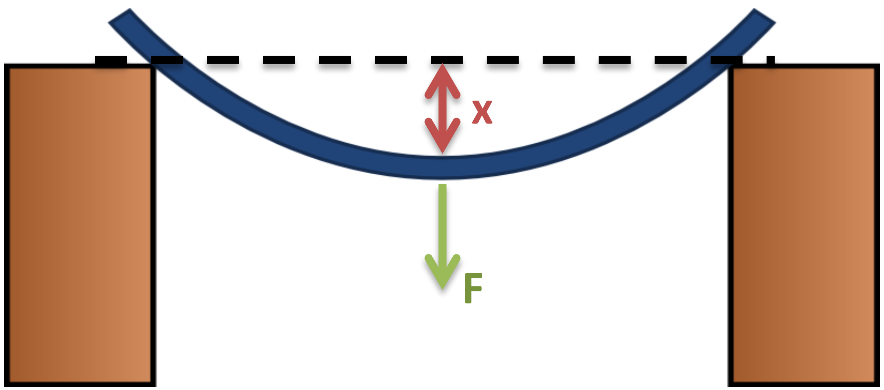
Cardboard- 250 g/m²

Cardboard is more resistant than bond paper for the same bridge structure.

Bond Paper- 75 g/m²

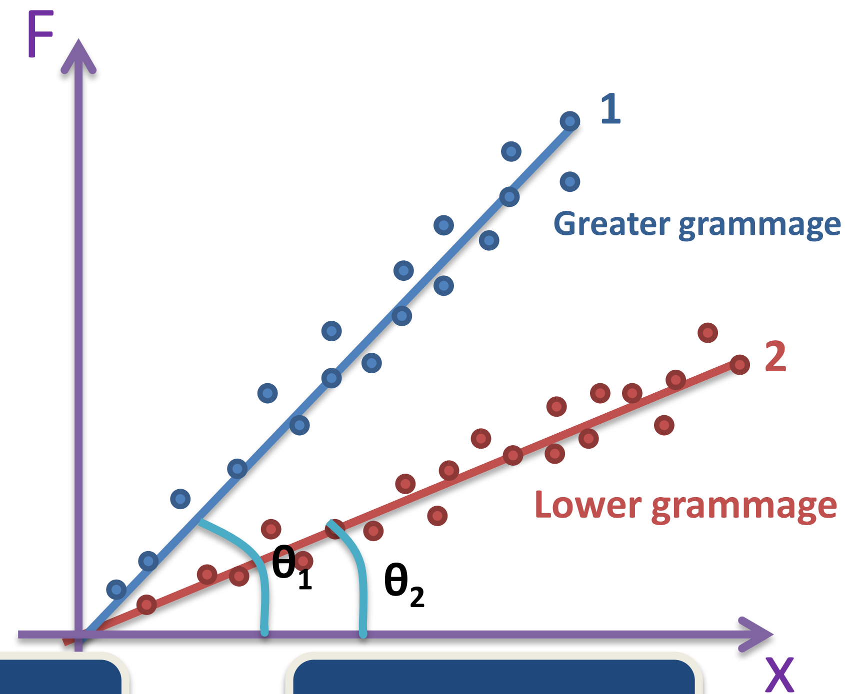
Strength of Materials - Paper

- Hooke's Law analogy:



$$F = kx$$

Increases with the resistance



$tg\theta_1 > tg\theta_2$ → $k_1 > k_2$ → Greater grammage = greater resistance

Strength of Materials - Paper

- Flexural rigidity:

Tension from the torque required to bend the structure

Thickness

$$\sigma_f = E.J = E\,\frac{t^3}{12}$$

Young Modulus

Second moment of inertia

General View

- Higher grammage
- Load supported increases

Specifications

- Each bridge geometry has a grammage limit
- Load supported increases with grammage until limit, then decreases

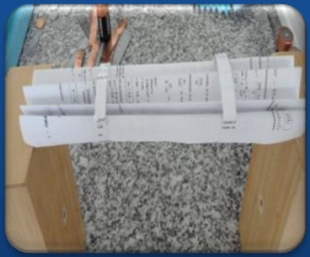
Bridge Types

Plane	• Falls due to it’s own weight
Rectangular	• Vertical segments which tend to prevent the collapse of the bridge in buckling
Tubular	• Continually distributes the weight horizontally and vertically
Fanfolded	• Weight is distributed in various horizontal and vertical spots • The segments tend not to suffer deformation to the sides
Trussed	• Provides full or partial annulment of the compression stress

Bridge Types - Accessories

Holding bands

- Used to prevent bridge's distension

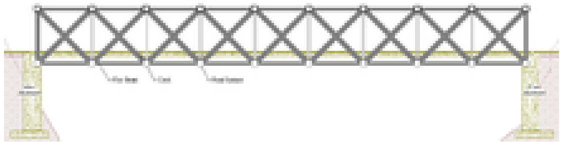


Tubes

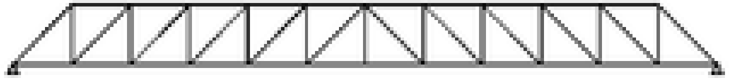
- By having a small cross-sectional area, are rigid, and when associated under the bridge, make it more resistant.

Bridge Types – Trussed Bridge

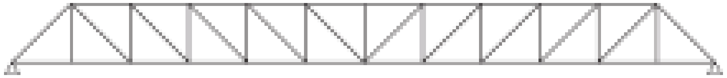
- Structure of connected elements forming triangular units.
- Typically straight, that may be stressed from tension and compression.
- External forces and reactions are considered to act only on the nodes.



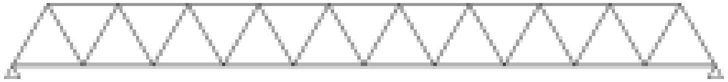
Brown Truss



Howe Truss

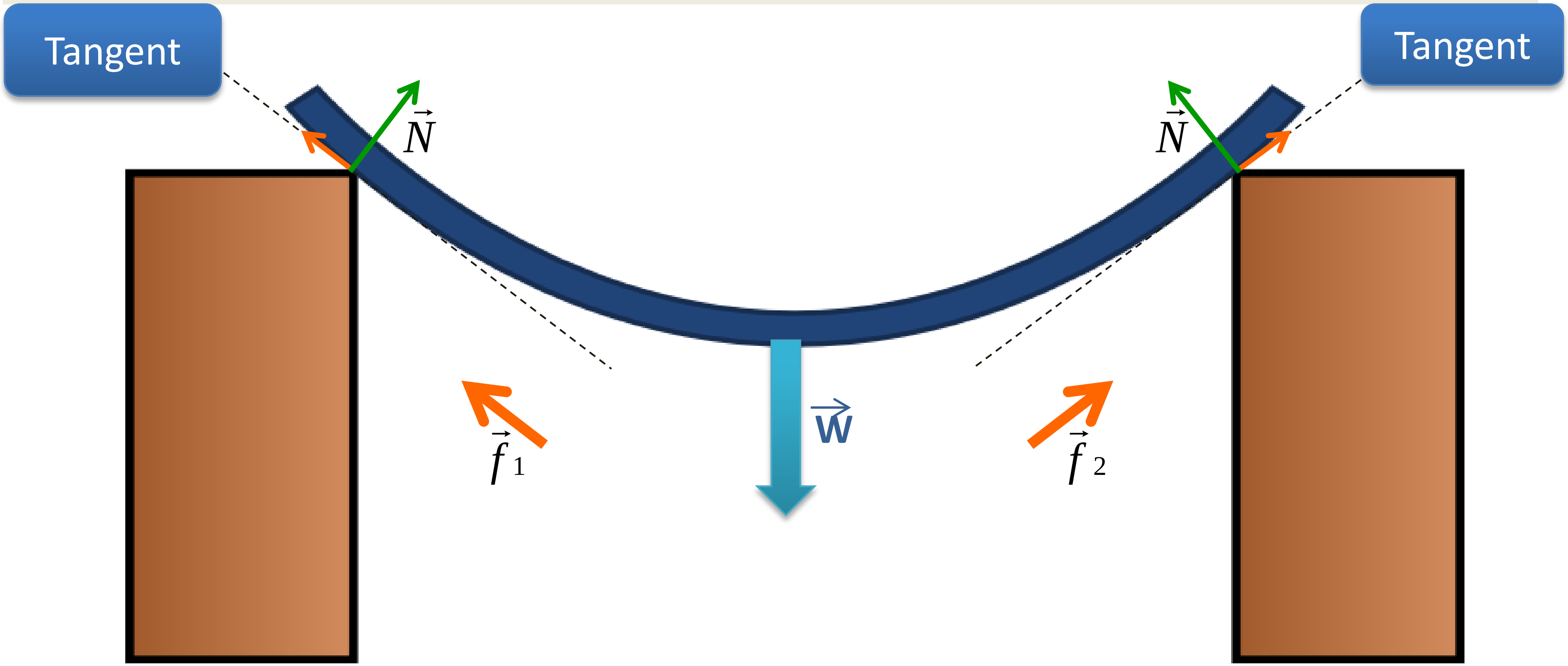


Pratt Truss



Warren Truss

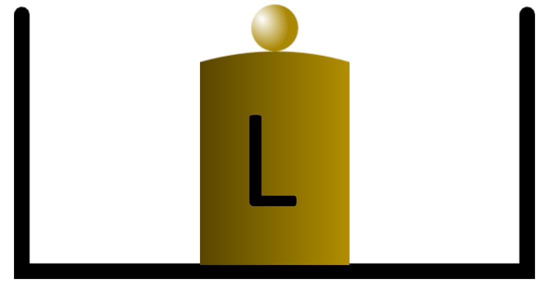
Force Distribution – Plane Bridge

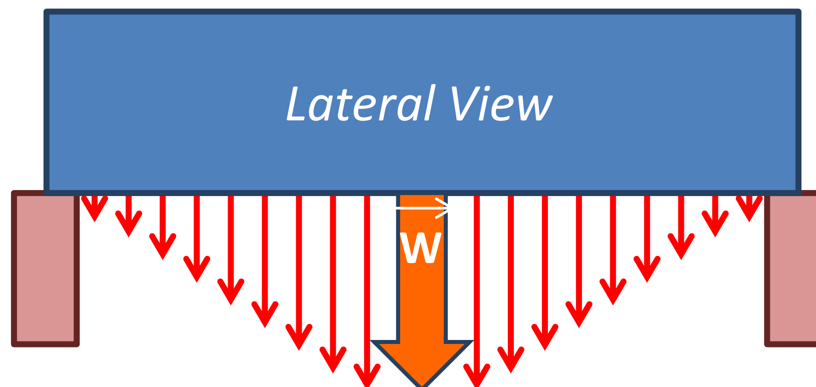


$$\vec{f}_1 + \vec{f}_2 < \vec{W}$$

Bridge falls due to its own weight.

Force Distribution – Rectangular Bridge





Weight generates forces in the surface the paper

Greater resistance (as already shown)

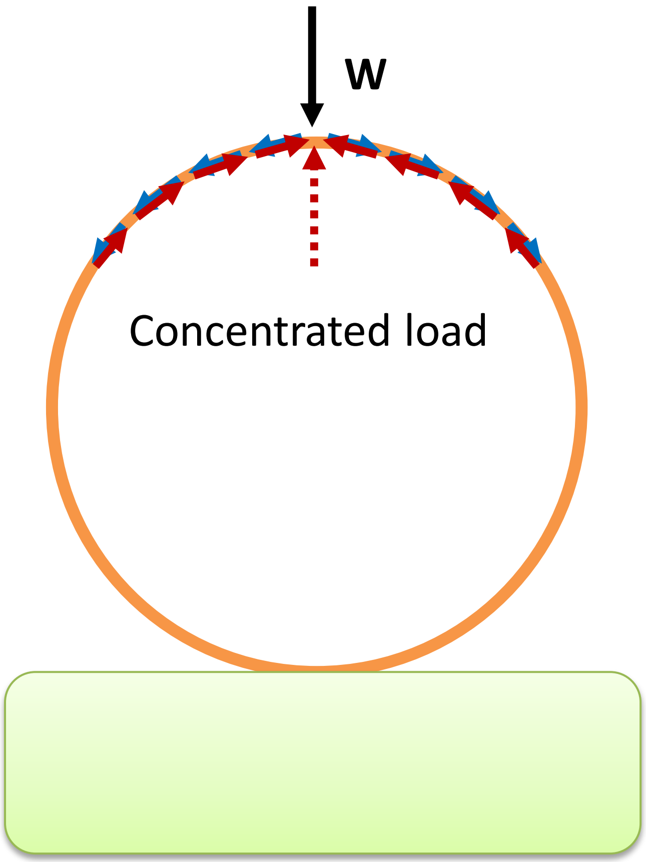
Force Distribution – Tubular Bridge

Smaller diameter

More layers of paper

Higher spring constant

More load supported

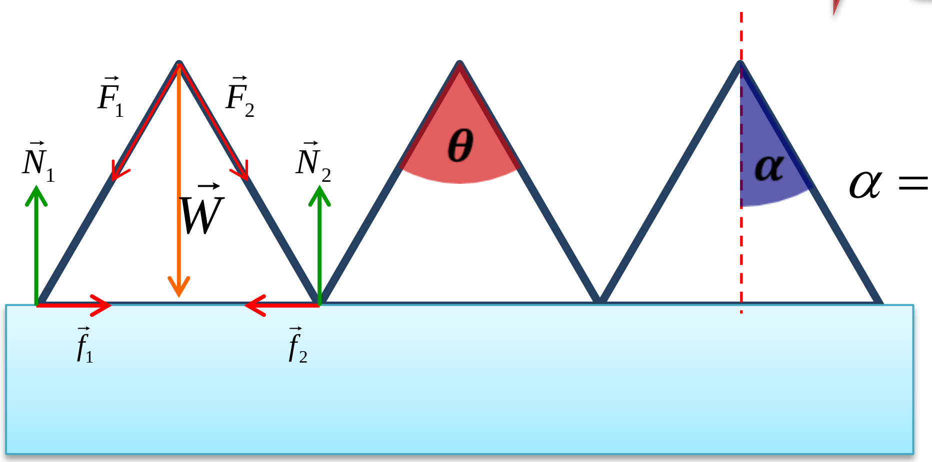


Force Distribution – Fanfolded Bridge

Series of triangular structures

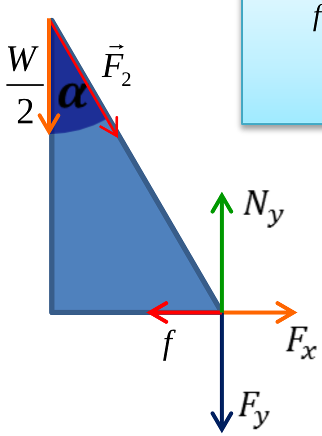
$$\sum \vec{F} = 0$$

Until the bridge collapses (max. load)



$$\alpha = \frac{\theta}{2}$$

Every opening angle has an optimal number of folds, as will be shown in experiments.



$$F_2 = \frac{W}{2\cos(\alpha)}$$

$$F_x = F_2.\sin(\alpha)$$

$$f = \mu_s.F_y$$

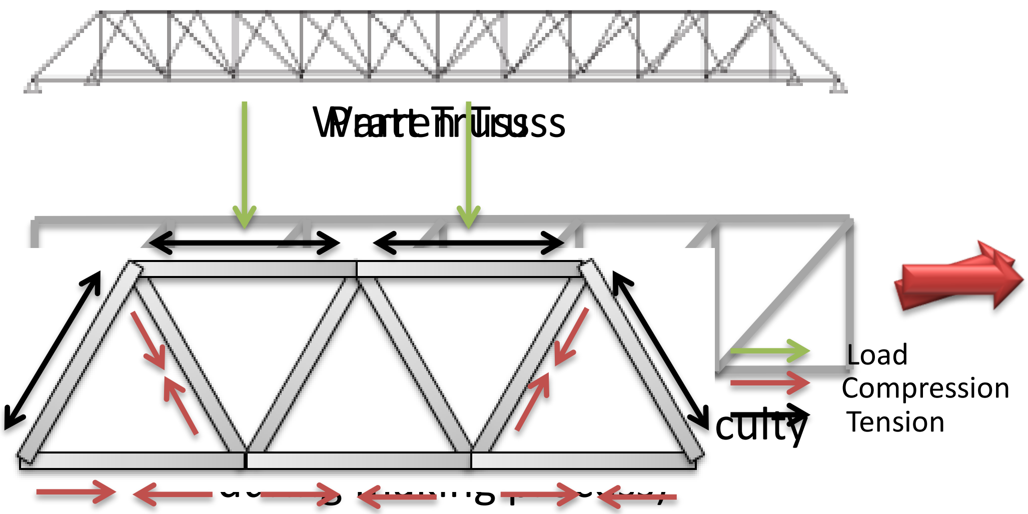
$$F_x = \frac{W}{2\cos(\alpha)}.\sin(\alpha)$$

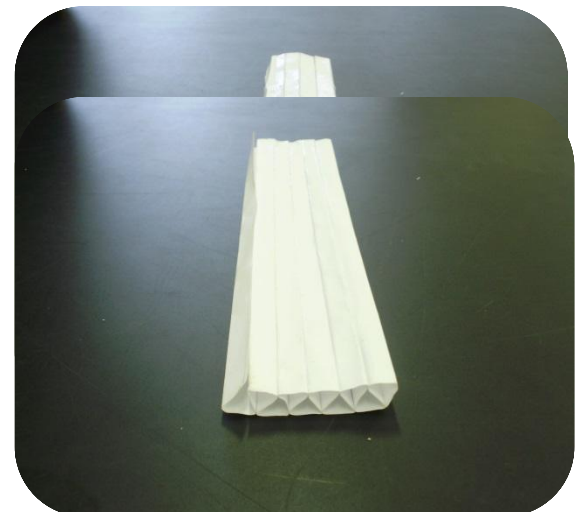
$$F_x = \frac{W}{2}.\tan(\alpha)$$

$$\mu_s > \tan(\alpha)$$

Force Distribution – Trussed Bridge

- Considering the most stable truss bridges, Warren and Pratt:





A4 sheet.

Offers more resistance since it provides the compression cancelling tension, balancing the entire structure

Equilateral triangles are very stable and uniform structures; they balance the force distribution better than any other truss

Force Distribution – Trussed Bridge

- Euler's equation for columns:

$$F = \frac{\pi^2 EJ}{(KL)^2}$$

K=1,0

$$\sigma = \frac{F}{A} = \frac{\pi^2 EJ}{\left(\frac{l}{r}\right)^2}$$

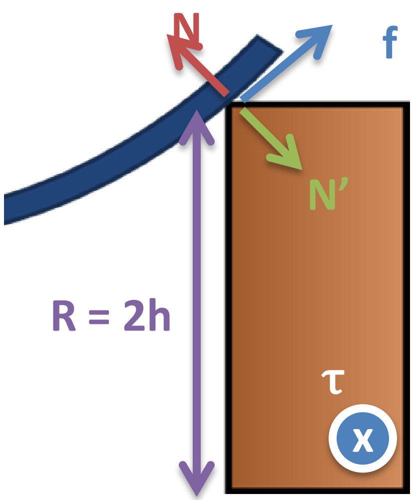
Must be low for maximum load supported

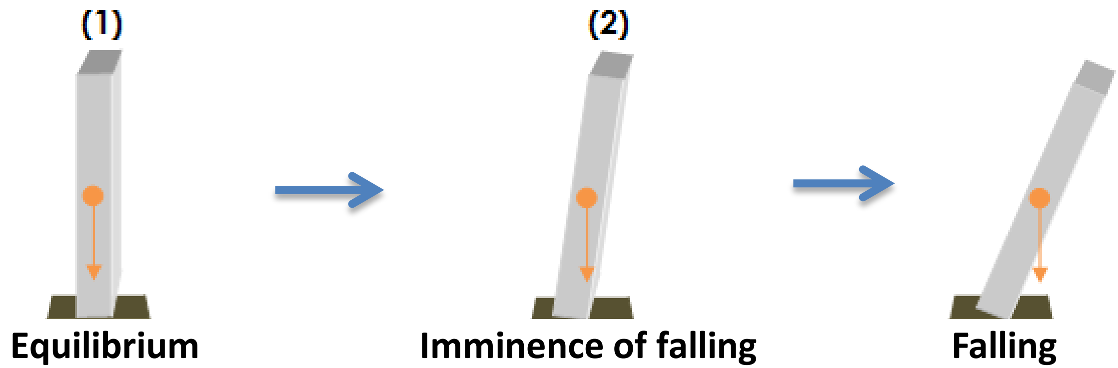
Slenderness ratio

- Optimization: **small triangles**
- Glue used: scholar
  - No chemical reactions with paper
  - More efficient distribution of tension along the bridge's joints
  - Represents ≈ **8%** of the paper bridge's mass

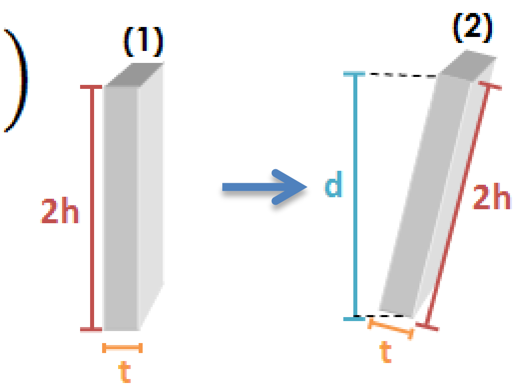
Torque on the supports

- Minimizing the influence of torque and activation energy





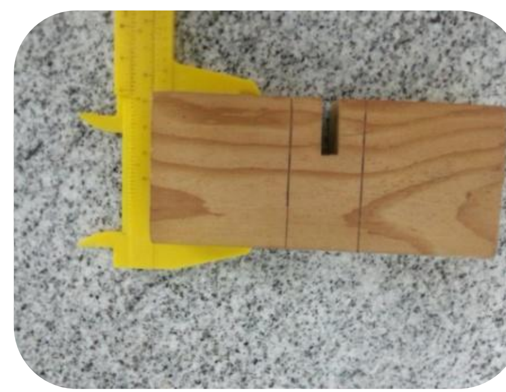
$$E_A = \frac{1}{2} mg \left(\sqrt{4h^2 + t^2} - 2h \right)$$



High thickness = more force required to fall over = no loss of contact

Material

- Types of paper:
  - Bond A4 (75g/m²)
  - Cardboard A4 (120 g/m²)
  - Cardboard A4 (250 g/m²)
  - Corrugated fiberboard A4 (237 g/m²)
  - Newsprint A4 (48 g/m²)
  - Wrapping tissue A4 (20 g/m²)
- Coins of different masses:
  - R$ 0.05: 4,10 g
  - R$ 0.10: 4,80 g
  - R$ 0.25: 7,55 g
  - R$ 0.50: 7,81 g
- Weights of different masses
- Sand
- Filler
- 2 supports for the bridge
- Wooden plank
- Glue
- Ruler (millimetres) (±0,5 cm)
- Scale (±0,05 g)

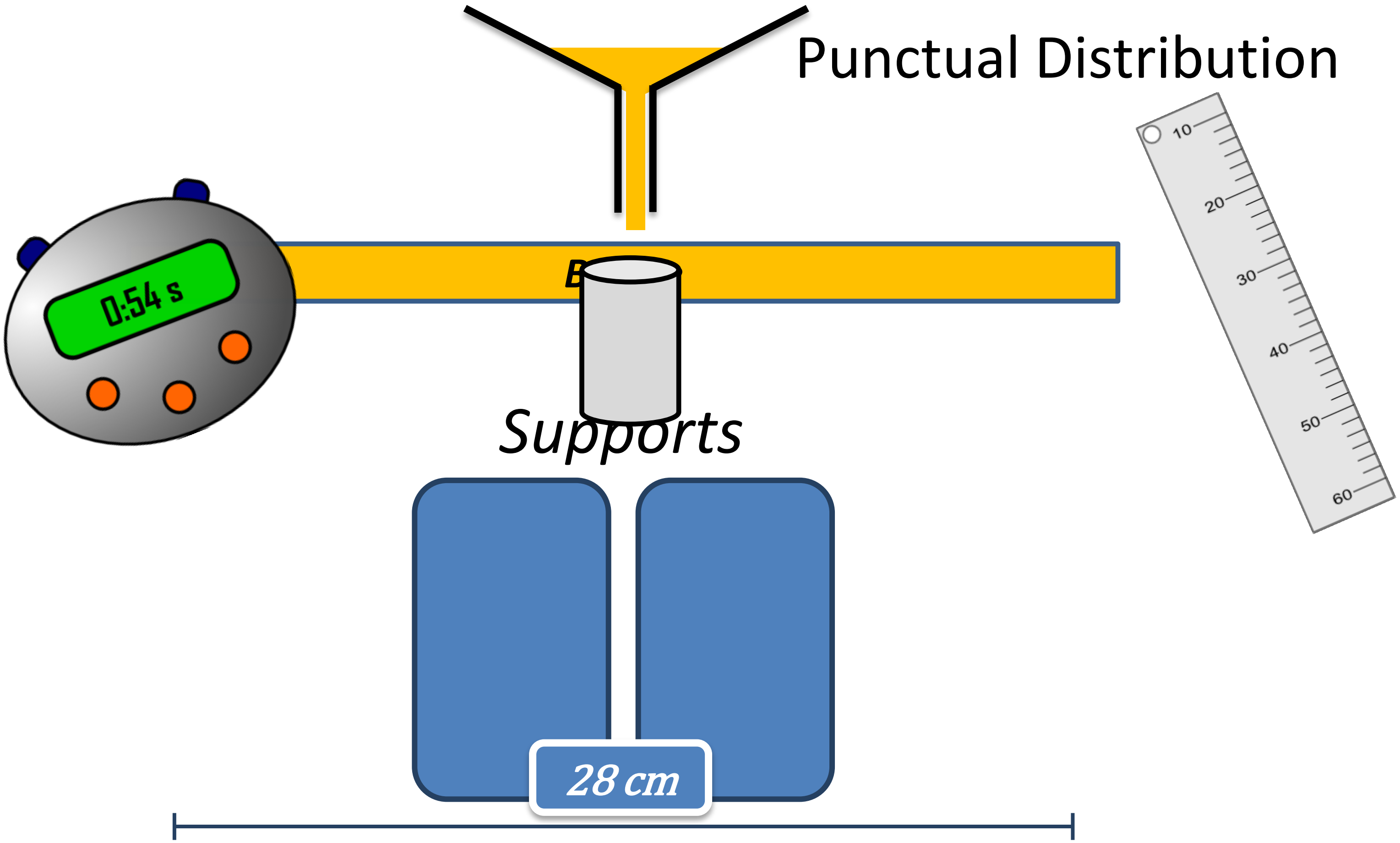


Experimental Description

- **Experiment 1:** Different weight arrangements.
- **Experiment 2:** Type of Bridge.
- **Experiment 3:** Variation of each type of bridge's configuration.
- **Experiment 4:** Accessories to increase the bridge's strength.
- **Experiment 5:** Variation of paper's characteristics.

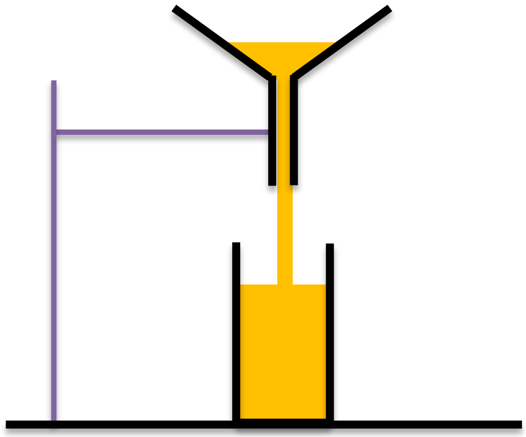
Experiment 1: Load Distribution

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper



Experiment 1: Load Distribution

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

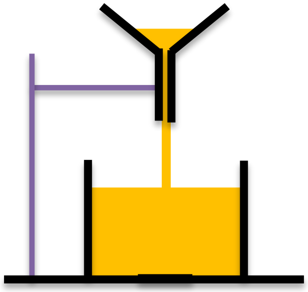


$$V = \frac{\rho \cdot Vol}{\Delta t}$$

$$V = 14{,}2\ \frac{g}{s}$$

Sand's density: 2,00g/ml
Total volume: 500 ml
Falling time: 70,18 s

Measuring

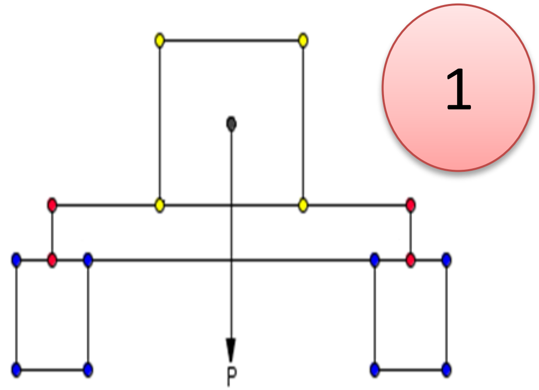


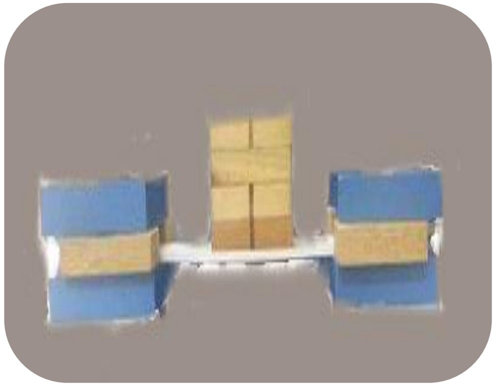
$$m = V \cdot \Delta t$$

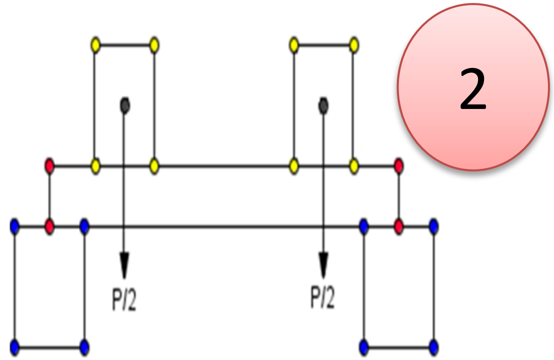
Experiment 1: Load Distribution

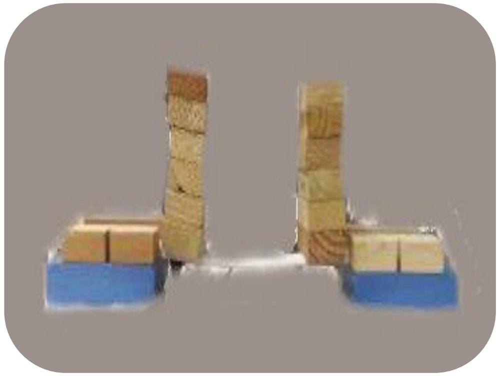
Uniform Distribution

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper



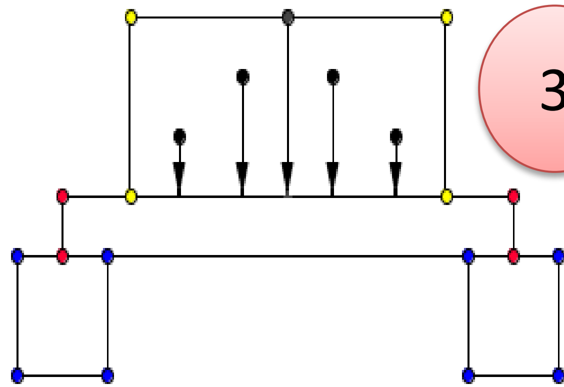






Experiment 1: Load Distribution

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper



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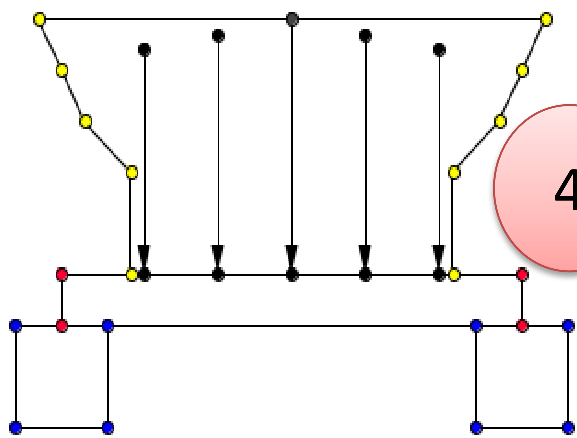

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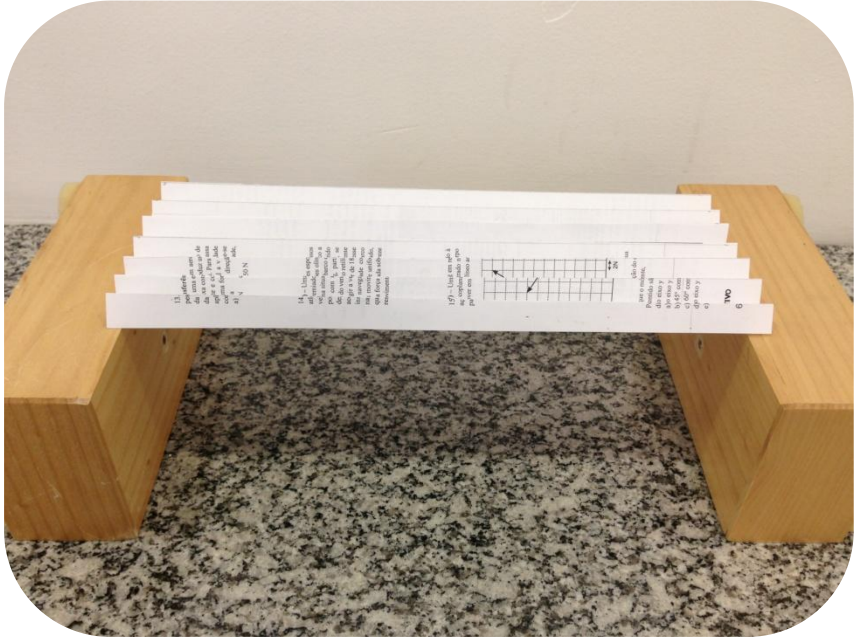
Experiment 1: Load Distribution

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

Standard bridge: fanfolded bridge ("accordion")



Note: number of folds= 13

Experiment 1: Load Distribution

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

Distribution	
Mode	*Load (g)*
3	796
4	961
5	1092

Optimization: uniform distribution (4)

Experiment 2: Bridge types

Plane

Rectangular

Tubular

Fanfolded

Trussed

Standard paper: bond (75 g/m^2)

Experiment 2: Bridge types

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

Type of bridge	Load (g)
Plane	0
[illegible]	[illegible]
Tubular	208
Fanfolded	225
Triangular	1562

Optimization: triangular bridge (5)

Experiment 3: Bridge parameters

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

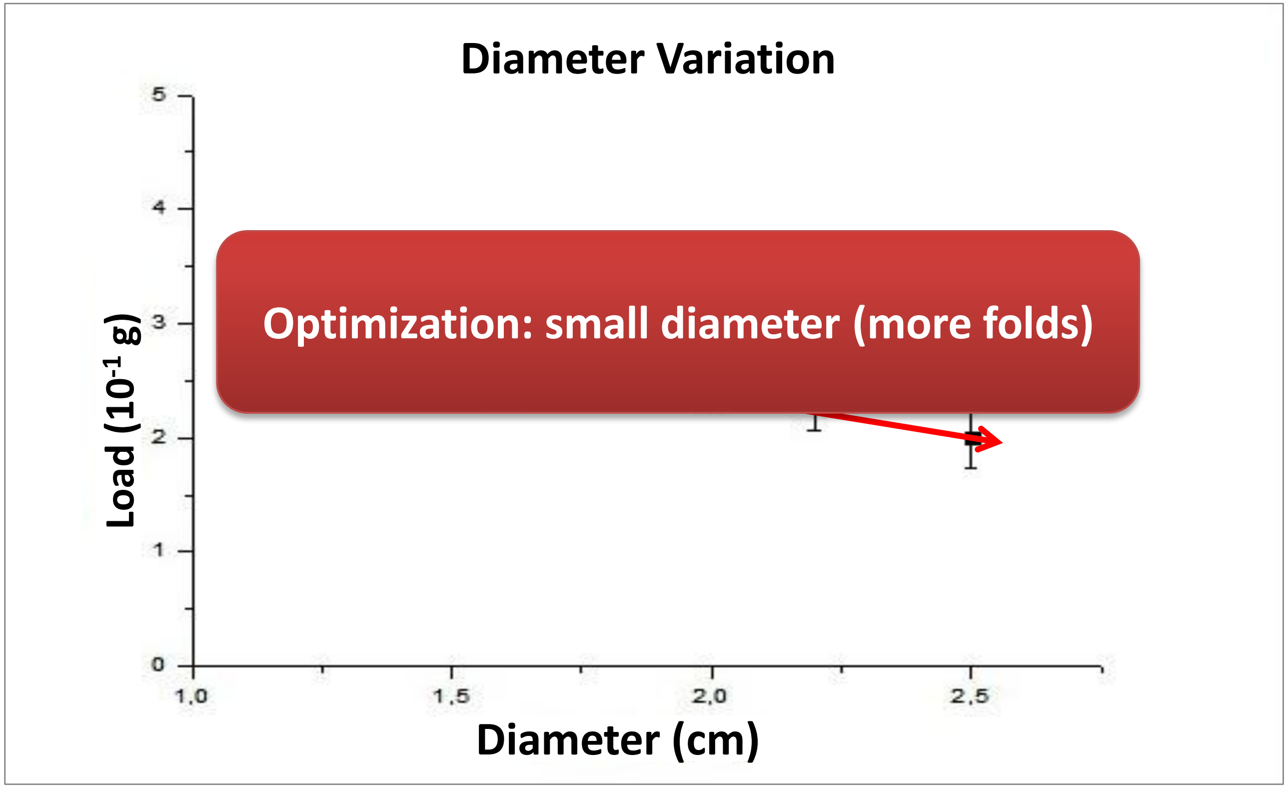
1. Plane bridge: number of folds

No folds → 2 folds → 3 folds

Experiment 3: Bridge parameters

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

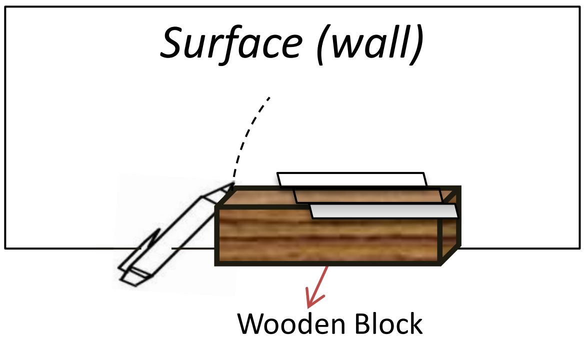
2. Tubular bridge: diameter

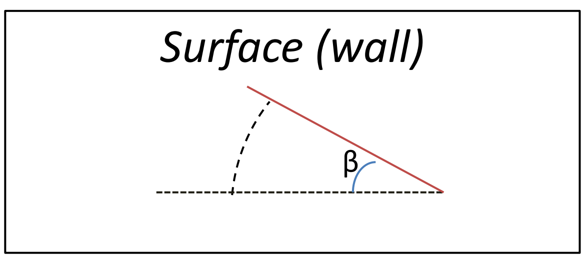


Experiment 3: Bridge parameters

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

Using the inclined plane method to determine the wood-paper static friction coefficient:





$$\mu_s = \tan(\beta)$$

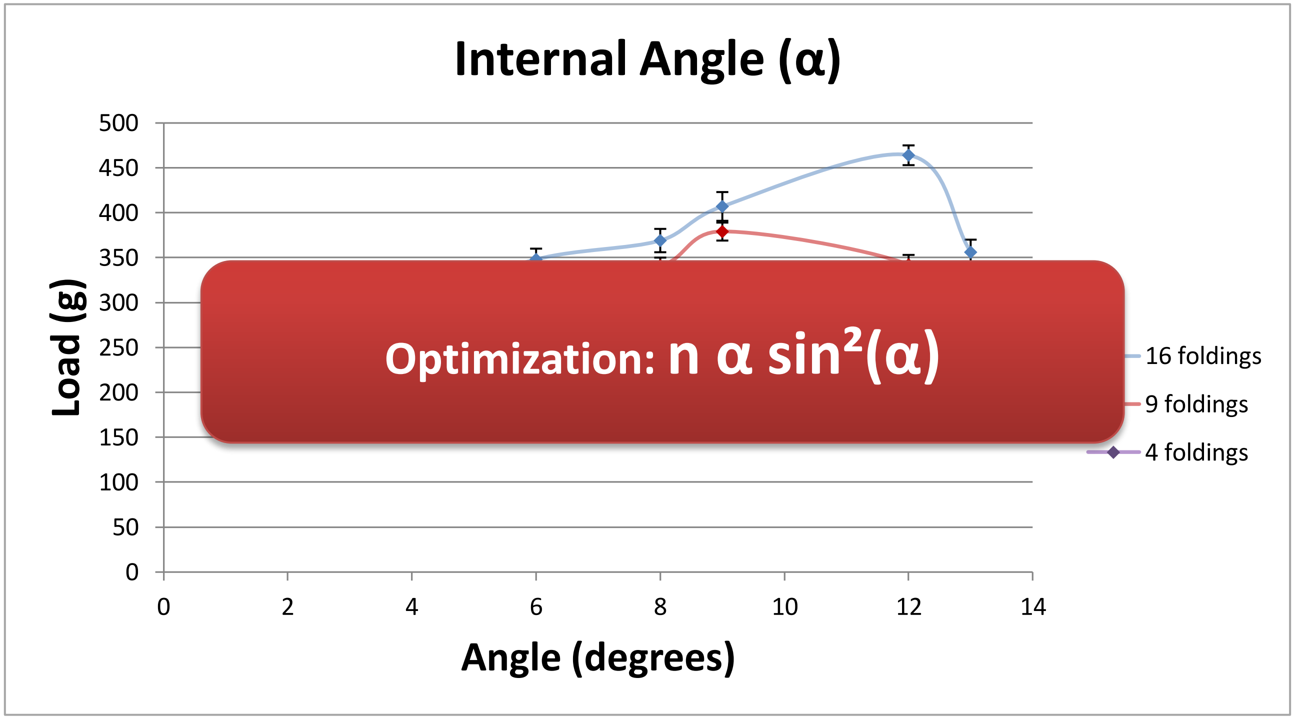
Measured Angles	
1	13°
2	12°
3	12°
4	15°
5	12°
6	13°
7	12°
8	12°
Average	**12,6°**
Standart Deviation	1°

$$\mu_s > \tan(\alpha) \therefore \alpha < 12^\circ$$

Experiment 3: Bridge parameters

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

3. Fanfolded bridge: internal angle

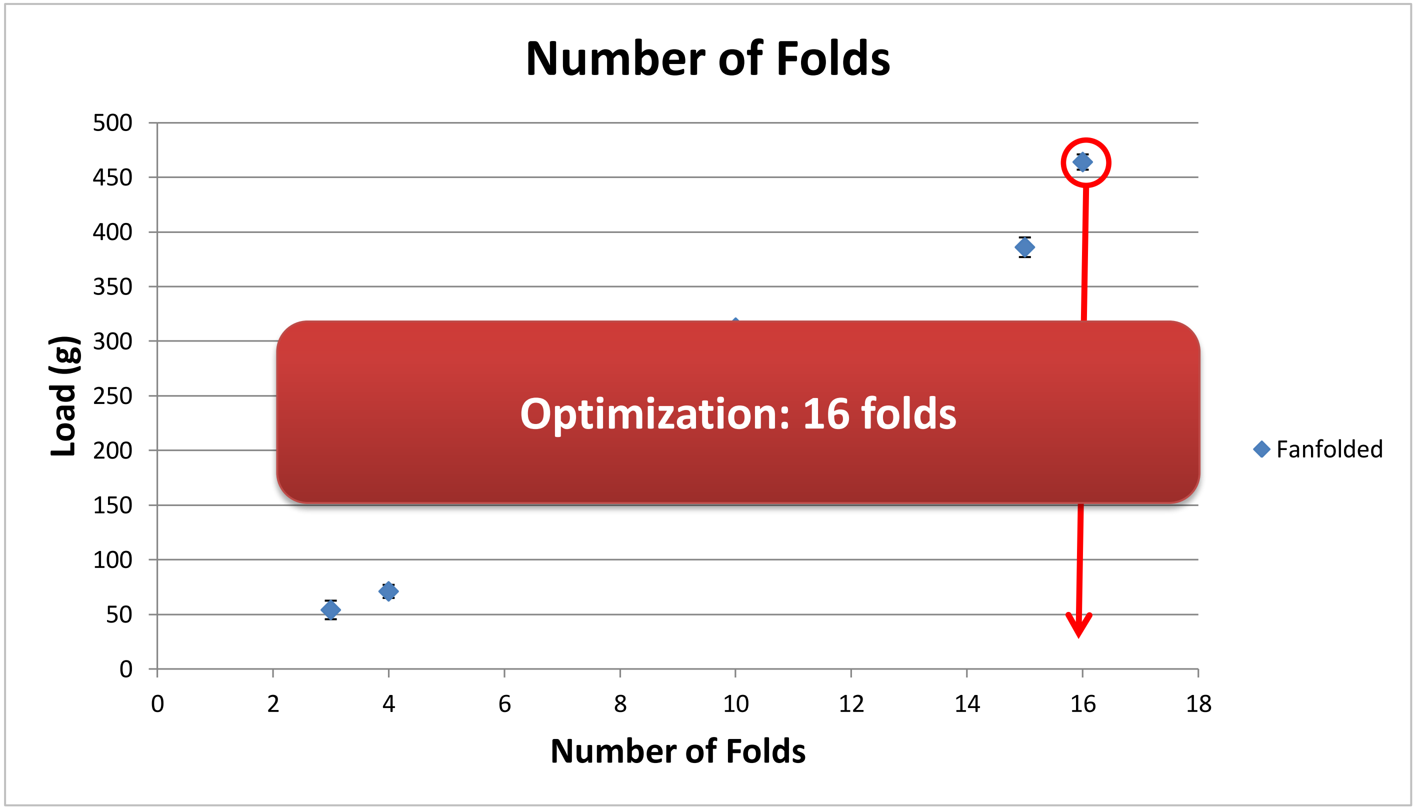


Optimization: n α sin²(α)

Each angle has an optimal number of folds

Experiment 3: Bridge parameters

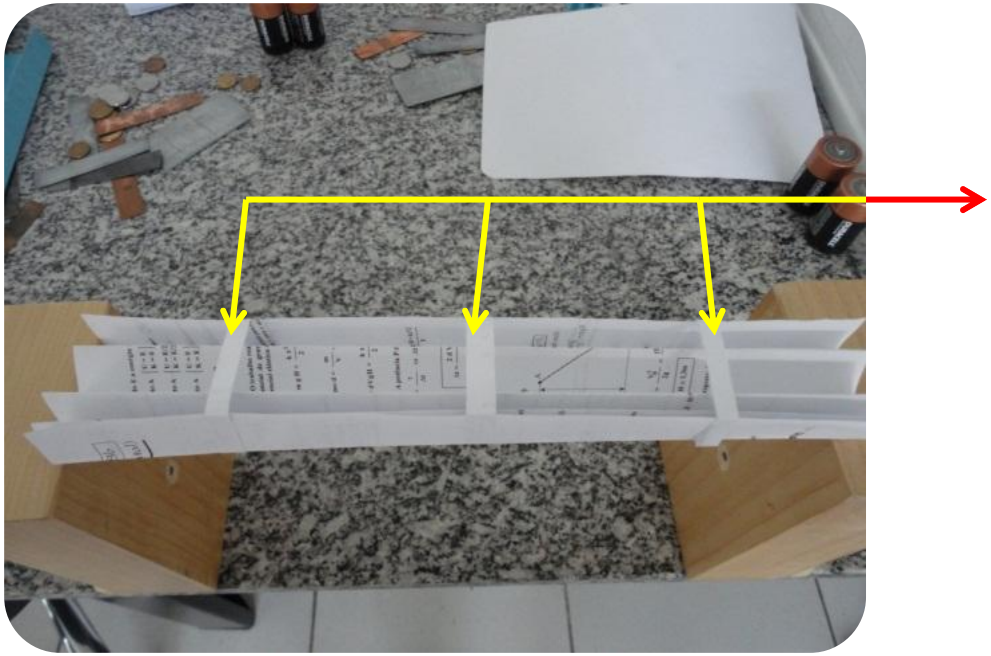
- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper



Experiment 4: Accessories used to increase strength

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

1. Fanfolded bridge: holding bands

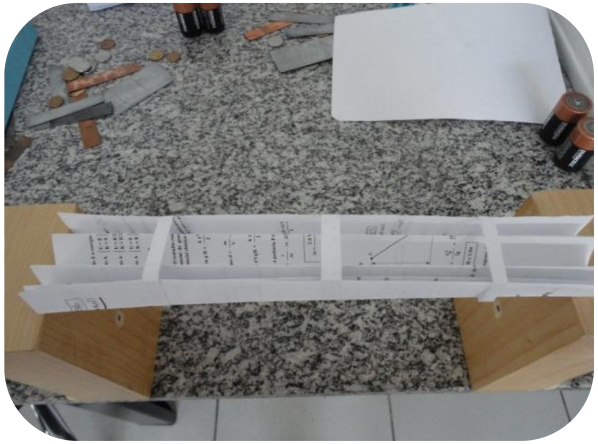


Removed bands from the same sheet

Experiment 4: Accessories used to increase strength

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

1. Fanfolded bridge: holding bands

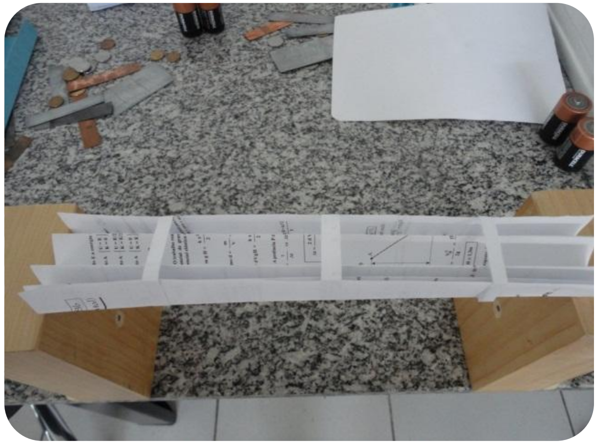


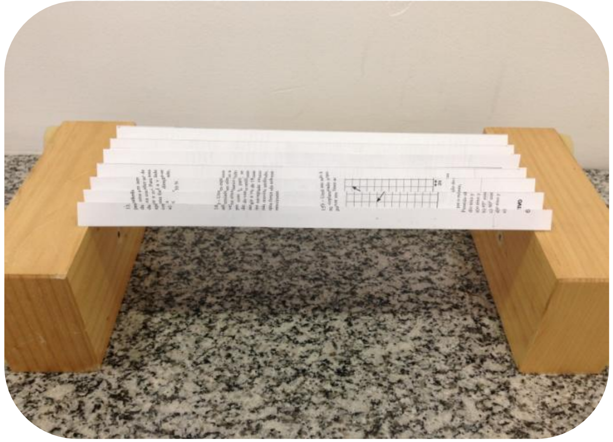
Supported nearly 386g

Experiment 4: Accessories used to increase strength

Weight Arrangement
Bridge
Parameters
Accessories
Paper

1. Fanfolded bridge: holding bands





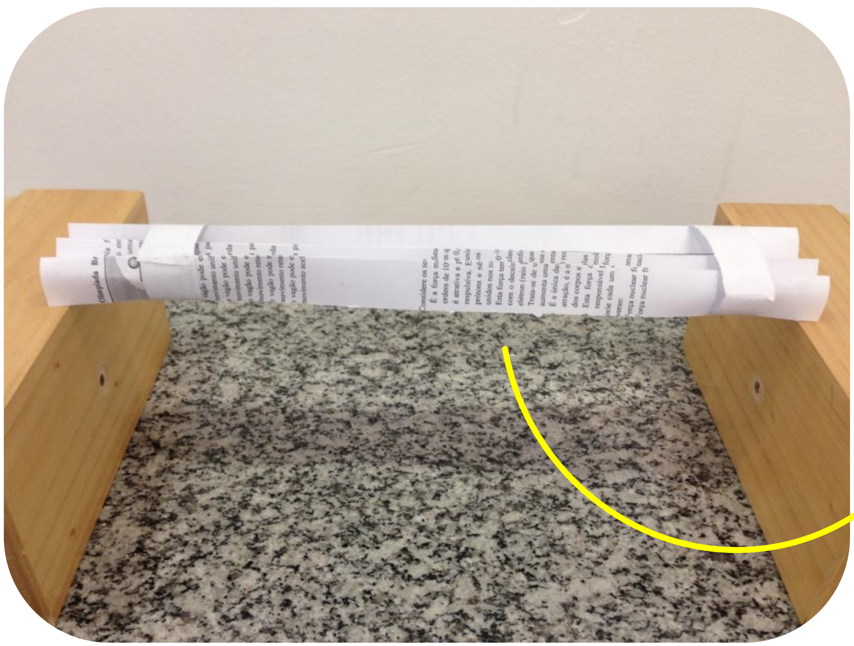
386g

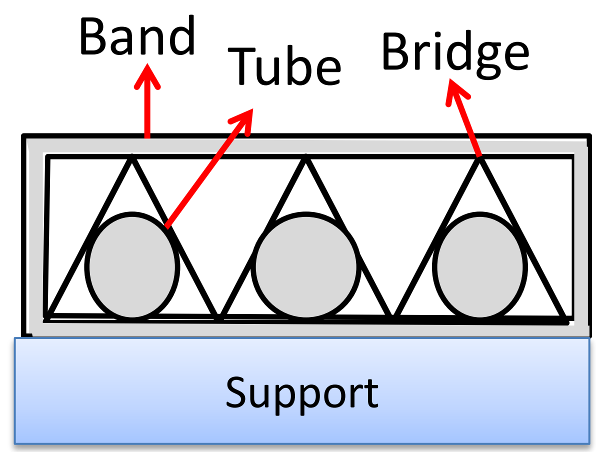
225g

Experiment 4: Accessories used to increase strength

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

2. Fanfolded bridge: holding bands + tubes



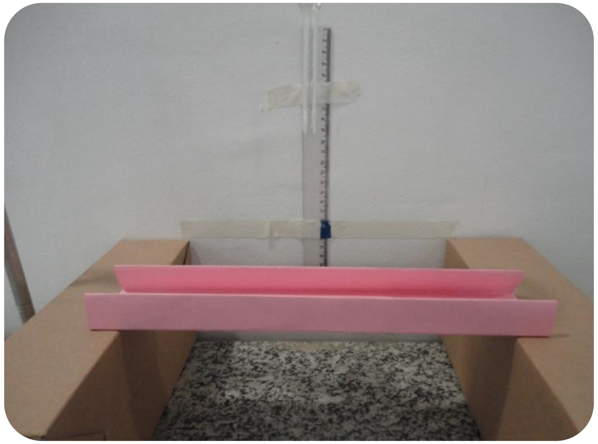


Supported nearly 1098g

Experiment 5: Paper Variation

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

1. Rectangular bridge:



Cardboard (120g/m²)

Supported nearly 415g

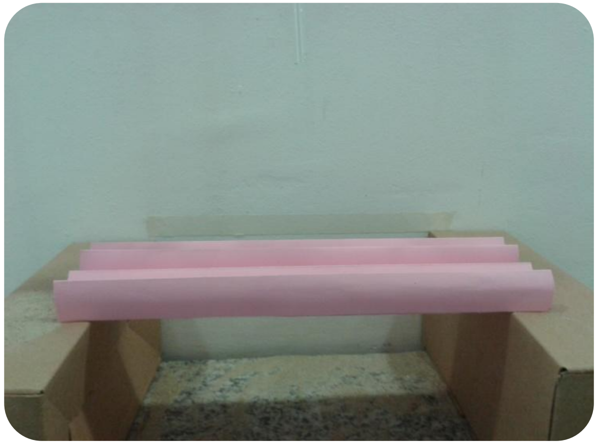
Cardboard (250g/m²)

Supported nearly 690g

Experiment 5: Paper Variation

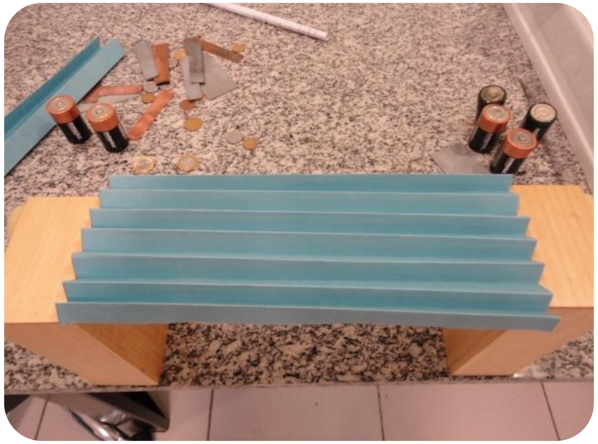
- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

2. Fanfolded bridge: *Cardboard (120g/m²)*



Cardboard (120g/m²)

Supported nearly 525g



Cardboard (250g/m²)

Supported nearly 447g

Experiment 5: Paper Variation

2. Fanfolded bridge:

Corrugated fiberboard (237g/m²)

Supported nearly 732g

Newsprint (48 g/m²)

Supported nearly 30g

Wrapping tissue (20 g/m²)

Supported nearly 12g

Experiment 5: Paper Variation

- Weight Arrangement
- Bridge
- Parameters
- Accessories
- Paper

Rectangular Bridge	
Grammage	*Load*
7[illegible]	[illegible]
120	415
250	345

Fanfolded Bridge	
Grammage	*Load*
20	12
[illegible]	30
[illegible]	225
120	524
237	732
250	447

Optimization: high grammage

Conclusion

Weight Arrangement	Bridge	Accessories	Paper
			
Uniform weight distribution	Trussed bridge	Holding bands and tubes	Corrugated fiberboard

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Thank you!