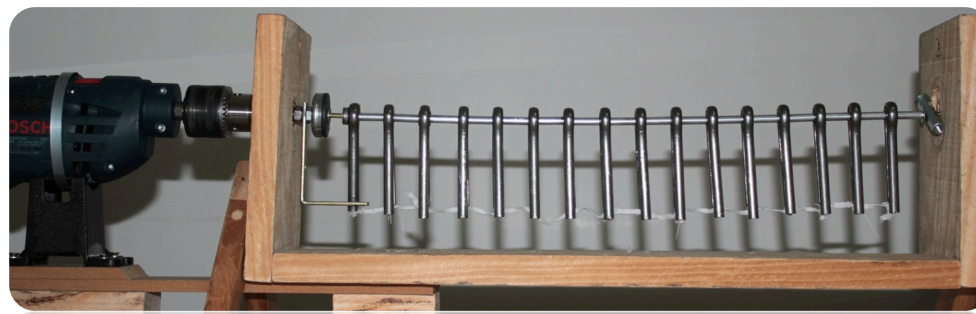
Team of Brazil

Problem 4
Soliton



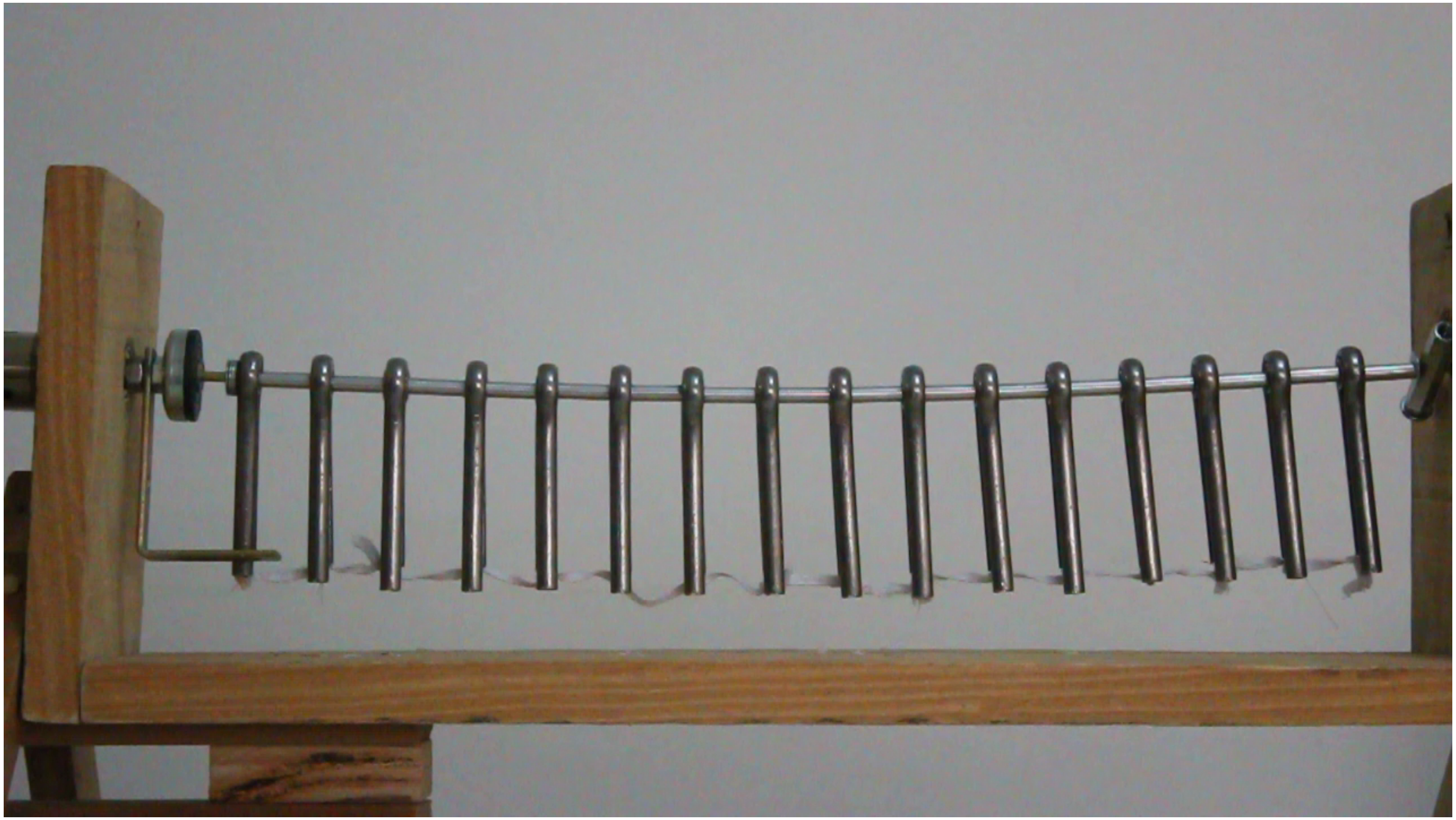
reporter:

Liara Guinsberg

Problem 4
Soliton

A chain of similar pendula is mounted equidistantly along a horizontal axis, with adjacent pendula being connected with light strings. Each pendulum can rotate about the axis but can not move sideways. Investigate the propagation of a deflection along such a chain. What is the speed for a solitary wave, when each pendulum undergoes an entire 360^o revolution?

Example of the phenomena



Introduction

Theoretical formulation

- Definition of soliton
- Examples
- Linear approach
- Klein-Gordon
- Sine-Gordon
- Soliton solutions

Experiments

- String and spring solitons
- Analysis of the data
- Parameter variations

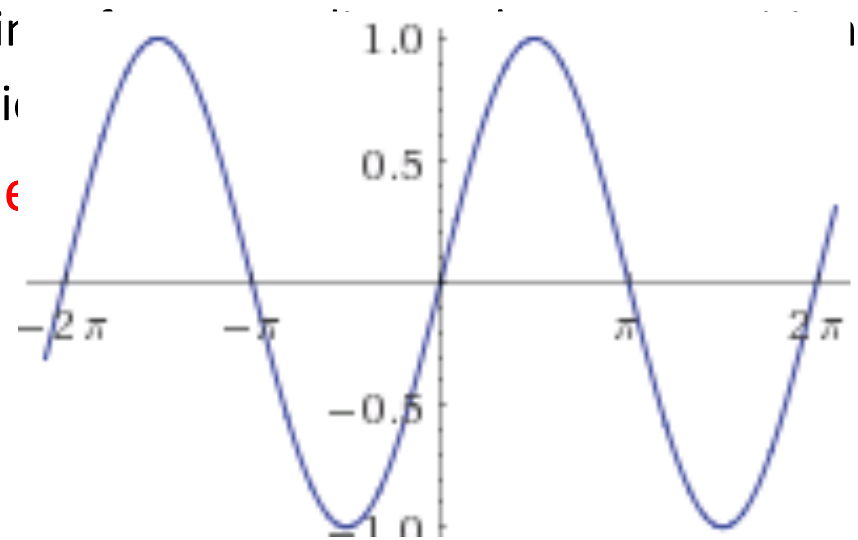
Comparison between the theory and the experiments

Definition

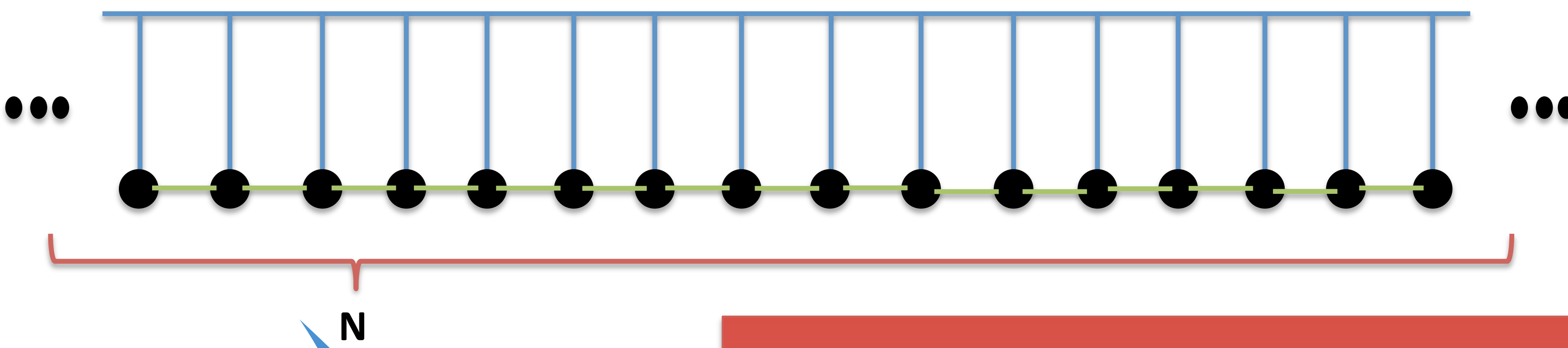
- Solitons are very stable "waves"
- They behave particle like
    - Non-linear equations
    - Dispersion
    - Collisions
- Non-linear term compensates dispersion

Linear waves

- Suppose, initially, that our problem behaves like linear sinusoidal wave.
- Problems from this supposition:
  - The waves ir … principle
  - The dispersi…
  - Our proble…



How to solve such equations?

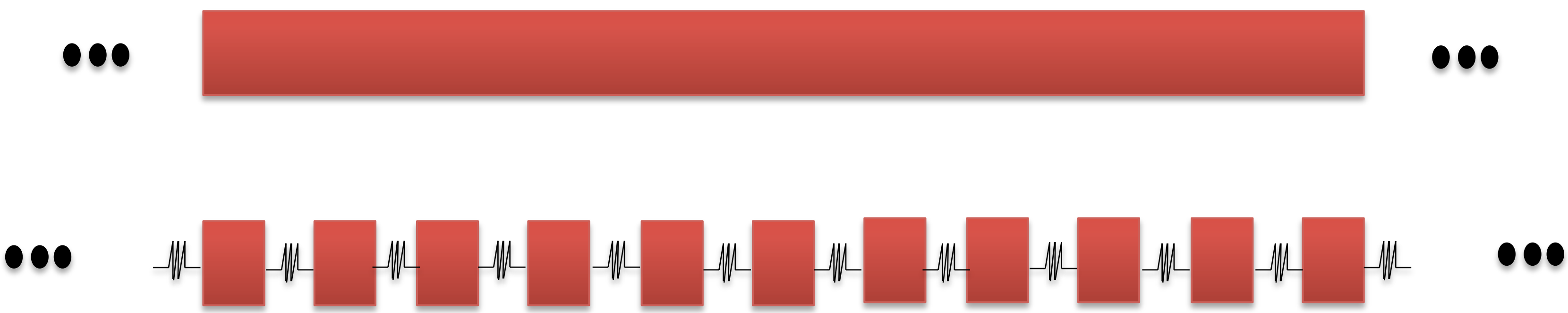


N non-linear coupled equations

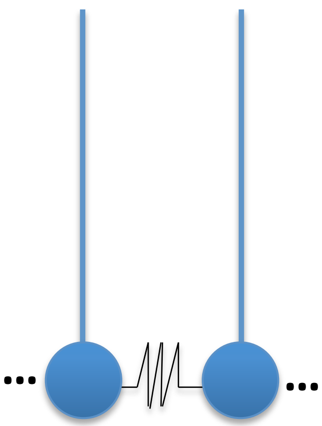
Hard to solve

Rope

- Suppose we have infinite oscillators
- We now have the continuum limit!
    - Rope!

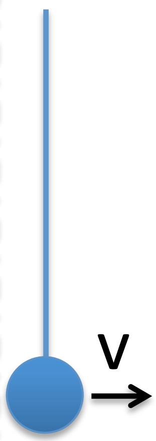


- Coupling between pendula



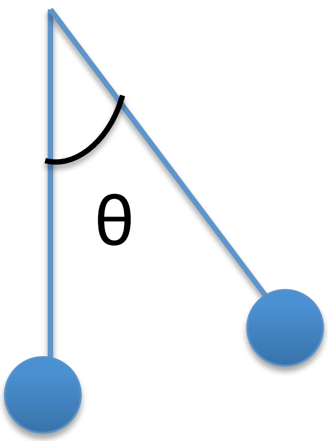
$$V \propto \left(\frac{\partial\theta}{\partial x}\right)^2$$

$$Eq. of\ motion \propto -\frac{\partial^2\theta}{\partial x^2}$$



$$V \propto \left(\frac{\partial\theta}{\partial t}\right)^2$$

$$Eq. of\ motion \propto -\frac{\partial^2\theta}{\partial t^2}$$



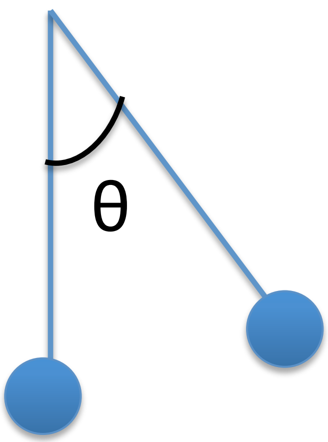
$$V \propto \frac{\theta^2}{2}$$

$$Eq. of\ motion \propto \theta$$

$$V \propto \cos\theta$$

$$Eq. of\ motion \propto \sin\theta$$

Klein-Gordon



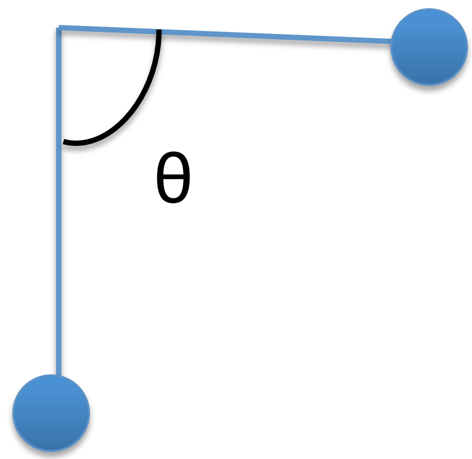
$$V \propto \frac{\theta^2}{2}$$

$$Eq.of\ motion \propto \theta$$

- Works fine for small angles
- It's linear

$$\frac{\partial^2}{\partial t^2}\psi - c_0^2\,\nabla^2\psi + {\omega_0}^2\psi = 0$$

Sine-Gordon



$$V \propto \cos\theta$$

$$Eq.of\ motion \propto \sin\theta$$

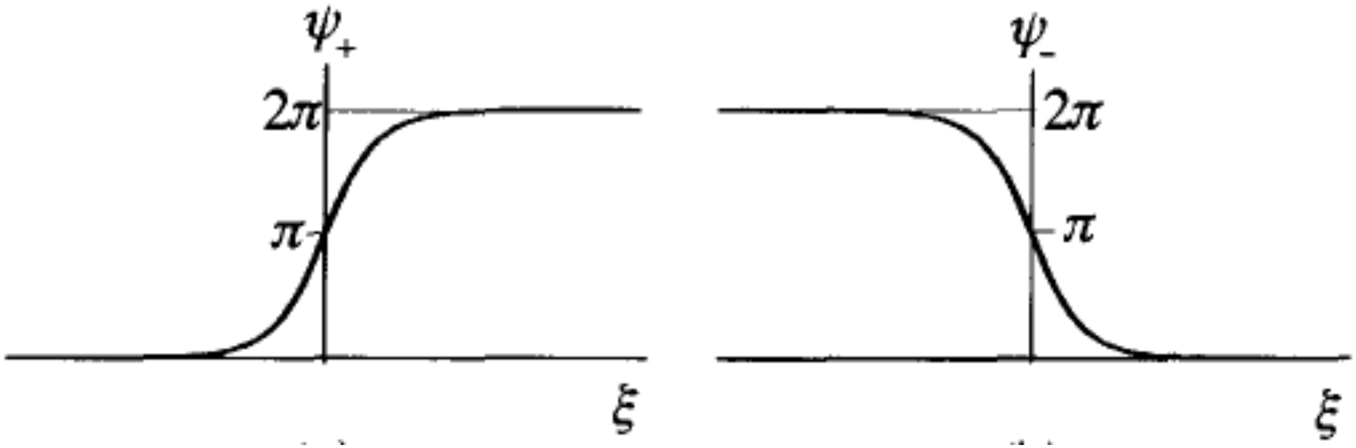
- Takes in account the whole gravitational potential
- It's well known for soliton solutions
- It's non-linear

$$\frac{\partial^2}{\partial t^2}\psi - c_0^2\,\nabla^2\psi + {\omega_0}^2\sin\psi = 0$$

Solution

- Complicated, but exact
- The angle between the pendula goes according to the function

$$\psi(\xi) = 4\arctan e^{\pm\gamma\xi}$$



Variations - theoretical

- We can analyze the factors that affect the rotation, the period and the velocity

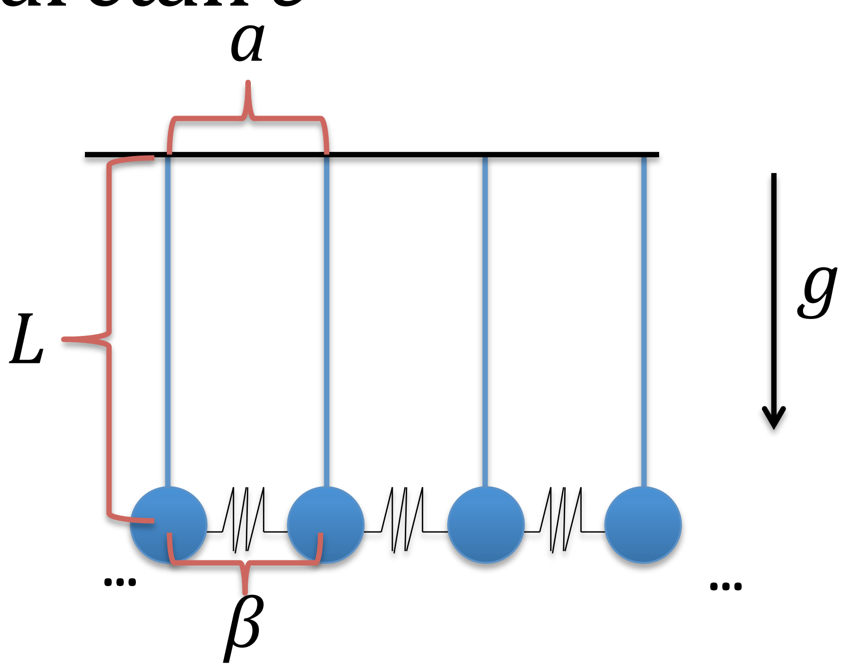
$$\psi(\xi) = 4\arctan e^{\pm\gamma\xi}$$

- Defining two parameters:

$$c_0^2 = \frac{a^2\beta}{I}$$

$$\omega_0^2 = \frac{mgL}{I}$$

$$\frac{\partial^2\psi}{\partial t^2} - c_0^2\frac{\partial^2\psi}{\partial x^2} + \omega_0^2\sin\psi = 0$$

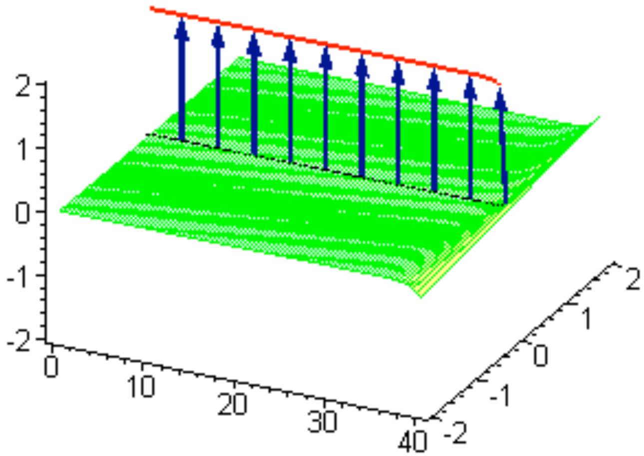


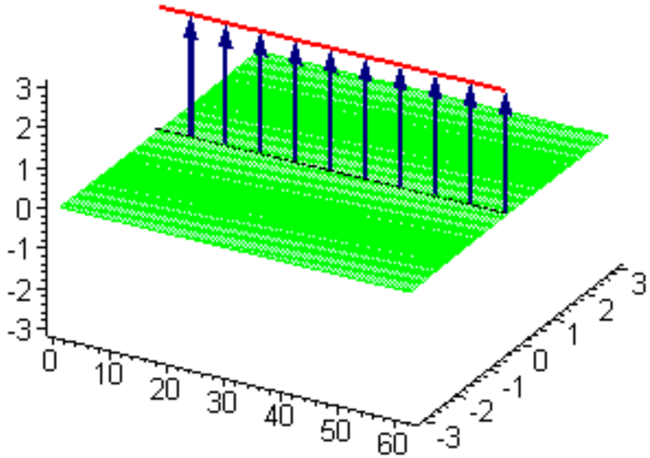
What about the velocity?

$$\psi = 4Arctan\,\exp\left[\pm\frac{\omega_0}{c_0}\frac{vt - s_0}{\left(1-\frac{v}{c_0}^2\right)^{\frac{1}{2}}}\right]$$

- Now we can see an analog, the sound speed for the material medium we are using, defined as c_0.
  - That means that the maximum velocity in such a system is defined by geometrical and elastic properties.

$$c_0^2 = \frac{a^2\beta}{I}$$

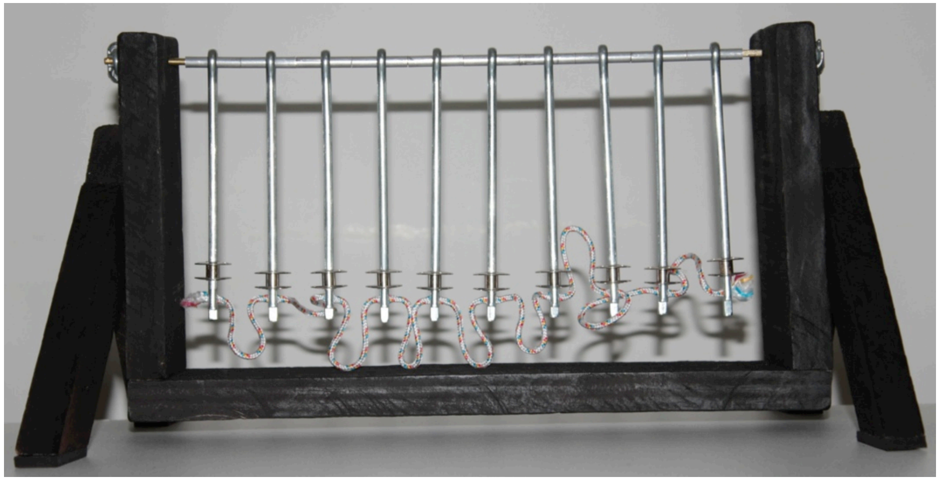


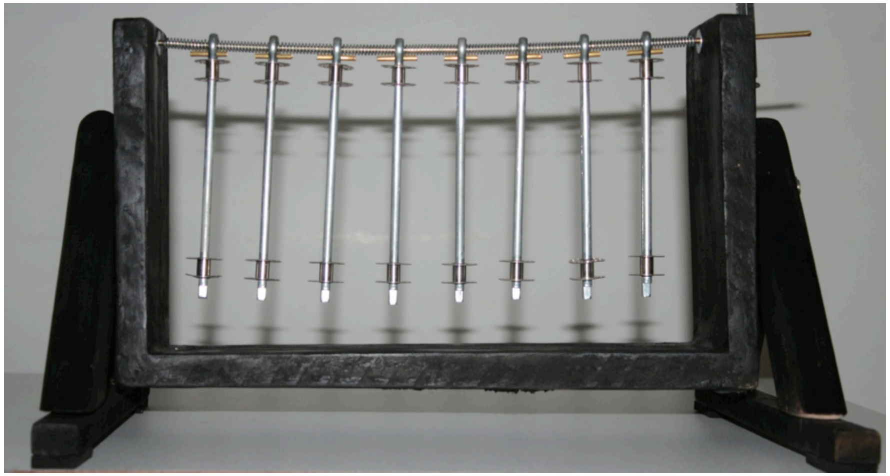


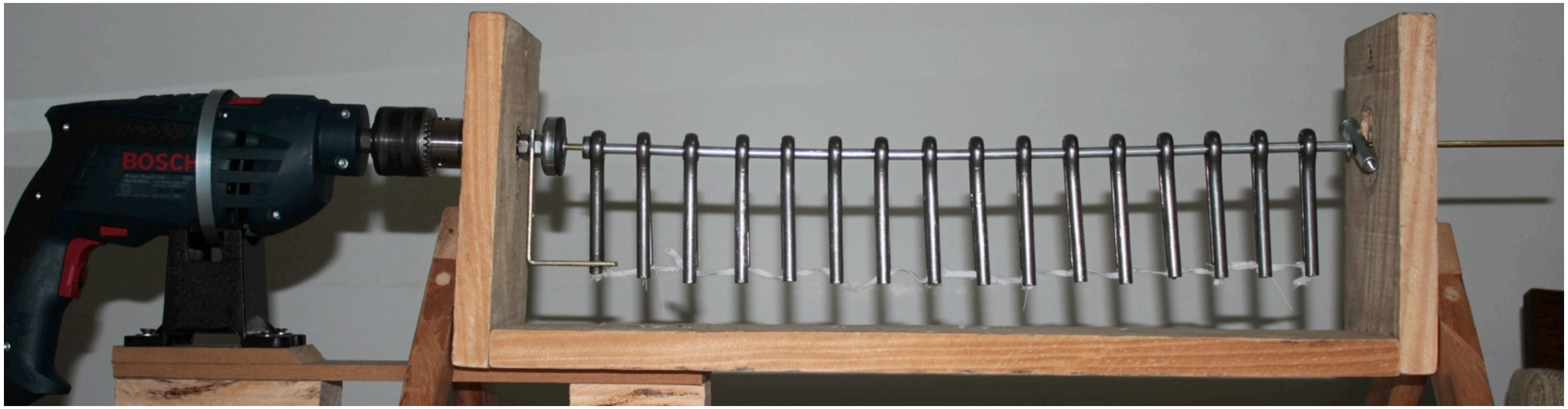
http://en.wikipedia.org/wiki/Soliton

Sine-Gordon simulations

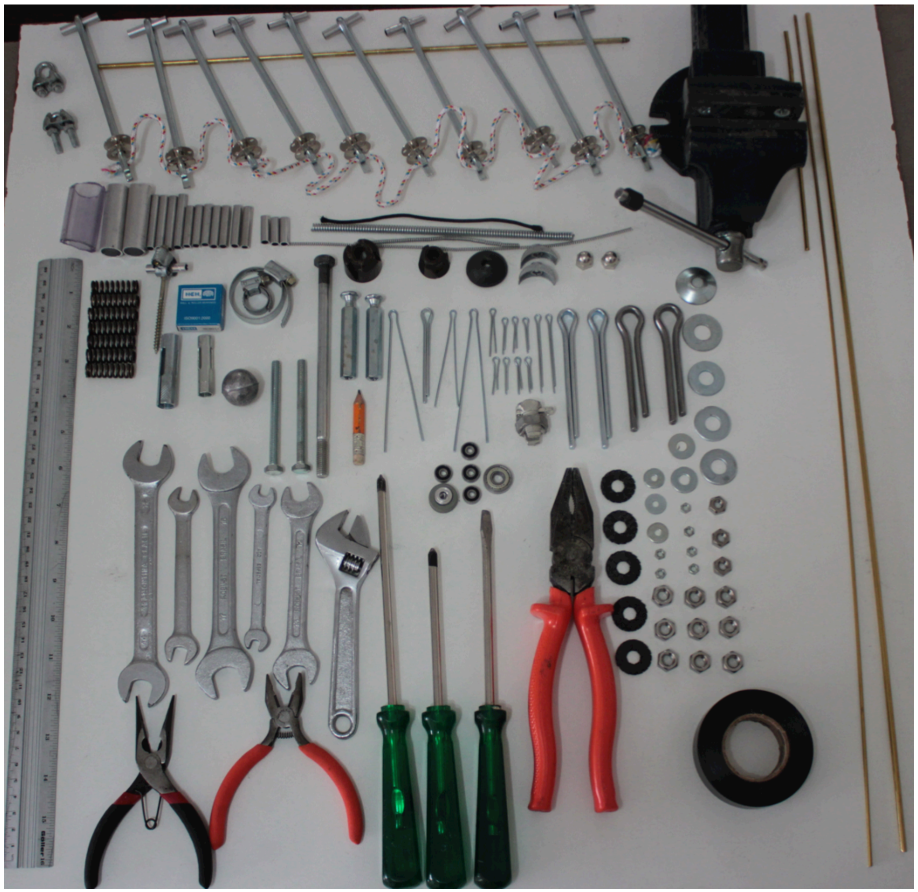
Experiments







Materials



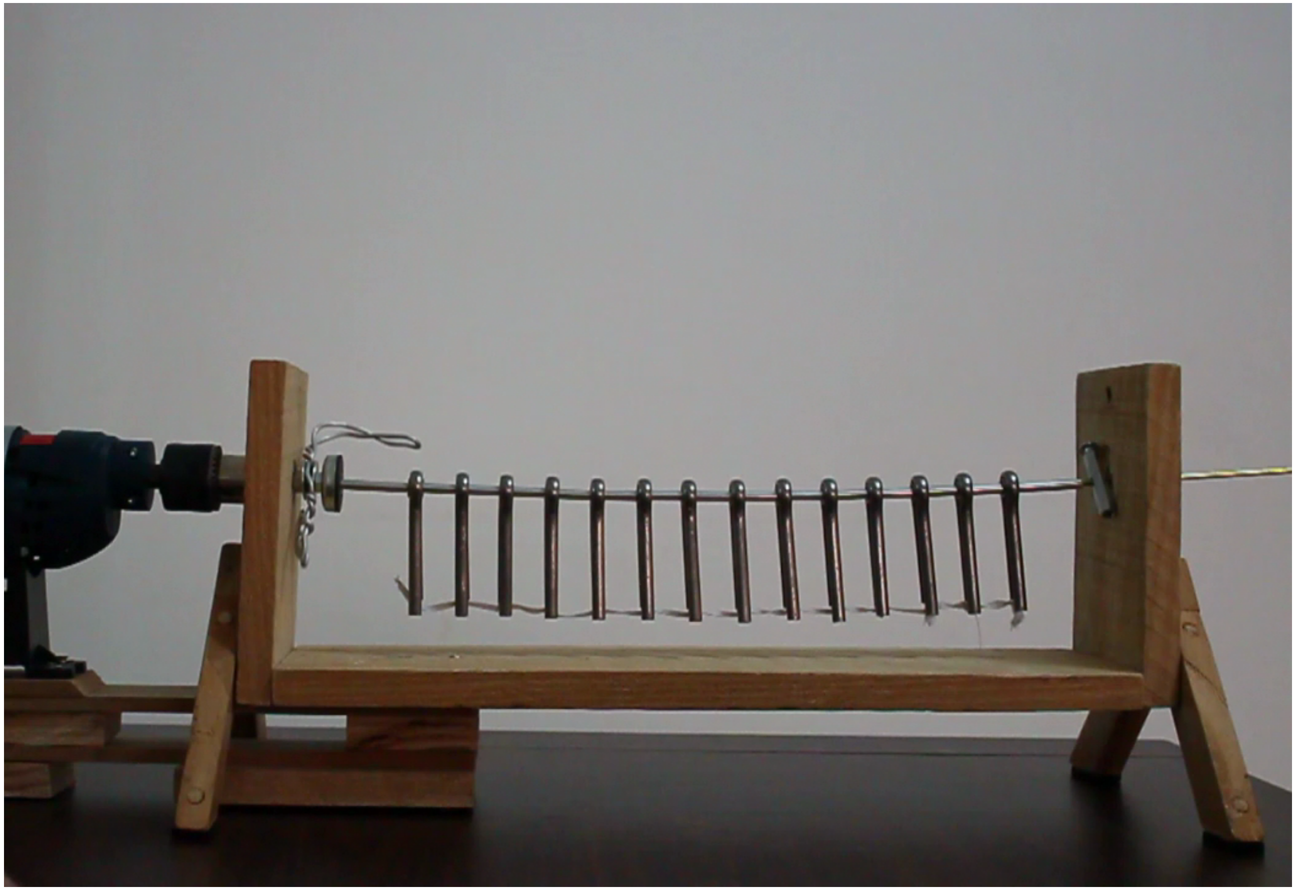
- Vise
- Flat nose pliers
- Wheel wrench
- Washer
- Cotters (many sizes)
- Rods
- Ball bearings (many sizes)
- Screwdrivers
- Ruler
- Scale
- Tubes
- Screws

Experimental setup

Variations

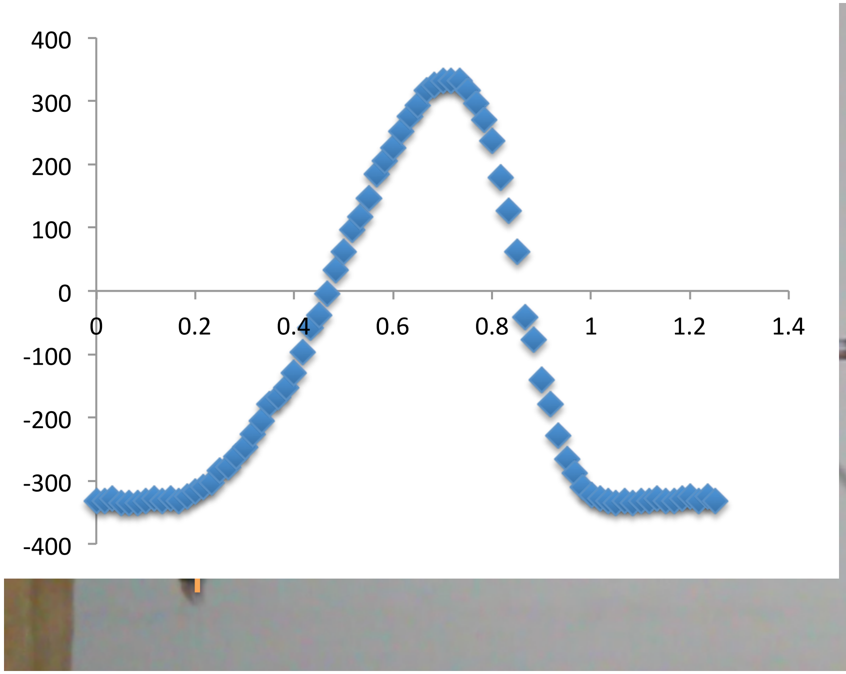

REAL
5 mm x 10 m
Nº 8
10R
REAL
3,5 mm x 10 m
Nº 6
72% ALGODÃO
28% ELASTODIENO
REAL

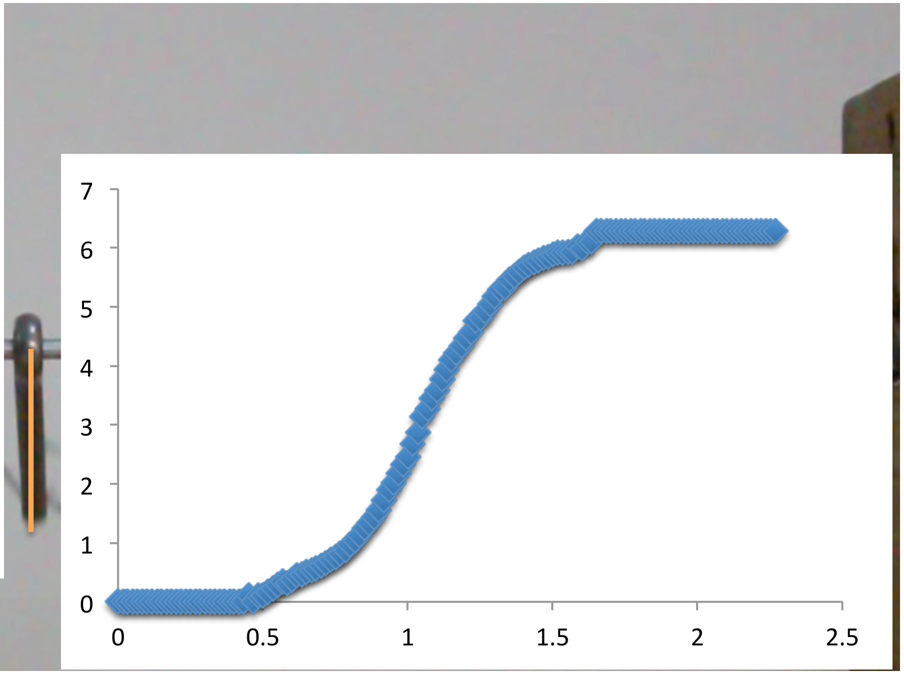

BOSCH
GSB 16 RE
Professional



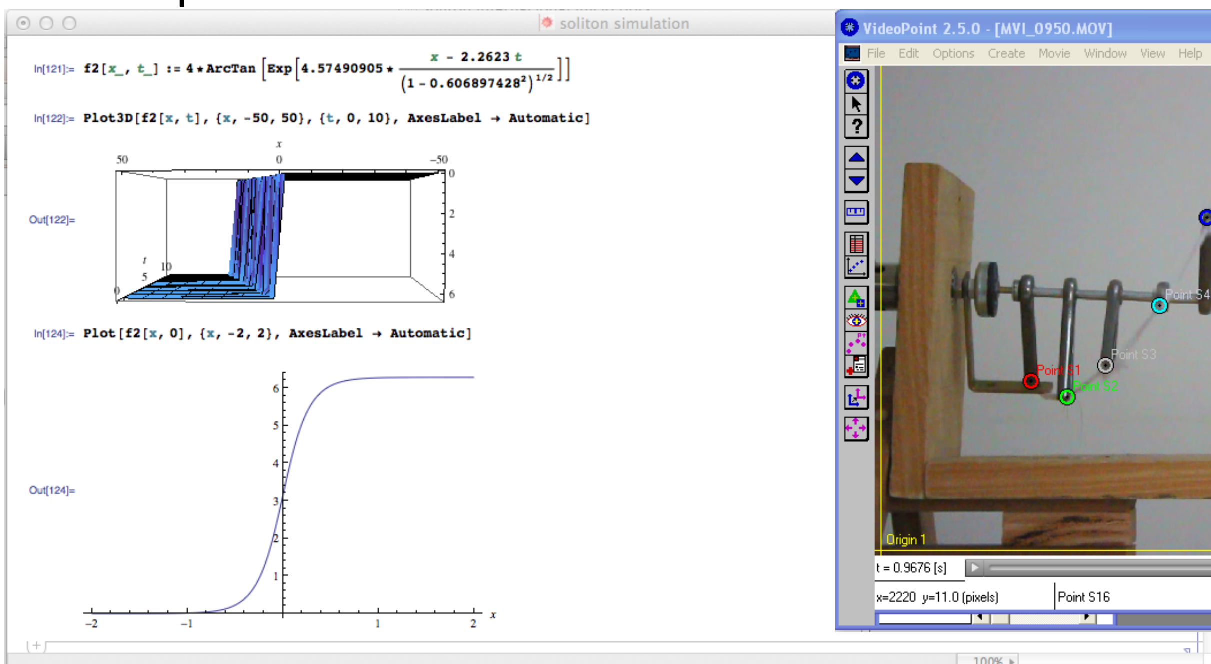
Analyzing videos

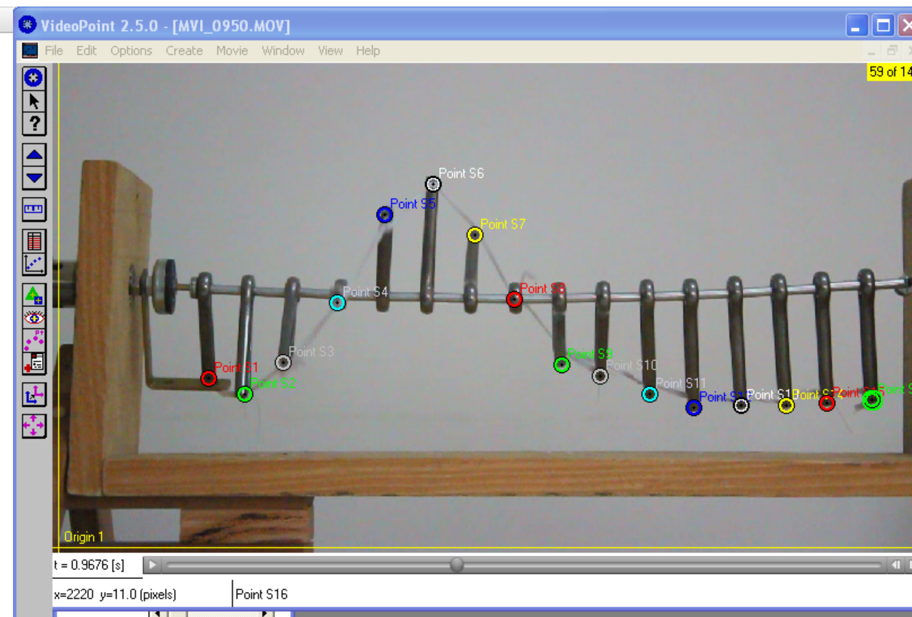
- For the video analysis, we used Video Point, measuring the relative height of each pendula in relation to the axis:





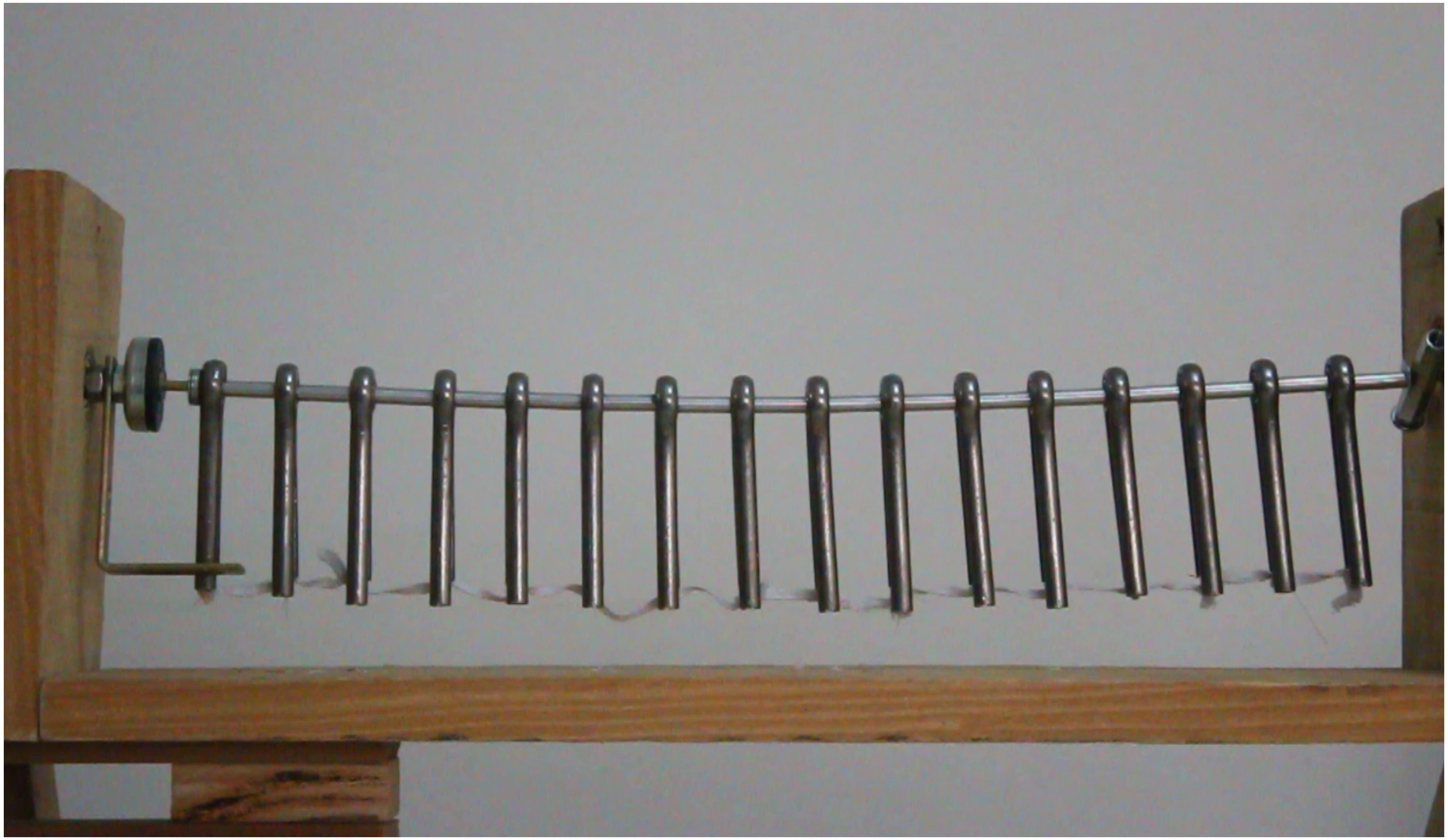
- Soliton simulations:
  - We used Mathematica for plotting the graphs and varying the parameters.





Experiment 1

- Angular velocity
- Pendula mass
- Elastic properties
- Pendula size
- String pendula
- Spring solitons



Pendulum specifications

Pendulum specifications	
Size	$9.00\ 10^{-2}$ m
Mass	$5.00\ 10^{-2}$ kg
Inertia moment	$4.96\ 10^{-5}$ kg m^2
Initial angular velocity	7.12 rad s^{-1}
Linear velocity	0.44 m s^{-1}
Number of pendulums	16
Distance between pendulums	$3.00\ 10^{-2}$ m
Torsion coefficient	$1.71\ 10^{-1}$ Nm rad^{-1}

Angular velocity

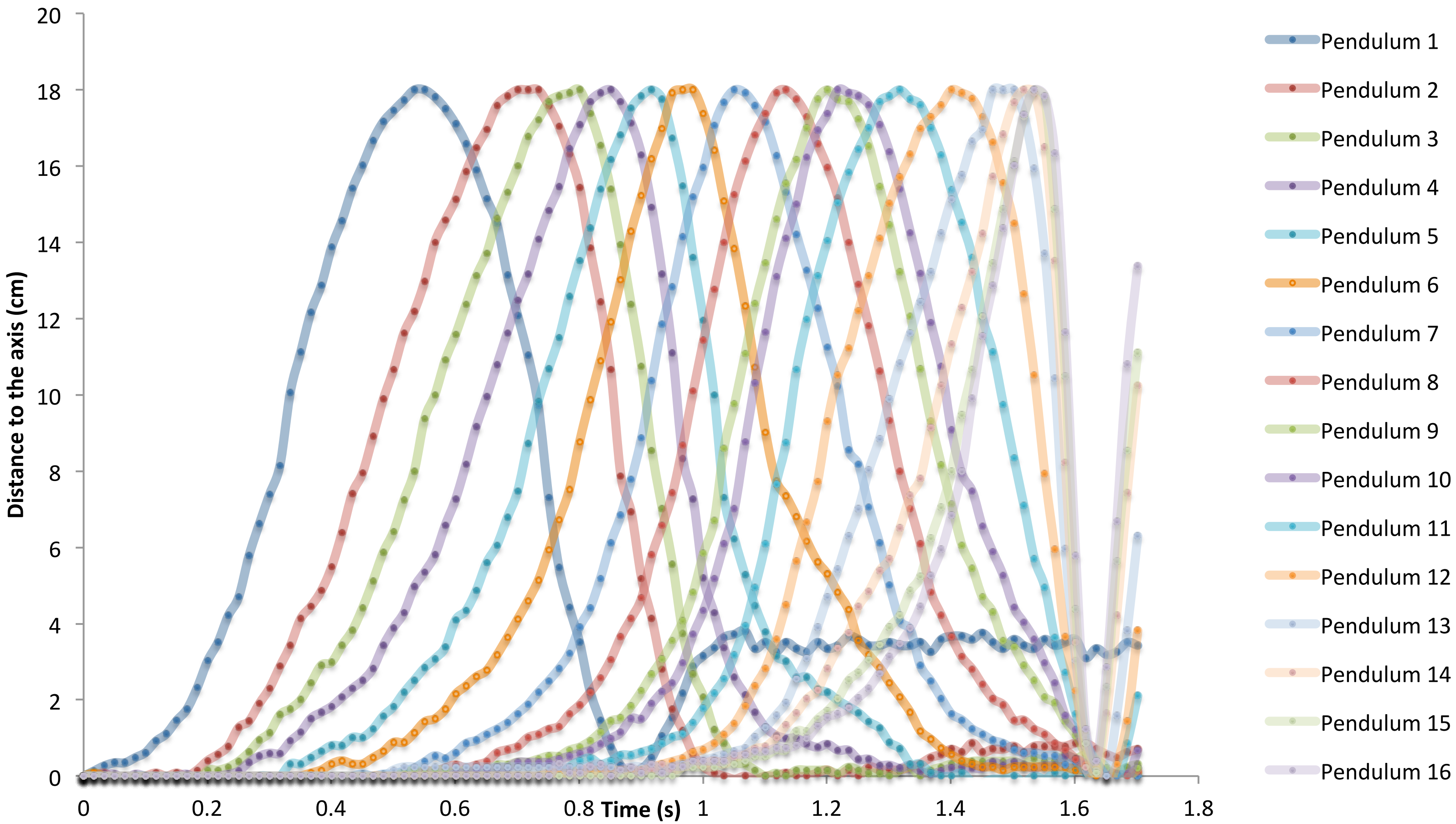
Pendula mass

Elastic properties

Pendula size

String pendula

Spring solitons



Angular velocity

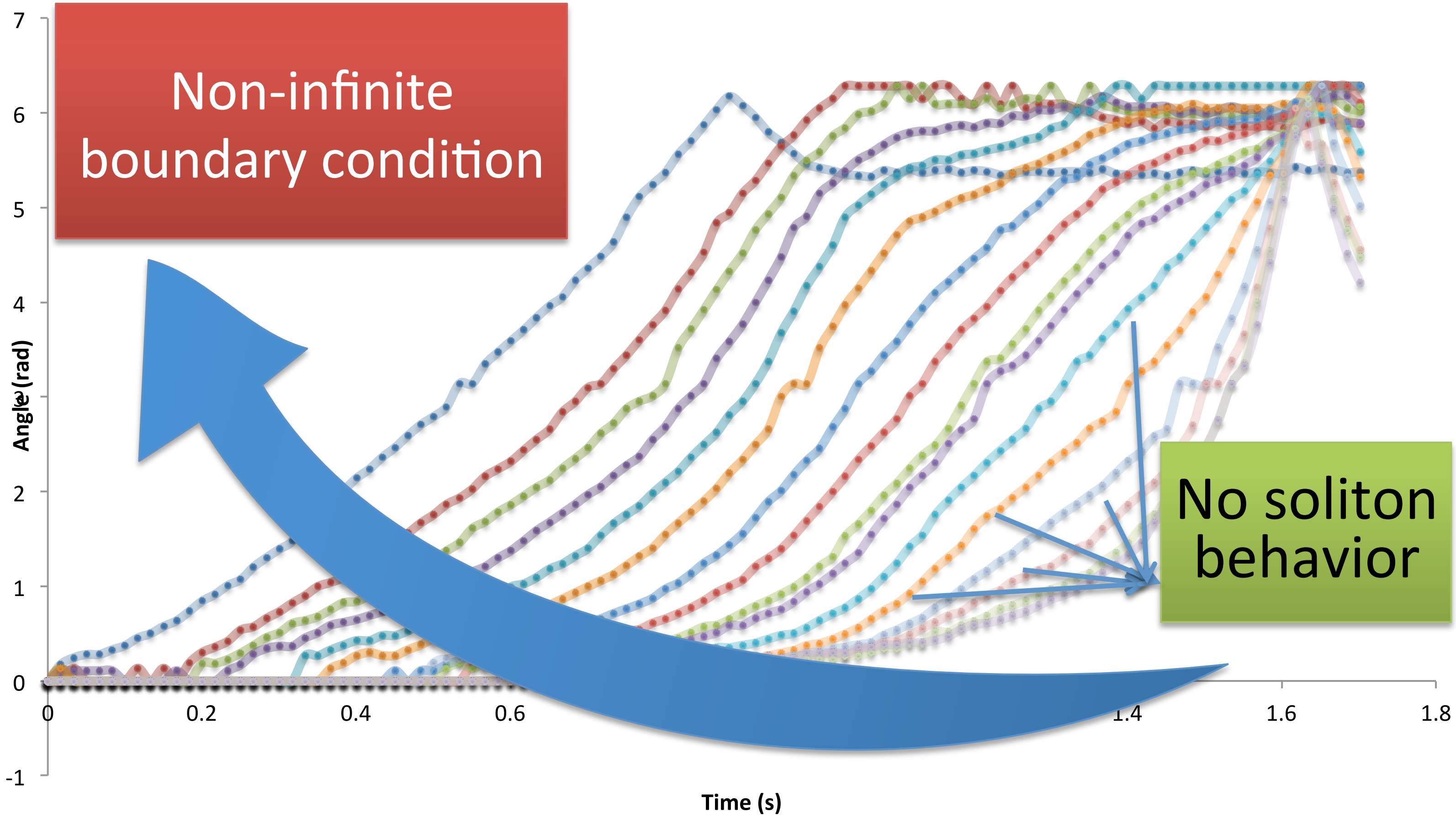
Pendula mass

Elastic properties

Pendula size

String pendula

Spring solitons



Graph

- Angular velocity
- Pendula mass
- Elastic properties
- Pendula size
- String pendula
- Spring solitons

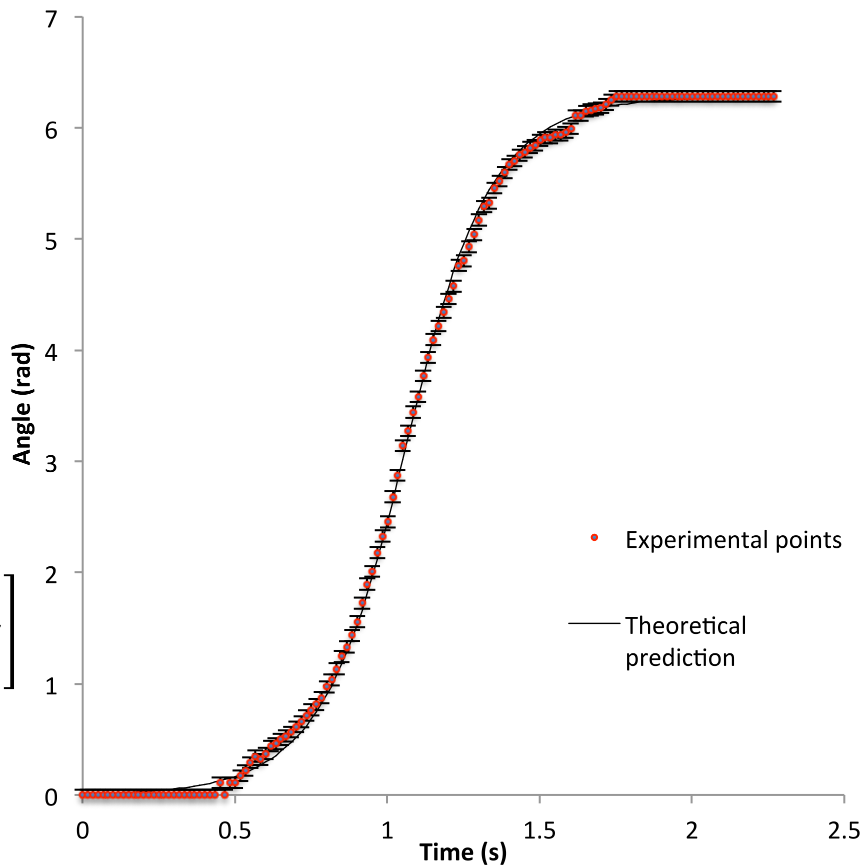
$$\omega_0 = 34.0 \pm 0.6\, s^{-1}$$

$$c_0 = 2.48 \pm 0.04\, m * s^{-1}$$

$$\psi = 4 Arctan \exp\left[\pm \frac{\omega_0}{c_0} \frac{vt - s_0}{\left(1 - \frac{v}{c_0}^2\right)^{\frac{1}{2}}}\right]$$

$$\psi = 4 Arctan \exp\left[\pm \frac{34.0}{2.48} \frac{0.44\, t}{(1 - 0.36^2)^{\frac{1}{2}}} - 2\pi\right]$$

$$\psi = 4 Arctan \exp[5.65\, t - 2\pi]$$



Angular velocity

Pendula mass

Elastic properties

Pendula size

String pendula

Spring solitons

Pendulum specifications	
Size	$9.00\ 10^{-2}$ m
Mass	$5.00\ 10^{-2}$ kg
Inertia moment	$4.96\ 10^{-5}$ kg m^2
Initial angular velocity	3.18 rad s^{-1}
Linear velocity	0.64 m s^{-1}
Number of pendulums	16
Distance between pendulums	$3.00\ 10^{-2}$ m
Torsion coefficient	$1.71\ 10^{-1}$ Nm rad^{-1}

Graph

Angular velocity

Pendula mass

Elastic properties

Pendula size

String pendula

Spring solitons

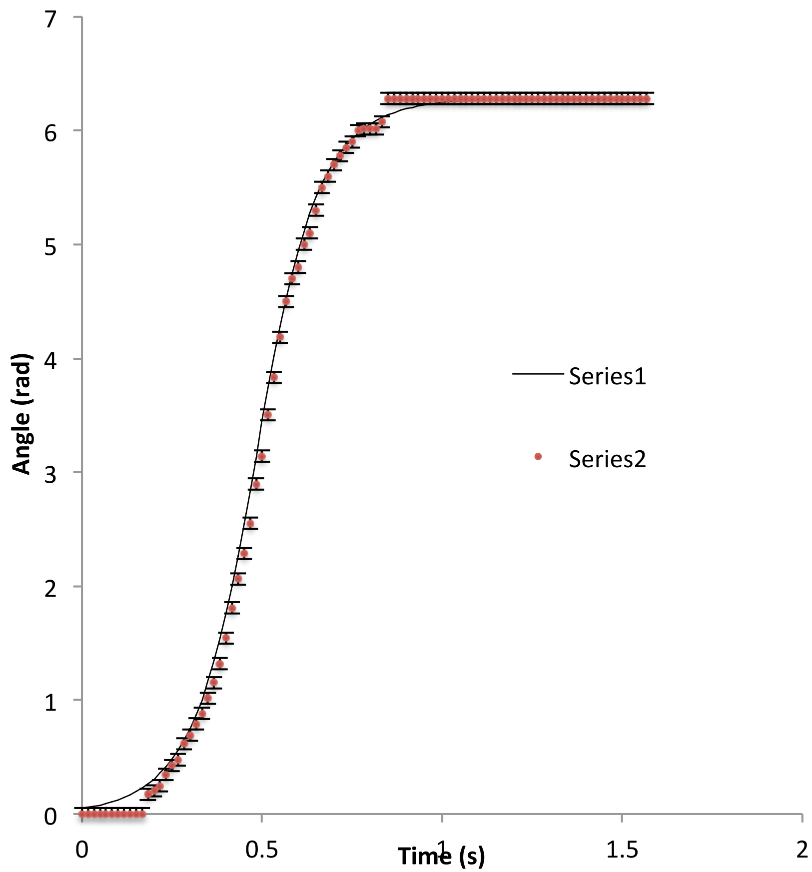
$$\omega_0 = 34.0 \pm 0.6\, s^{-1}$$

$$c_0 = 2.48 \pm 0.04\, m * s^{-1}$$

$$\psi = 4Arctan\, \exp\left[\pm \frac{\omega_0}{c_0} \frac{vt - s_0}{\left(1 - \frac{v}{c_0}^2\right)^{\frac{1}{2}}}\right]$$

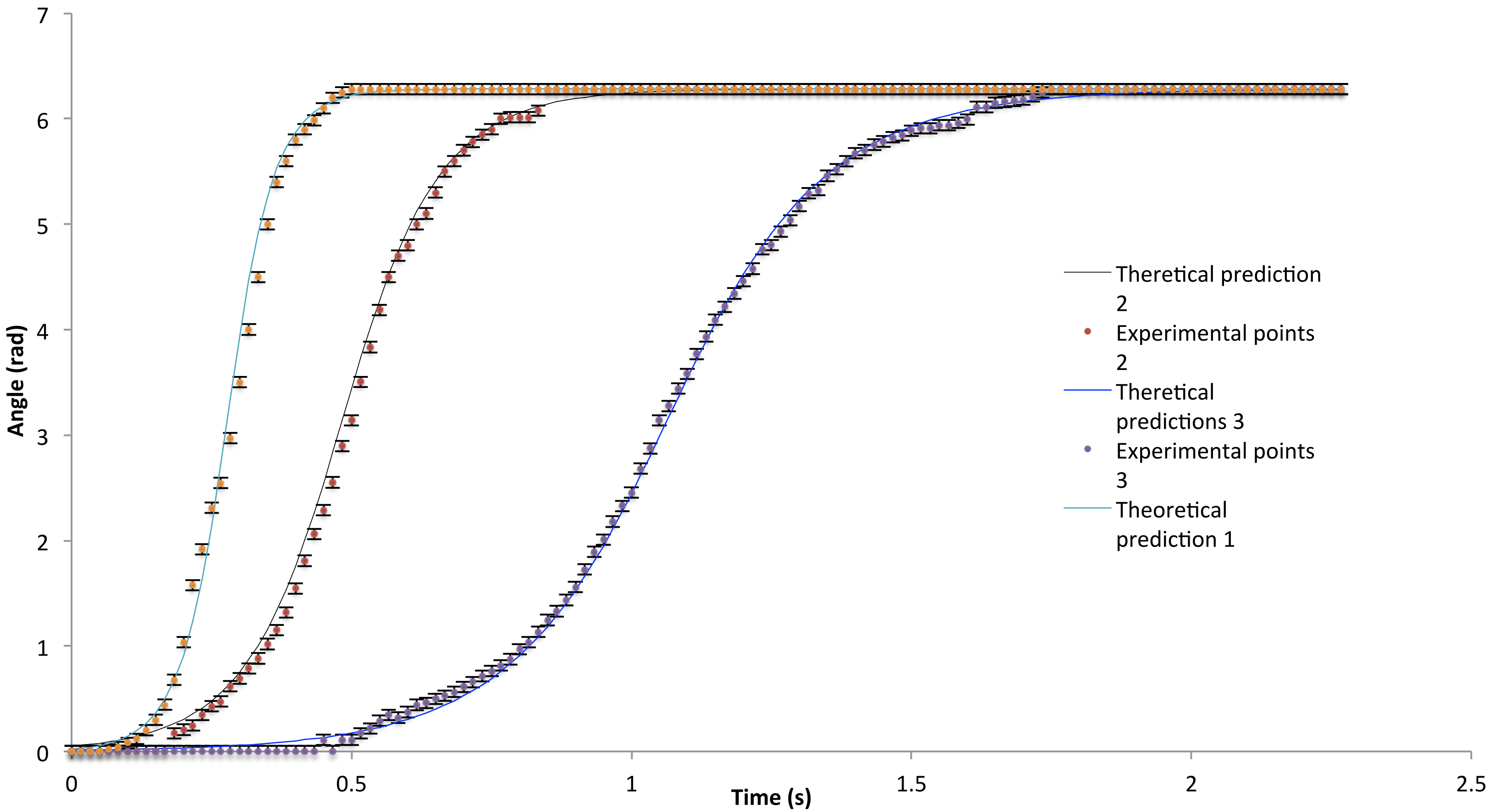
$$\psi = 4Arctan\, \exp\left[\pm \frac{34.0}{2.48} \frac{0.64\, t}{(1 - 0.36^2)^{\frac{1}{2}}} - 2\pi\right]$$

$$\psi = 4Arctan\, \exp[5.65\, t - 2\pi]$$



Comparison between the velocities

- Angular velocity
- Pendula mass
- Elastic properties
- Pendula size
- String pendula
- Spring solitons



Mass distribution variation

Angular velocity

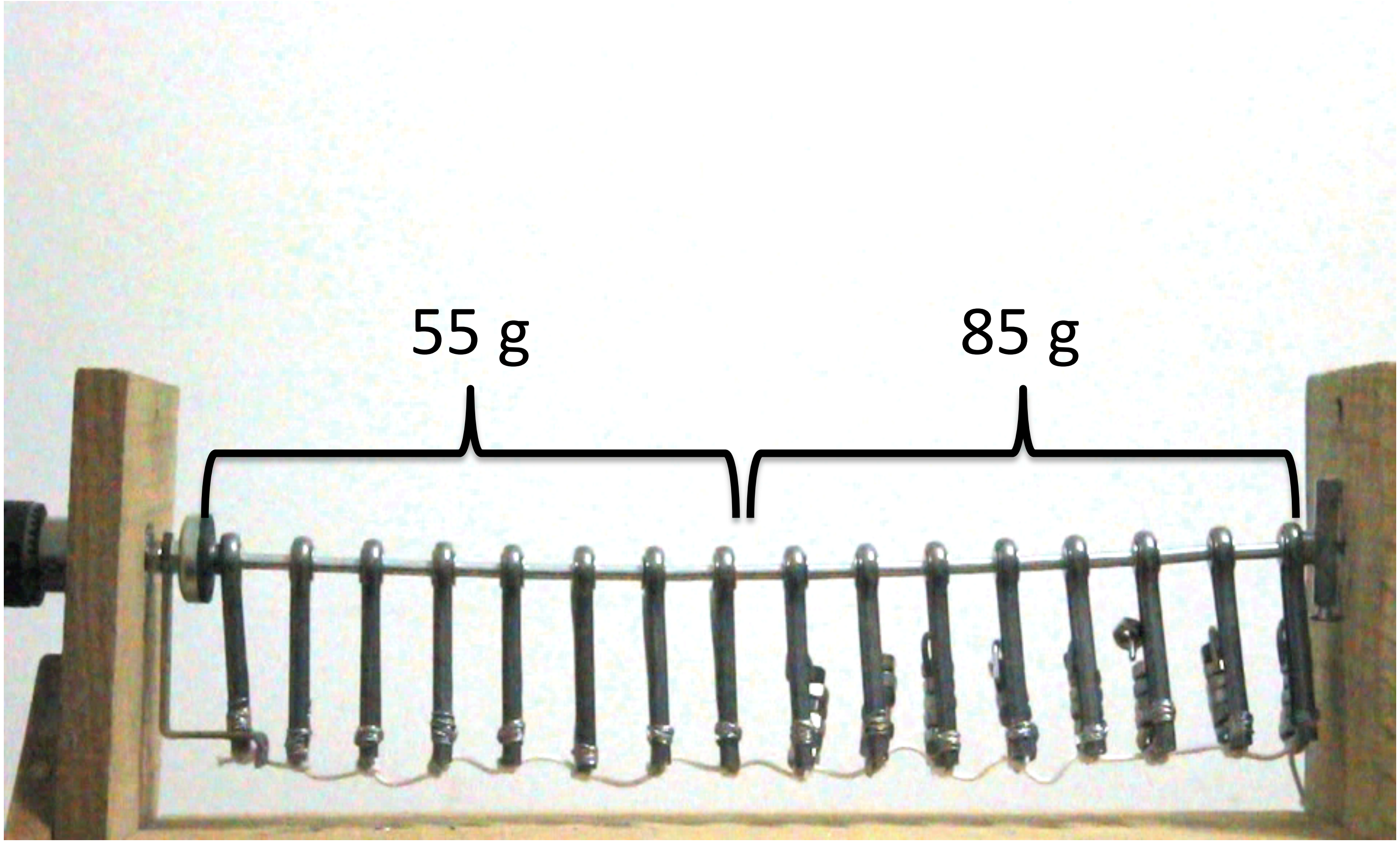
Pendula mass

Elastic properties

Pendula size

String pendula

Spring solitons



Angular velocity

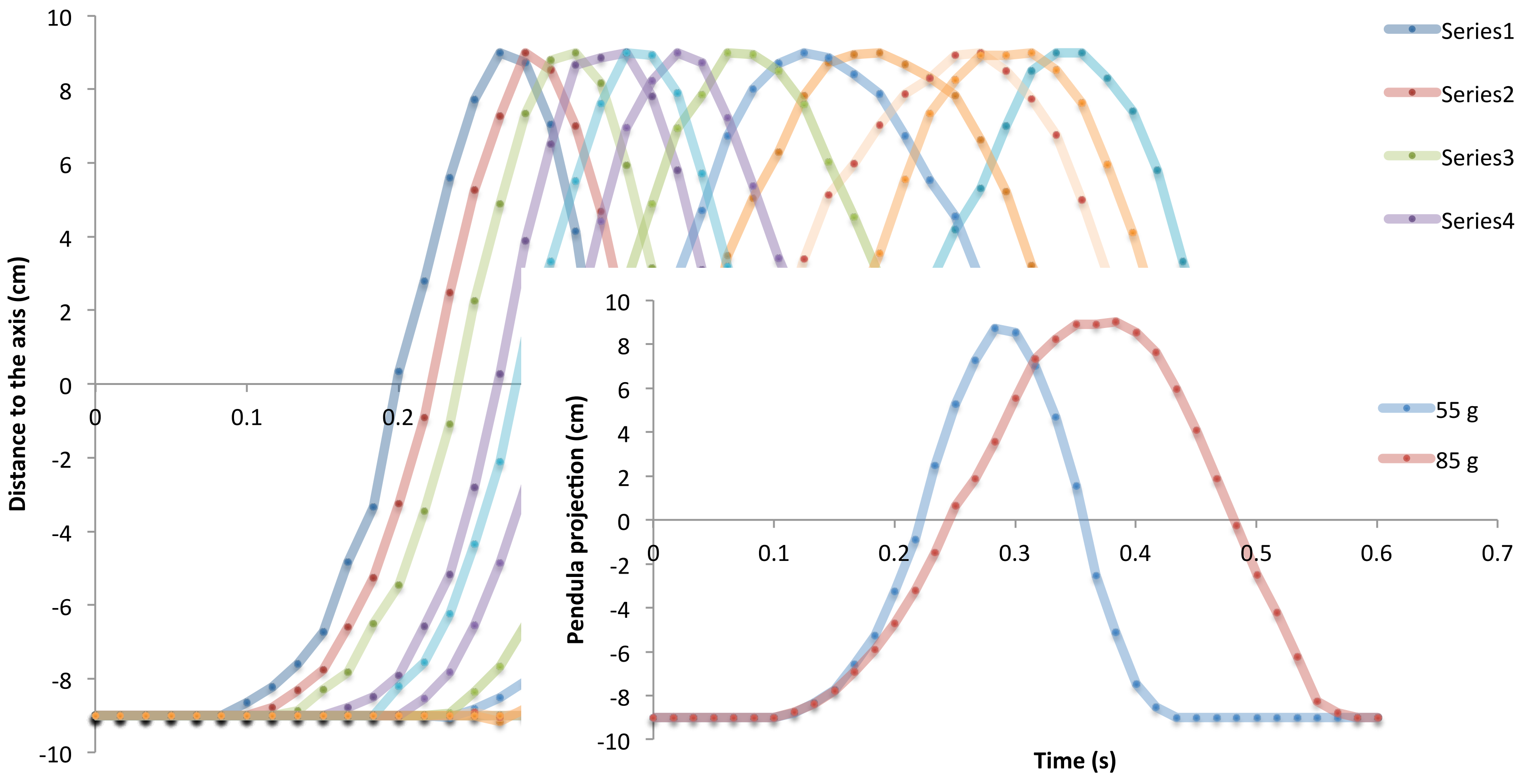
Pendula mass

Elastic properties

Pendula size

String pendula

Spring solitons



Angular velocity

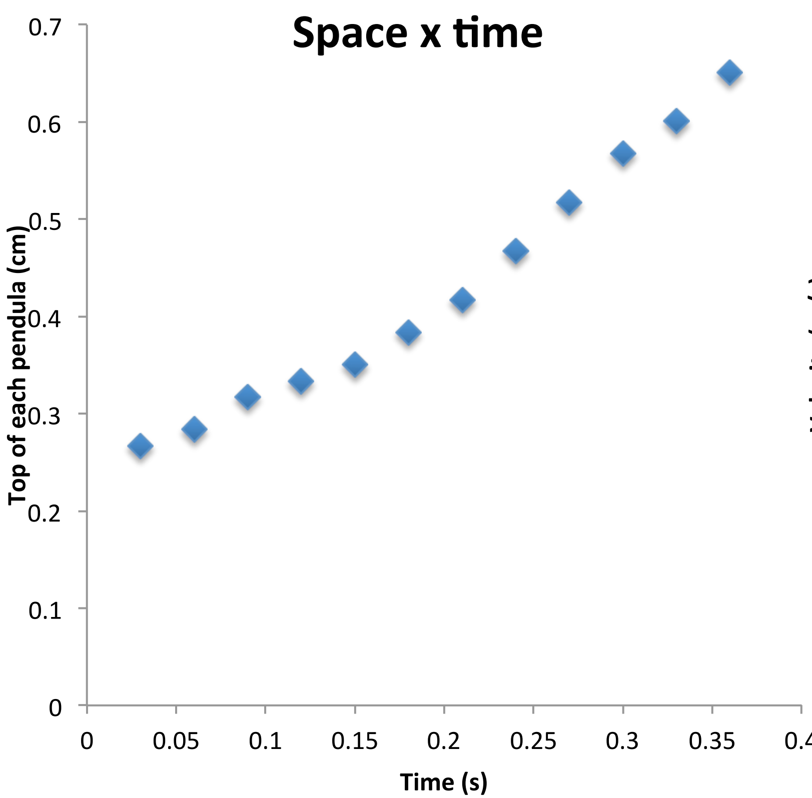
Pendula mass

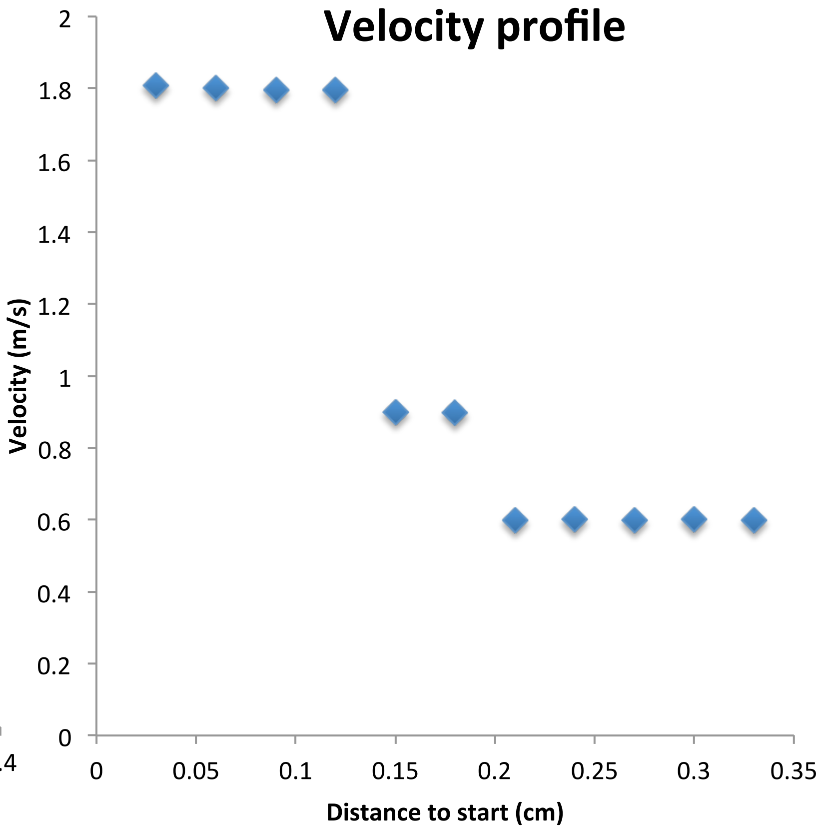
Elastic properties

Pendula size

String pendula

Spring solitons





Angular velocity

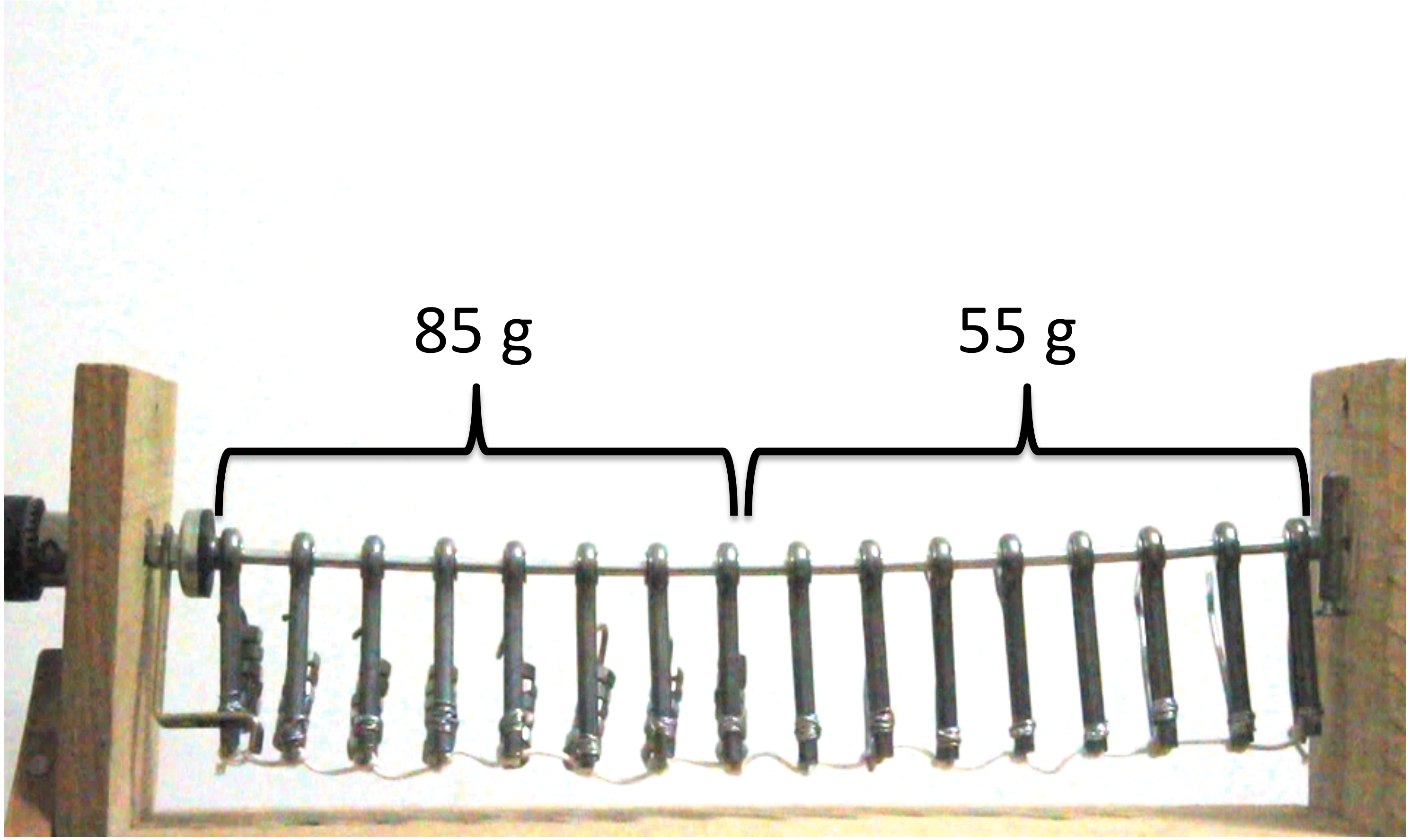
Pendula mass

Elastic properties

Pendula size

String pendula

Spring solitons



Angular velocity

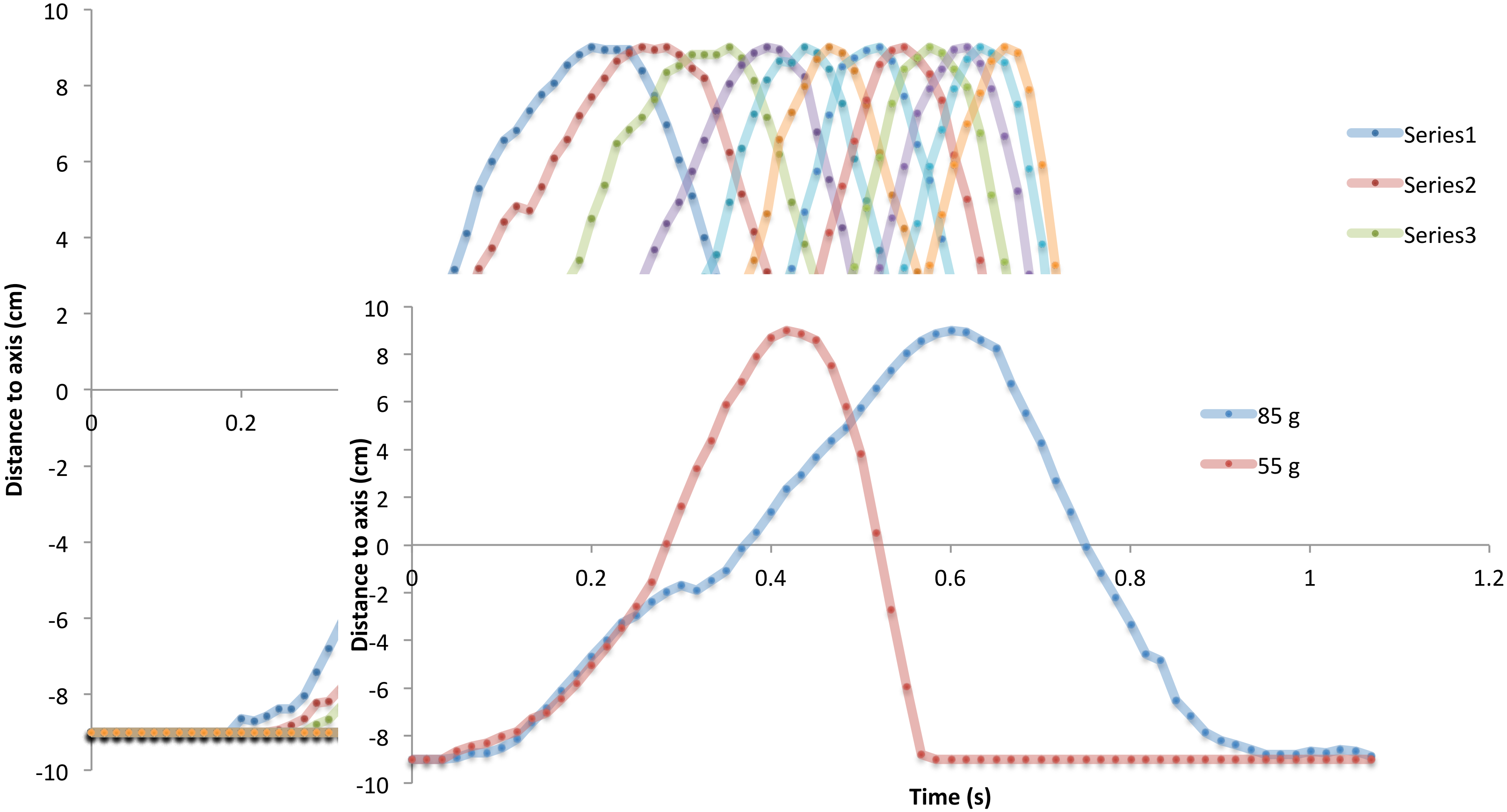
Pendula mass

Elastic properties

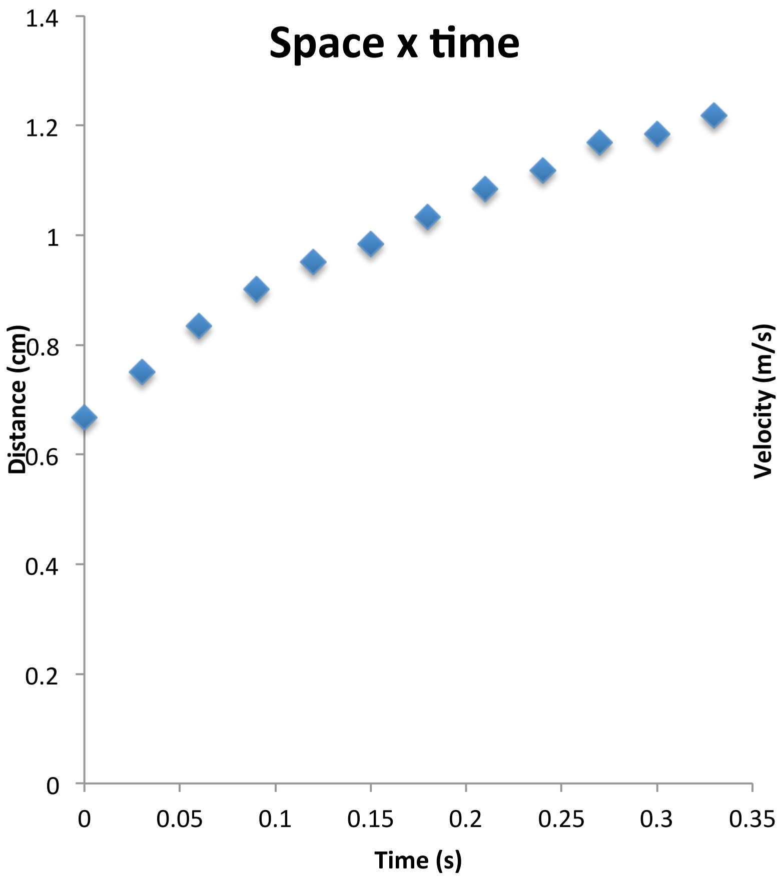
Pendula size

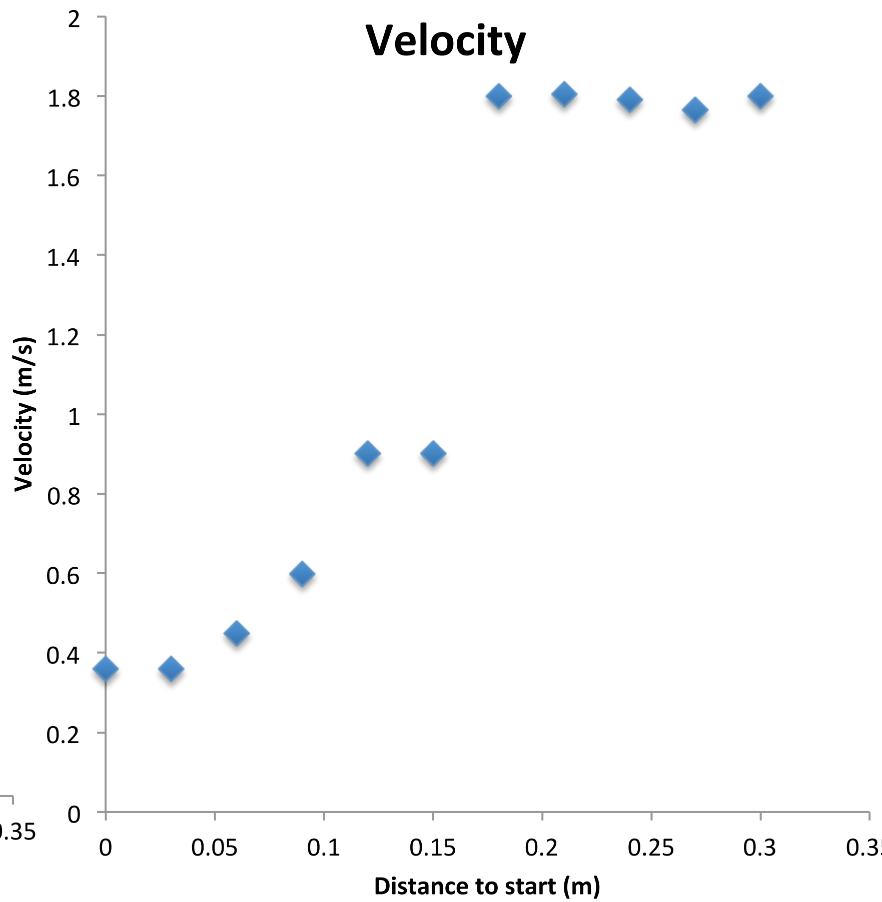
String pendula

Spring solitons



- Angular velocity
- **Pendula mass**
- Elastic properties
- Pendula size
- String pendula
- Spring solitons





Varying the string kind (elastic properties)

Angular velocity

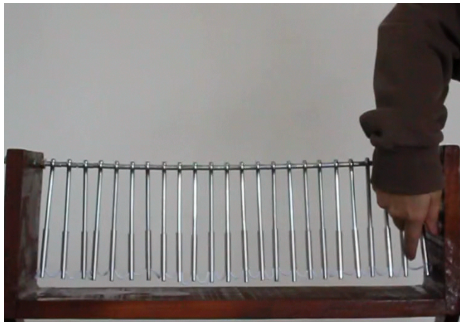
Pendula mass

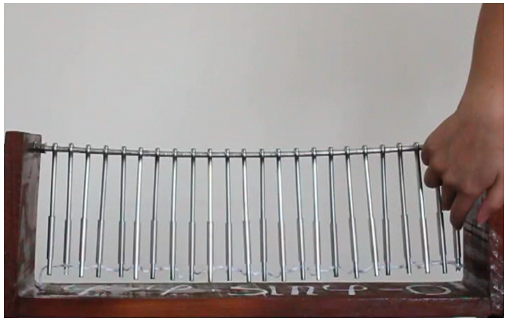
Elastic properties

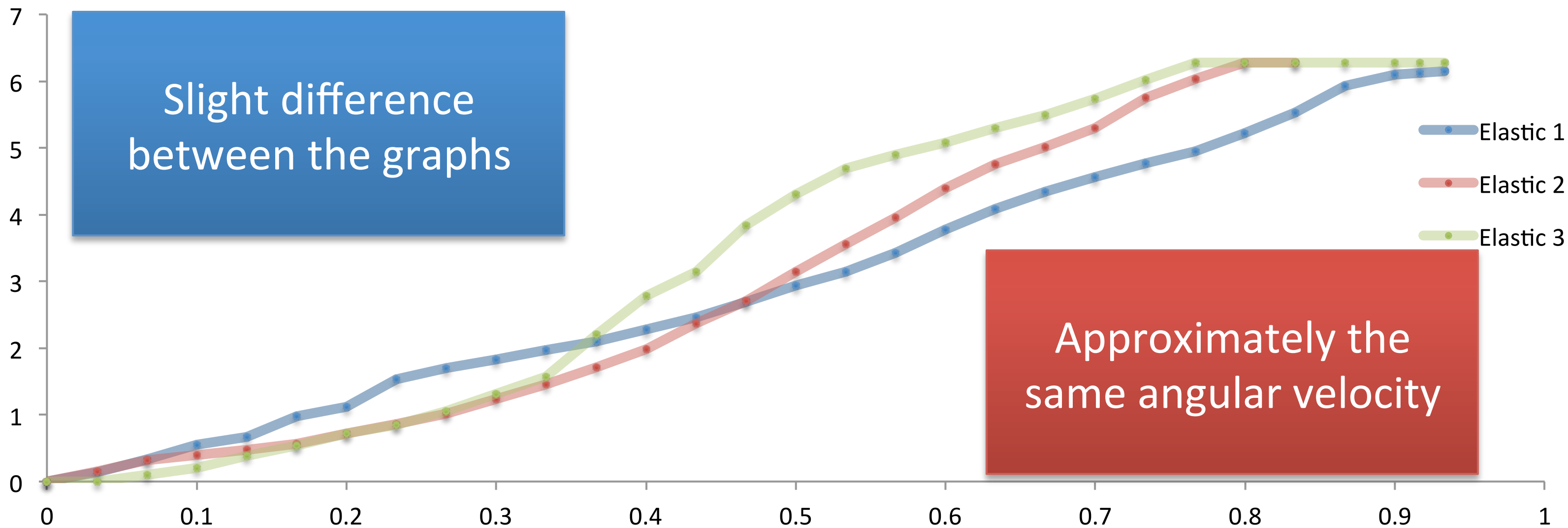
Pendula size

String pendula

Spring solitons







- Angular velocity
- Pendula mass
- Elastic properties
- Pendula size
- String pendula
- Spring solitons

Angular velocity

Pendula mass

Elastic properties

Pendula size

String pendula

Spring solitons

Pendulum specifications	
Size	$8.00\ 10^{-1}$ m
Mass	$2.00\ 10^{-1}$ kg
Inertia moment	$3.12\ 10^{-3}$ kg m^2
Initial angular velocity	3.92 rad s^{-1}
Linear velocity	0.40 m s^{-1}
Number of pendulums	25
Distance between pendulums	$3.00\ 10^{-2}$ m
Torsion coefficient	3.25 Nm rad^{-1}

Angular velocity

Pendula mass

Elastic properties

Pendula size

String pendula

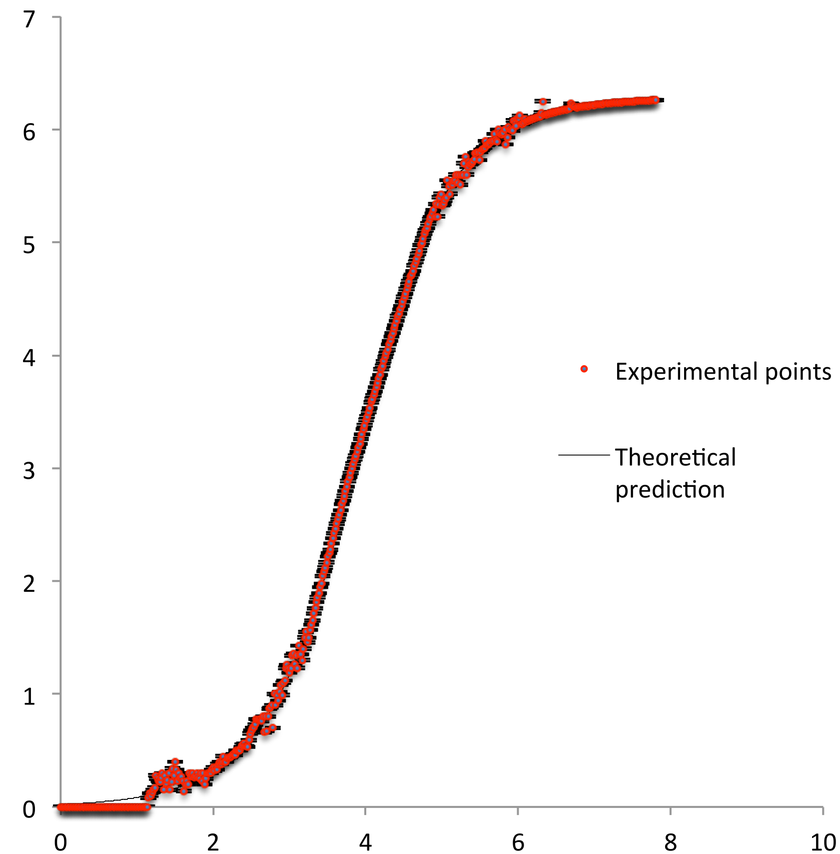
Spring solitons

$$\omega_0 = 21.8 \pm 0.6\ s^{-1}$$

$$c_0 = 6.50 \pm 0.04\ m * s^{-1}$$

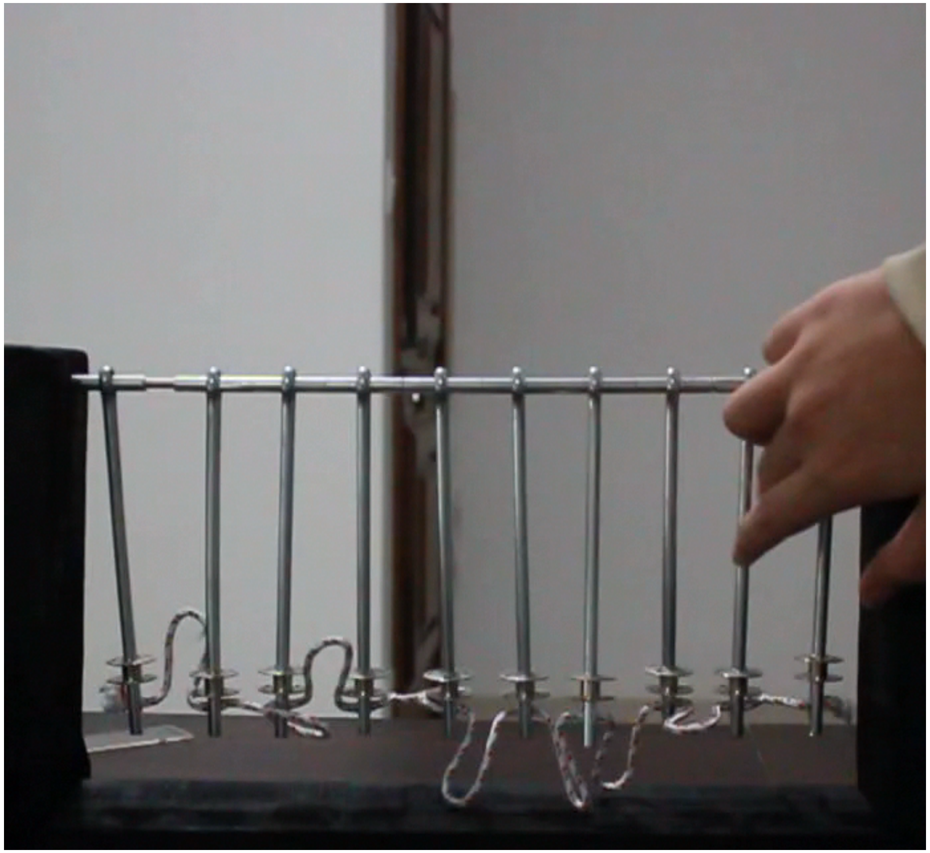
$$\psi = 4Arctan\,\exp\left[\pm\frac{\omega_0}{c_0}\frac{vt - s_0}{\left(1-\frac{v}{c_0}^2\right)^{\frac{1}{2}}}\right]$$

$$\psi = 4Arctan\,\exp[1.34\,t - 5.2]$$



- Angular velocity
- Pendula mass
- Elastic properties
- Pendula size
- String pendula
- Spring solitons

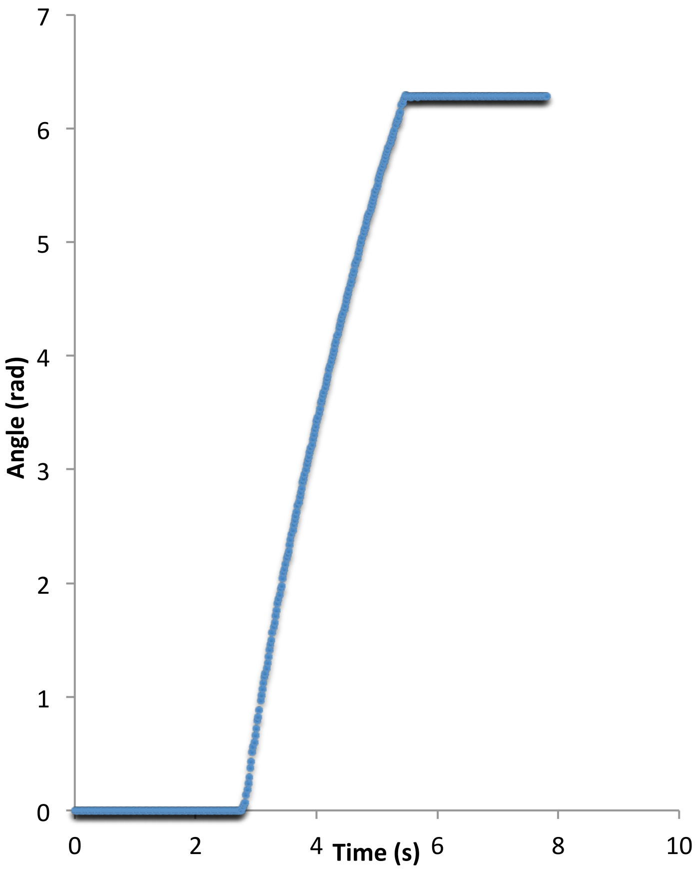
- Pendulums connected by non-elastic strings
- 10 pendulums
- Distance of 3 cm between two pendulums
- Mass of 30 g



Spring pendula

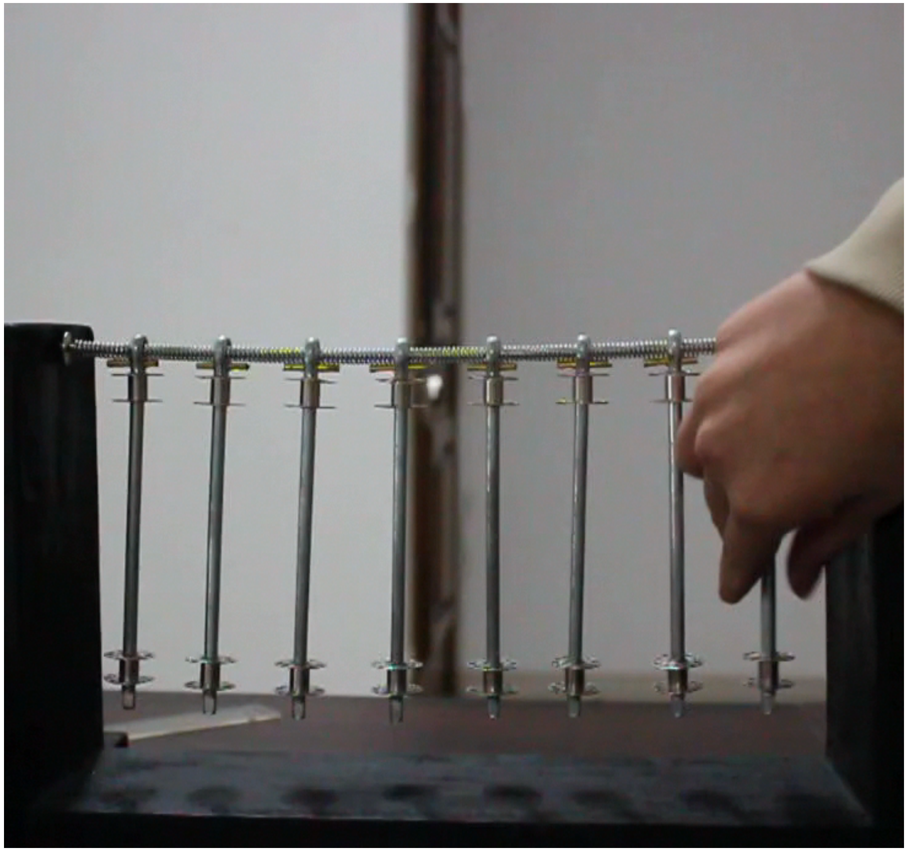
- Angular velocity
- Pendula mass
- Elastic properties
- Pendula size
- String pendula
- Spring solitons

- Non-elastic string connecting the pendulums
- It's not a soliton.
- The angles between the pendulums are constant
- The β coefficient goes to zero

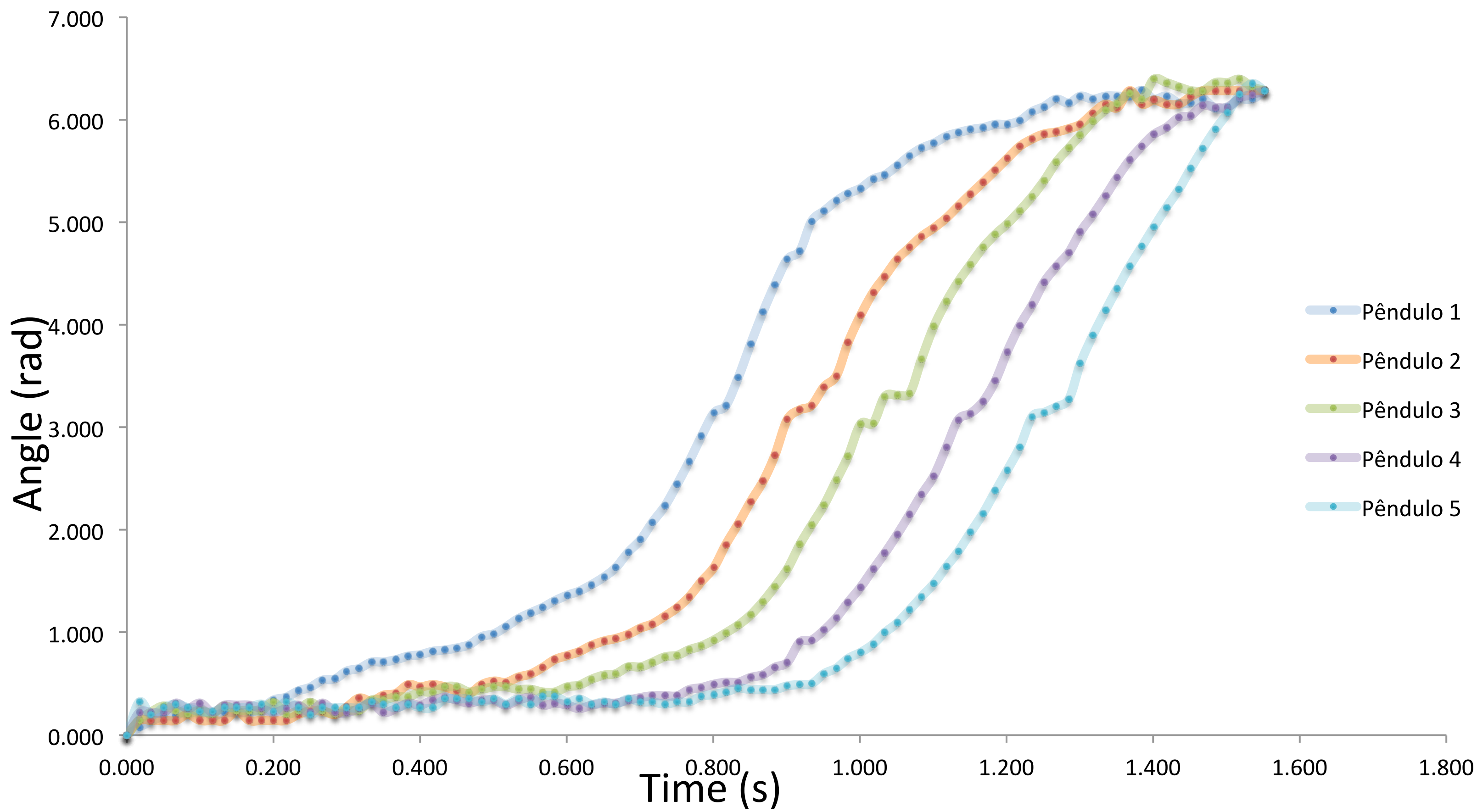


- Angular velocity
- Pendula mass
- Elastic properties
- Pendula size
- String pendula
- **Spring solitons**

- Spring connecting the pendulums
- 8 pendulums
- Distance of 3 cm between two pendulums
- Mass of 30 g
- Many turns because of the low friction



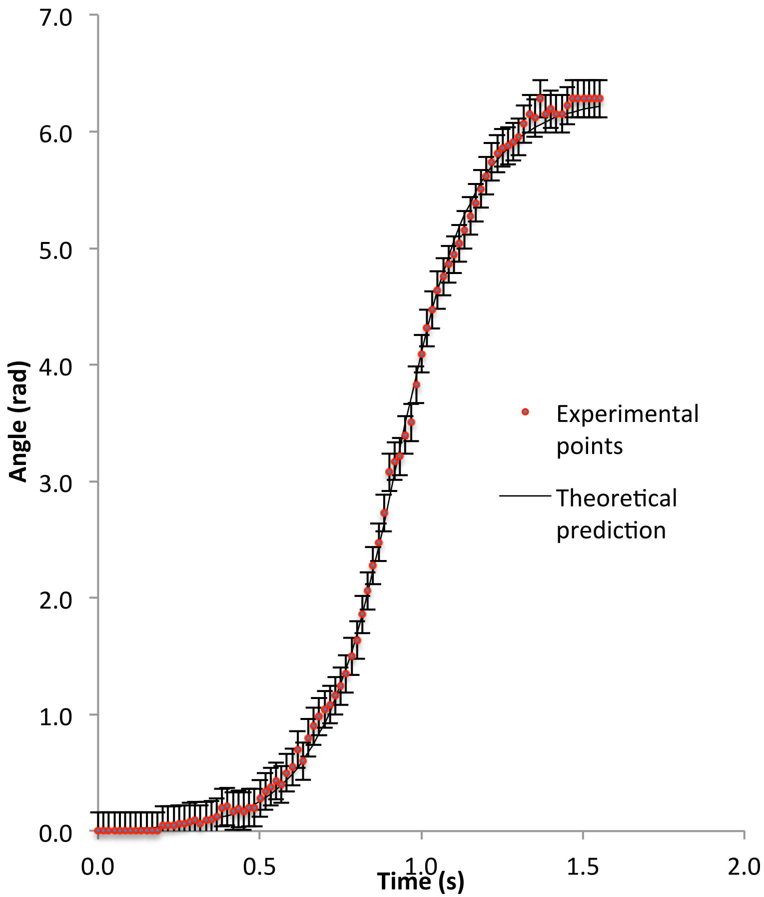
- Angular velocity
- Pendula mass
- Elastic properties
- Pendula size
- String pendula
- Spring solitons



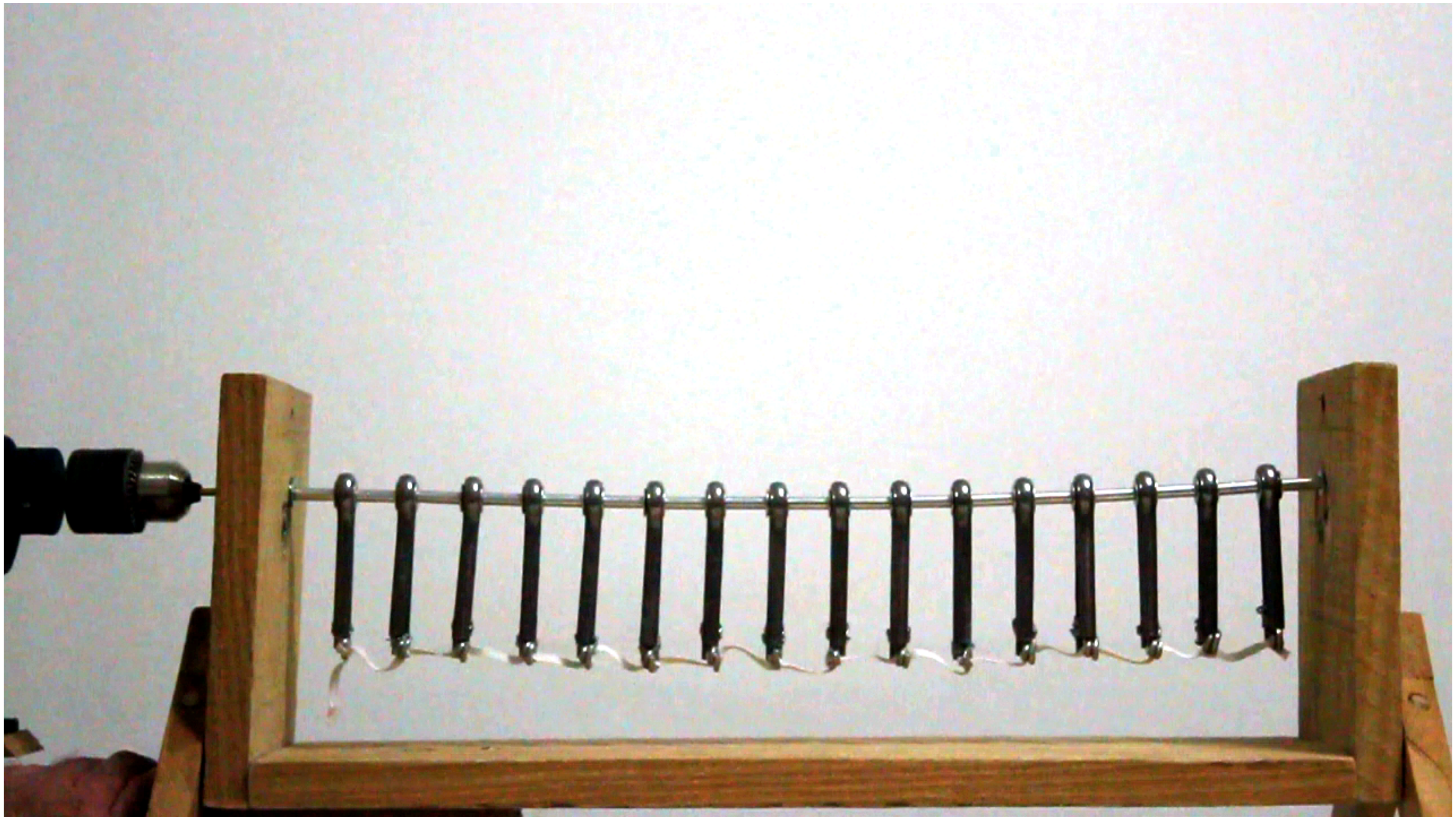
- Angular velocity
- Pendula mass
- Elastic properties
- Pendula size
- String pendula
- Spring solitons

$$Linear\ velocity = 0.34\ m\ s^{-1}$$

$$\psi = 4Arctan \exp[6.5\ t - 2\pi]$$



Infinite motion



Conclusion

- Sine-Gordon explains our problem well
- We can predict the wave form
- The angle goes according to our equations
- We have many kinds of analog solitons, like the spring ones, the string ones, quantum solitons and many others.

References

- Tristability in the pendula chain, Ramaz Khomeriki1,2, Jerome Leon
- Discrete breathers in a forced-damped array of coupled pendula: Modeling, Computation and Experiment, J. Cuevas, L.Q English, P.G. Kevrekidis, M. Anderson
- Falaco sólitons, Cosmic strings in a swimming pool, R. M. Kiehn
- Five lectures on soliton equations, Edward Frenkel
- Classical Dynamics, A contemporary approach, Jorge V., José, Eugene J., Saletan

Thank you!

Appendices

Lagrangians

- 1 particle: $L = T - V = \frac{m\dot{x}^2}{2} - \frac{kx^2}{2}$
- Euler-Lagrange: $\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = 0$
- MHS: $\ddot{x} + \frac{k}{m}x = 0$
- N particles, density of Lagrangian:

$$\mathcal{L} = \left(\dot{q}^2 - v_0^2\left(\frac{dq}{dx}\right)^2 - m^2q^2\right) \quad (1-D) \qquad \frac{\partial^2 q}{\partial t^2} - v_0^2\frac{\partial^2 q}{\partial x^2} + m^2 q = 0$$

- Klein-Gordon!

Klein-Gordon

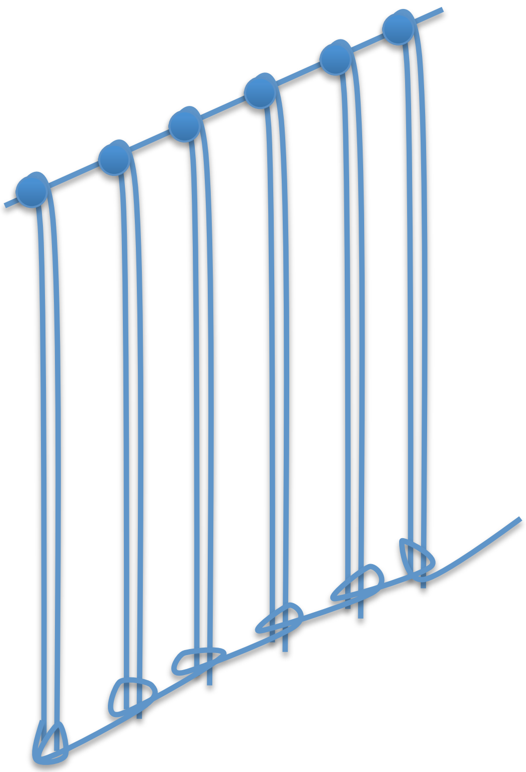
- Behaves like an wave equation
- Becomes a wave when m is zero
- Has double time and space derivatives

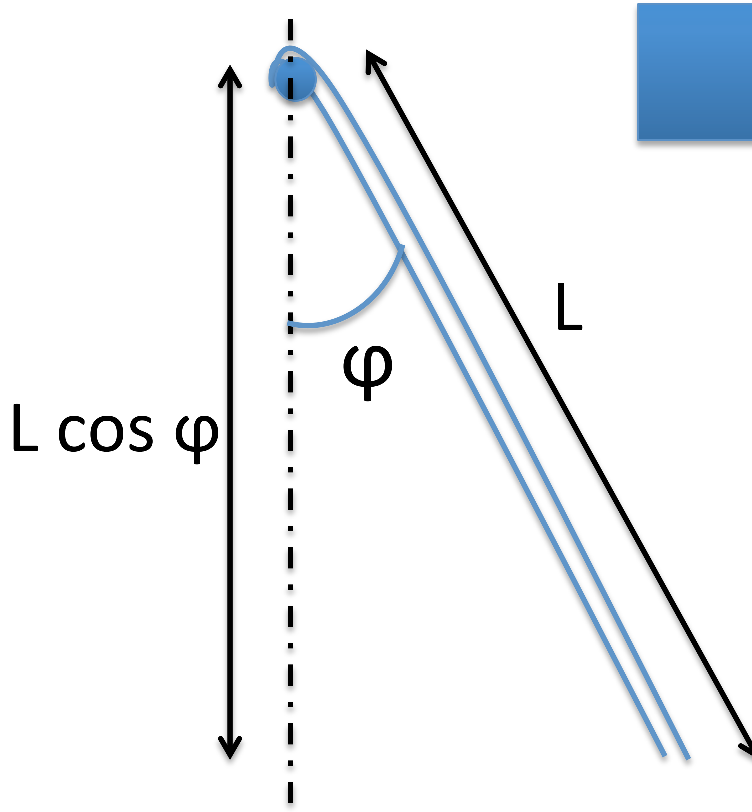
$$\frac{\partial^2}{\partial t^2}\psi - \mathrm{v}_0^2\nabla^2\psi + m^2\psi = 0$$

$$\frac{\partial^2}{\partial t^2}\psi - v_0^2\,\nabla^2\psi + m^2\psi = 0$$

Reasons

- Klein-Gordon works fine for small angles
- For 360° we have to take in count the gravitational potential
- We cannot approximate sin theta to theta
- We now have sine-Gordon!



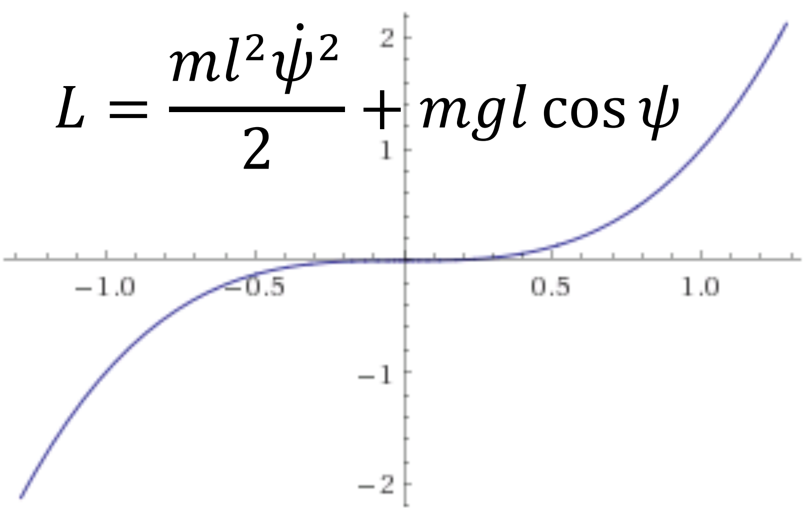


Potential energy

~Transition slide~

- The sine term implies in a potential energy with an even power, stable.
- Fits very well the expected phenomena.

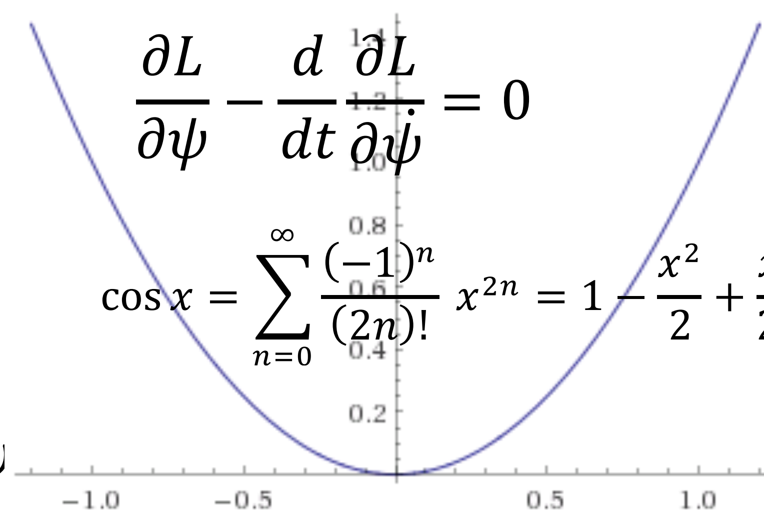
$$L = \frac{ml^2\dot{\psi}^2}{2} + mgl\cos\psi$$



$$\frac{\partial L}{\partial \psi} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\psi}} = 0$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \cdots$$

$$\cdots \quad \frac{\partial^2}{\partial x^2}\psi$$



Sine-Gordon

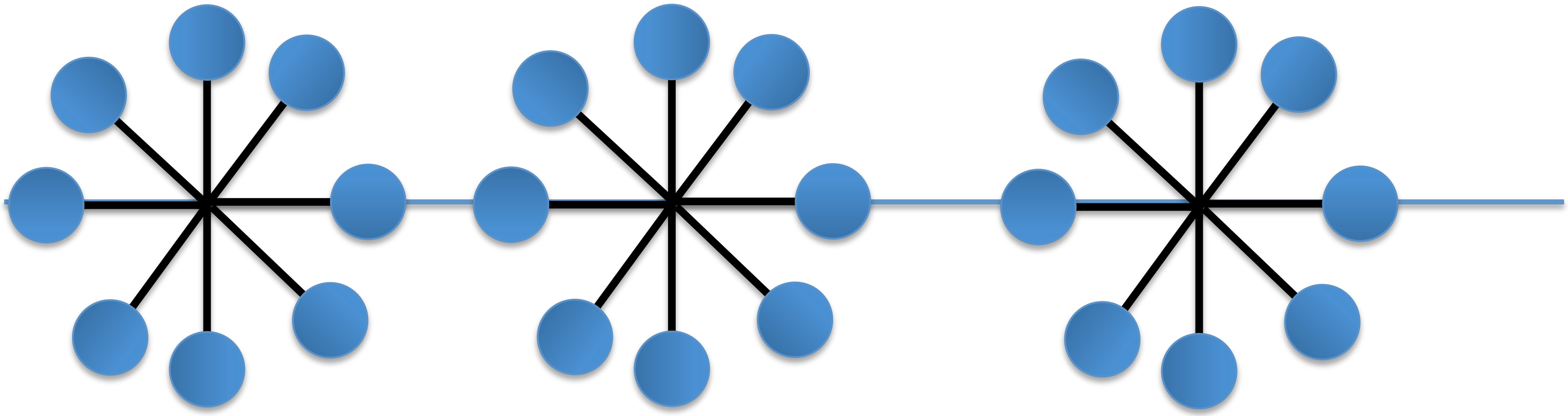
$$\varphi_{tt} - v_0^2 \varphi_{xx} + m^2 \sin\varphi = 0$$

- It's well known for the soliton solution
- Give us the wave form.

Solução

- Complicada, mas exata
- O ângulo entre os pêndulos segue a função

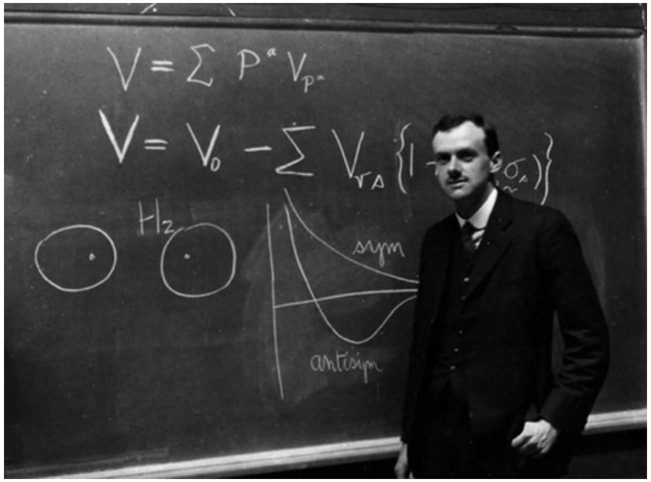
$$\psi(\xi) = 4 \arctan e^{\pm\gamma\xi}$$



Dirac equation

$$\left(\beta mc^2 + \sum_{k=1}^{3} \alpha_k p_k c\right)\psi(x,t) = i\hbar \frac{\partial \psi(x,t)}{\partial t}$$

In particle physics, the Dirac equation is a relativistic wave equation formulated by British physicist Paul Dirac in 1928. It describes fields corresponding to elementary spin-½ particles (such as the electron) as a vector of four complex numbers (a bispinor), in contrast to the Schrödinger equation which described a field of only one complex value.



Scalar and pseudo-scalar fields

- A field that is an invariant in respect to any Lorentz transformation is called scalar, in contrast to vectorial or tensorial fields. The quantization of the scalar filed is the zero spin, like bosons.
- No fundamental scalar field was observed in nature, even thought Higgs' boson can prove the fist example. However, scalar fields appear in the descriptions of many physical phenomena in effective field theory. One of the examples is the pion, that is a pseud-scalar, that means that's not an invariant in respect to a pair of transformations, that invert the spatial directions, distinguishing it from a real pair invariant.

Deduction

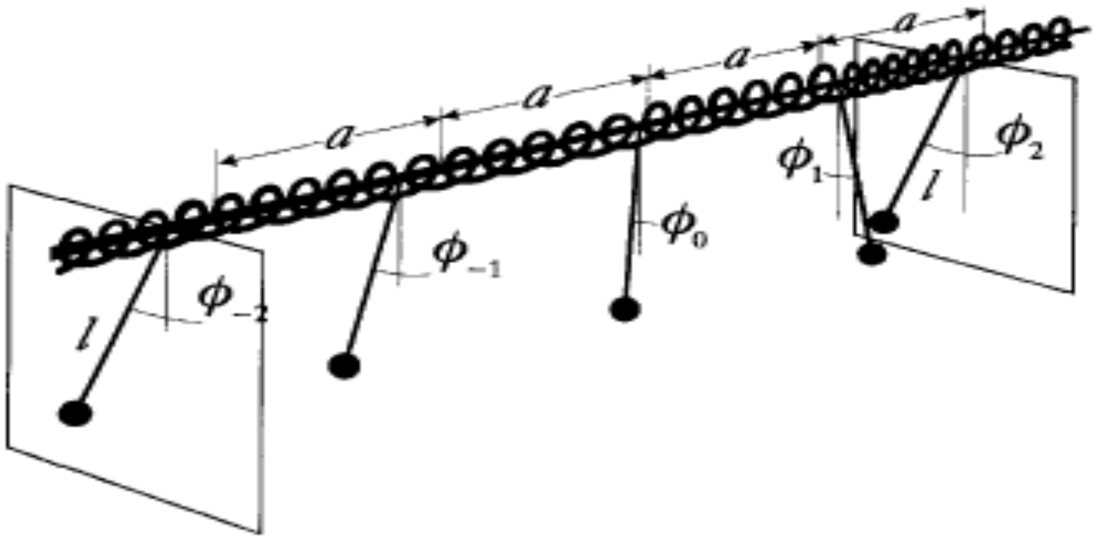


FIGURE 9.1
A chain of pendula coupled through springs. a is the separation between the equilibrium positions of the pendula. The deviation of the nth suspension point from its equilibrium positon is x_n.

At equilibrium, that is, when the pendula hang straight down (all the $\phi_j = 0$), the distance between the points of suspension is a. In the figure the points of suspension have moved in proportion to the angles ϕ_j and the springs are correspondingly stretched and compressed.

$$\overline{j=-n} \quad \Big(\quad j=-n \qquad\qquad j=-n \quad \Big)$$

where the first term is the kinetic energy and the second, in braces, is the potential, consisting of a gravitational and an elastic term. The constant k in the elastic term is the force constant of the springs (assumed to be the same all along the rod). The gravitational term is an *external* force on the system, whereas the elastic term is an *internal* force, the interaction between the particles. If $g = 0$, there is no external force and the system becomes essentially the one in Section 4.2.3. If $k = 0$, there is no elastic term, and the system becomes a chain of noninteracting pendula.

We write the Euler–Lagrange equations only for particles numbered $-(n-1) \leq j \leq n-1$. In the final analysis n will go to infinity, so neglecting the two end pendula will make no difference. After factoring out $m\lambda^2$, the EL equations are

$$\ddot{\phi}_j - \omega^2[(\phi_{j+1} - \phi_j) - (\phi_j - \phi_{j-1})] + \Omega^2 \sin\phi_j = 0, \quad -(n-1) \leq j \leq n-1, \tag{9.2}$$

where $\omega^2 = k\beta^2/m\lambda^2$ and $\Omega^2 = gl/\lambda^2$. This is a set of coupled second-order ordinary differential equations for the ϕ_j, something like those of Section 4.2.3 but now nonlinear (because of the sine term).

The continuum limit is obtained by replacing each pendulum of Eq. (9.1) by s others of the same length l and passing to the limit $s \to \infty$. The s new pendula have masses $\Delta m = m/s$ and they are distributed at displacements $\Delta x = a/s$ along the rod, so that the mass per unit length along the rod, that is, the linear mass density $\rho \equiv \Delta m/\Delta x = m/a$, is independent of s. Each of the reduced-mass pendula is labeled by the coordinate x along the rod of its point of suspension at equilibrium: ϕ_j is rewritten as $\phi(x)$. The spring remains the same, so if the constant is k for a length a, it is sk for a length a/s. Then Y, defined by $Y\lambda^2 \equiv ak\beta^2 = (a/s)(sk\beta^2)$, is also independent of s. The Lagrangian for the reduced-mass pendula is then

$$L = \frac{m}{2s}\lambda^2 \sum \dot{\phi}(x)^2 - \left\{ \frac{m}{s} gl \sum [1 - \cos\phi(x)] + \frac{1}{2} sk\beta^2 \sum [\phi(x + \Delta x) - \phi(x)]^2 \right\}$$

$$= \sum \Delta x \left\{ \sum \frac{1}{2}\rho\lambda^2\dot{\phi}^2(x) - \rho\Omega^2\lambda^2[1 - \cos\phi(x)] - \frac{1}{2}Y\lambda^2 \left(\frac{\phi(x+\Delta x) - \phi(x)}{\Delta x} \right)^2 \right\}. \quad (9.3)$$

After factoring out $\rho\lambda^2\Delta x \equiv (m/s)\lambda^2$, the corresponding EL equation becomes

$$\ddot{\phi}(x) - \frac{Y}{\rho} \frac{[\phi(x + \Delta x) - 2\phi(x) + \phi(x - \Delta x)]}{(\Delta x)^2} + \Omega^2 \sin\phi(x) = 0. \quad (9.4)$$

Because this result is independent of s, it is possible to pass to the $s \to \infty$ limit with no difficulty. In that limit $\Delta x \to 0$, and

$$\lim_{\Delta x \to 0} \left\{ \frac{[\phi(x+\Delta x) - 2\phi(x) + \phi(x-\Delta x)]}{(\Delta x)^2} \right\}$$
$$= \lim_{\Delta x \to 0} \left\{ \frac{1}{\Delta x} \left[\frac{\phi(x+\Delta x) - \phi(x)}{\Delta x} - \frac{\phi(x) - \phi(x-\Delta x)}{\Delta x} \right] \right\}$$
$$= \frac{\partial^2 \phi(x,t)}{\partial x^2},$$

where we write $\phi(x,t)$ since ϕ for each x depends on t. Hence the EL equation reads

$$\frac{\partial^2 \phi}{\partial t^2} - v^2 \frac{\partial^2 \phi}{\partial x^2} + \Omega^2 \sin \phi(x) = 0, \qquad (9.5)$$

with $v^2 = Y/\rho$. This is the *one-dimensional real sine–Gordon equation* (we will call it simply the sine–Gordon, or sG, equation). The function $\phi(x,t)$ is called the *wave function* or *field function* or just the *field*.

In the $s \to \infty$ limit the equation (9.3) for the Lagrangian becomes an integral:

$$L = \int \rho\lambda^2 \left\{ \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}v^2\left(\frac{\partial\phi}{\partial x}\right)^2 - \Omega^2(1 - \cos\phi) \right\} dx . \qquad (9.6)$$

The integrand

$$\mathcal{L}\left(\phi, \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial t}\right) \equiv \rho\lambda^2 \left\{ \frac{1}{2}\left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{1}{2}v^2\left(\frac{\partial \phi}{\partial x}\right)^2 - \Omega^2(1 - \cos\phi) \right\} \qquad (9.7)$$

is called the *Lagrangian density* for the sG equation. Then $L = \int \mathcal{L}\, dx$.

Solutions of the sG equation will be discussed in Section 9.4.

To find the first such solution, try to solve (9.102) with a function of the form (this is called making an *ansatz*)

$$\phi(x,t) = \psi(x - ut) \equiv \psi(\xi), \tag{9.103}$$

where u is some constant velocity (because x and t are dimensionless, so is u, which is now measured in units of v). The first question is whether such solutions can be found and, if they can, for what values of u.

With this ansatz (9.102) becomes

$$(1 - u^2)\psi''(\xi) = \sin\psi(\xi), \tag{9.104}$$

$$(1-u^2)\psi''(\xi) = \sin\psi(\xi), \tag{9.104}$$

where the prime denotes differentiation with respect to ξ. Before obtaining the soliton solutions, we explain how they arise. Think of ψ as a generalized coordinate q in a one-freedom particle dynamical system and of ξ as the time. We will call this the *auxiliary* dynamical system. Then (9.104) can be obtained from the one-freedom auxiliary Lagrangian

$$L_\psi = \frac{1}{2}\psi'^2 - \gamma^2\cos\psi,$$

where $\gamma = (1-u^2)^{-1/2}$ is a Lorentz contraction factor that reflects the relativistic nature of the sG equation. The potential energy of the auxiliary system is $V_\psi(\psi) = \gamma^2\cos\psi$. discussed in Chapter 1 [around Eq. (1.49) and again in Section 1.5.1; Fig. 1.5 is essentially a graph of $-V_\psi$]. Recall that originally the sG equation was derived as the continuum limit of a one-dimensional chain of pendula connected by springs; $\psi \equiv \varphi = 0$ represents the pendula hanging straight down, and $\psi = 2\pi$ represents them also hanging straight down but having undergone one revolution about their support. If only one revolution is under consideration, ψ varies only from 0 to 2π. Within this range, the auxiliary system has unstable fixed points at $\psi = 0$ and $\psi = 2\pi$ and a stable one at $\psi = \pi$.

The energy first integral of the auxiliary system (not to be confused with the actual energy in the sG field) is

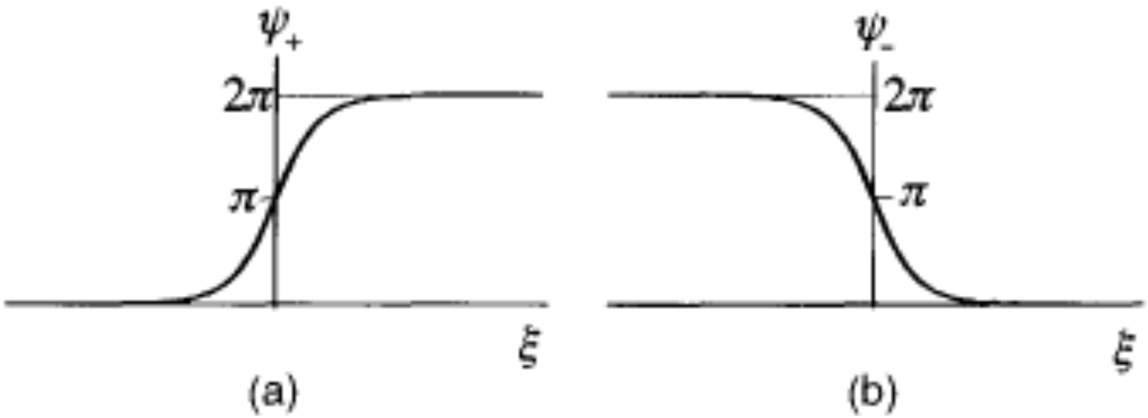
$$E_\psi \equiv \gamma^2 C = \frac{1}{2}\psi'^2 + \gamma^2 \cos\psi, \qquad (9.105)$$

$$\int_{\pi}^{\psi(\xi)} \frac{d\psi}{\sin \psi/2} \equiv 2 \ln(\tan \psi/4) = \pm 2\gamma\xi,$$

or

$$\psi(\xi) = 4 \arctan\{e^{\pm\gamma\xi}\}, \qquad (9.107)$$

which is what is actually plotted in Fig. 9.4. The solution is called a soliton if the positive sign is chosen in the exponent and an *antisoliton* if the negative sign is chosen. Solitons can be obtained for all values of u such that $u^2 < 1$.

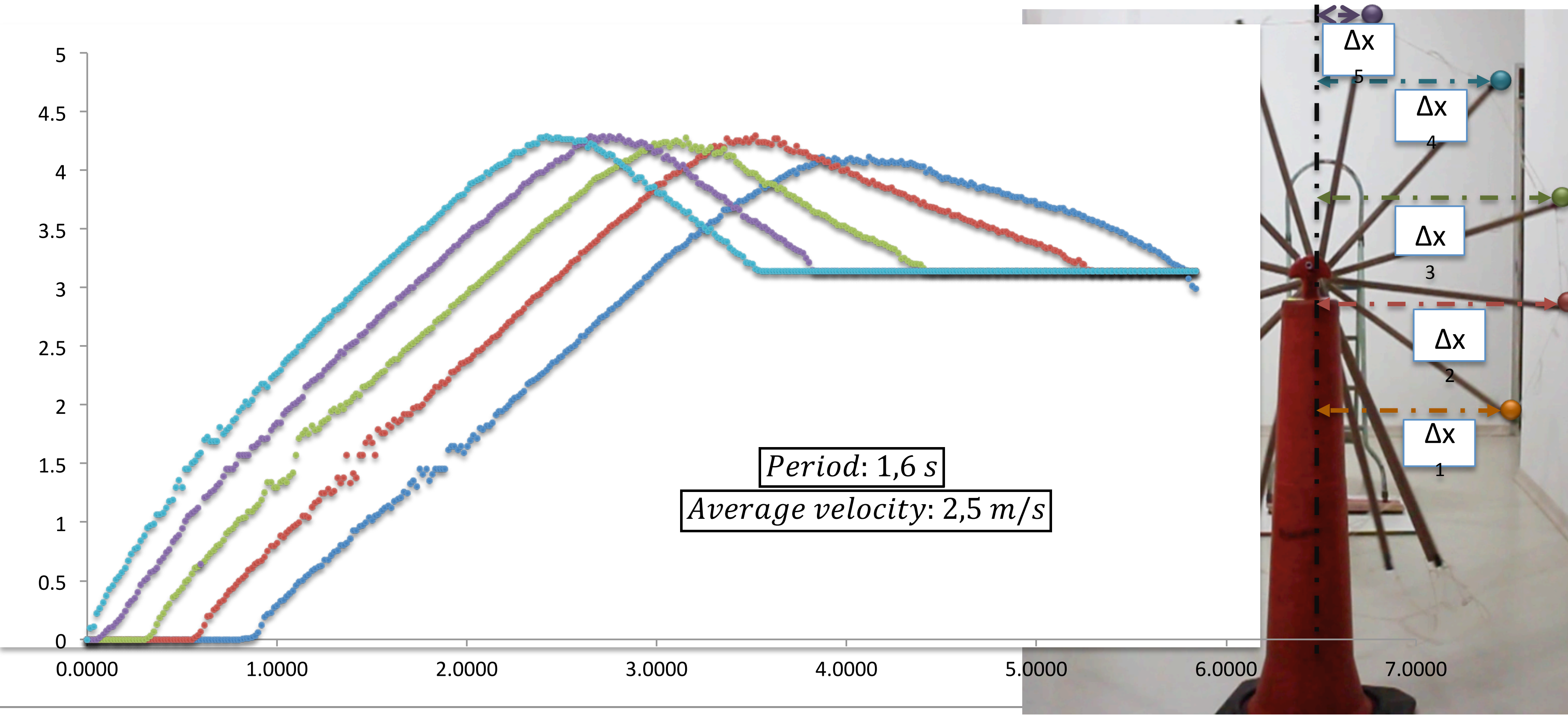


Rewritten in terms of x and t the solitons are

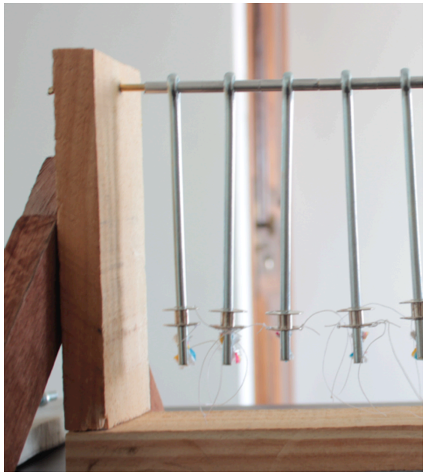
$$\phi(x,t) = 4\arctan\left\{e^{\pm\gamma(x-ut)}\right\}, \tag{9.108}$$

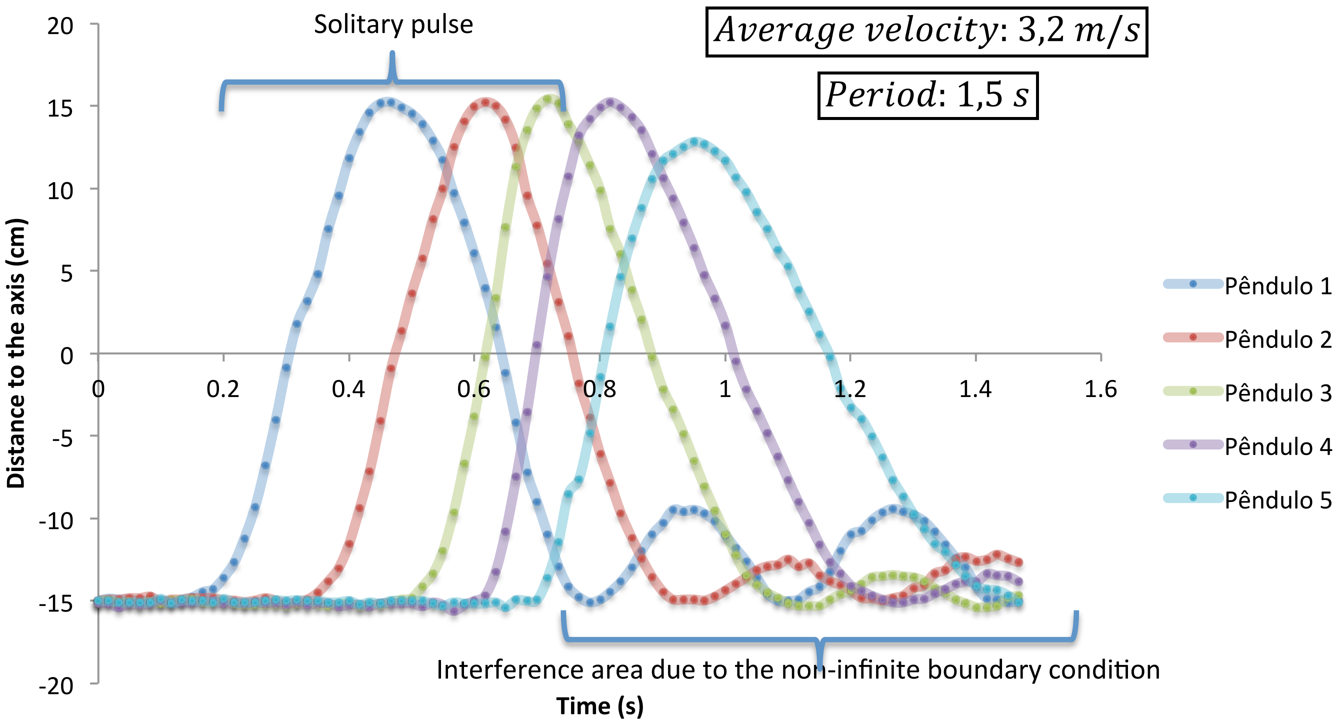
which means that the graph of Fig. 9.4(a) or (b) moves in the positive or negative x direction, depending on whether u is positive or negative. We emphasize that nonlinearity implies that multiples of these solutions are not themselves solutions; even $\arctan\{e^{\pm\gamma(x-ut)}\}$ is not a solution of the sG equation.

Big pendula

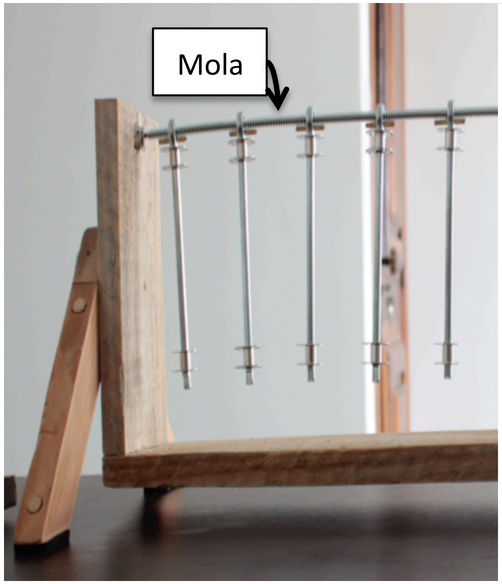

5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0.0000
1.0000
2.0000
3.0000
4.0000
5.0000
6.0000
7.0000
$Period$: 1,6 s
$Average\ velocity$: 2,5 m/s
Δx_5
Δx_4
Δx_3
Δx_2
Δx_1

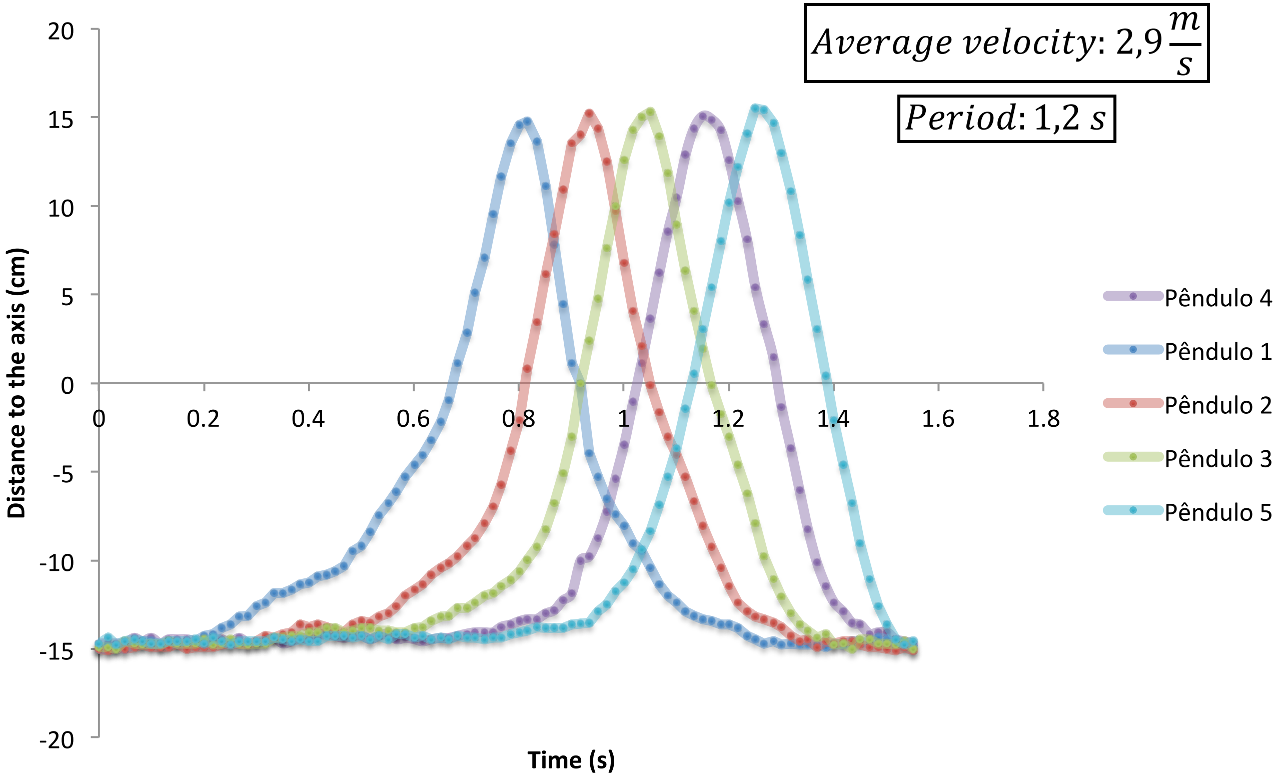
Small string pendula





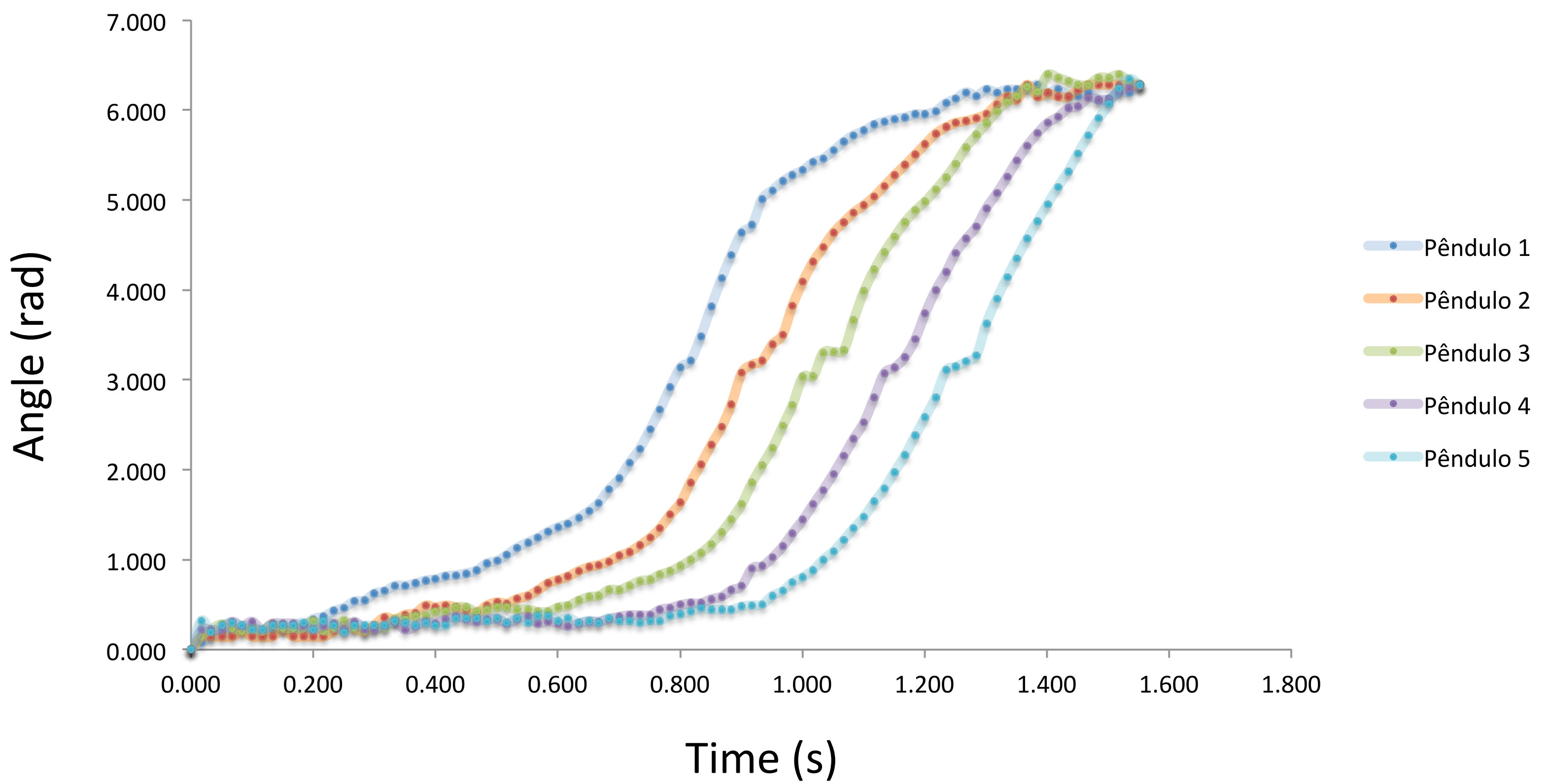
Spring pendula



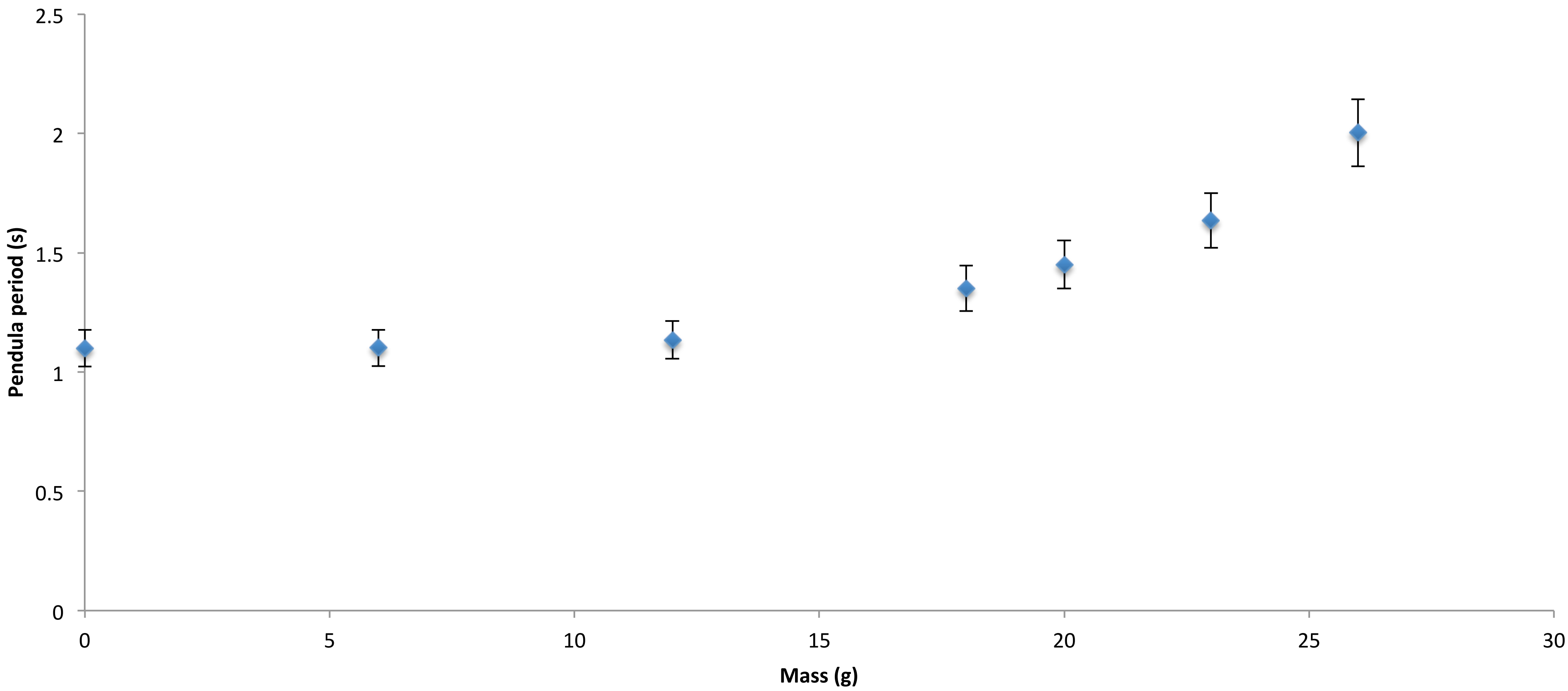


$Average\ velocity: 2{,}9\,\frac{m}{s}$

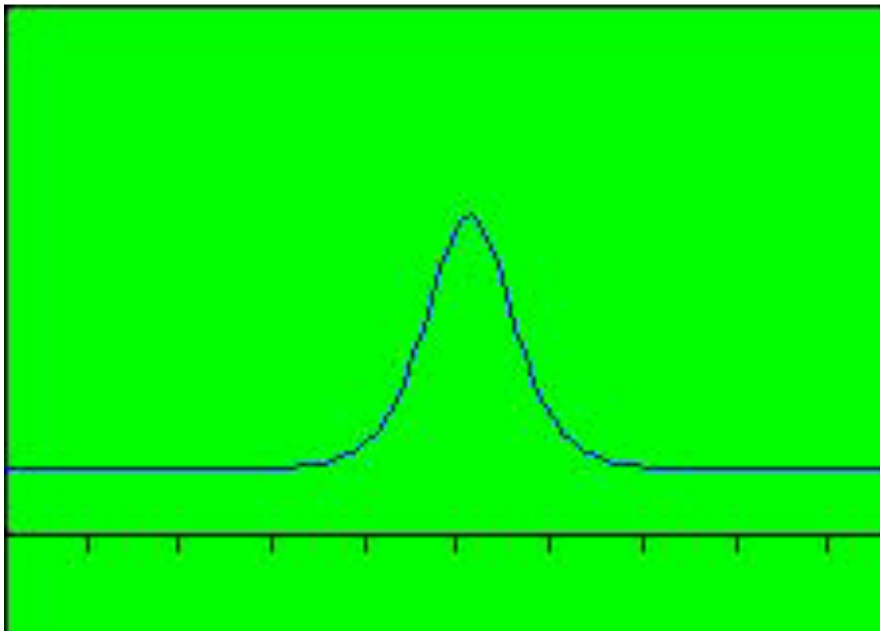
$Period: 1{,}2\ s$


7.000
6.000
5.000
4.000
3.000
2.000
1.000
0.000
Angle (rad)
0.000
0.200
0.400
0.600
0.800
1.000
1.200
1.400
1.600
1.800
Time (s)
Pêndulo 1
Pêndulo 2
Pêndulo 3
Pêndulo 4
Pêndulo 5

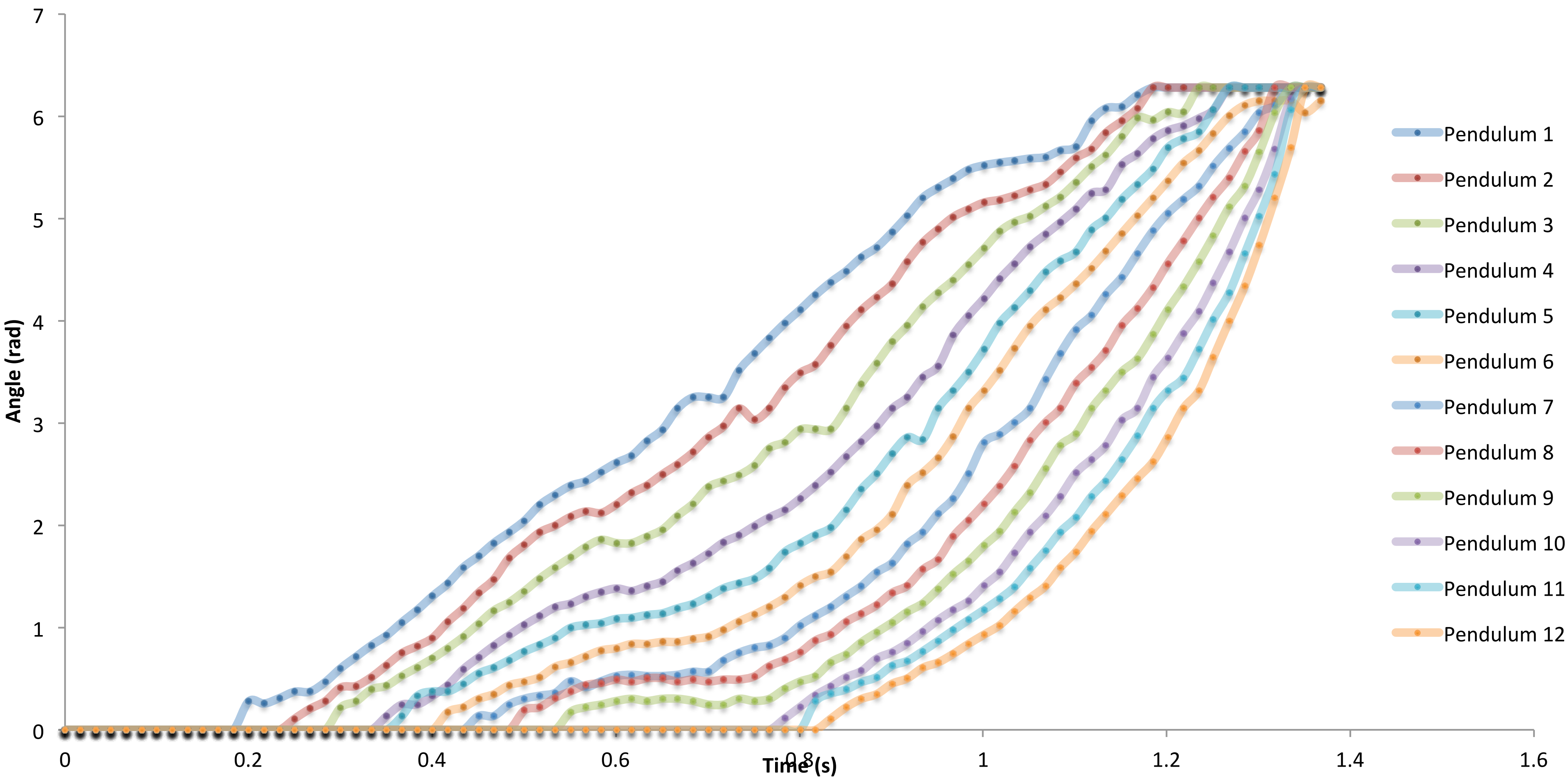
Mass variation



KdV

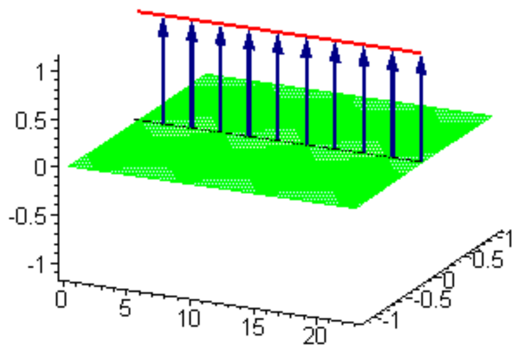


http://www.ma.hw.ac.uk/~chris/soliton1.mpg


Angle (rad)
Time (s)
0
1
2
3
4
5
6
7
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
Pendulum 1
Pendulum 2
Pendulum 3
Pendulum 4
Pendulum 5
Pendulum 6
Pendulum 7
Pendulum 8
Pendulum 9
Pendulum 10
Pendulum 11
Pendulum 12

Breathers

- Angular velocity
- Pendula mass
- Pendula size
- String pendula
- Spring solitons
- Breathers



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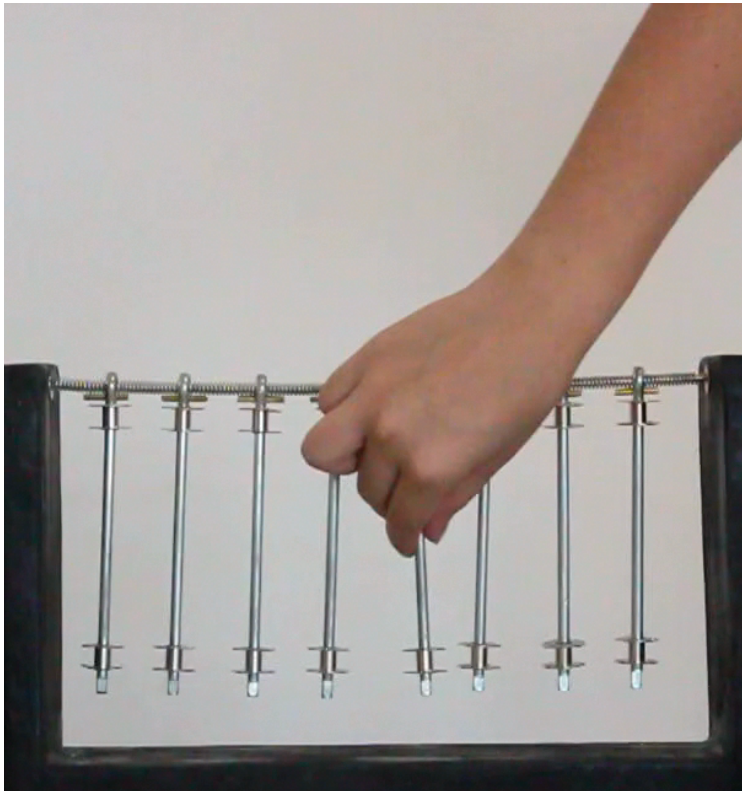


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Mass variation

- Angular velocity
- Pendula mass
- Pendula size
- String pendula
- Spring solitons

Mass of each pendula	Number of screws
55 g	0
60 g	1
65 g	2
70 g	3
75 g	4
80 g	5

Same angular velocity

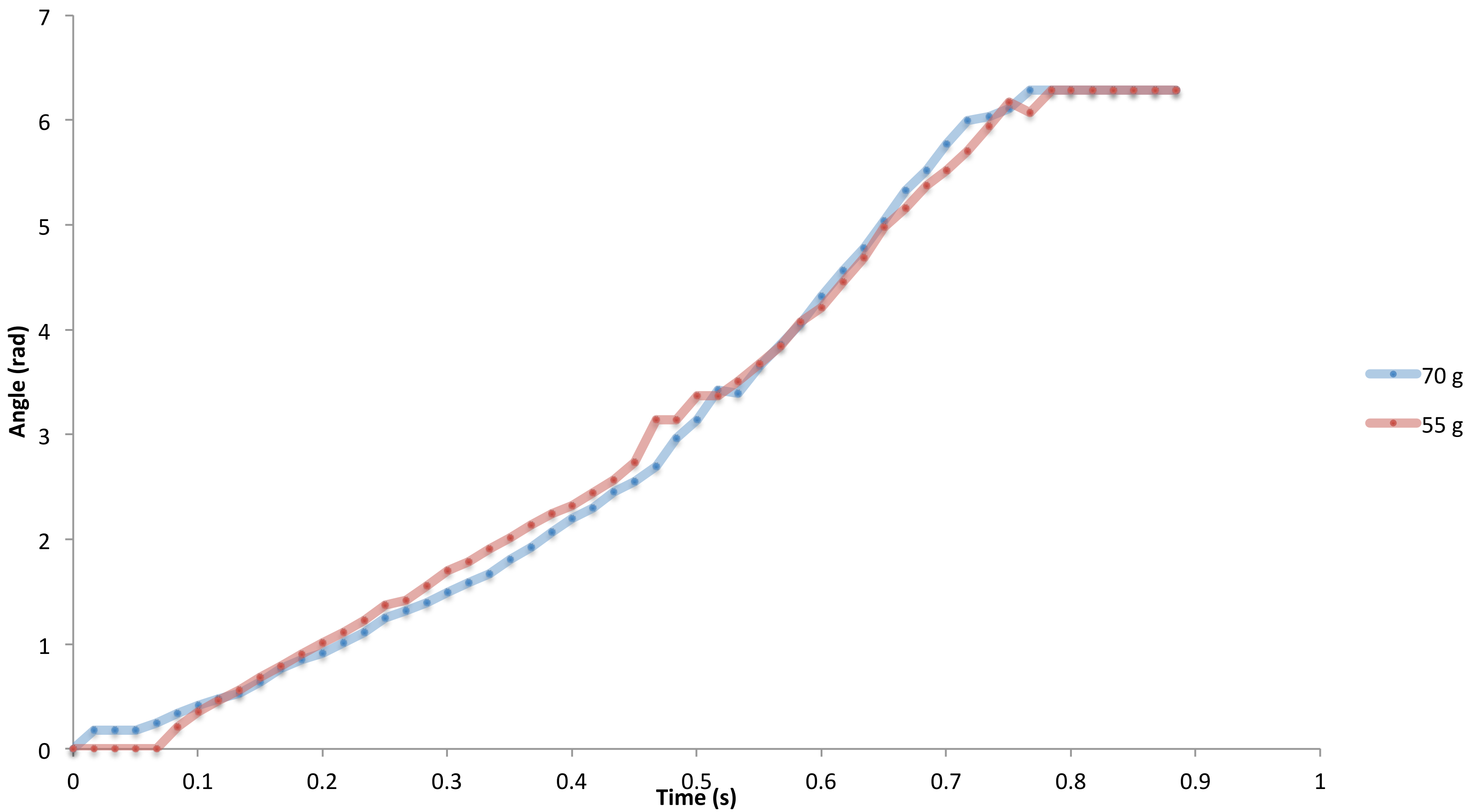
Angular velocity

Pendula mass

Pendula size

String pendula

Spring solitons



Different angular velocities

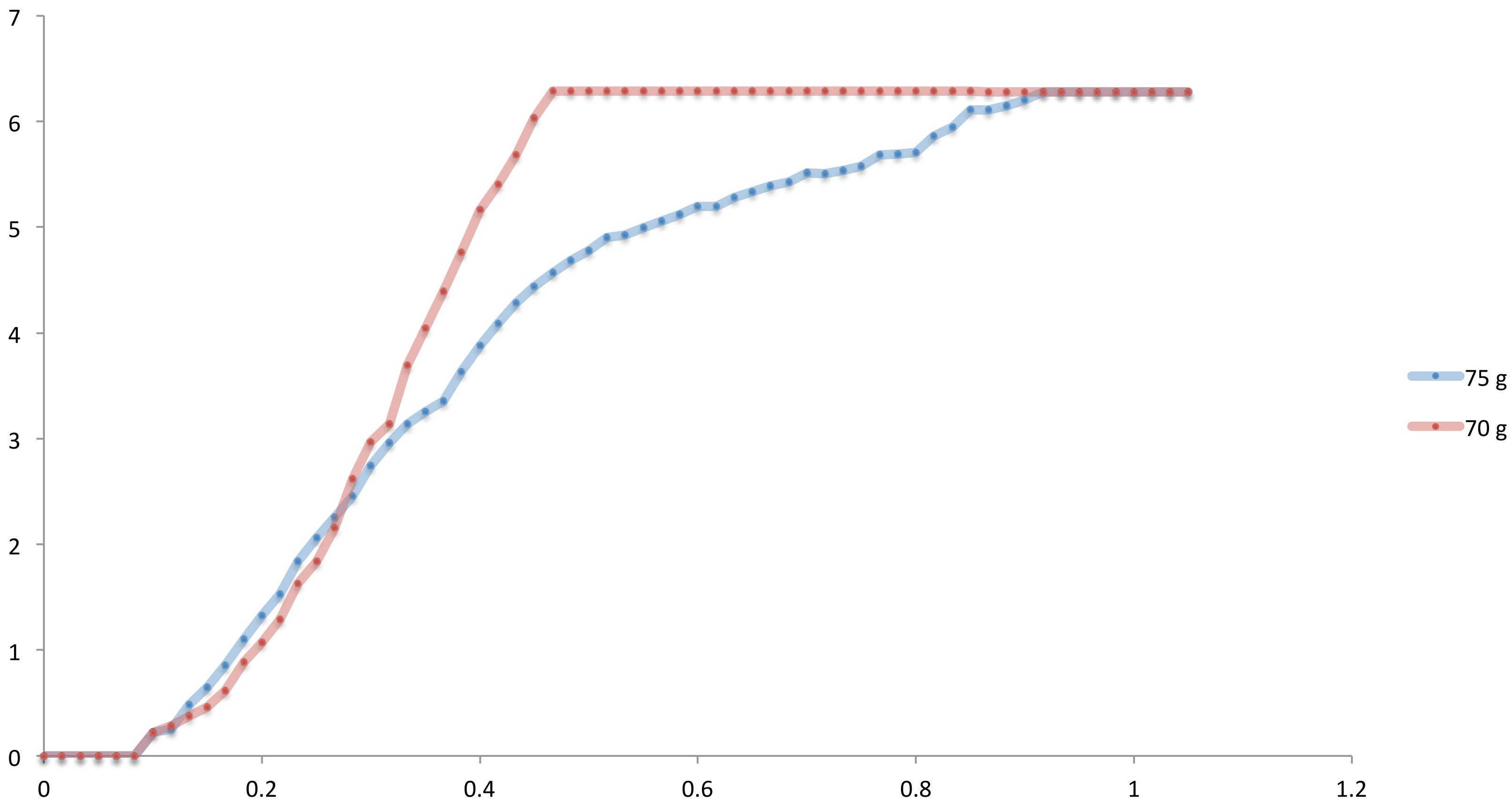
Angular velocity

Pendula mass

Pendula size

String pendula

Spring solitons



High precision scale