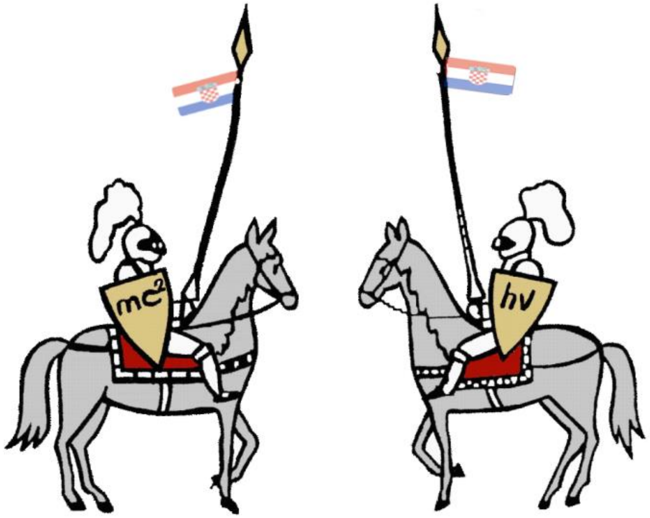
IYPT 2013

TEAM OF CROATIA

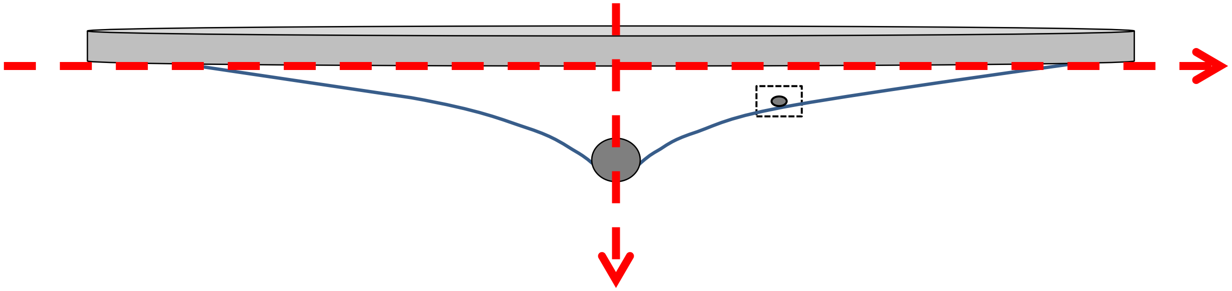
2. ELASTIC SPACE

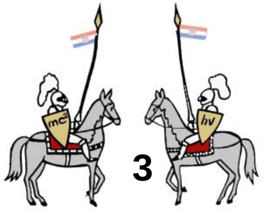
Reporter: Domagoj Pluščec



Problem

The dynamics and apparent interactions of massive balls rolling on a stretched horizontal membrane are often used to illustrate gravitation. Investigate the system further. Is it possible to define and measure the apparent “gravitational constant” in such a “world”?

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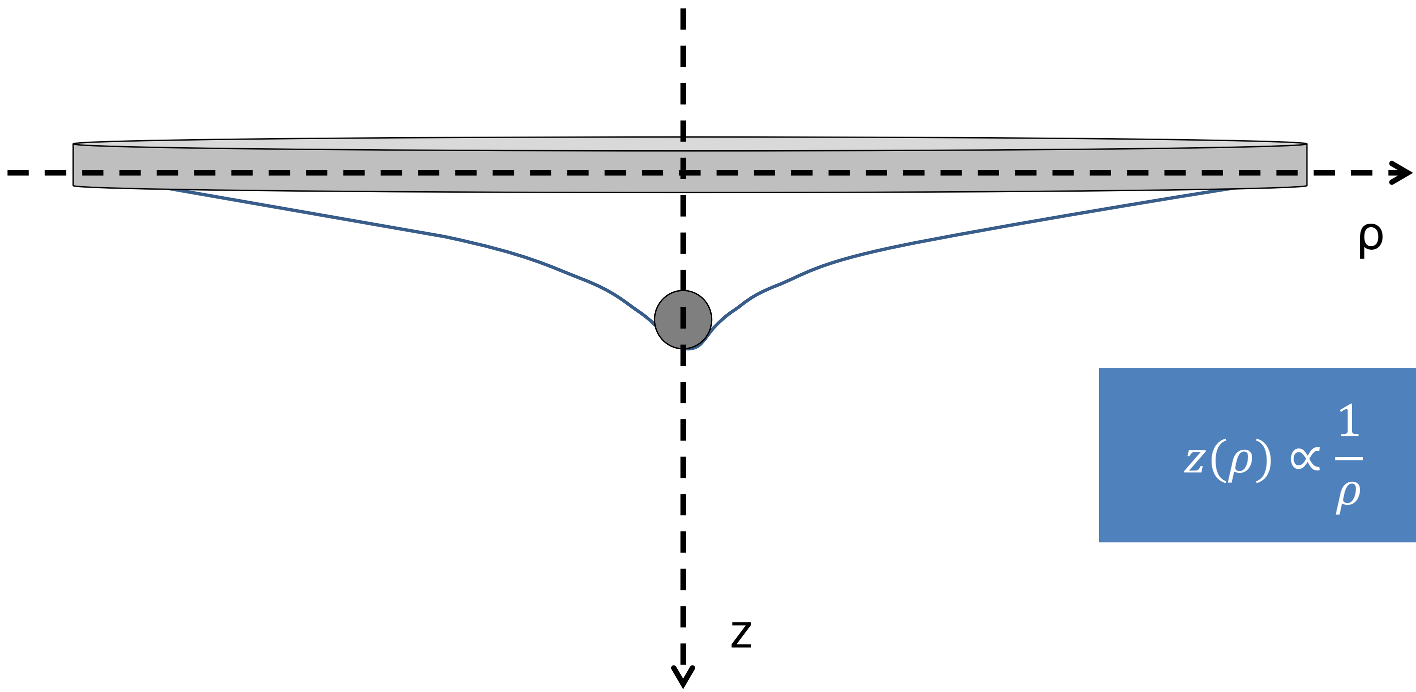
Summary

- Hypothesis
- Theory
  - Fabric shape
  - Test ball
  - Kepler’s laws
- Apparatus and methods
- Results
  - Fabric shape
  - Gravitational constant
  - Orbit
  - Energy and angular momentum
- Conclusion

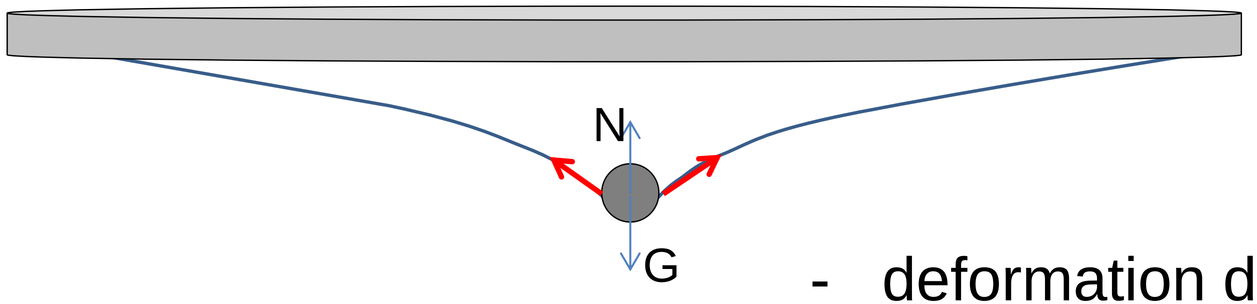
Hypothesis

It was assumed that the shape of the curve behaves like a gravitational potential and that the form should be inversely proportional to ρ



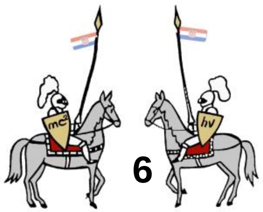
$$z(\rho) \propto \frac{1}{\rho}$$

Fabric shape and force on the ball



- deformation depends on:
  1. The coefficient of elasticity
  2. The mass of the ball
  3. The position of the ball on the fabric

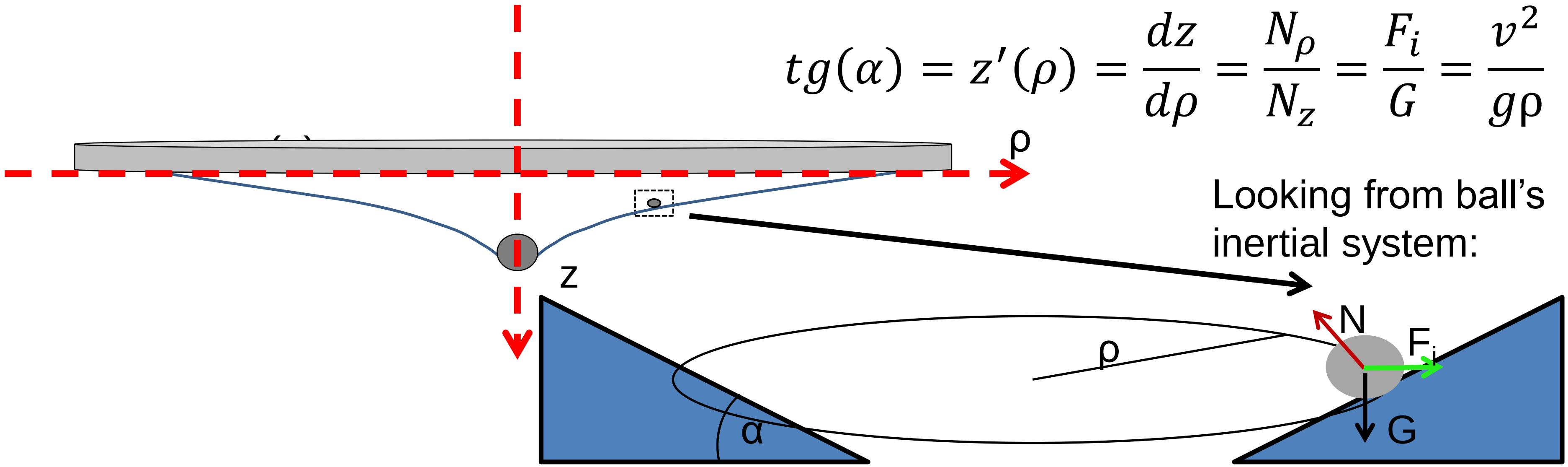
Shape can’t be calculated analyticaly because of big deformations
→ it was determined experimentaly



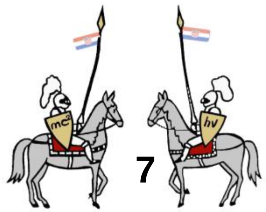
Test ball – circular orbit

Test ball – small mass → negligibile deformation

$$tg(\alpha) = z'(\rho) = \frac{dz}{d\rho} = \frac{N_\rho}{N_z} = \frac{F_i}{G} = \frac{v^2}{g\rho}$$



	Elastic membrane	Universe
Force	$F = -mg\frac{dz}{d\rho}$	$F = -m\frac{\gamma M}{\rho^2}$
Speed	$v = \sqrt{g\rho\frac{dz}{d\rho}}$	$v = \sqrt{\rho g}$



Test ball – energy

- Energy of the system:

$$E_{tot} = E_k + E_p + W_{fr}$$

Kinetic energy:

$$E_k = \frac{1}{2}(I_{cm}\omega^2 + mv^2) = \frac{7}{10}mv^2$$

Ball's moment of inertia:

$$I_{cm} = \frac{2}{5}mr^2$$

Potential energy:

$$E_p = mgz(\rho)$$

Approximation – friction is constant

$$\overrightarrow{F_{fr}} = -\mu mg\frac{\vec{v}}{v}$$

$$dW = F_{fr}ds = -\mu mg\frac{\vec{v}}{v}ds$$

$$dW = -\mu mg\frac{\vec{v}}{v}v(t')dt'$$

$$W_{fr}(t) = \mu mg\int_0^t v(t')dt'$$

Friction that does work on the ball is occuring when ball deformates the fabric when traveling on it (assumption μ is relatively small)

$$E = \frac{1}{2}\left(\frac{7}{5}mv^2\right) + \mathrm{mgz}(\rho) + \mu mg\int_0^t v(t')dt$$

Test ball – angular momentum

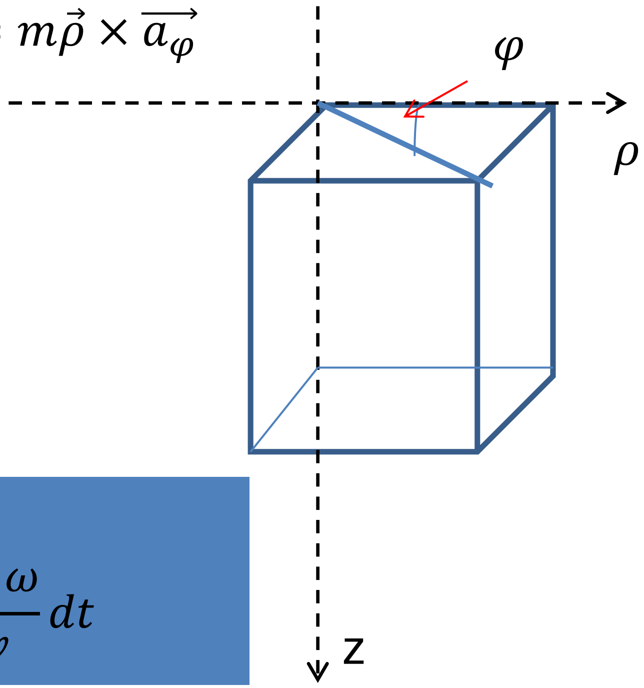
$$\vec{L} = m\vec{\rho} \times \vec{v} = m\vec{\rho} \times \overrightarrow{v_\rho} + m\vec{\rho} \times \overrightarrow{v_\varphi} = m\vec{\rho} \times \overrightarrow{v_\rho}$$

$$L = m\rho^2\omega$$

$$\frac{d\vec{L}}{dt} = m\vec{v} \times \vec{v} + m\vec{\rho} \times \vec{a} = m\vec{\rho} \times \overrightarrow{a_\rho} + m\vec{\rho} \times \overrightarrow{a_\varphi} = m\vec{\rho} \times \overrightarrow{a_\varphi}$$

$$\overrightarrow{a_\varphi} = \frac{\overrightarrow{F_{fr}}}{m} = -\mu g \frac{\vec{v}}{v}$$

$$\frac{d\vec{L}}{dt} = -\mu m g \frac{\omega\rho^2}{v}\vec{k} = -\mu g \frac{L}{v}\vec{k}$$



$$L(t) = L_0 + \Delta L(t)$$

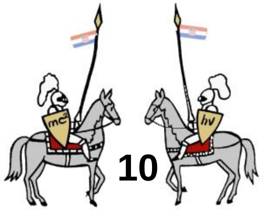
$$L(t) = m{\rho_0}^2\omega_0 - \mu m g \int_0^t \frac{\rho^2\omega}{v} dt$$

L – angular momentum
ω – angular velocity

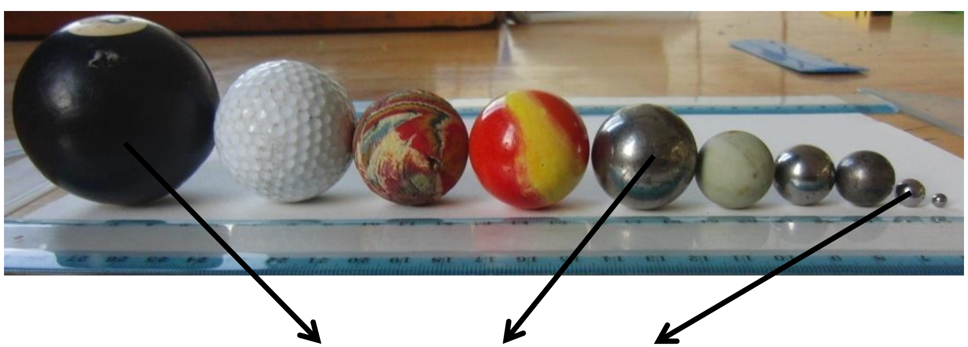
Kepler's laws

1. The orbit of every planet is an ellipse with the Sun at one of the two foci.
2. A line joining a planet and the Sun sweeps out equal areas during equal intervals of time. ($\vec{L} = const$)
3. The square of the orbital period of a planet is proportional to the cube of average distance from the sun.

Universe	Elastic membrane
$T^2 = \frac{4\pi^2\rho^3}{\gamma M}$	$T^2 = \frac{4\pi^2\rho}{g\frac{dz}{d\rho}}$
$\frac{dL}{dt} = 0$	$\frac{dL}{dt} = -\mu g \frac{L}{v}$



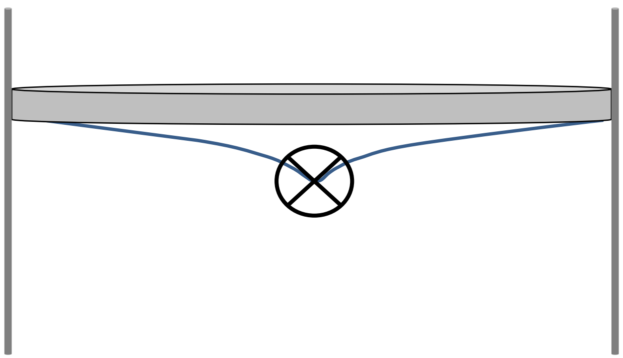
Apparatus

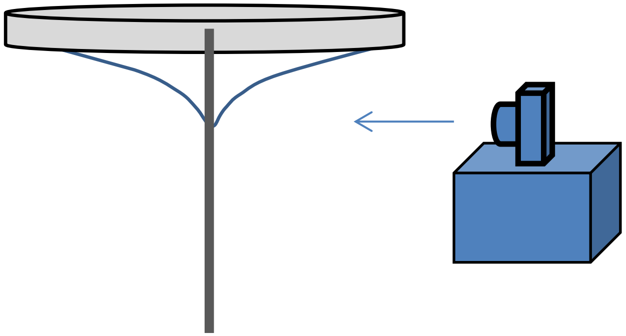


Balls that we used

- Bicycle wheel
  $(R = 32.2\ cm)$
- Elastic fabric
- Stands
- Tubular spirit level
- Meter
- Camera
- Balance
- Computer software (ImageJ, IrfanView, self-made programs for numerical analysis)

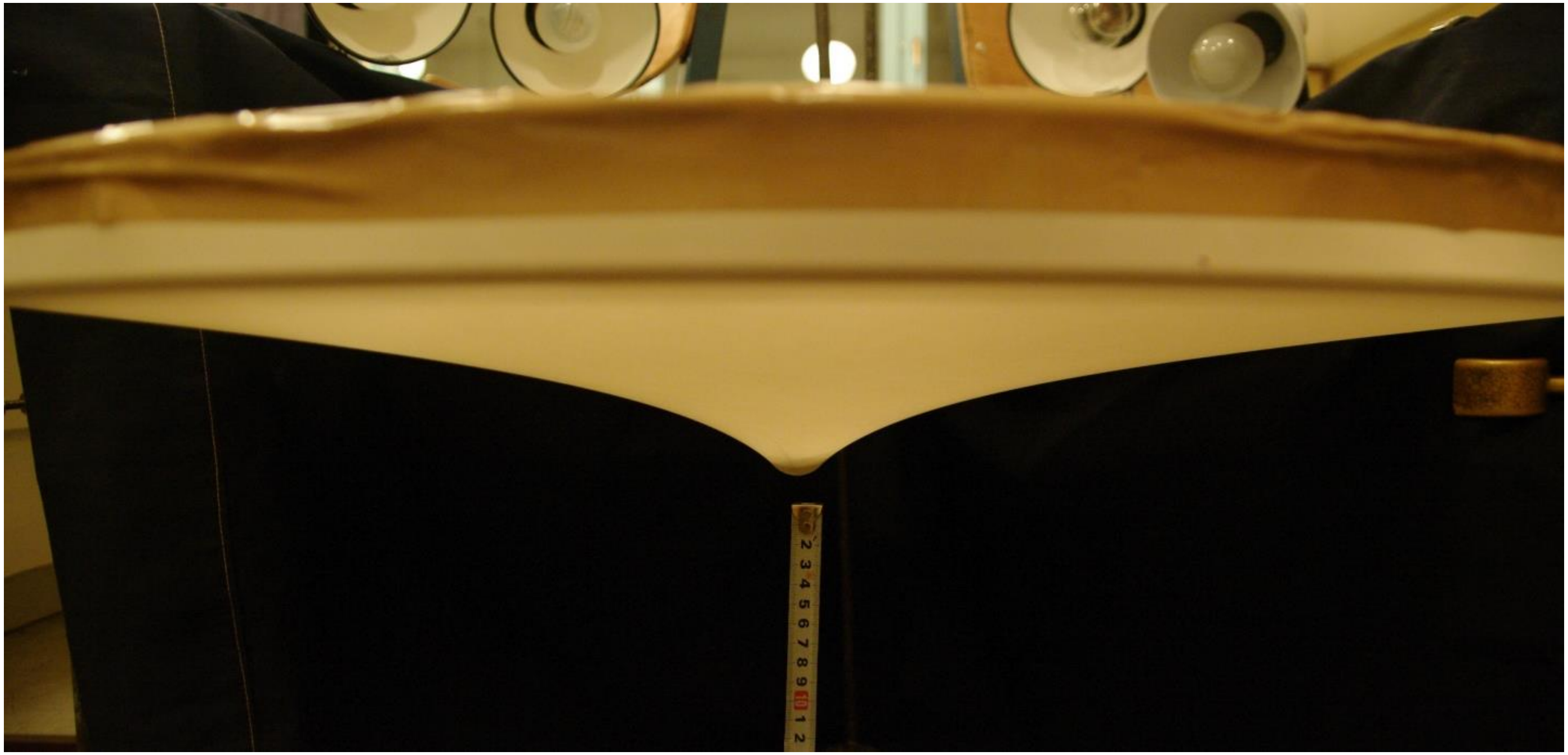
Method – fabric shape



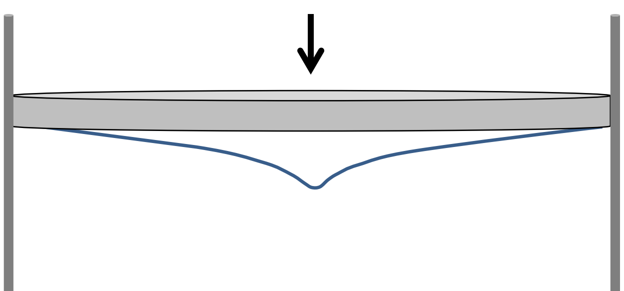


1) Massive ball was placed in the middle of the wheel
2) Camera was perpendicular to the lowest point of elastic membrane
3) Coordinates of the fabric ends were determined by computer program ImageJ

Example of shape’s photography

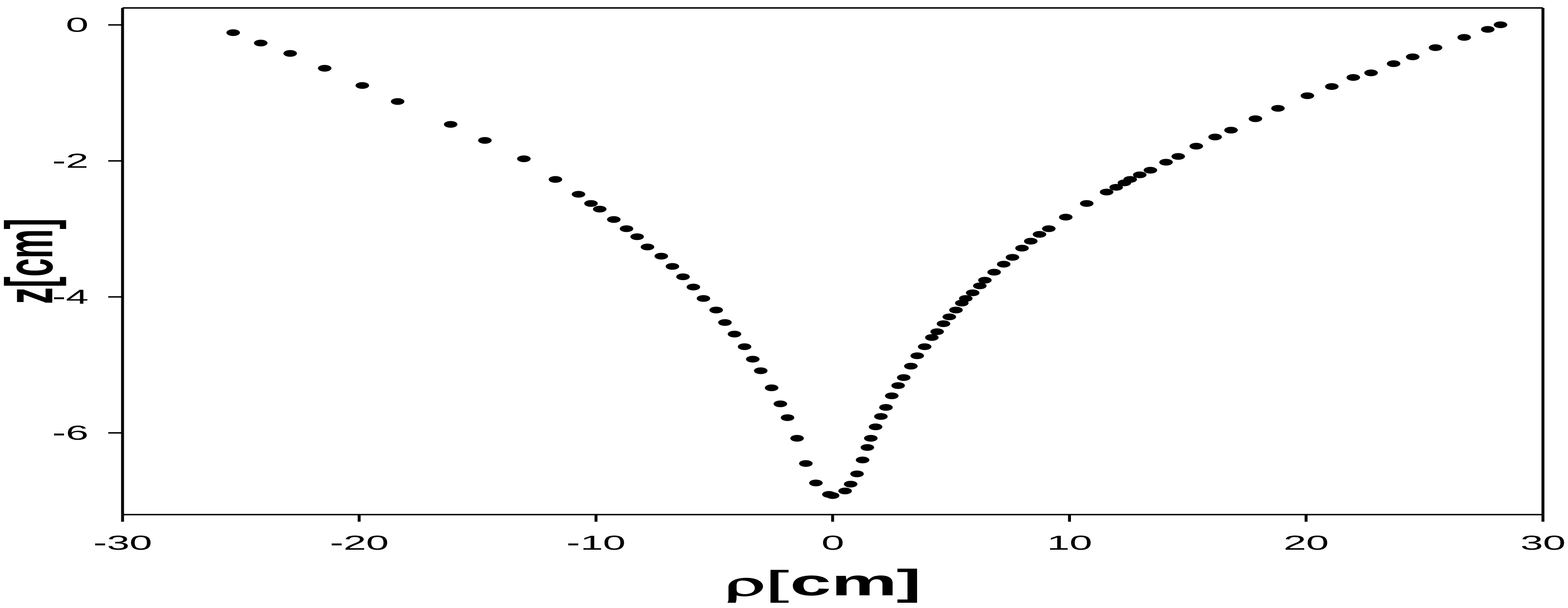


Method – motion of the balls



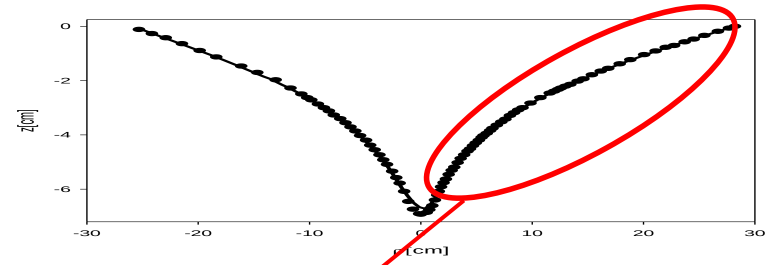
- Massive ball – center of the wheel
- Small ball ejected from the channel with initial speed
- Camera was perpendicular to the plane of the wheel
- Coordinates of the balls were obtained by video analysis (120-240)fps

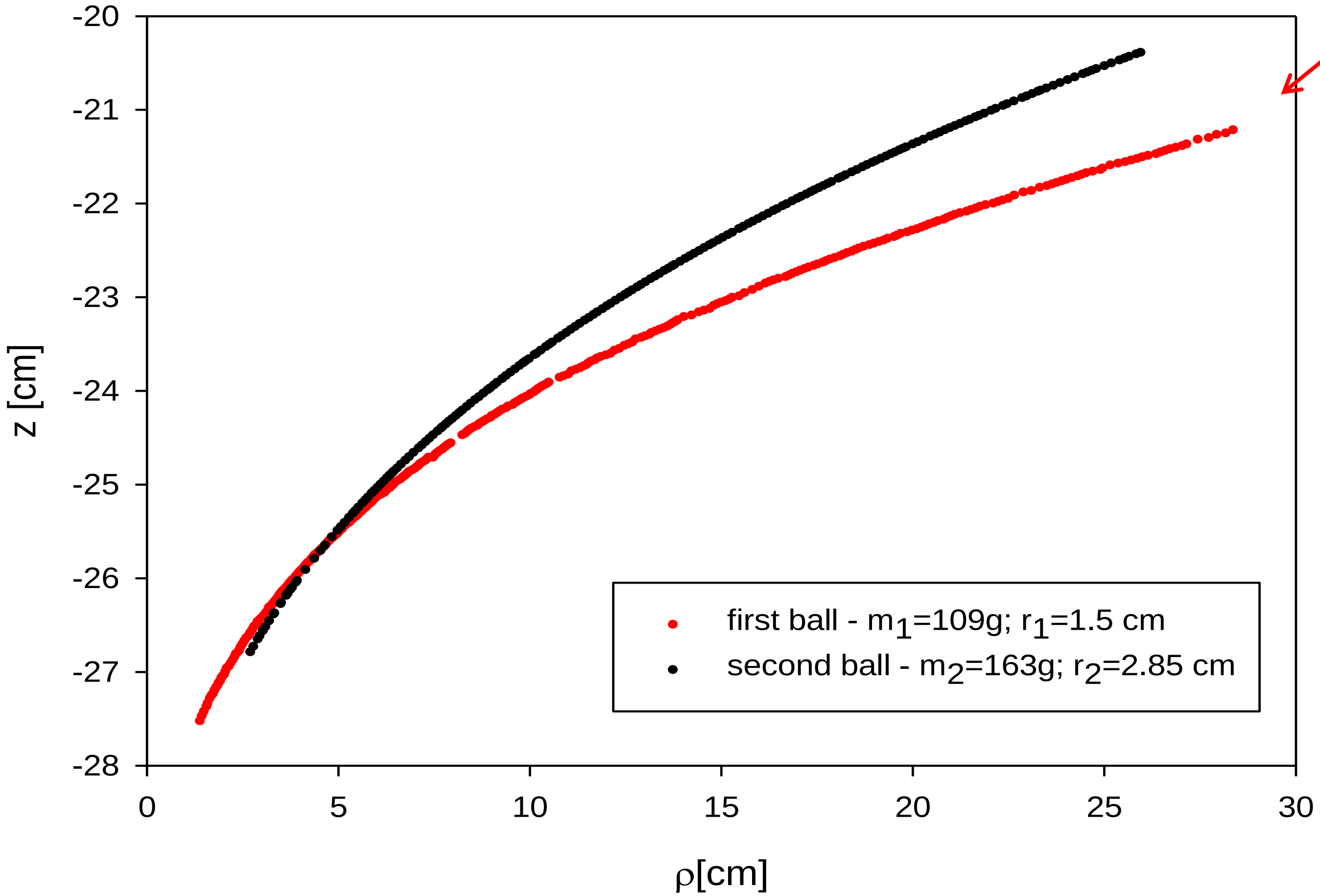
Fabric shape



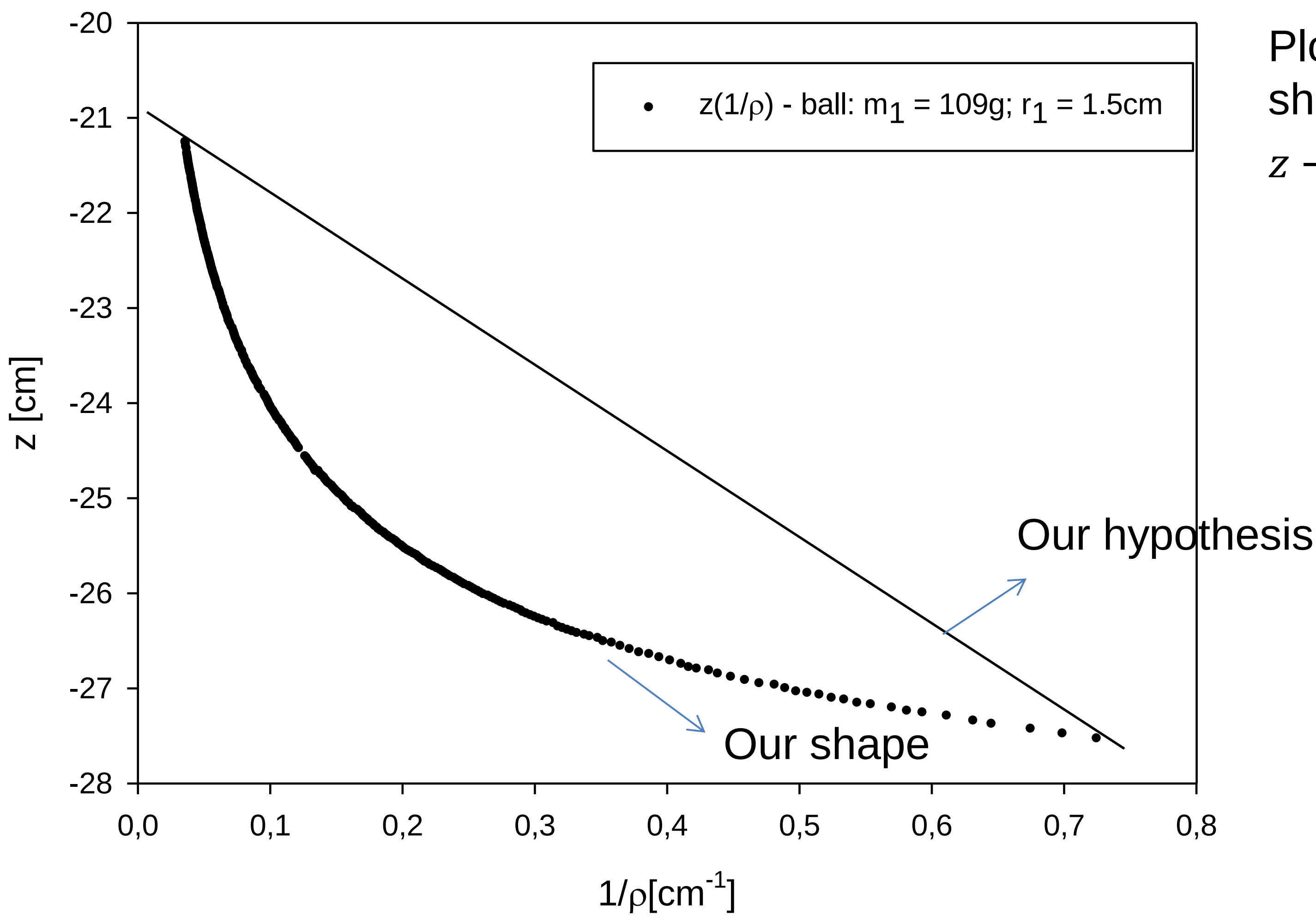
Ball: m_1=109g; r_1=1.5 cm

Fabric shape



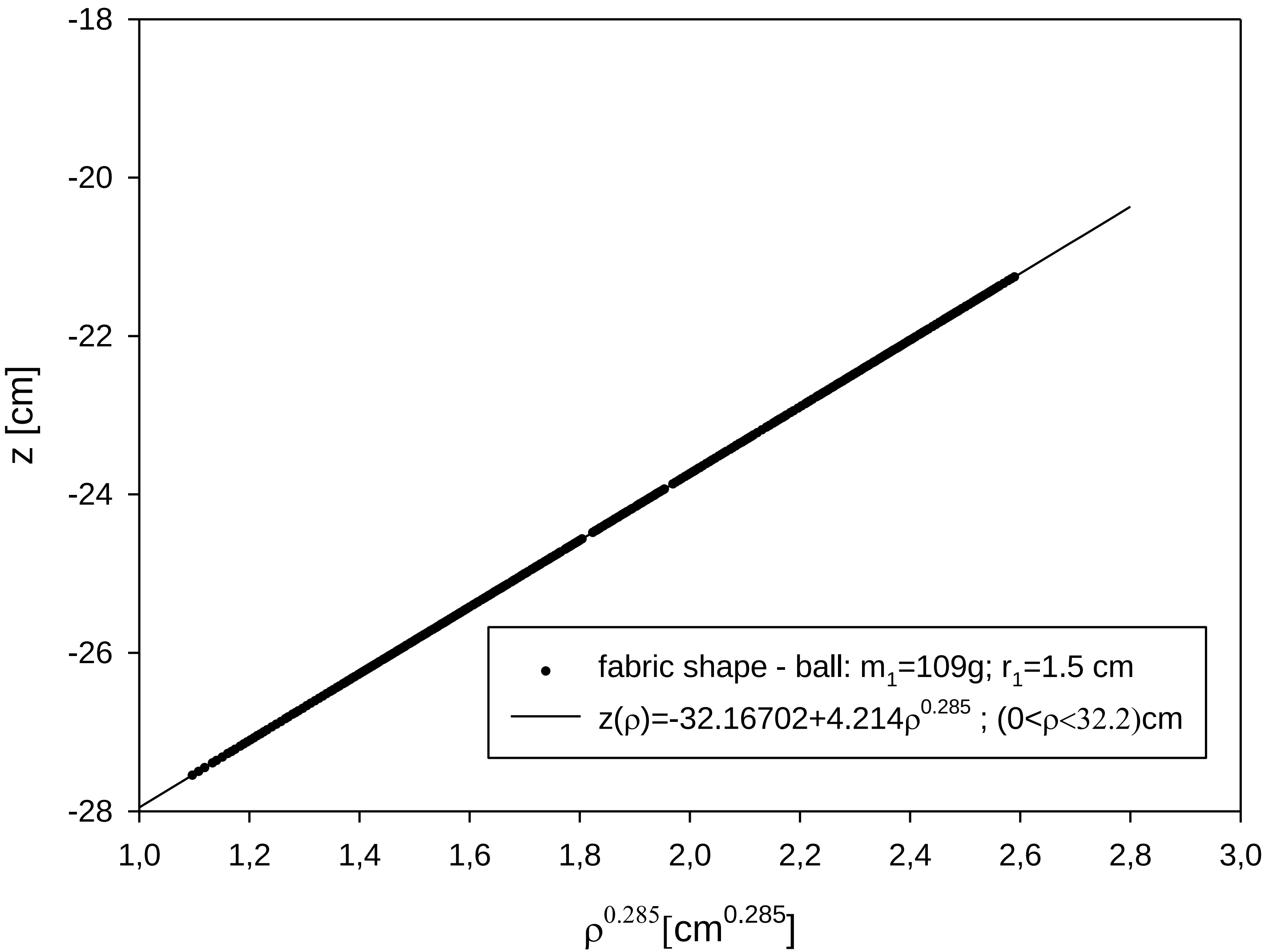


Test – is our potential proportional to Kepler's potential ?



Plot of our fabric shape dots on $z - \frac{1}{\rho}$ graph

Function of our shape



Function that describes fabric shape in form:

$$z(\rho) = A + B\rho^{p}$$

Numerically calculated A, B, p so that they best fit to measured data

„Gravitational constant”

Elastic membrane

$$z(\rho) = A + B\rho^{p}$$
$$U(\rho) = U_0 + mgB\rho^{p}$$

$$F = -\nabla U(\rho) = -mg\frac{dz}{d\rho} = -mg\frac{pB}{\rho^{1-p}}$$

→ constant in our system:

$$\Gamma = pgB$$

A B p

$$z(\rho) = -32.16702 + 4.214\rho^{0.285}$$

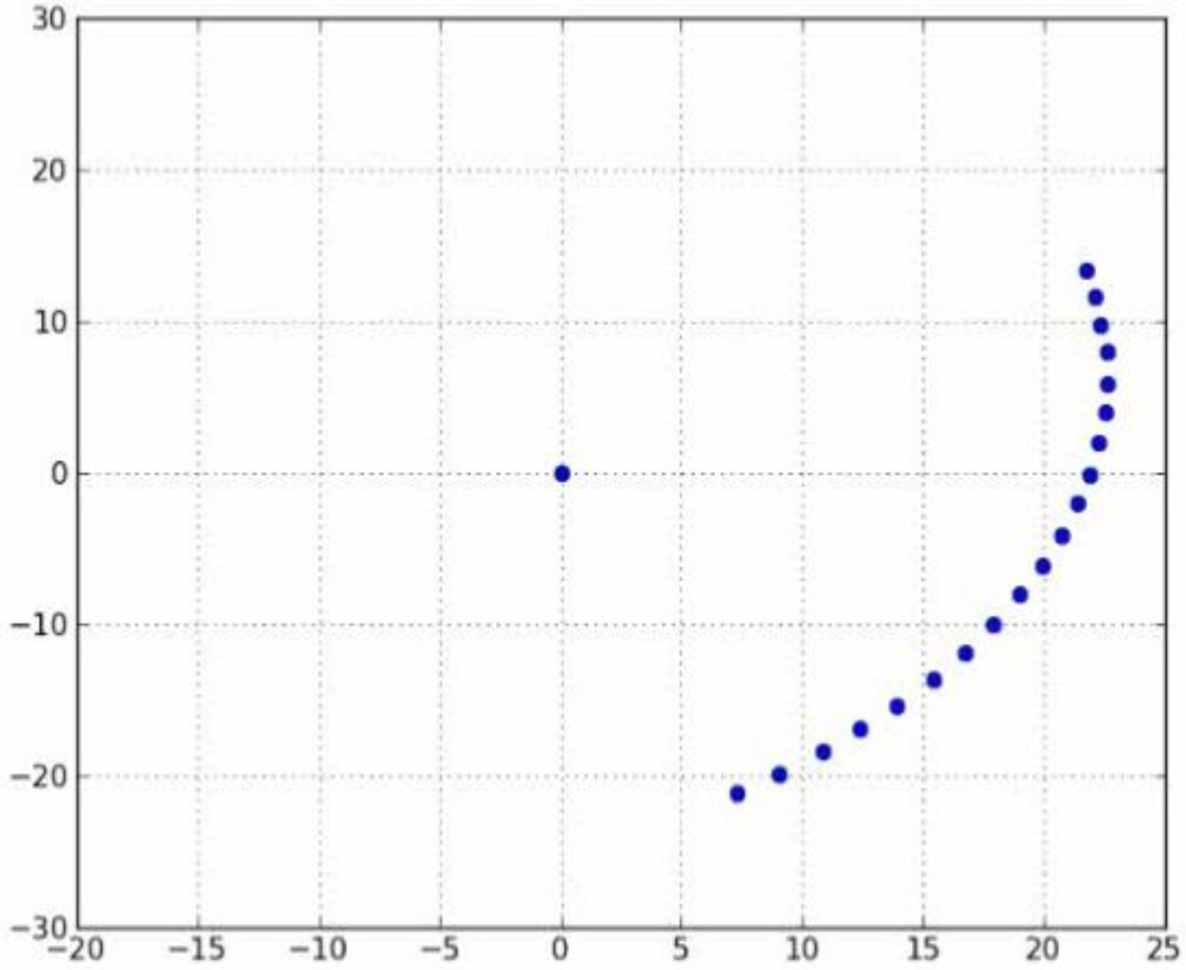
$$\Gamma = 11.782\, m^{1.715} s^{-2}$$

Universe

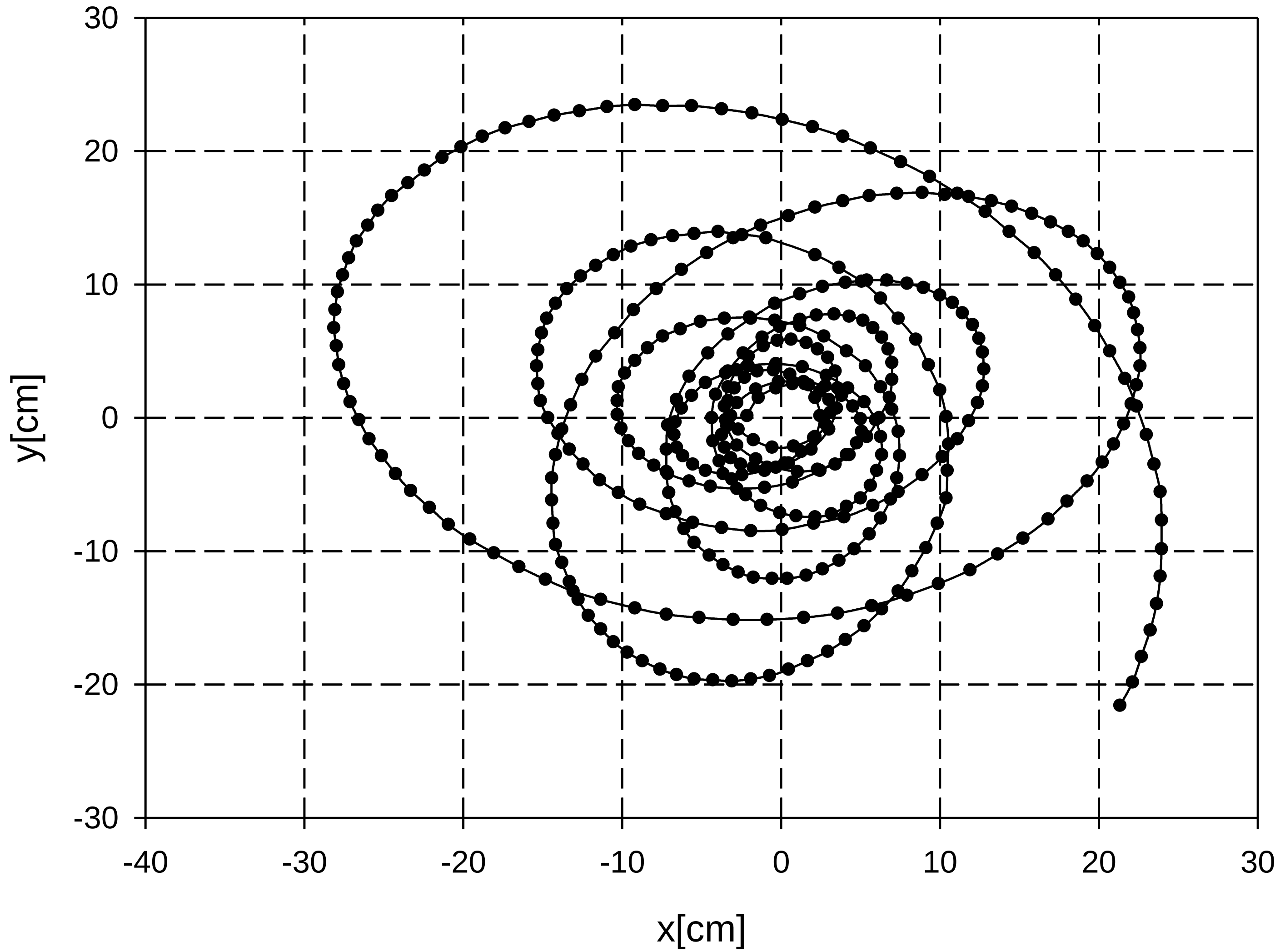
$$U(\rho) = \frac{\gamma M}{\rho}$$
$$F = -\gamma\frac{mM}{\rho^{2}}$$

$$\gamma = 6.6742 * 10^{-11} m^{3} kg^{-1} s^{-2}$$

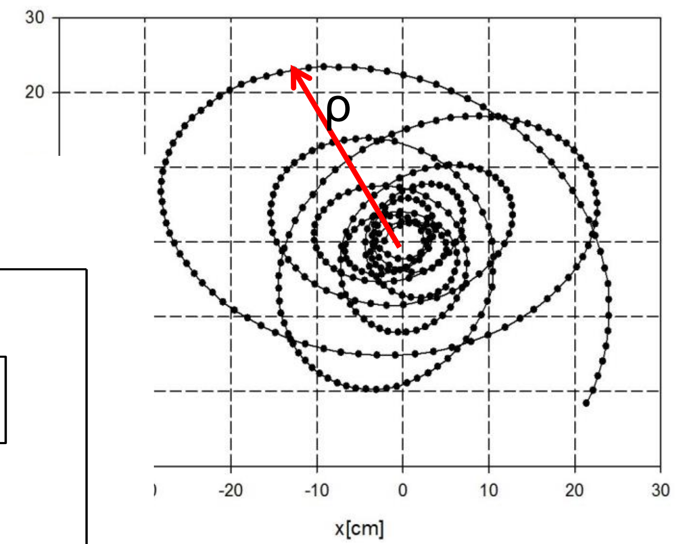
Trajectory animation

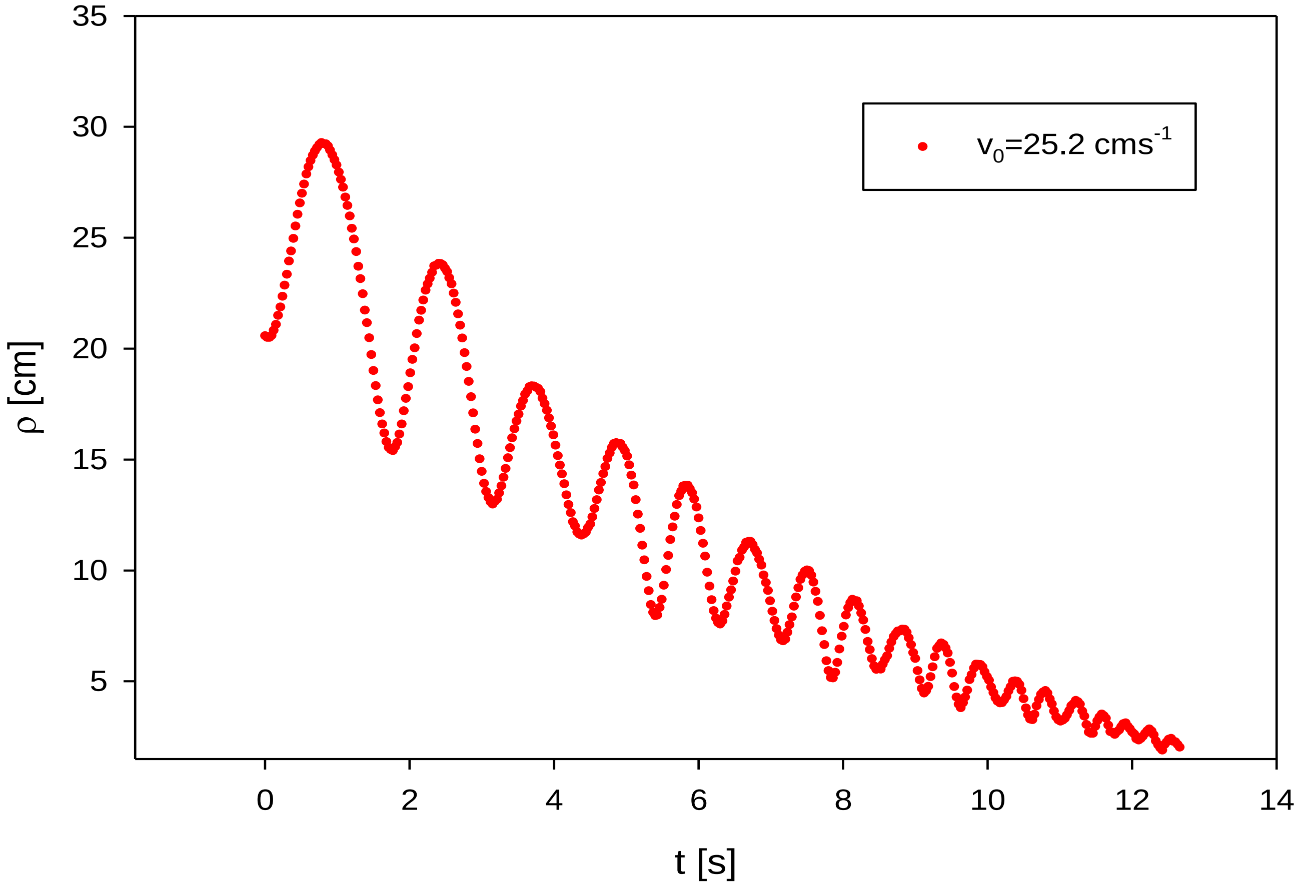


Trajectory

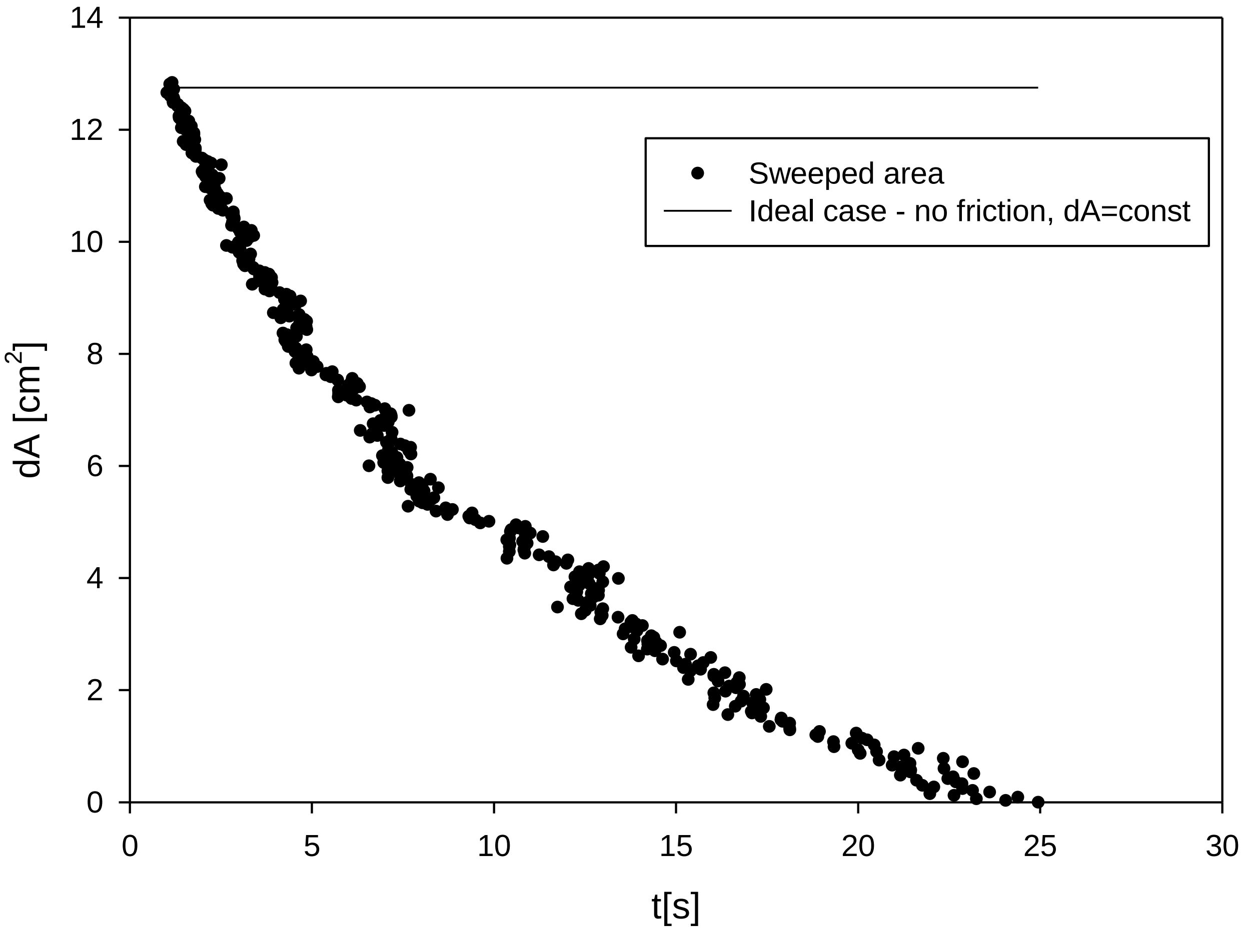


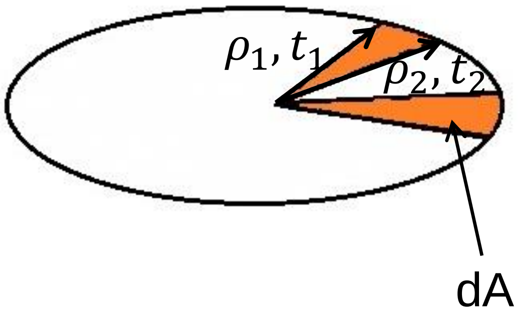
Ball distance from center vs time





Second Kepler's law



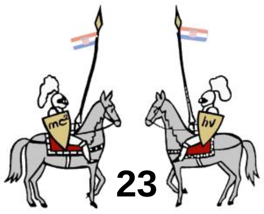


Calculated from coordinates of the ball

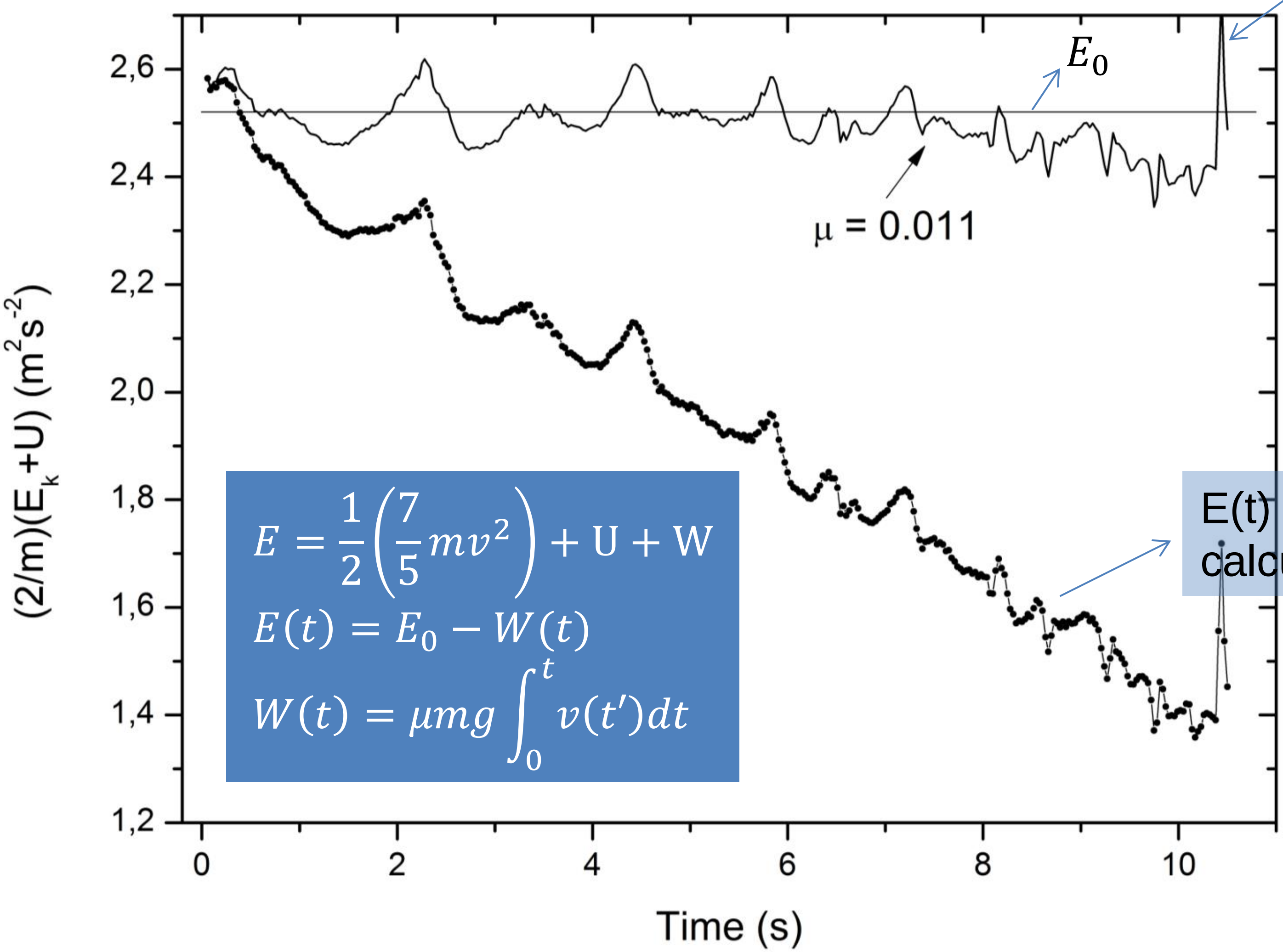
$$dA = \frac{1}{2}\rho_1\rho_2 \sin(\measuredangle(\rho_1\rho_2))$$

$$t = t_2$$

dA – sweeped area in time interval dt (0.03 s)



Energy



$$E = \frac{1}{2}\left(\frac{7}{5}mv^2\right) + \mathrm{U} + \mathrm{W}$$

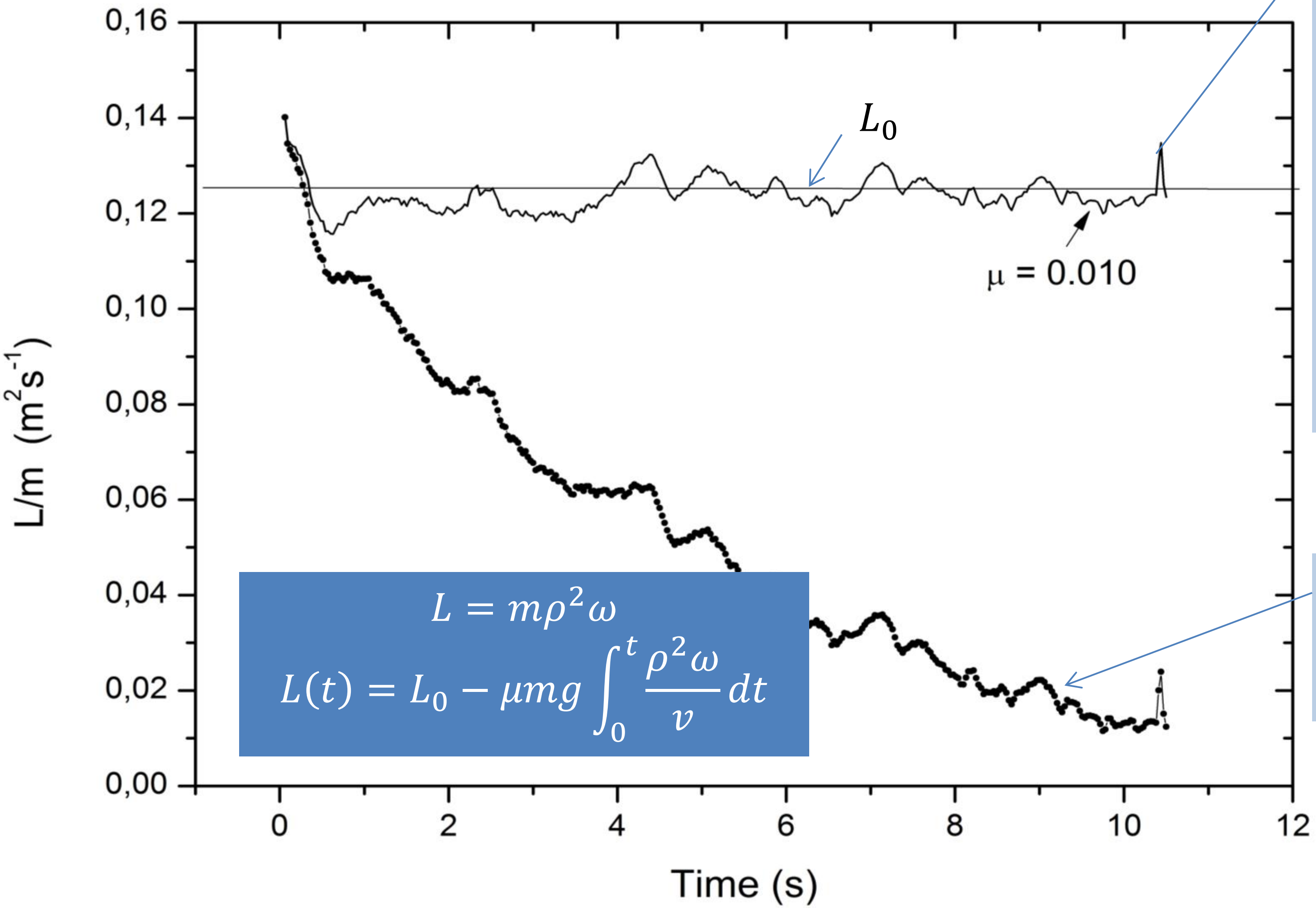
$$E(t) = E_0 - W(t)$$

$$W(t) = \mu mg \int_0^t v(t')dt$$

Numerically adjusting μ →show's that if there weren't friction the ball would have the same energy (maximal deviation 8%)

E(t) numerically calculated from a video

Angular momentum



$$L = m\rho^2\omega$$

$$L(t) = L_0 - \mu m g \int_0^t \frac{\rho^2\omega}{v}\,dt$$

Numerically adjusting μ →show's that if there weren't friction the ball would have same angular momentum (maximal deviation 12%)

L(t) numerically calculated from video

Conclusion

- We have determined the potential of the system by experimentaly determing shape of our membrane
- Potential in our system isn’t proportional to Kepler’s potential
- We have calculated constant of our system which is not analogous to the gravitational constant because we couldn’t determine dependence of deformation to the mass
- We have a non-central force (friction) in system
  - Second Kepler’s law isn’t valid in our case

Conclusion – comparison between universe and elastic membrane

	Universe	Elastic membrane
Potential	$U(\rho) = \frac{\gamma M}{\rho}$	$U(\rho) = U_0 + mgB\rho^p$
Force	$F = -m\frac{\gamma M}{\rho^2}$	$F = -mg\frac{dz}{d\rho}$
Circular orbit speed	$v = \sqrt{g\rho}$	$v = \sqrt{g\rho\frac{dz}{d\rho}}$
Period of an orbit	$T^2 = \frac{4\pi^2\rho^3}{\gamma M}$	$T^2 = \frac{4\pi^2\rho}{g\frac{dz}{d\rho}}$
Angular momentum	$L = m{\rho_0}^2\omega_0$ $L = const$	$L(t) = m{\rho_0}^2\omega_0 - \mu mg\int_0^t \frac{\rho^2\omega}{v}dt$
Energy	$E_{tot} = E_k + E_p$	$E = \frac{1}{2}\left(\frac{7}{5}mv^2\right) + \mathrm{mgz(\rho)} + \mu mg\int_0^t v(t')dt$

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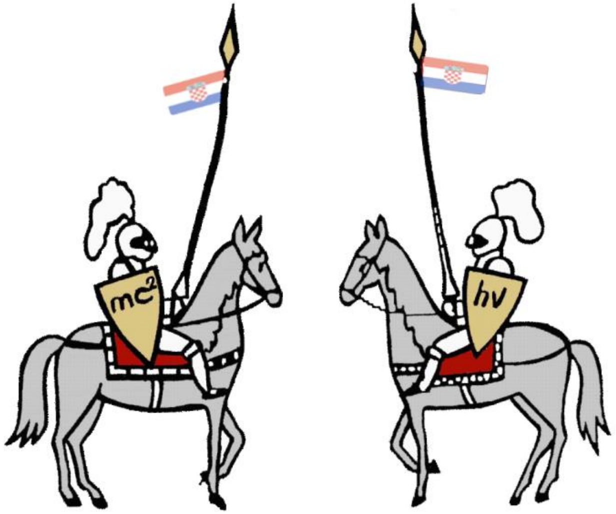
TEAM OF CROATIA

THANK YOU



Period of an orbit

$$F_i = -F_{cp}$$

$$mg\frac{dz}{d\rho} = m\omega^2\rho$$

$$g\frac{dz}{d\rho} = \frac{4\pi^2}{T^2}\rho$$

$$T^2 = \frac{4\pi^2\rho}{g\frac{dz}{d\rho}}$$