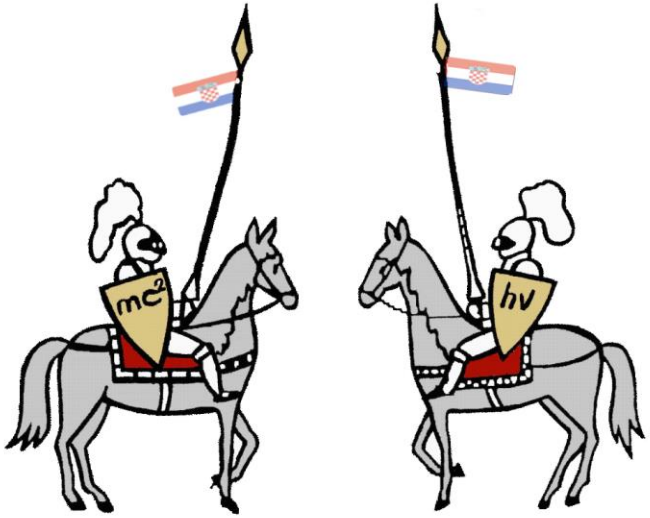
IYPT 2013

TEAM OF CROATIA

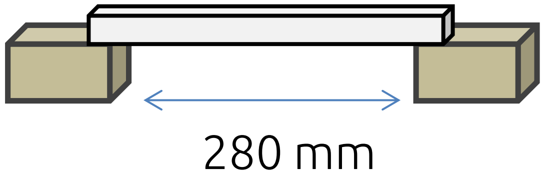
INVENT YOURSELF

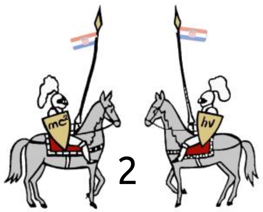
Reporter: Domagoj Pluščec



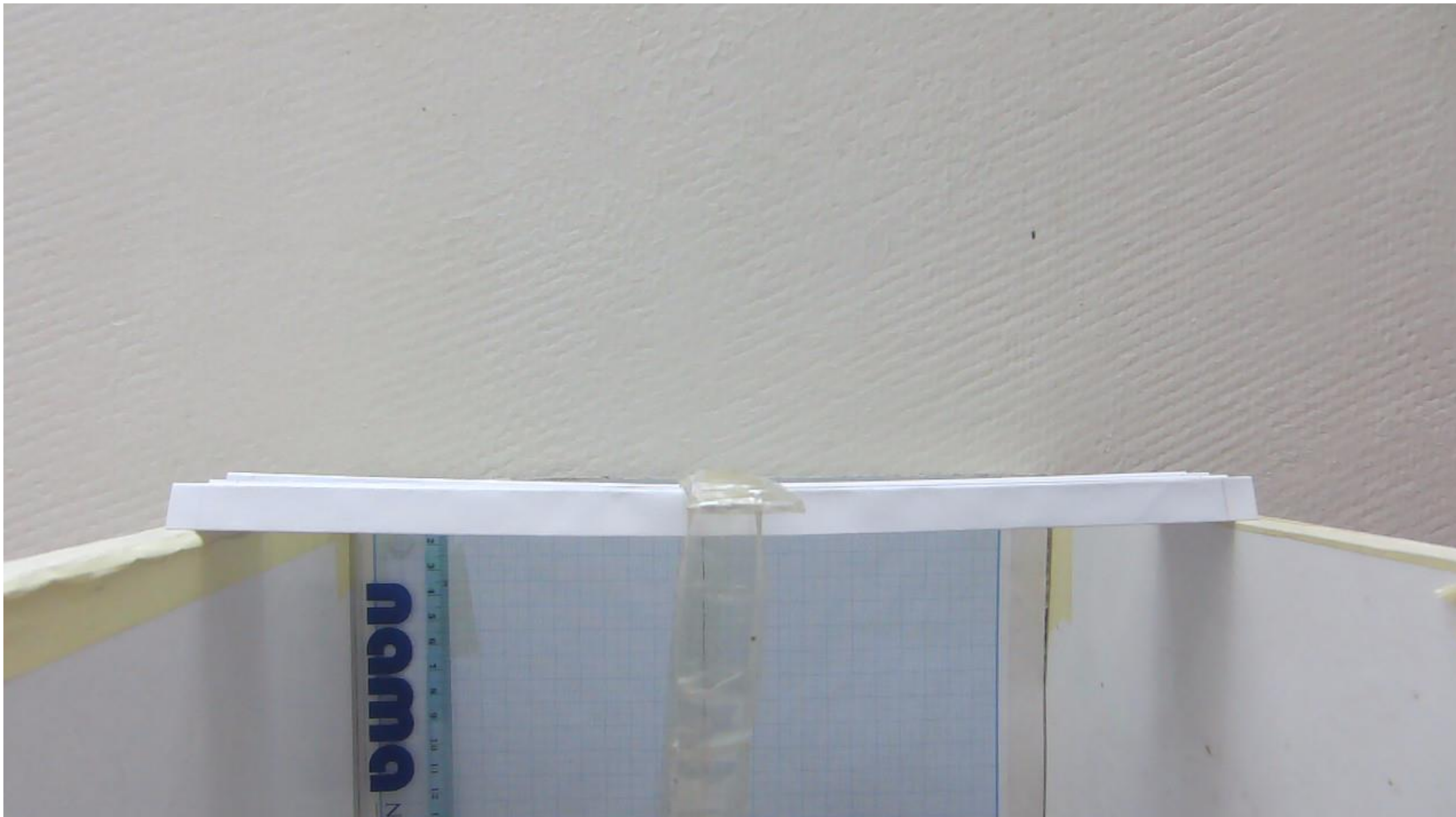
Problem

„It is more difficult to bend a paper sheet, if it is folded 'accordion style' or rolled into a tube. Using a single A4 sheet and a small amount of glue, if required, construct a bridge spanning a gap of 280 mm. Introduce parameters to describe the strength of your bridge, and optimise some or all of them."





Collapse of the bridge

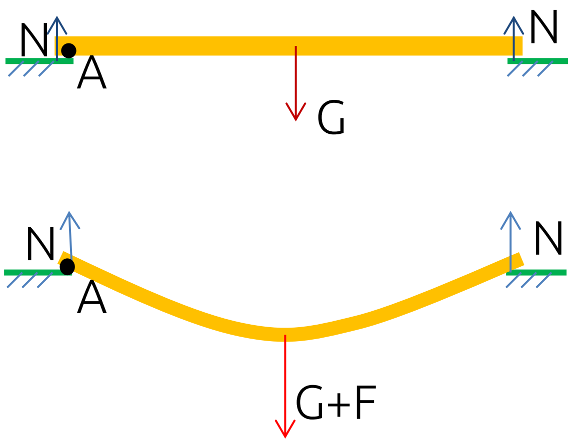


Summary

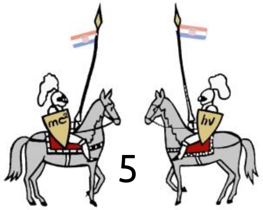
- Theory
  - Equilibrium
  - Stress
  - Buckling
  - Strength of the bridge
- Experiment
  - Models
  - Apparatus
  - Method
- Results
- Conclusion

Equilibrium conditions

- $\sum \vec{F} = 0$
- $\sum \vec{\tau} = 0$



- For unloaded bridge
  - $2N - G = 0$
  - Point A: $G \cdot \frac{l}{2} - Nl = 0$
- For loaded bridge
  - $2\mathrm{N} - (\mathrm{G} + \mathrm{F}) = 0$
  - Point A: $(G + F) \cdot \frac{l}{2} - Nl = 0$



Basic characteristics

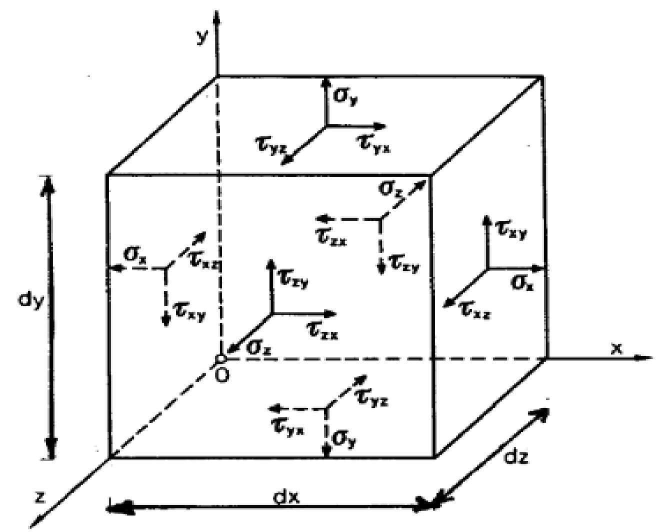
- Paper
  - orthotropic (orthogonally anisotropic)
    - mechanical properties equal in certain directions of the fibers
  - 80 gm^{-2}; (210x297)mm

- When the force is applied it leads to appearance of internal forces in material

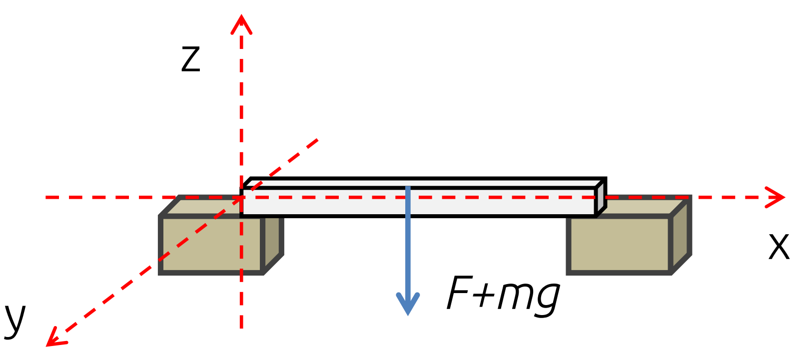
Stress

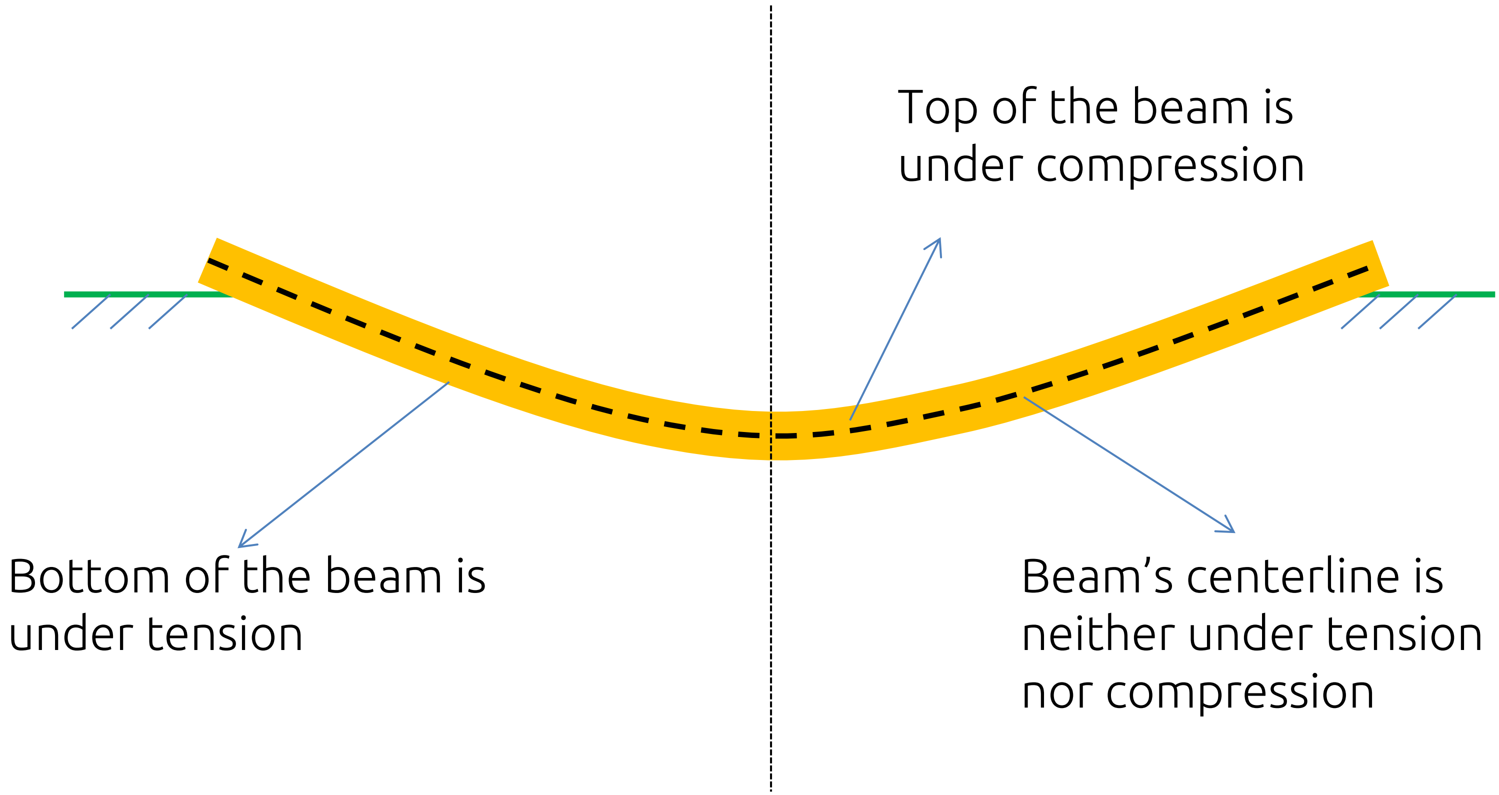
- Two types of stress: normal (σ) and shear stress (τ)

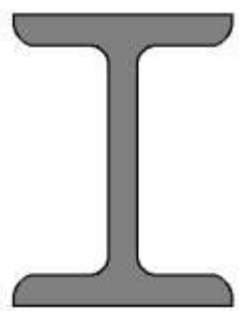
- The Cauchy stress tensor - total stress at the point:
  - $[\sigma_{ij}] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$



- Normal stress
  - $\sigma_z = \frac{F_z}{A} = \frac{M}{I_y} z$





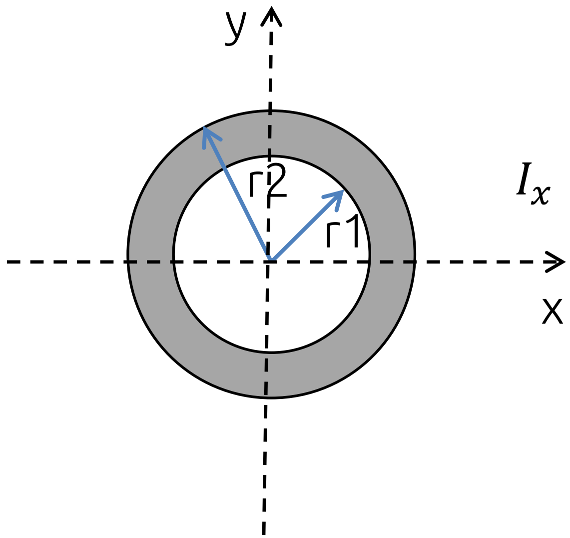


That is why I-cross section is used in construction of complex building structures

Area moment of inertia

- Geometrical property of an area
  - reflects how its points are distributed with regard to an arbitrary axis

- Definition
  - $I_x = \int_A y^2 dA = \int\int_A y^2 dx dy$

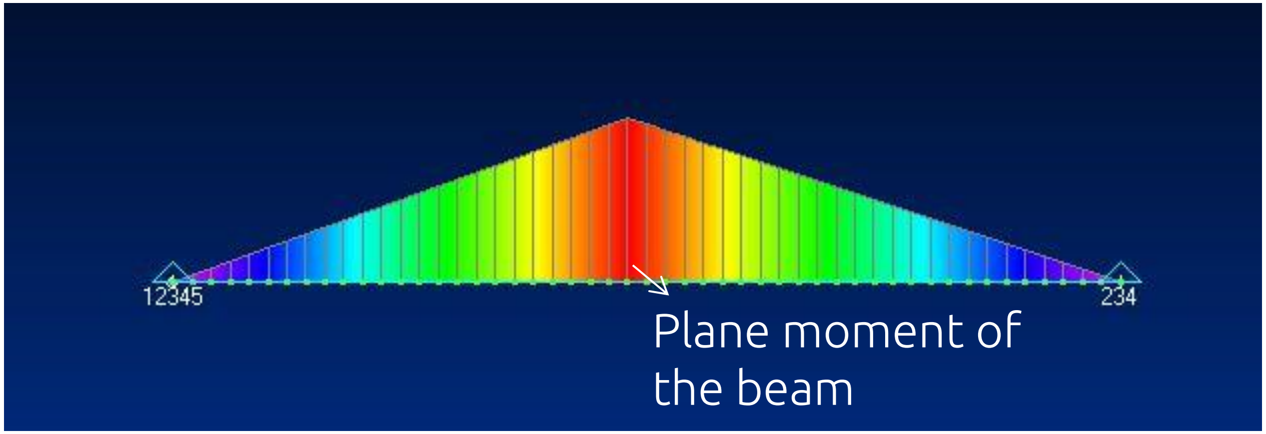


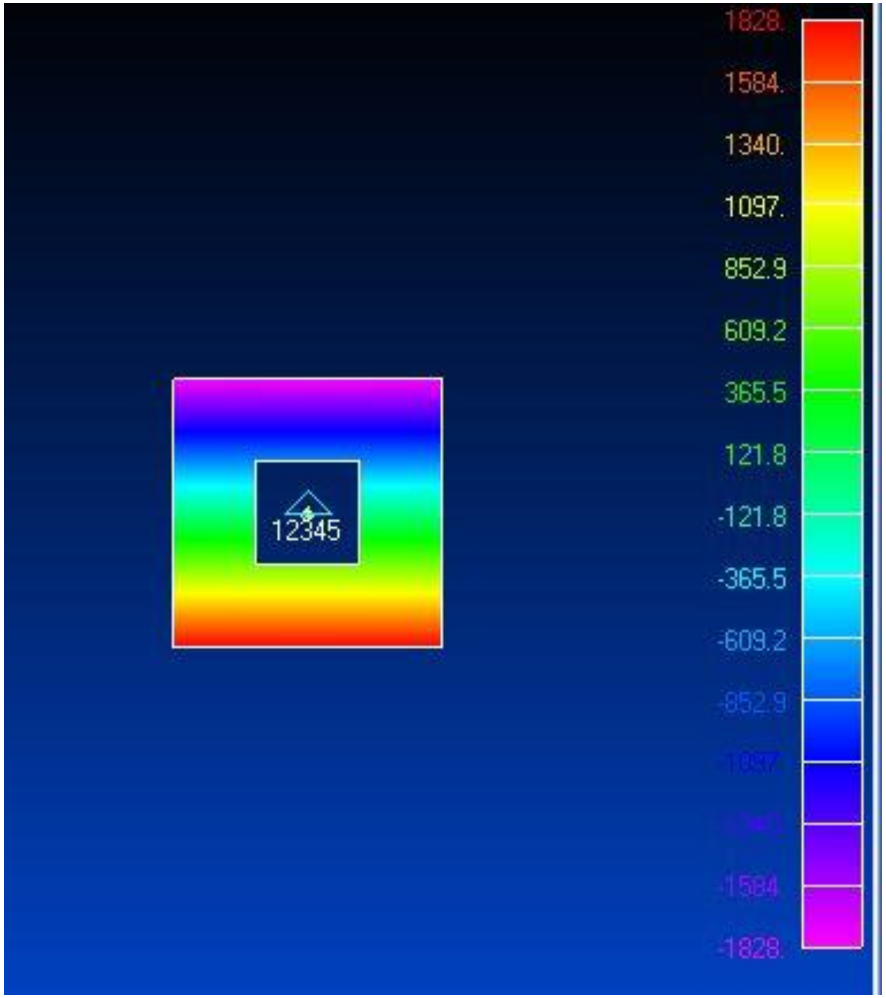
$$I_x = \frac{\pi}{4}({r_2}^4 - {r_1}^4)$$

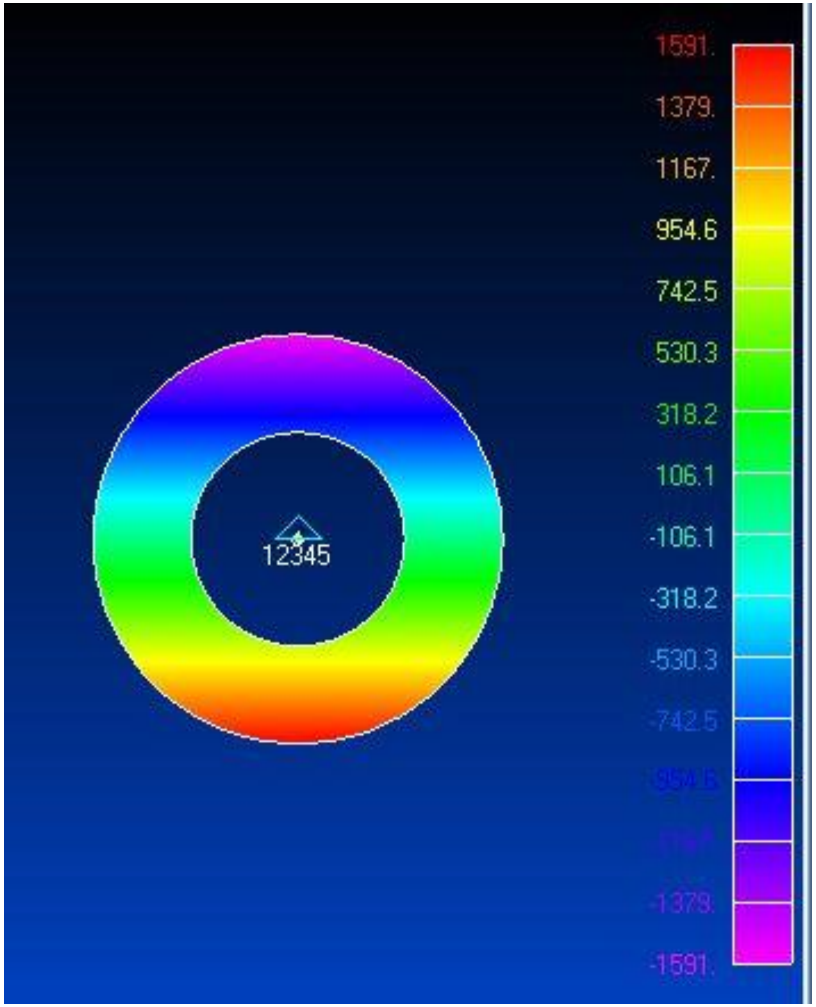
Simulation of stresses in the material

- Software FEMAP – Finite Element Modeling And Postprocessing (numerically solves differential equation)

- Necessary to define:
  - Material
  - Constraints
  - Shape of the bridge
  - Load

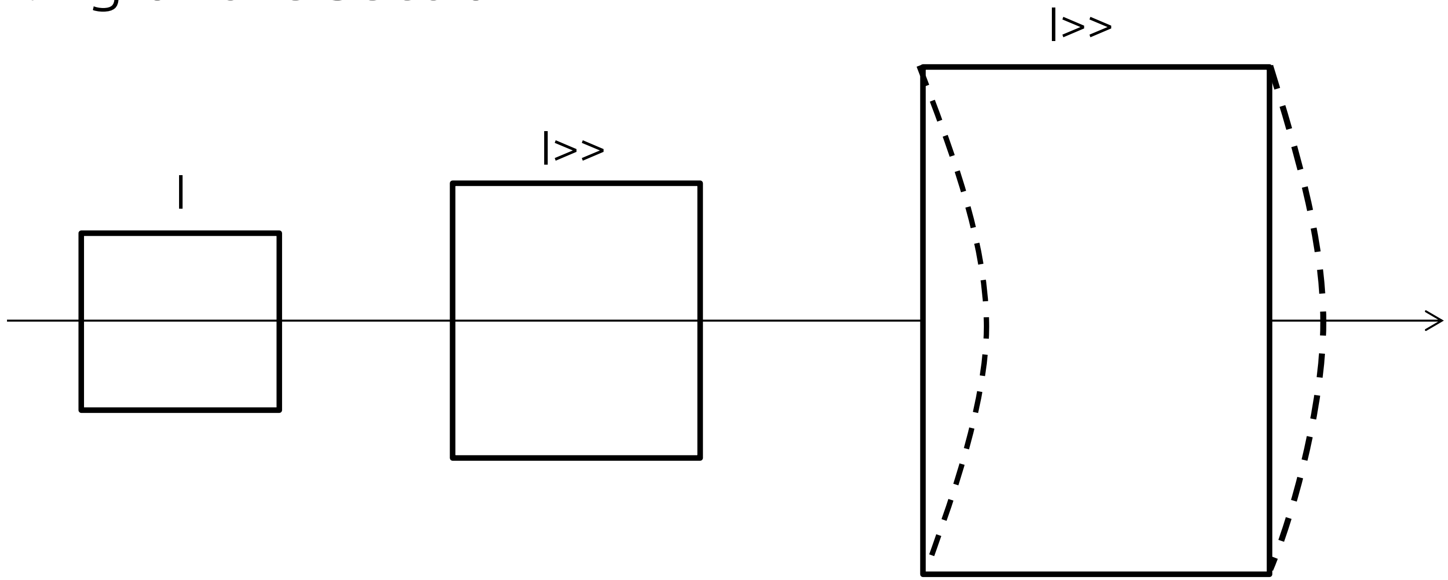



1828.
1584.
1340.
1097.
852.9
609.2
365.5
121.8
-121.8
-365.5
-609.2
-852.9
-1584.
-1828.
12345


1591.
1379.
1167.
954.6
742.5
530.3
318.2
106.1
-106.1
-318.2
-530.3
-742.5
-1379.
-1591.
12345

Buckling

- When area moment of inertia is too large it comes to buckling of the section



- It is necessary to find the optimal ratio between the area moment of inertia and stiffness of the bridge (stifness changes with number of windings)

Definition of the "strength" of the bridge

- Strength - ability to carry a load without causing failure
- Rigidity - resistance to structural deformation (change in shape and volume)
- Stability - the ability of construction and its elements that under given load retain the original form of elastic equilibrium

Definition of the "strength" of the bridge

- The maximum force prior to the plastic deformation

*

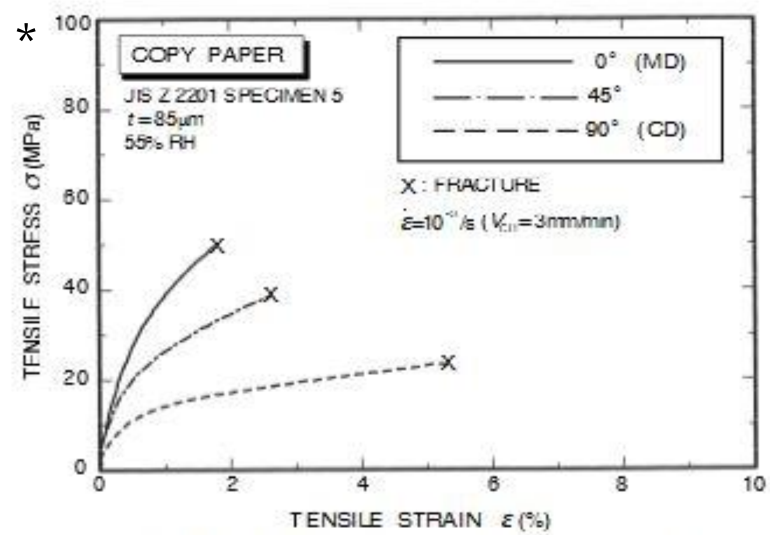
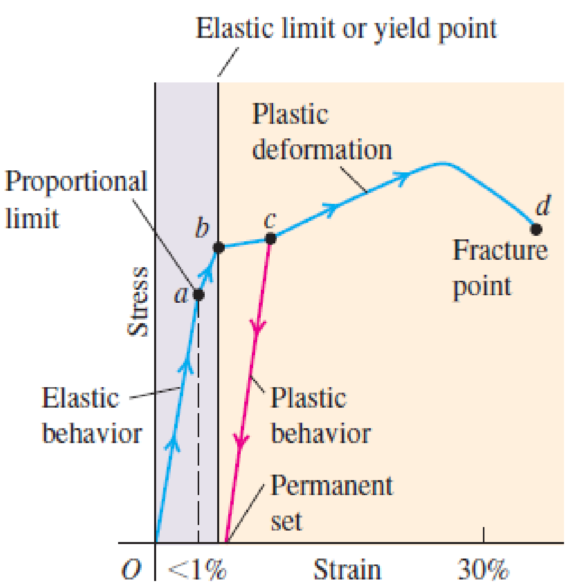


Fig. 6 Effect of test direction on tensile stress-strain curves for copy paper



- If plastic deformation doesn't occur – maximum force before the collapse of the bridge

* Włodzimierz Szewczyk, Krzysztof Marynowski, Wiktorian Tarnawski, An Analysis of Young's Modulus Distribution in the Paper Plane

Definition of the model

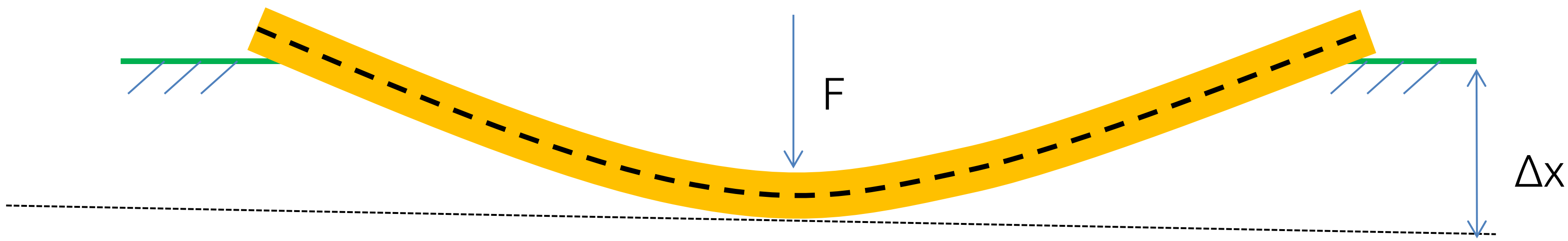
- Model - basic shape of the bridge
  - Defined by as little as possible parameters

„bi-triangle”

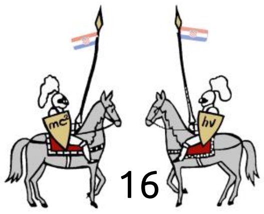
Model	The range of the area moment of inertia [cm^4]
Basic models	
Triangle	0.14 – 5.19
Rectangle	0.40 – 12.26
Circle	0.35 – 11.21
Accordion	0.11 – 4.41
U-shape	- model discarded
Complex models	

Isosceles trapezoid
„Bi-triangle”
„Accordion with base”

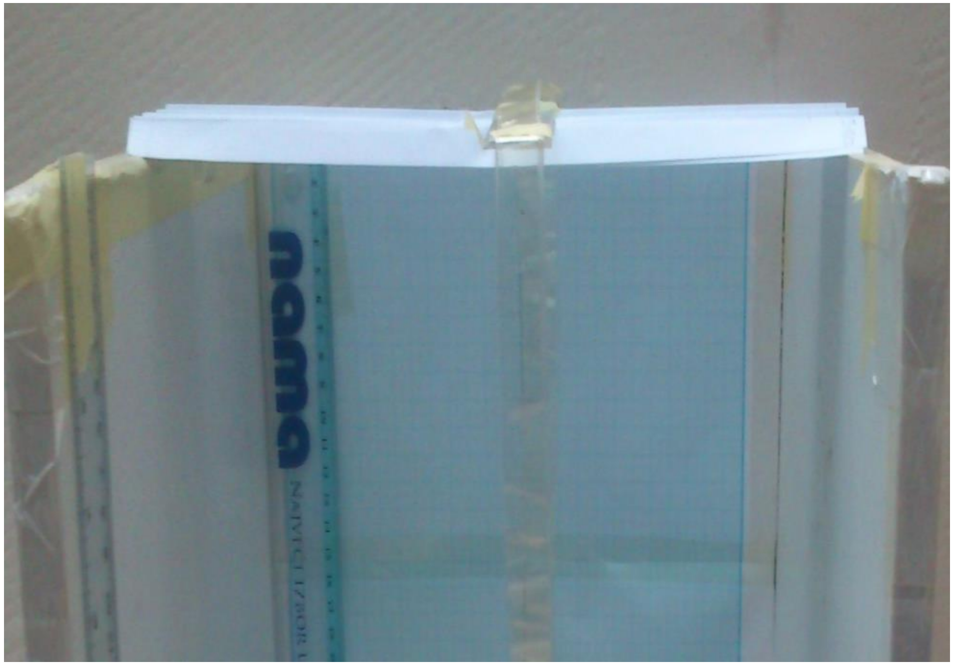
Experiment



We measured:
Δx – vertical displacement while applying some force F
(Δx is proportional to the radius and angle of curvature of the bridge)



Aparatus

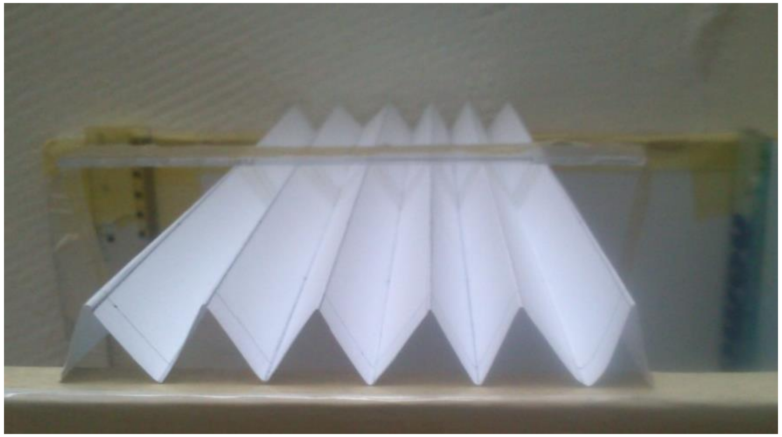


- Stands
- Caliper
- Scale
- Ruler
- Software (FEMAP)
- A4 paper and glue
- Mirror

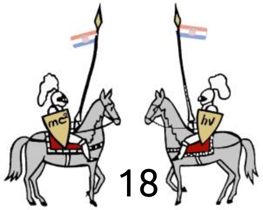
In preparing of the setup we wanted to :

- Reduce possible vibration of the environment
- Reduce the possibility of parallax by using a mirror
- That the force on the bridge is perpendicular to the plane in which bridge lies

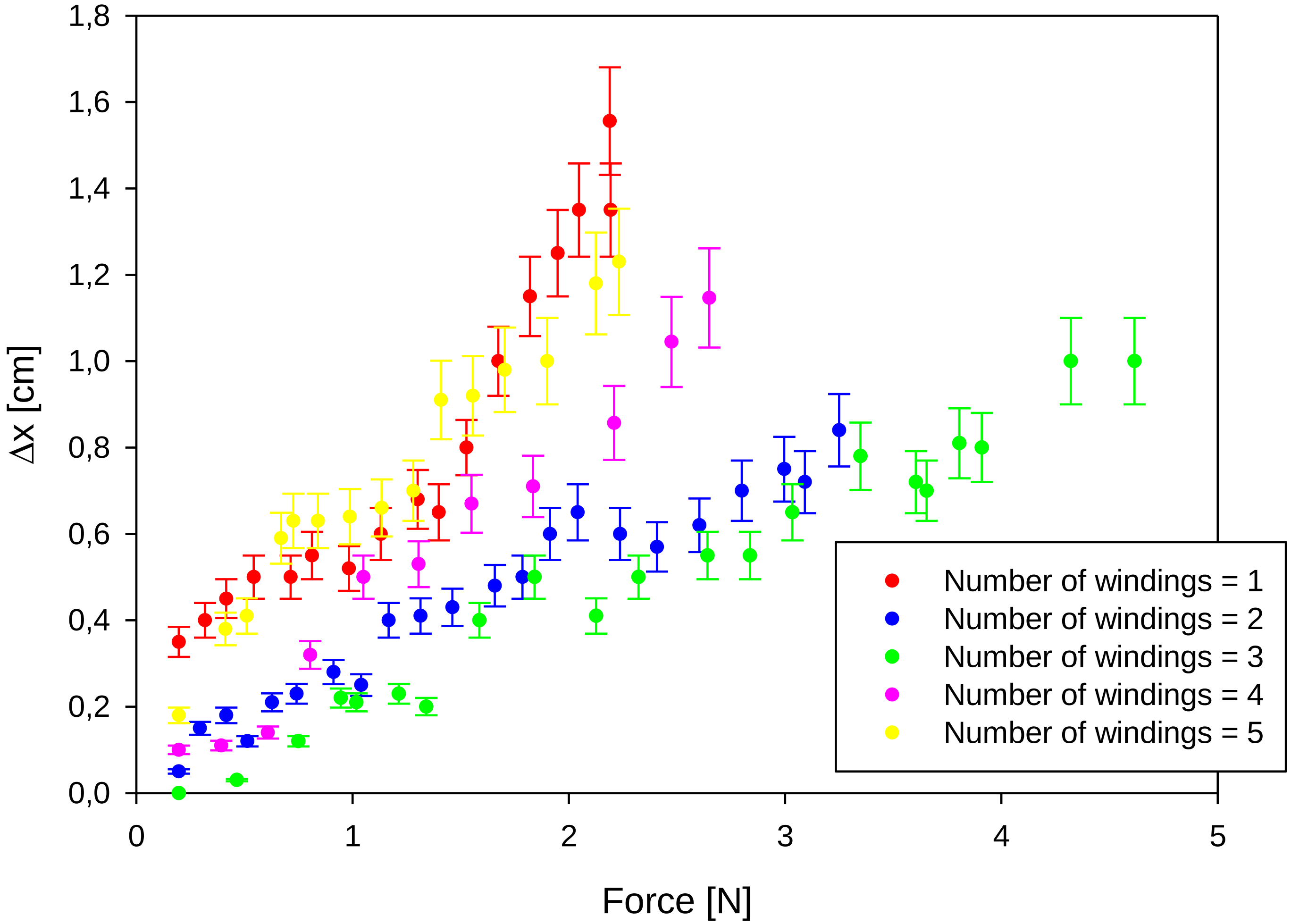
Method

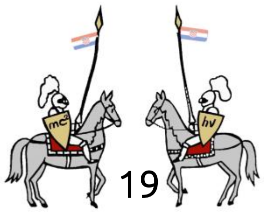


- Bridge was constructed
- Bridge was simetrically positioned on the setup
- Bridge was loaded in a way that the force was perpendicular to the plane of the bridge
- Vertical displacement was measured
- We checked if there was plastic deformation of the bridge

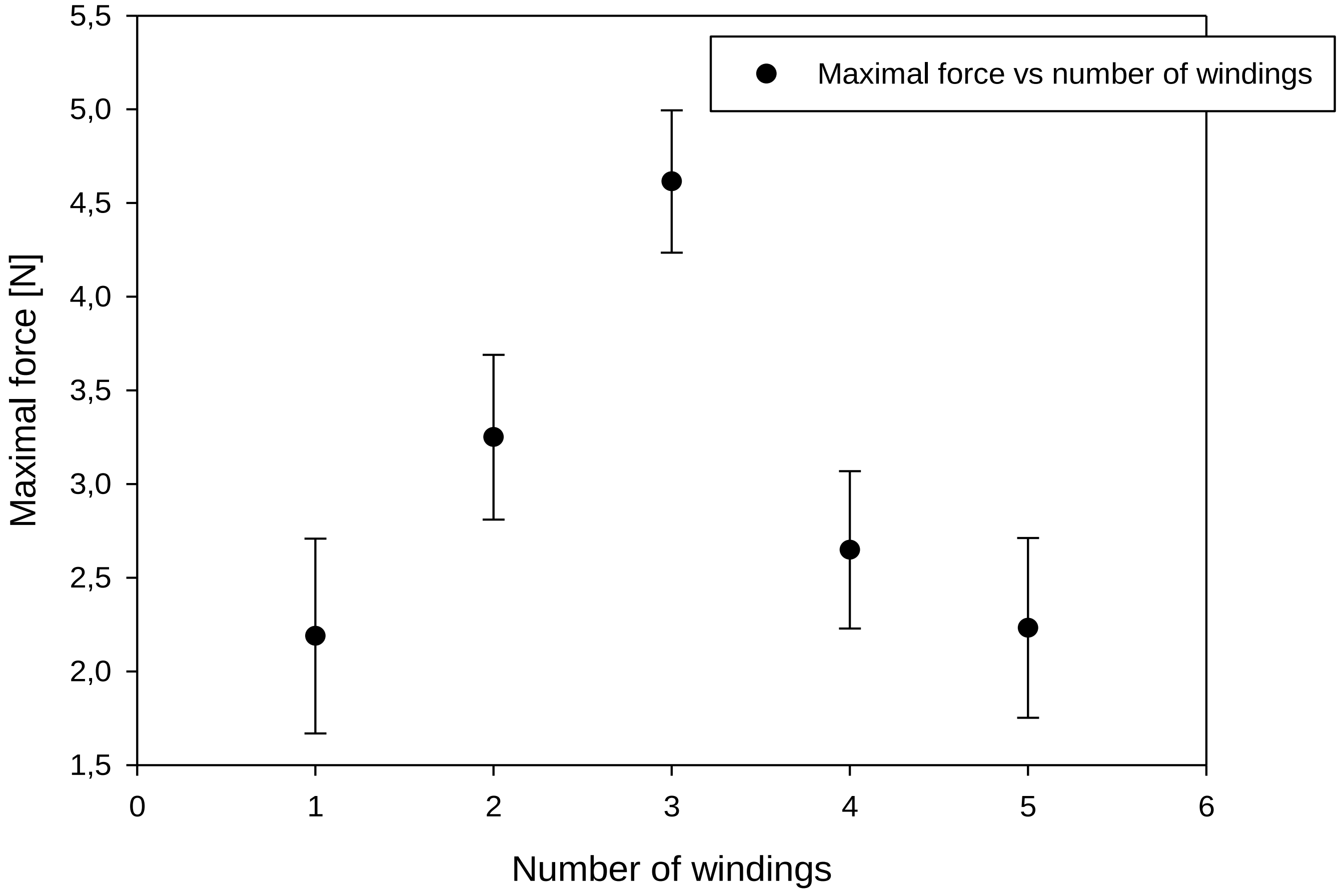


Circle – force vs vertical displacement

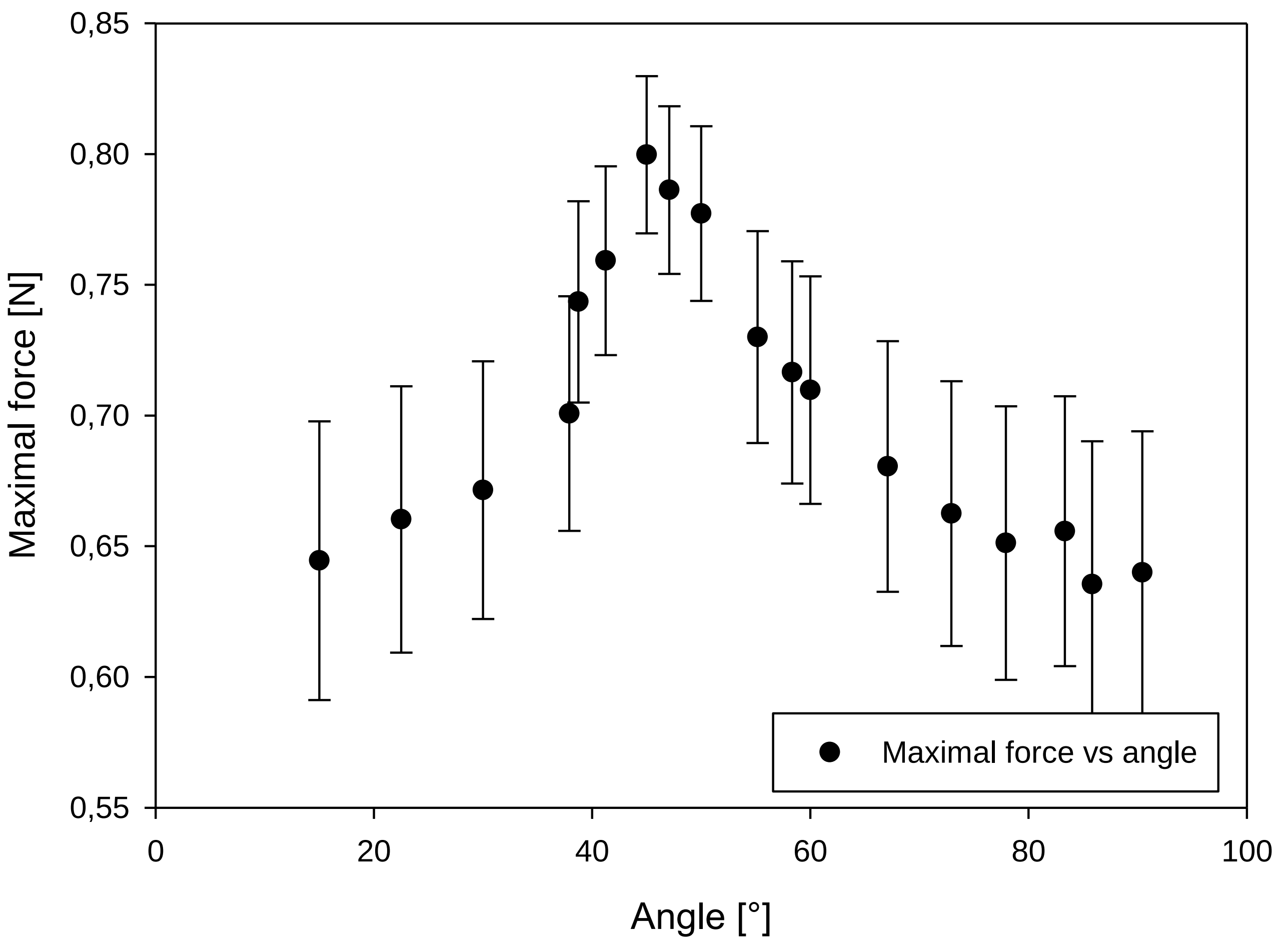


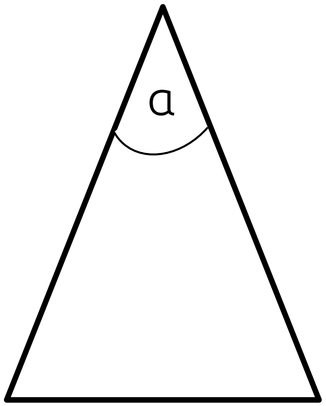


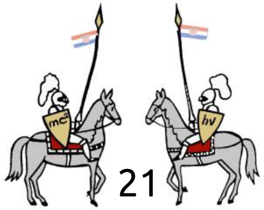
Circle – maximal force vs number of windings



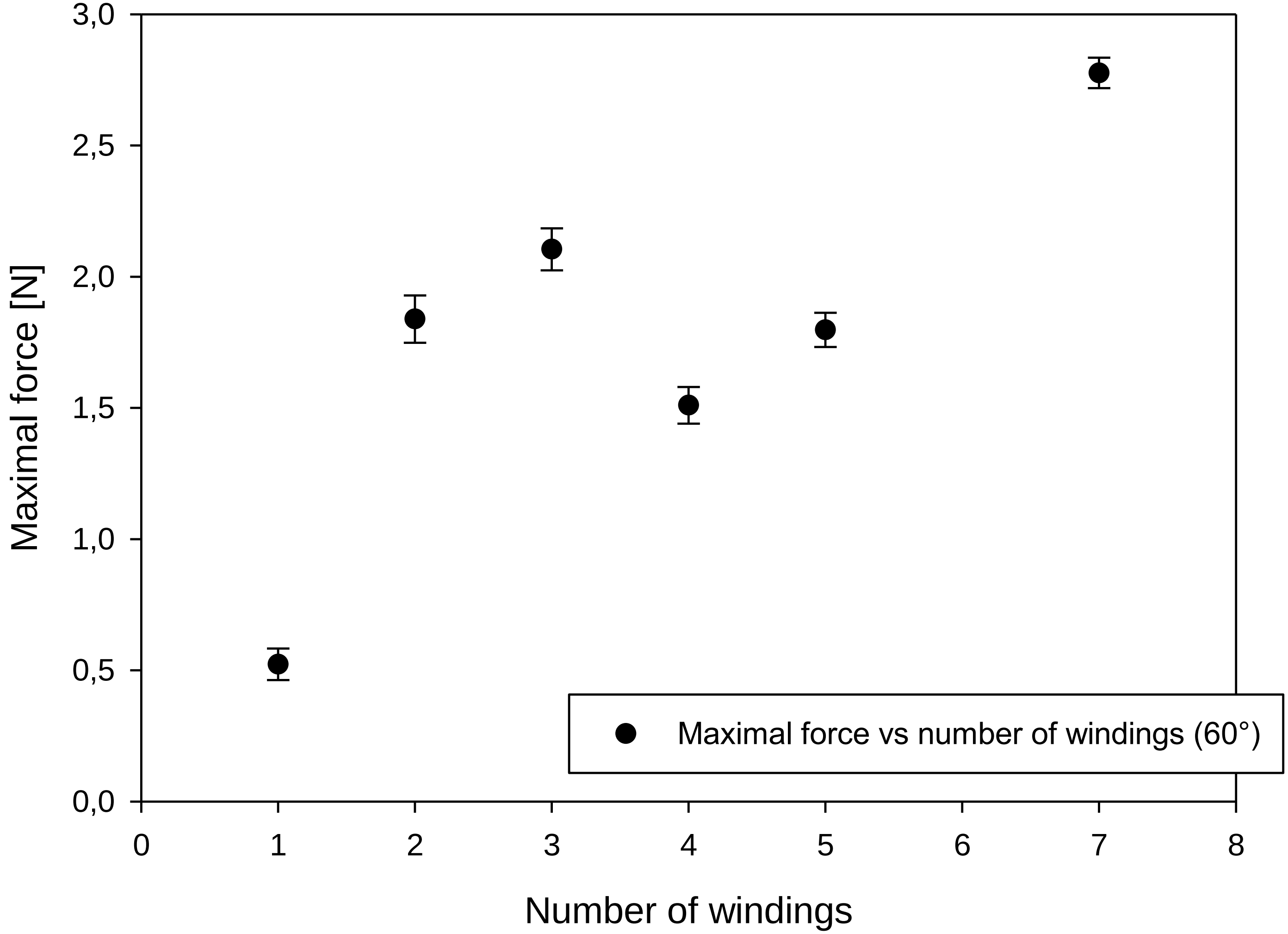
Triangle – maximal force vs angle

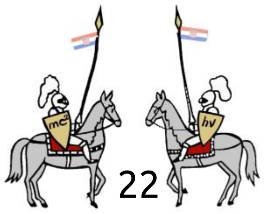




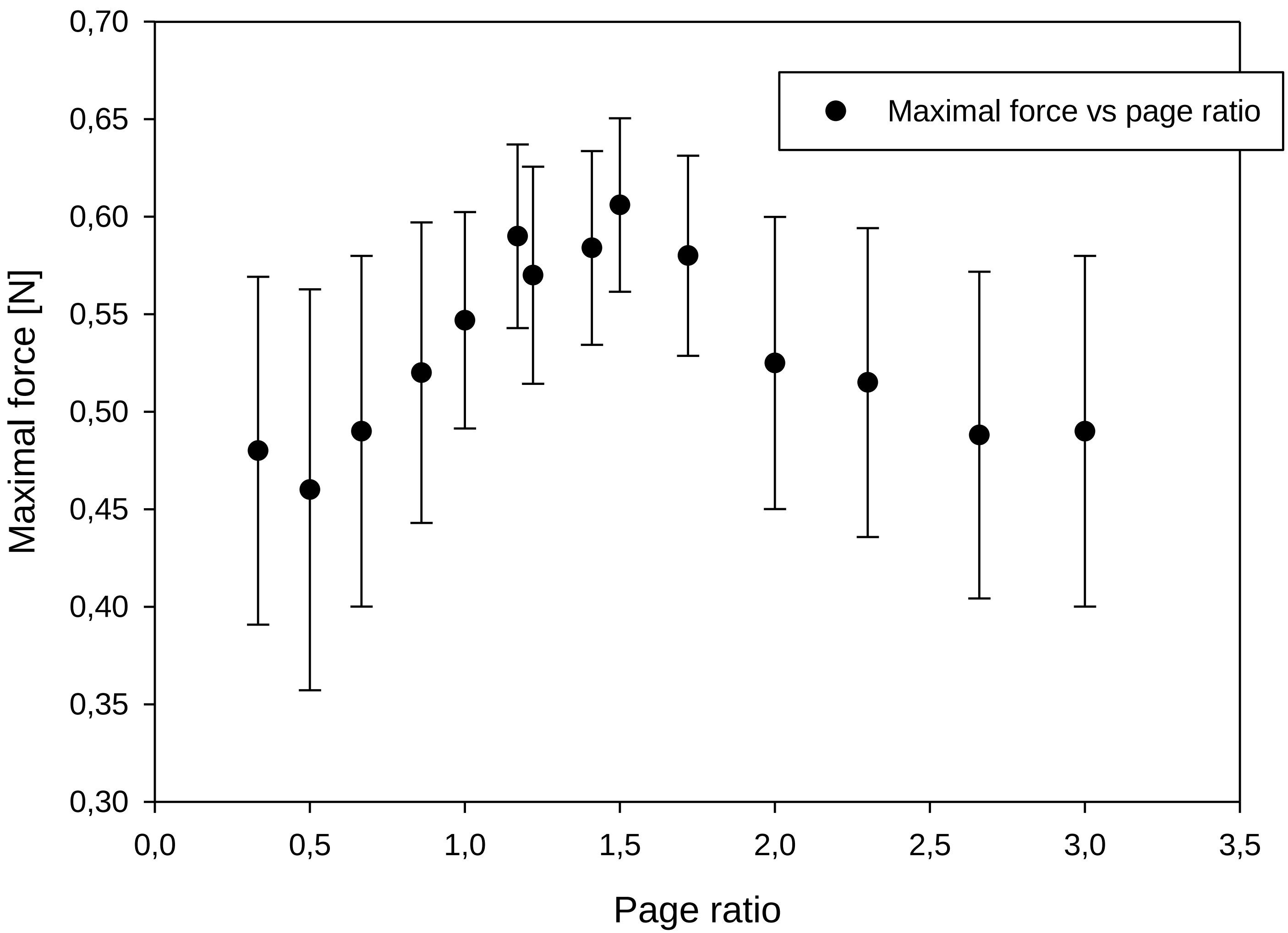


Triangle – maximal force vs number of windings

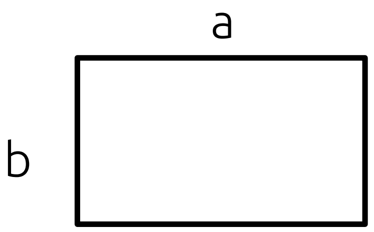




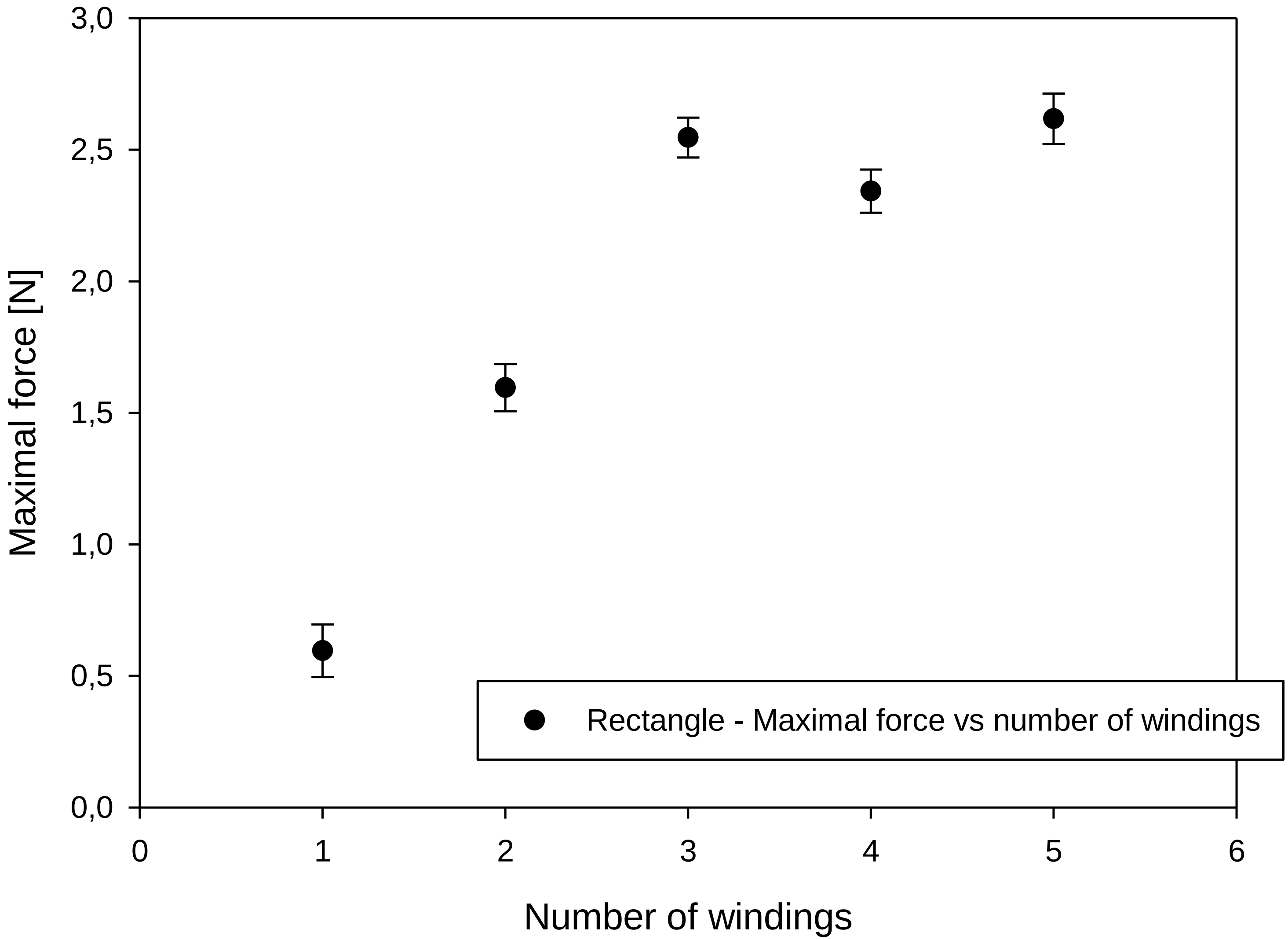
Rectangle – maximal force vs page ratio



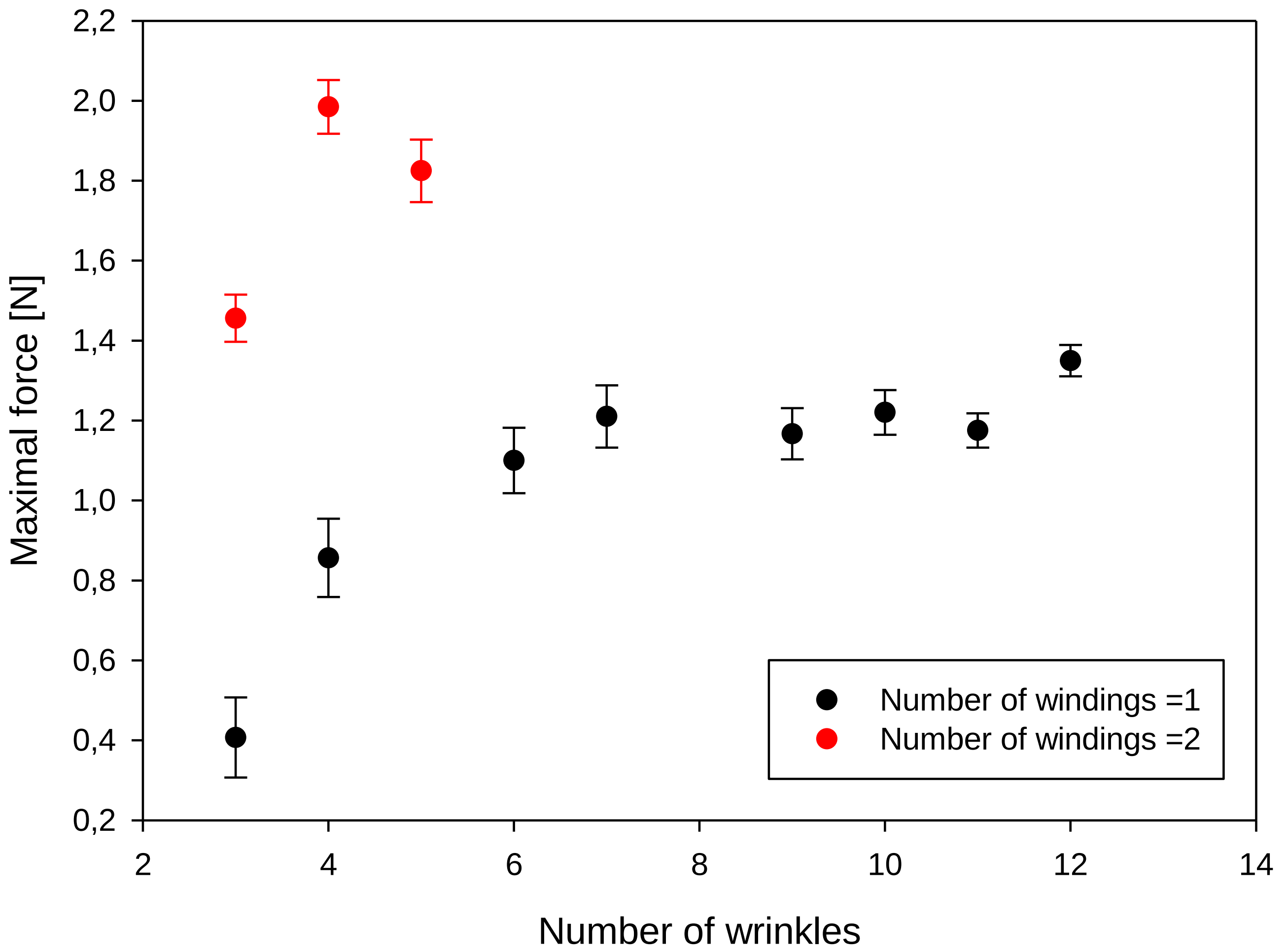
$$Page\ ratio \quad \frac{a}{b}$$

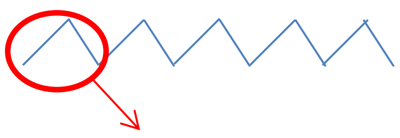


Rectangle – Maximal force vs number of windings



Accordion – maximal force vs number of winkles





Optimization

- Complex models
  - Bigger number of parameters
  - Design based on conclusions from the measurements on basic models
- Optimal model – maximal force before plastic deformation or before collapse of the bridge

Maximal force for every model

Model	Maximal force[N]
Circle	4.6
Triangle	2.8
Rectangle	2.6
Accordion	2.0
Isosceles trapezoid	2.5
„Bi-triangle”	1.4
„Accordion with base”	2.3

Conclusion

- Strenght was defined trought parameters:
  - Area moment of inertia
  - Stifness (number of windings)
- We have defined and analysed 8 models
- Models were analysed experimentaly
- We have theoreticaly explained what happens to the bridge when force acts on it
- Optimal model is bride with circular cross section wit windings (inner radius 11.1 cm)

Literature

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THANK YOU