Problem #17

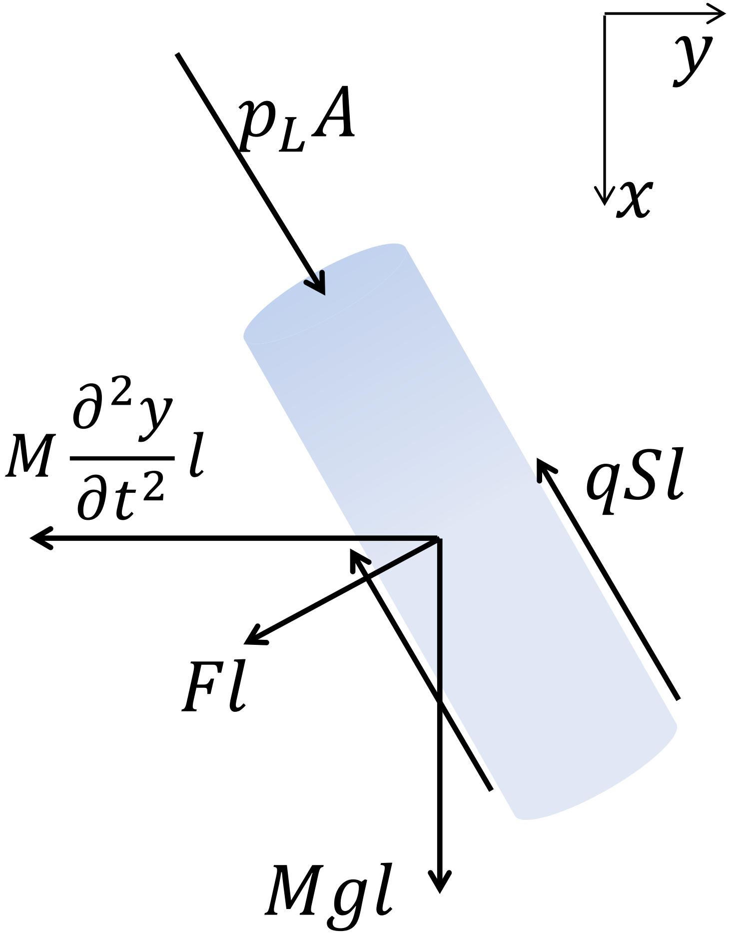
Fire Hose

Mingyu Kang

Team Korea

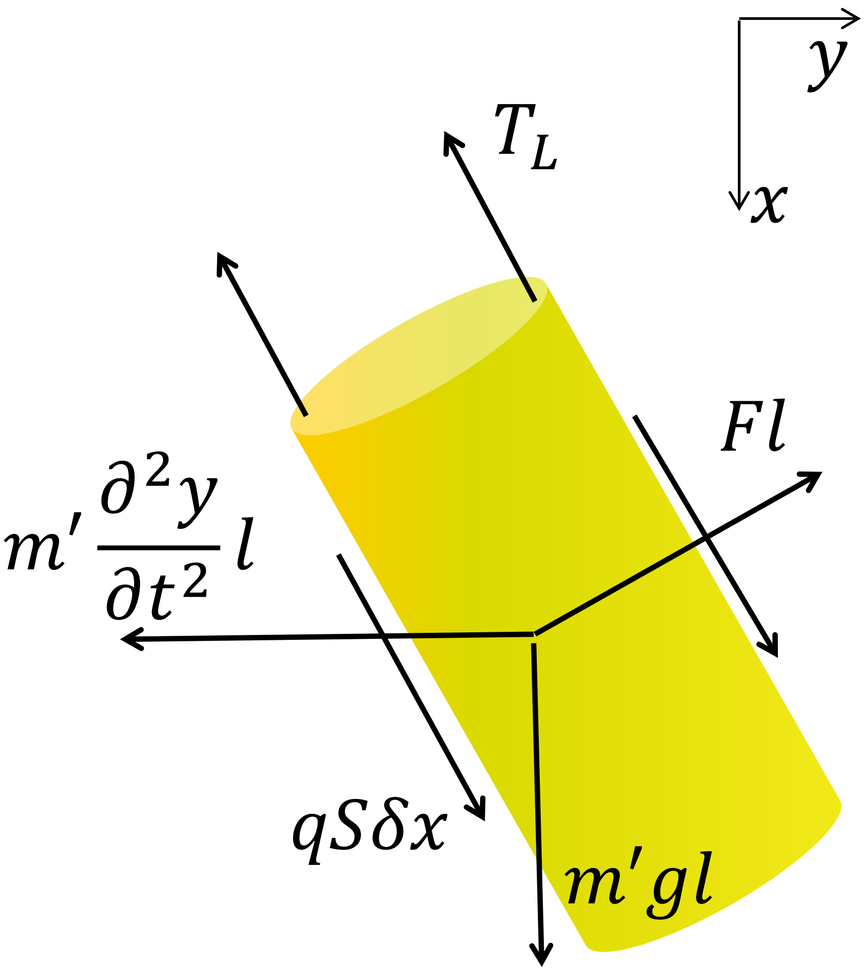
Phenomenon

Force Analysis: Fluid inside the nozzle



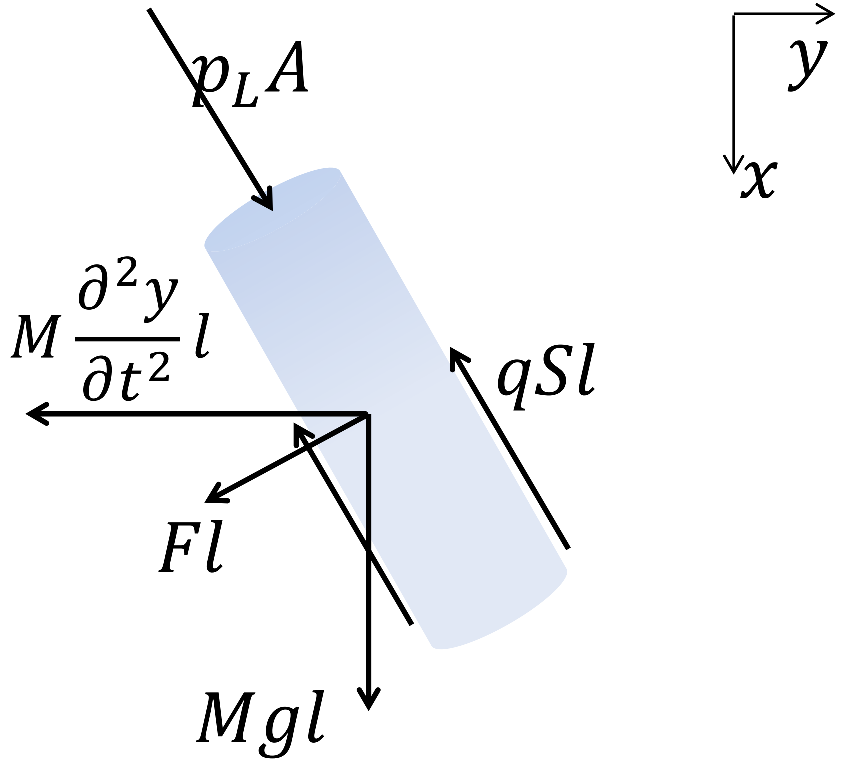
p_L: Internal pressure at the end of the hose
l: Length of the nozzle
M: Mass of fluid per unit length
F: Force per unit length between tube and fluid
q: Shear stress on the tube
S: Internal perimeter
A: Internal cross-section area

Force Analysis: Nozzle



T_L: Tension force at the end of the hose
m': Mass of nozzle per unit length
F: Force per unit length between nozzle and fluid
q: Shear stress on the tube
S: Internal perimeter
l: Length of the nozzle

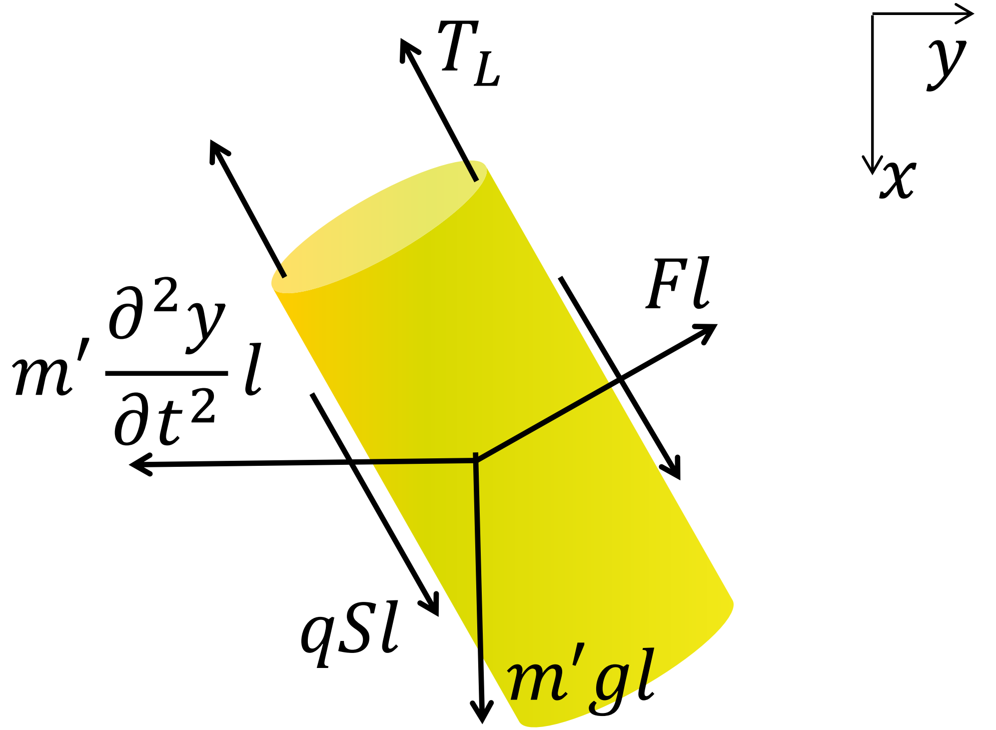
Force Equation



x: $$\frac{p_L A}{l} - qS + Mg + F\frac{\partial y}{\partial x} = 0$$

y: $$F + M\frac{\partial^2 y}{\partial t^2} - \frac{A}{l}\left(p_L \frac{\partial y}{\partial x}\right) + qS\frac{\partial y}{\partial x} = 0$$

Force Equation

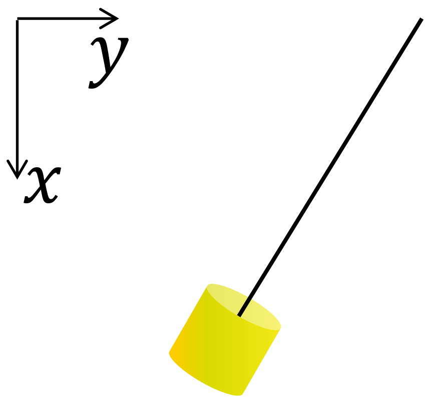


x:
$$-\frac{T_L}{l} + qS + m'g - F\frac{\partial y}{\partial x} = 0$$

y:
$$F - m'\frac{\partial^2 y}{\partial t^2} - \frac{1}{l}\left(T_L \frac{\partial y}{\partial x}\right) + qS\frac{\partial y}{\partial x} = 0$$

Combined equation

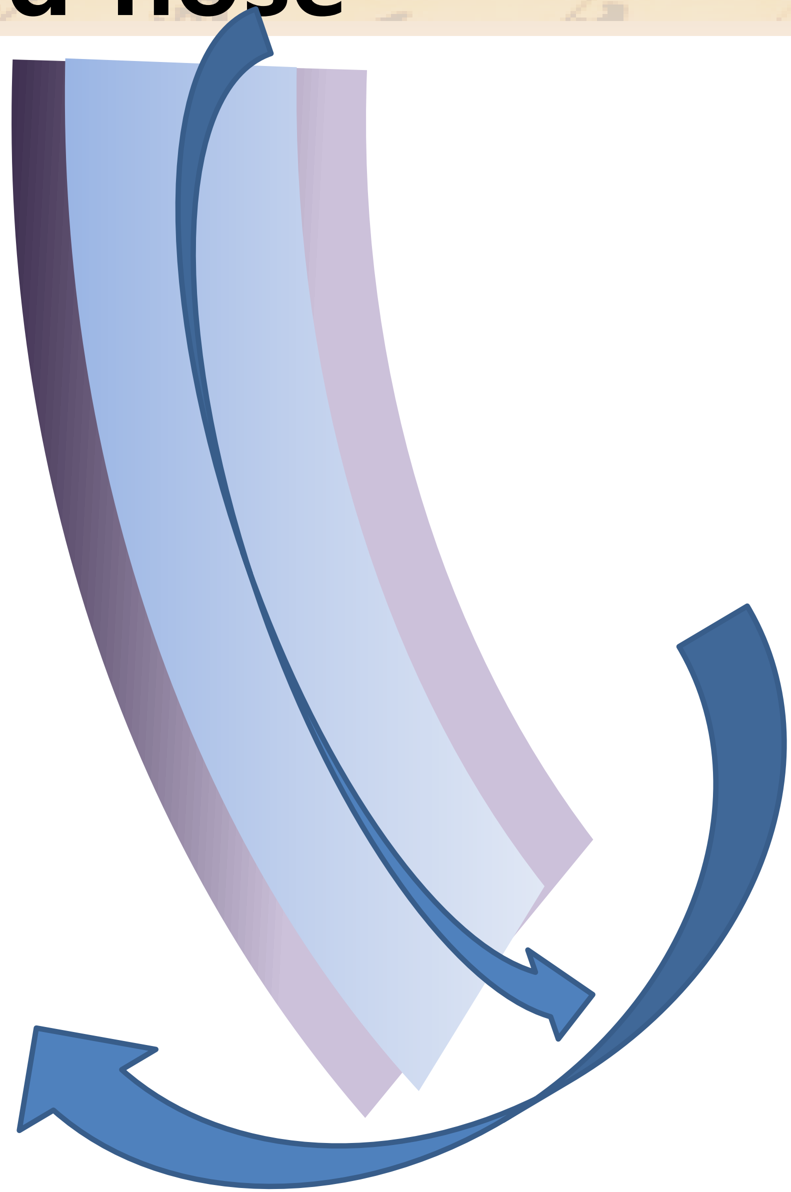
$$\frac{\partial^2 y}{\partial t^2} + g\frac{\partial y}{\partial x} \approx \frac{\partial^2 y}{\partial t^2} + g\sin\theta = 0$$



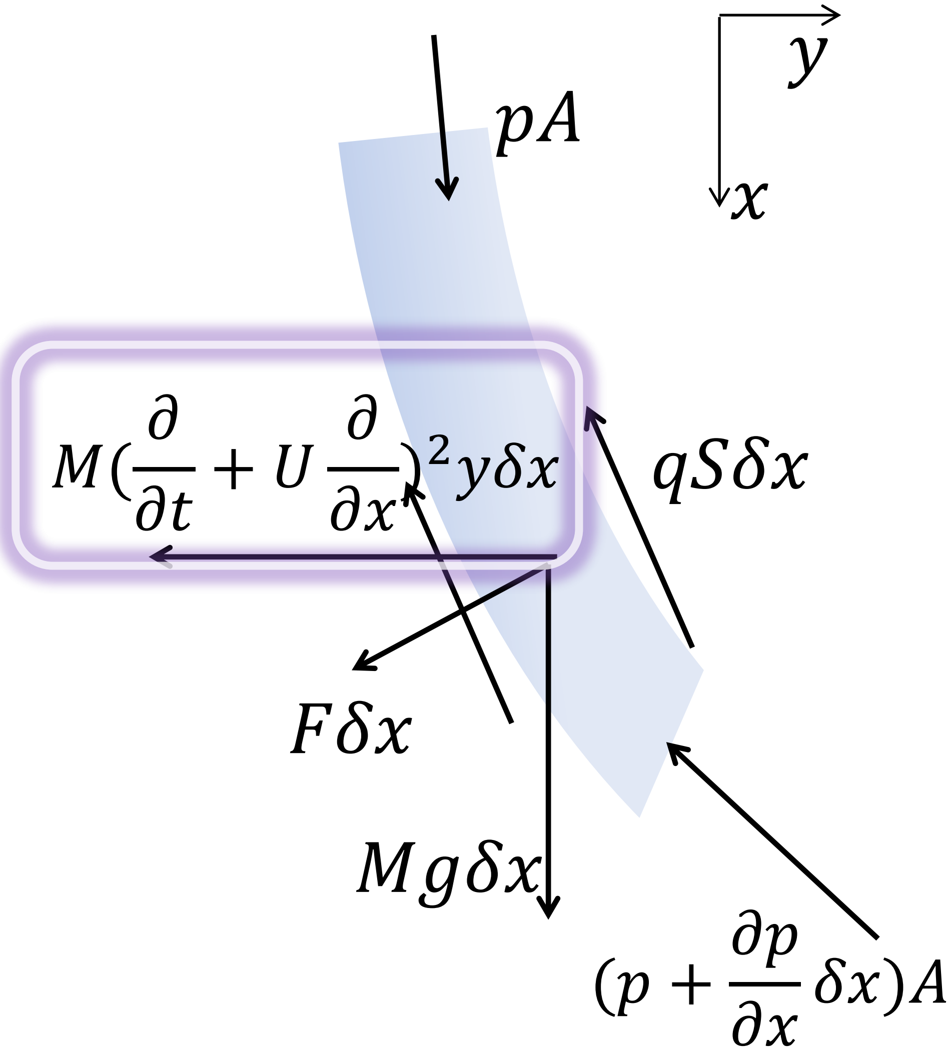
$$\frac{\partial y}{\partial x} \approx 0$$

$$T_L - p_L A = (M + m')gl$$

Difference between nozzle and hose

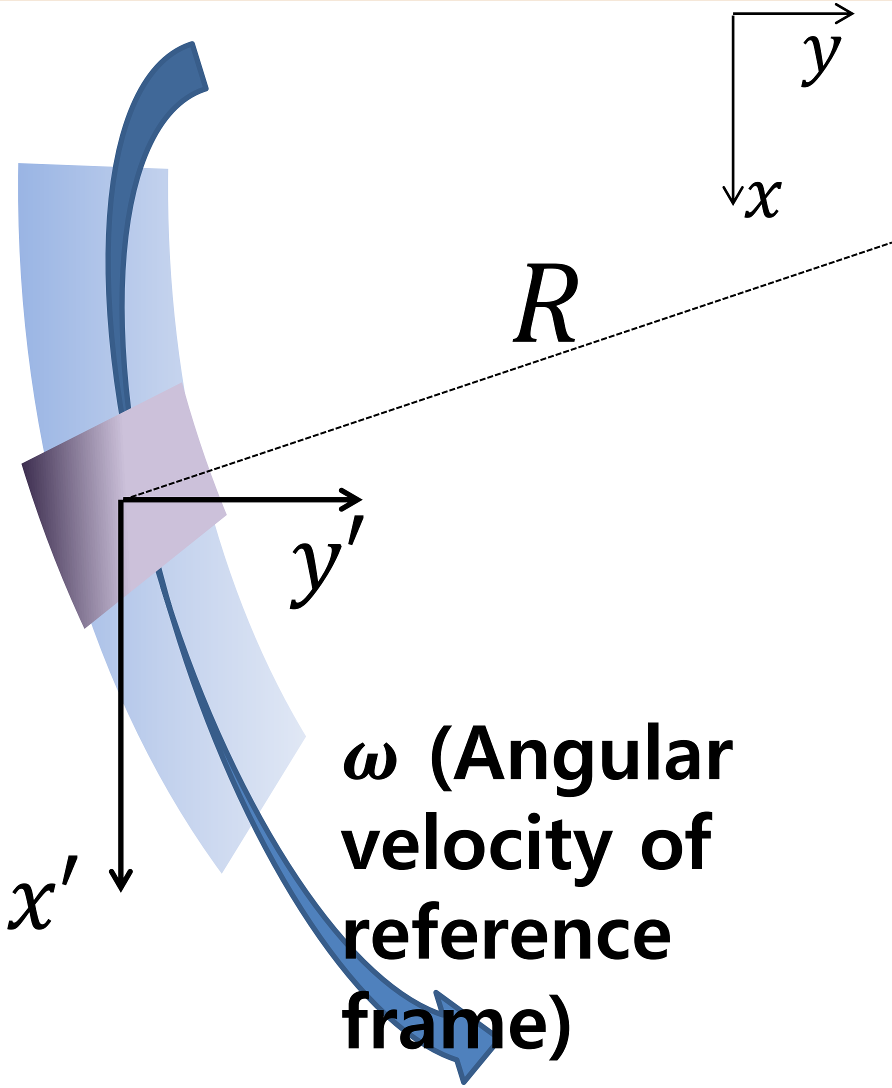


Force Analysis: Fluid



p: Internal pressure
M: Mass of fluid per unit length
U: Flow velocity
F: Force per unit length between tube and fluid
q: Shear stress on the tube
S: Internal perimeter
A: Internal cross-section area

Non-Inertial Reference Frame



$$a_{iniertial} = M\frac{\partial^2 y}{\partial t^2}\delta x$$

$$a_{centripetal} = M\frac{U^2}{R}\delta x$$

$$\kappa = \frac{1}{R} \approx \frac{\partial^2 y}{\partial x^2}$$

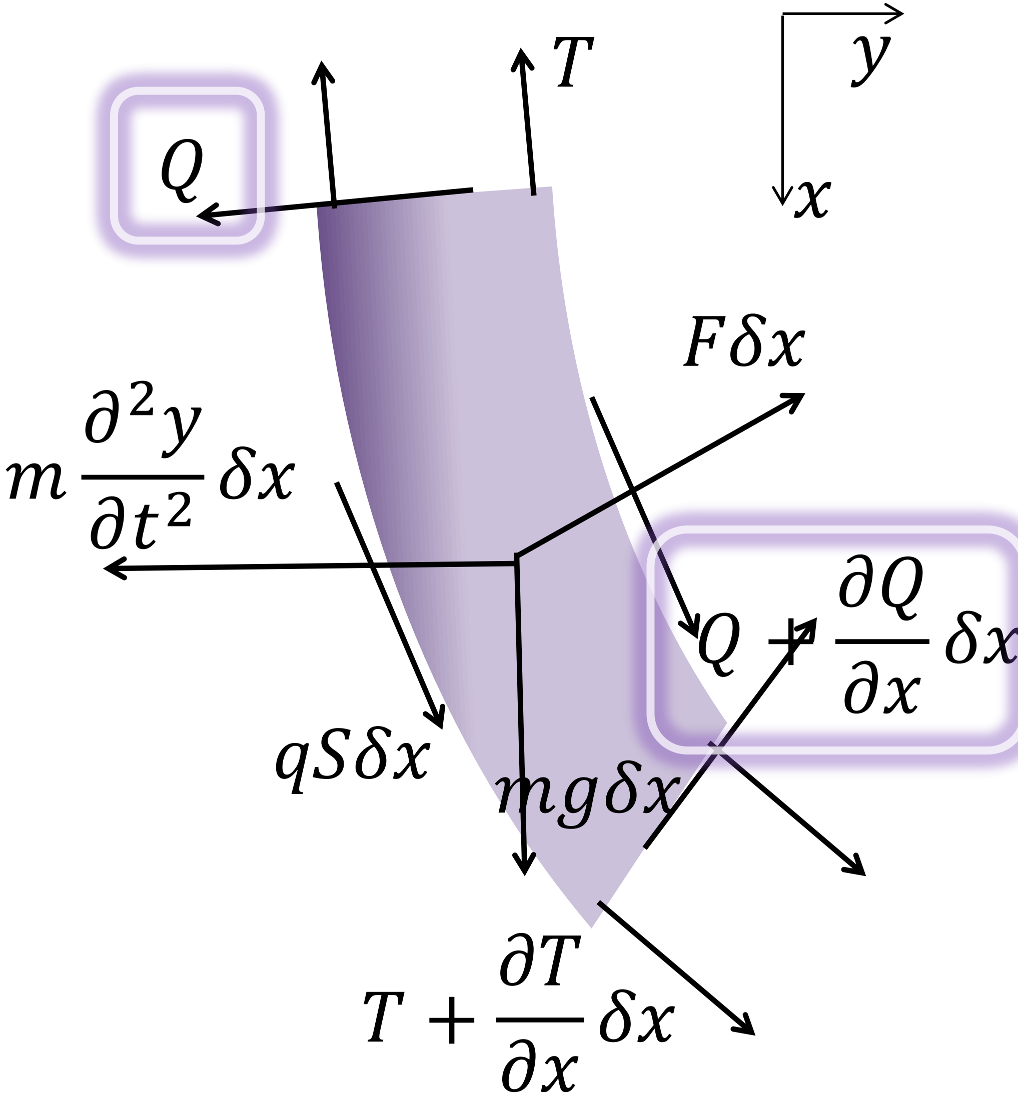
$$\therefore a_{centripetal} = MU^2\frac{\partial^2 y}{\partial x^2}\delta x$$

$$a_{coriolli} = 2\omega {v_x}'$$

$$= 2\frac{U}{R}\frac{\partial x}{\partial t} = 2U\frac{\partial^2 y}{\partial x \partial t}$$

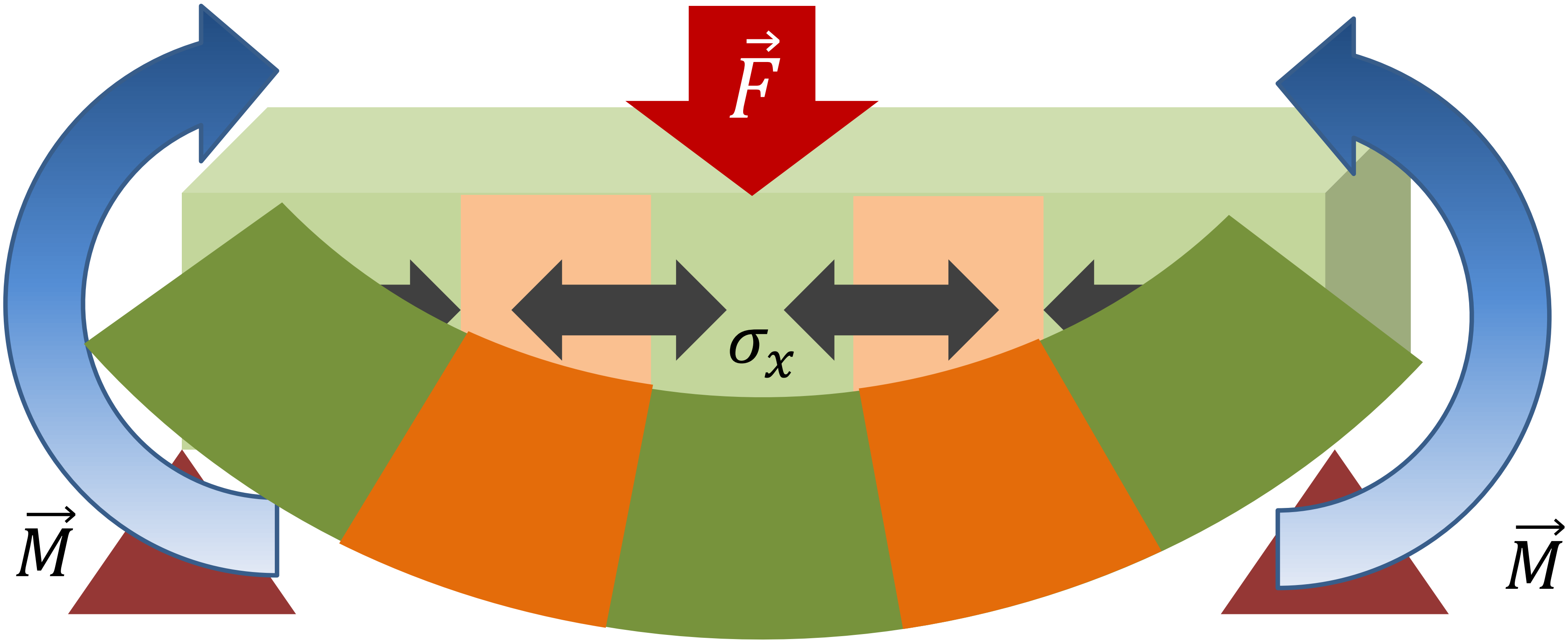
$$\boxed{a = M\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2 \delta x}$$

Force Analysis: Hose



T: Tension force
U: Flow velocity
m: Mass of hose per unit length
F: Force per unit length between tube and fluid
q: Shear stress on the tube
S: Internal perimeter
Q: Transverse shear force on the tube

Bending Moment

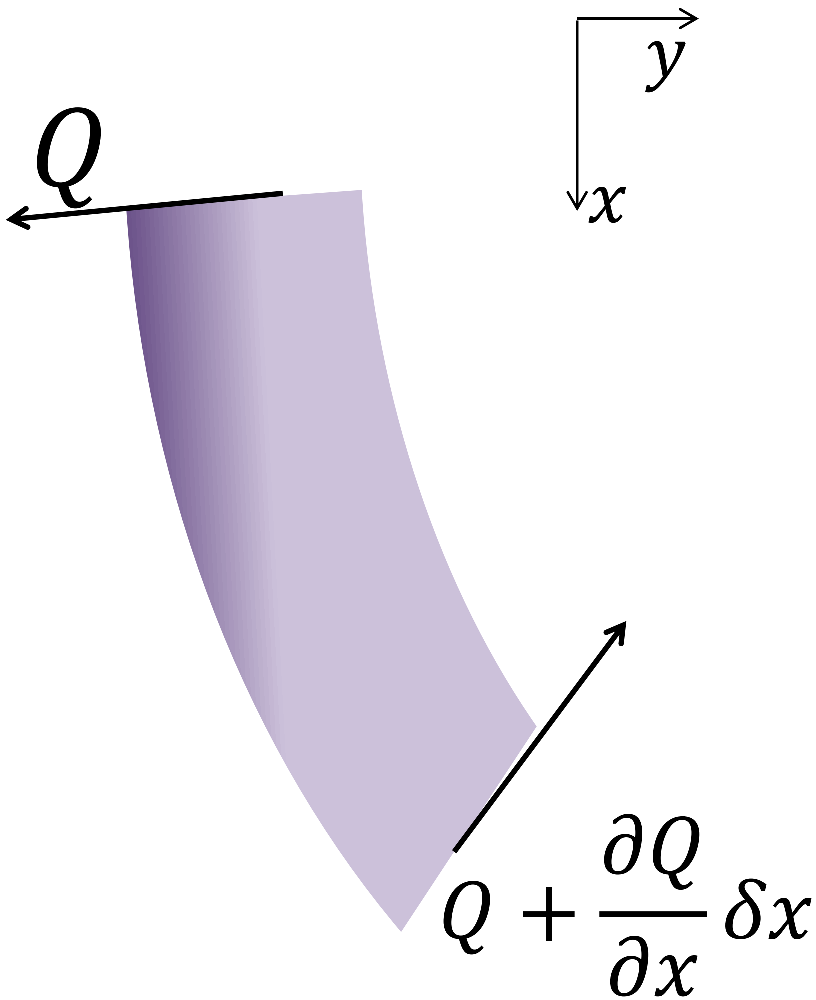


Bending Moment

$$M = \int_A \sigma_x y \, dA = \kappa \underline{EI}$$

Rigidity

Transverse shear force on the hose

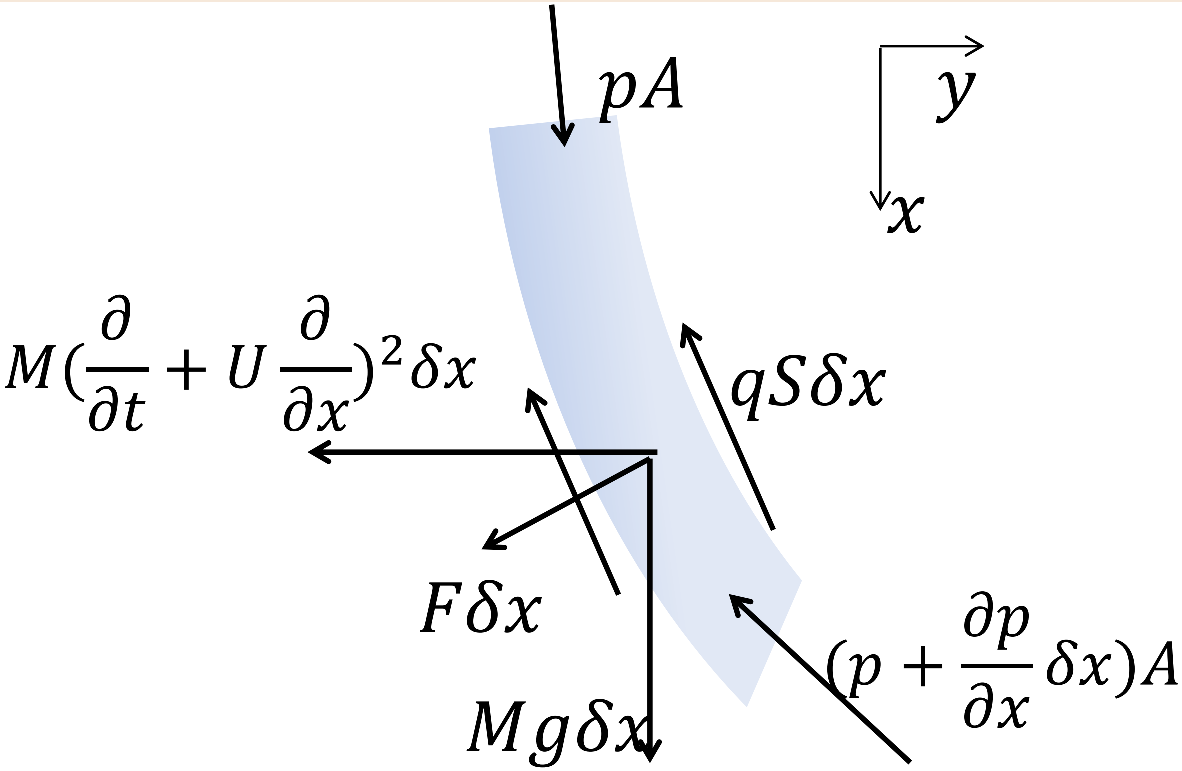


$$M = \kappa EI = EI\frac{\partial^2 y}{\partial x^2}$$

$$Q = -\frac{\partial}{\partial x}M = -EI\frac{\partial^3 y}{\partial x^3}$$

$$\boxed{\therefore\ \frac{\partial Q}{\partial x}\delta x = -EI\frac{\partial^4 y}{\partial x^4}\delta x}$$

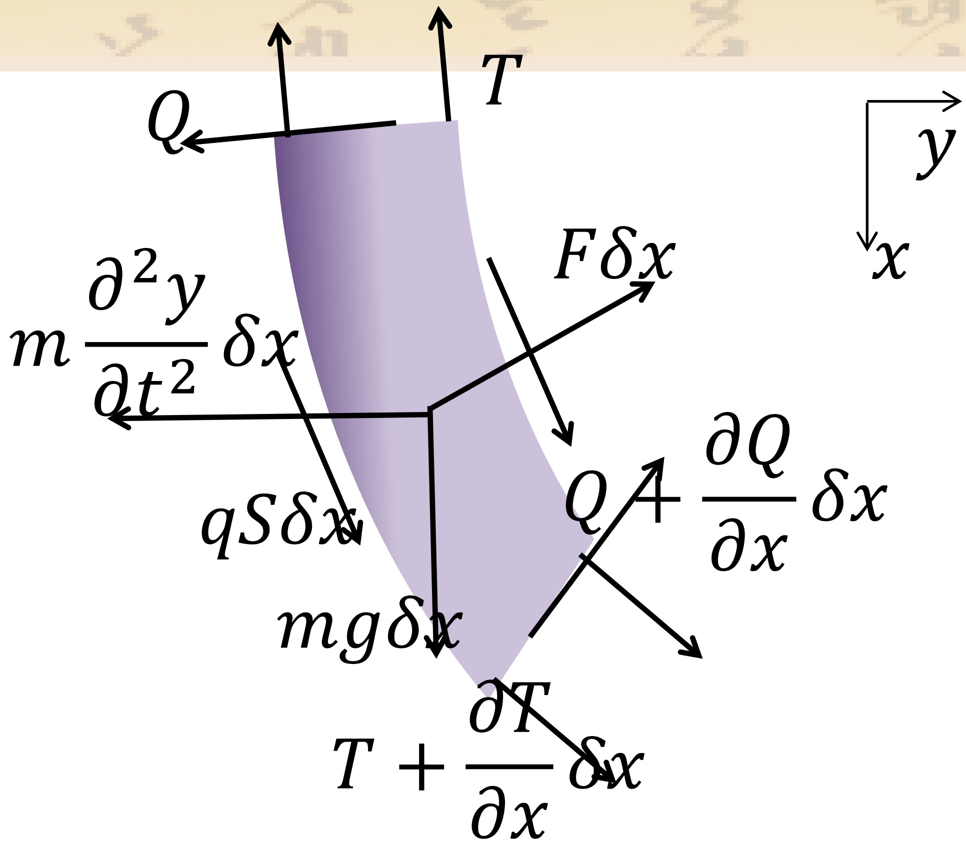
Force equation



x: $$-A\frac{\partial p}{\partial x} - qS + Mg + F\frac{\partial y}{\partial x} = 0$$

y: $$F + M(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x})^2 y + A\frac{\partial}{\partial x}\left(p\frac{\partial y}{\partial x}\right) + qS\frac{\partial y}{\partial x} = 0$$

Force equation



x:
$$\frac{\partial T}{\partial x}+qS+mg-F\frac{\partial y}{\partial x}=0$$

y:
$$-EI\frac{\partial^4 y}{\partial x^4}+F-m\frac{\partial^2 y}{\partial t^2}+\frac{\partial}{\partial x}\left(T\frac{\partial y}{\partial x}\right)+qS\frac{\partial y}{\partial x}=0$$

Combined Equation

For the end point & $T_L - p_L A = (M + m')gl$

$$\frac{\partial^2 y}{\partial t^2} = -\frac{EI}{M+m}\frac{\partial^4 y}{\partial x^4} - \frac{M}{M+m}U^2\frac{\partial^2 y}{\partial x^2} - 2\frac{M}{M+m}U\frac{\partial^2 y}{\partial x \partial t} - g\frac{\partial y}{\partial x}$$

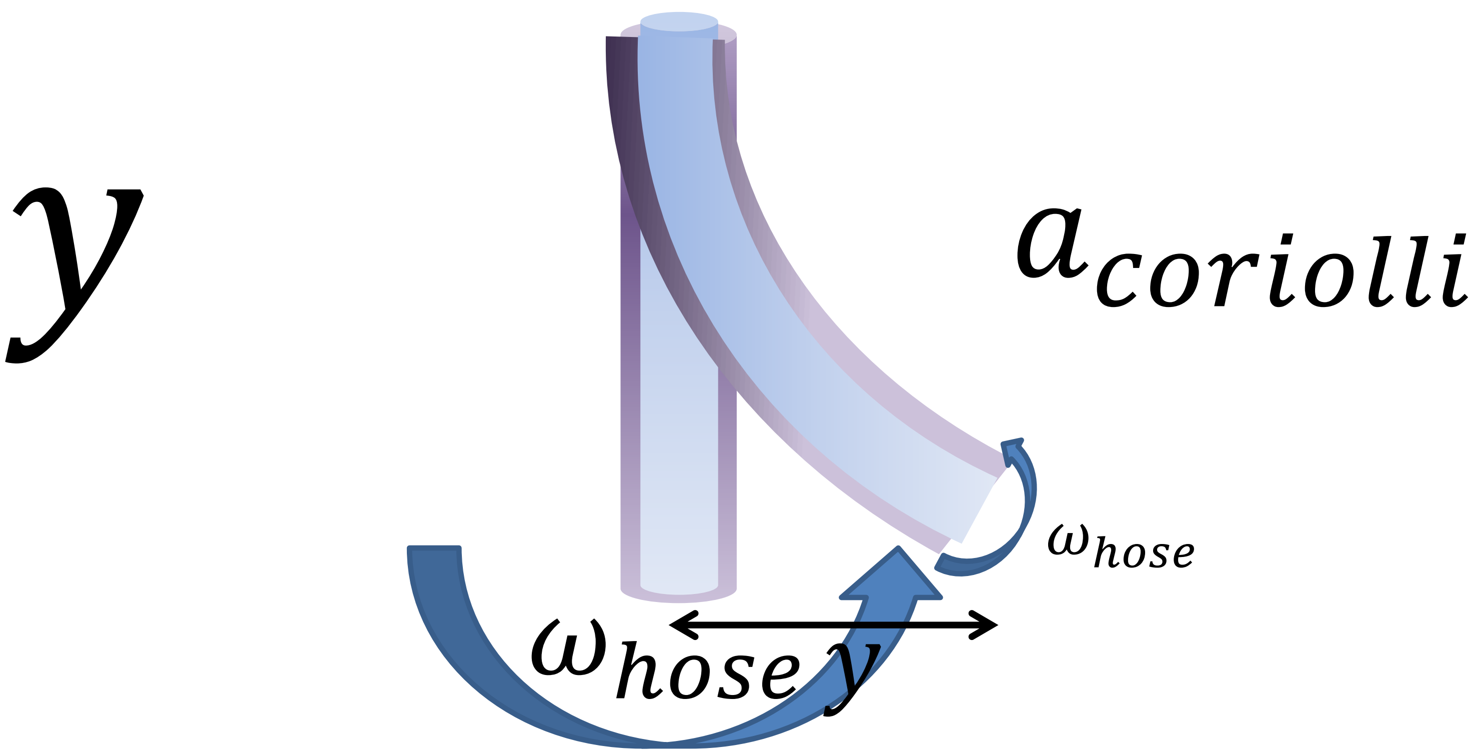
Flexural Restoring Force | **Centrifugal Force** | **Coriolis Force** | **Gravity**

Restoring force?

Coriolis force is not a restoring force!

$$a_{coriolli} = -2\frac{M}{M+m}U\frac{\partial^2 y}{\partial x \partial t}$$

$$= -2\frac{M}{M+m}U\frac{\partial}{\partial t}\left(\frac{\partial y}{\partial x}\right) = -2\frac{M}{M+m}U\omega_{hose}$$



Restoring forces are…

$$\frac{\partial^2 y}{\partial t^2} = -\frac{EI}{M+m}\frac{\partial^4 y}{\partial x^4} - \frac{M}{M+m}U^2\frac{\partial^2 y}{\partial x^2} - 2\frac{M}{M+m}U\frac{\partial^2 y}{\partial x \partial t} - g\frac{\partial y}{\partial x}$$

Flexural Restoring Force

Centrifugal Force

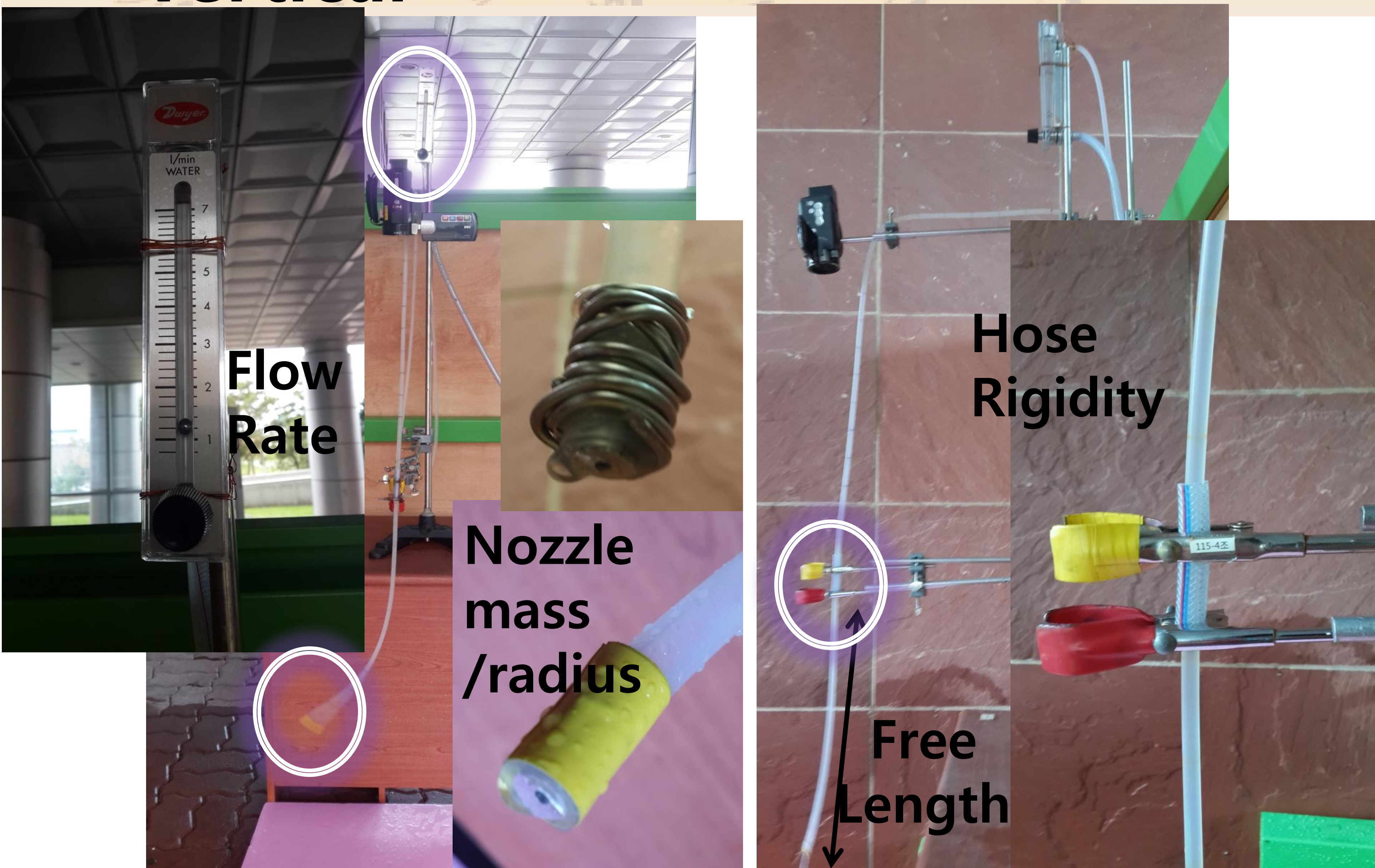
Coriolis Force

Gravity

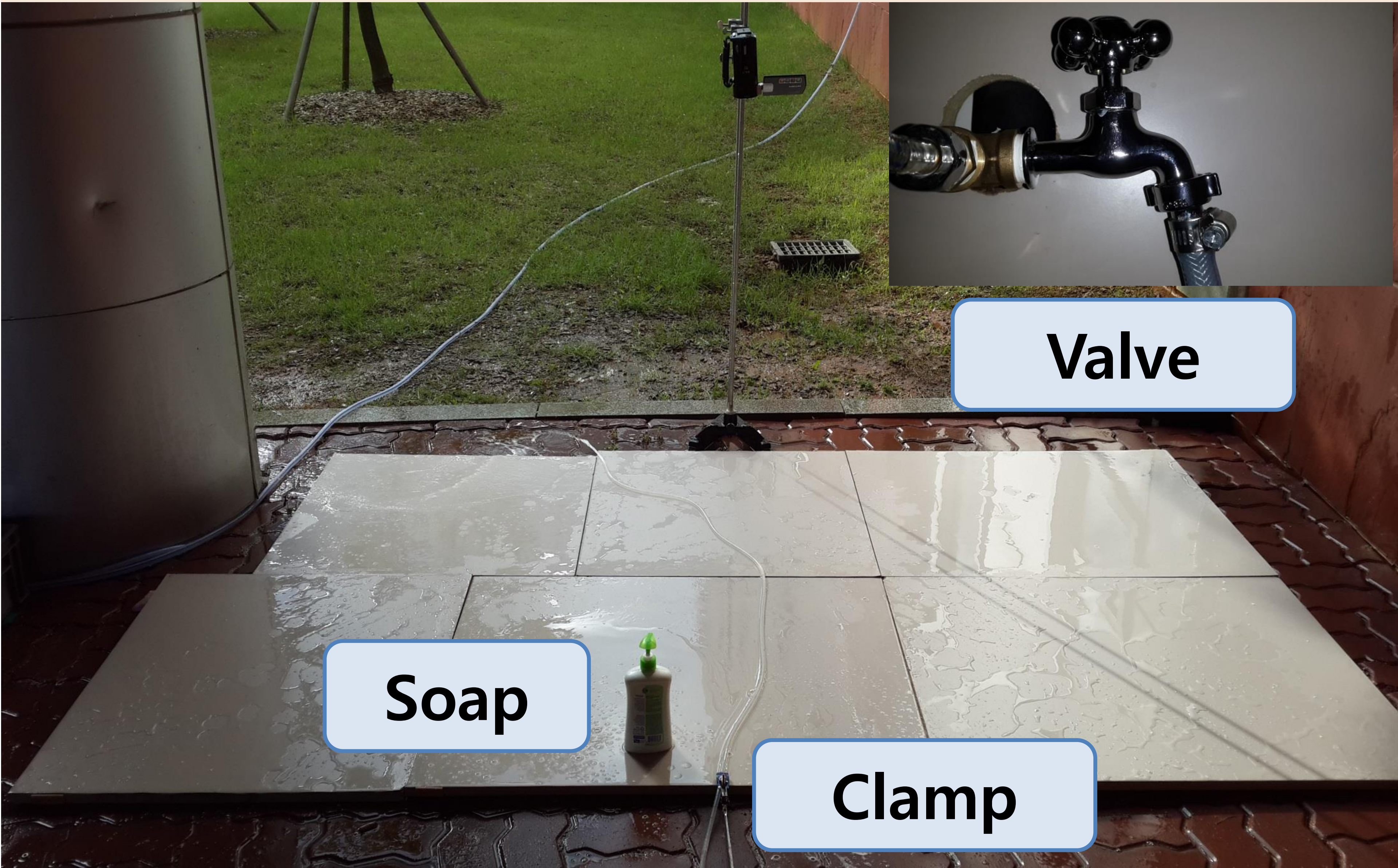
Restoring force!

T

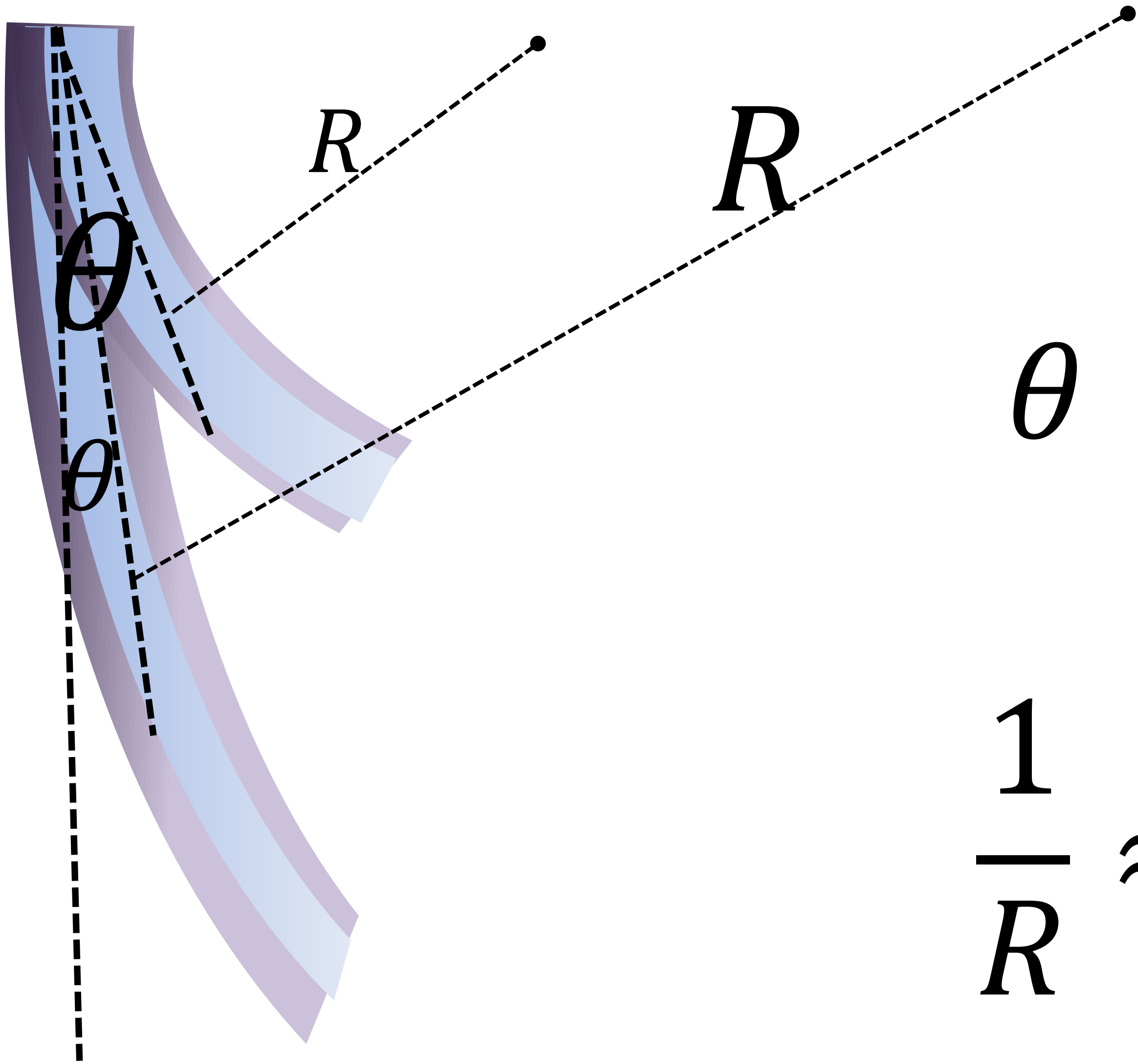
Experimental Setting - Vertical



Experimental Setting - Horizontal



Free length – Oscillation period Tendency



$$\theta \approx \frac{dy}{dx}$$

$$\frac{1}{R} \approx \frac{d^2y}{dx^2}$$

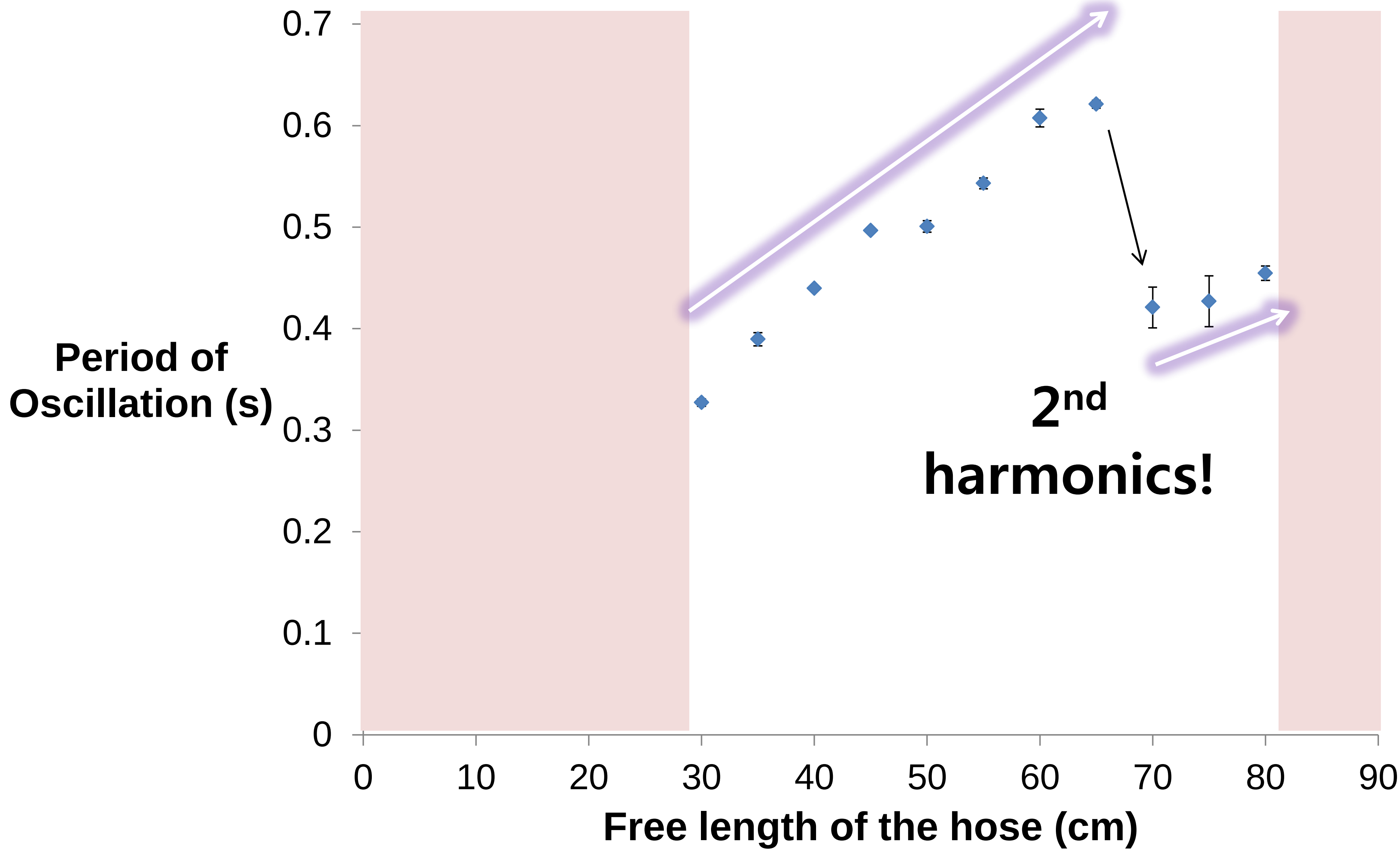
Free length – Oscillation period Tendency

$$\frac{\partial^2 y}{\partial t^2} = -\frac{EI}{M+m}\frac{\partial^4 y}{\partial x^4} - \frac{M}{M+m}U^2\boxed{\frac{\partial^2 y}{\partial x^2}} - 2\frac{M}{M+m}U\frac{\partial^2 y}{\partial x \partial t} - g\boxed{\frac{\partial y}{\partial x}}$$

Flexural Restoring Force	Centrifugal Force	Coriolis Force	Gravity

T

Free length – Oscillation period Graph (Vertical)

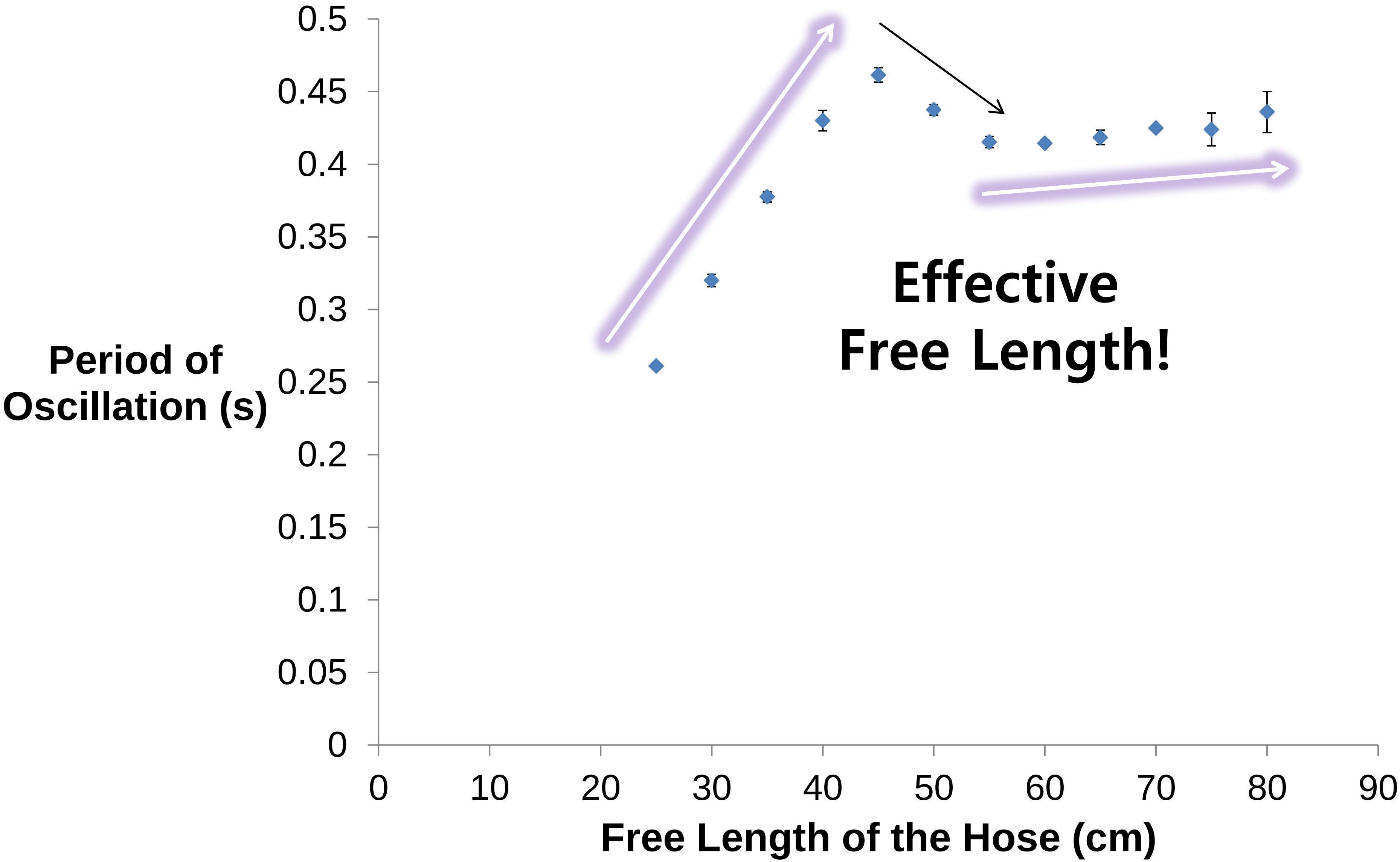


Modes of Oscillation

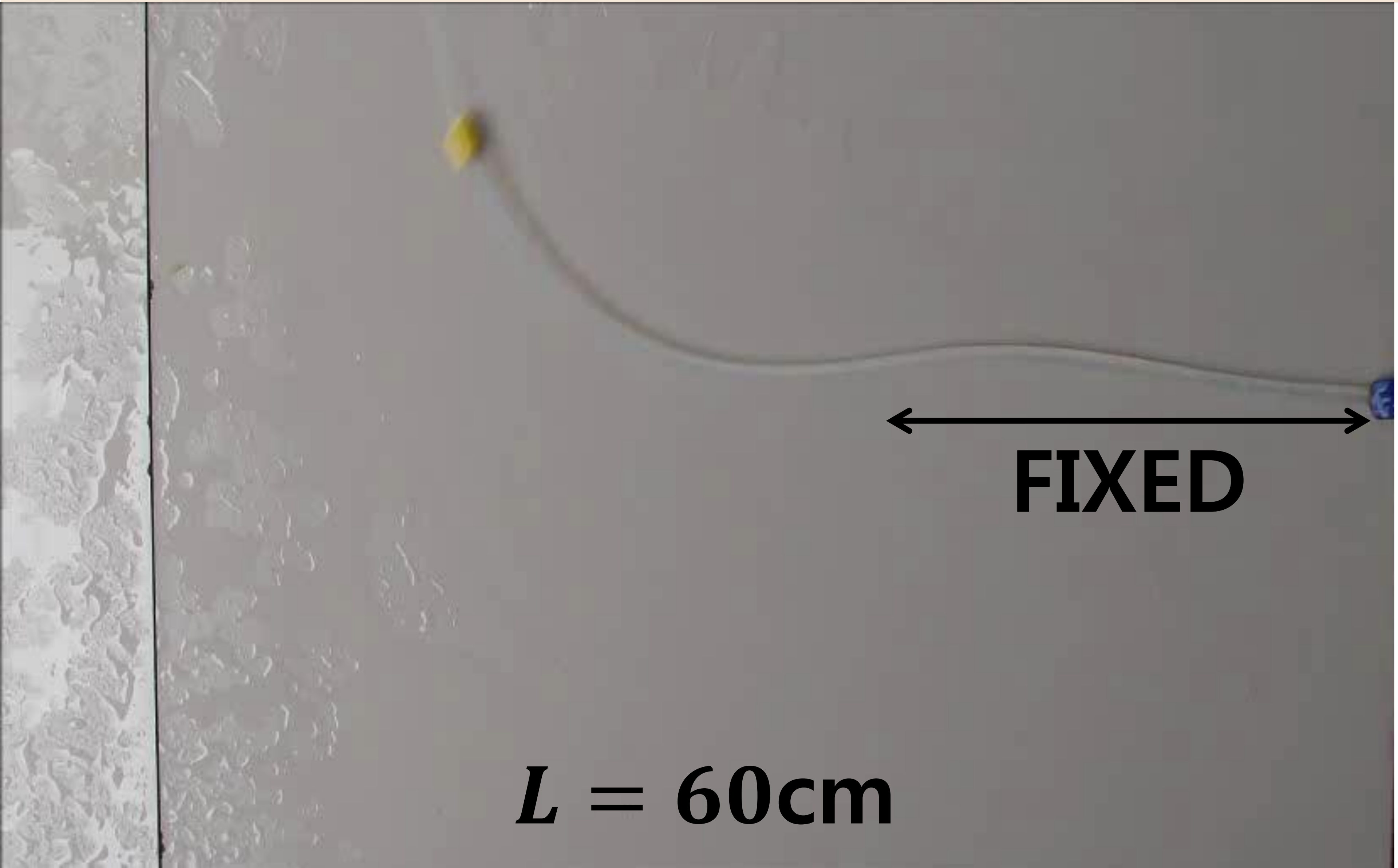
30cm

70cm

Free length – Oscillation period Graph (Horizontal)



Effective Free Length



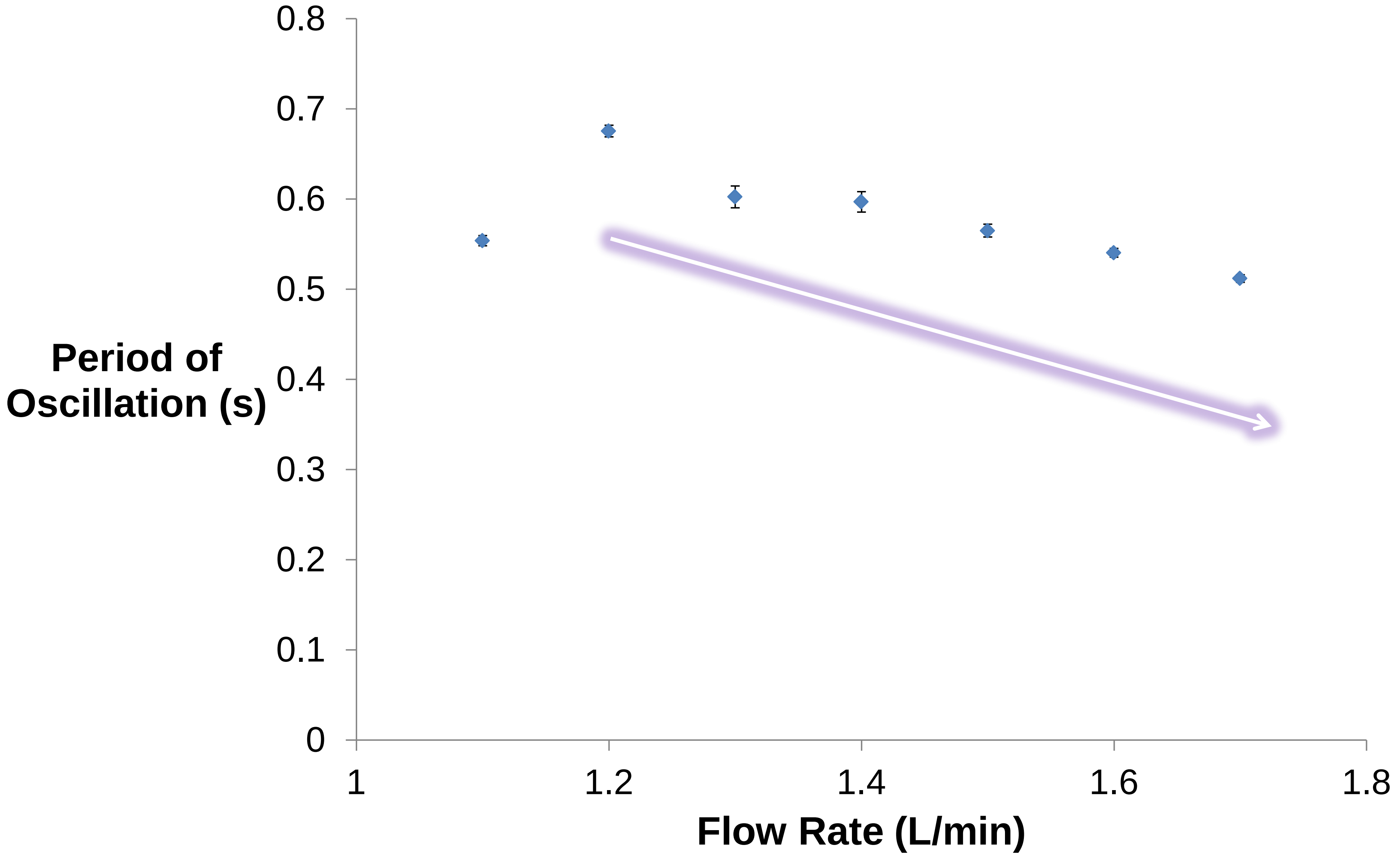
Flow rate – Oscillation period Tendency

$$\frac{\partial^2 y}{\partial t^2} = -\frac{EI}{M+m}\frac{\partial^4 y}{\partial x^4} - \frac{M}{M+m}\boxed{U^2}\frac{\partial^2 y}{\partial x^2} - 2\frac{M}{M+m}U\frac{\partial^2 y}{\partial x \partial t} - g\frac{\partial y}{\partial x}$$

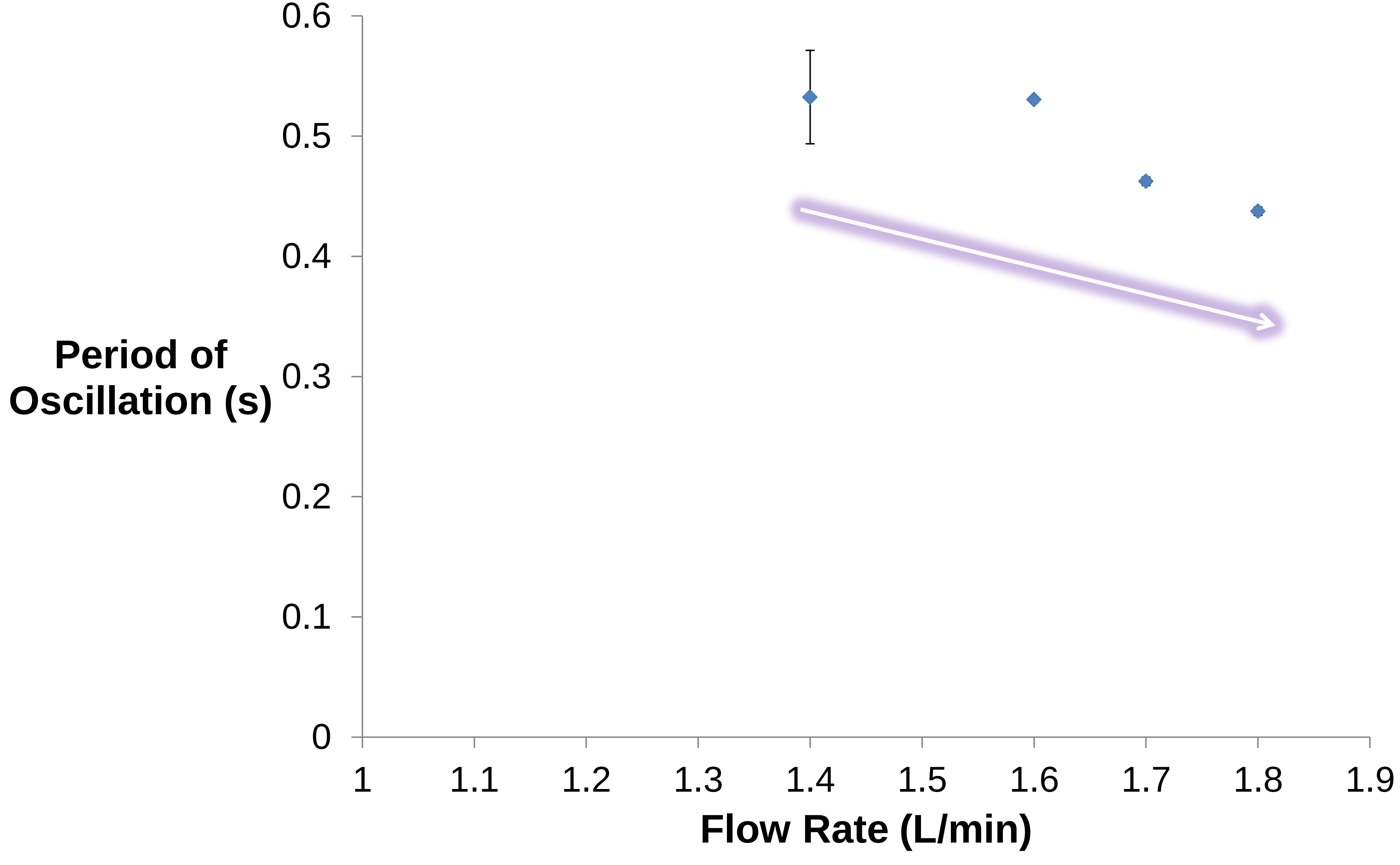
Flexural Restoring Force	Centrifugal Force	Coriolis Force	Gravity

T

Flow rate – Oscillation period Graph (Vertical)



Flow rate – Oscillation period Graph (Horizontal)



Nozzle mass– Oscillation period Tendency

$$\frac{\partial^2 y}{\partial t^2} = -\frac{EI}{M+\boxed{m}}\frac{\partial^4 y}{\partial x^4} - \frac{M}{M+\boxed{m}}U^2\frac{\partial^2 y}{\partial x^2} - 2\frac{M}{M+m}U\frac{\partial^2 y}{\partial x \partial t} - g\frac{\partial y}{\partial x}$$

Flexural Restoring Force

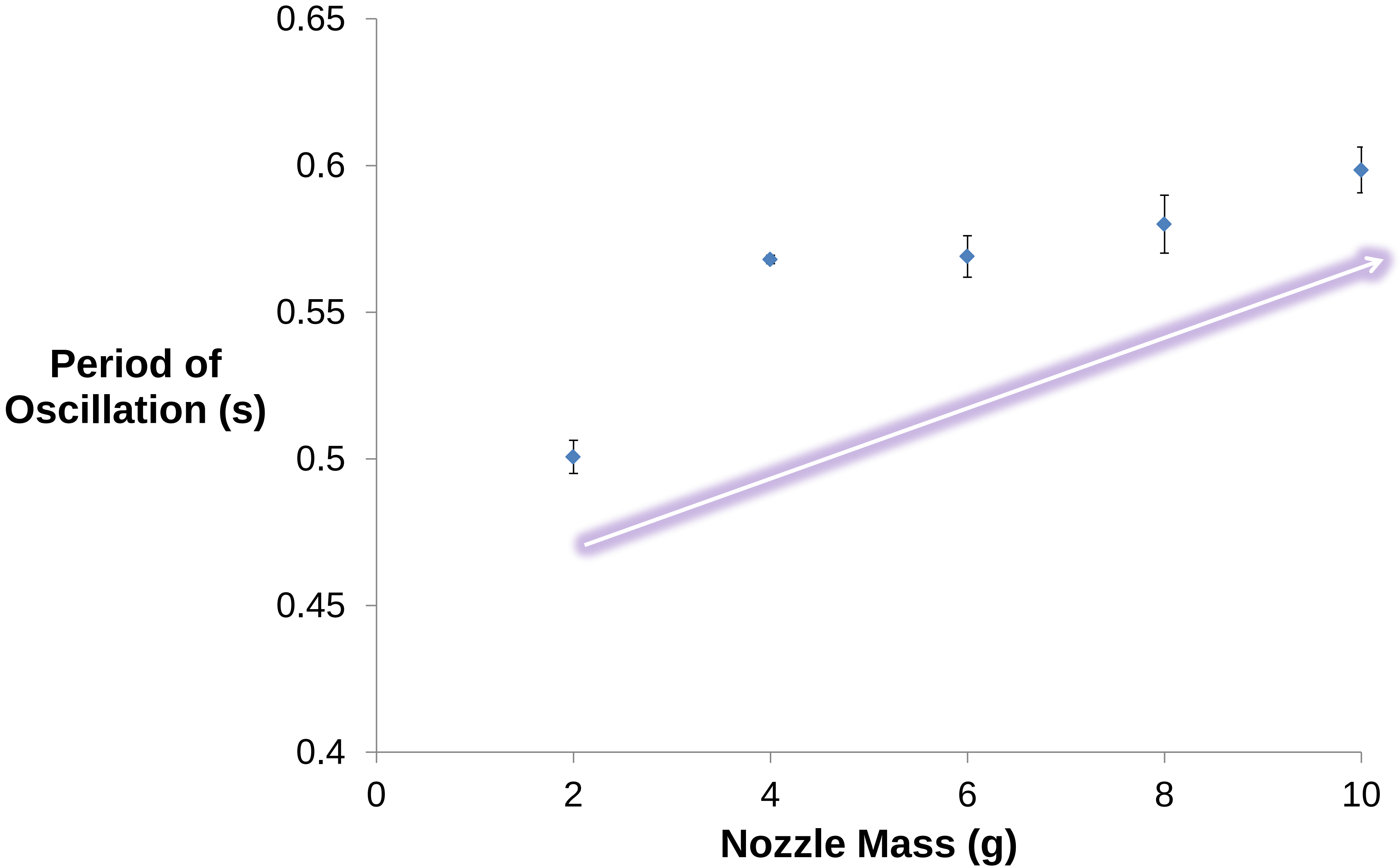
Centrifugal Force

Coriolis Force

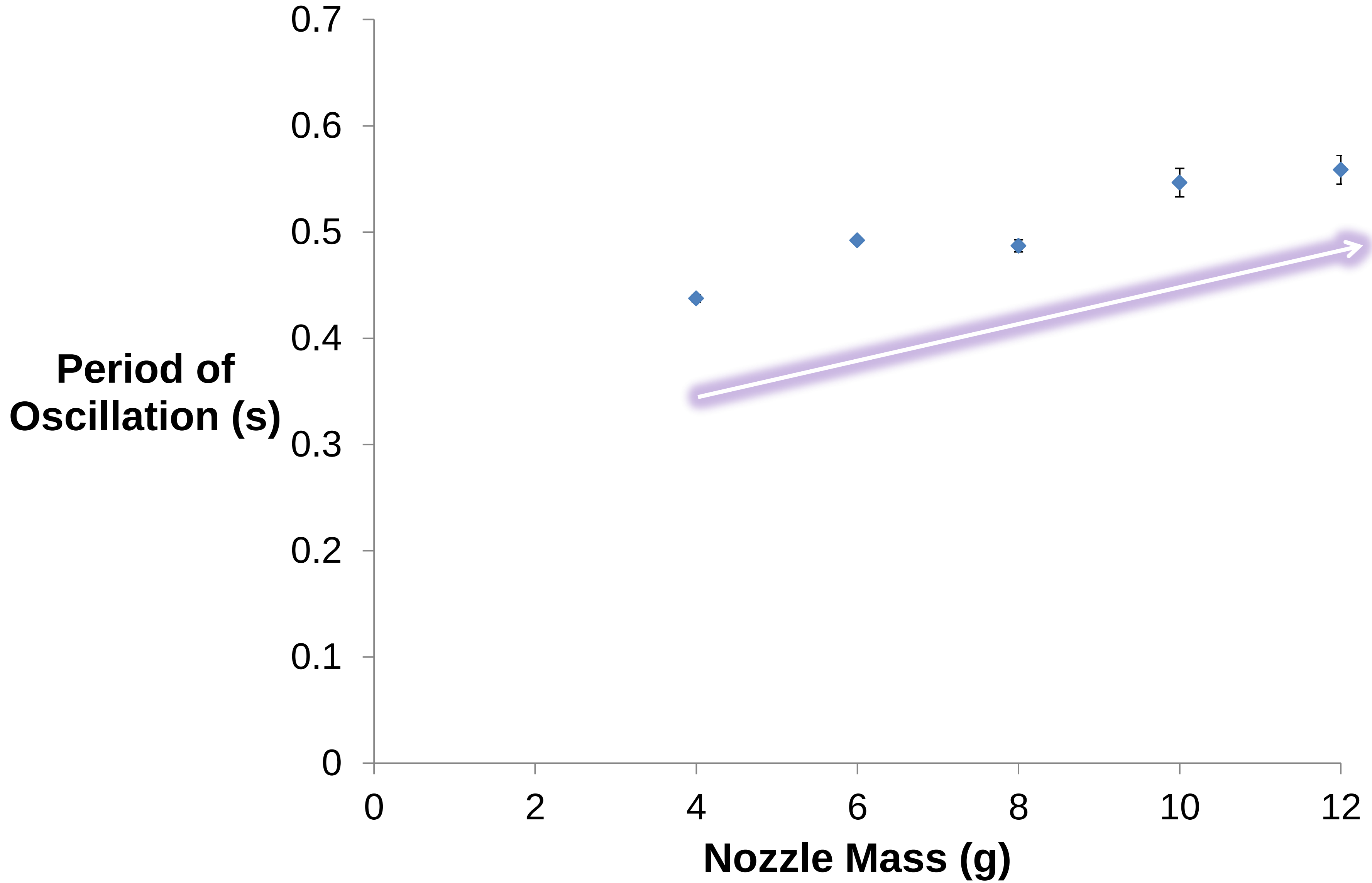
Gravity

T

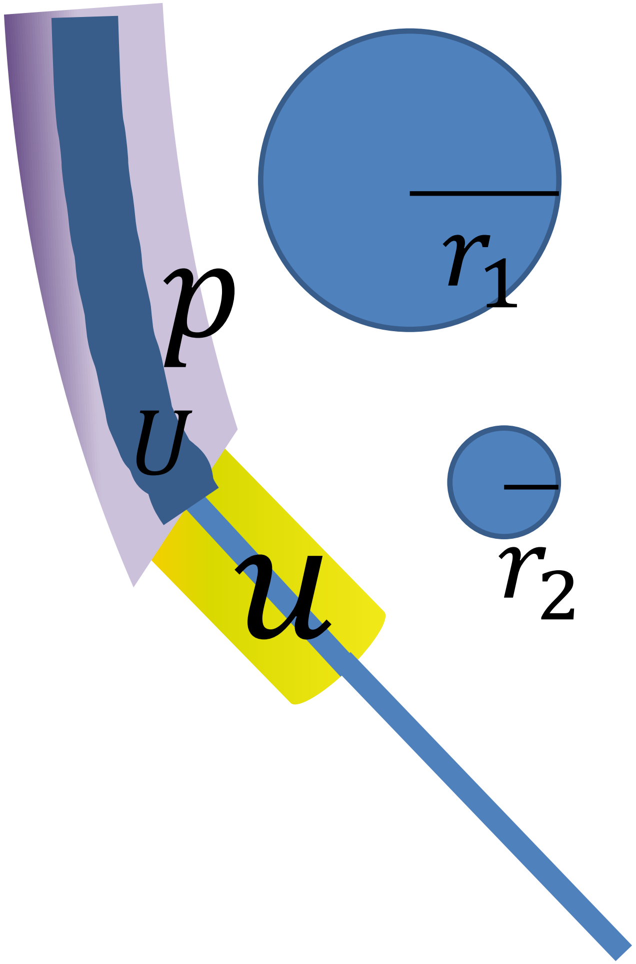
Nozzle mass – Oscillation period Graph (Vertical)



Nozzle mass – Oscillation period Graph (Horizontal)



Nozzle radius – Oscillation period Tendency



$$p + \frac{1}{2}\rho U^2 = \frac{1}{2}\rho u^2$$

$${r_1}^2 U_L = {r_2}^2 U_l$$

$$U_L = \sqrt{\frac{2p}{\rho\left(\frac{{r_1}^2}{{r_2}^2} - 1\right)}}$$

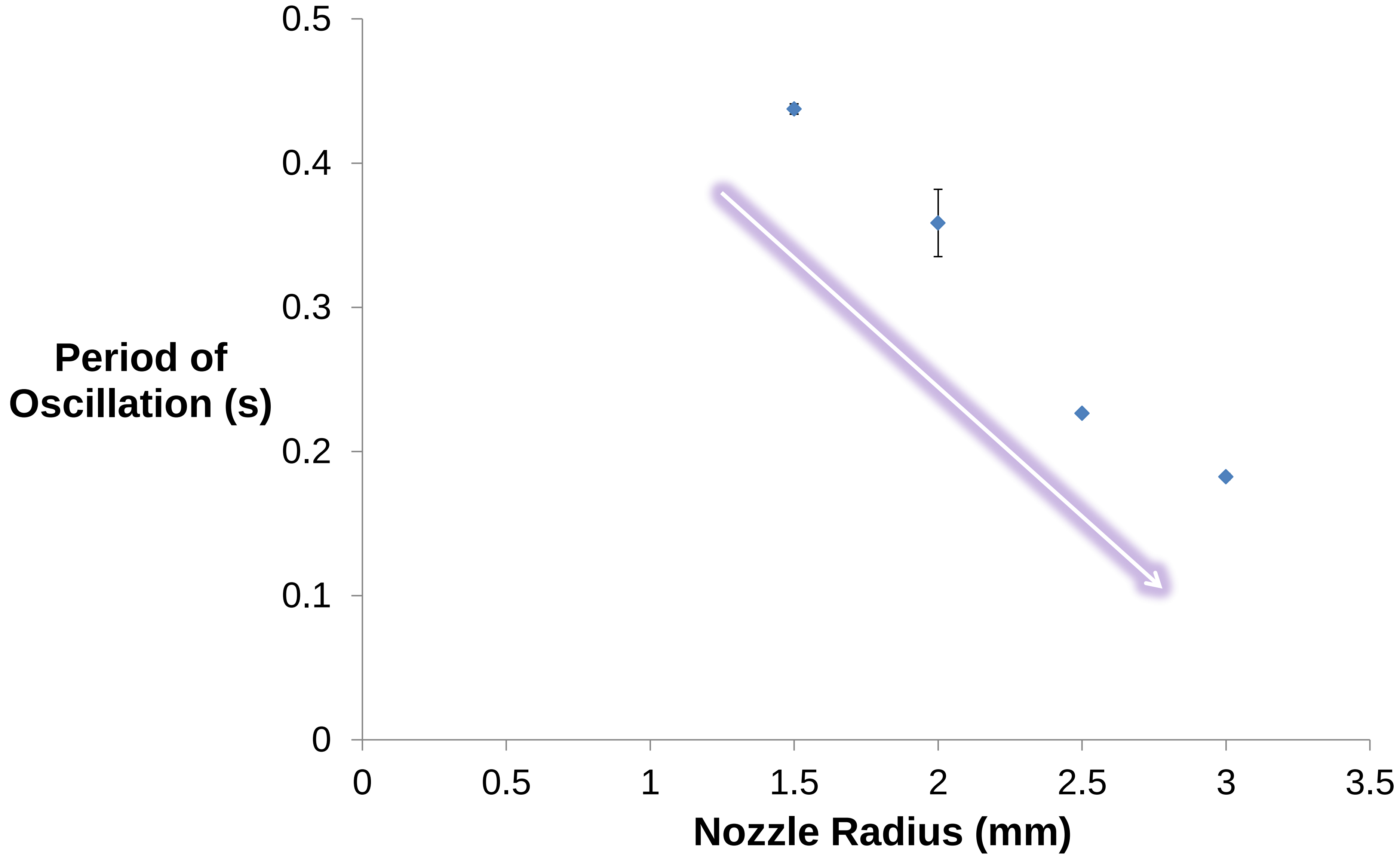
Nozzle radius – Oscillation period Tendency

$$\frac{\partial^2 y}{\partial t^2} = -\frac{EI}{M+m}\frac{\partial^4 y}{\partial x^4} - \frac{M}{M+m}\boxed{U^2}\frac{\partial^2 y}{\partial x^2} - 2\frac{M}{M+m}U\frac{\partial^2 y}{\partial x \partial t} - g\frac{\partial y}{\partial x}$$

Flexural Restoring Force	Centrifugal Force	Coriolis Force	Gravity

T

Nozzle radius – Oscillation period Graph (Horizontal)



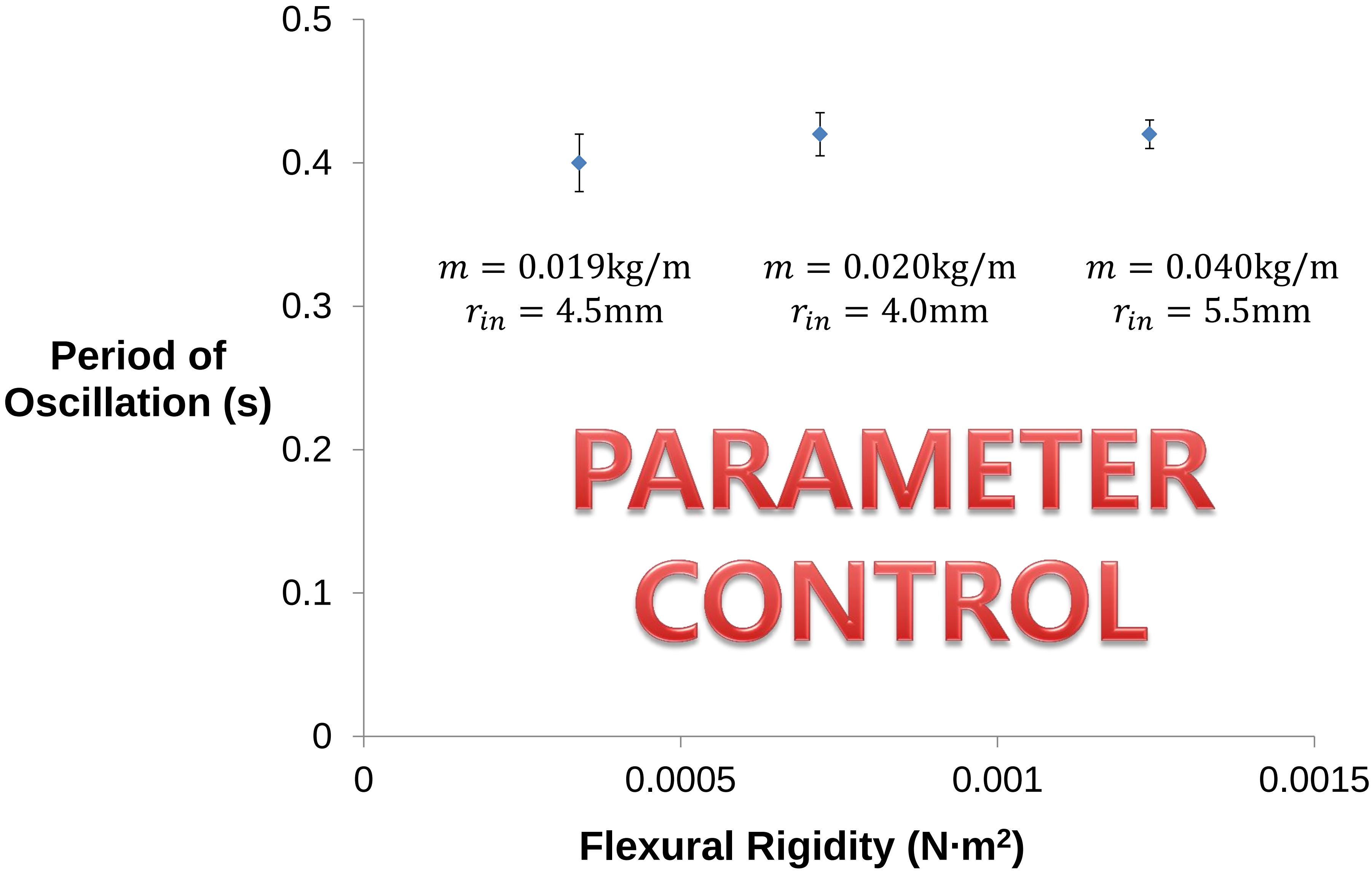
Hose Rigidity– Oscillation period Tendency

$$\frac{\partial^2 y}{\partial t^2} = -\frac{\boxed{EI}}{M+m}\frac{\partial^4 y}{\partial x^4} - \frac{M}{M+m}U^2\frac{\partial^2 y}{\partial x^2} - 2\frac{M}{M+m}U\frac{\partial^2 y}{\partial x \partial t} - g\frac{\partial y}{\partial x}$$

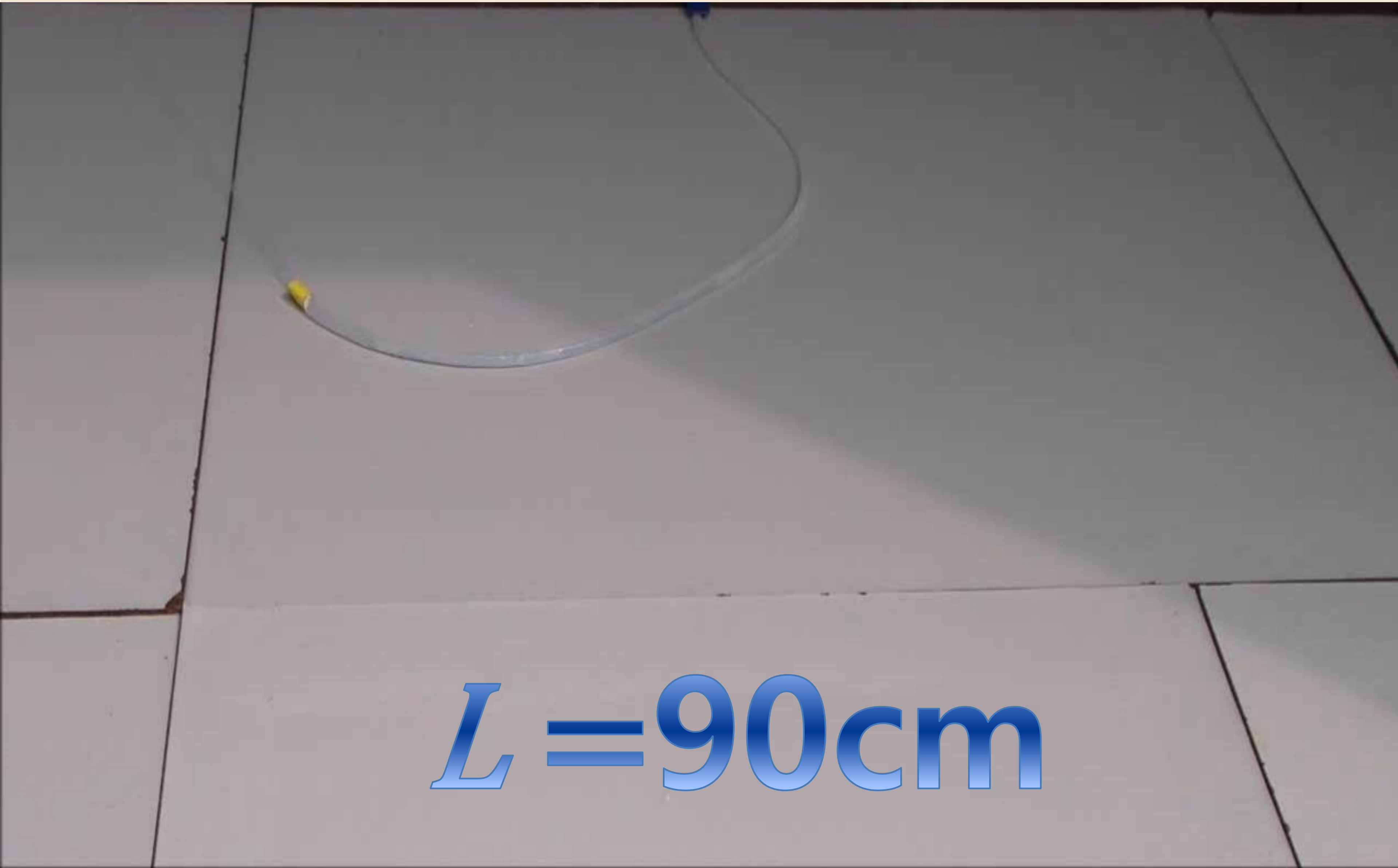
Flexural Restoring Force	Centrifugal Force	Coriolis Force	Gravity

T

Hose Rigidity– Oscillation period Graph (Vertical)

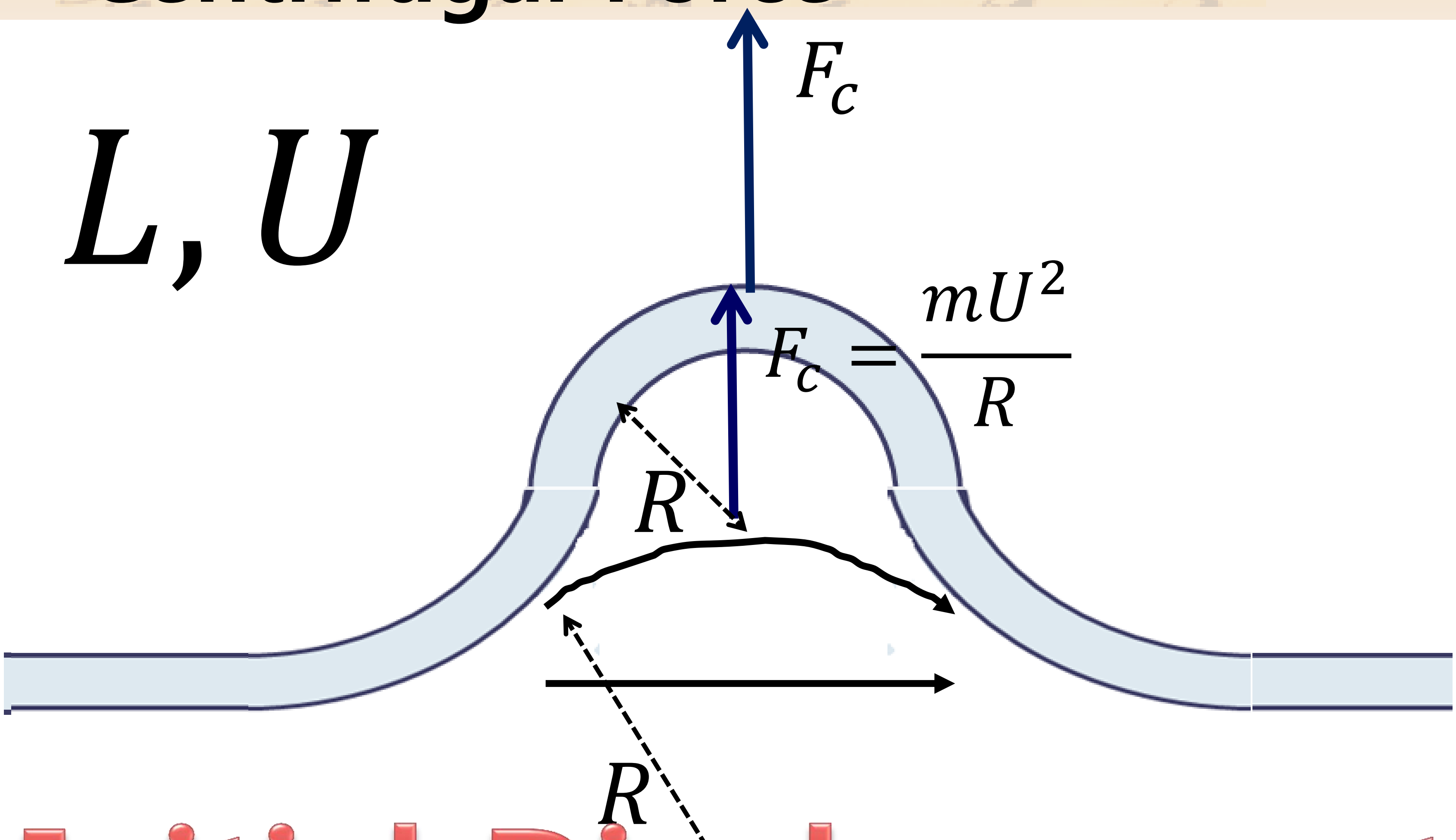


Why does instable motion occur?



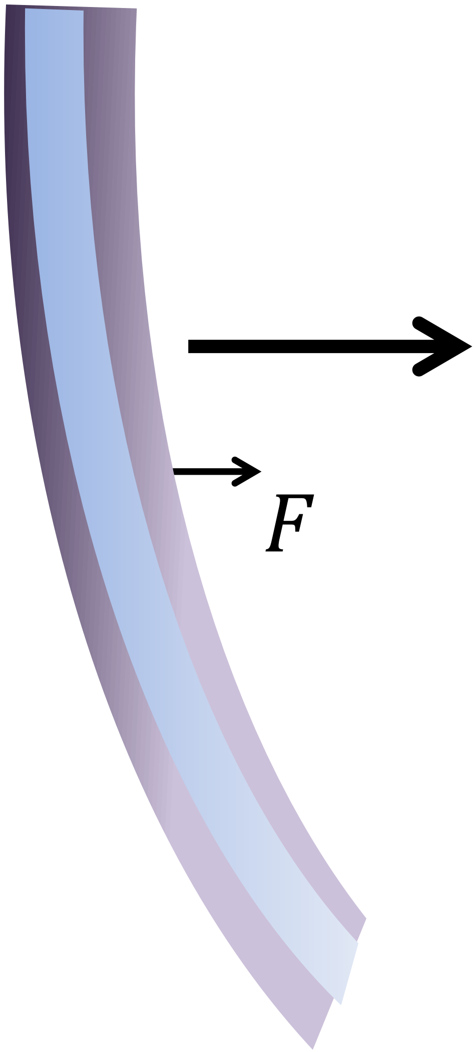
Instability 1 : Centrifugal Force

$$L, U$$



Initial Displacement

Instability 2: Friction



μ_k

Discontinuity On Time

Conclusion

Parameters : T

Free Length of the Hose

Flow Rate

Nozzle Mass

Nozzle Radius

Rigidity of the Hose

Instability

Centrifugal Force

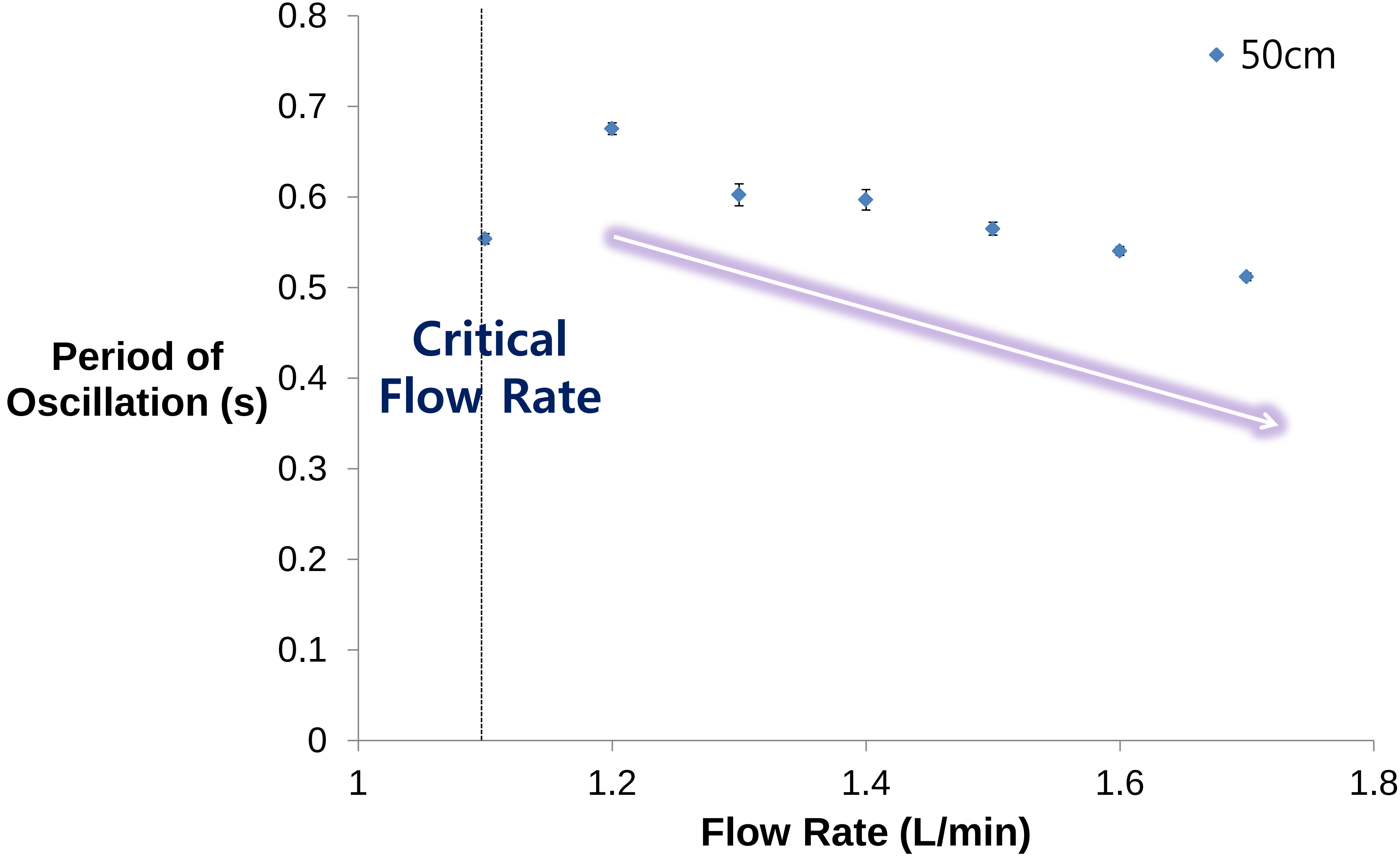
Friction

Thank You

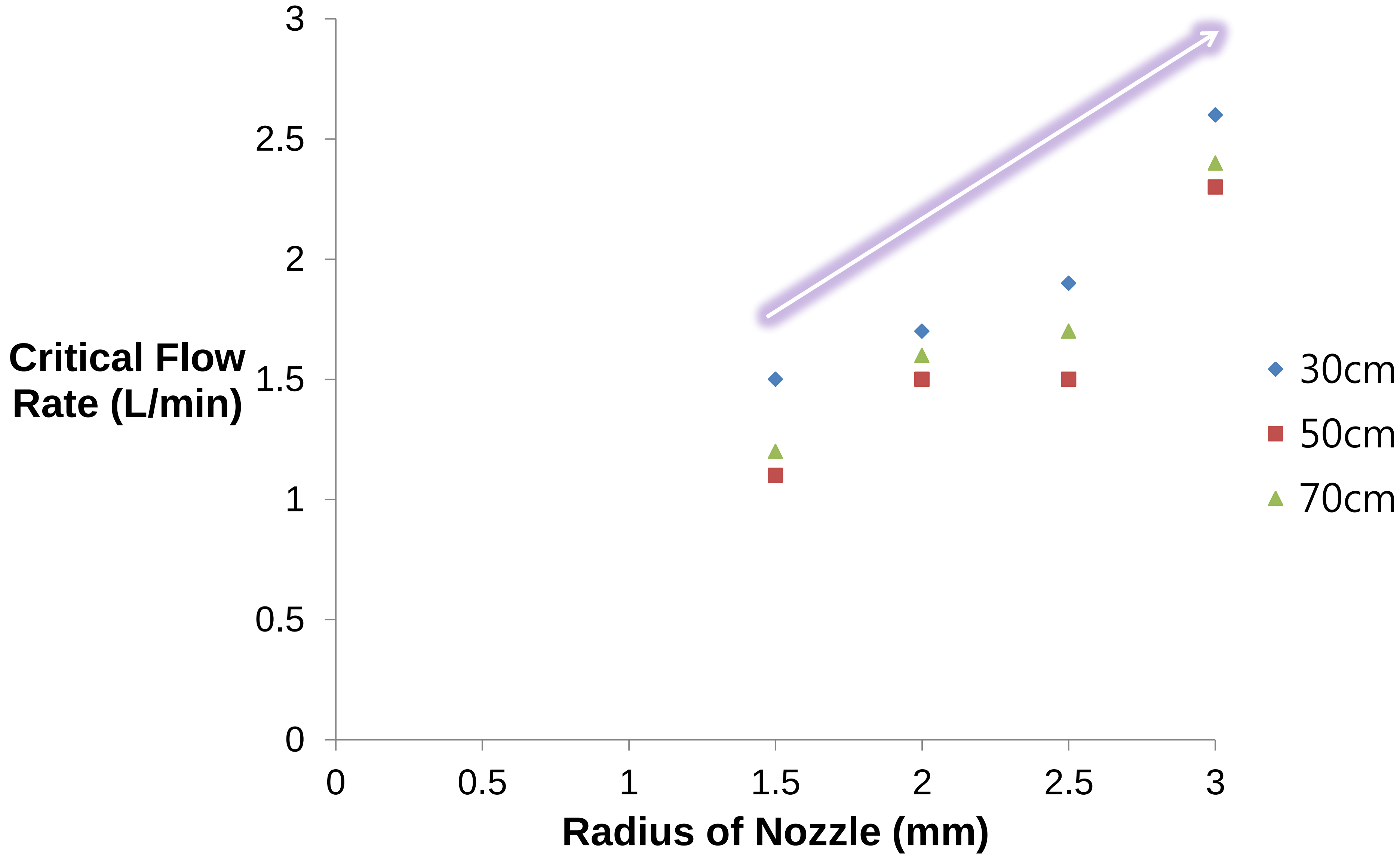
Problem statement

Consider a hose with a **water jet** coming from its **nozzle**. Release the hose and observe its subsequent **motion**. Determine the **parameters** that affect this motion.

Flow rate – Oscillation period Graph



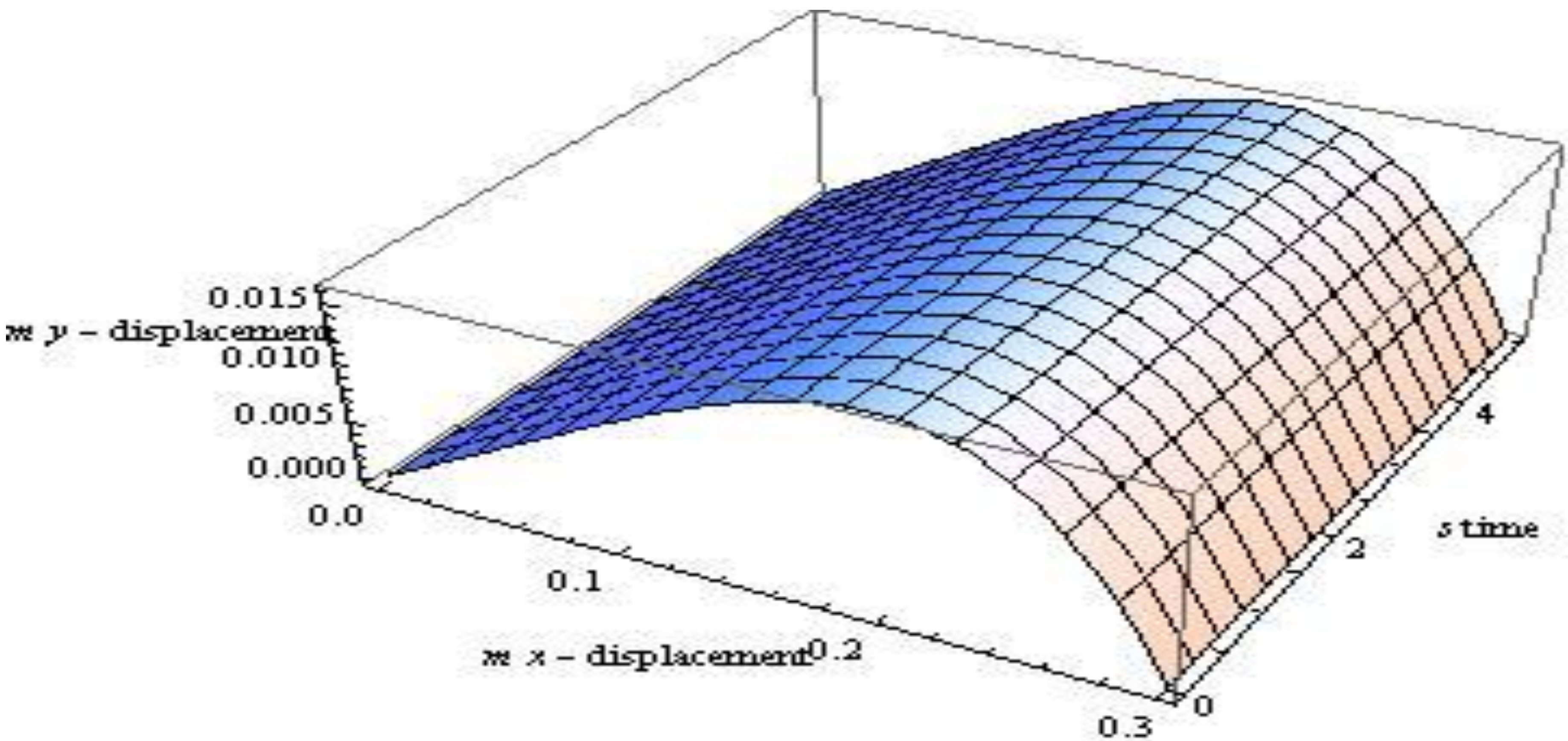
Nozzle radius – Critical flow rate Graph



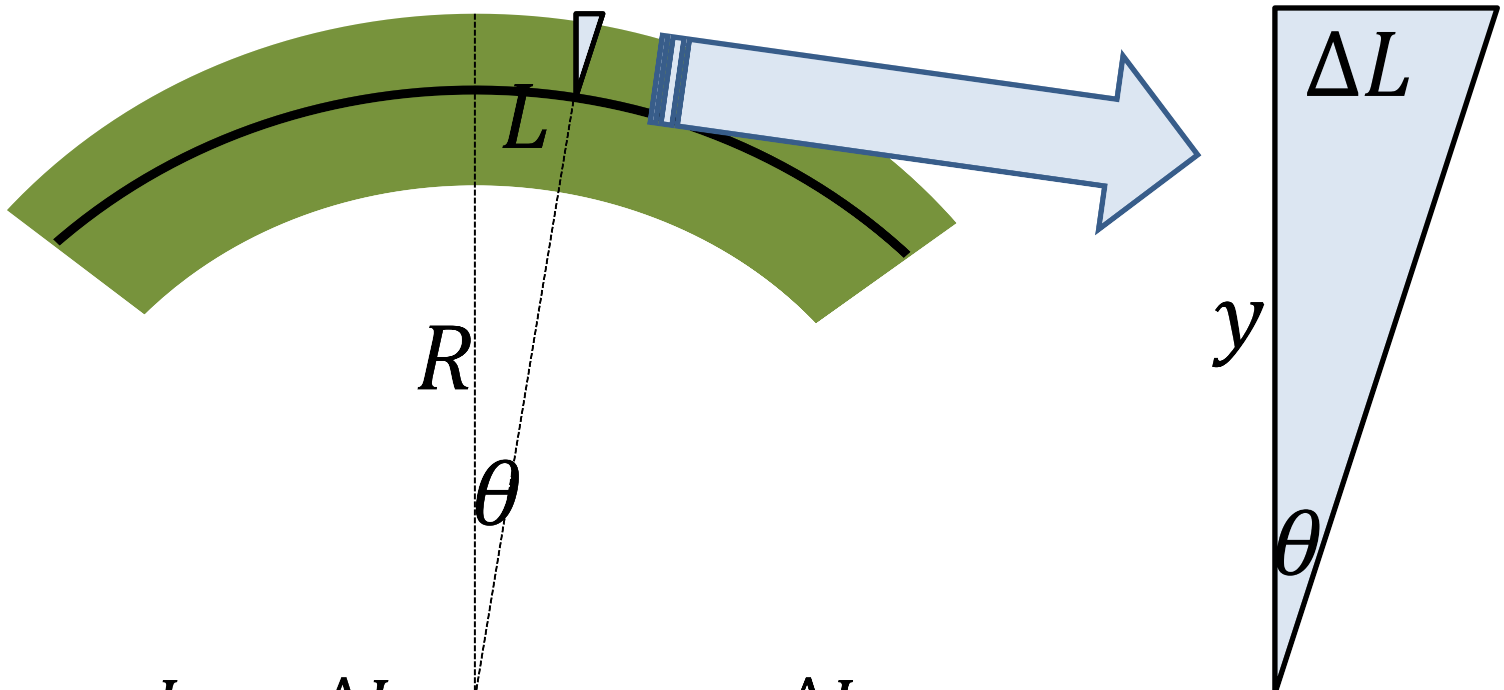
Solving the Differential Equation

$$y(0,t) = 0$$

$$\frac{\partial}{\partial x} y(x,t) \mid_{x=0} = \frac{\partial^2}{\partial x^2} y(x,t) \mid_{x=0} = \frac{\partial^3}{\partial x^3} y(x,t) \mid_{x=0} = 0.01$$



Bending Moment and Curvature



$$\theta = \frac{L}{R} = \frac{\Delta L}{y}\ . \quad y = R\frac{\Delta L}{L} = R\varepsilon\ . \quad \sigma = E\varepsilon = E\,\frac{y}{R}$$

$$M = \frac{E}{R}\int y^2 dA = \underline{\kappa EI}$$

Rigidity

Combined Equation – Length term included

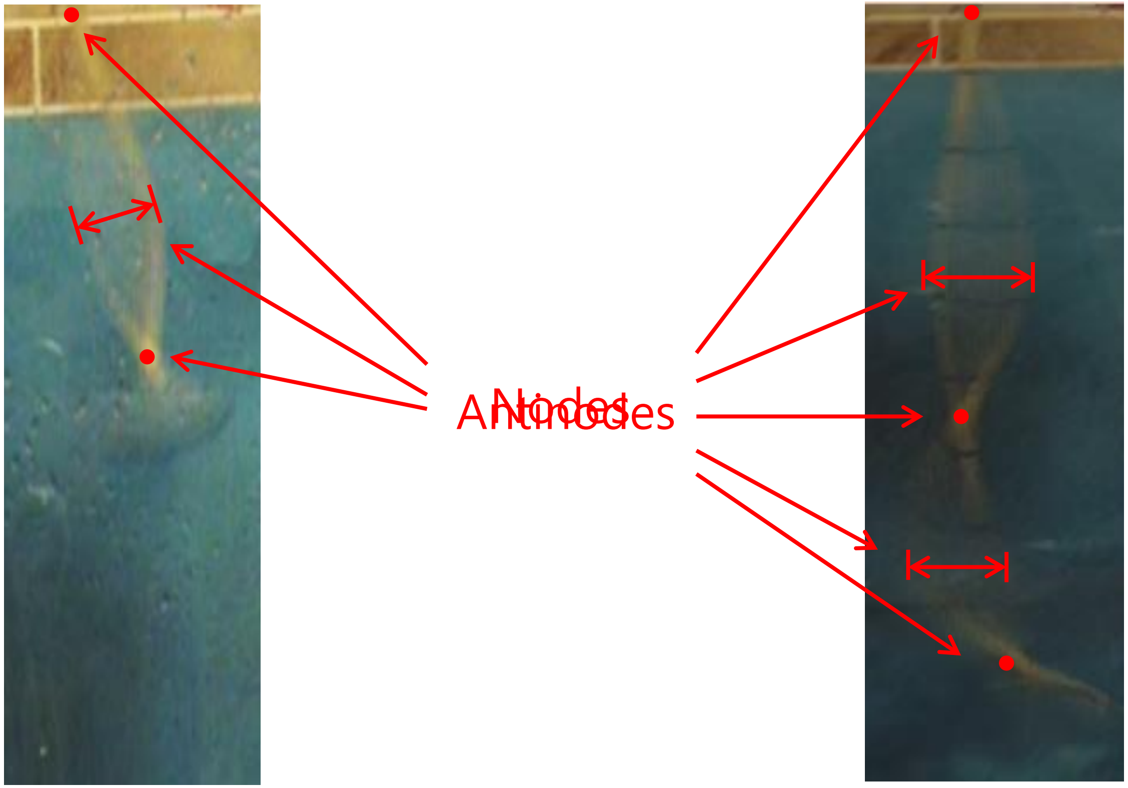
$$T_L - p_L A = (M + m')gl)$$

$$\frac{\partial^2 y}{\partial t^2} = -\frac{EI}{M+m}\frac{\partial^4 y}{\partial x^4} - \left(\frac{M}{M+m}U^2 + (L + l - x)g\right)\frac{\partial^2 y}{\partial x^2} - 2\frac{M}{M+m}U\frac{\partial^2 y}{\partial x \partial t} - g\frac{\partial y}{\partial x}$$

Pressure & Tension term Included!

Different oscillation modes in same length

Nodes and Antinodes



1st harmonic oscillation

2nd harmonic oscillation