Problem #5

Levitation

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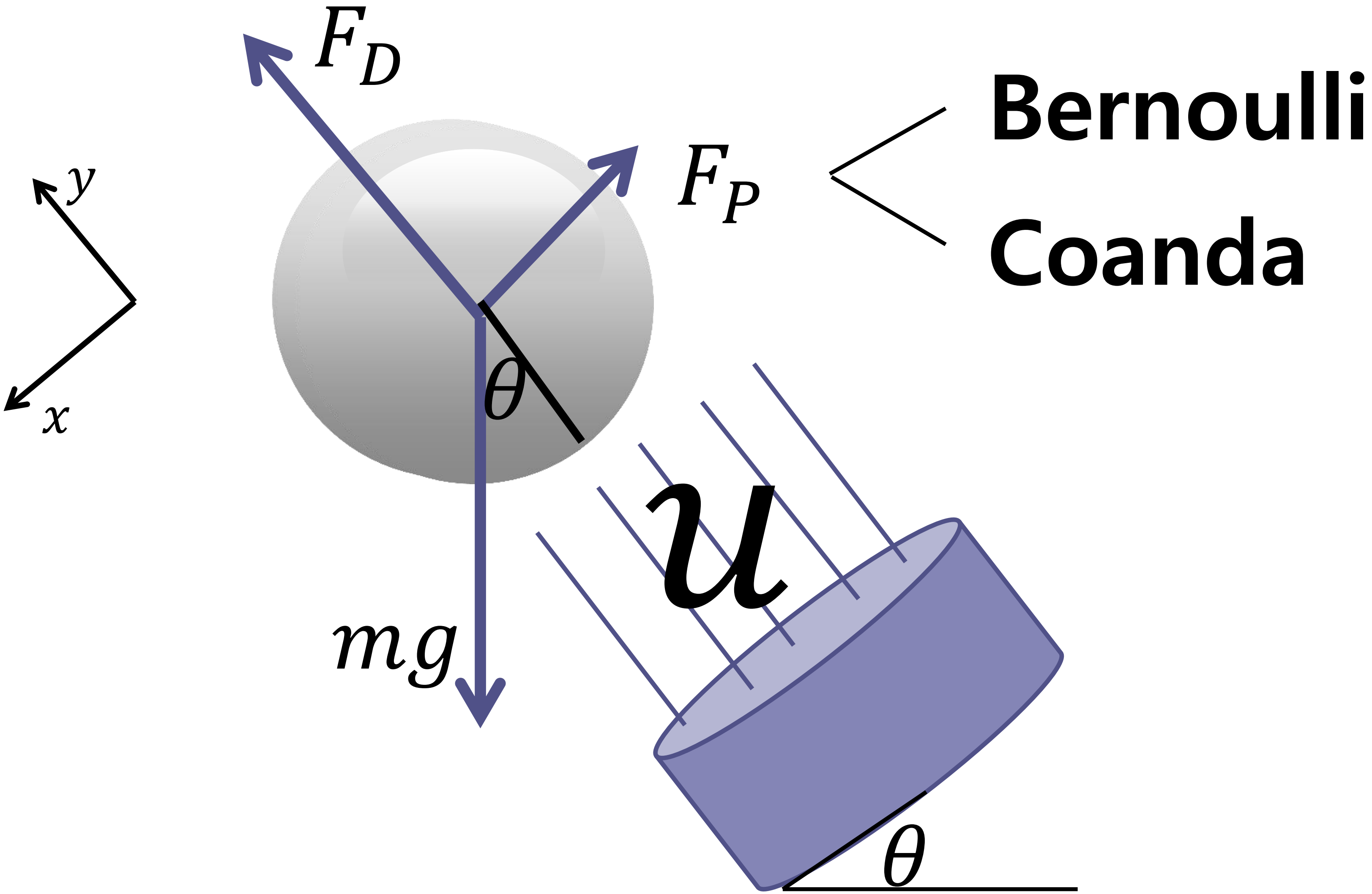
Team Korea

Phenomenon

- Investigate the effect
  - Oscillation

- Produce the maximum angle of tilt

Force Analysis



Flow Chart

Airstream Velocity Profile

Momentum flow conservation

Gaussian curve distribution

Pitot tube experiment

y-Oscillation

Drag force

Gravity

Oscillation period

y-Equilibrium Point

x-Oscillation

Bernoulli effect

Coanda effect (Boundary layer thickness)

Oscillation period

x-Equilibrium Point

Optimization (Maximum angle of tilt)

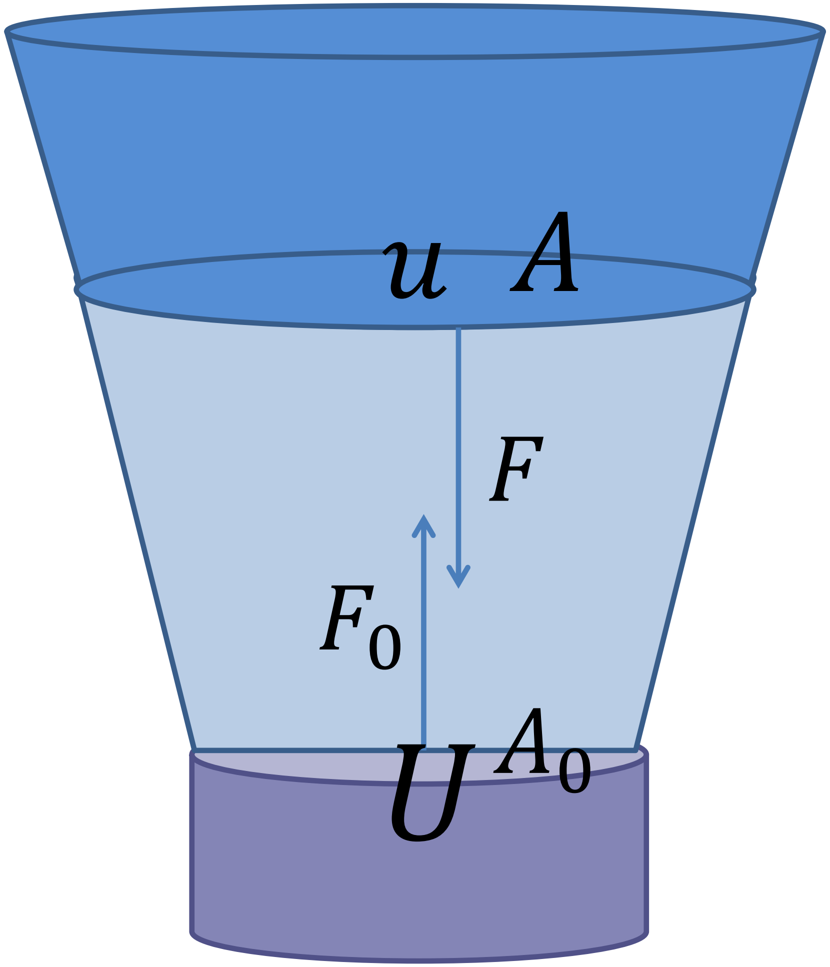
Force equilibrium

Airstream velocity

Mass of ball

Free Airstream Velocity Profile

Momentum flow conservation of free jet



$$F = \frac{dp}{dt}$$

Free jet:

$$\sum F_{ext} = F - F_0 = 0$$

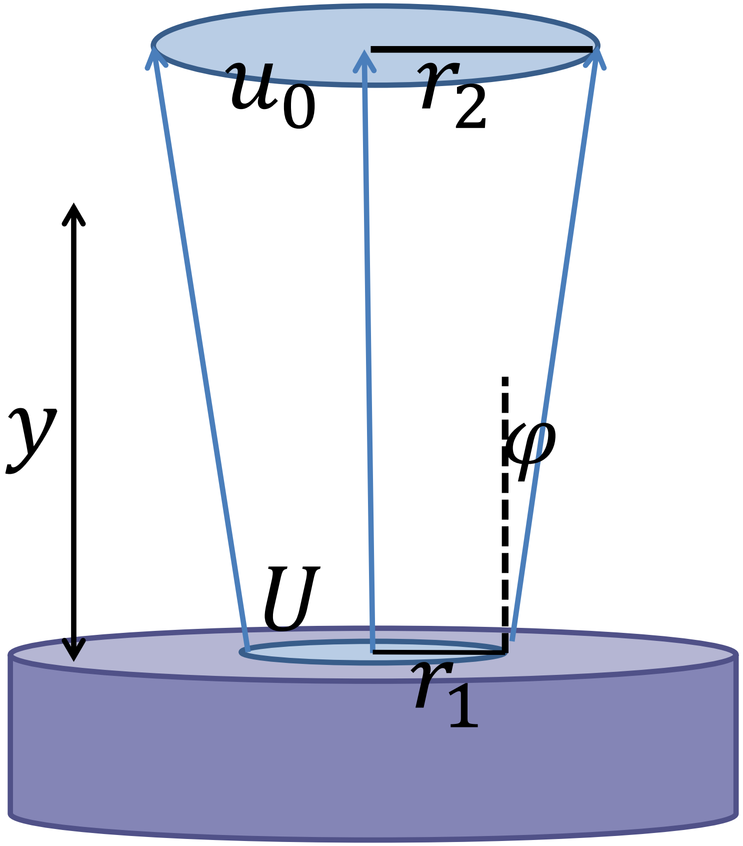
∴ Momentum flow $\frac{dp}{dt}$ is a CONSERVED quantity

$$F = \frac{dp}{dt} = \rho \frac{dV}{dt} u = \rho A u^2$$

$$F_0 = \rho A_0 U^2$$

Velocity at the Center

Assume that the velocity is even at the narrow area of the center.



By momentum flow conservation

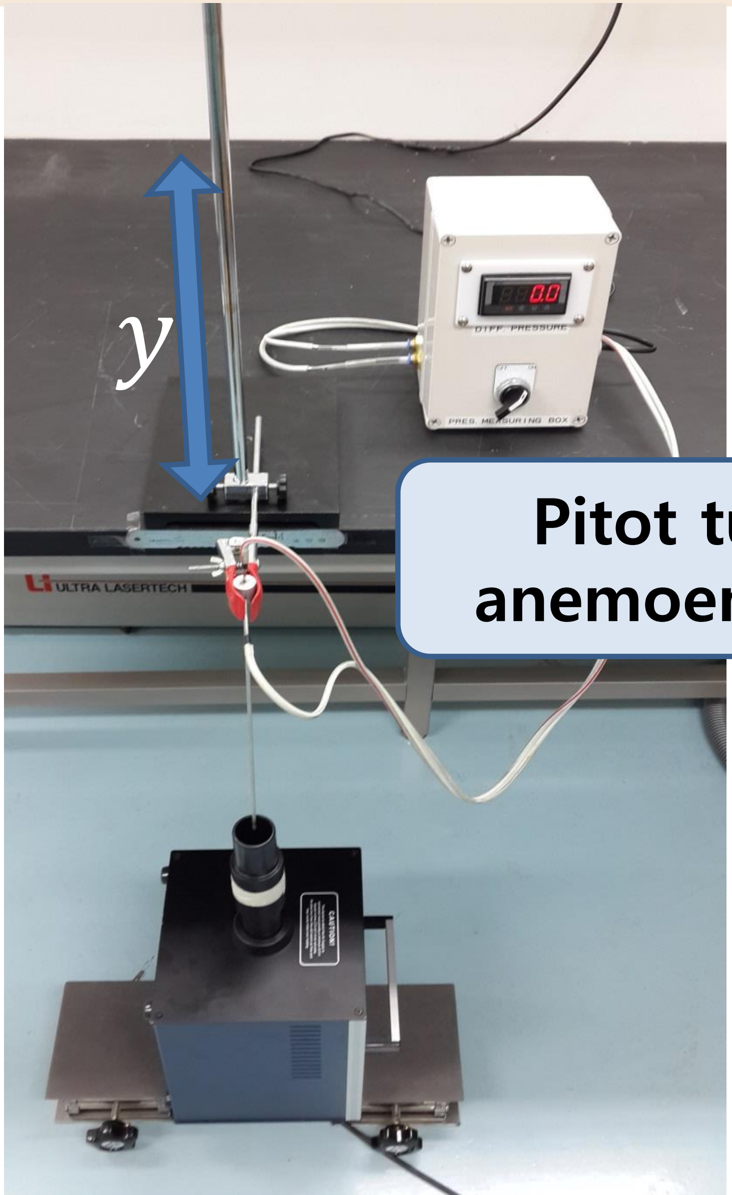
$$\rho Q_0 U = \rho Q u_0$$

$$\rho(\pi {r_1}^2)U^2 = \rho(\pi {r_2}^2){u_0}^2$$

$$r_1 = r_2 - y\tan\varphi$$

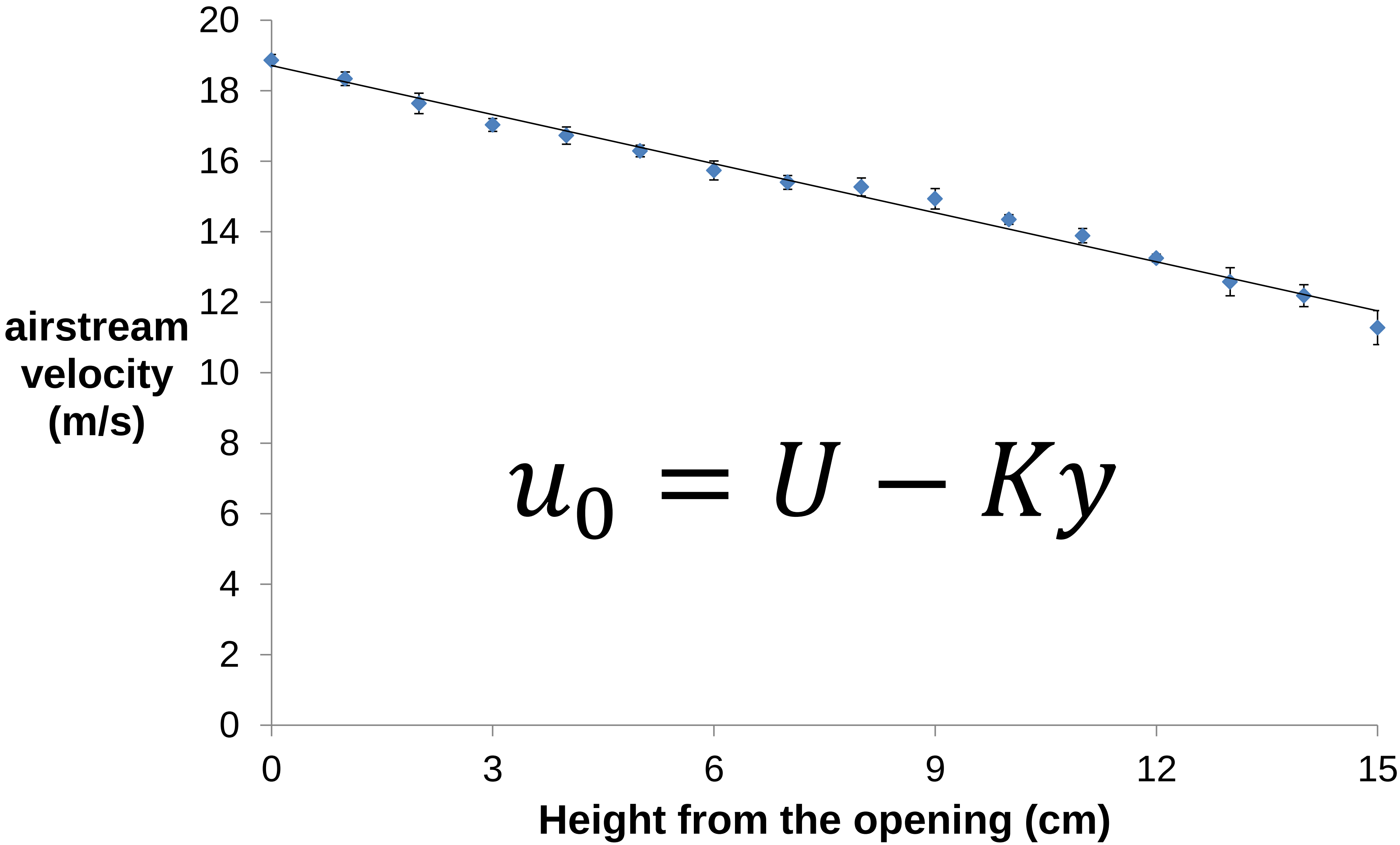
$$\therefore\ u_0 = U - \frac{U\tan\varphi}{r_2}y$$

Experiment: Velocity at the Center



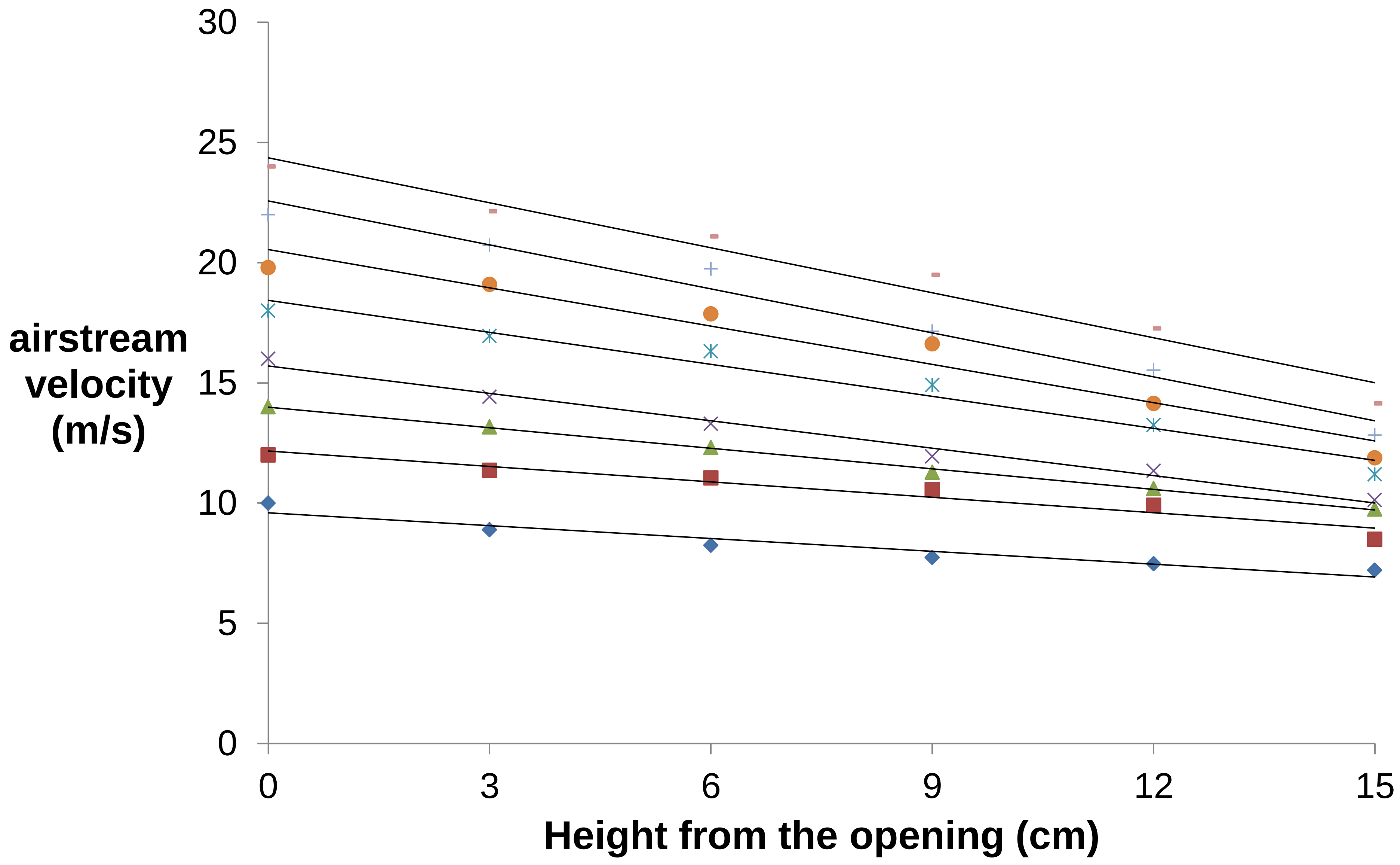
Pitot tube anemoemeter

Result



$$u_0 = U - Ky$$

Result

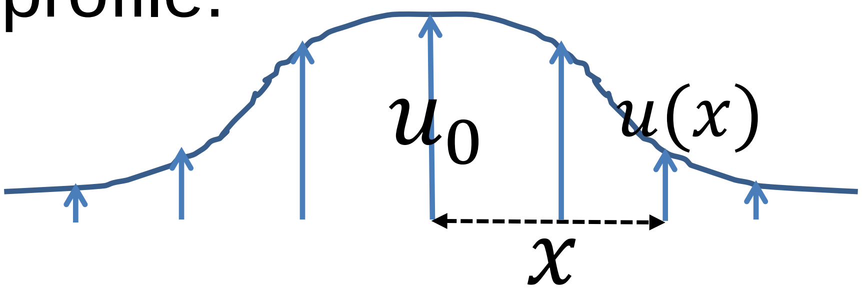


airstream velocity (m/s)
30
25
20
15
10
5
0
0
3
6
9
12
15
Height from the opening (cm)

Velocity Profile: Gaussian Curve

Assume Gaussian curve profile:

$$u(x) = u_0 e^{-\frac{x^2}{2c^2}}$$



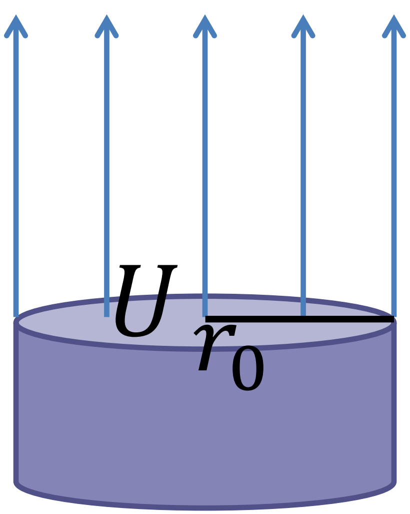
By momentum flow conservation

$$\rho U^2 \cdot \pi {r_0}^2 = \rho \int u(x)^2 dA$$

$$= \rho {u_0}^2 \int_0^\infty e^{-\frac{x^2}{c^2}} \cdot 2\pi x dx$$

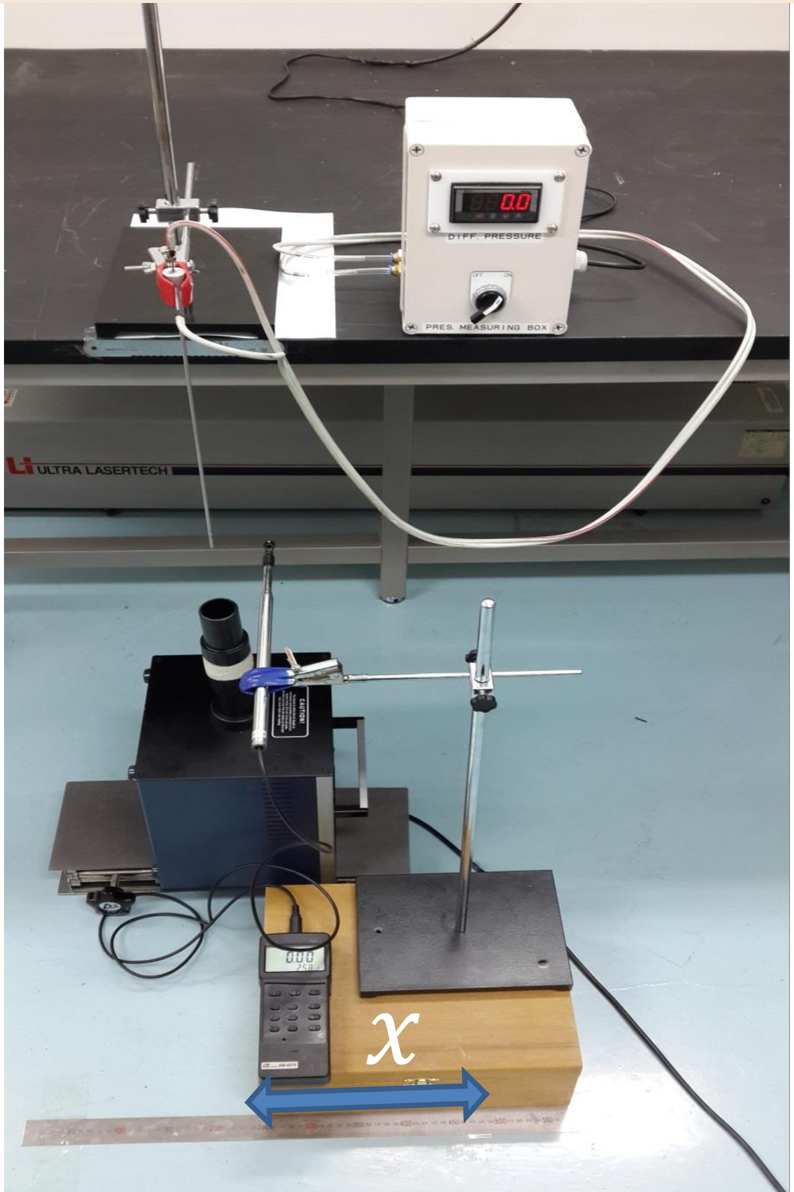
$$= \rho {u_0}^2 \cdot \pi c^2$$

$$\therefore c = \frac{r_0 U}{u_0}$$

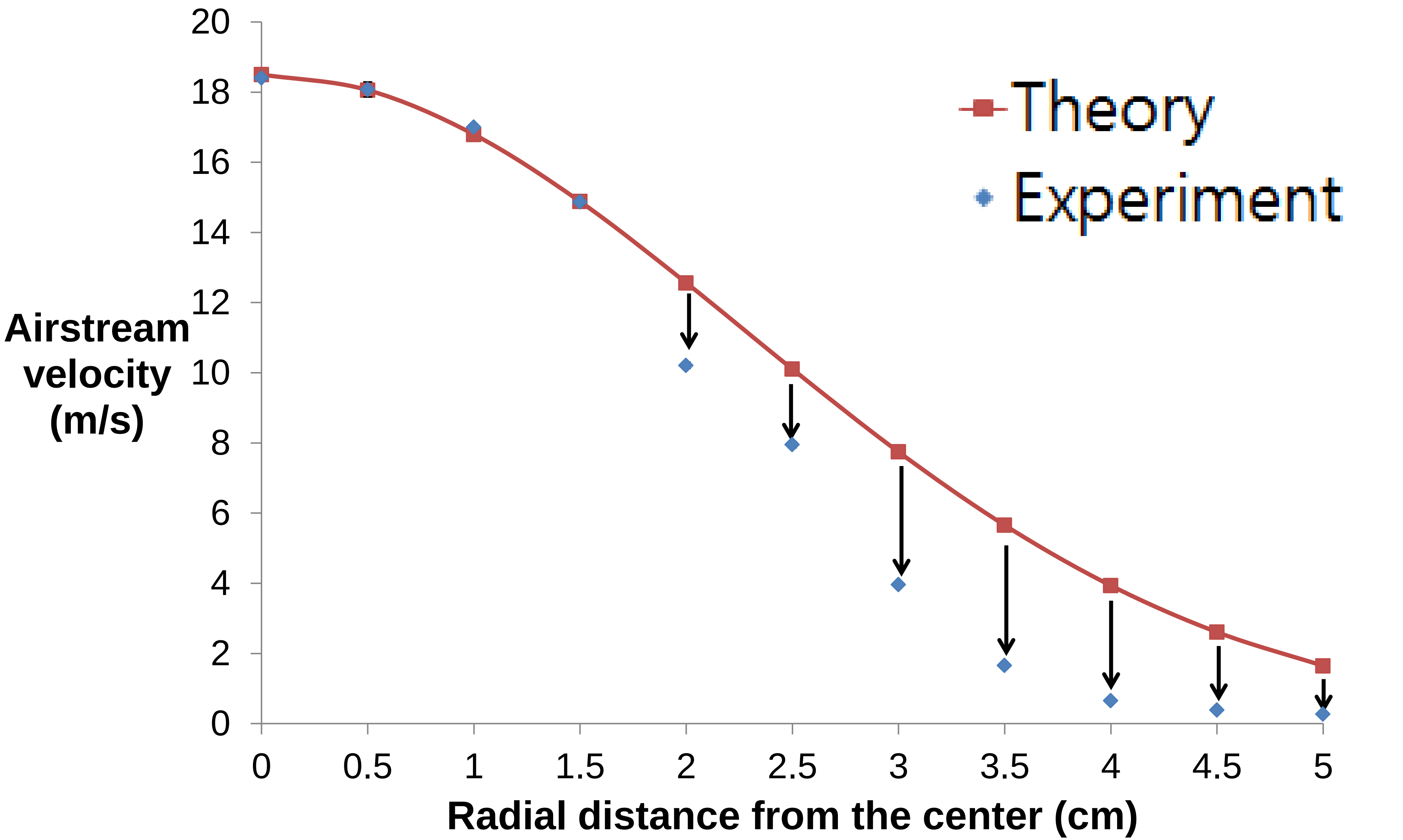


$$u(x, y) = (U - Ky) e^{-\frac{1}{2{r_0}^2}\left(1-\frac{K}{U}y\right)^2 x^2}$$

Experiment: Velocity Profile



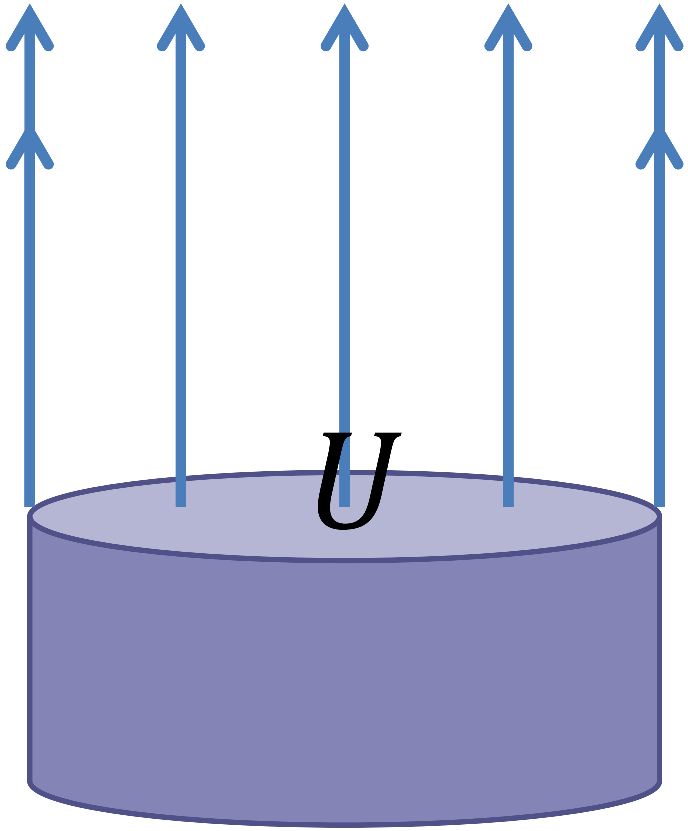
Result



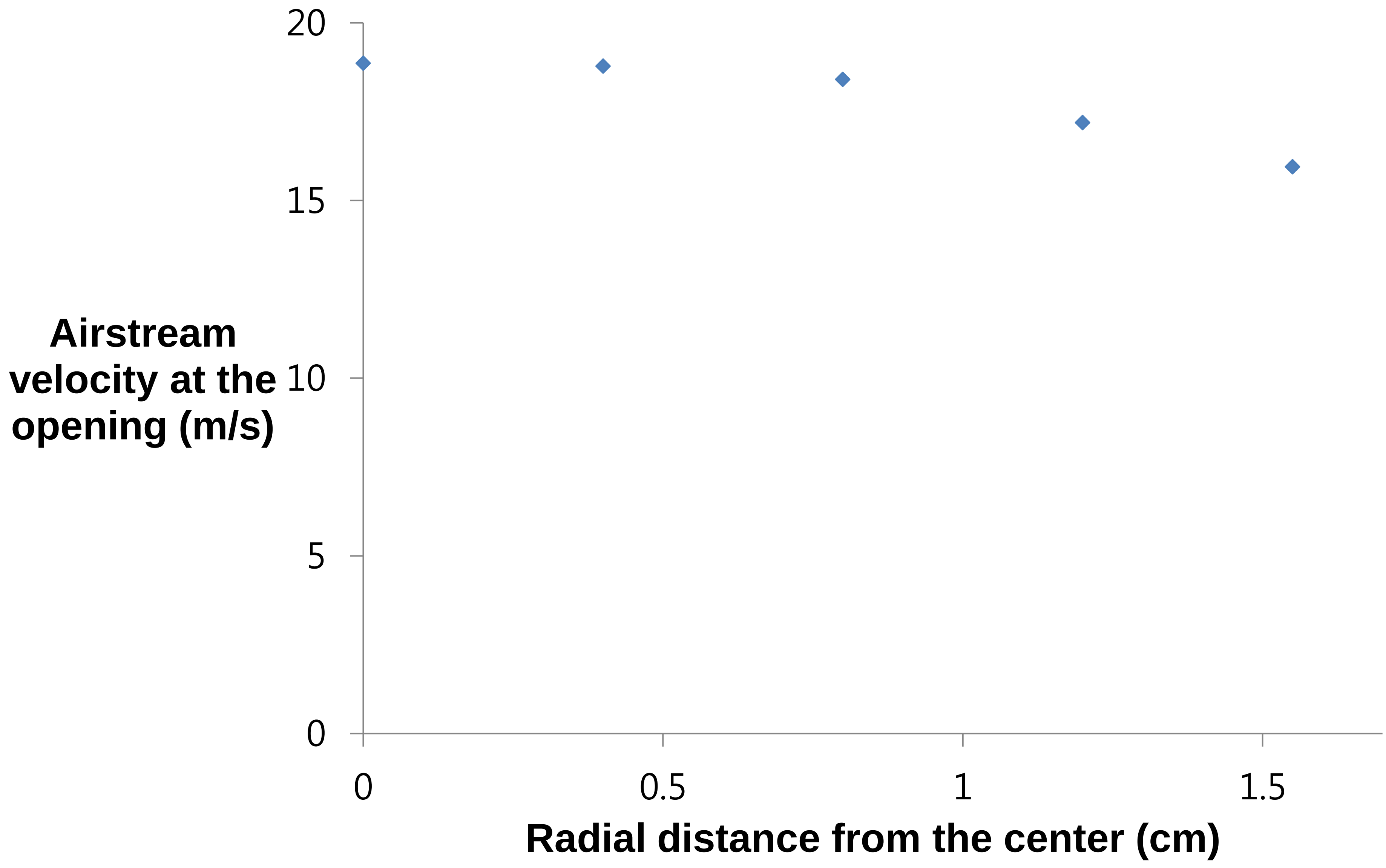
Theory
Experiment
Airstream velocity (m/s)
0
2
4
6
8
10
12
14
16
18
20
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Radial distance from the center (cm)

Error

Airstream velocity at the opening is weaker at the edge!

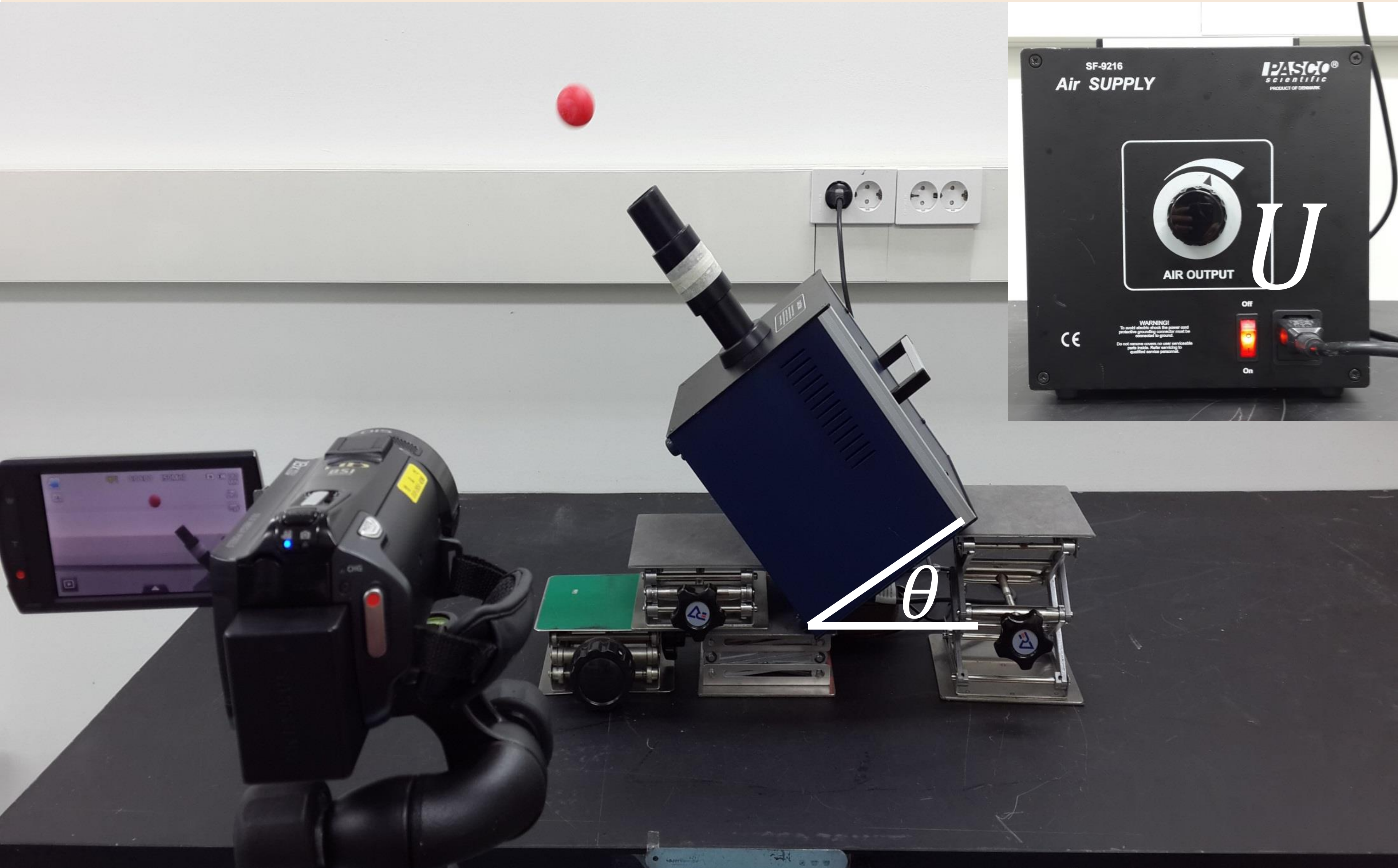


Velocity at the Opening



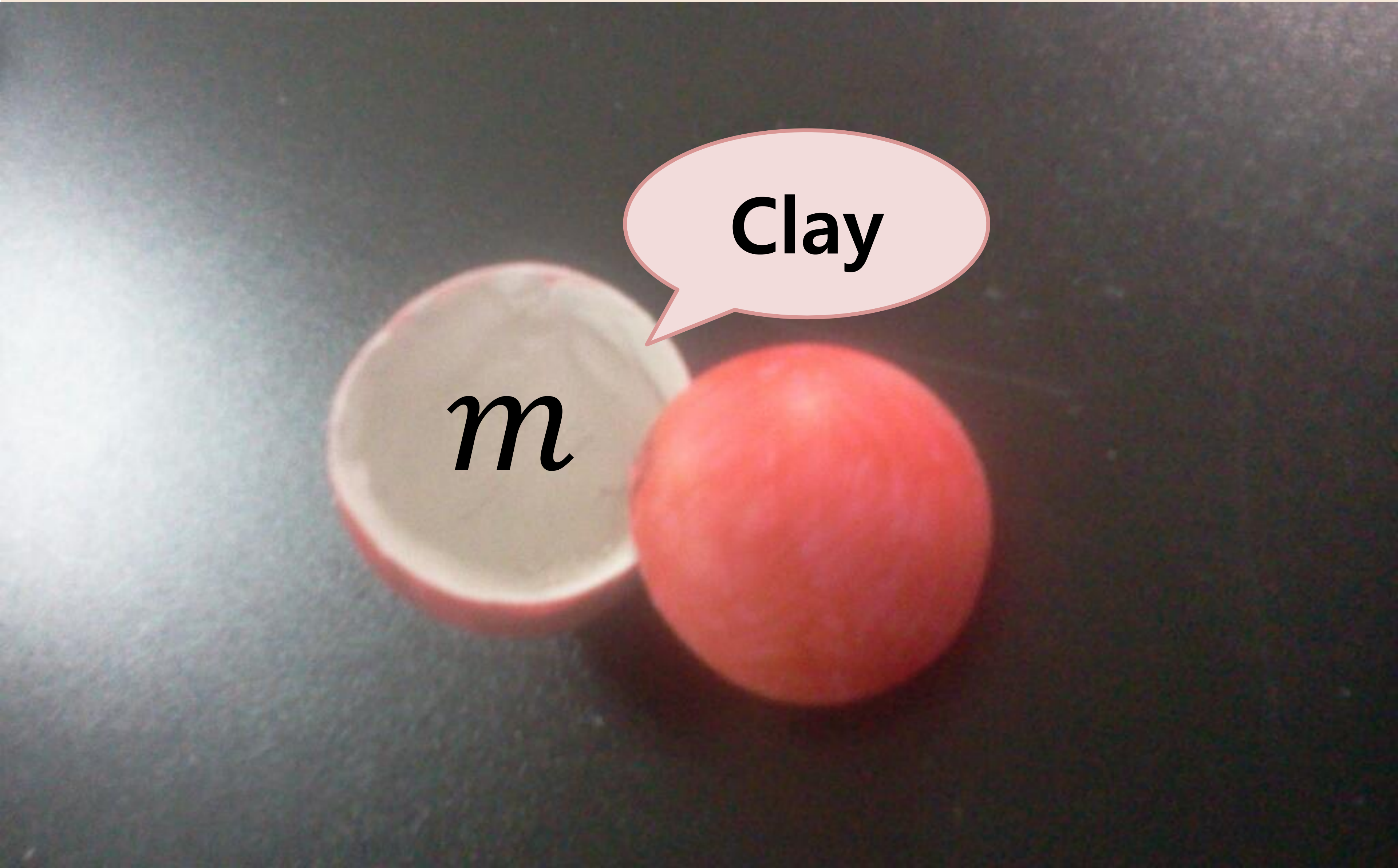
Experiment

Experimental Setting



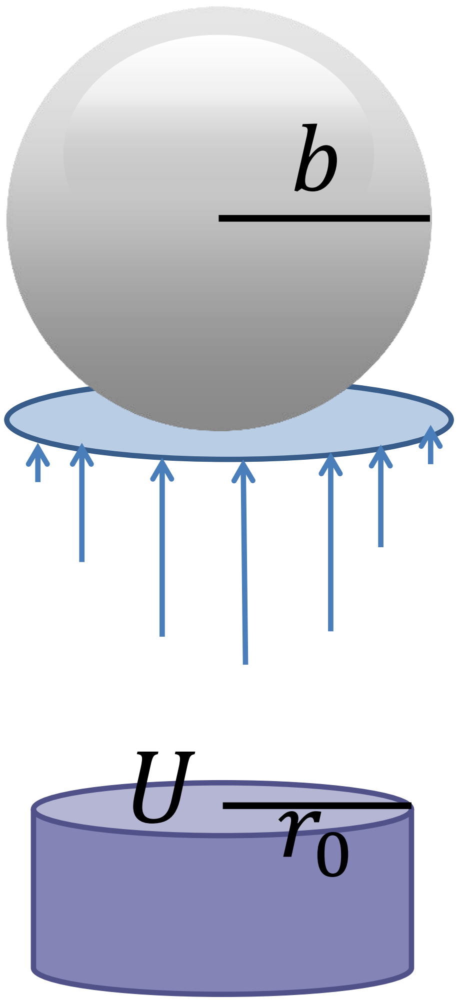
JAVA Image Processing

Controlling the Mass of the Ball



Drag Force and y- Oscillation

Drag Force



$$u(x) = (U - Ky)e^{-\frac{1}{2{r_0}^2}(1-\frac{K}{U}y)^2x^2}$$

$$F_D = \int dF_D = \int \frac{1}{2}\rho C_D u^2 dA$$

$$= \frac{1}{2}\rho C_D (U - Ky)^2 \int_0^b e^{-\frac{1}{{r_0}^2}(1-\frac{K}{U}y)^2x^2} \cdot 2\pi x dx$$

$$= \frac{1}{2}\pi\rho C_D U^2 {r_0}^2 (1 - e^{-\frac{b^2}{{r_0}^2}\left(1-\frac{K}{U}y\right)^2})$$

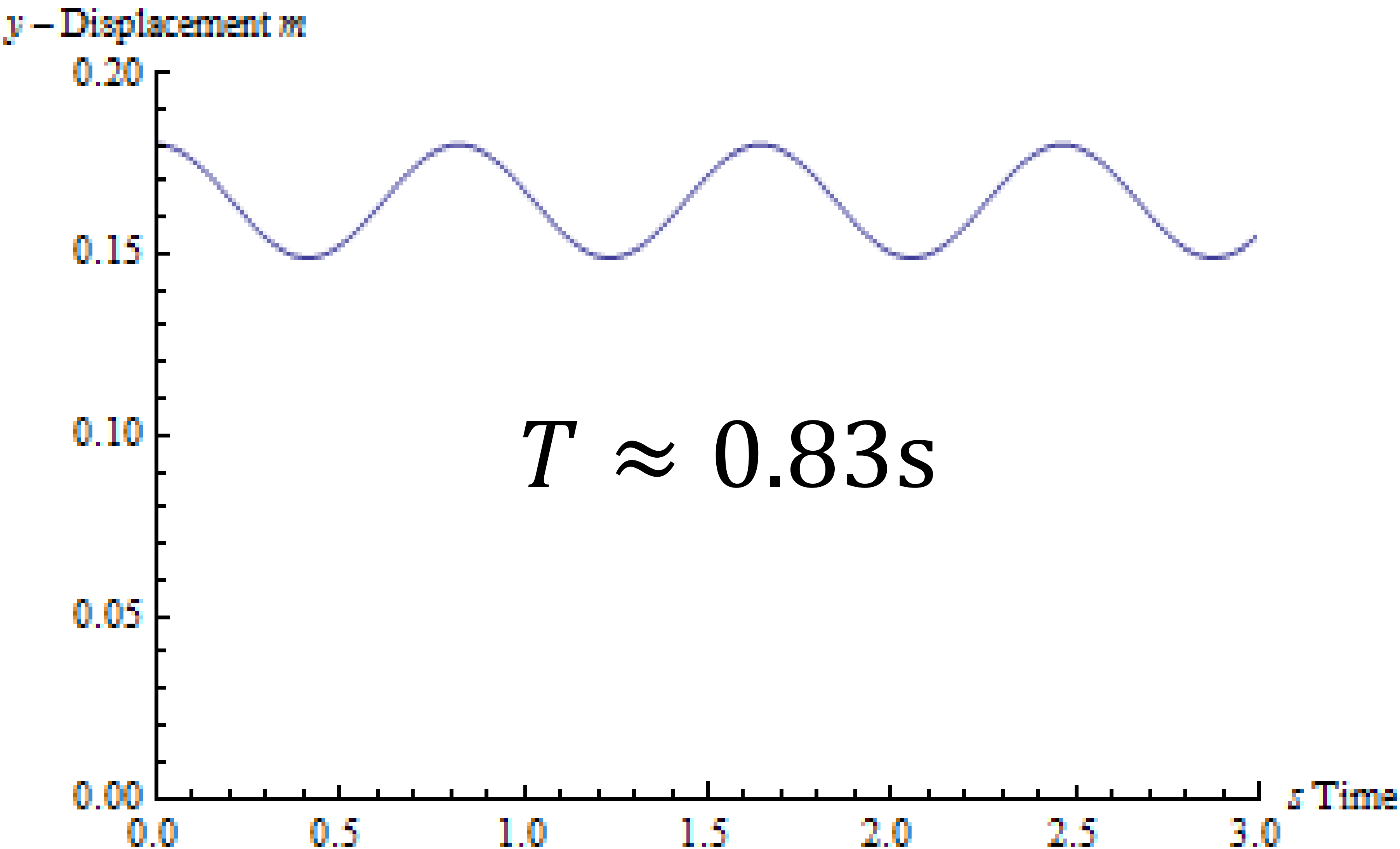
y- Oscillation Model

$$m\ddot{y} = F_D - mg$$

$$\ddot{y} = \frac{\pi\rho C_D U^2 {r_0}^2}{2m}\left(1 - e^{-\frac{b^2}{{r_0}^2}\left(1-\frac{K}{U}y\right)^2}\right) - g$$

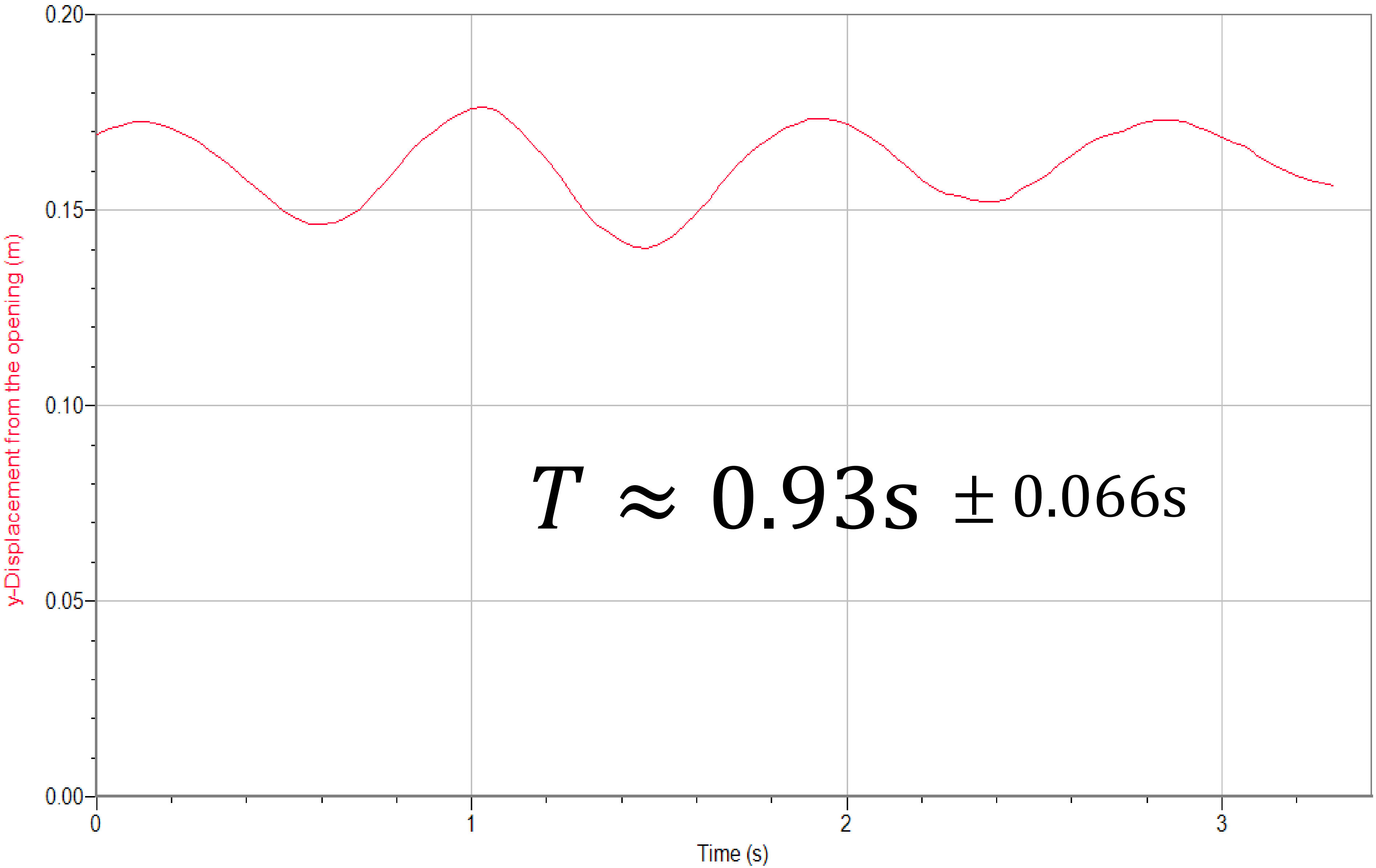
$U = 16\mathrm{m/s},\ K = 38.04\mathrm{s}^{-1},\ y(0) = 0.18\mathrm{m},\ \ y'(0) = 0$

Theory: y- Oscillation



$$T \approx 0.83s$$

Experiment: y- Oscillation



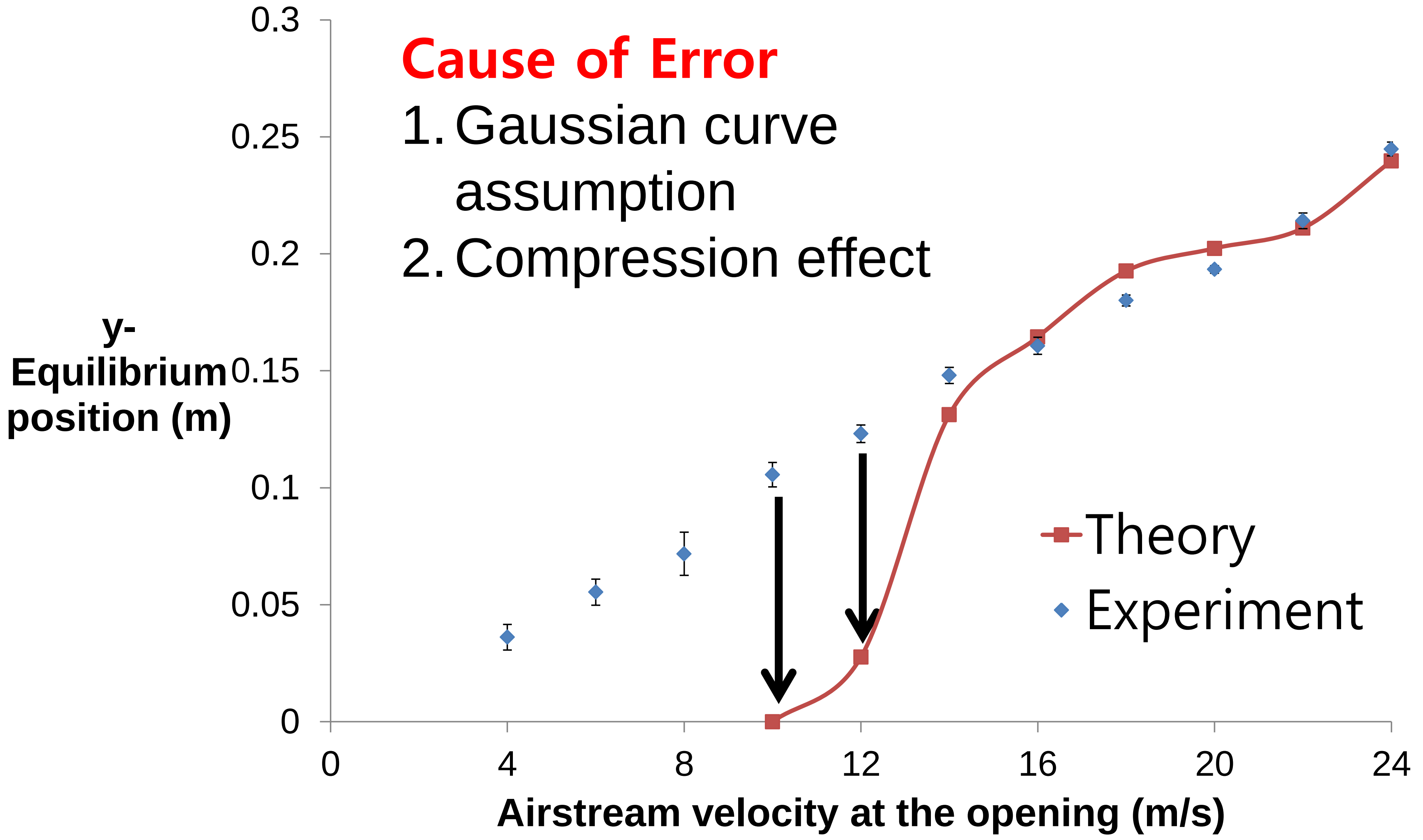
$$T \approx 0.93s \pm 0.066s$$

Theory: y-Equilibrium Position

$$m\ddot{y} = F_D - mg = 0$$

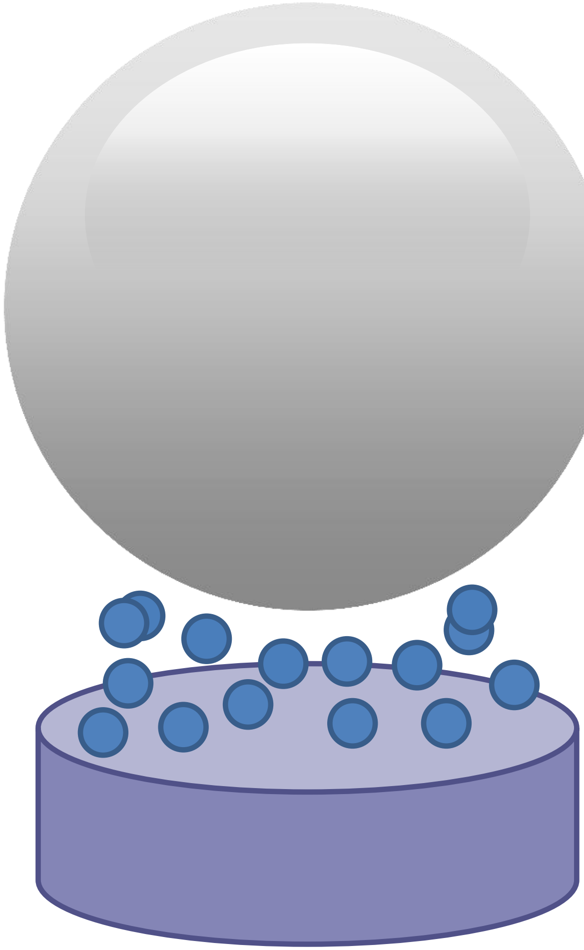
$$\frac{1}{2}\pi\rho C_D U^2 {r_0}^2 \left(1 - e^{-\frac{b^2}{{r_0}^2}\left(1-\frac{K}{U}y_{eq}\right)^2}\right) - mg = 0$$

Experiment:
y-Equilibrium Position



Compression of the Airstream

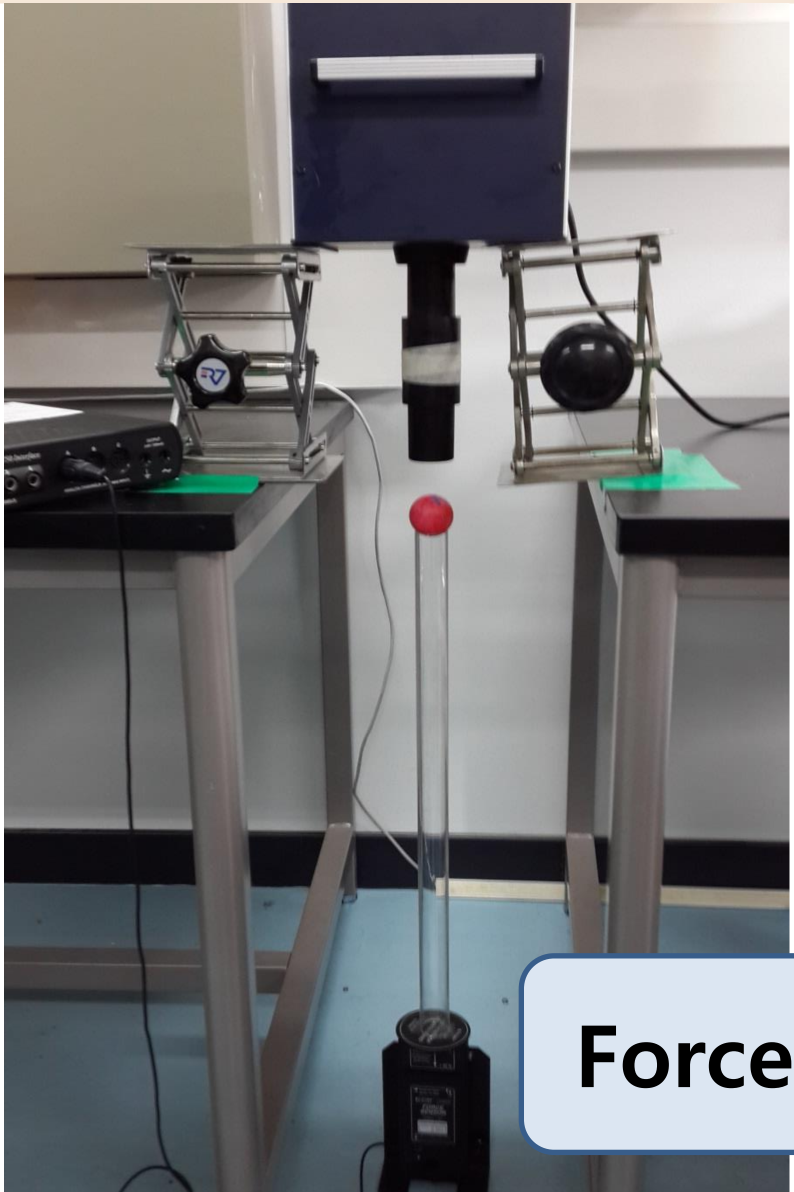
When y-displacement is low……



$$A_{eff}$$

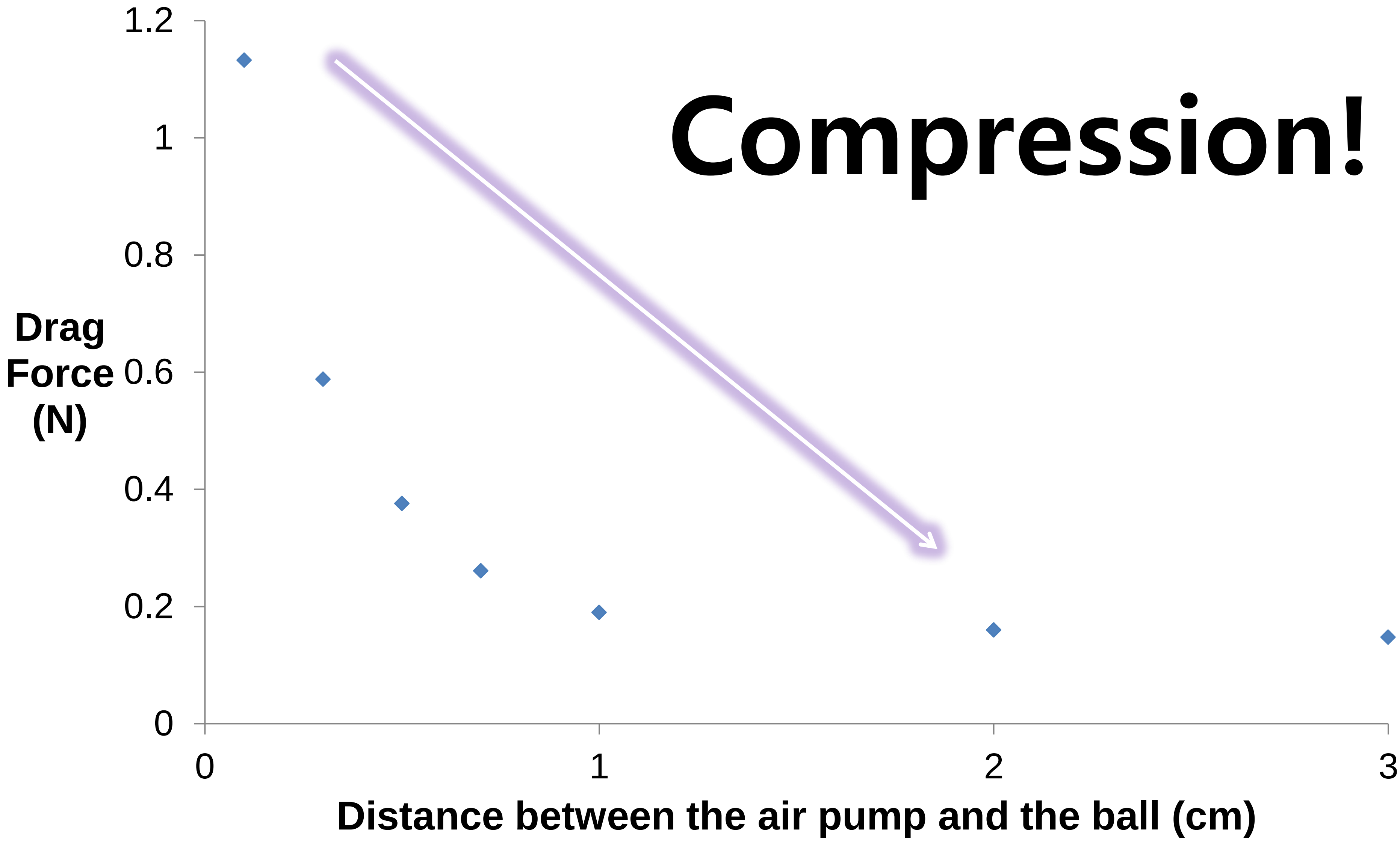
$$u_{eff}$$

Experiment Setting: Compression of the Airstream



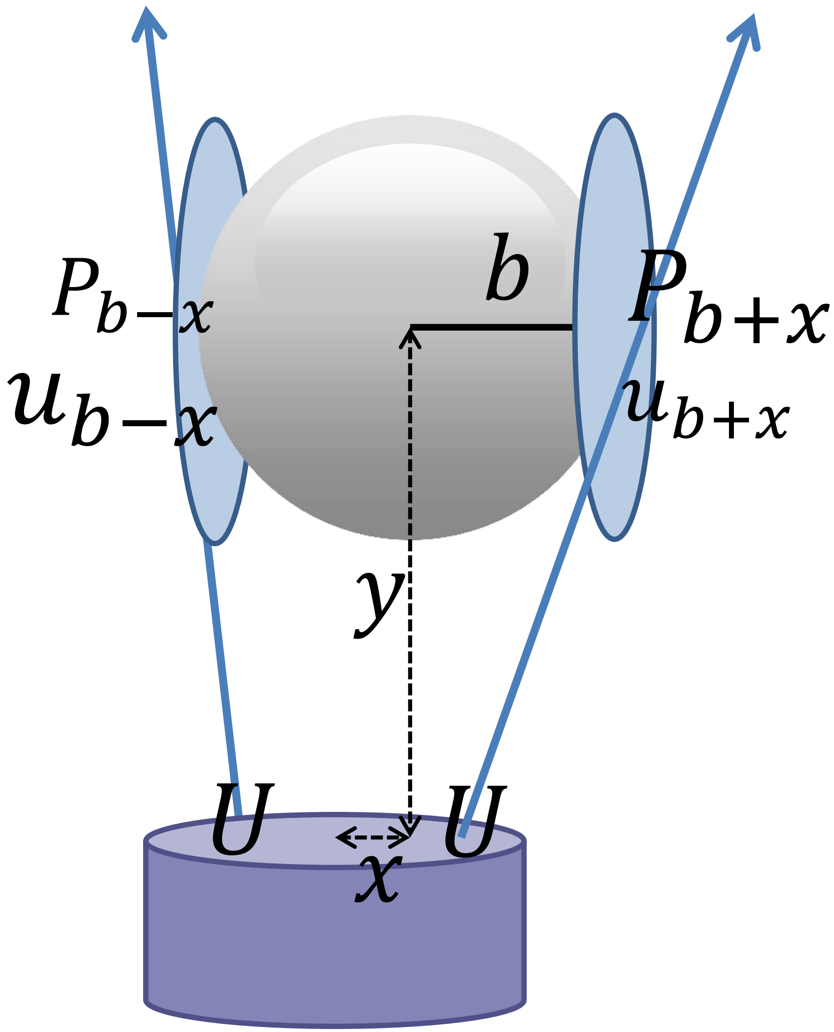
Force sensor

Experiment Result: Compression of the Airstream



Pressure Force and x- oscillation

Bernoulli Effect



$$P_0 + \frac{1}{2}\rho U^2 = P_{b+x} + \frac{1}{2}\rho {u_{b+x}}^2$$

$$P_0 + \frac{1}{2}\rho U^2 = P_{b-x} + \frac{1}{2}\rho {u_{b-x}}^2$$

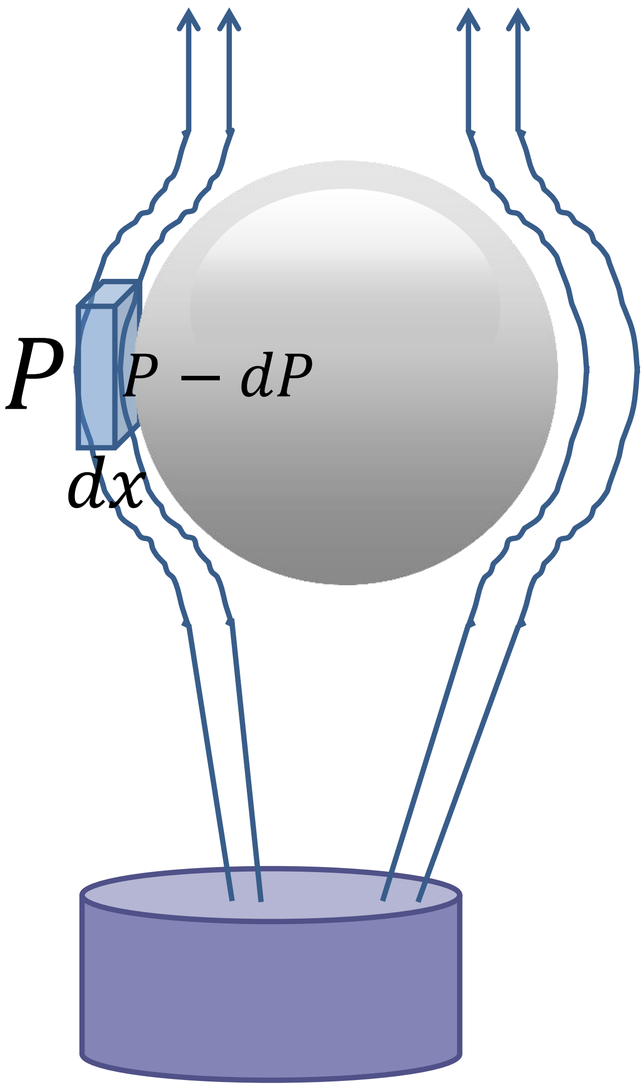
$$F_{bernoulli} = \pi b^2 (P_{b-x} - P_{b+x})$$

$$= -\pi b^2 \frac{\rho}{2} ({u_{b-x}}^2 - {u_{b+x}}^2)$$

Bernoulli Effect

$$F_{bernoulli} = -\frac{\pi b^2}{2}\rho(U - Ky)^2(e^{-\left(1-\frac{K}{U}y\right)^2\left(\frac{b-x}{r_0}\right)^2} - e^{-\left(1-\frac{K}{U}y\right)^2\left(\frac{b+x}{r_0}\right)^2})$$

Coanda Effect



$$dF = -AdP = -\frac{dP}{dx}Adx$$

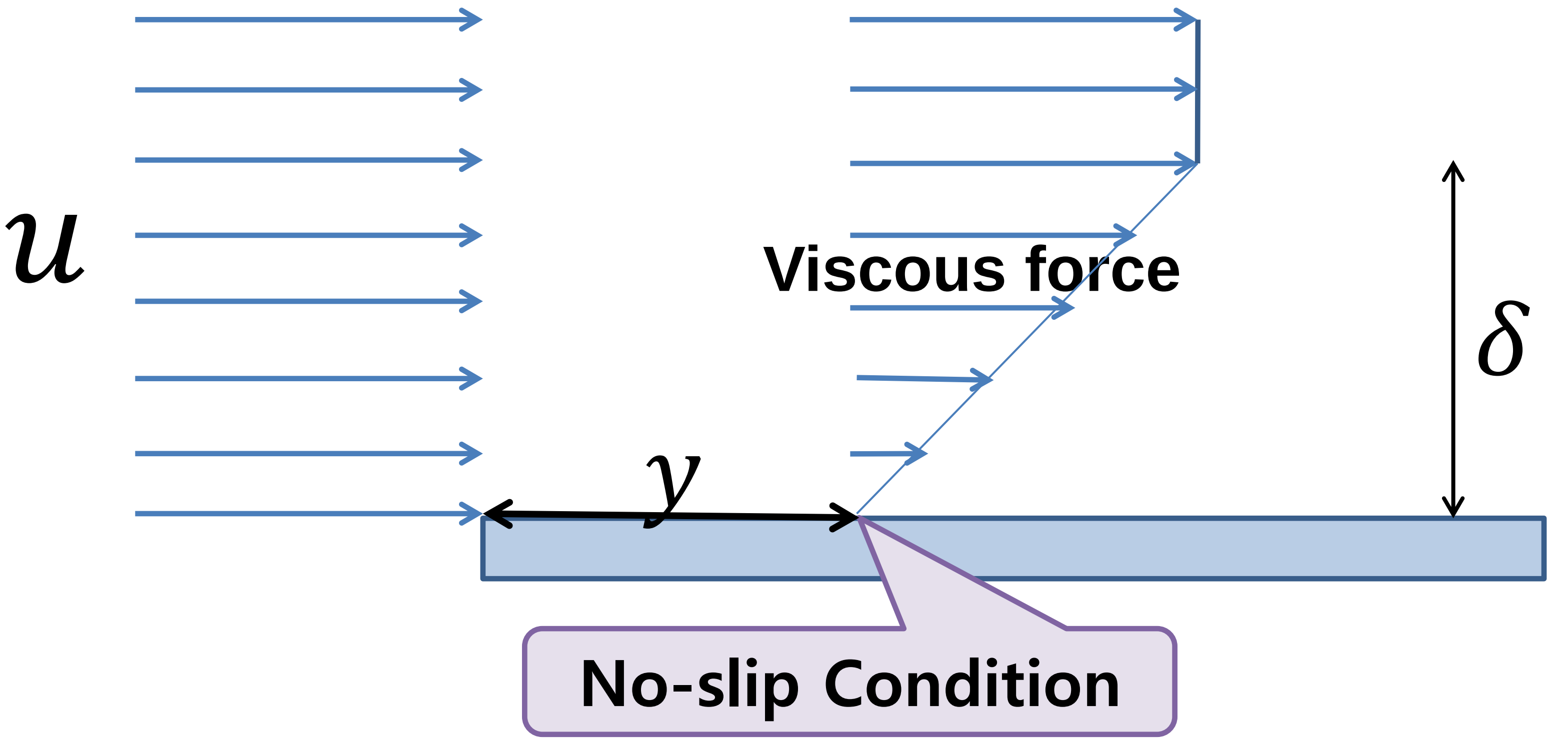
$$dF = adm = a\rho Adx$$

$$\frac{dP}{dx} = -\rho a$$

$$a = -\frac{u^2}{b}$$

$$\therefore \frac{dP}{dx} = \rho\frac{u^2}{b}$$

How thick is dx?



Boundary Layer Thickness

$$\delta \approx 0.382y/{Re_y}^{0.2}$$

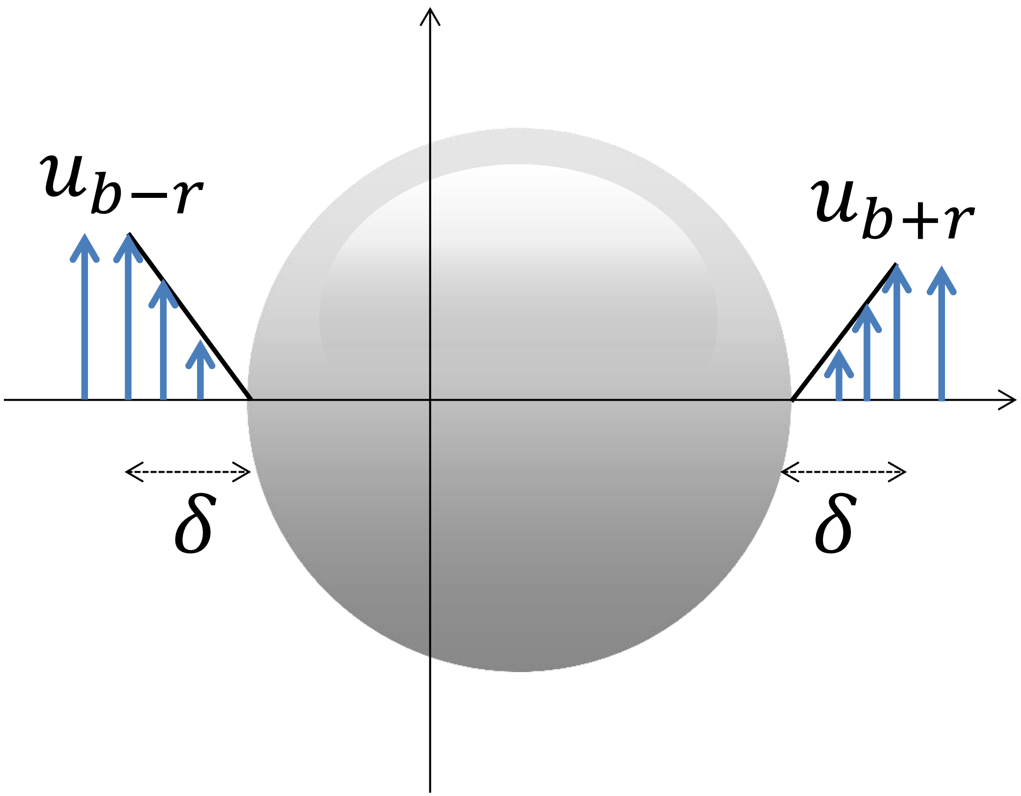
$$Re_y = \frac{\rho u}{\mu} y \quad \left(\frac{\mu}{\rho}: \text{kinematic viscosity} \right)$$

Coanda Effect

$U = 16\text{m/s}: \quad \delta \approx 1.2\text{mm}$

$$\frac{dP}{dx} = \rho \frac{u^2}{b}$$

$$u = u_{b+x} \frac{x}{\delta}$$



$$\Delta P_{right} = \frac{\rho {u_{b+x}}^2}{b\delta^2} \int_{\delta}^{0} x^2\, dx$$

$$= -\frac{\rho\delta}{3b} {u_{b+x}}^2$$

$$\Delta P_{left} = -\frac{\rho\delta}{3b} {u_{b-x}}^2$$

Coanda Effect

$U = 16\text{m/s}: \quad \delta \approx 1.2\text{mm}$

$$
\begin{aligned}
F_{coanda} &= \pi b^2(\Delta P_{left} - \Delta P_{right}) \\
&= -\pi b^2 \cdot \frac{\rho\delta}{3b}({u_{b-x}}^2 - {u_{b+x}}^2) \\
&= \frac{2\delta}{3b} F_{bernoulli} \\
&\approx 0.04 F_{bernoulli}
\end{aligned}
$$

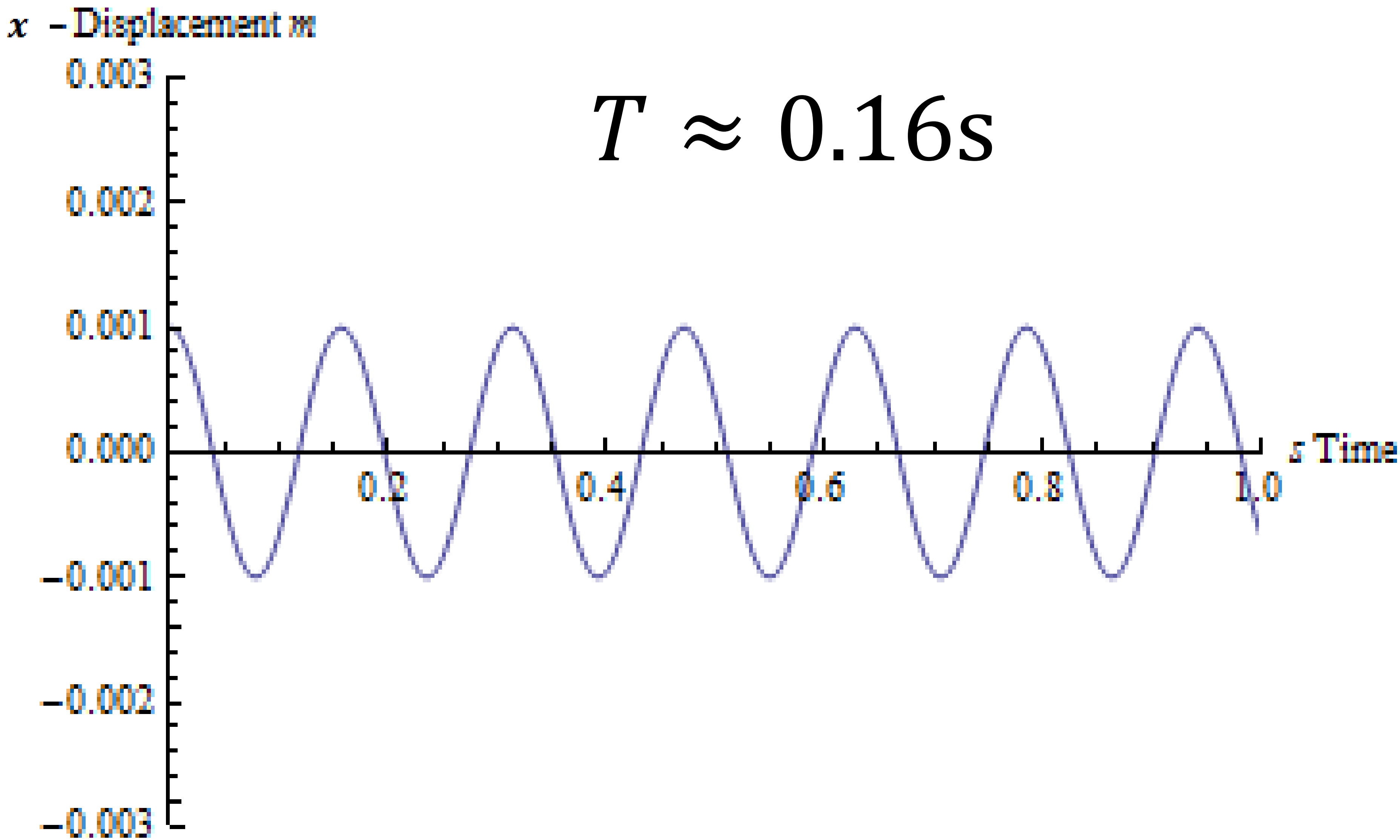
Bernoulli effect is the major one!

x-Oscillation Model

$$\ddot{x} = -1.04\frac{\pi b^2}{2m}\rho(U - Ky)^2\left(e^{-\left(1-\frac{K}{U}y\right)^2\left(\frac{b-x}{r_0}\right)^2} - e^{-\left(1-\frac{K}{U}y\right)^2\left(\frac{b+x}{r_0}\right)^2}\right)$$

$U = 16\text{m/s},\ K = 38.04\text{s}^{-1},\ x(0) = 0.001\text{m},\ x'(0) = 0$

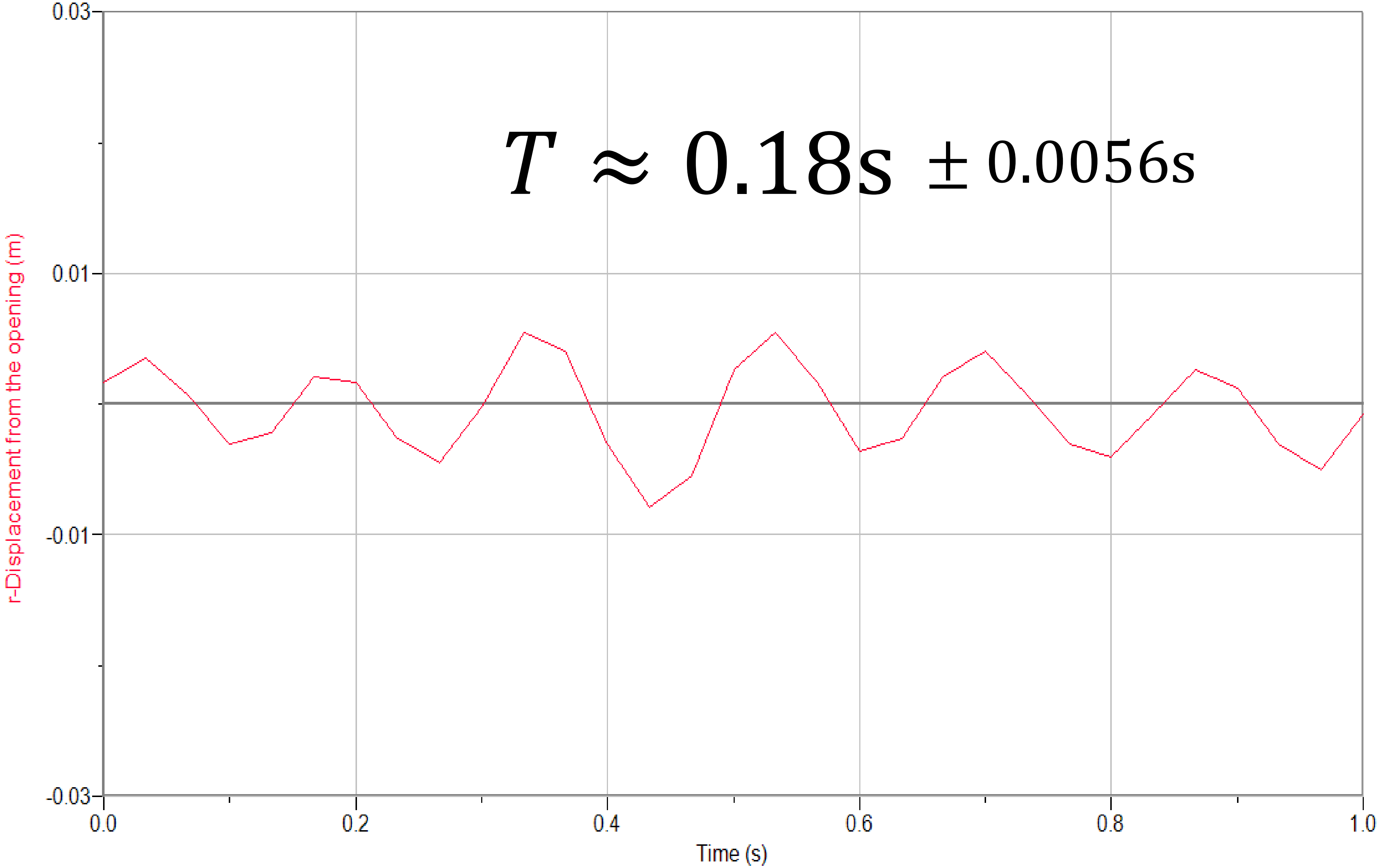
Theory: x-Oscillation



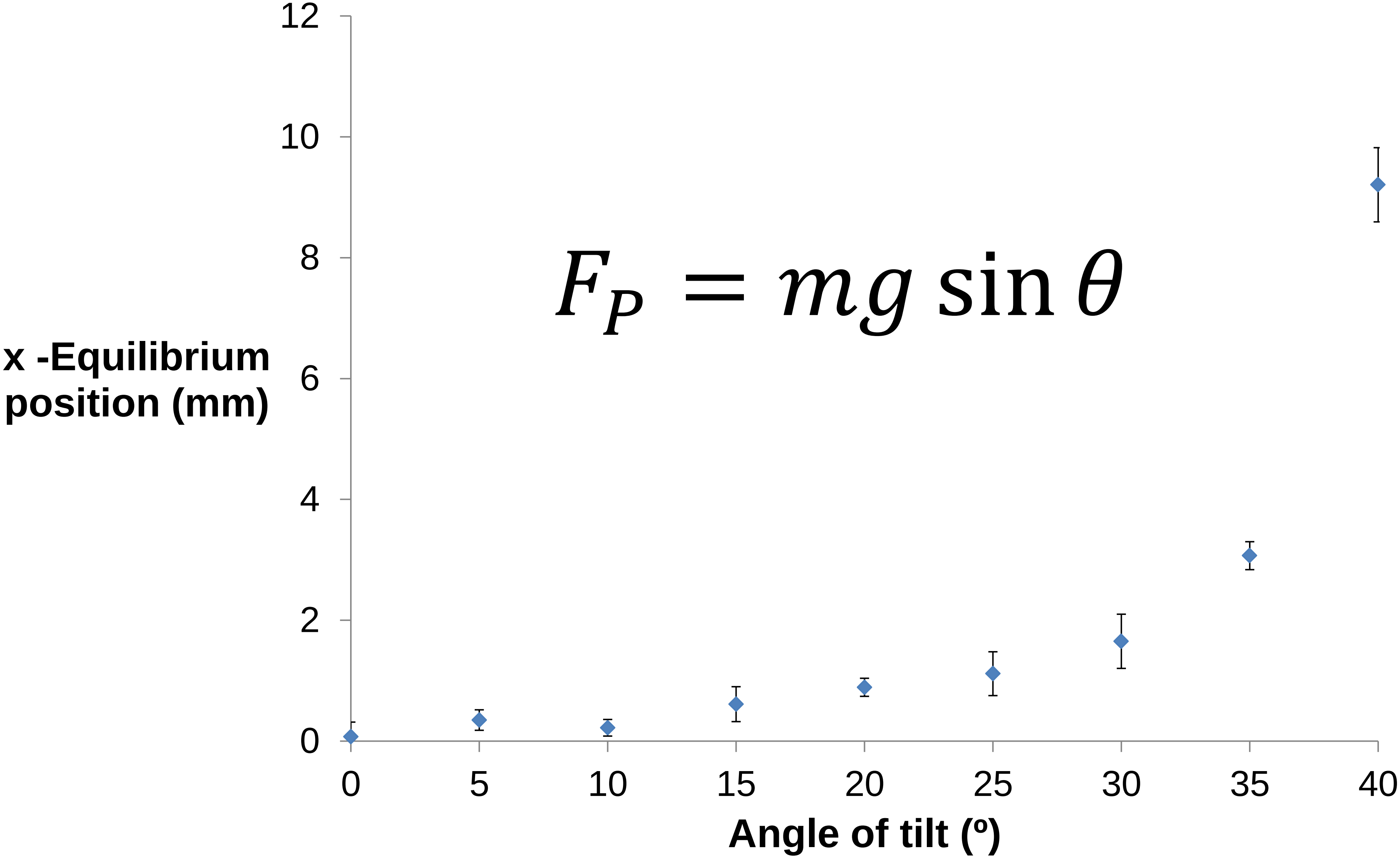
$$T \approx 0.16s$$

Experiment: x-Oscillation

$$T \approx 0.18s \pm 0.0056s$$



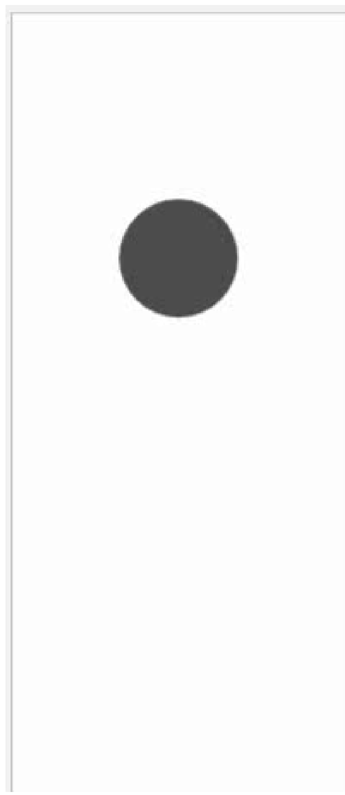
Experiment:
x-Equilibrium Position



$$F_P = mg\sin\theta$$

Motion of the Ball

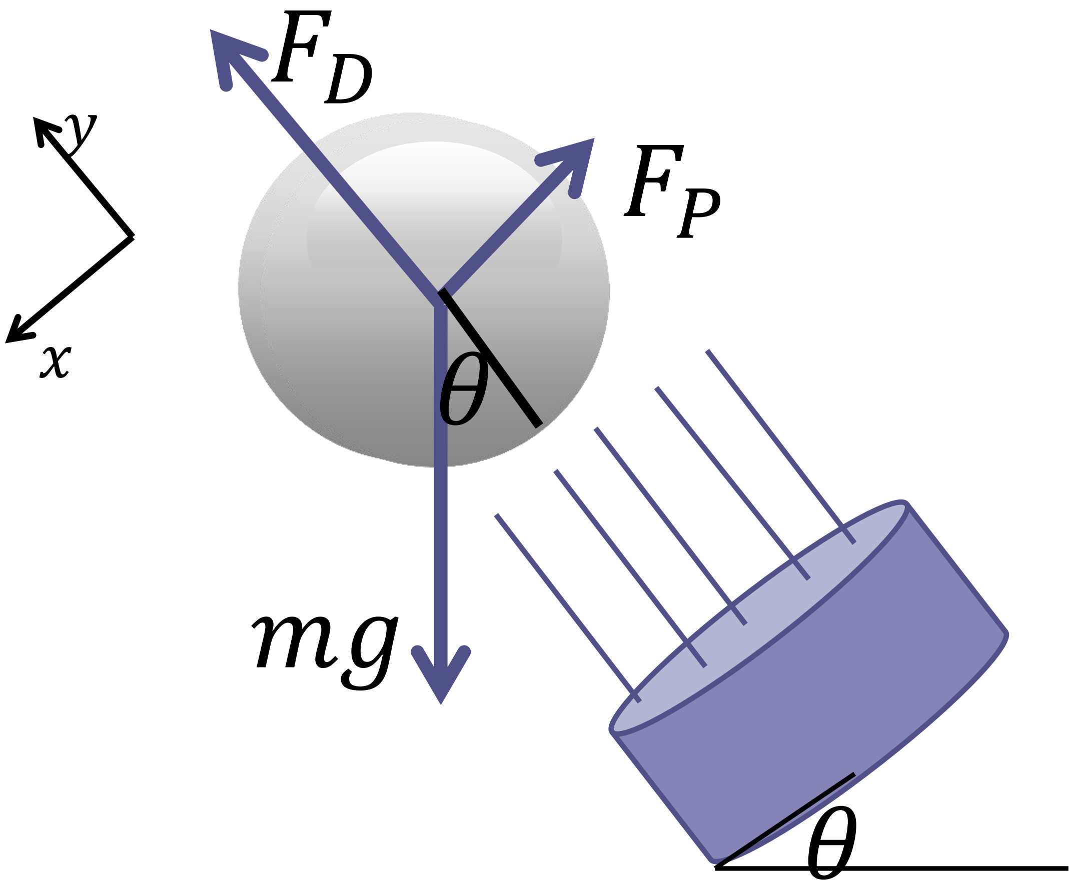
Theory



Experiment

Optimization

Theory: Optimization



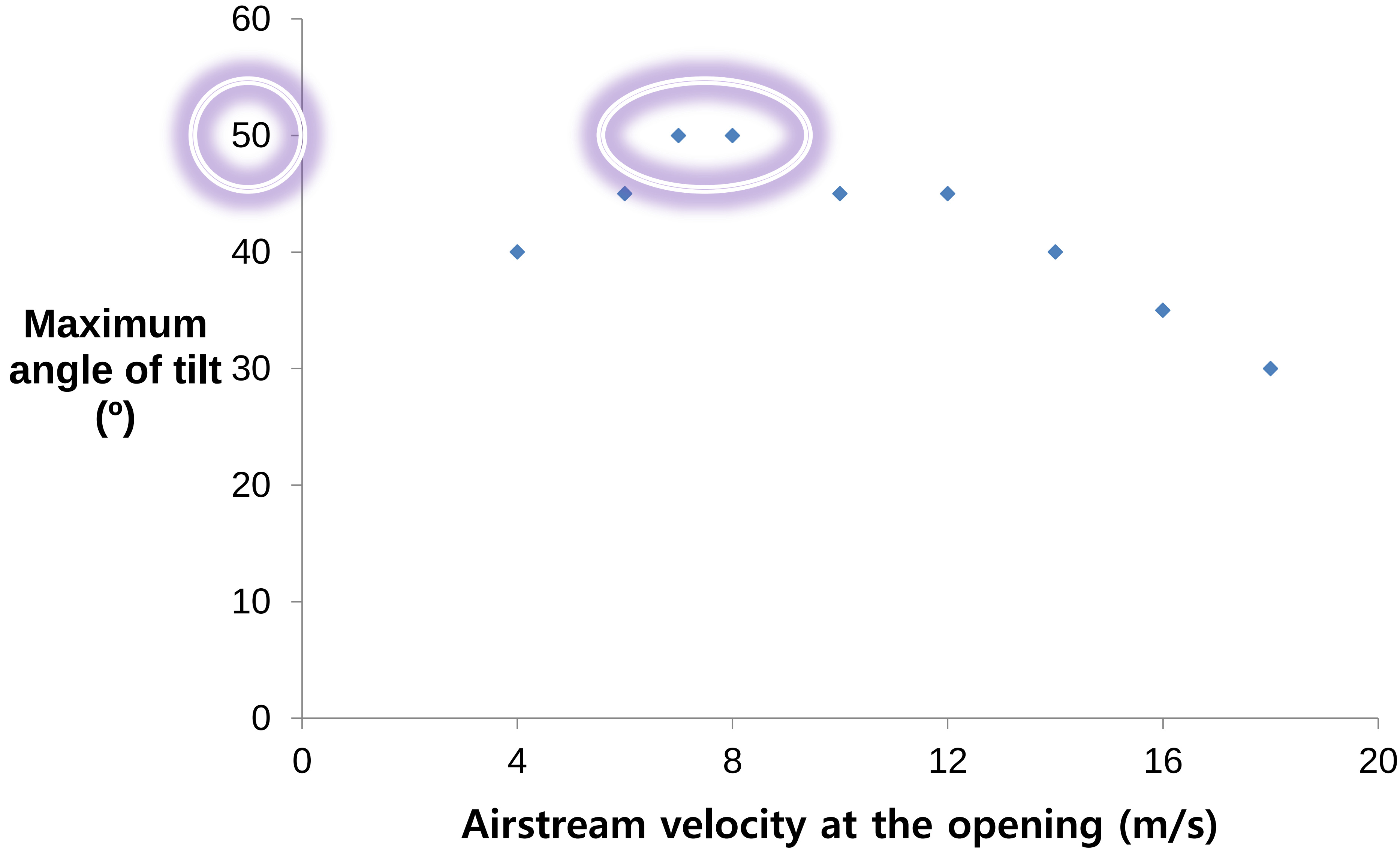
$$F_D = mg\cos\theta$$
$$F_P = mg\sin\theta$$
$$\frac{F_P}{F_D} = \tan\theta$$

y, x that achieves

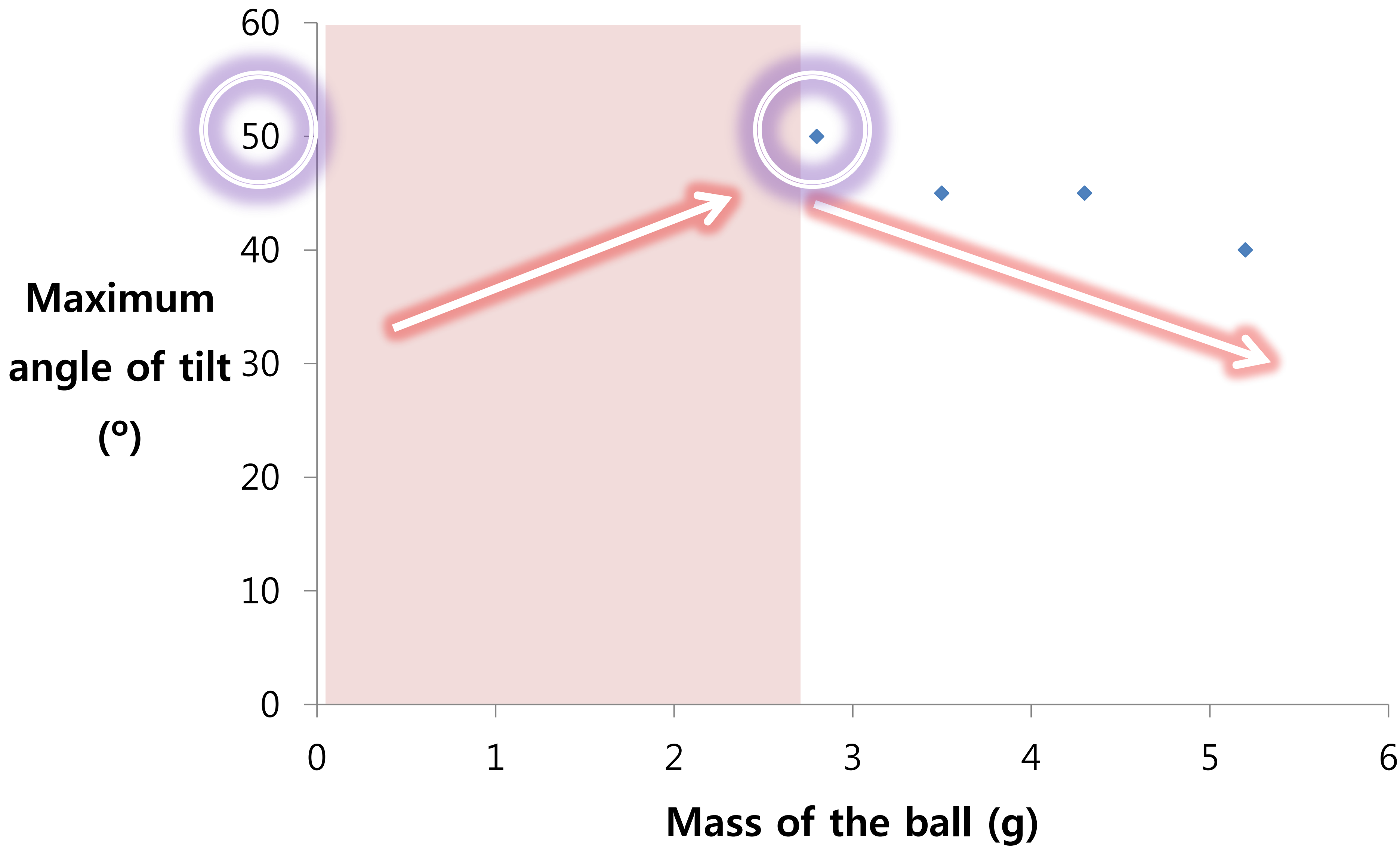
$$\max\left(\frac{F_P}{F_D}\right)$$

$\Rightarrow$ **OPTIMIZATION!**

Airstream velocity optimization



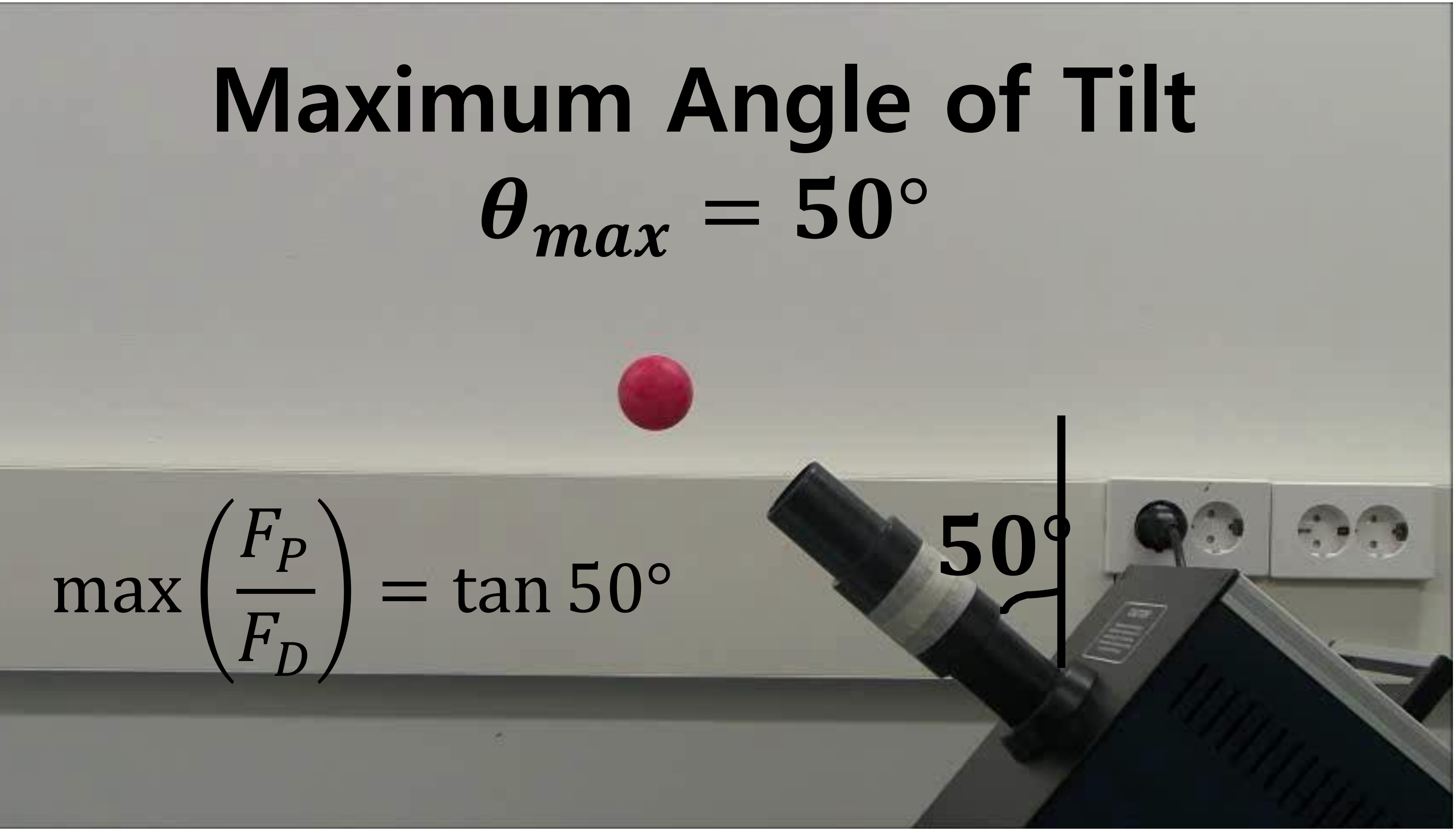
Mass of the ball Optimization



Experiment: Optimization

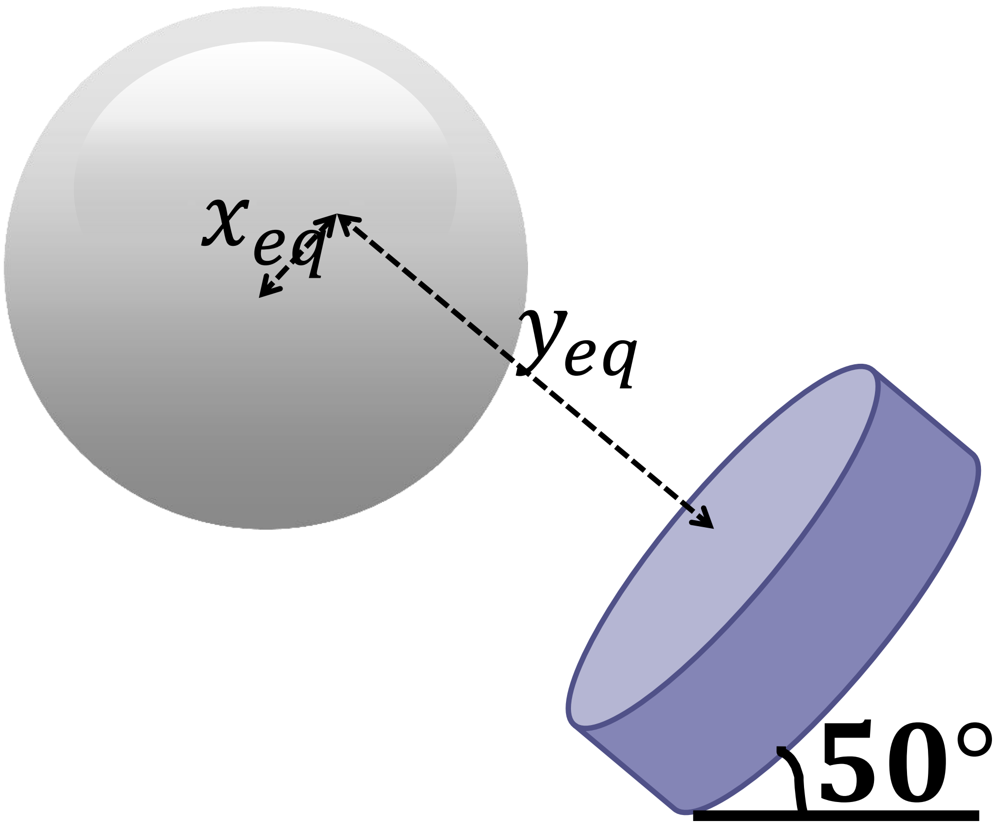
Maximum Angle of Tilt

$$\theta_{max} = 50°$$



$$\max\left(\frac{F_P}{F_D}\right) = \tan 50°$$

Experiment: Optimization



$$U = 7\sim8\mathrm{m/s}$$

$$m = 2.8\mathrm{g}$$

$$y_{eq} = 8.4\mathrm{cm}$$

$$x_{eq} = 1.3\mathrm{cm}$$

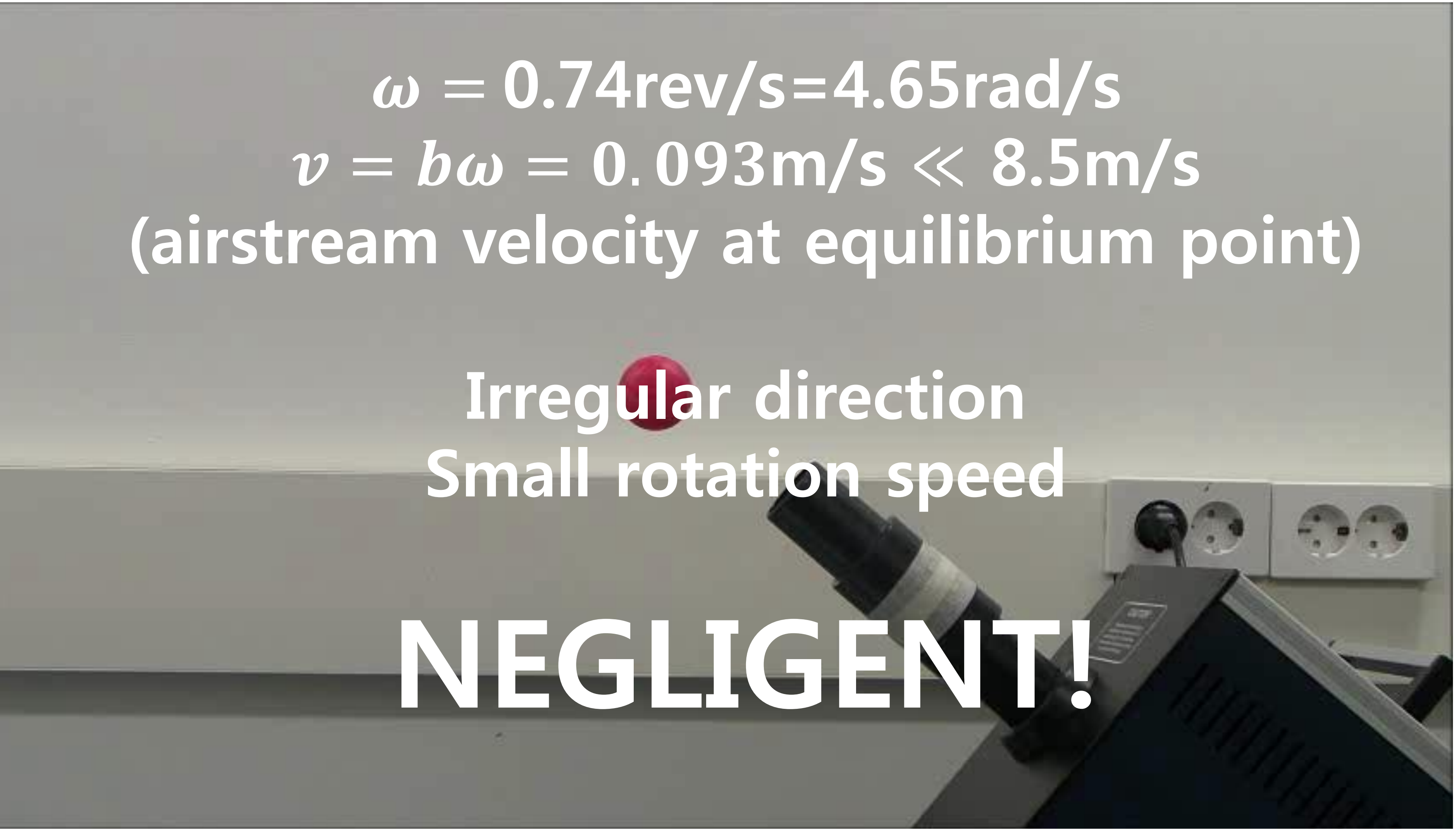
$$\max\left(\frac{F_P}{F_D}\right) = \tan 50°$$

Conclusion

Velocity Profile	Gaussian Curve & Momentum flow Conservation
y- Oscillation	Drag Force F_D
x- Oscillation	Bernoulli + Coanda Pressure Force F_P
Optimization	$\max(F_P/F_D)$, 50°

Thank You!

Spin of the ball



$\omega = 0.74\text{rev/s} = 4.65\text{rad/s}$

$v = b\omega = 0.093\text{m/s} \ll 8.5\text{m/s}$

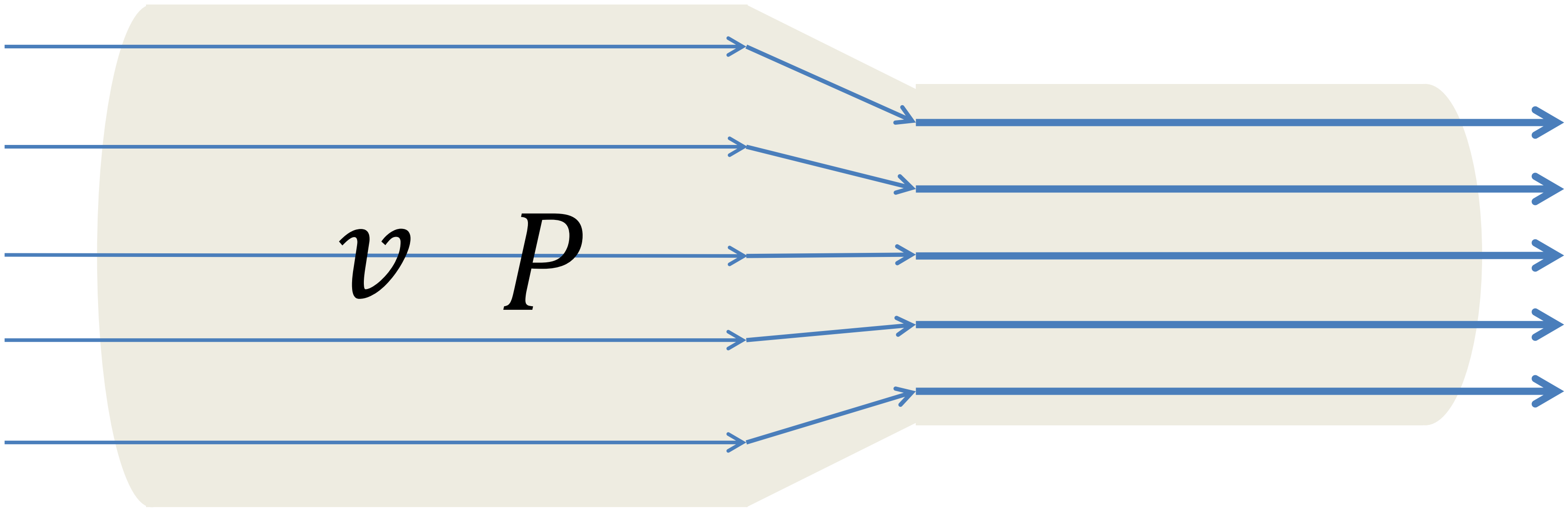
(airstream velocity at equilibrium point)

Irregular direction

Small rotation speed

NEGLIGENT!

Turbulence and Bernoulli Effect

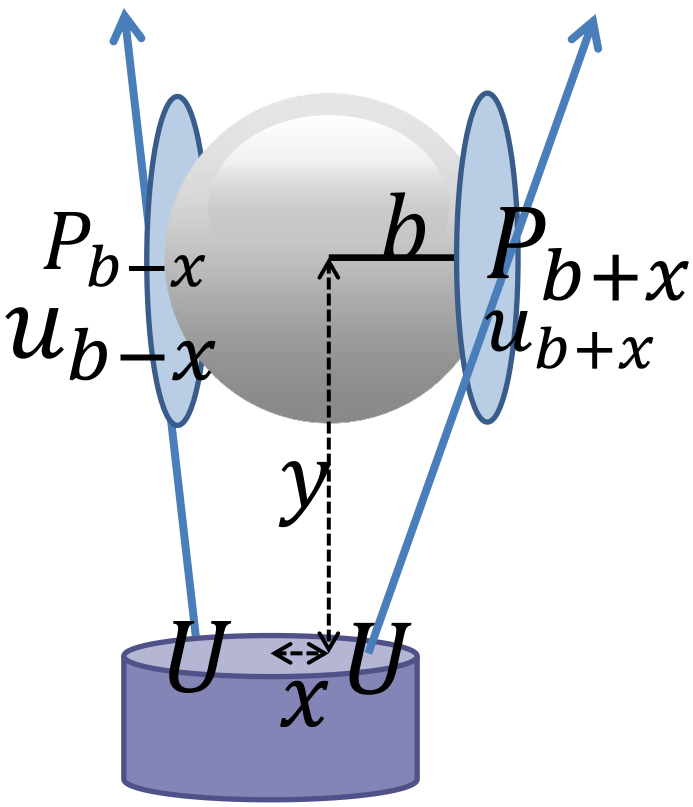


$$P + \frac{1}{2}\rho v^2 = \text{constant}$$

Throughout the field!

Assumption: $\nabla \times \vec{v} = 0$ (Irrotational)

But in my model...



$$P_0 + \frac{1}{2}\rho U^2 = P_{b+x} + \frac{1}{2}\rho {u_{b+x}}^2$$
$$P_0 + \frac{1}{2}\rho U^2 = P_{b-x} + \frac{1}{2}\rho {u_{b-x}}^2$$

Bernoulli equation applied in a SINGLE streamline

Although cause of error, still VALID in turbulent situations!

Ignoring gravity term in Bernoulli equation

$$\rho g h \qquad \frac{1}{2}\rho v^2$$

8.5m/s at 8cm height

$$gh = 0.784\ {m^2}/{s^2}$$

$$\frac{1}{2}v^2 = 75.9\ {m^2}/{s^2}$$

Reynolds number

$$Re = \frac{\rho U L}{\mu}$$

$\frac{\mu}{\rho}$ =(kinematic viscosity of air at 27℃)=1.57×10^{-5}m^2/s

$$U = 8.5\text{m/s}$$

$$L = 0.04\text{m}$$

$$Re = 2.7 \times 10^4$$

$$C_D \approx 0.5$$

Conservative Drag Force

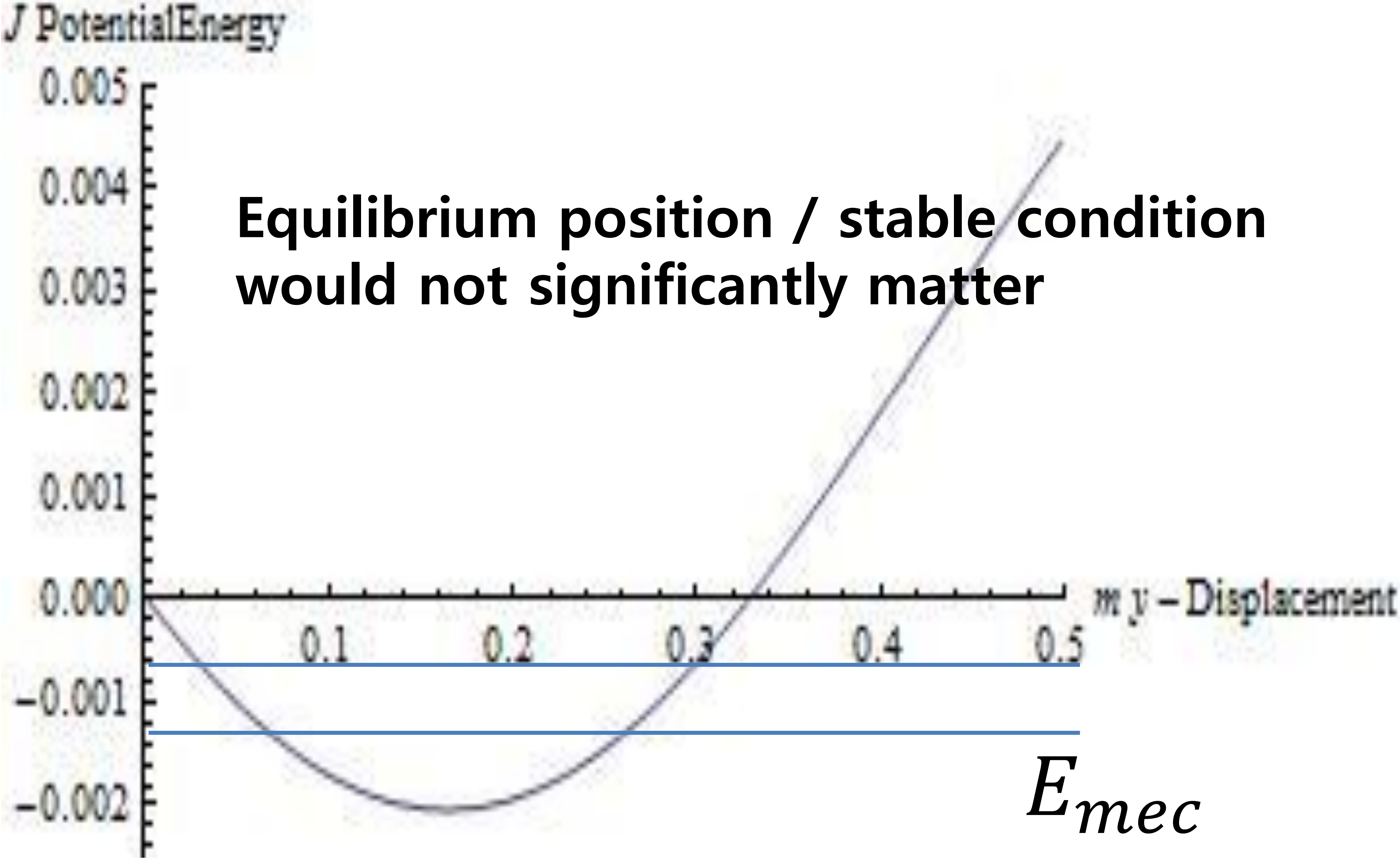
Under assumptions that…

1. Velocity of the ball can be ignored compared to velocity of the airstream
2. Drag force is a decreasing function of height

We can apply 'drag force field'

just like gravitational field

Starting point of the ball does not matter!



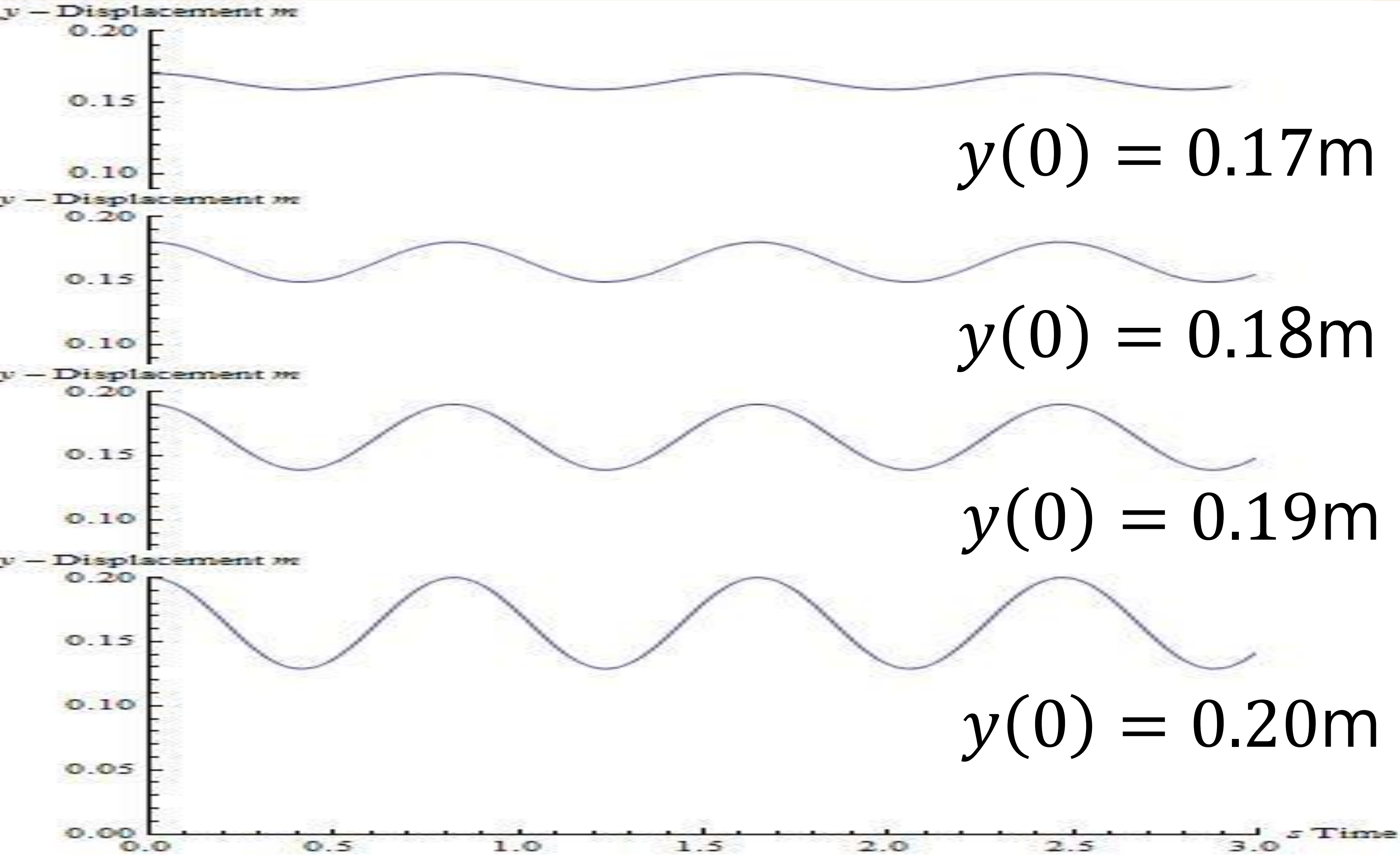
Equilibrium position / stable condition would not significantly matter

E_{mec}

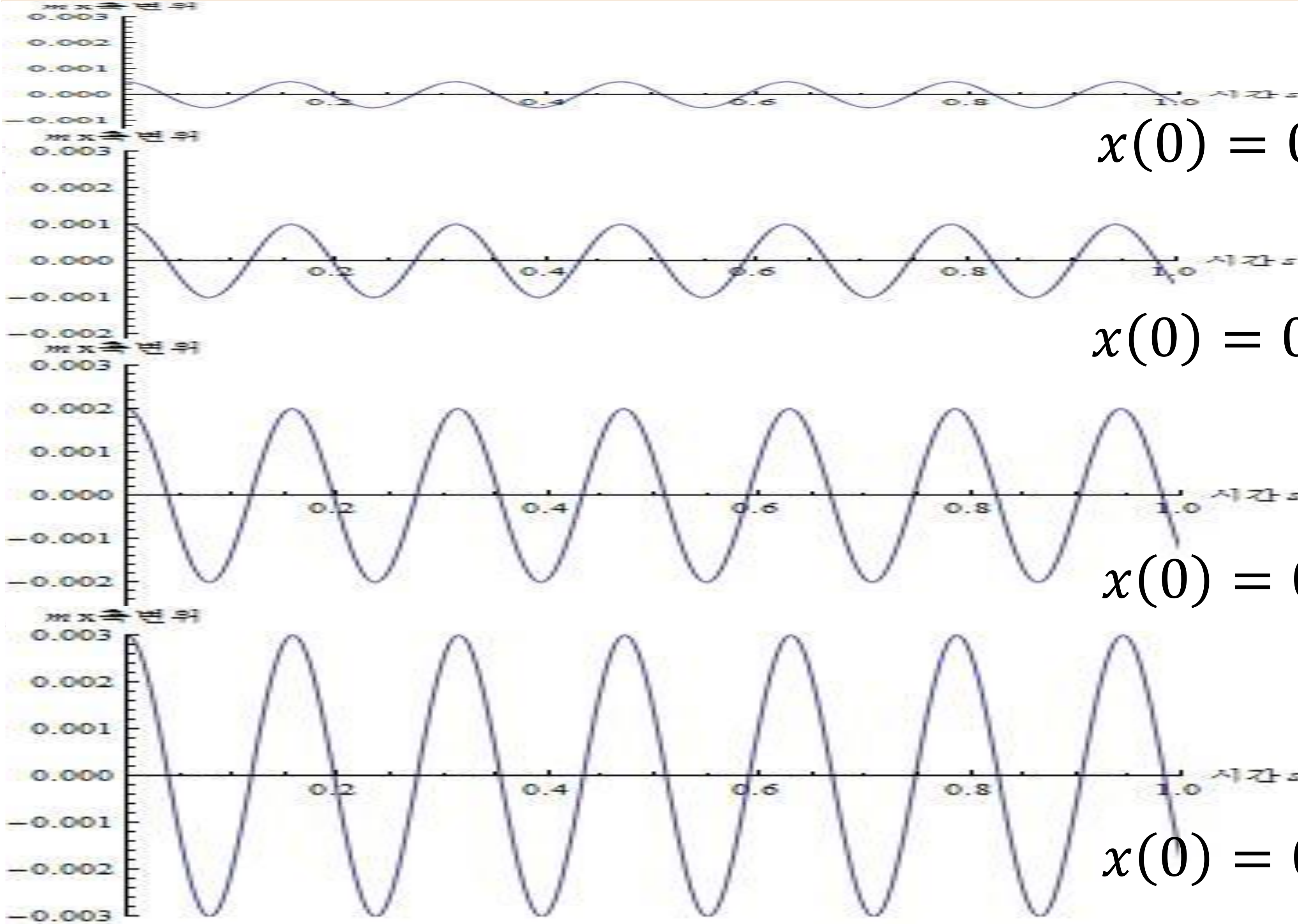
How measure maximum angle of tilt?

- Ten times of trial
- When stably levitated for more than ten seconds, PASS!

Period and Amplitude: *y*-Oscillation



Period and Amplitude: x-Oscillation



$x(0) = 0.0005\text{m}$

$x(0) = 0.001\text{m}$

$x(0) = 0.002\text{m}$

$x(0) = 0.003\text{m}$

Drag force change due to motion of the ball

$$u_{ball,y} = (y_0 - y_{eq})\left({}^{2\pi}\!/_{T_y}\right) \approx 0.10\text{m/s}$$

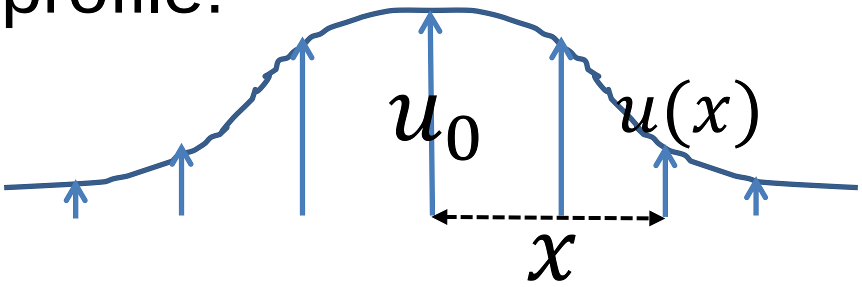
$$u_{ball,x} = x_0\left({}^{2\pi}\!/_{T_x}\right) \approx 0.04\text{m/s}$$

$$U = 16\text{m/s}$$

NEGLIGIBLE!

Gaussian Curve Assumption

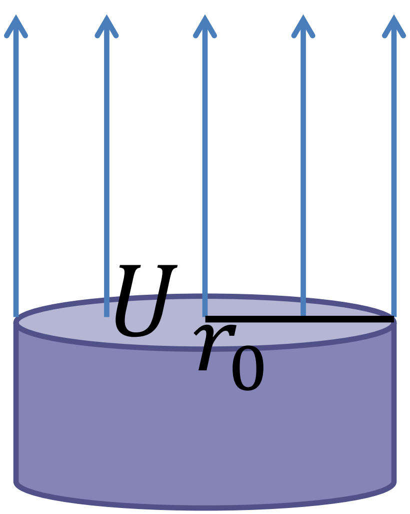
Assume Gaussian curve profile:



$$u(x) = u_0 e^{-\frac{x^2}{2c^2}}$$

$$\frac{du(x)}{dx} = -\frac{u_0}{c^2} x e^{-\frac{x^2}{2c^2}}$$

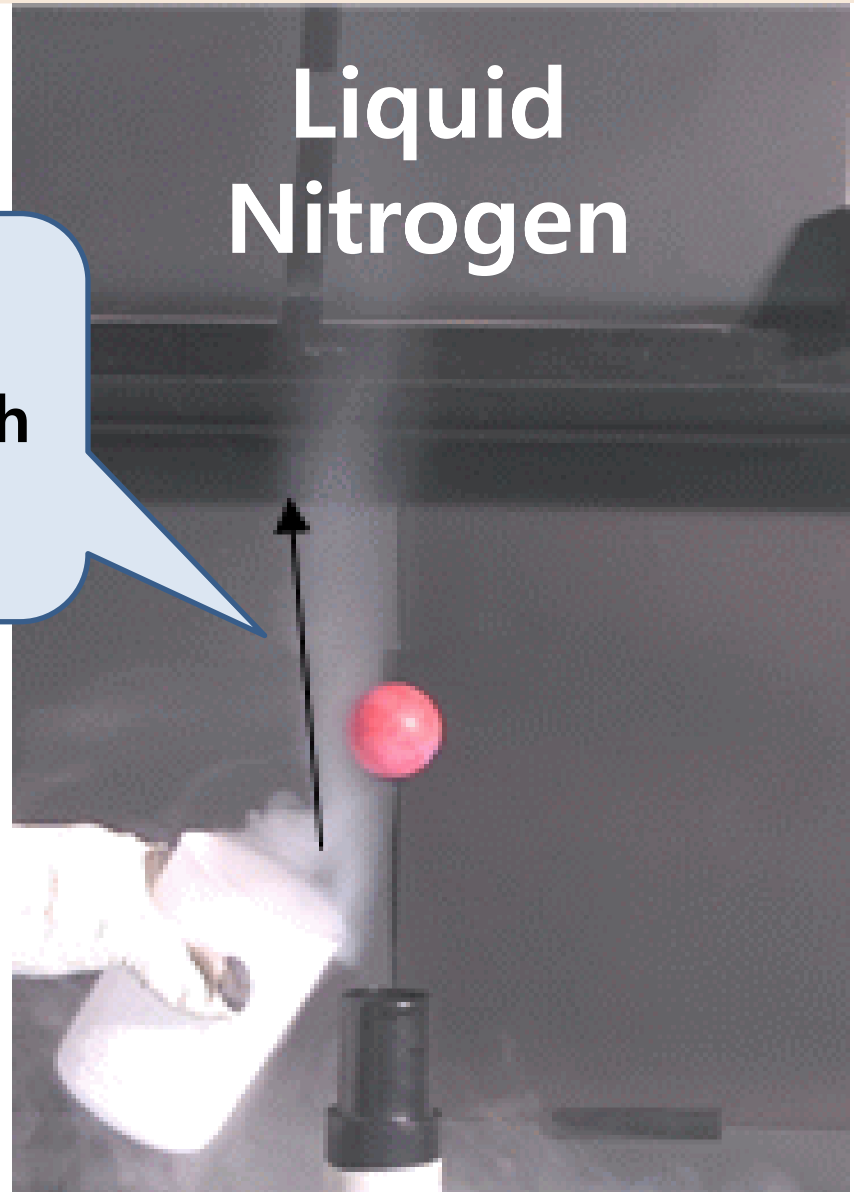
$$= -\frac{1}{c^2} x u(x)$$



$$\frac{du(x)}{dx} \propto -x u(x)$$

Experimental proof: Coanda < Bernoulli

Outer layer:
No interaction with
the ball surface



JAVA code

```
public void obtainFilteredImage()
{
        for (int w=1; w<=width-1; w++)
        {
                for (int h=1; h<=height-1; h++)
                {

                if (red[w][h]>50 && red[w][h]-blue[w][h]>20 && red[w][h]-
green[w][h]>20)
                {
                copyImg.setRGB(w,h,RED.getRGB());
        }


                else
                {
                copyImg.setRGB(w,h,WHITE.getRGB())
        }
        }
 }
```

JAVA code

```
public double[] center()
{
        int xsum, ysum, count = 0;;;
        double[] c = new double[2];
        for (int w=1; w<=width-1; w++){
                for (int h=1; h<=height-1; h++){
                if(new Color(copyImg.getRGB(w,h)).equals(new Color(255,0,0)) ){
                                        xsum += w;
                                        ysum += h;
                                        count += 1;
                                }
                        }
                }
                c[0] = xsum/count;
                c[1] = ysum/count;
                return c;
        }
```

JAVA code

```
public static double rotateX(double x, double y)
		{
			double xReal = x-referX;
			double yReal = referY-y;

			return xReal*Math.cos(deg)+yReal*Math.sin(deg);
		}

		public static double rotateY(double x, double y)
		{
			double xReal = x-referX;
			double yReal = referY-y;

			return yReal*Math.cos(deg)-xReal*Math.sin(deg);
		}
```