Invent yourself

Ivan Chaika
Roman Doronin
Vitaliy Matiunin
Aleksandr Severinov
Vladislav Tumanov

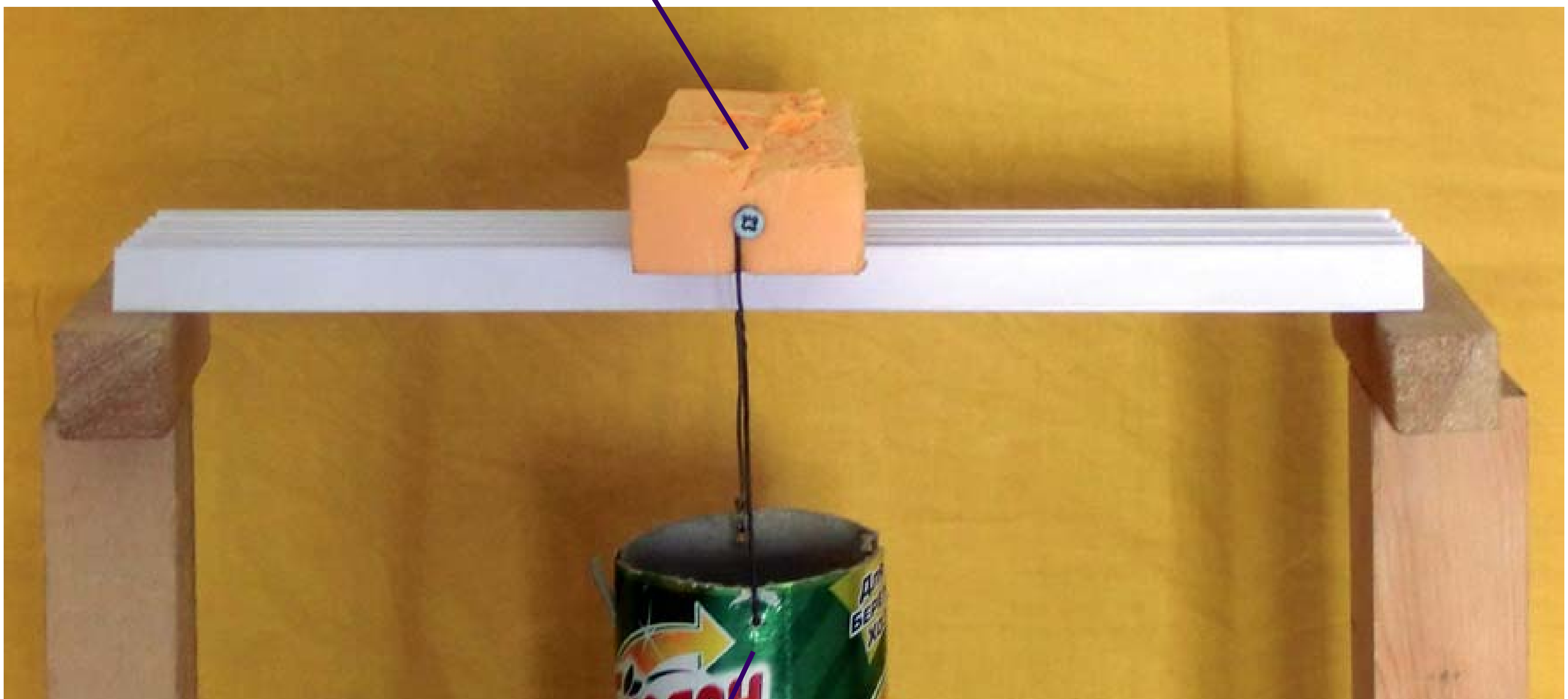
Russia
IYPT

The problem

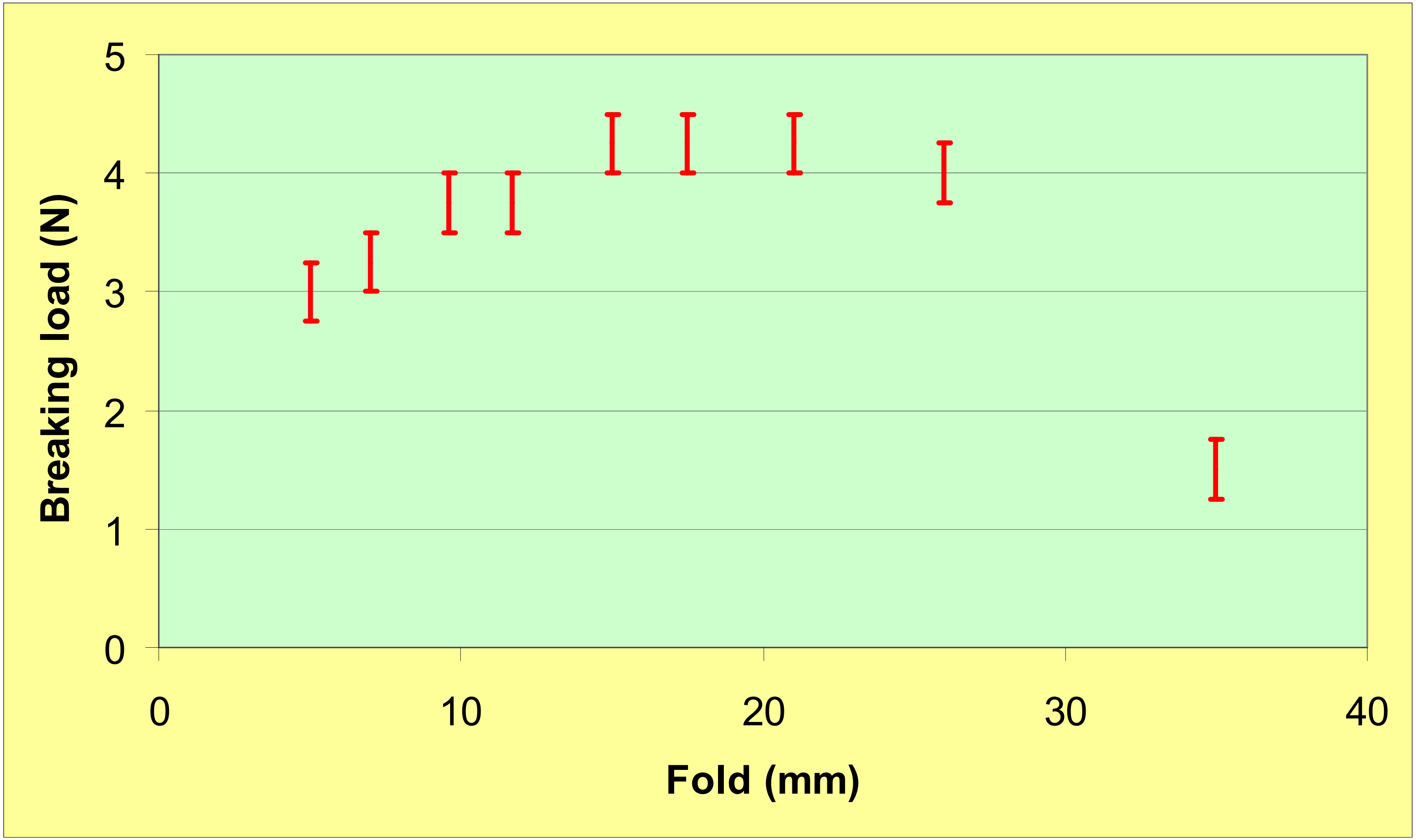
It is more difficult to bend a paper sheet, if it is folded "accordion style" or rolled into a tube. Using a single A4 sheet and a small amount of glue, if required, construct a bridge spanning a gap of 280 mm. Introduce parameters to describe the strength of your bridge, and optimize some or all of them.

Accordion style bridges

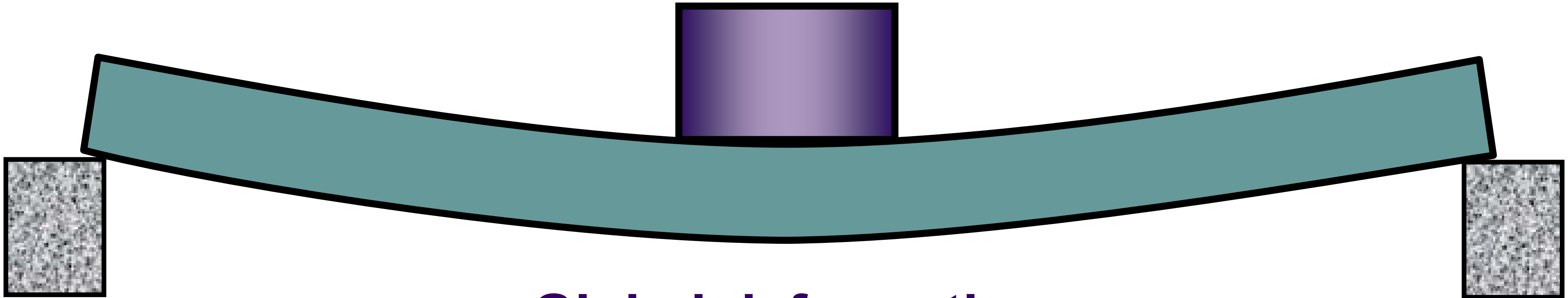
Bridge testing



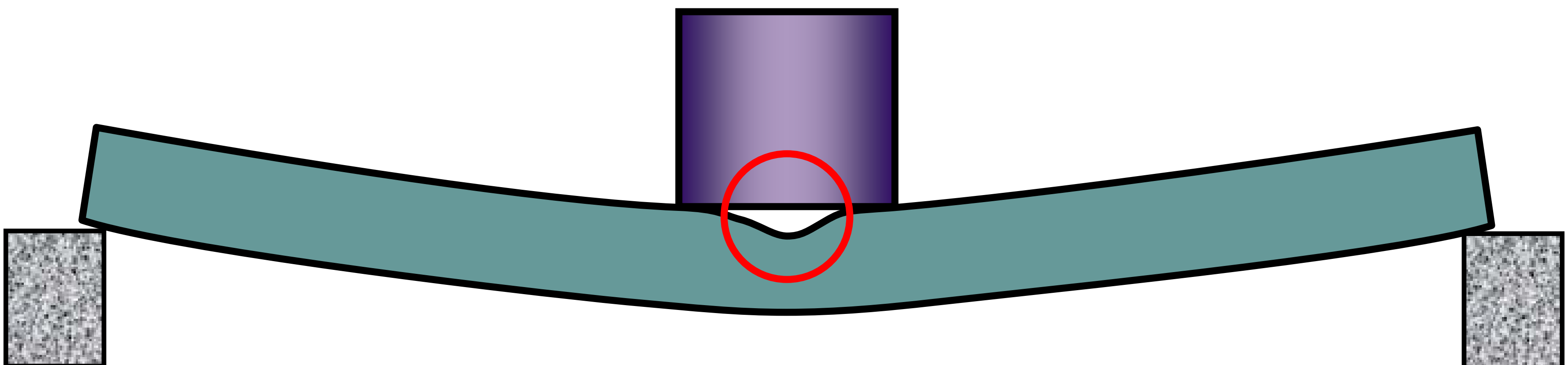
Test results



Loading of thin-walled constructions



Global deformation



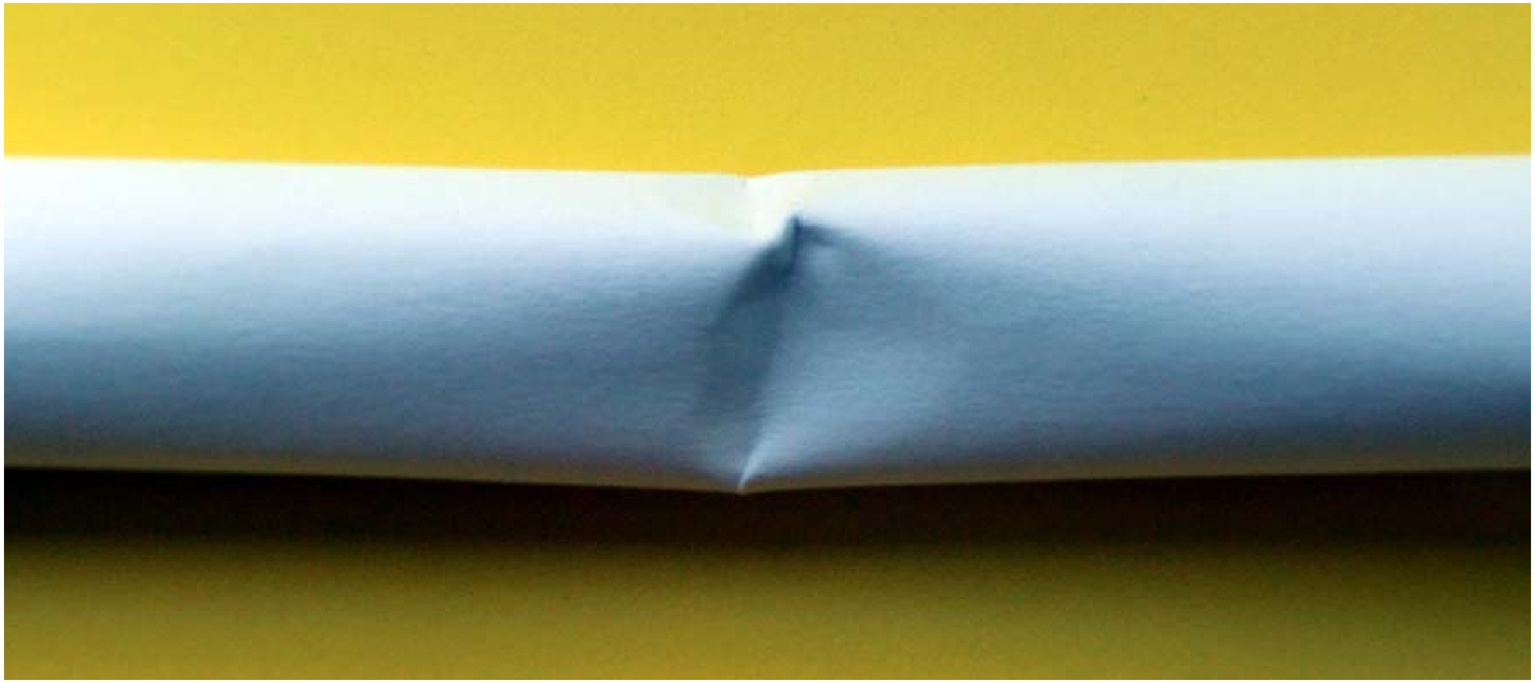
Local buckling

Tube bridges

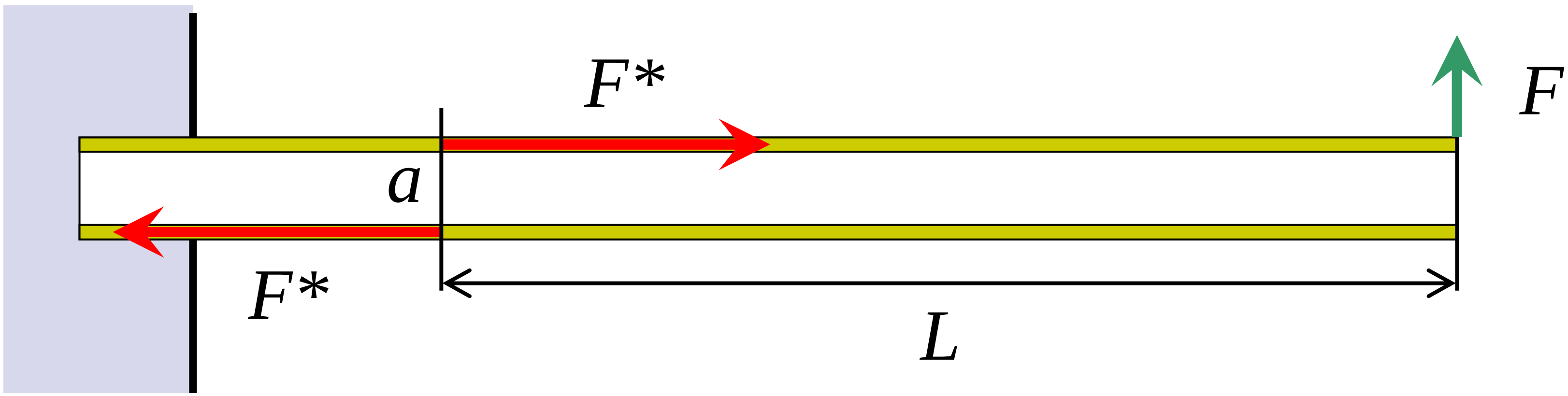
Tube overhead crane

How the tube breaks

- The tube usually breaks in the midspan.
- The tube is wrinkled at the top due to local buckling.



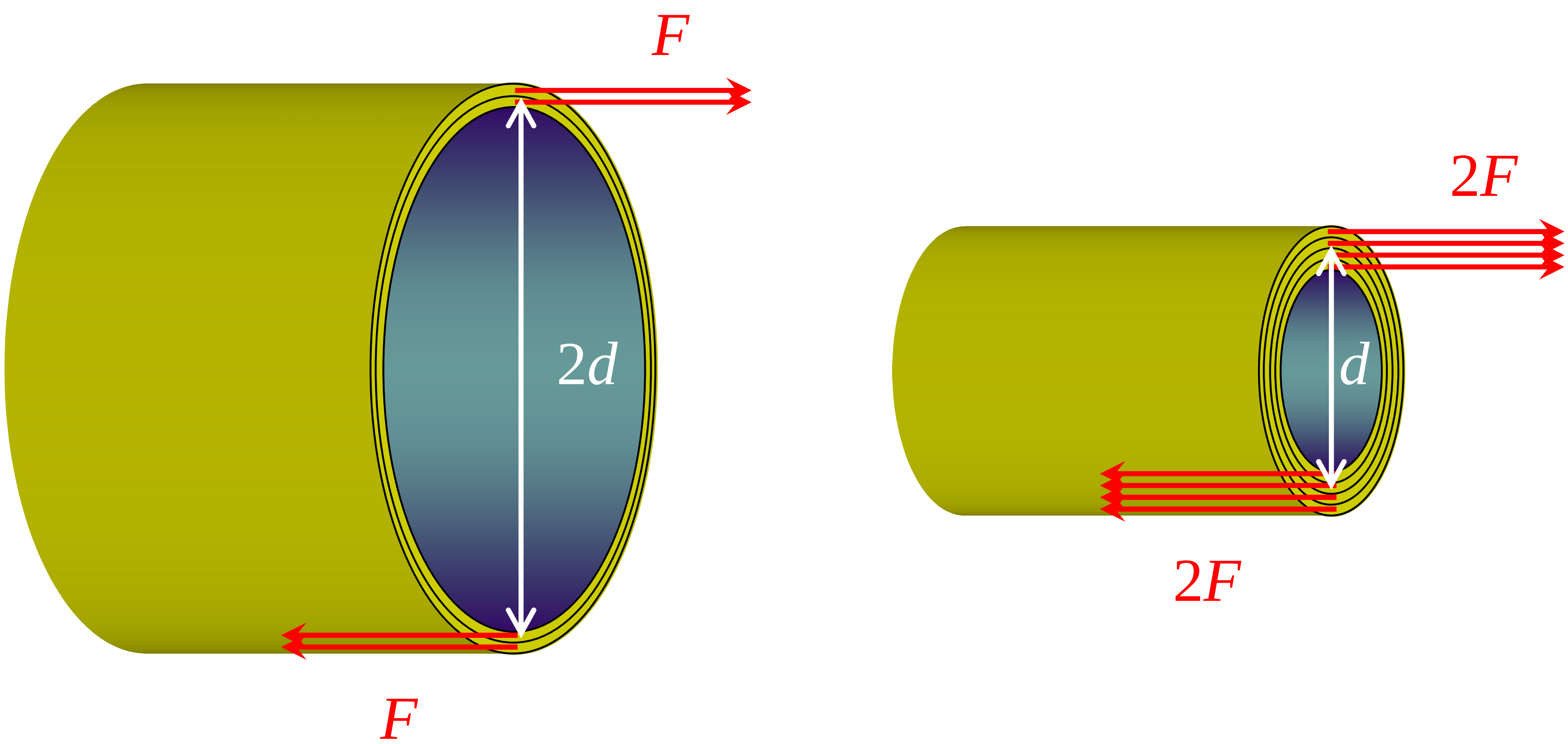
Role of torques



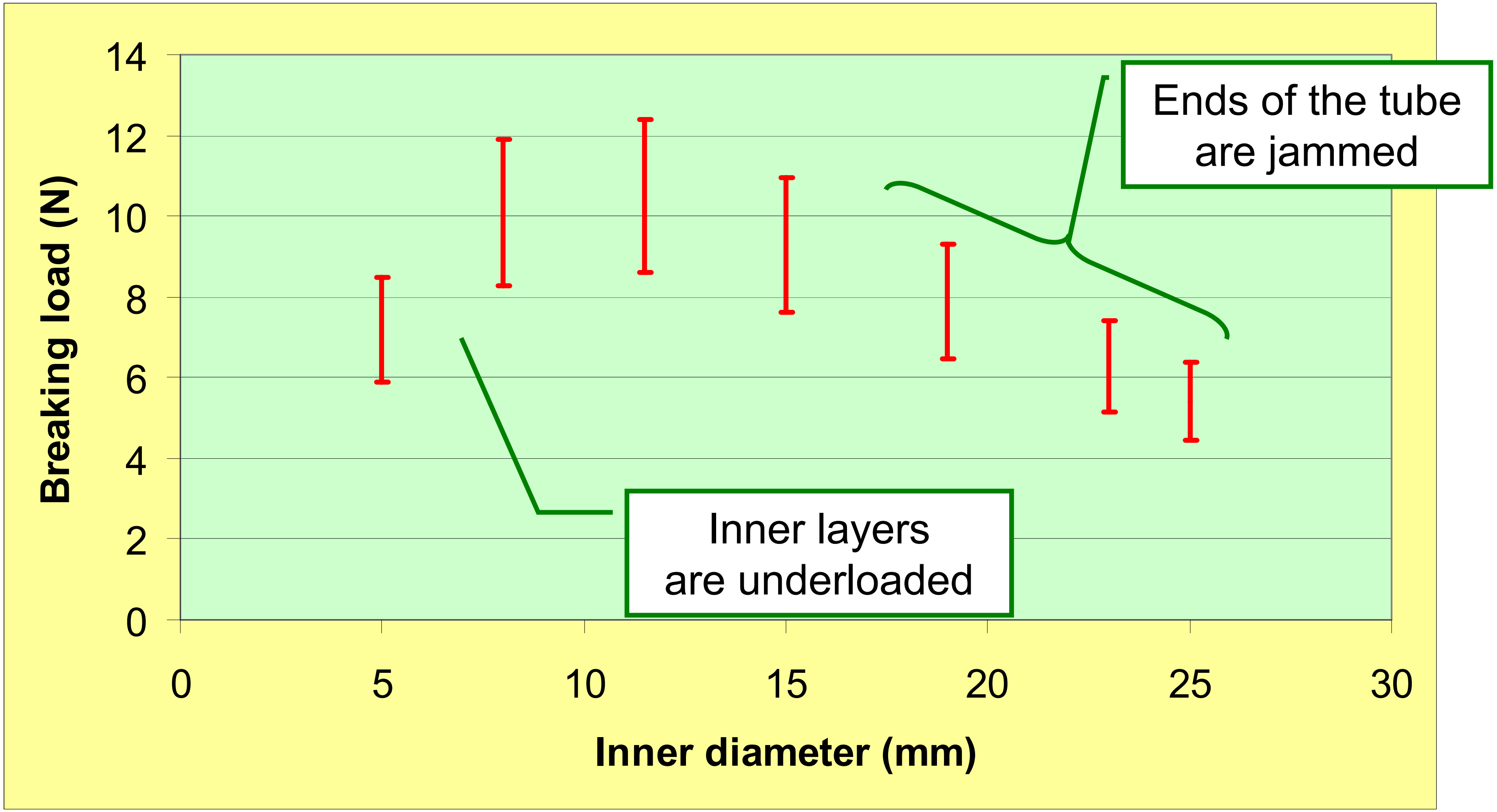
- Force F applied to the arm L produces torque $F \cdot L$.
- This torque is balanced by elastic forces of tension and compression in the tube walls: $F \cdot L = F^* \cdot a$.
- Critical compression at the top of the tube leads to a local buckling.

Theoretical prediction

We expect that the breaking load is independent on the diameter of the tube.

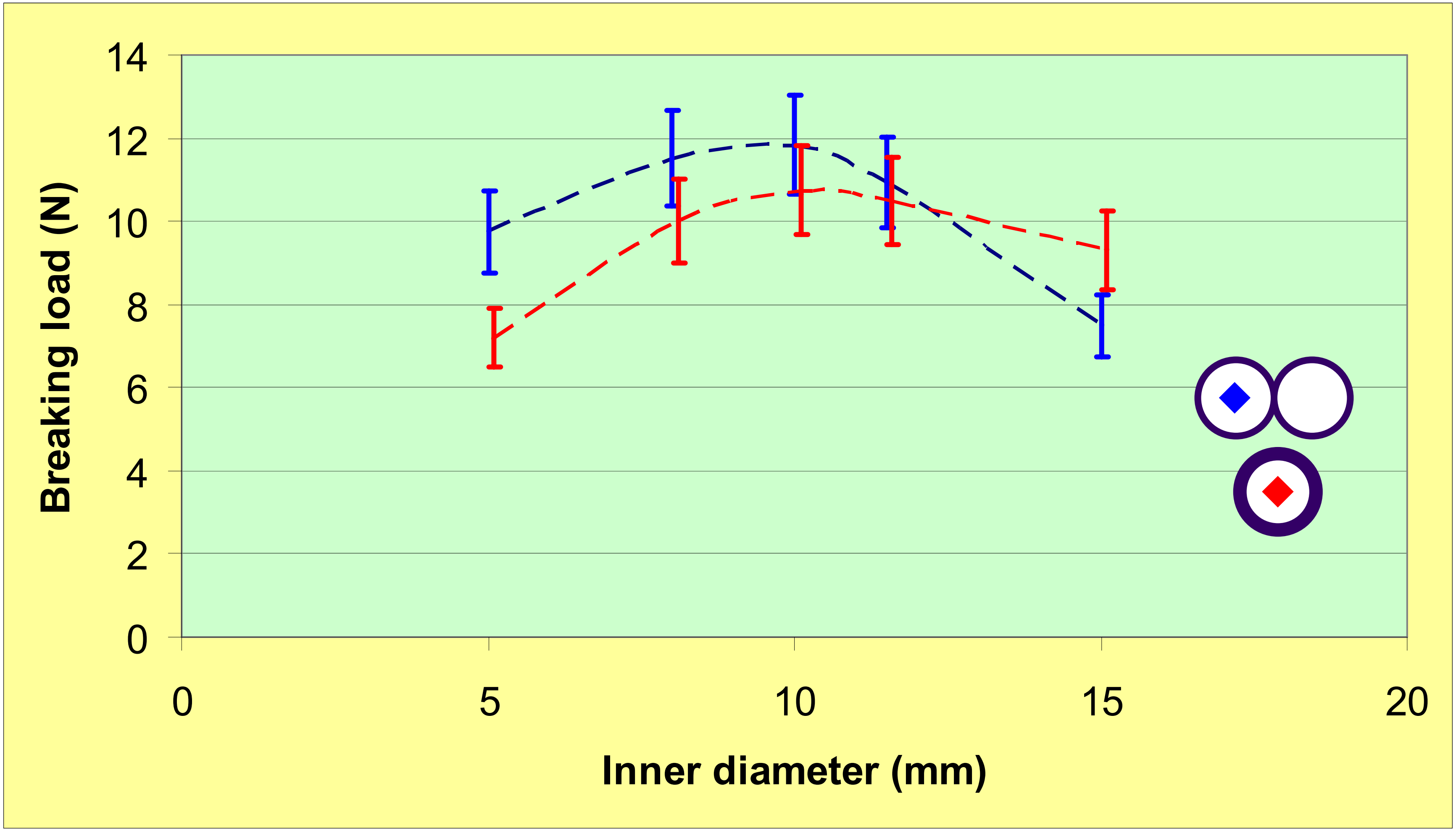


Test results

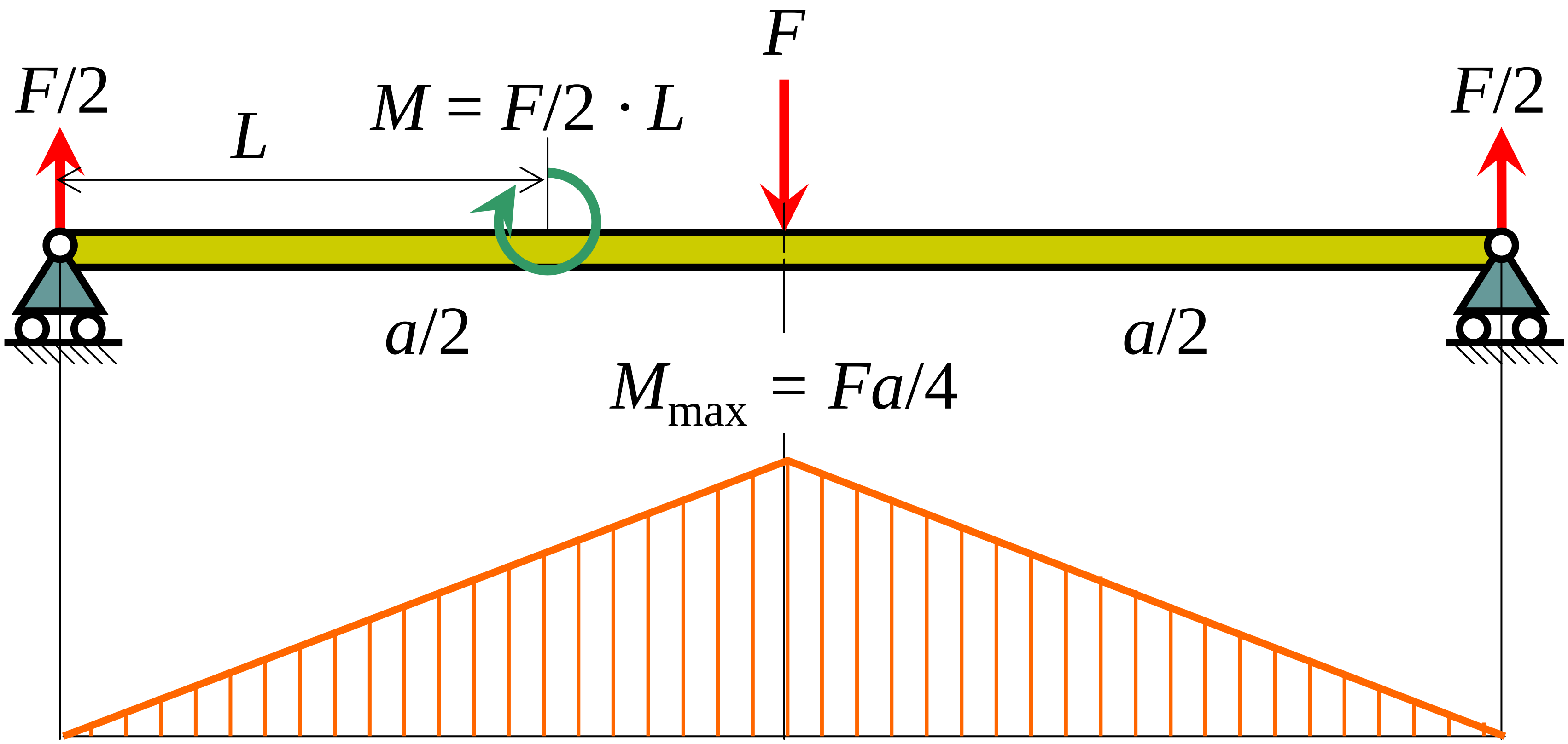


A4 sheet of paper curled into a tube 297 mm in length.
For each tube diameter 10 trials were conducted.

Testing of twin tubes



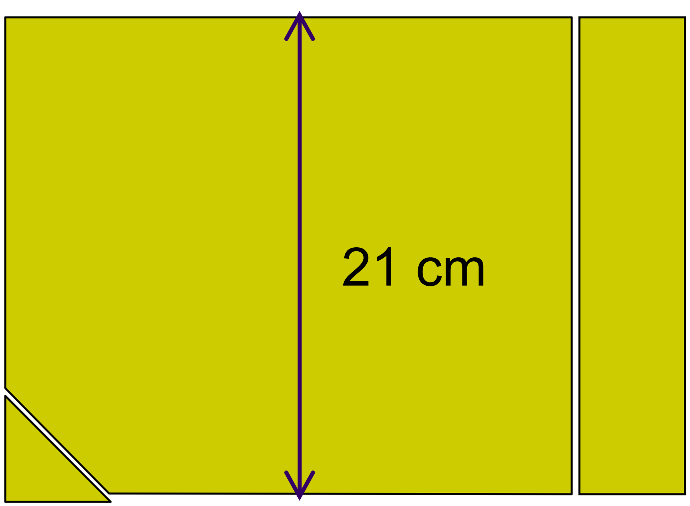
Distribution of torques

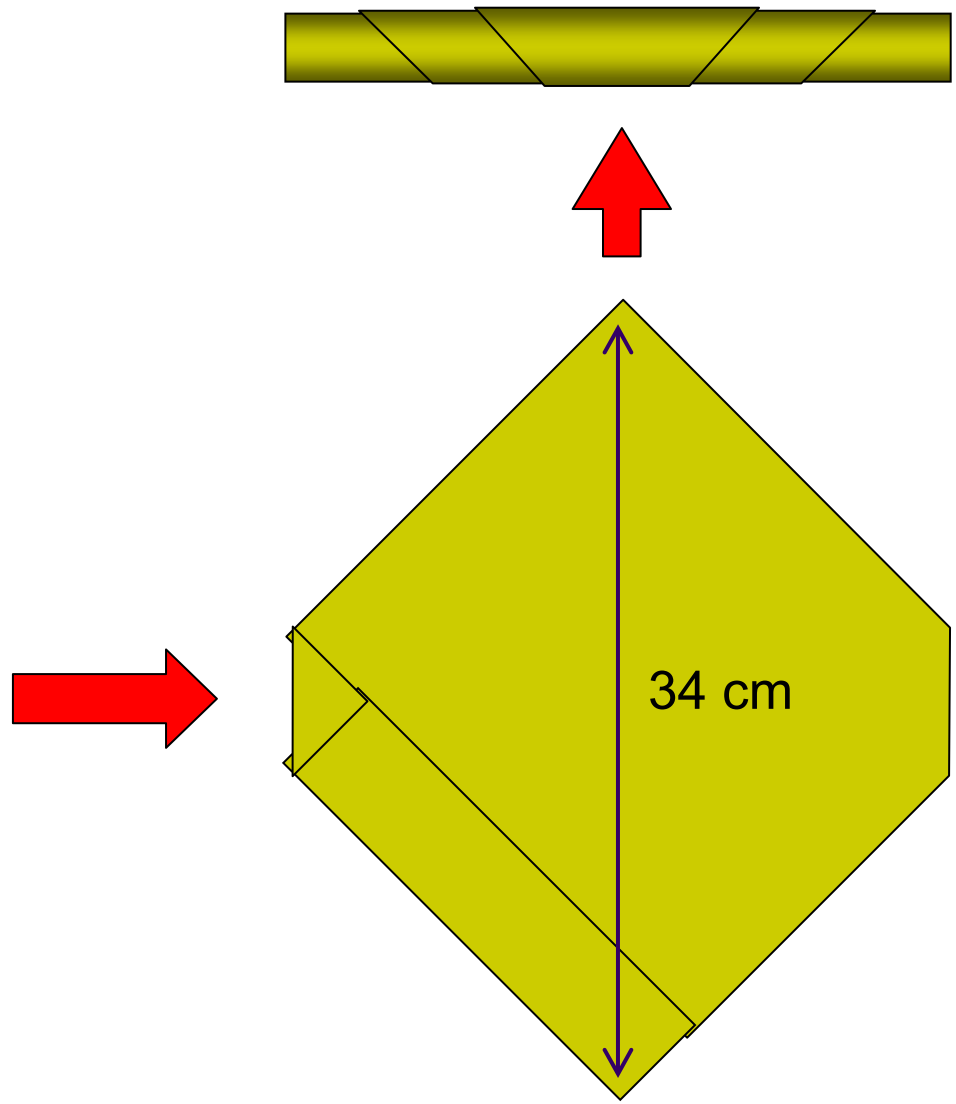


The most dangerous place of the beam is its midspan.
So it is desirable to strengthen the midspan more than the ends.

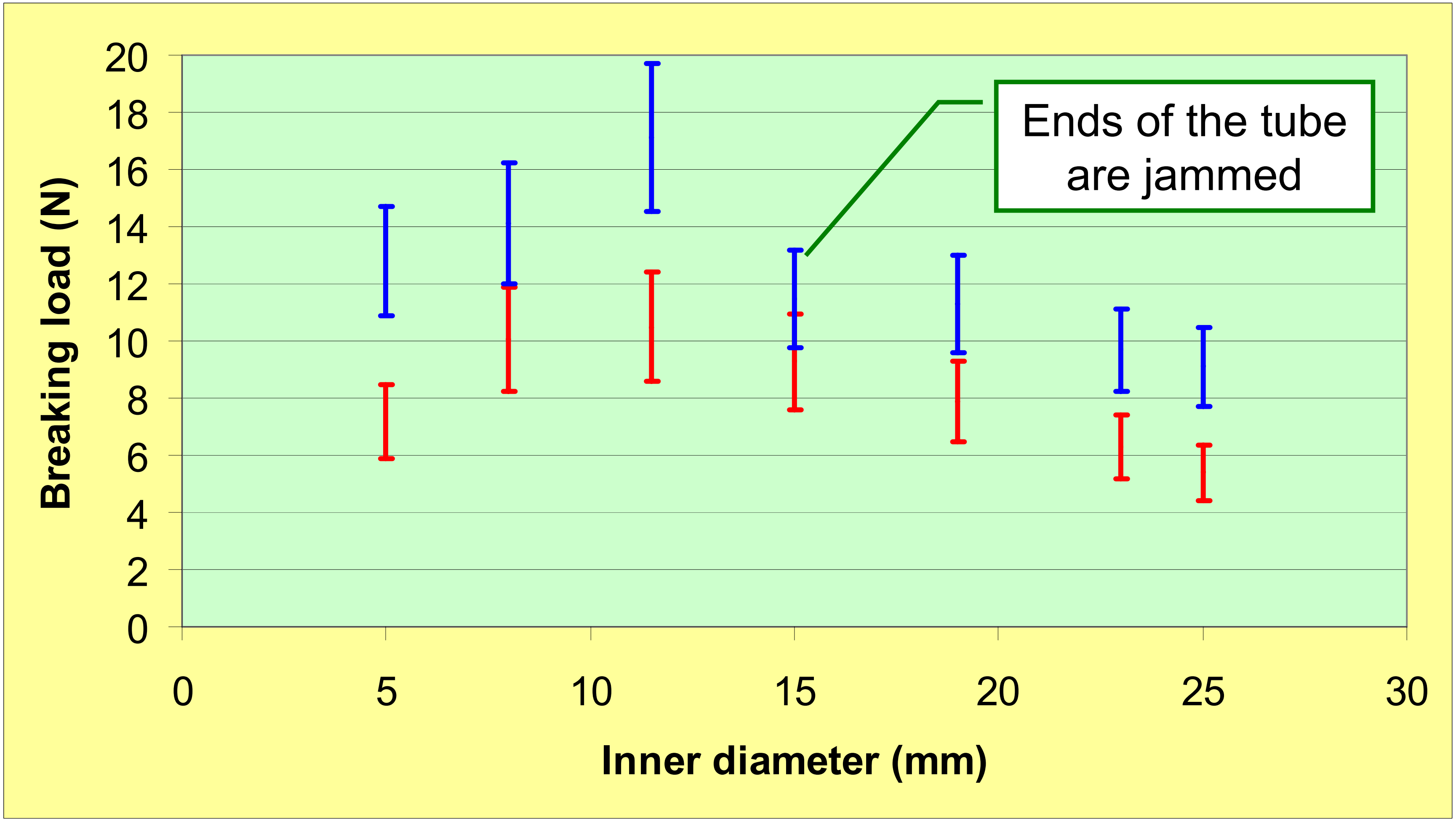
How to strength the midspan

34 : 21 = 1,6
Expected to increase
the breaking load
by 1.6 times.





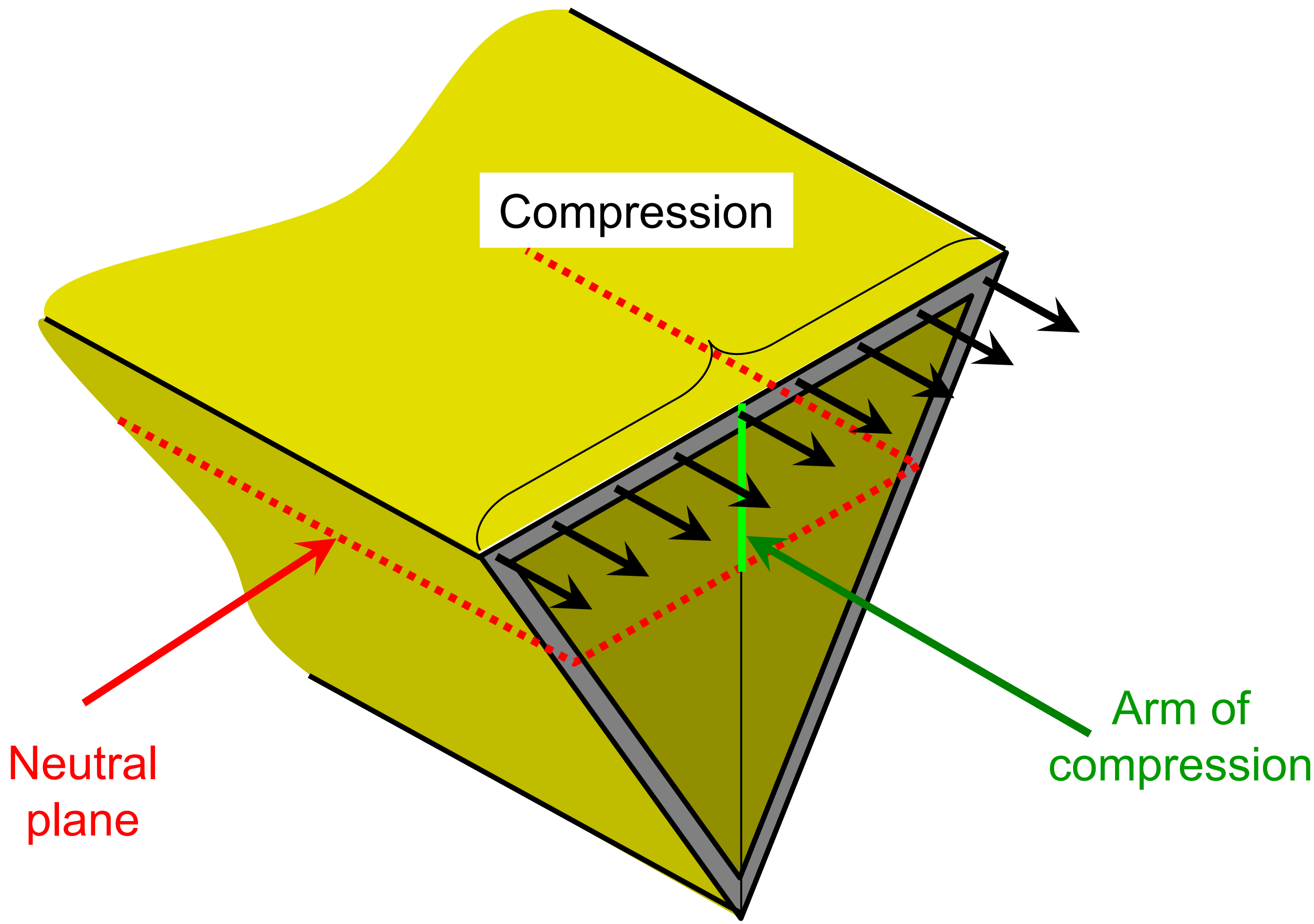
Testing of strengthening tubes



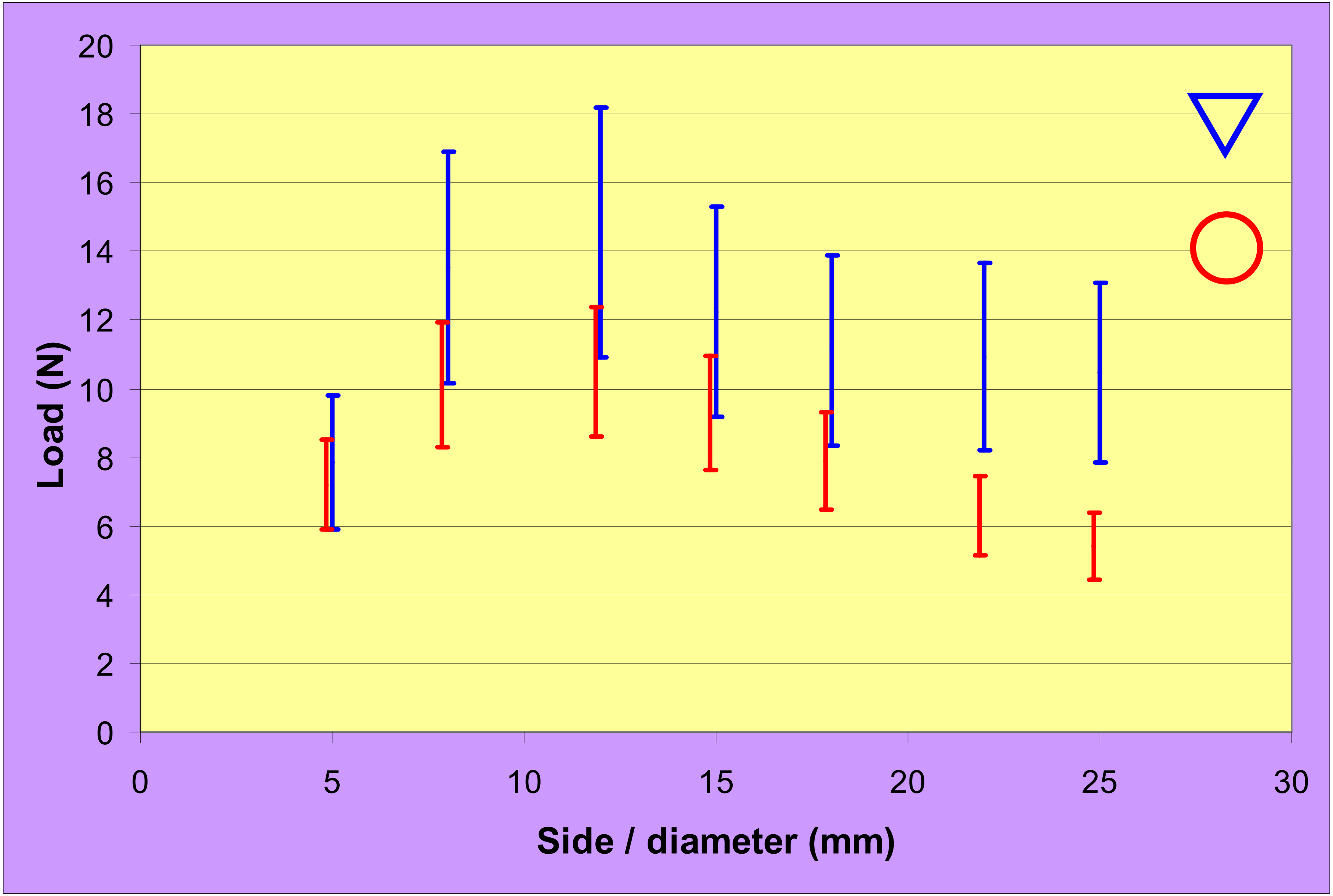
Red segments — tubes of constant section
Blue segments — tubes with midspan strengthening

Triangular beam bridges

Why triangular beam is stronger than round?



Test results

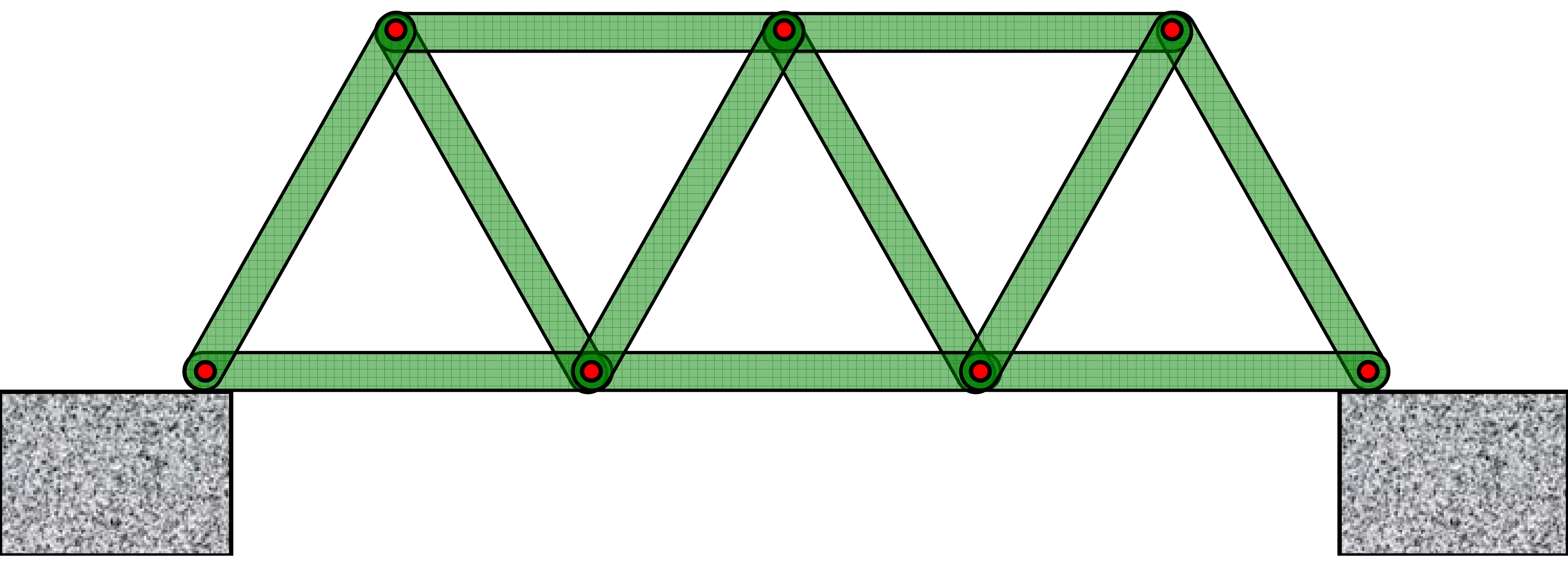


Truss bridges

Truss bridge in Novosibirsk

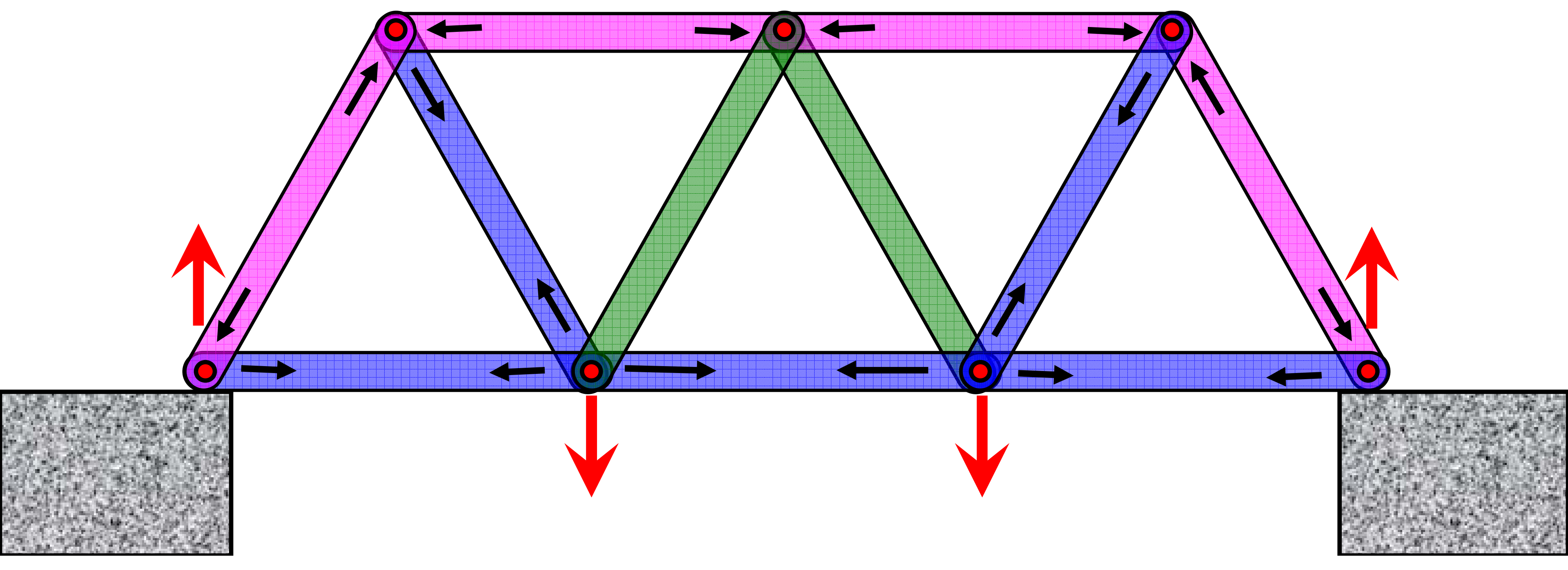
Properties of truss structure

- A truss structure consists of **straight members** connected at **nodes.**
- Trusses are composed of triangles because of the structural stability of that shape.



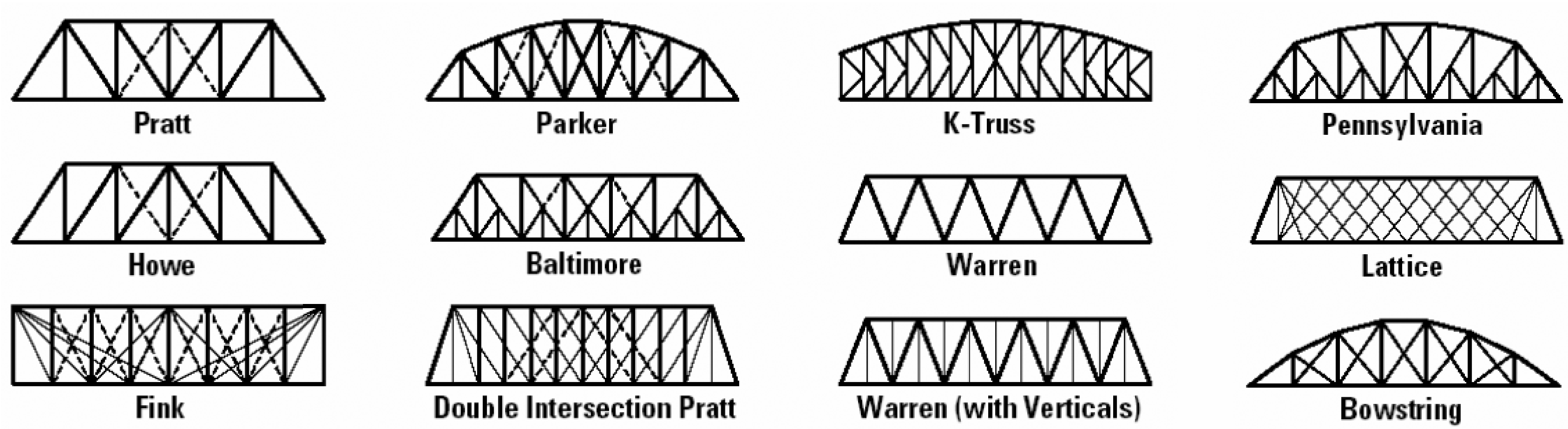
Properties of truss structure

- **External forces and reactions** in trusses act such a way that truss members are only in **tension** or in **compression**.
- Torques in truss structures are excluded.

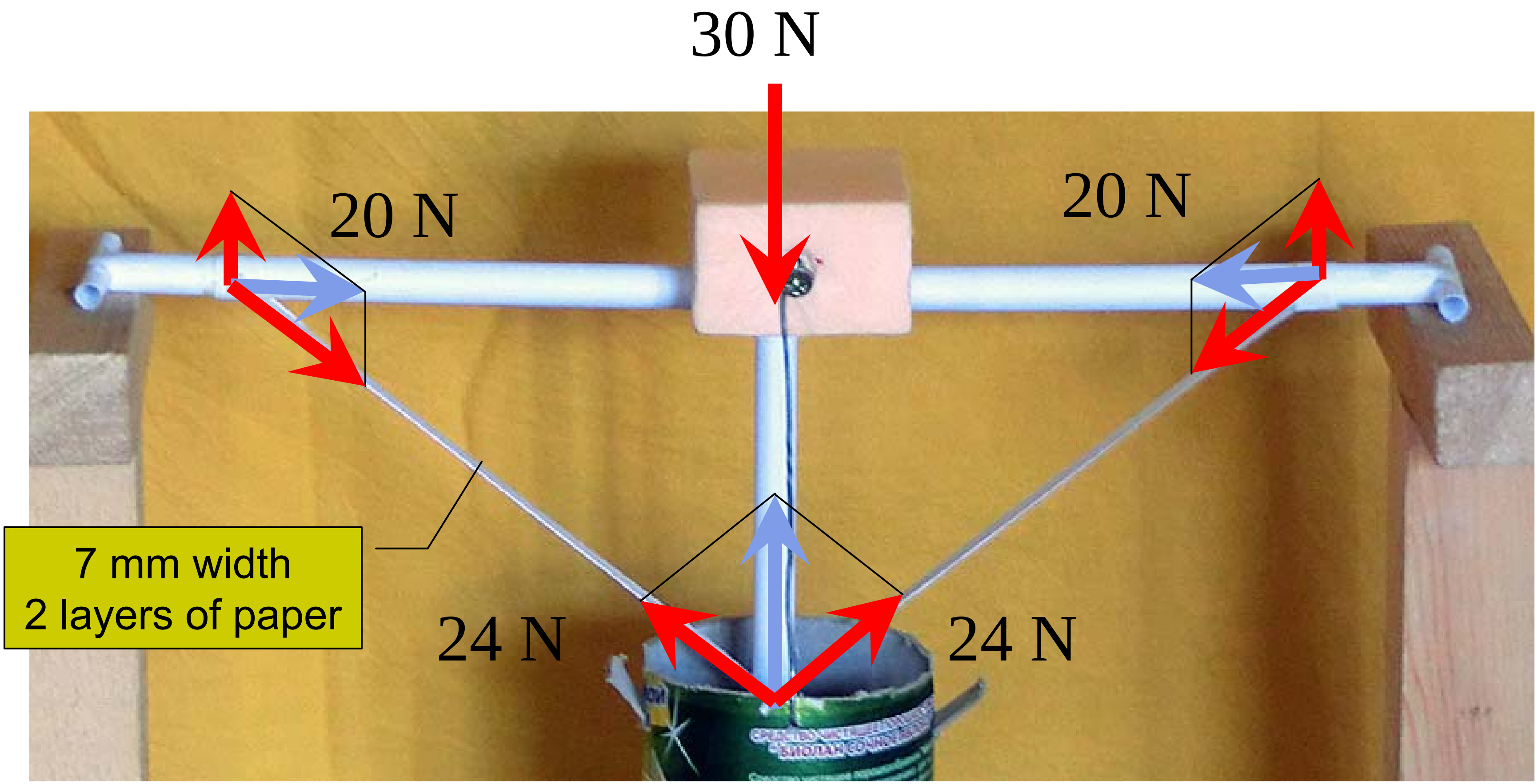


Truss structure for paper bridges

- Paper is enough strong in tension.
- Short paper tubes are sufficiently strong in compression.
- **So a truss structure may be enough good for paper bridges.**

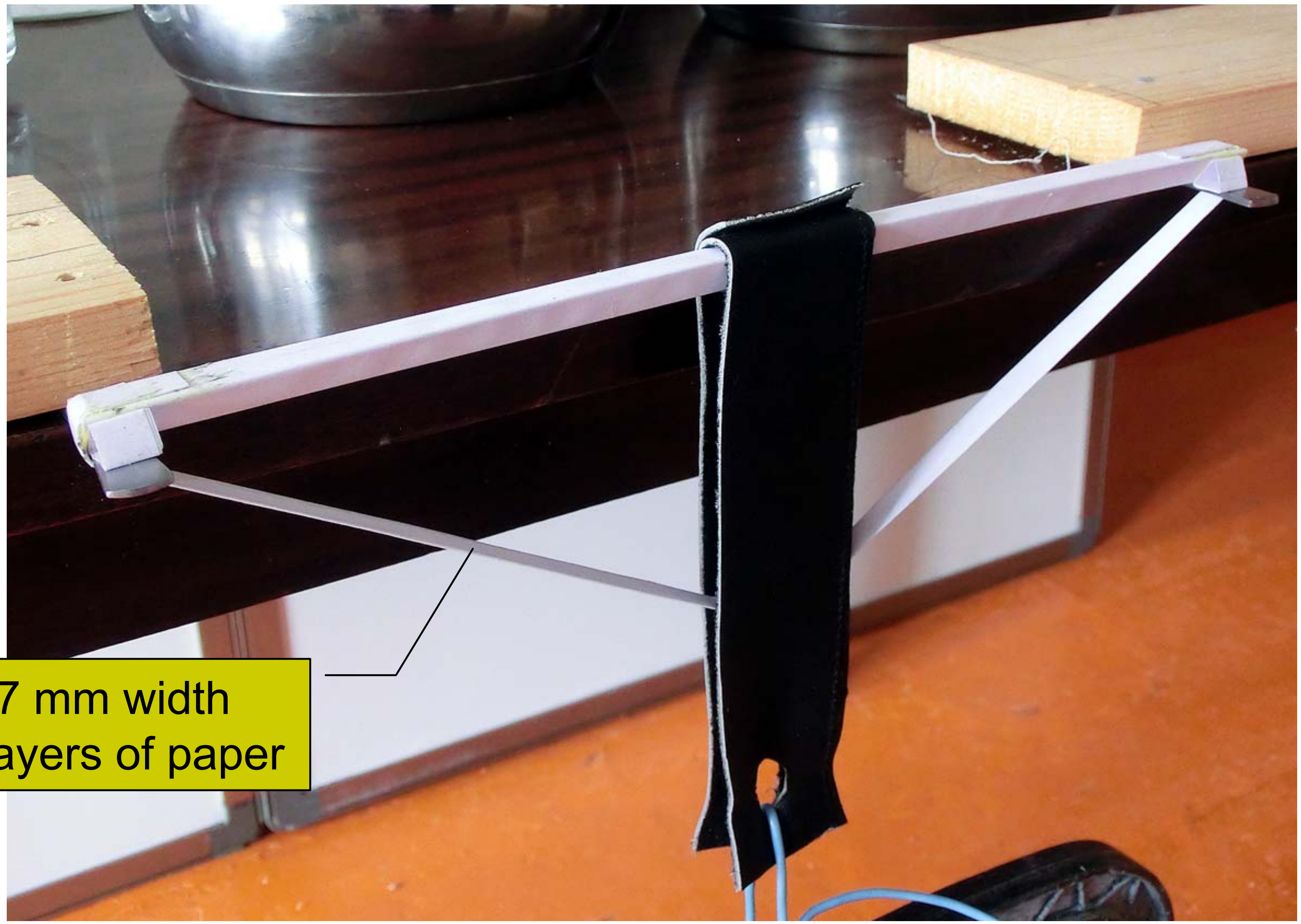


“Inverted king-post truss”



6 bridges of this type were tested.
In 3 trials horizontal beam bent (**32, 30, 31 N**).
In 3 trials a bracing broken (**30.5, 32.5, 28.5 N**).

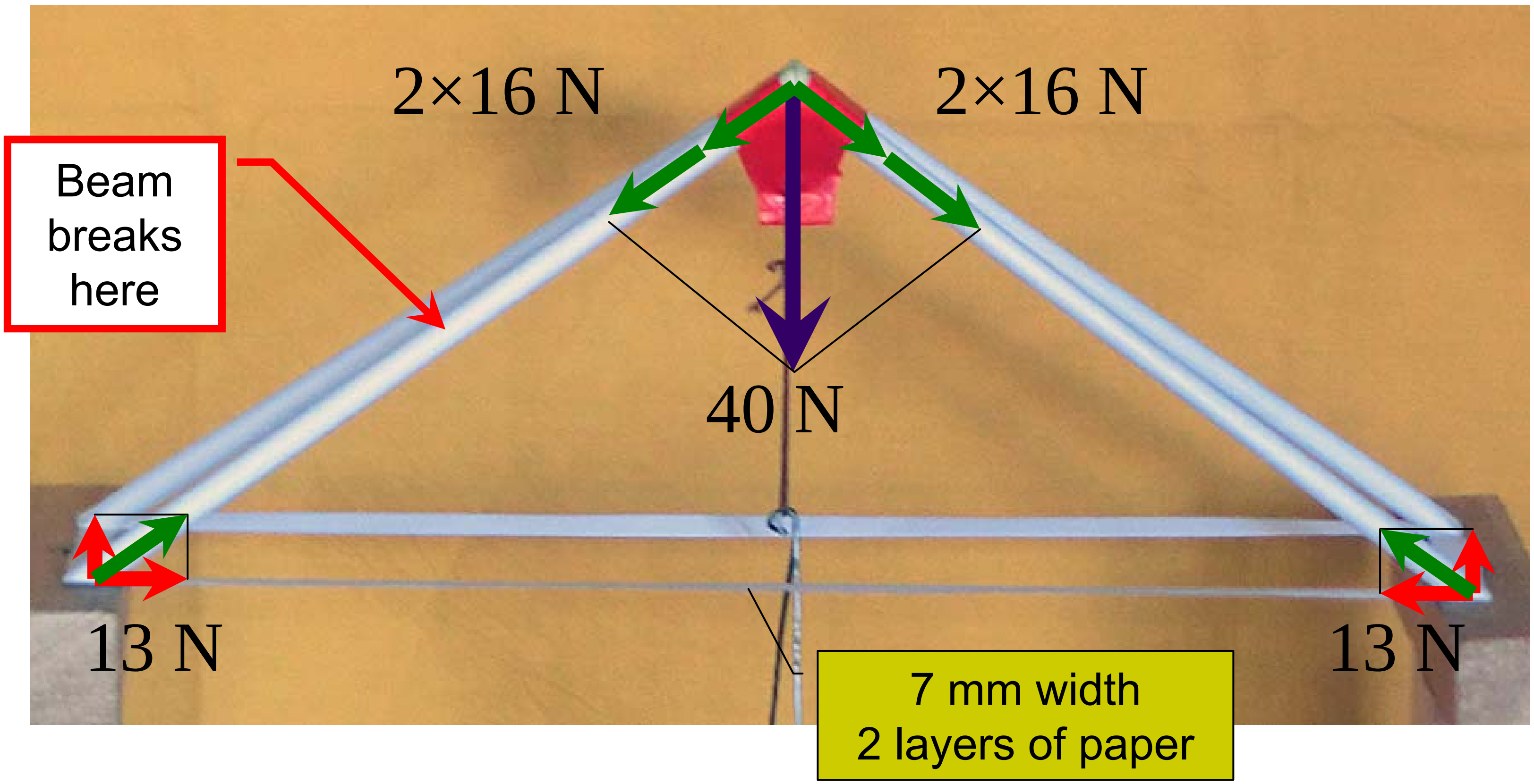
"King-post truss" with triangular beam



7 mm width
4 layers of paper

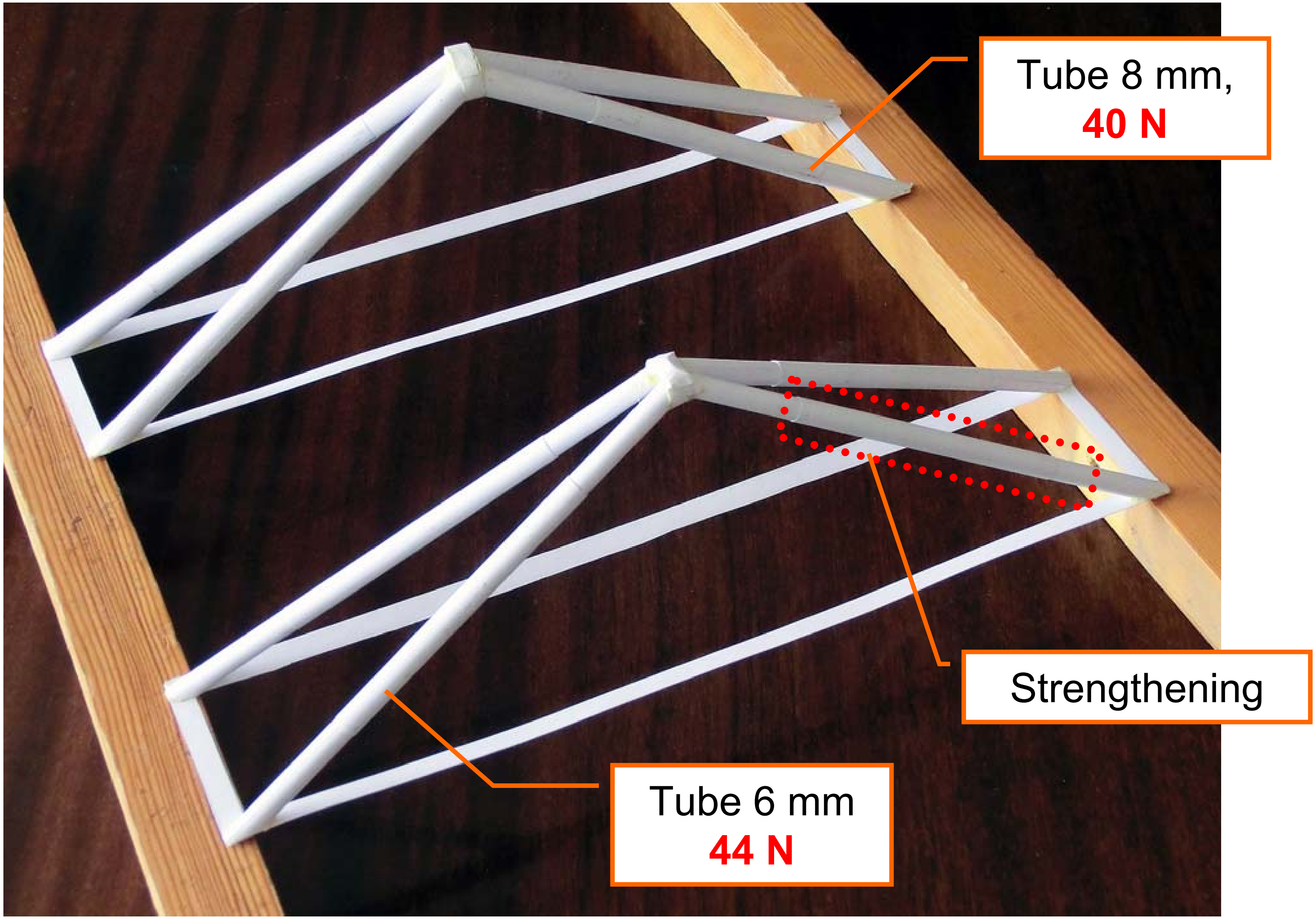
6 bridges of this type were tested. Breaking load was **102, 61, 42, 66, 59, 56, 47, 61 N.** The beam was usually bending "sideways".

Arch truss



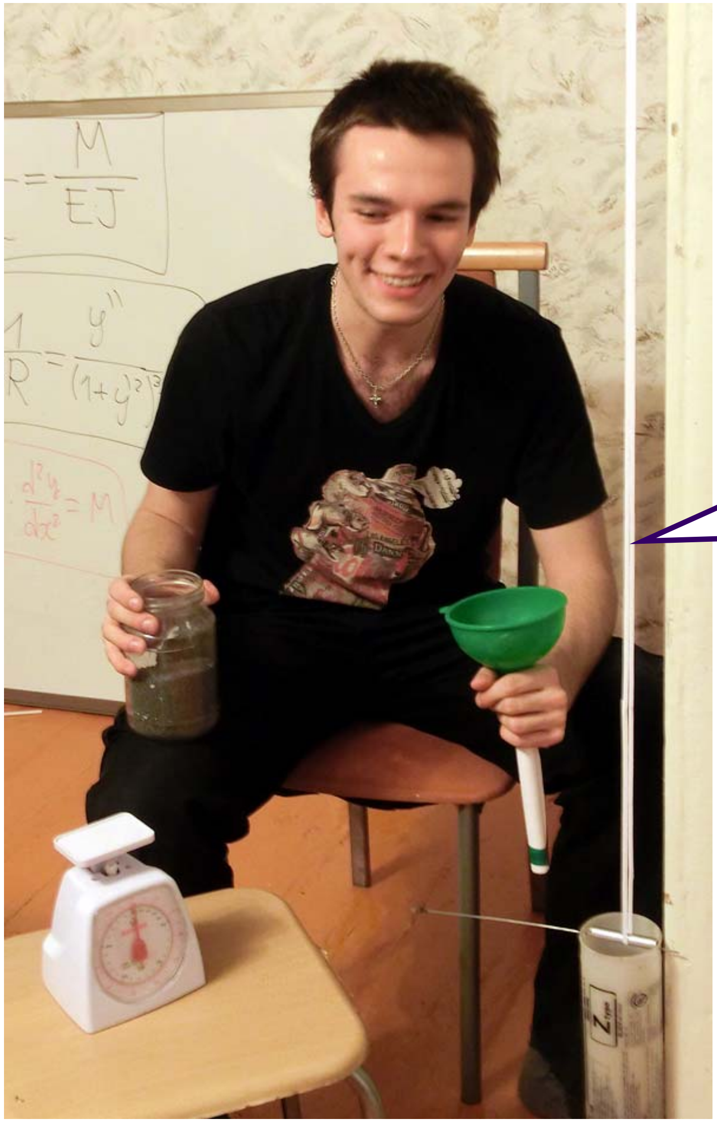
Four bridges broke under the load of **31, 32, 32.3, 34 N**.

Arch truss with strengthening



Paper tensile test

Testing procedure



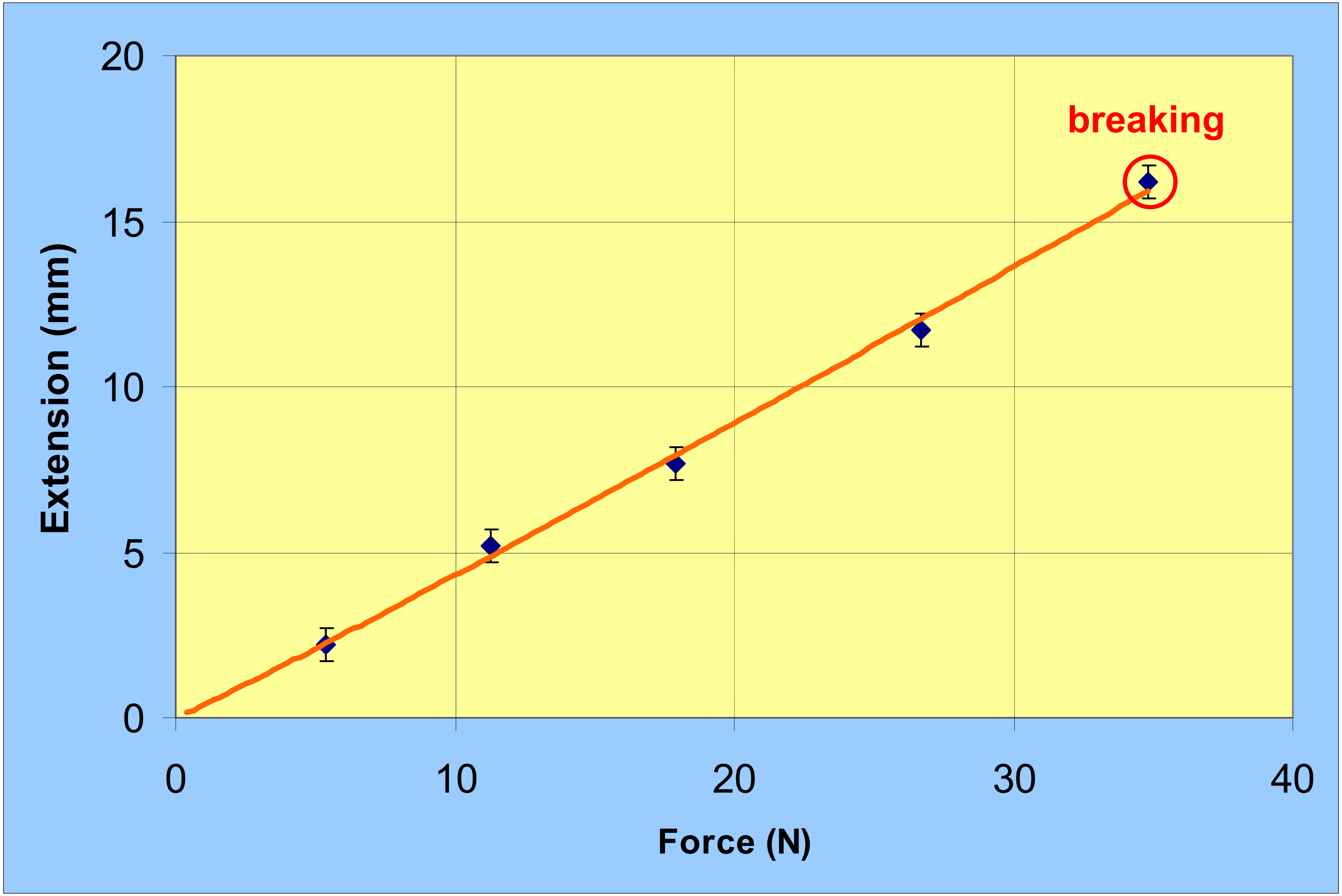
Paper twin bend

Length 207 cm

Width 2 × 5 mm

Thickness 0,1 mm

Testing results



Material properties

stress

Young's modulus

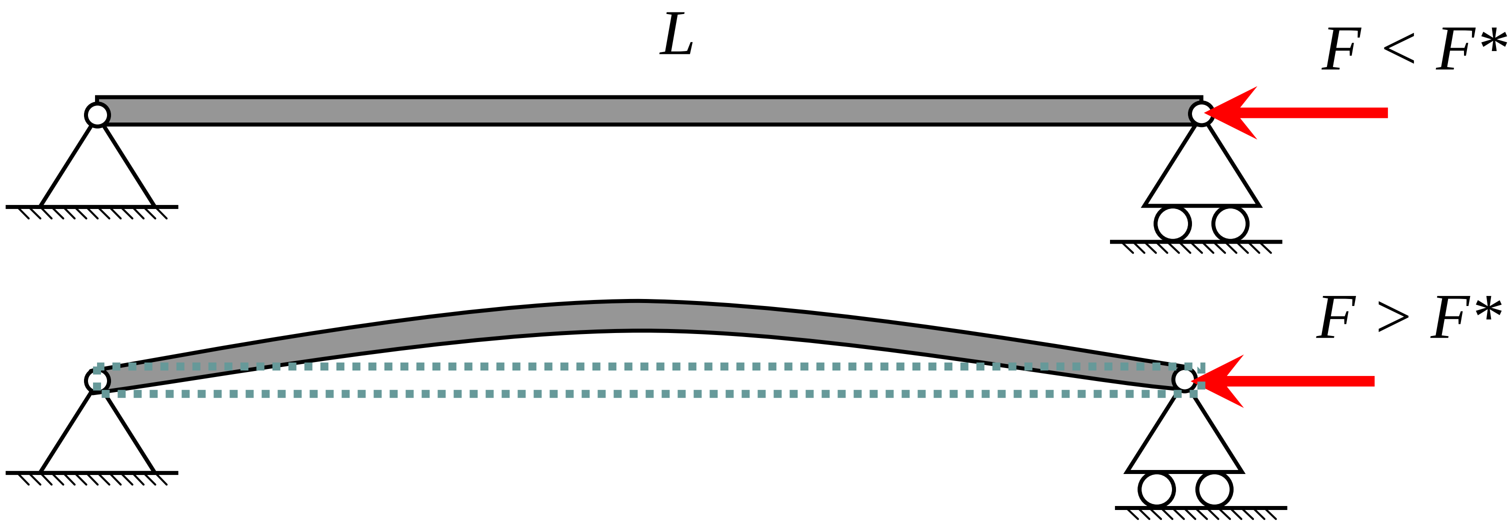
$$E = \frac{\sigma}{\varepsilon} = \frac{F:S}{\Delta l : l}$$

strain

- Young's modulus ***E* = 5·10^9 N/m^2.**
- Tensile strength **3500 N/m.**

Euler rods instability

Euler's formula



$$F^* = \frac{\pi^2 EJ}{L^2}$$

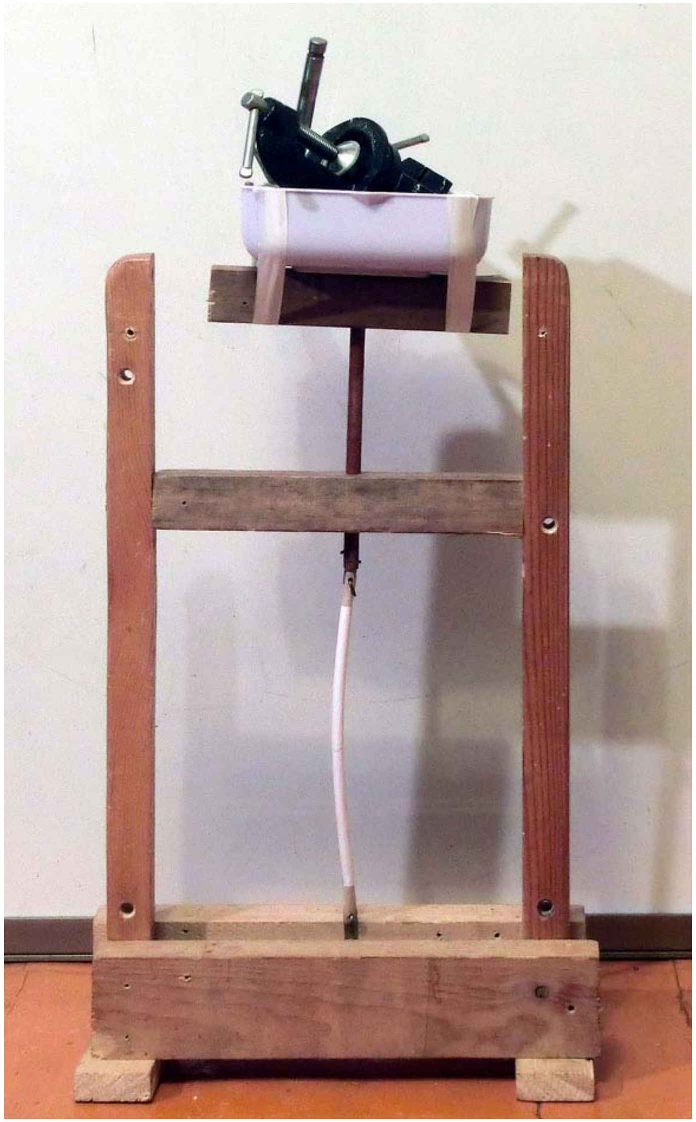
Area moment of inertia

When $F > F^*$, the axial load work exceeds the energy of elastic deformation of the rod, and the loss of stability becomes energetically favorable.

Calculation of the critical load

- Tube length ***L* = 15 cm.**
- Tube inner radius ***r* = 3 mm.**
- Width of a sheet **74 mm** → 4 layers of paper.
- Thickness of the wall **δ = 0,4 mm.**
- Area moment of inertia **$J = \pi r^3 \delta = 4 \cdot 10^{-11}$ m^4.**
- Flexural rigidity **EJ = 0,2 N·m^2.**
- Calculated critical load **F^* = 100 N.**

Testing of tubes stability



- 8 tubes were tested.
- Global buckling occurs with a load **18–22 N.**
- Local buckling occurs with a load **22–26 N.**
- Tubes loose stability under the load **5–6 times less than *F** .**

Summary

Conclusions

- The members of paper bridges have thin walls, so those bridges are broken due to **local buckling** of their walls.
- In **a truss structure** all elements work on compression or tension, and do not work on bending. As a result, truss paper bridges can withstand the great load.
- The best design of our paper bridges withstands the load **~ 60 N.**

References

- Timoshenko S.P. (1953) *History of the strength of materials*.
- Gordon J.E. (1978) *Structures, or why things don’t fall down*.

Thank you for your attention!