SLOVAKIA
IYPT '13

16

Hoops

Kamila Součková

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An elastic hoop is pressed against a hard surface and then suddenly released. The hoop can jump high in the air.

Investigate how the height of the jump depends on the relevant parameters.

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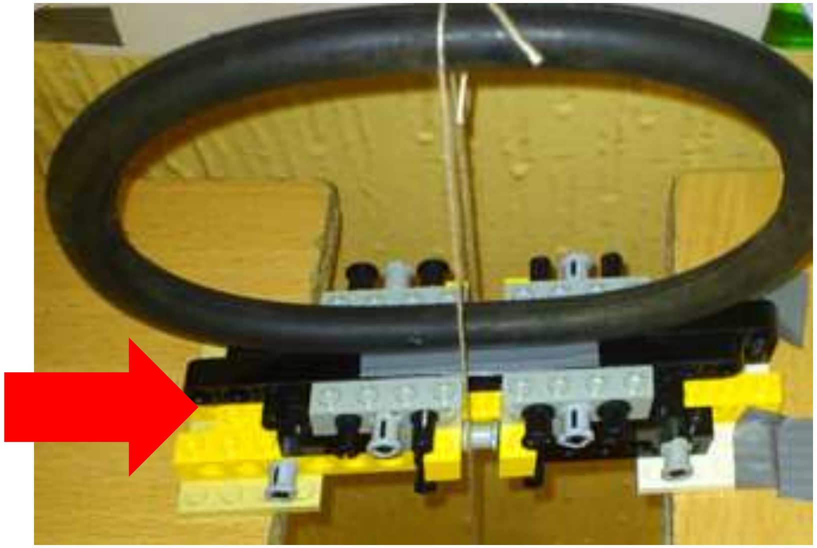

stiff plastic & point force

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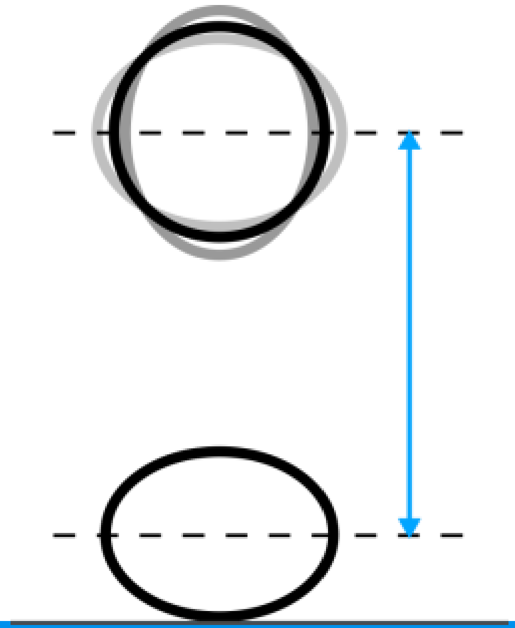

stiff plastic & point force

Δ height of center of mass

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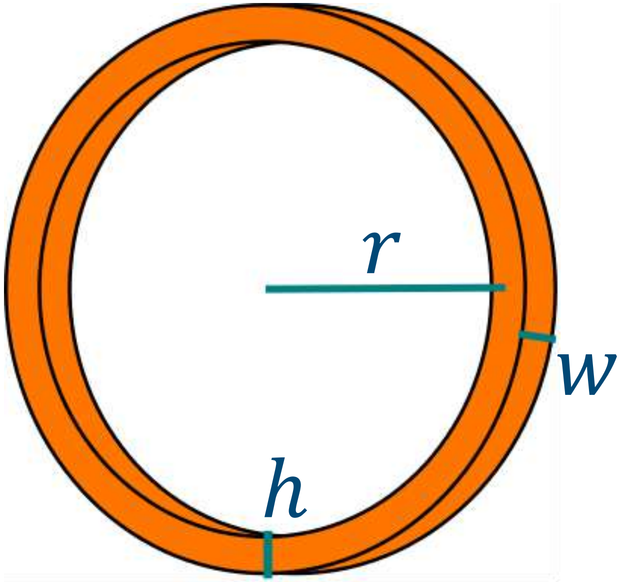
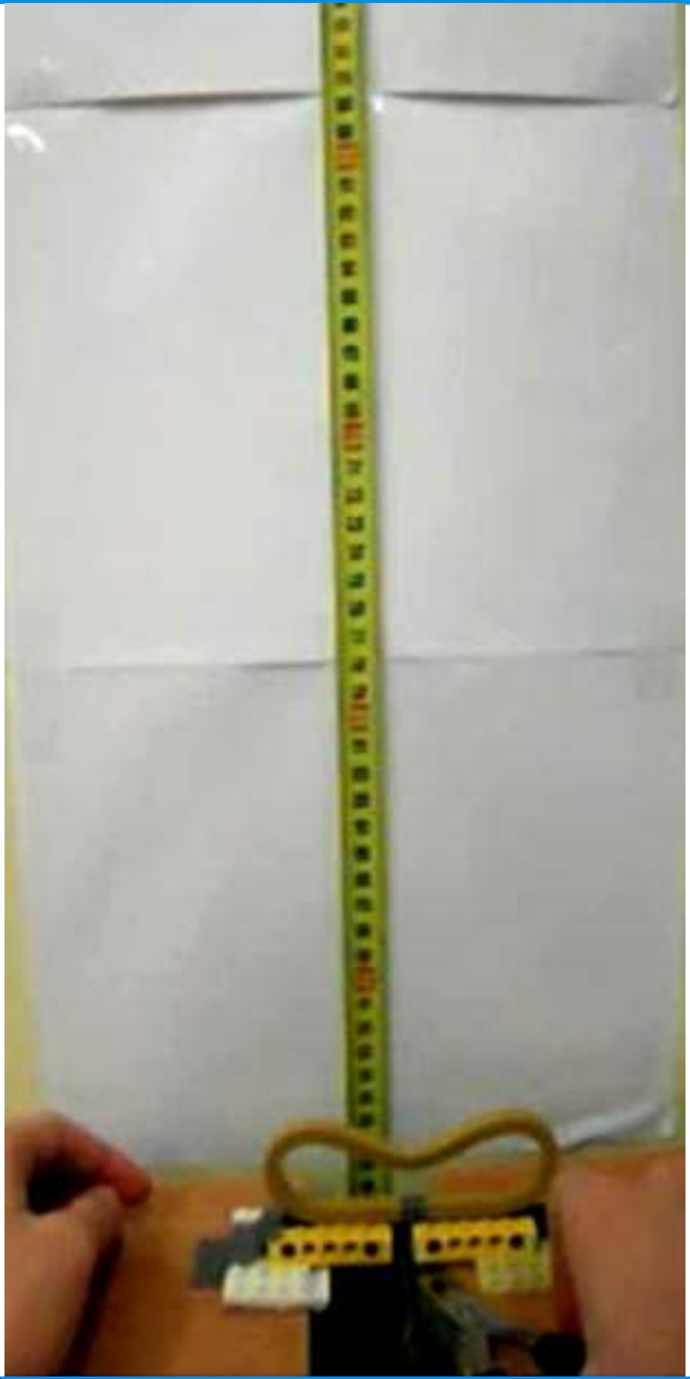


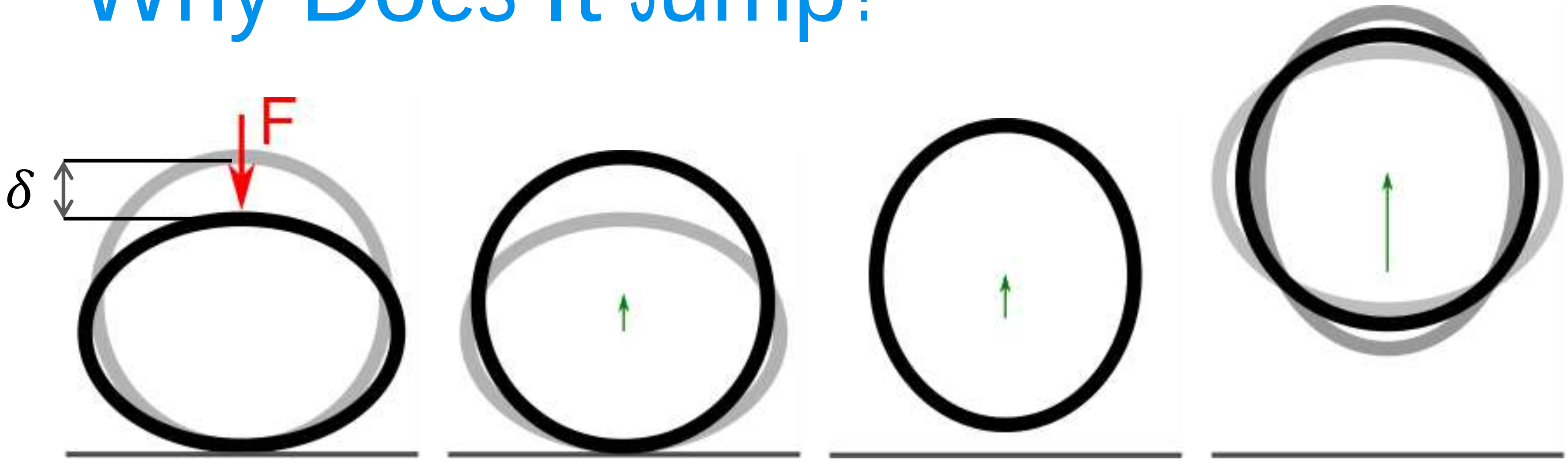
stiff plastic & point force

Δ height of center of mass

material properties, geometry



Why Does It Jump?

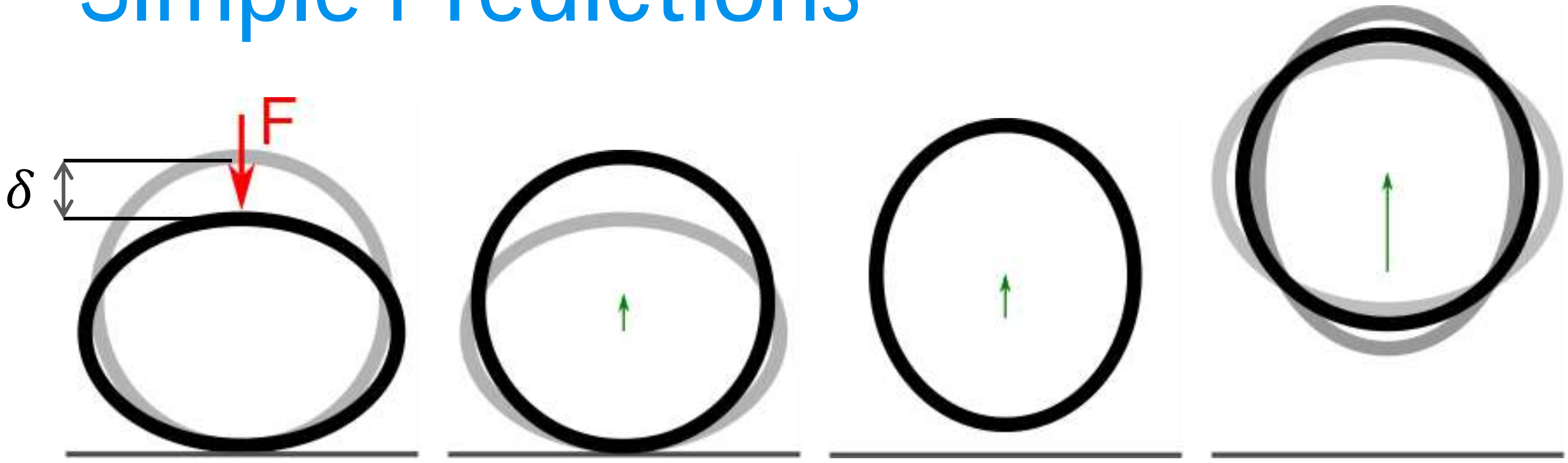


F ⇒ deformation   original shape

$$E_{def} \rightarrow E_{kin} + E_{vib}$$

$$E_{kin} \rightarrow E_{pot}$$

Simple Predictions

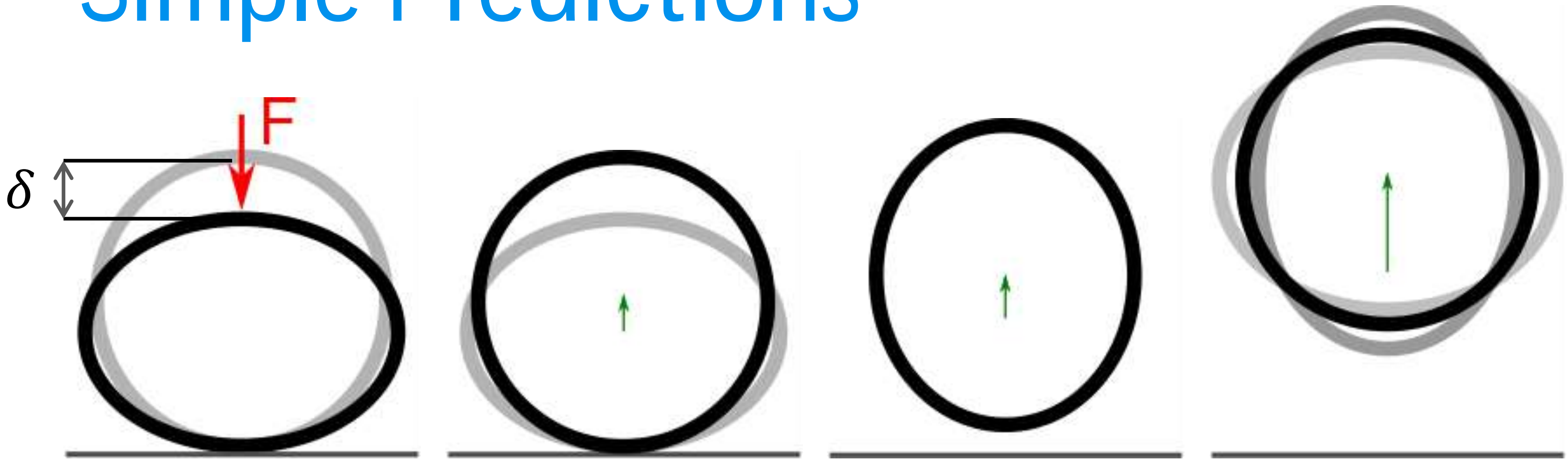


small deformations – $F \propto \delta$

$$h \propto E_{BENDING} = \int F \, \mathrm{d}\delta \propto \delta^2 \propto F^2$$

$$h \propto F^2$$

Simple Predictions

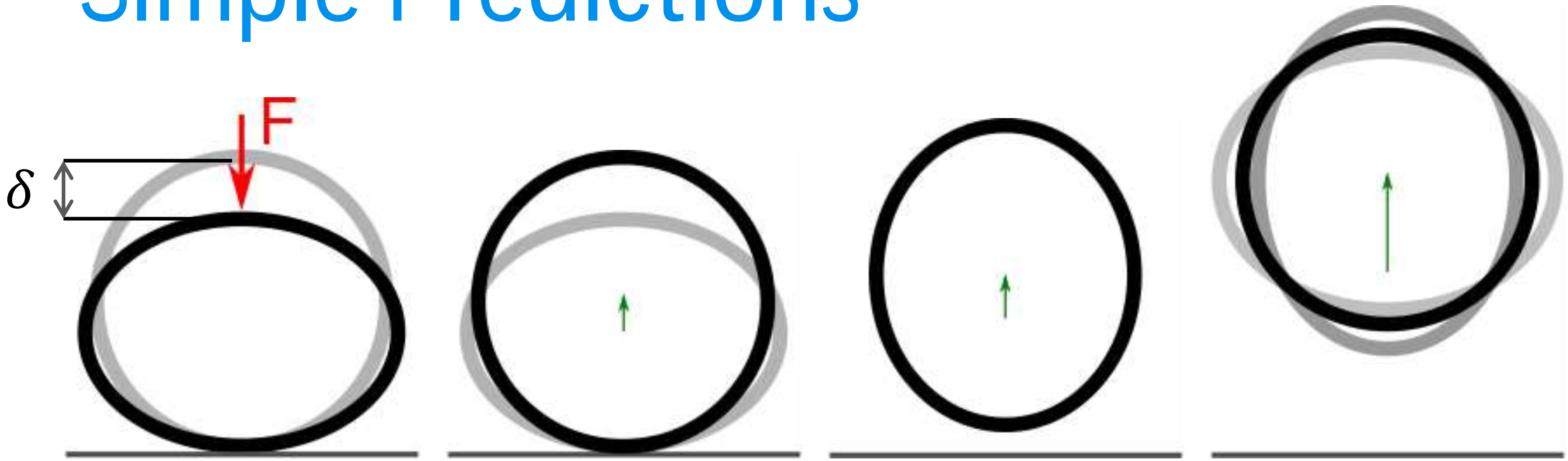


$$E_{BENDING} = \oint_{PERIMETER} \frac{IY}{2R_{CUR}^2} \mathrm{d}l$$

$$R_{CUR} \propto r_{HOOP} \Rightarrow E_{BENDING} \propto \frac{1}{r_{HOOP}}$$

$$h \propto \frac{E_{BENDING}}{m_{HOOP}} \propto \frac{1}{r_{HOOP}^2}$$

Simple Predictions



$$h \propto \frac{1}{r_{HOOP}^2}$$

$$h \propto F^2$$

Existing Work of Yang and Kim

Eunjin Yang and Ho-Young Kim

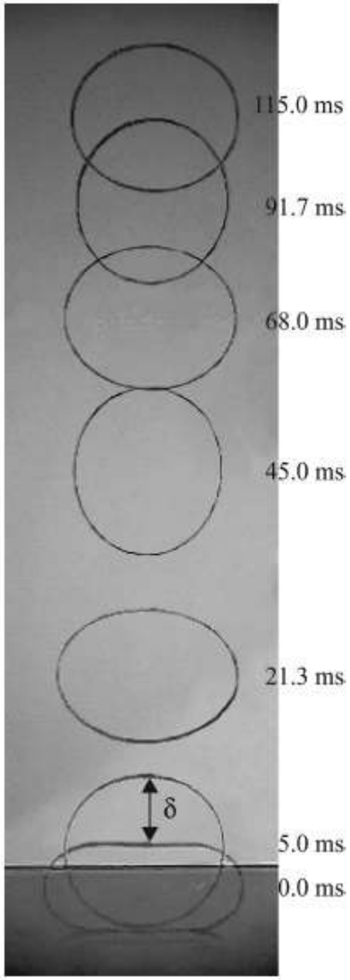
American Journal of Physics, Vol. 80, Issue 1 (January), p. 19 (2012)

Jumping hoops

Eunjin Yang and Ho-Young Kim[a)]
School of Mechanical and Aerospace Engineering, Seoul National University, Seoul 151-744, Korea

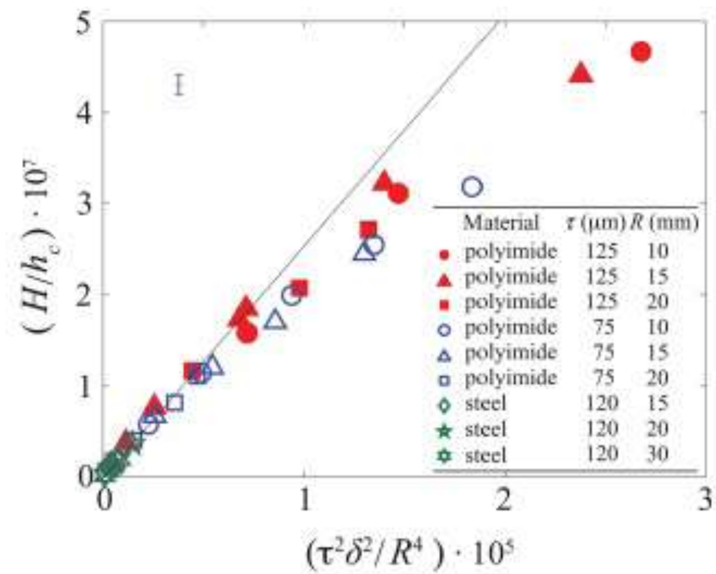
(Received 24 April 2011; accepted 17 August 2011)

We investigate the dynamics of an elastic hoop as a model of the jumps of small insects. During a jump the initial elastic strain energy is converted to translational, gravitational, and vibrational energy, and is dissipated by interaction with the floor and the ambient air. We show that the strain energy is initially divided into translational, vibrational, and dissipation energies with a ratio that is constant regardless of the dimension, initial deflection, and the properties of a hoop. This novel result enables us to accurately predict the maximum jump height of a hoop with known initial conditions and drag coefficient without resorting to a numerical computation. Our model reduces the optimization of the hoop geometry for maximizing the jump height to a simple algebraic problem.
[DOI: 10.1119/1.3633700]

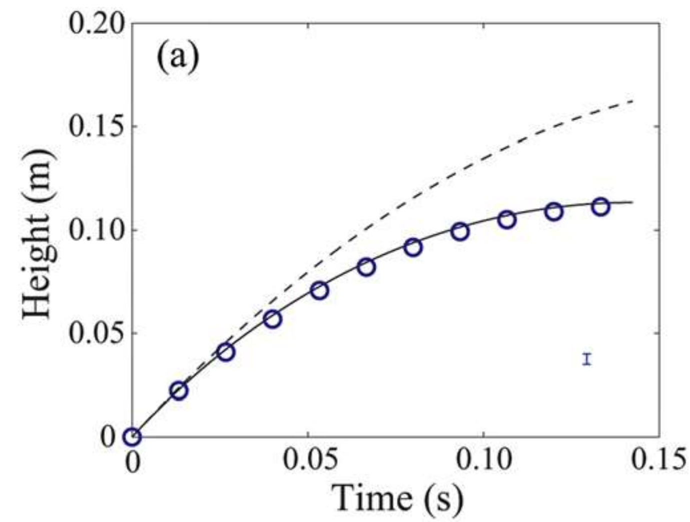


Summary of [Yang and Kim]

- theoretical prediction of the jump height
  - constant ratio of translational/vibrational energy



- discrepancies explained by air drag
  - no other energy losses considered

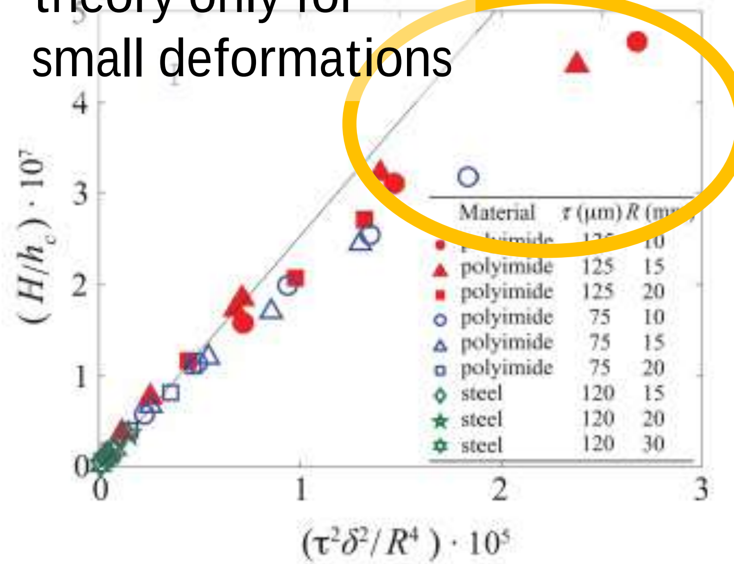


Place for Improvement

- theoretical prediction of the jump height

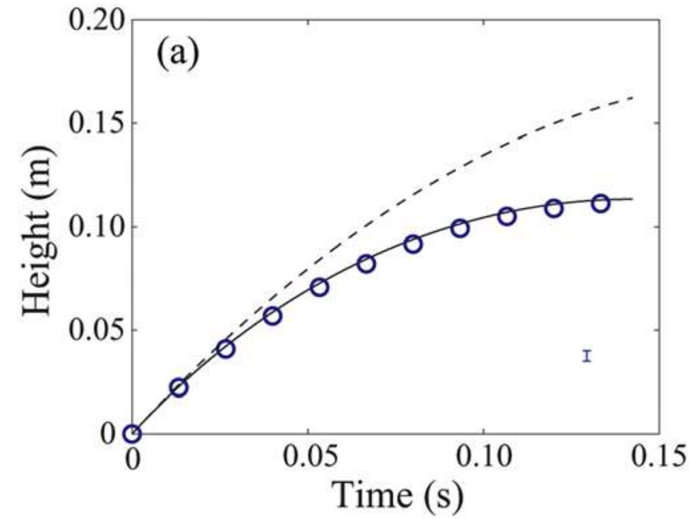
  + large deformations

theory only for small deformations



- discrepancies explained by air drag

  + damping in hoop

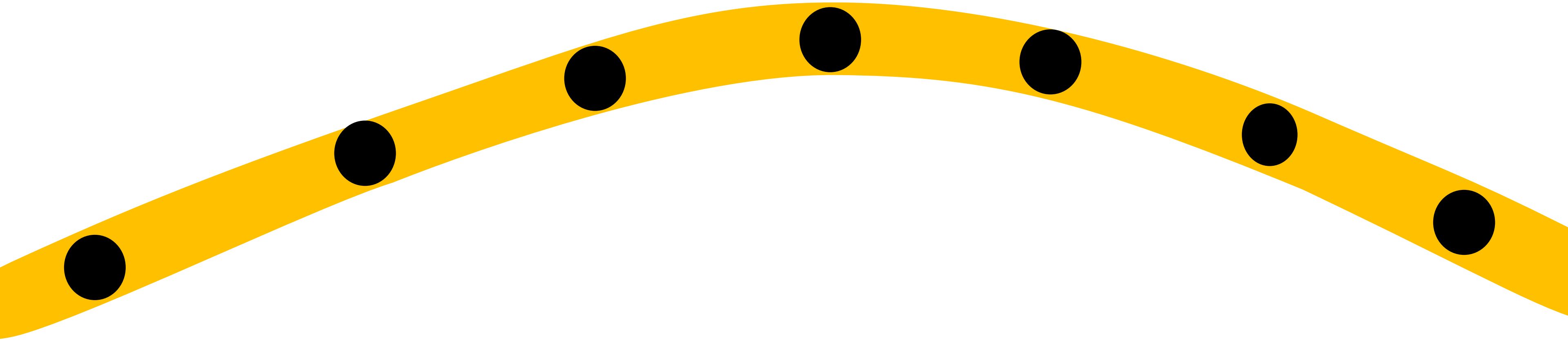


Menu of Our Presentation

1. switch to a numerical approach
2. predict jump height, including damping forces
3. determine importance of air drag/material losses

Numerical approach

- discretization → small point masses, Euler method



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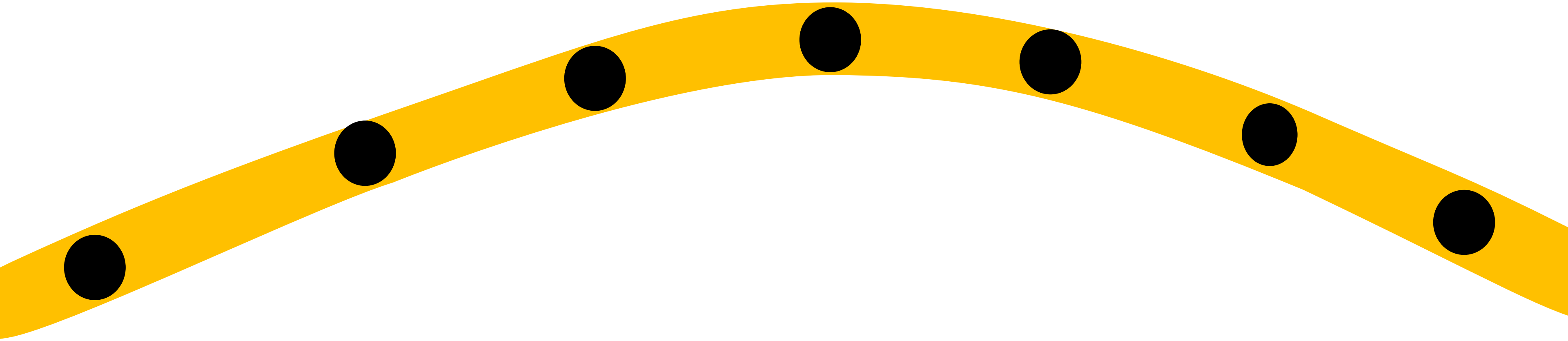
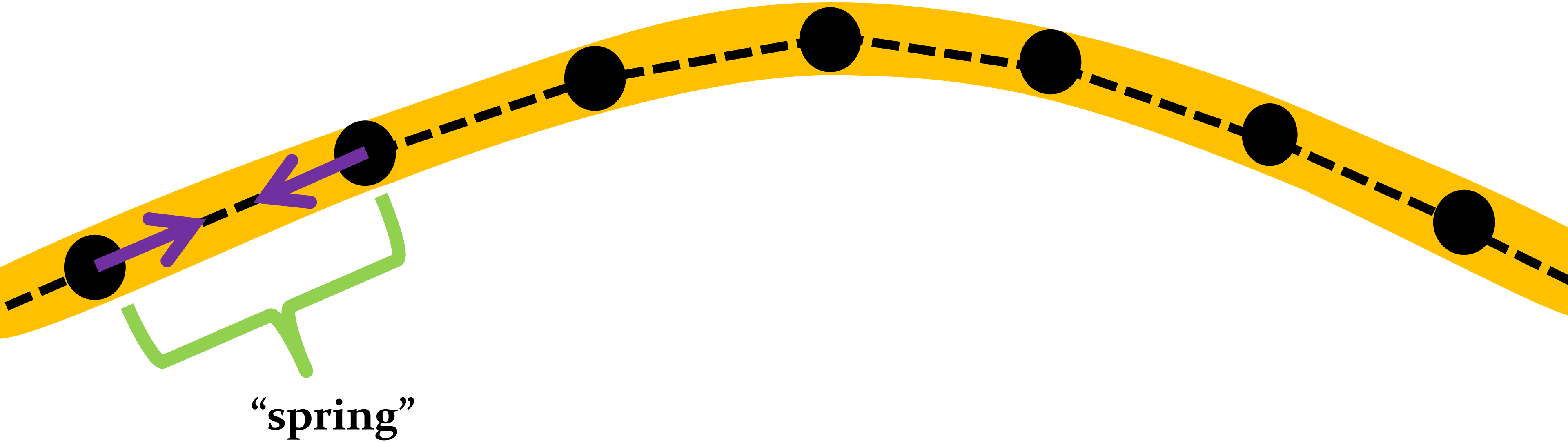


Elasticity

- discretization → small point masses, Euler method
  - elasticity: linear model – $\sigma = Y\epsilon$



Elasticity

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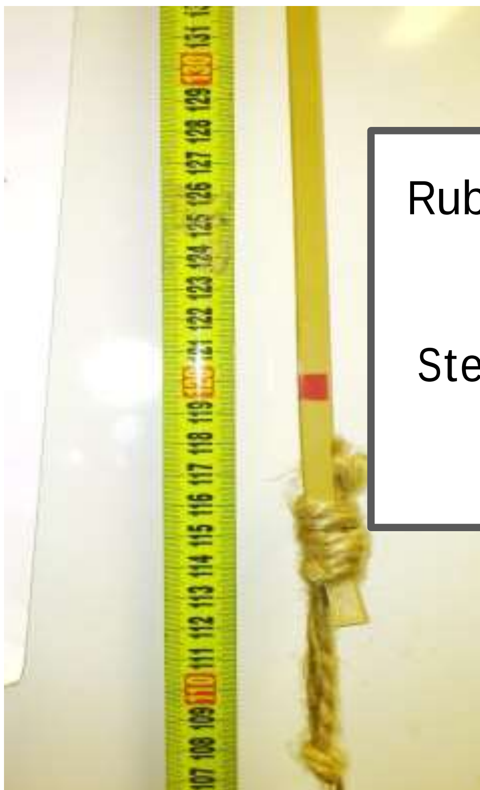
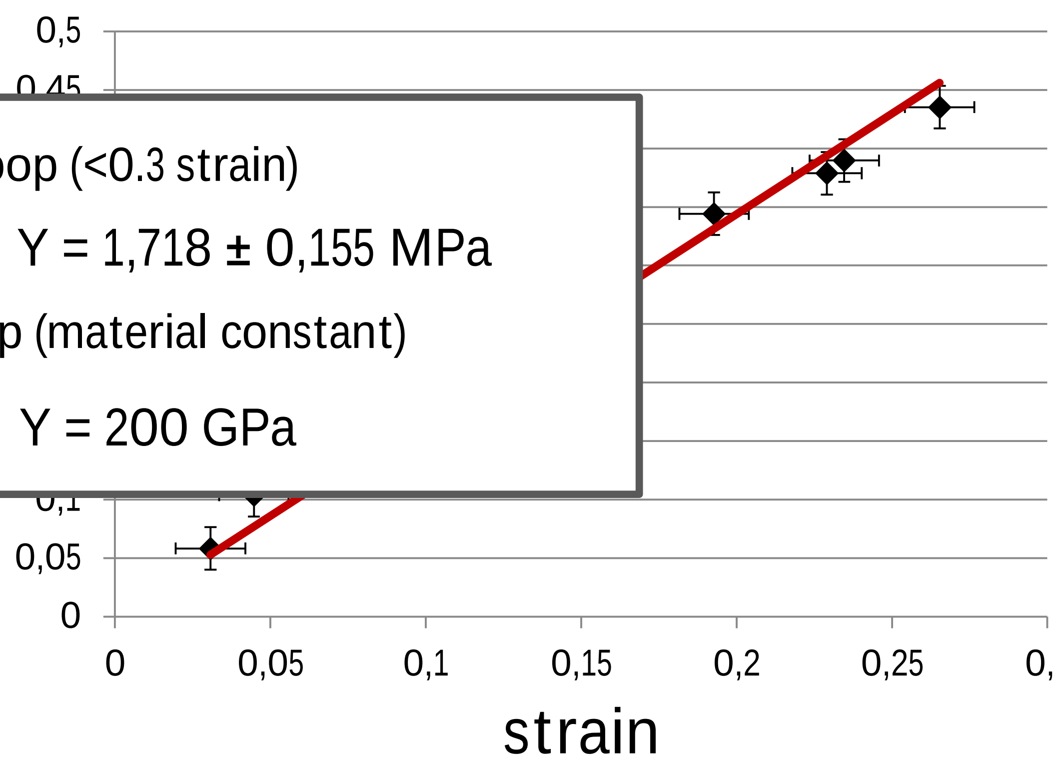




Rubber hoop (<0.3 strain)

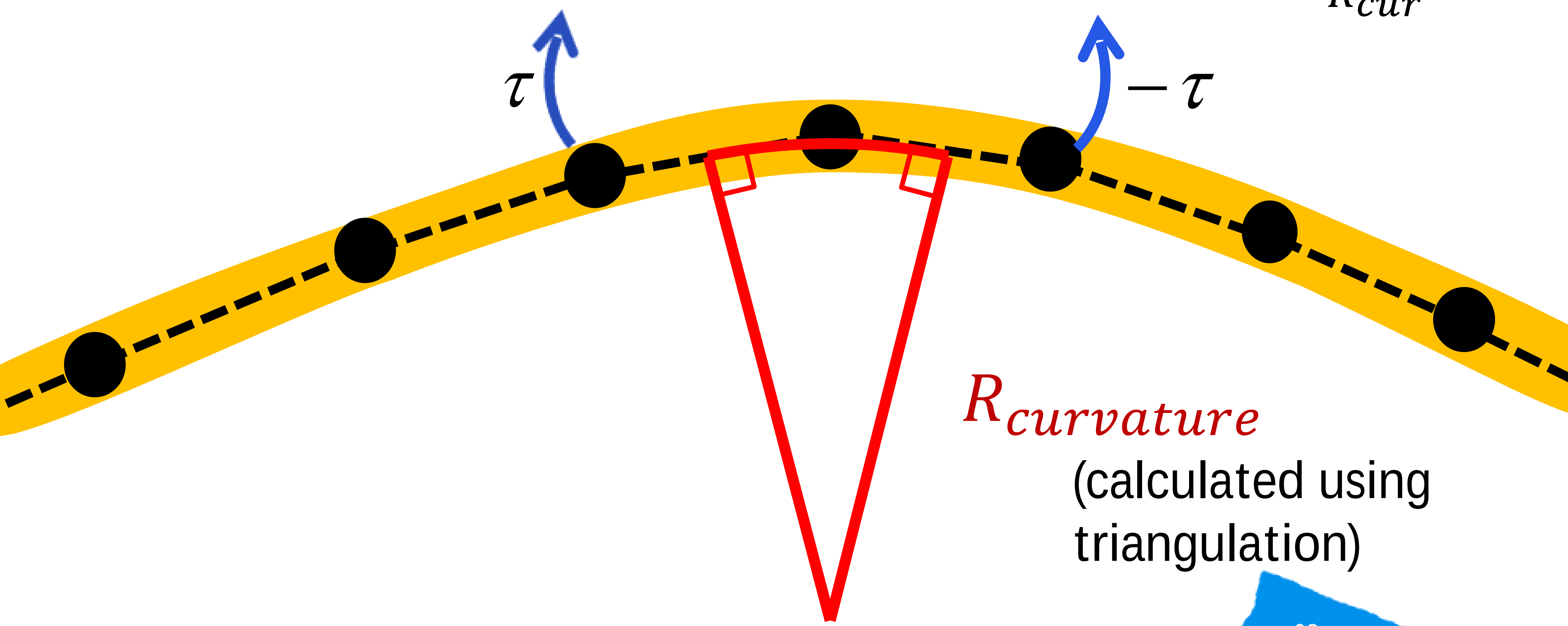
Y = 1,718 ± 0,155 MPa

Steel hoop (material constant)

Y = 200 GPa

Bending stiffness

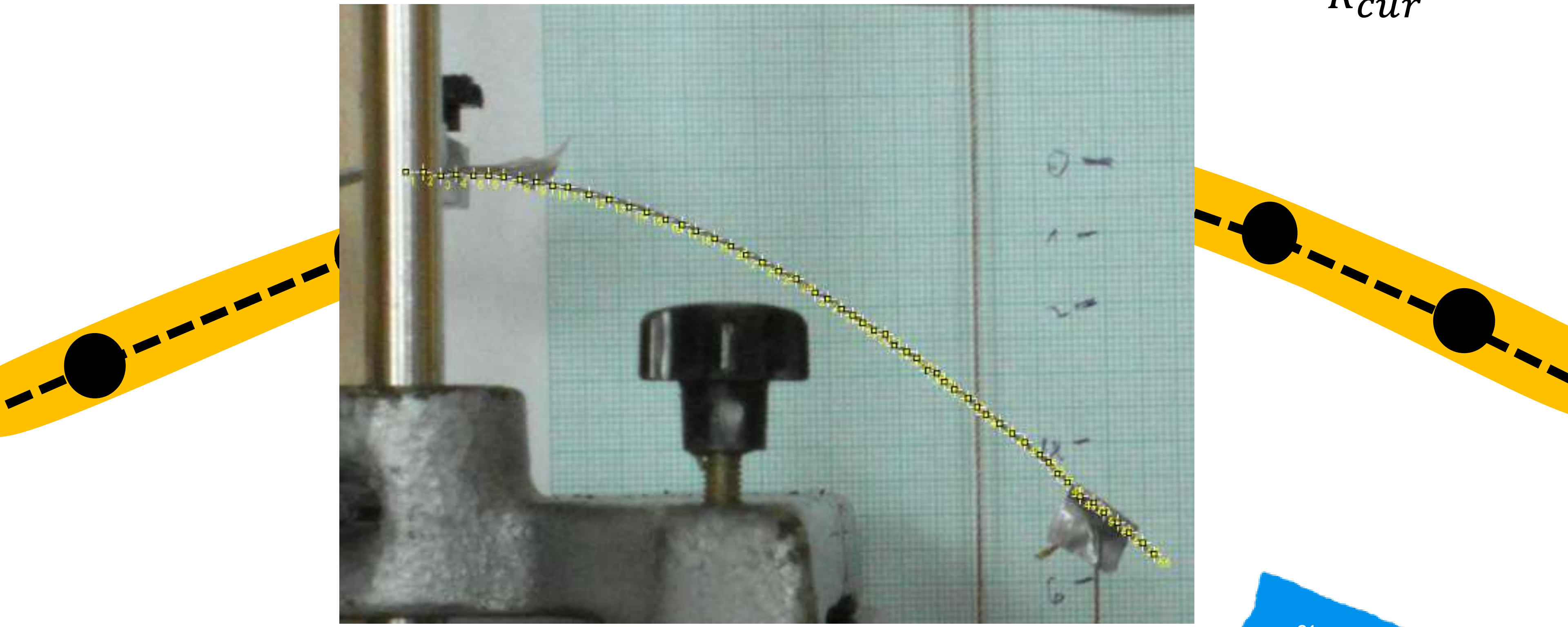
- discretization → small point masses, Euler method
  - elasticity: linear model – $\sigma = Y\epsilon$ ✓
  - bending stiffness: Euler beam theory – $\tau = \frac{IY}{R_{cur}}$



(calculated using triangulation)

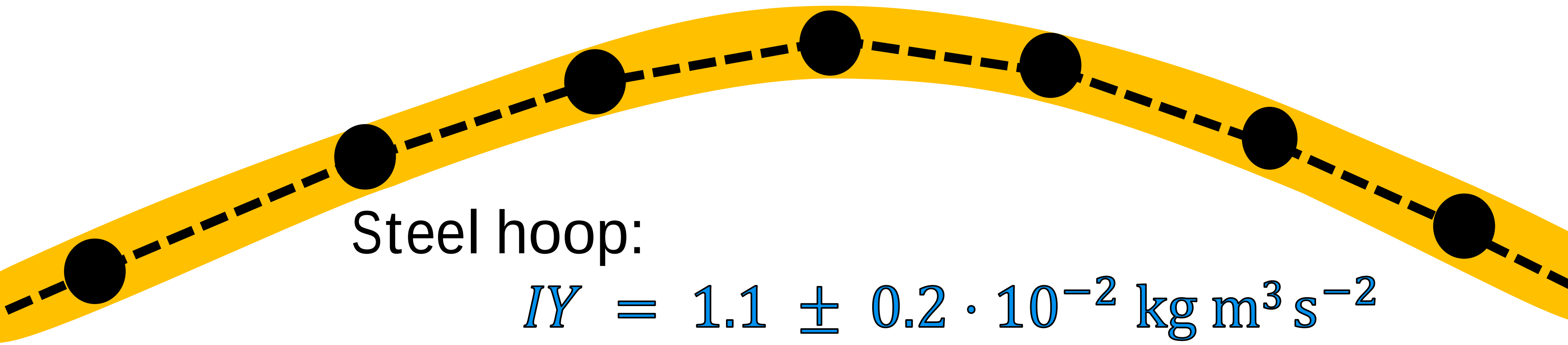
Bending stiffness

- discretization → small point masses, Euler method
  - elasticity: linear model – $\sigma = Y\epsilon$ ✓
  - bending stiffness: Euler beam theory – $\tau = \dfrac{IY}{R_{cur}}$



Bending stiffness: measured values

- discretization → small point masses, Euler method
  - elasticity: linear model – $\sigma = Y\epsilon$ ✓
  - bending stiffness: Euler beam theory – $\tau = \frac{IY}{R_{cur}}$ ✓



Steel hoop:

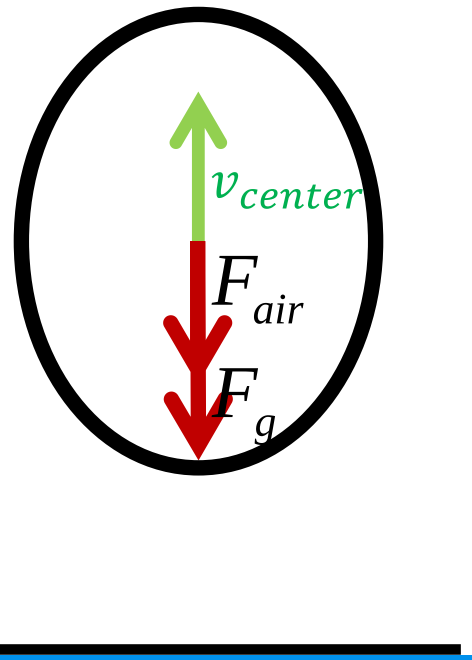
$$IY = 1.1 \pm 0.2 \cdot 10^{-2}\ \mathrm{kg\,m^3\,s^{-2}}$$

Rubber hoop (changing length of the strip):

$$IY = 1.2 \pm 0.1 \cdot 10^{-4}\ \mathrm{kg\,m^3\,s^{-2}}$$

Air drag

- discretization → small point masses, Euler method
  - elasticity: linear model – $\sigma = Y\epsilon$ ✓
  - bending stiffness: Euler beam theory – $\tau = \frac{IY}{R_{cur}}$ ✓
  - air drag: using velocity of the center of mass ✓



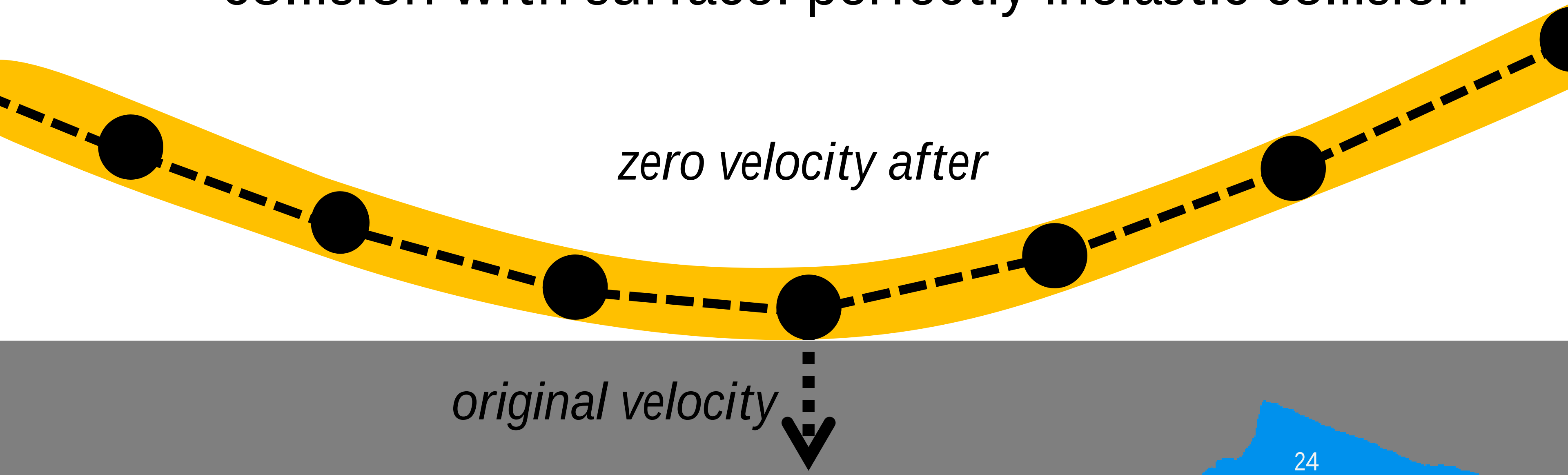
$$ma = -mg - \frac{1}{2} CS\rho v_{center}^2$$

$$C = 2.32$$

taken from [Yang and Kim] (thickness/radius<0.04, Reynolds number 150-1500)

Collision with hard surface

- discretization → small point masses, Euler method
  - elasticity: linear model – $\sigma = Y\epsilon$ ✓
  - bending stiffness: Euler beam theory – $\tau = \frac{IY}{R_{cur}}$ ✓
  - air drag: using velocity of the center of mass ✓
  - collision with surface: perfectly inelastic collision ✓



Simulation: Governing Equations

- elasticity: linear model

$$\vec{F_i} = \hat{d}_1 k\left(\left|\vec{d_1}\right| - l_{eq}\right) + \hat{d}_2 k\left(\left|\vec{d_2}\right| - l_{eq}\right)$$

$$\vec{d_1} = \overrightarrow{r_{i+1}} - \vec{r_i}$$
$$\vec{d_2} = \overrightarrow{r_{i-1}} - \vec{r_i}$$

$$k = \frac{SY}{l_{eq}}$$

- bending stiffness: Euler beam theory

$$\vec{c} = \vec{d_1} \times \vec{d_2}$$

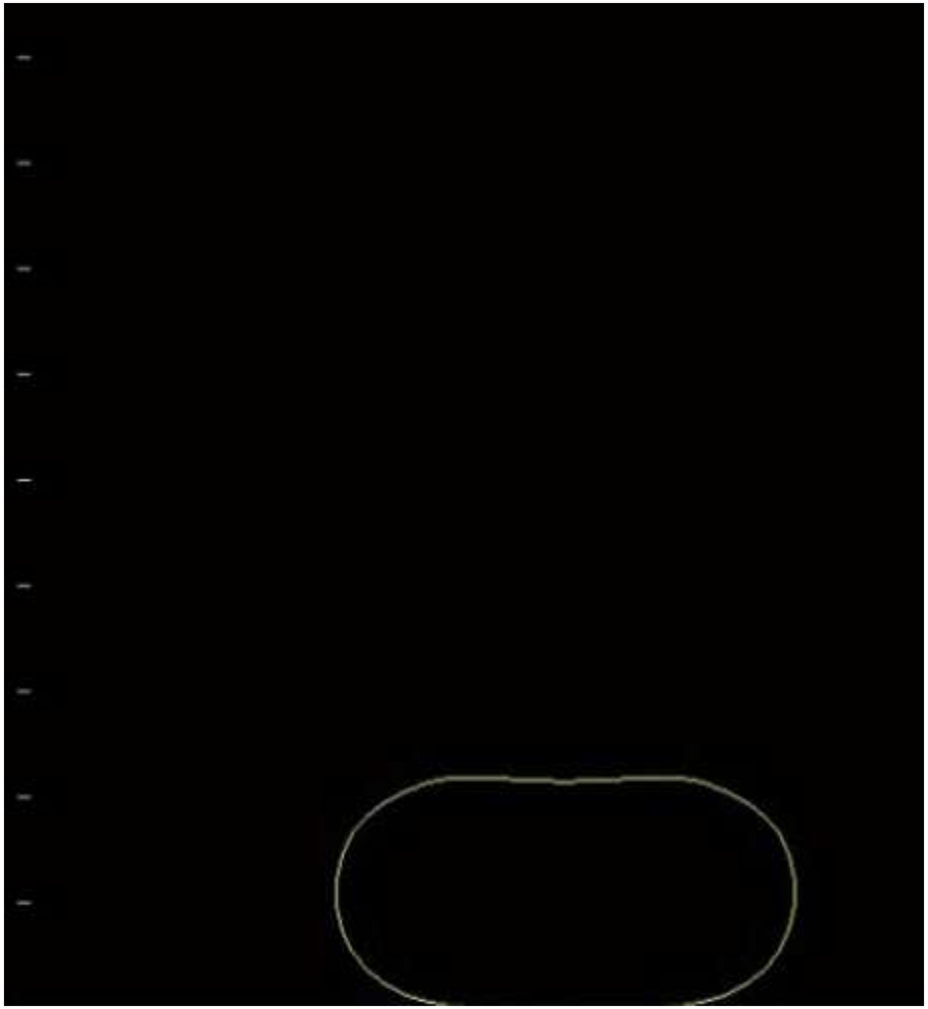
$$cur = 2\tan\left(\frac{1}{2}\arcsin\left(\frac{|\vec{c}|}{l_i^2}\right)\right)\frac{1}{l_i}$$

$$F_i = -IY\frac{cur}{l_i}\widehat{\left(\vec{d_1} + \vec{d_2}\right)}$$

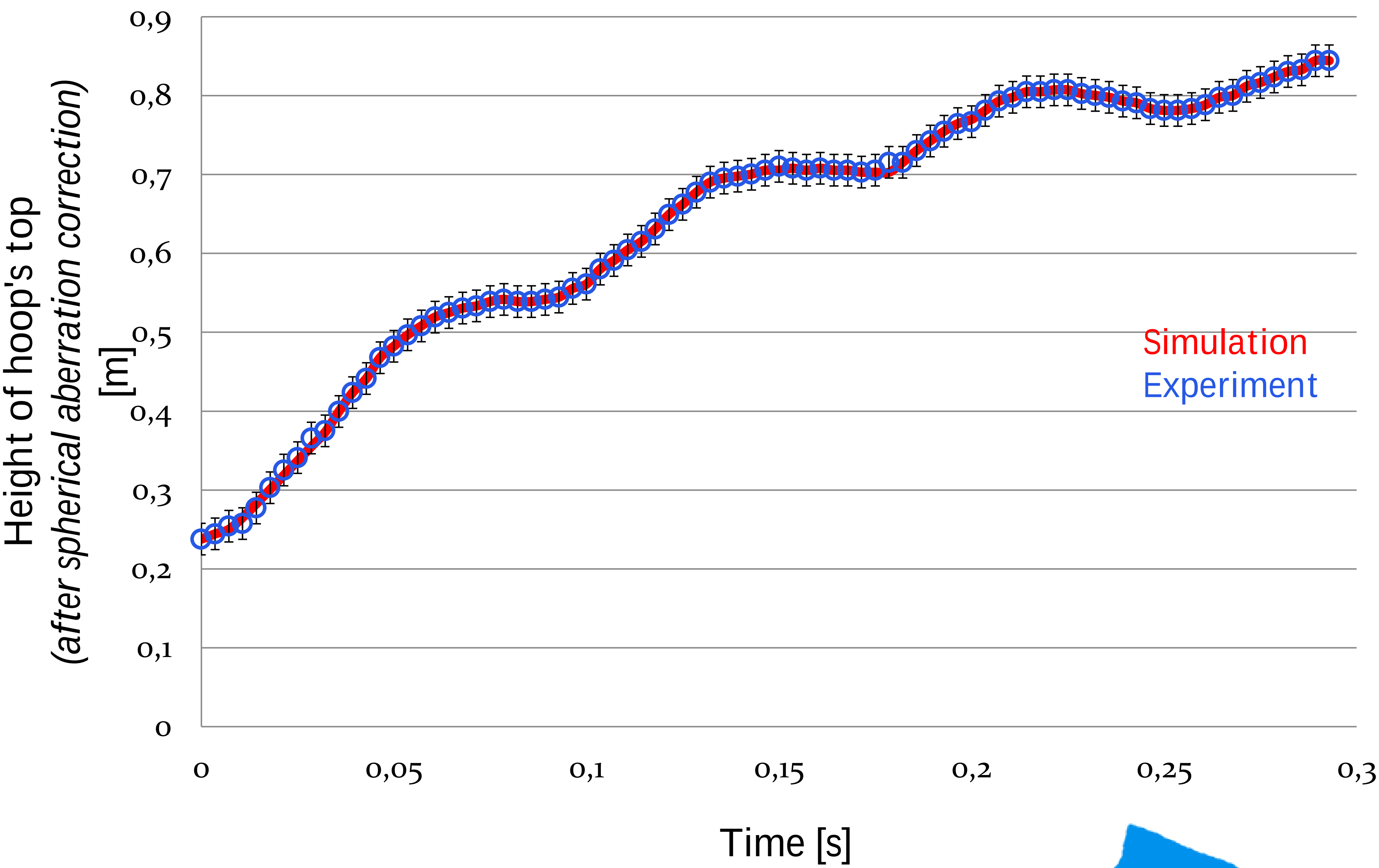
$$F_{i+1} = IY\frac{cur}{l_i}\widehat{\left(\vec{d_1}\right)}$$

$$F_{i-1} = IY\frac{cur}{l_i}\widehat{\left(\vec{d_2}\right)}$$

Comparison (Steel hoop)

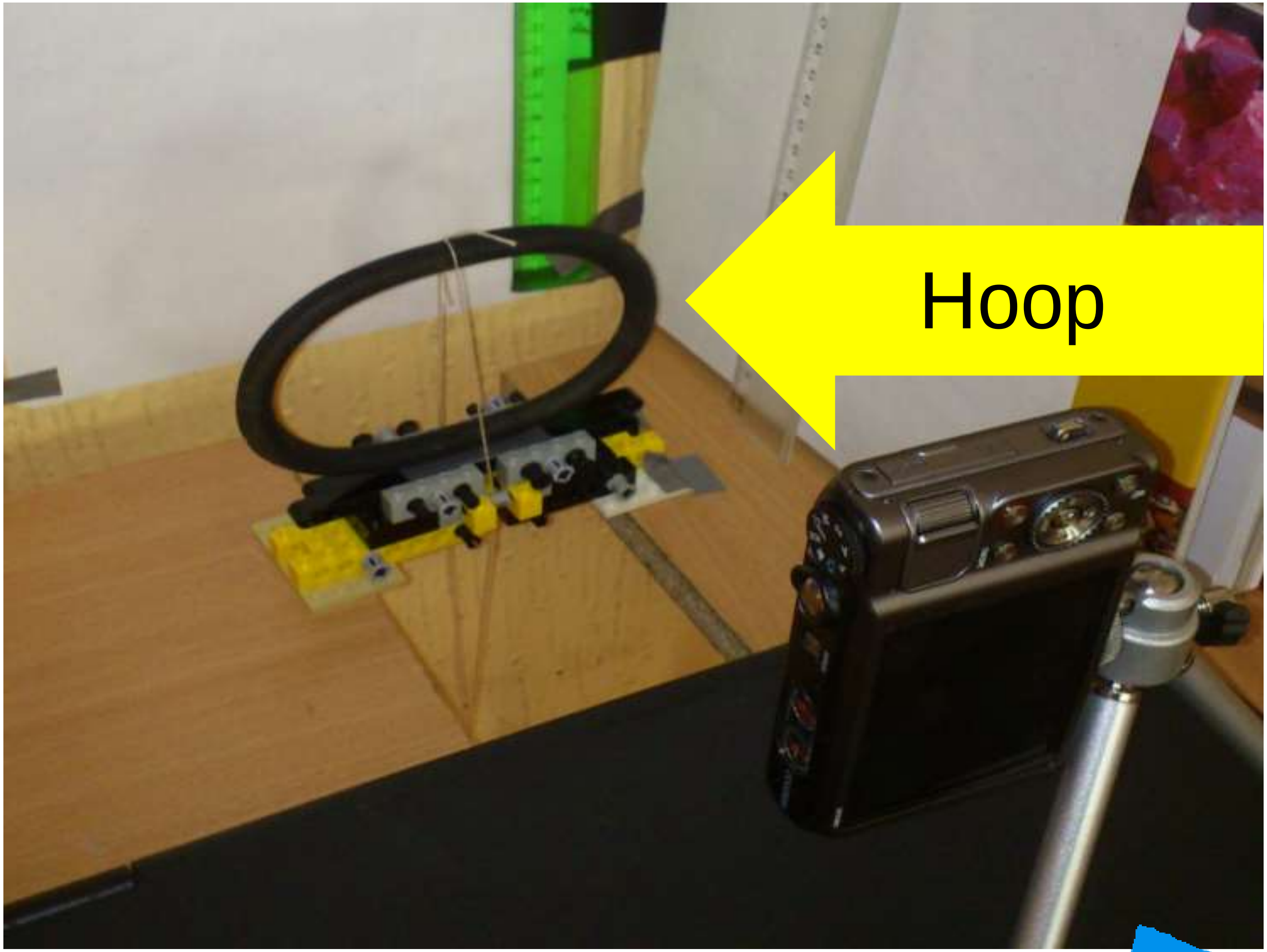


Height of hoop's top in time

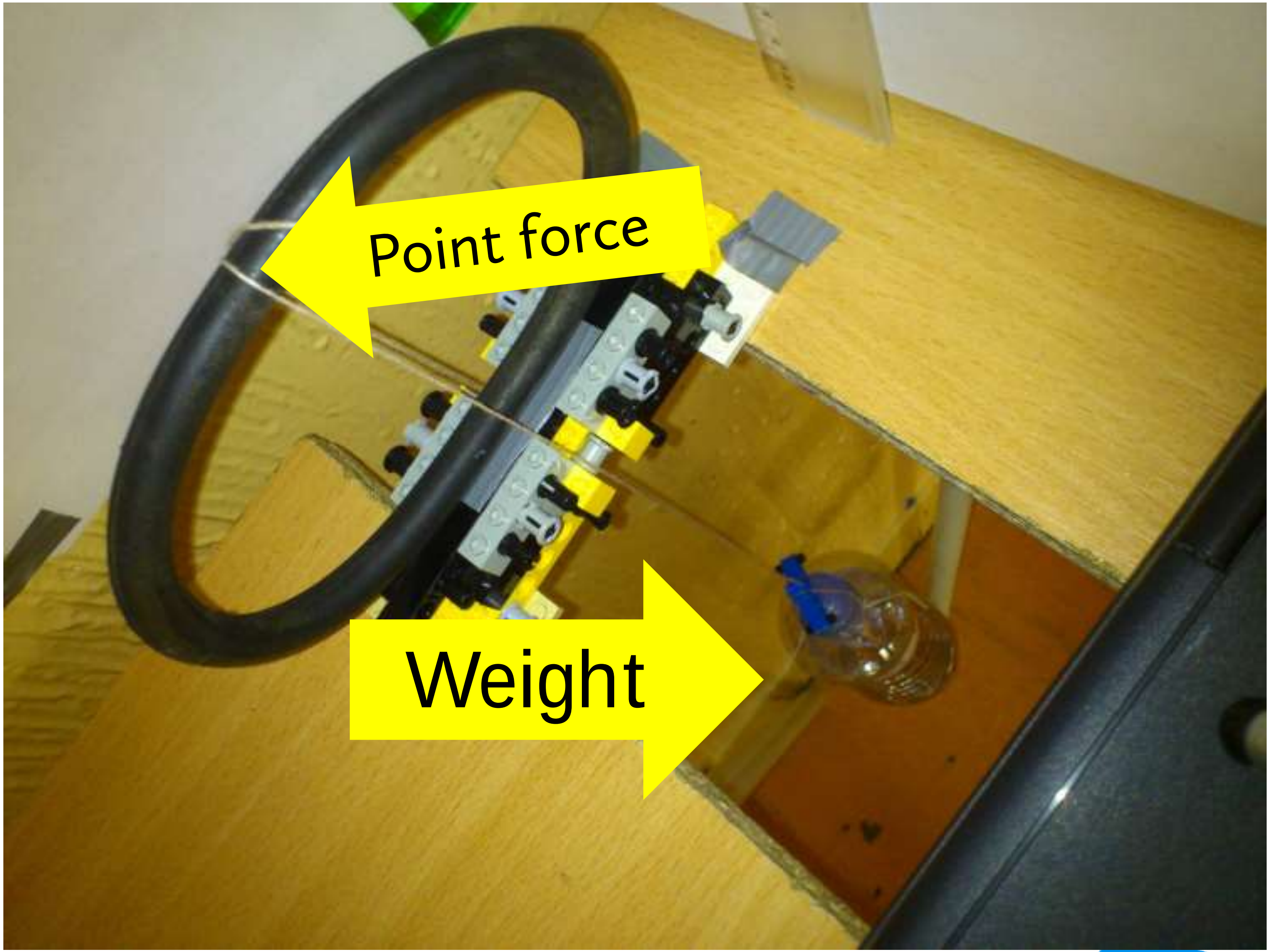


PARAMETER DEPENDENCES

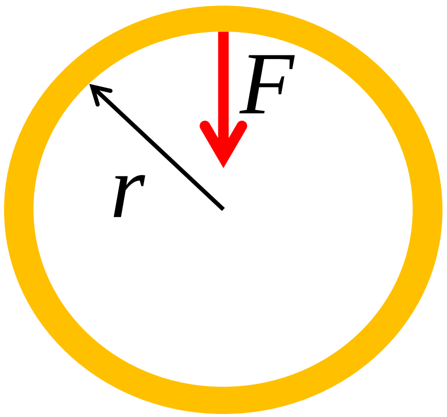
Apparatus



Apparatus

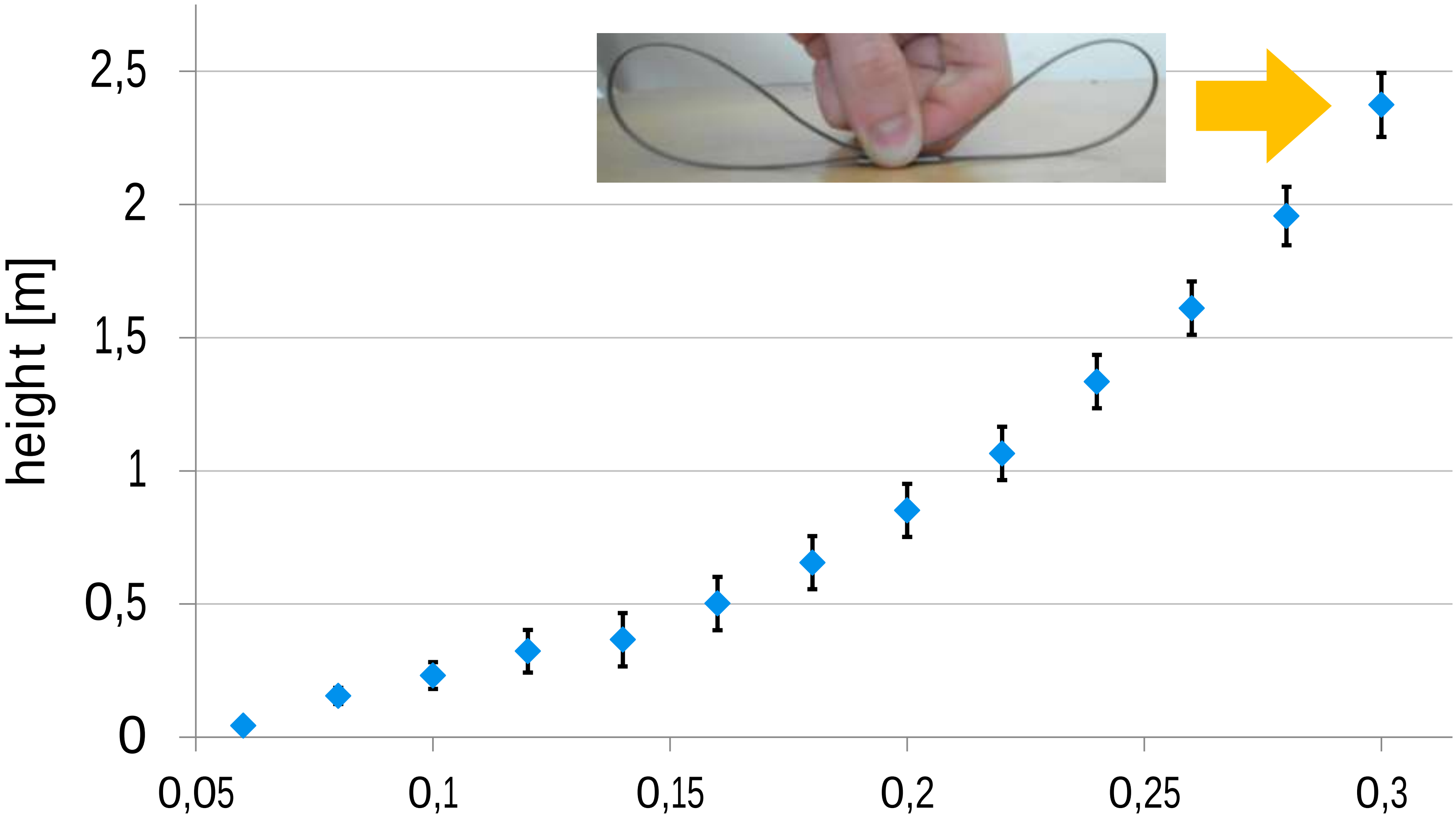


Experiments



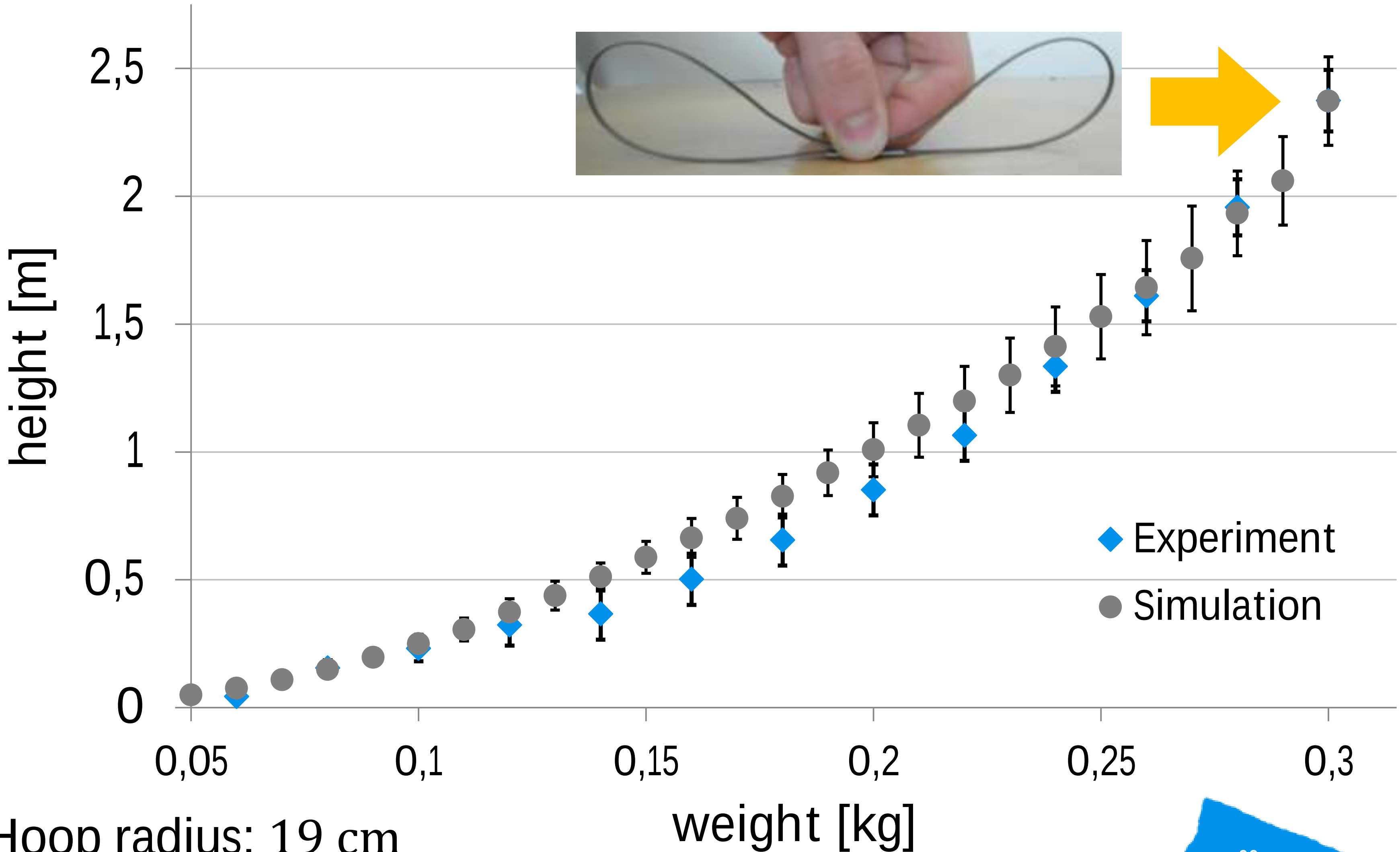
- material
  - changes: elasticity, bending stiffness
  - *rubber, steel*
- geometry, deformation
  - height of jump vs. weight attached (*rubber, steel*)
  - height of jumps vs. radius of the hoop (*steel*)

Steel hoop: jump height vs. mass applied



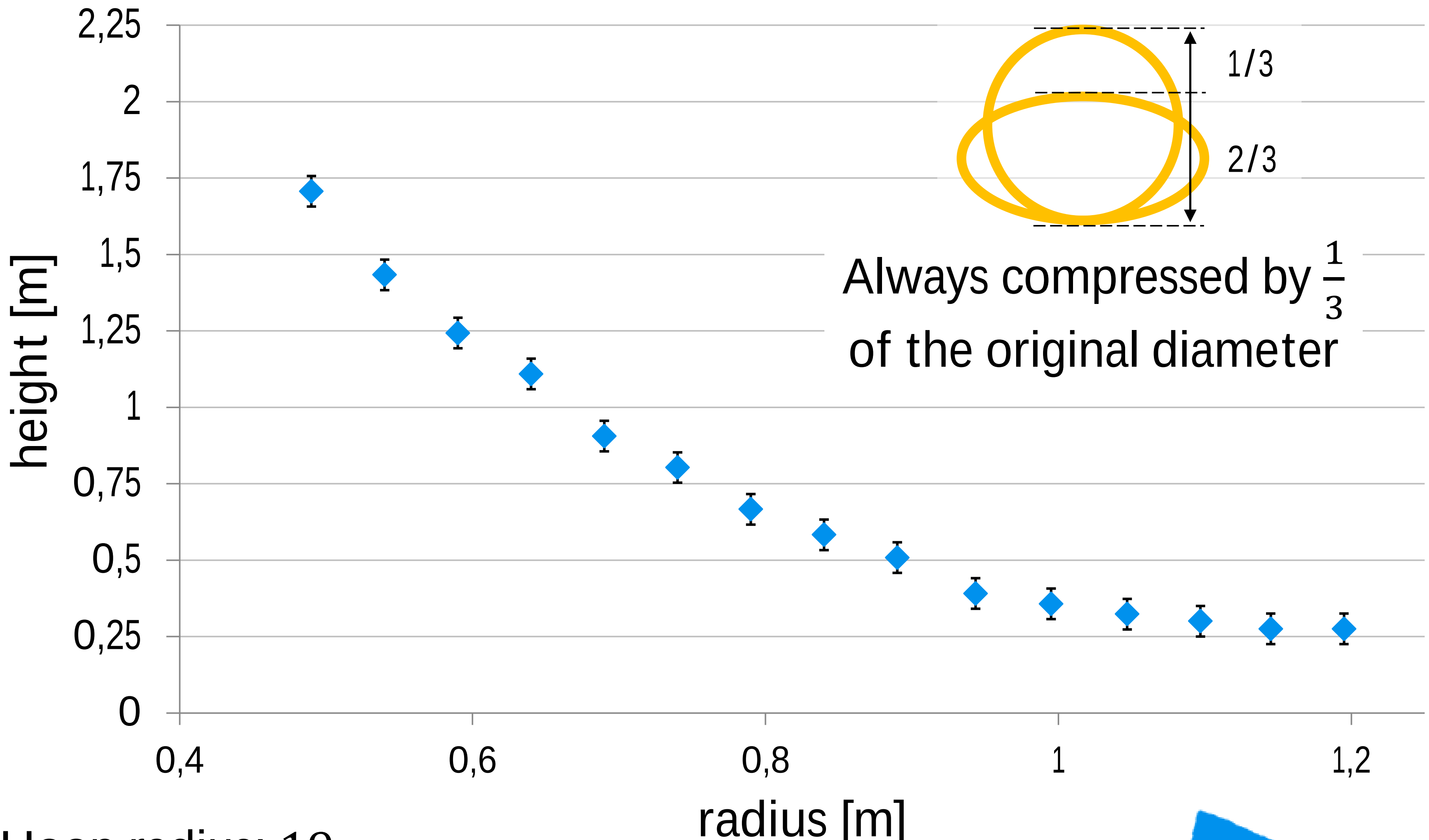
Hoop radius: 19 cm

Steel hoop: jump height vs. mass applied



Hoop radius: 19 cm

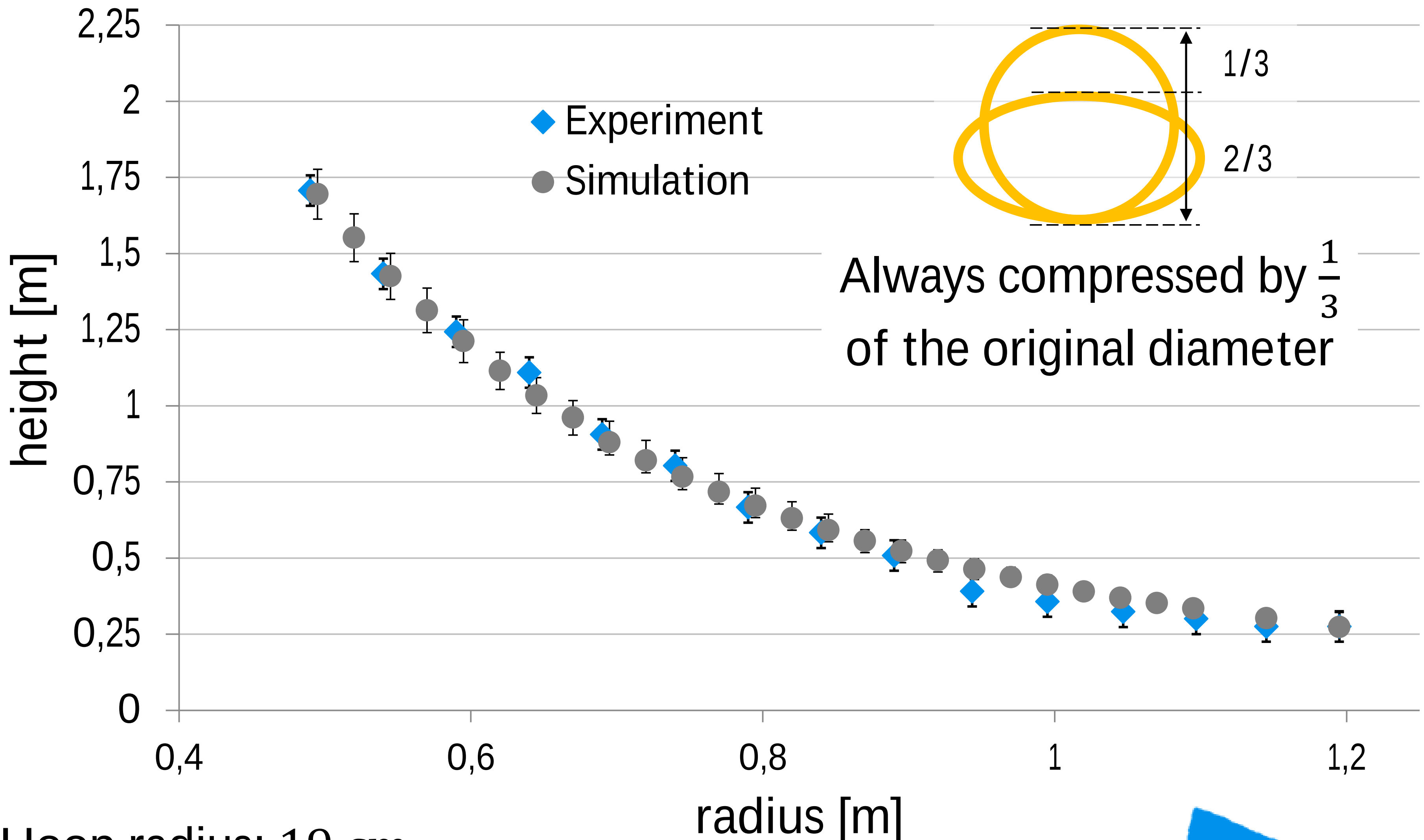
Steel hoop: jump height vs. hoop radius



Always compressed by $\frac{1}{3}$ of the original diameter

Hoop radius: 19 cm

Steel hoop: jump height vs. hoop radius

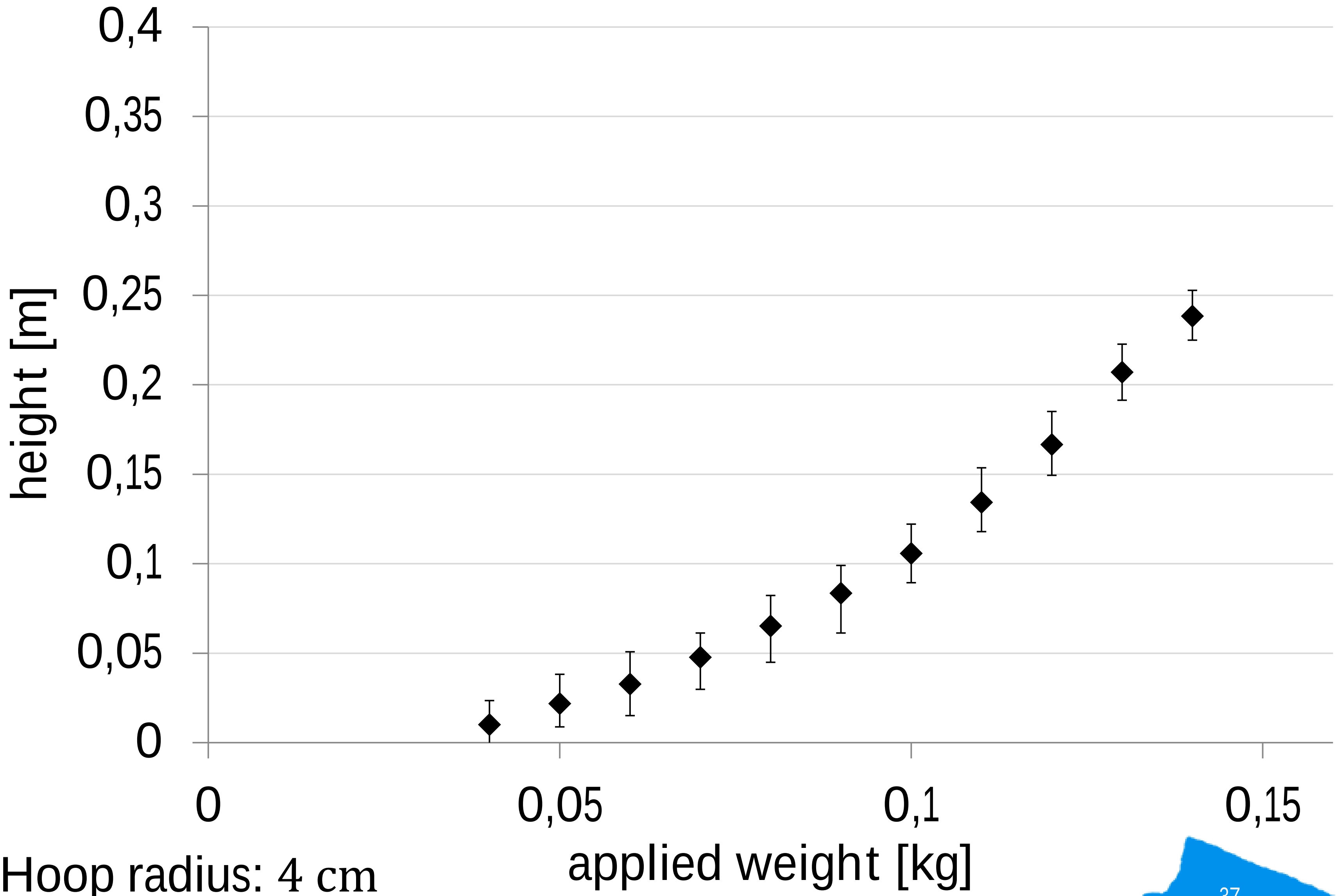


Always compressed by $\frac{1}{3}$ of the original diameter

Hoop radius: 19 cm

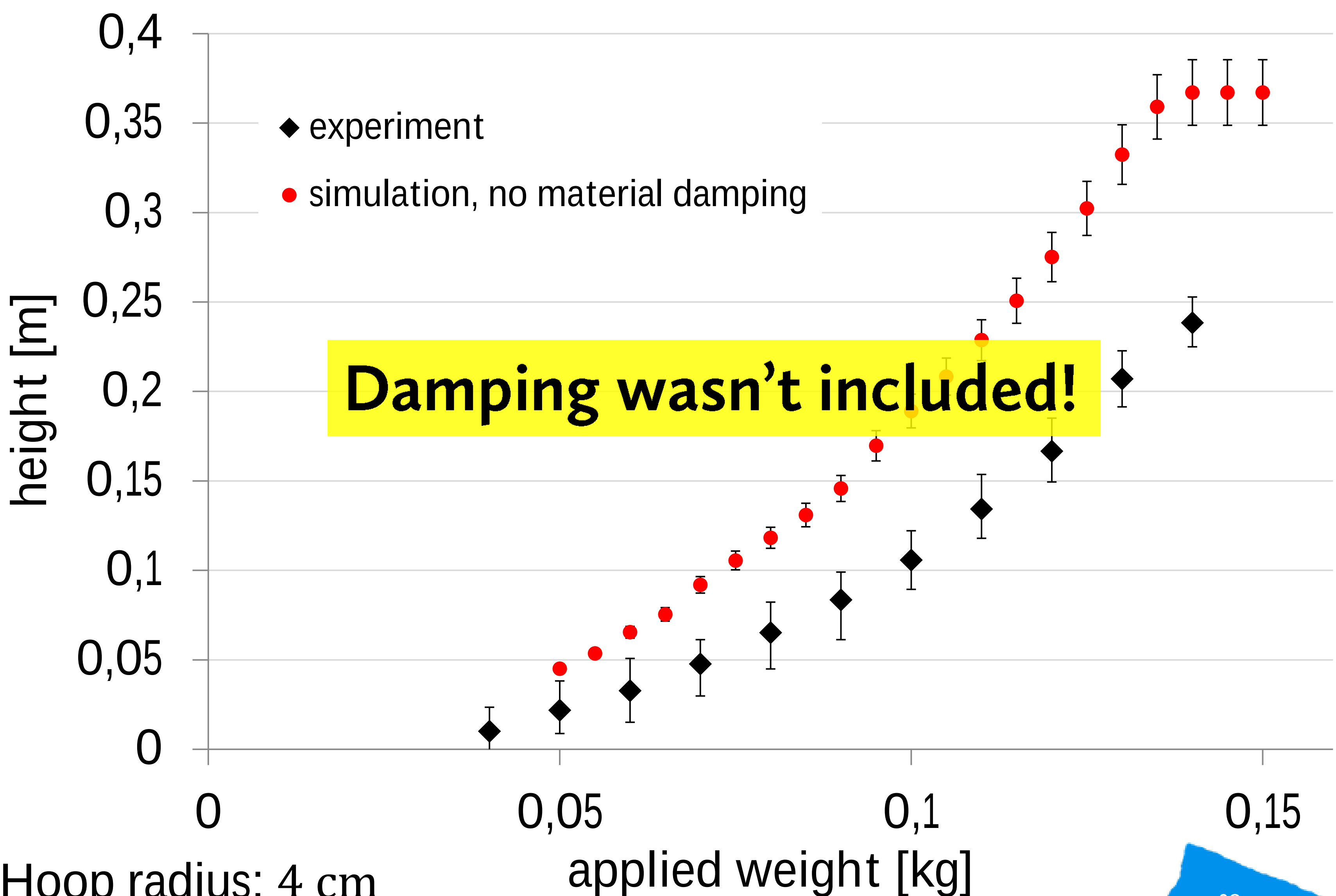
Rubber Hoop

Rubber Hoop: Jump Height vs. Mass Applied



Hoop radius: 4 cm

Rubber Hoop: Jump Height vs. Mass Applied

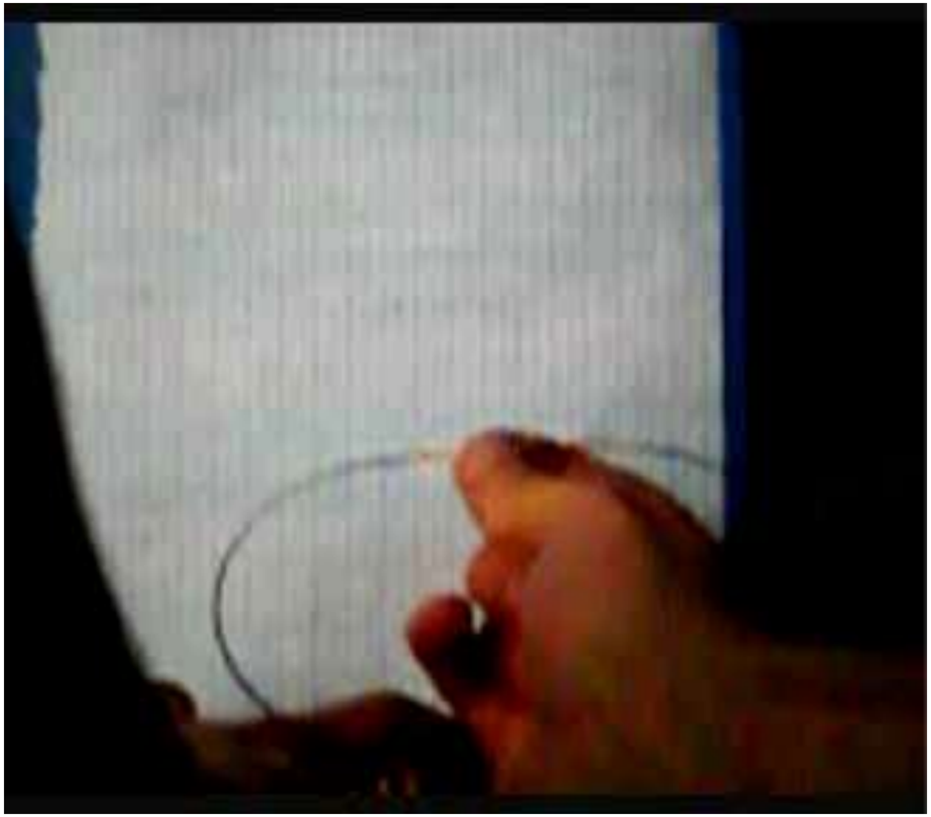


Hoop radius: 4 cm

Air Drag vs. Internal Damping

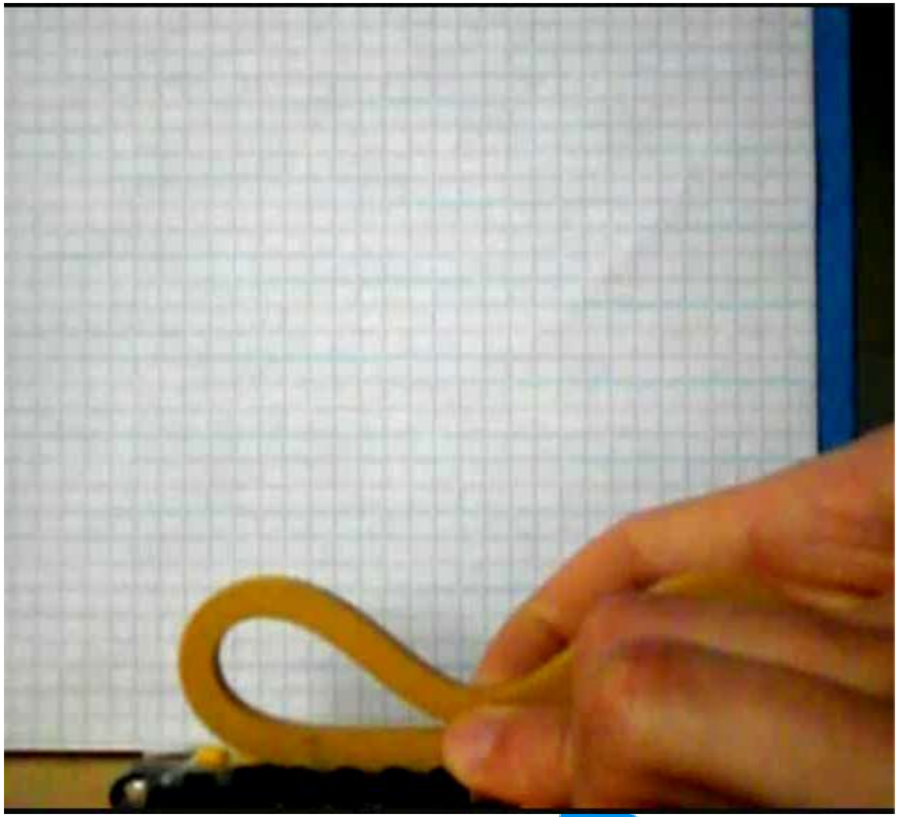
metal hoops

- *air drag* prevails



rubber hoops

- *material dissipation* of energy is significant



Damping

In mechanics of continuum: → Discretization

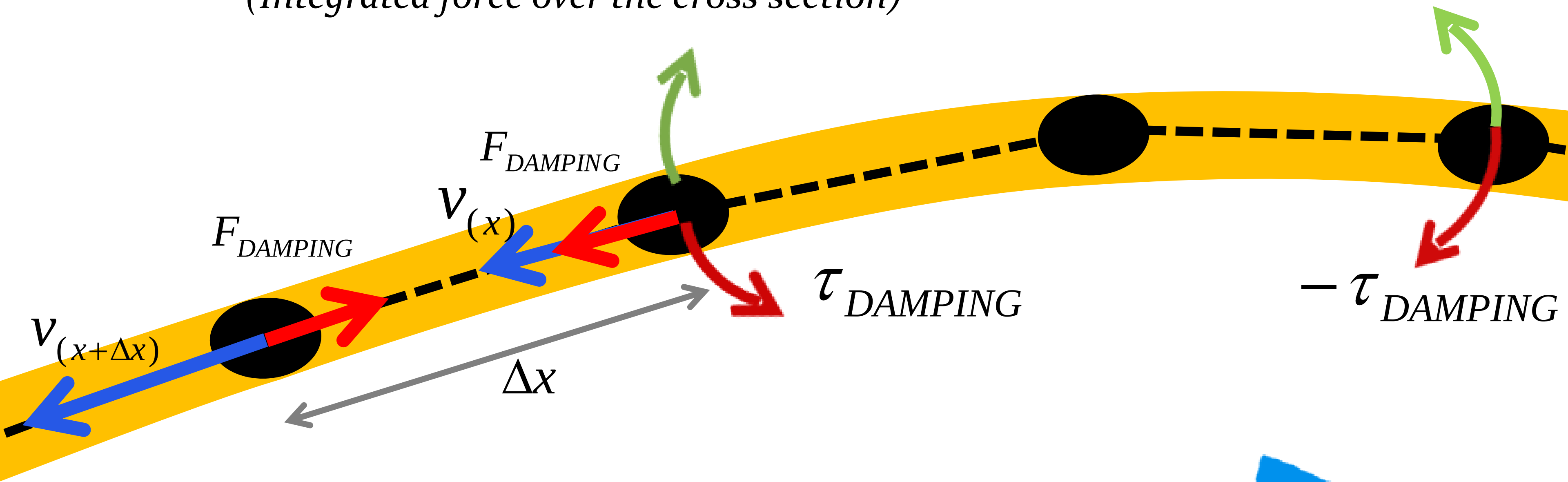
Translational motion

$$F_{DAMPING} = \frac{dv}{dx} SD \qquad F_{DAMPING} = \frac{v_{(x+\Delta x)} - v_{(x)}}{\Delta x} SD$$

Bending motion

$$\tau_{DAMPING} = \frac{DI}{R^2} \frac{dR}{dt} \qquad \tau_{DAMPING} = \frac{DI}{R^2} \frac{R_{(t+\Delta t)} - R_{(t)}}{\Delta t}$$

(Integrated force over the cross section)



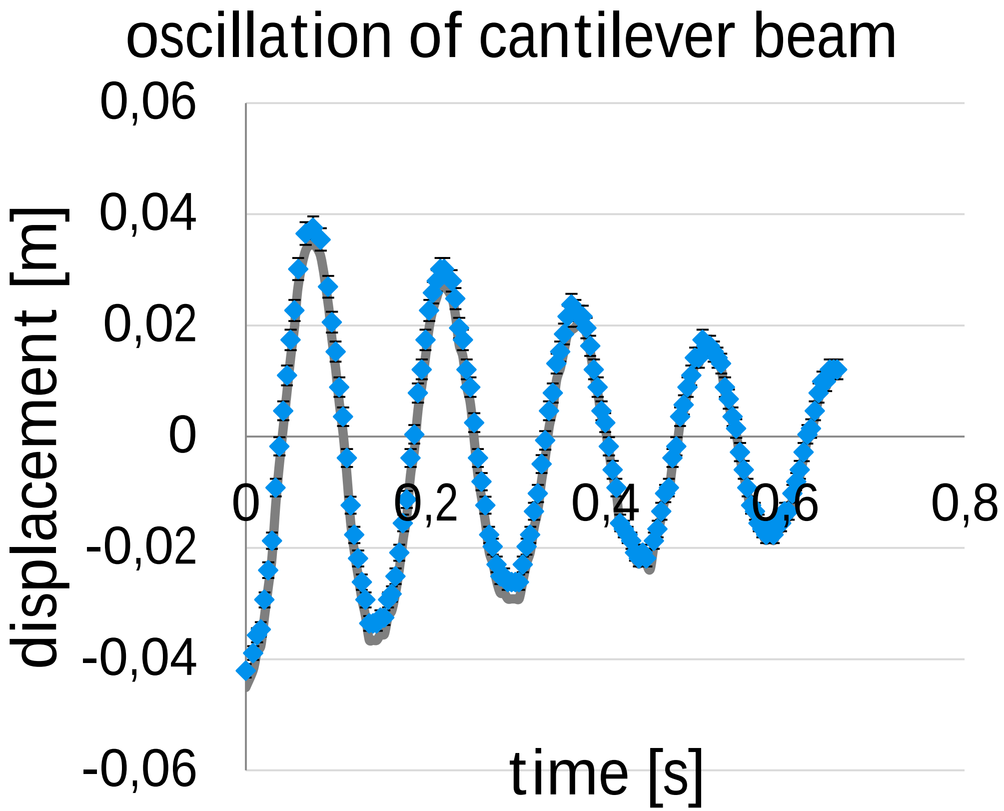
Damping

experimentally measured

oscillation of cantilever beam

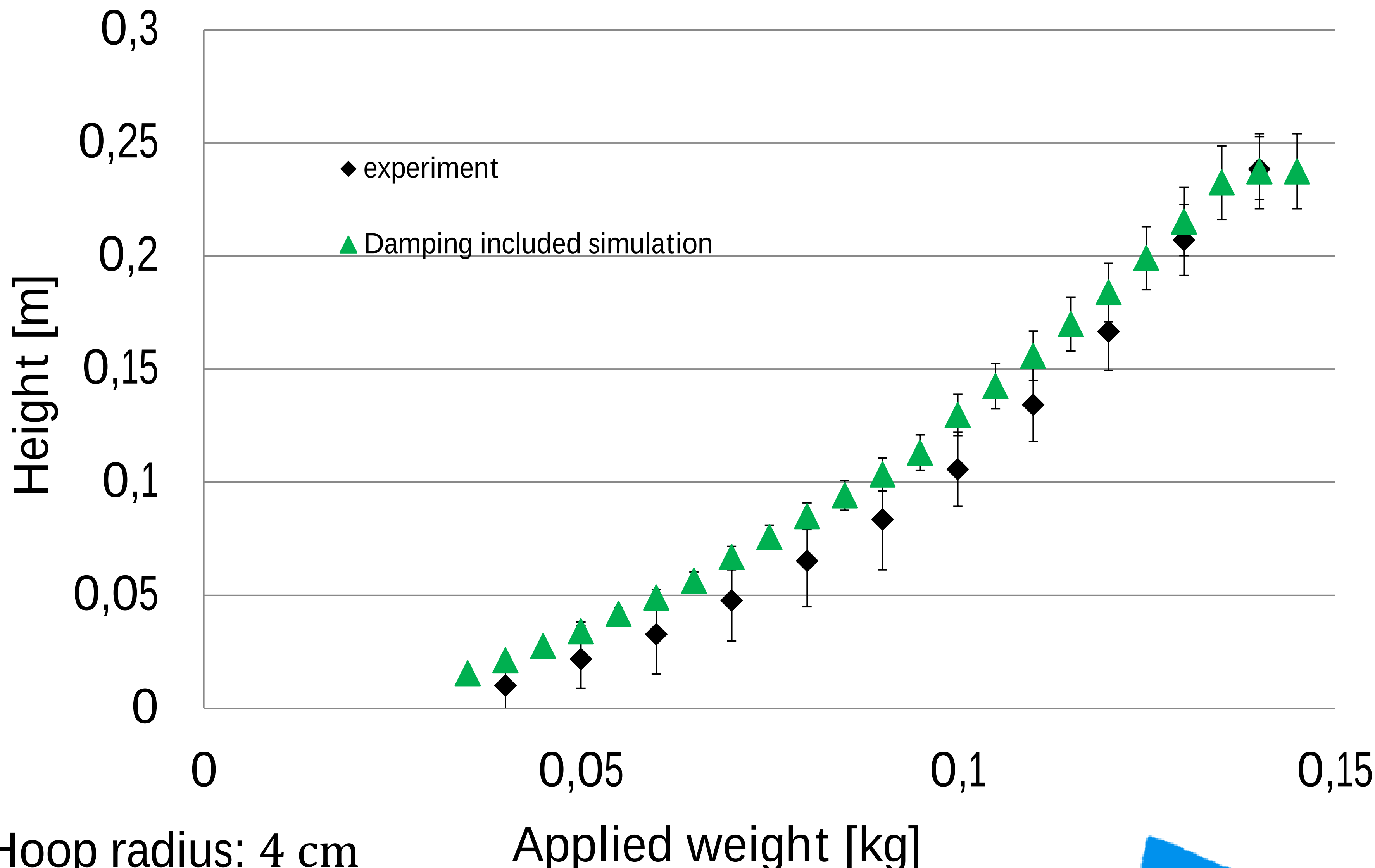
Damping

Experimentally measured



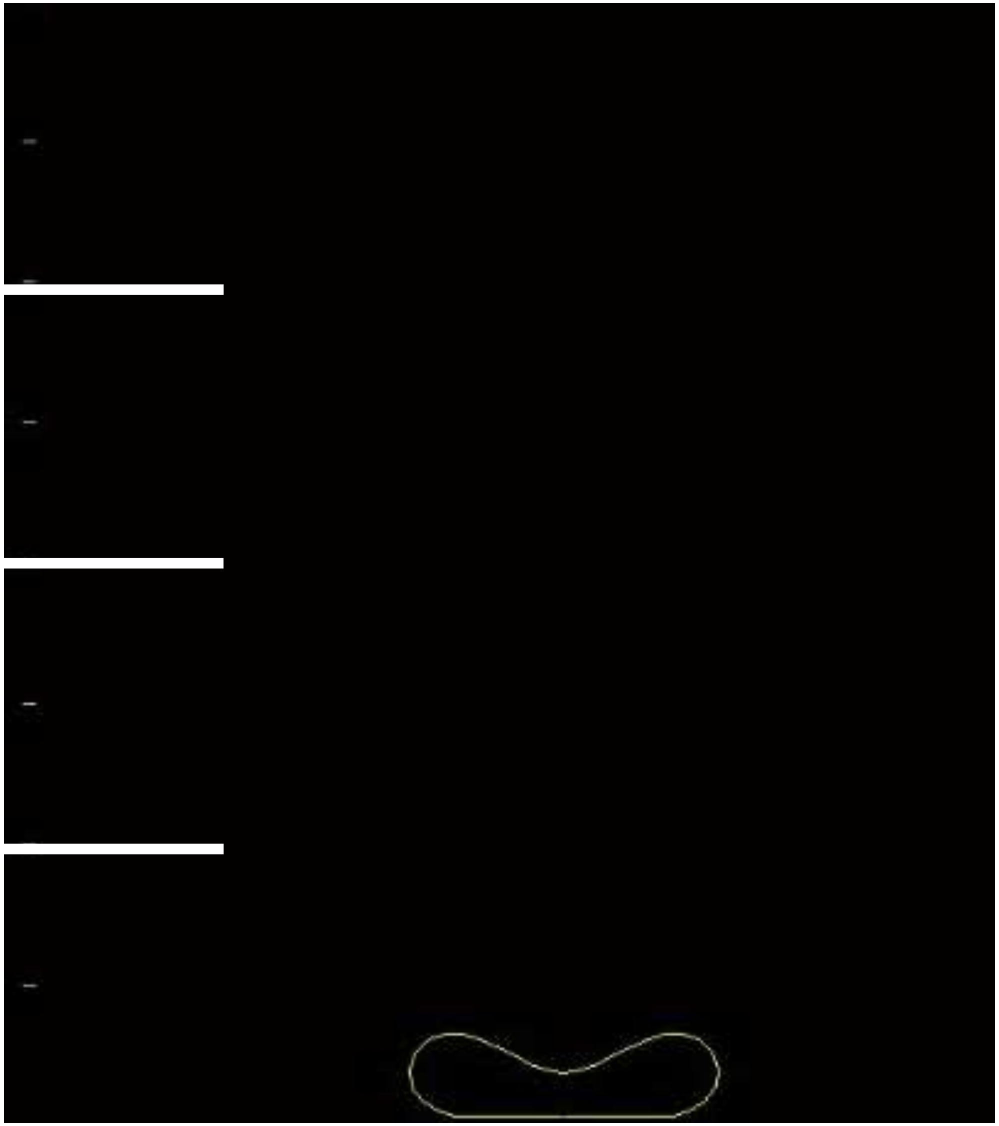
$$D = 3900\ \mathrm{kg\ m^{-1}\ s^{-1}}$$

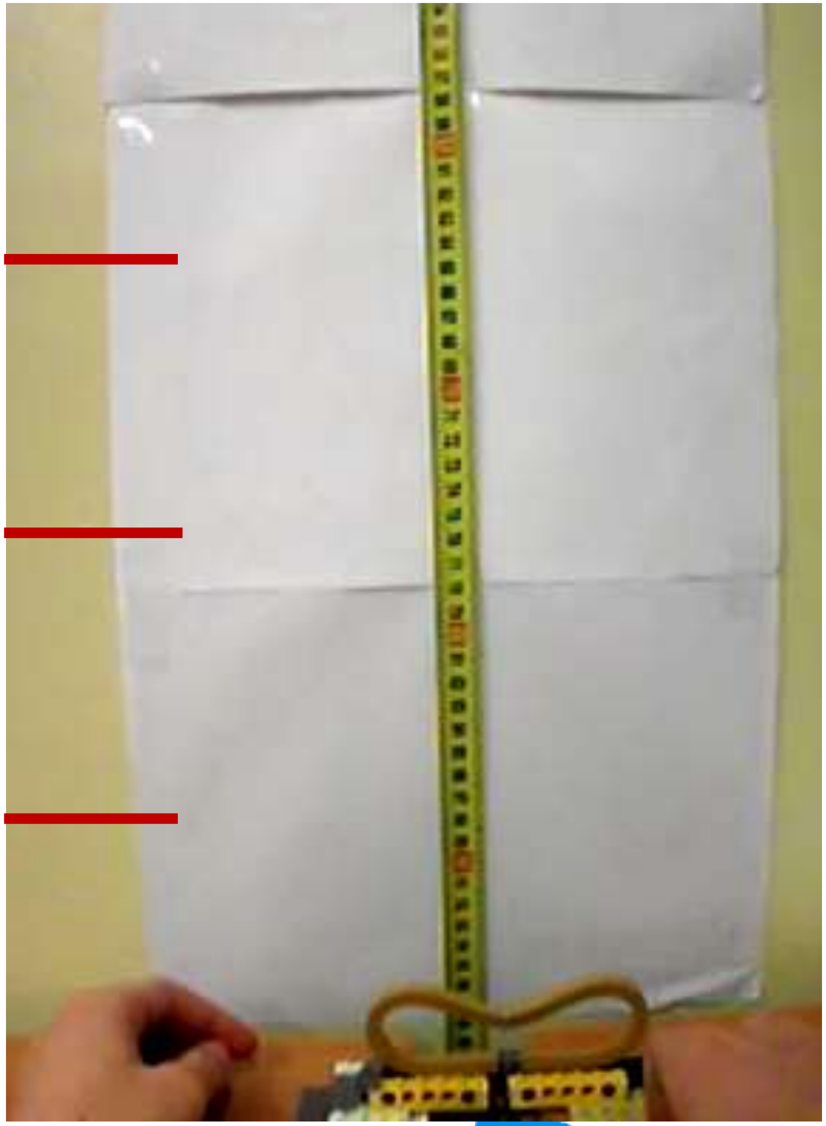
Rubber Hoop: Jump Height vs. Mass Applied



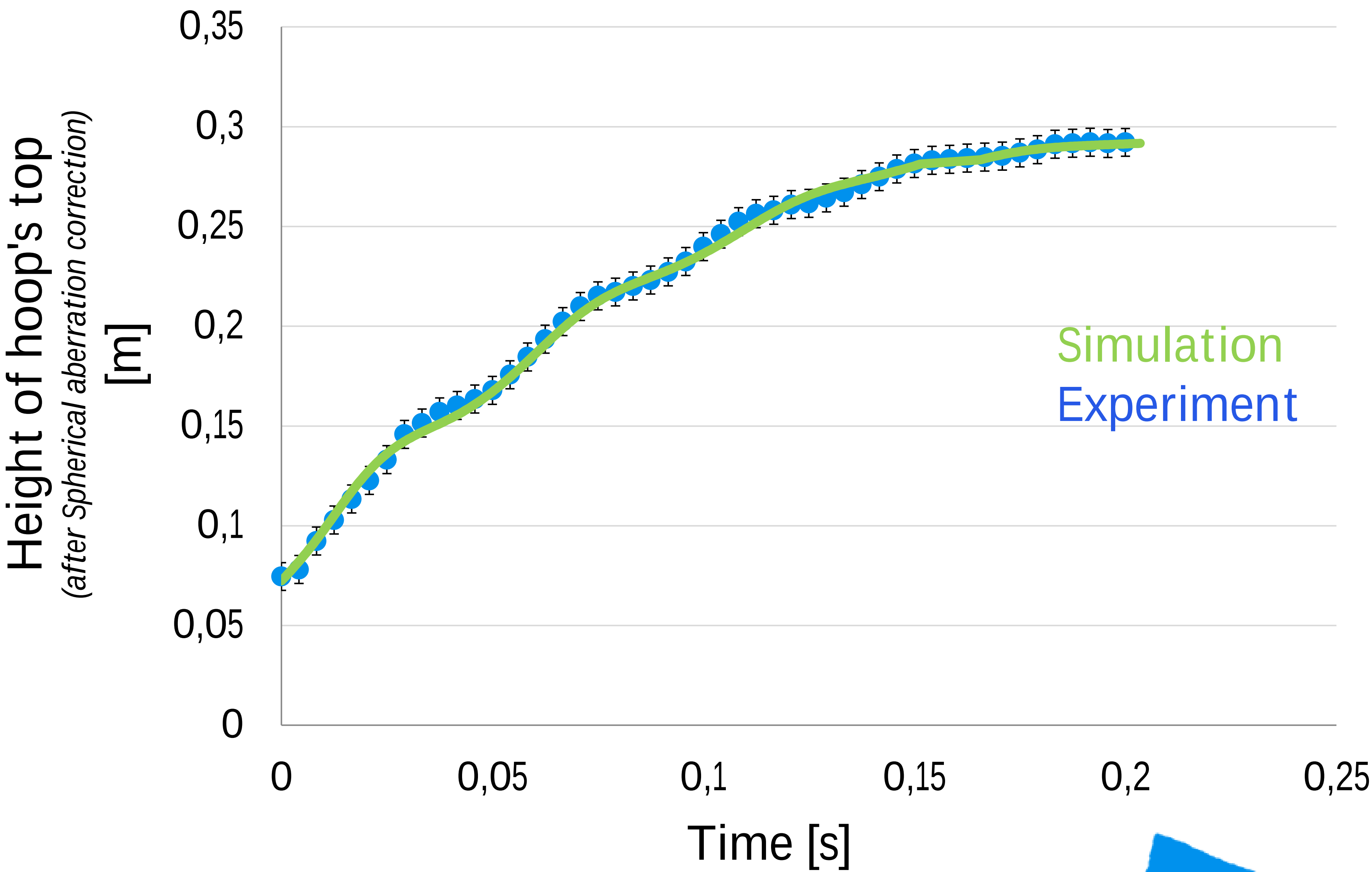
Hoop radius: 4 cm

Comparison (Rubber hoop)





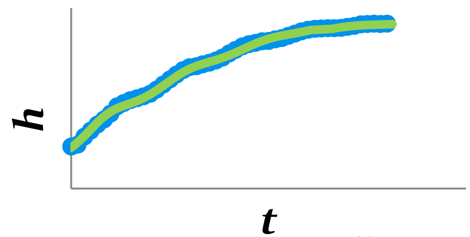
Height of Top in Time (Rubber hoop)



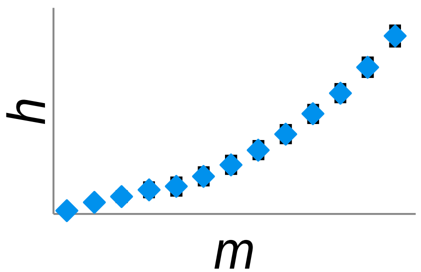
Conclusion

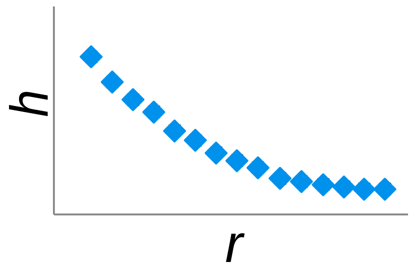
- simple predictions $h \propto \frac{1}{r_{HOOP}^2}$ $h \propto F^2$
- numerical model:

  - large deformations
  - complex model of energy dissipation (damping, air drag, collisions with surface)
  - proved by experiments



crucial for some materials





"Investigate how the height of the jump depends on the relevant parameters." ✓

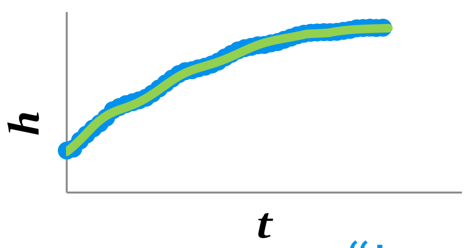
Thank you for your attention!

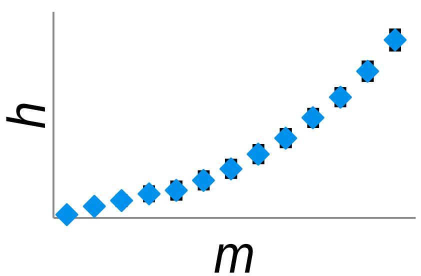
- simple predictions $h \propto \frac{1}{r_{HOOP}^2}$ $h \propto F^2$
- numerical model:
  - large deformations

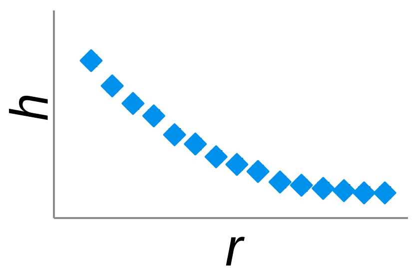
  - complex model of energy dissipation (damping, air drag, collisions with surface)

crucial for some materials

  - proved by experiments







"Investigate how the height of the jump depends on the relevant parameters." ✓

APPENDICES

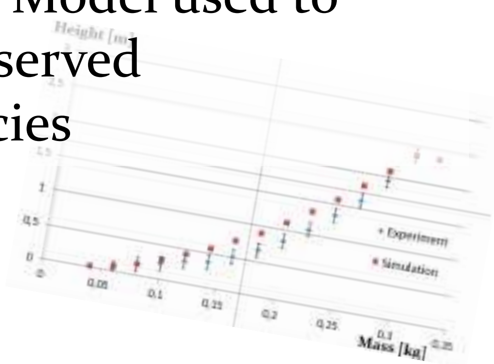
Comparison *Our model – Young-Kim*

- Model extended to large deformations

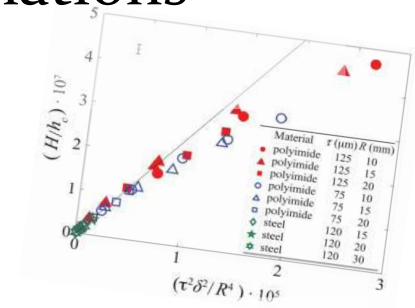
  - Complex model of dissipation energy (Damping, Air drag, Surface)

Crucial for some materials

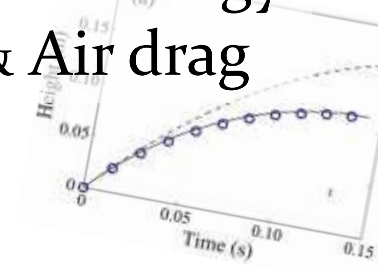
- Numerical Model used to explain observed dependencies



- Small deformations model only

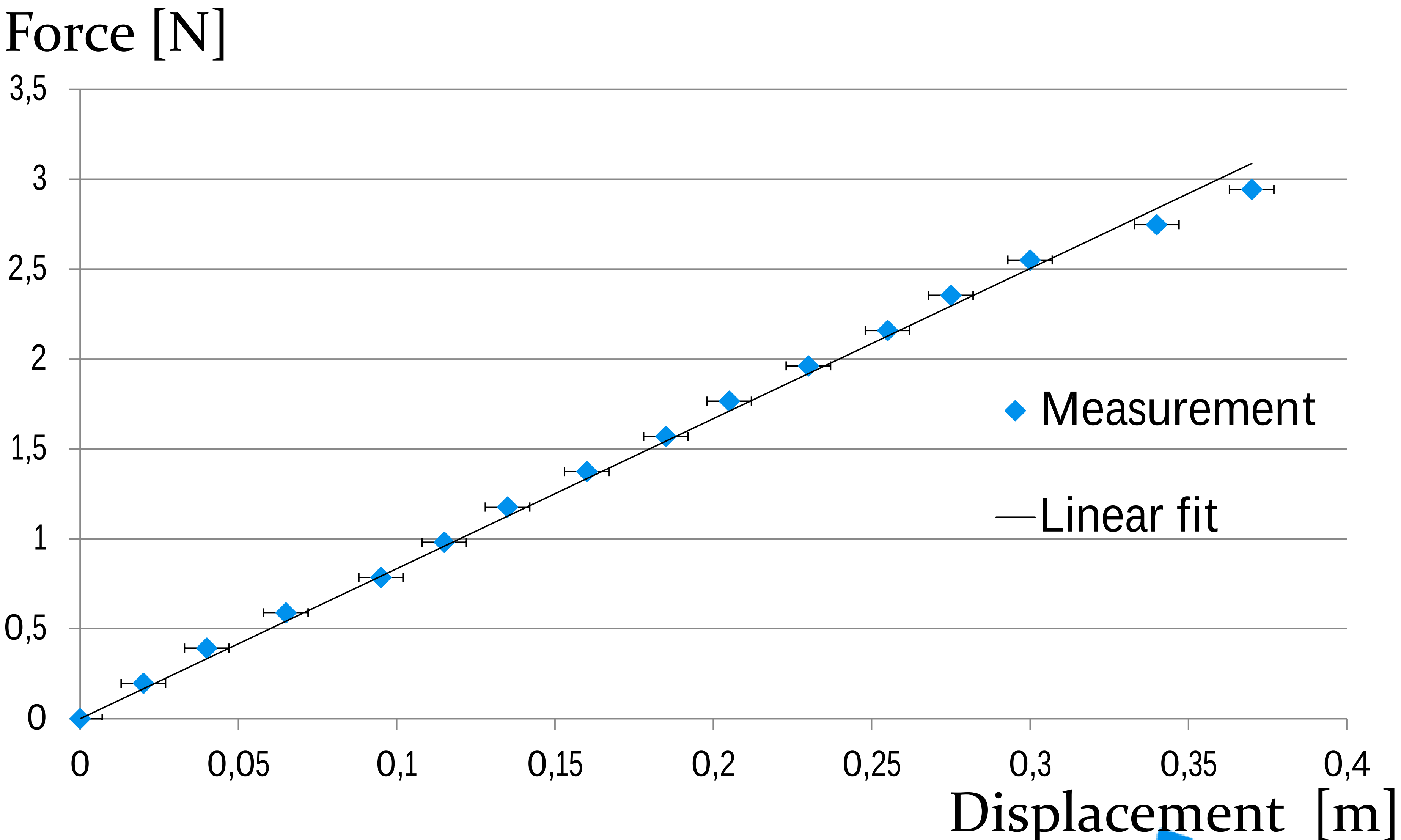


- Dissipation of the energy to the surface & Air drag

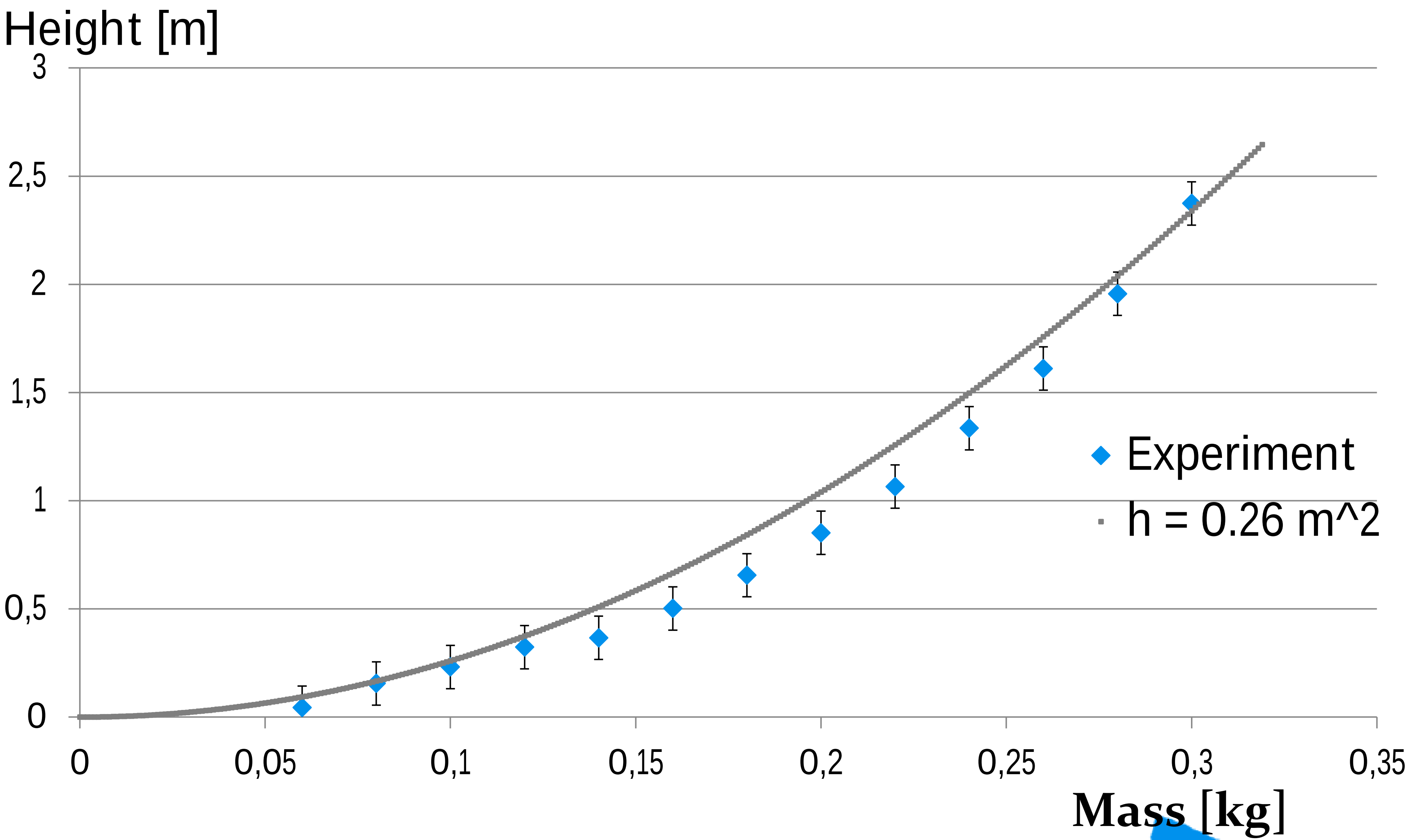


- Theoretical calculation
  Fixed ratio
  Initial def. Energy/Initial kinetic energy

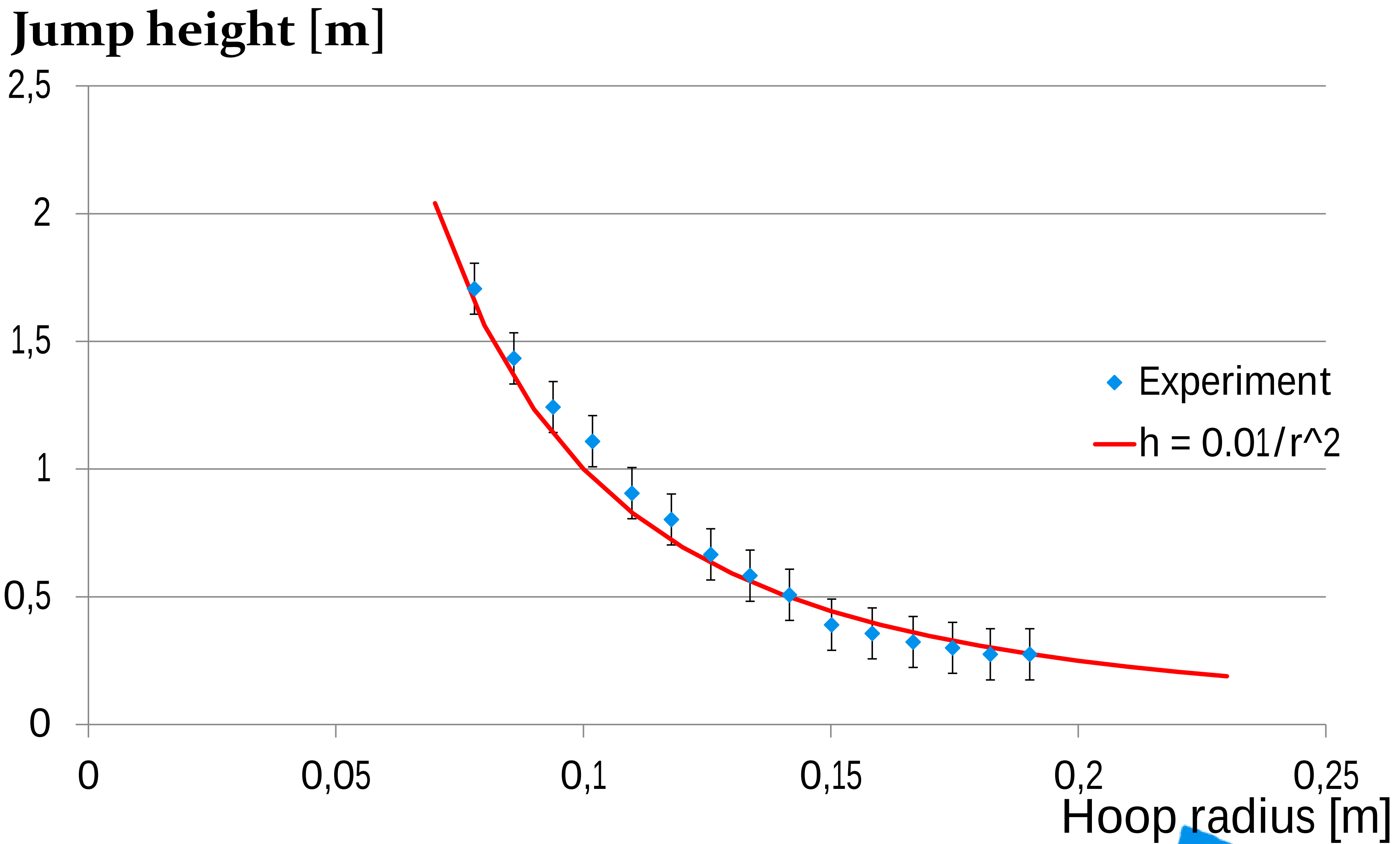
Force/top point displacement



Simple prediction result



Jump height/hoop radius



How to determine bending stiffness?

Equilibrium implies:

$$IY \frac{dw^4}{dx^4} = 0$$

Boundary condition:

$$w_{(0)} = 0 \quad w'_{(0)} = 0 \quad w''_{(L)} = 0$$

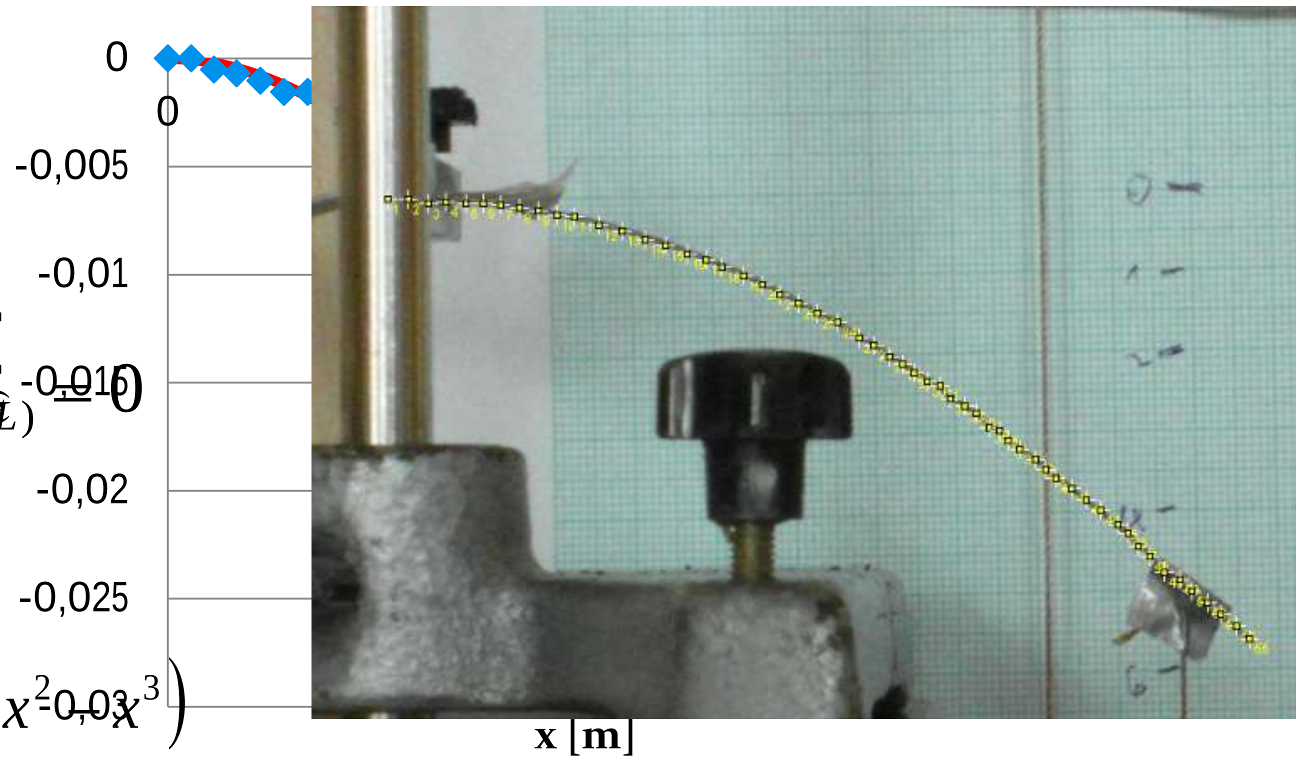
$$IYw'''_{(L)} = m_{weight} g$$

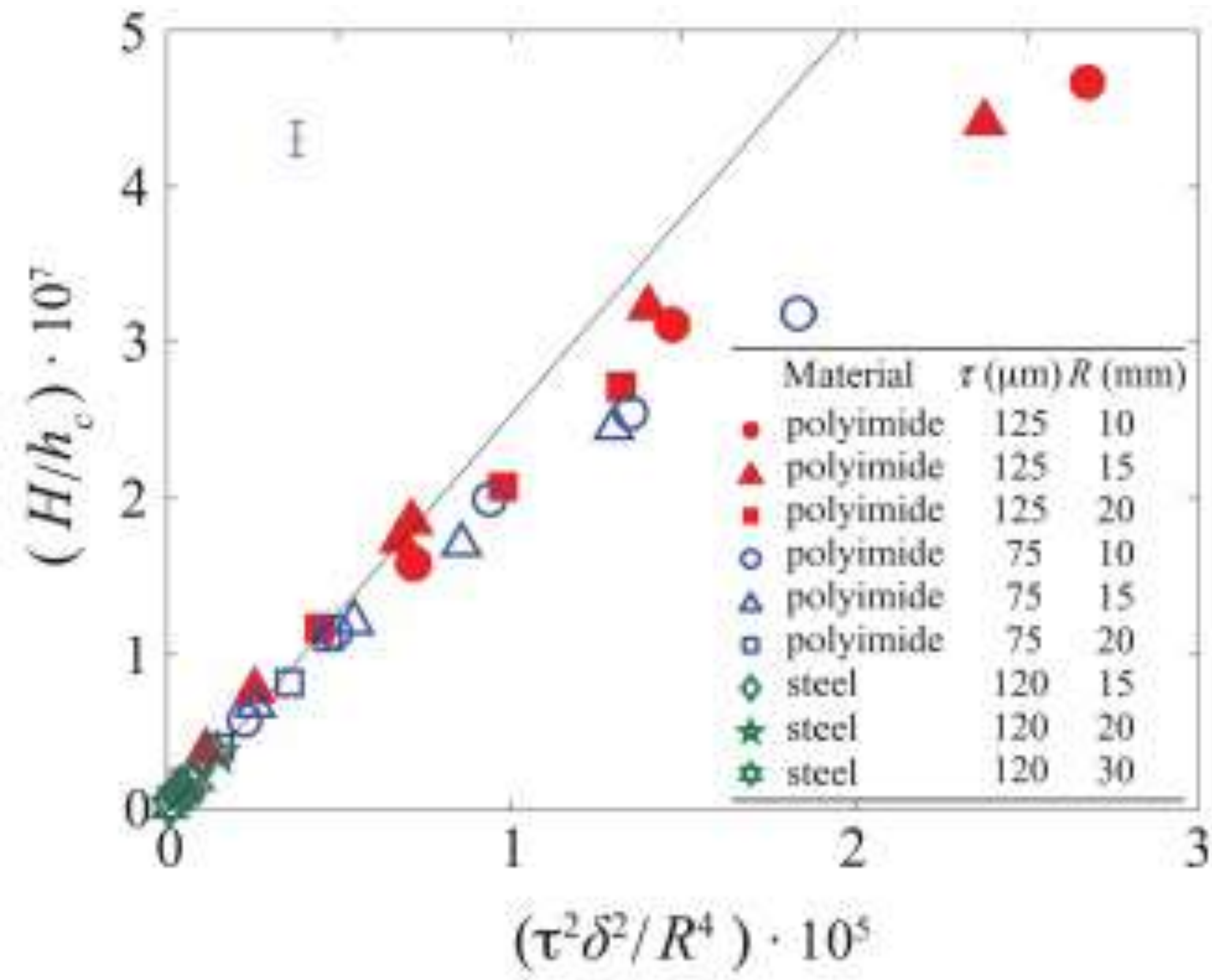
Shape:

$$w_{(x)} = \frac{-m_{weight} g}{6IY} \left(3Lx^2 - x^3\right)$$

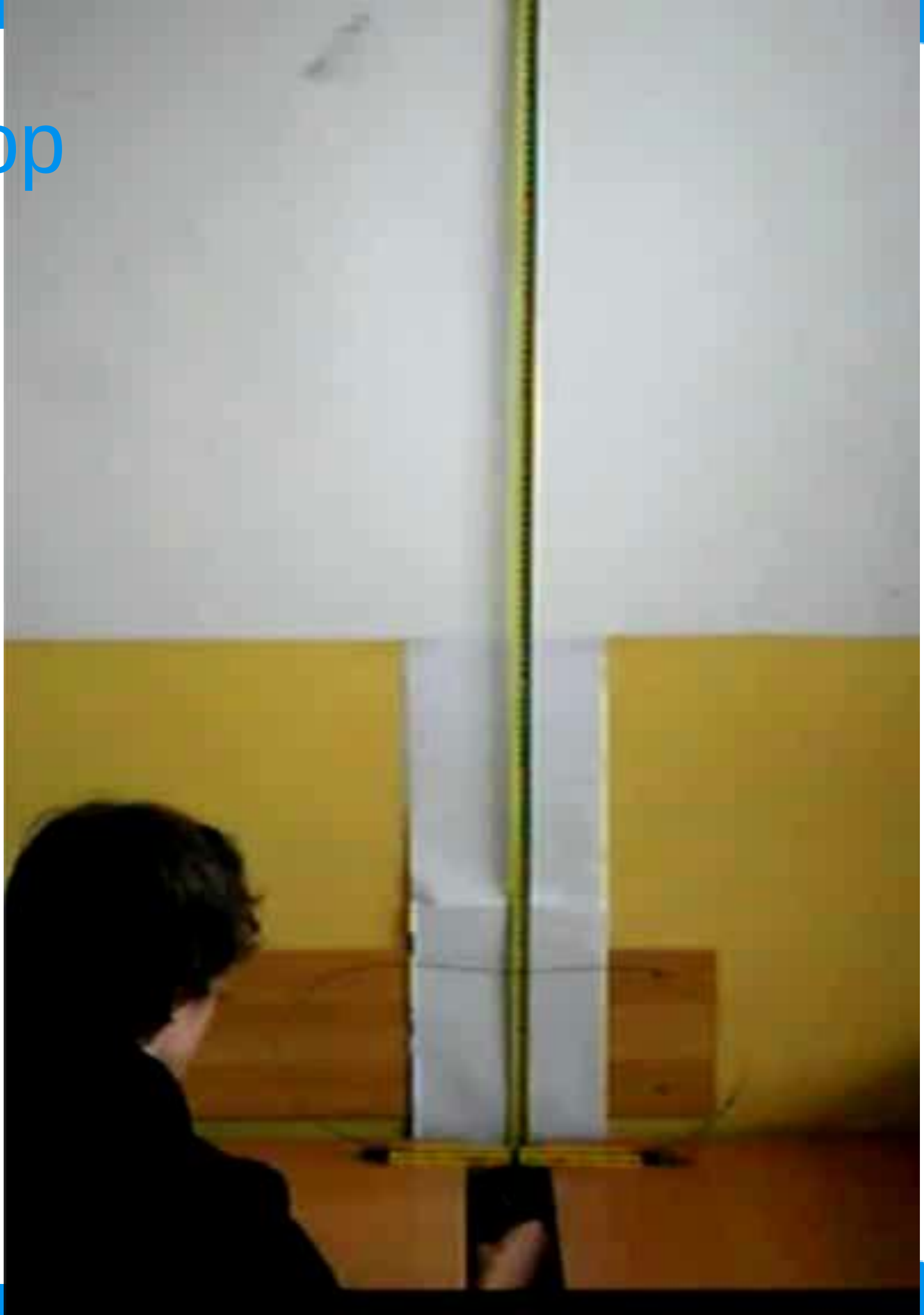
Changing weight → fitting **IY** from shape

→ average **IY + standard deviation**




5
4
3
2
1
0
0
1
2
3
$(H/h_c) \cdot 10^7$
$(\tau^2\delta^2/R^4) \cdot 10^5$
Material τ (μm) R (mm)
polyimide 125 10
polyimide 125 15
polyimide 125 20
polyimide 75 10
polyimide 75 15
polyimide 75 20
steel 120 15
steel 120 20
steel 120 30

Metal hoop

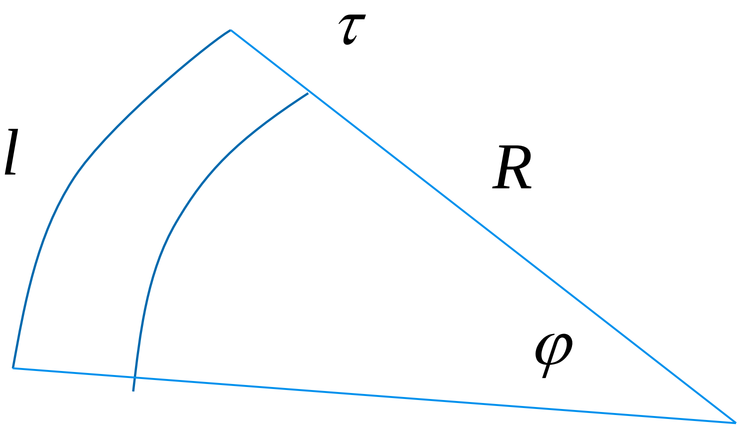


Energy of bending

$$E_l = \int_0^r \tau d\varphi = \int_0^r \frac{IY}{R}\frac{d\varphi}{dR}dR = \int_0^r \frac{IY}{R}\frac{-l}{R^2}dR = \frac{IYl}{2R^2}$$

$$\varphi = \frac{l}{R}$$

$$\frac{d\varphi}{dR} = -\frac{l}{R^2}$$



Young’s modulus of elasticity