15. Oil stars

If a thick layer of a viscous fluid (e.g. silicone oil) is vibrated vertically in a circular reservoir, symmetrical standing waves can be observed. ***How many lines of symmetry*** *are there in such wave patterns? Investigate and explain the* ***shape and behaviour*** *of the patterns.*

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Team Croatia

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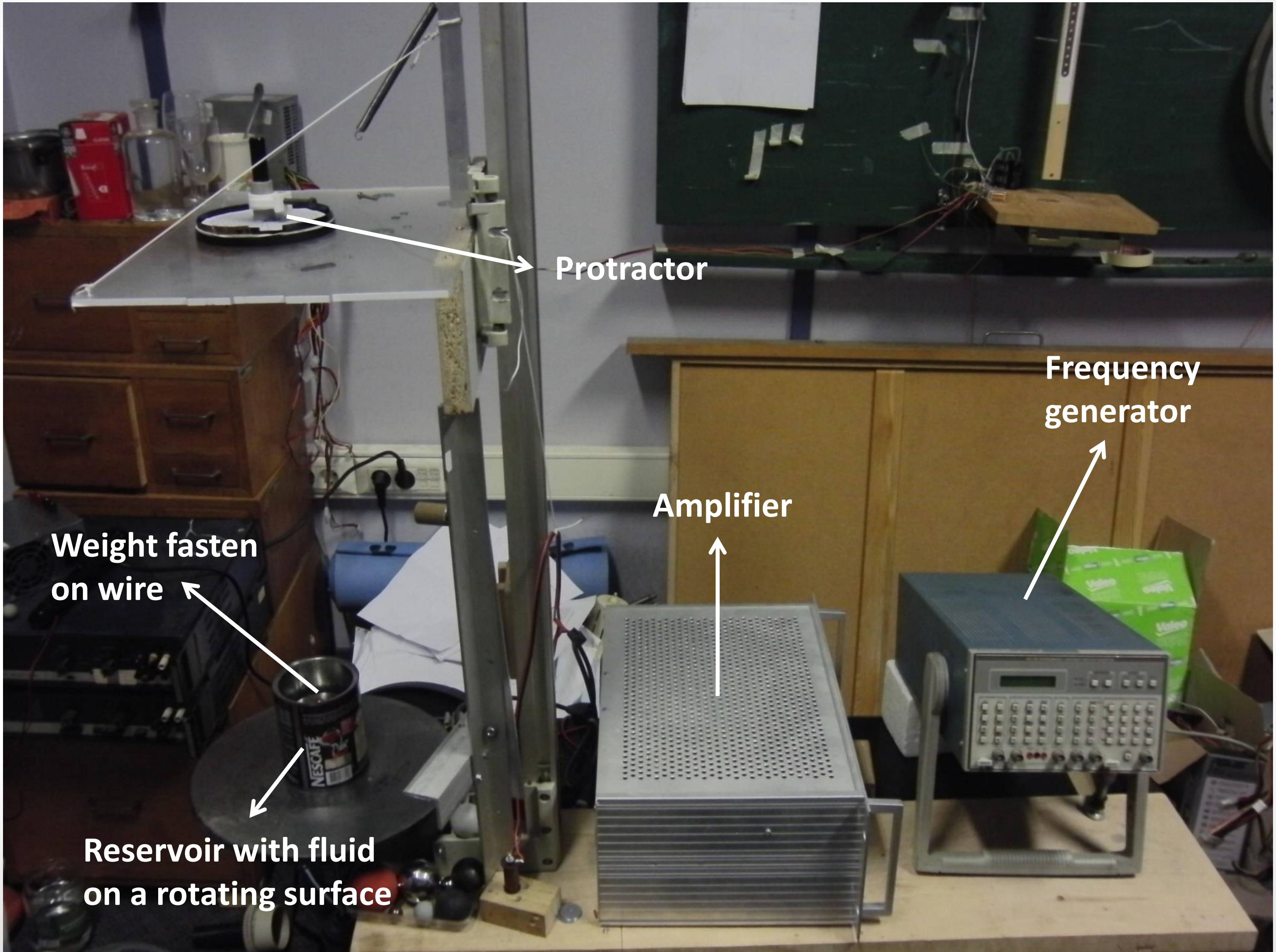
f=9Hz
A=1,75mm

Observations

- nonlinear standing waves in three dimensions– „searching”, sudden apperaring/disappearing of waves while changing frequency or amplitude by which resevoir is vibrated
- symmetries
- if volume of fluid in reservoir is increased, lower frequencies and amplitudes are needed to observe similar waves
- fluids of high viscosity should be used, otherwise there aren’t any results
- if fluids of higher viscosity are used, more complicated waves and symmetries are being observed

Measuring viscosity



Measuring viscosity

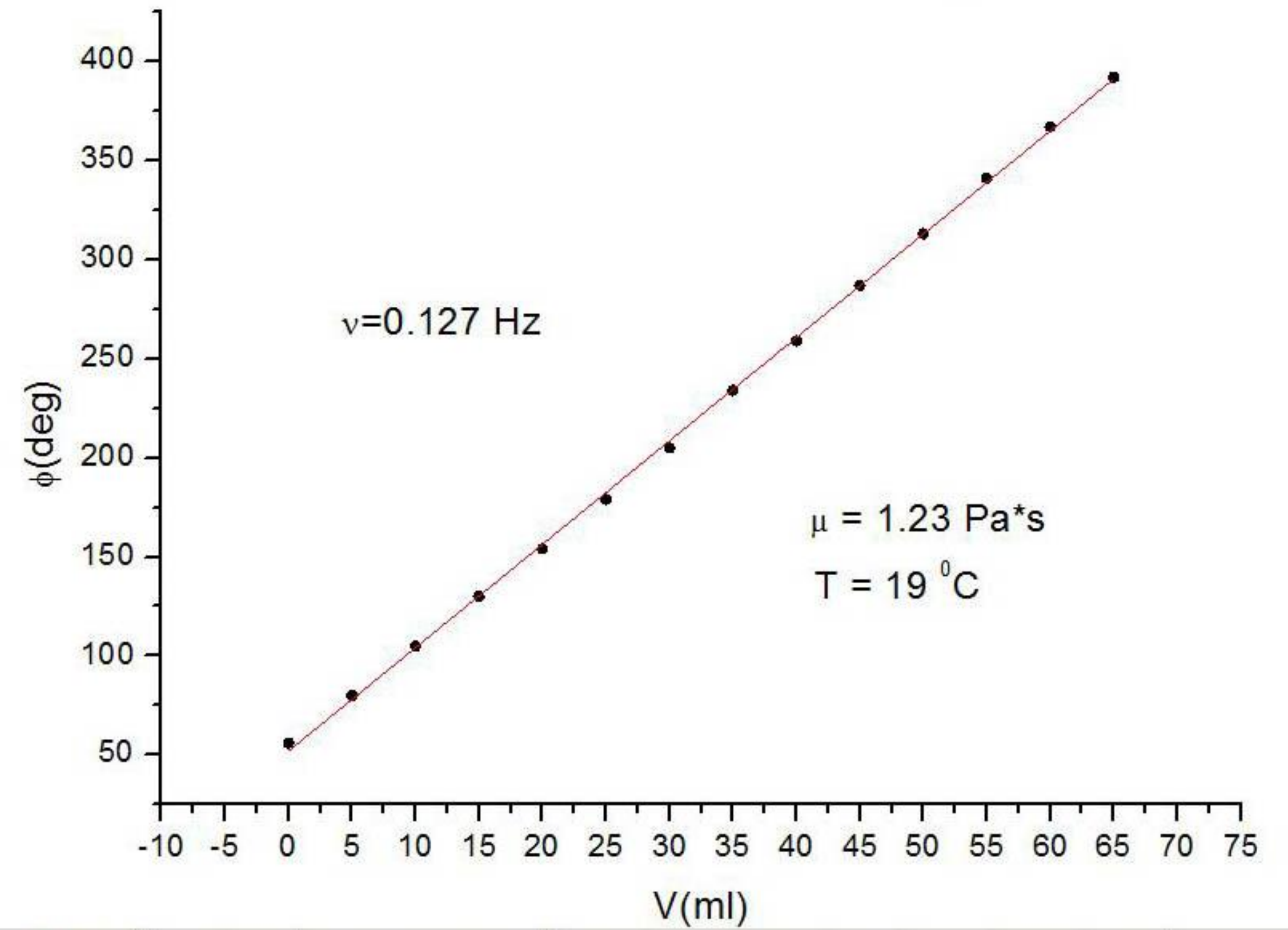
- glycerol and silicon oil (Newtonian fluids)
- knowing the measured angular displacement and frequency (determined with strobescope) while rotating reservoir, viscosity of fluid in reservoir was calculated
- While doing measurements, we were adding known volume (height) of the fluid in reservoir
- derived equasions:

$$\mu = \Delta\varphi v = \frac{\Delta\varphi}{\Delta V * k} ,$$

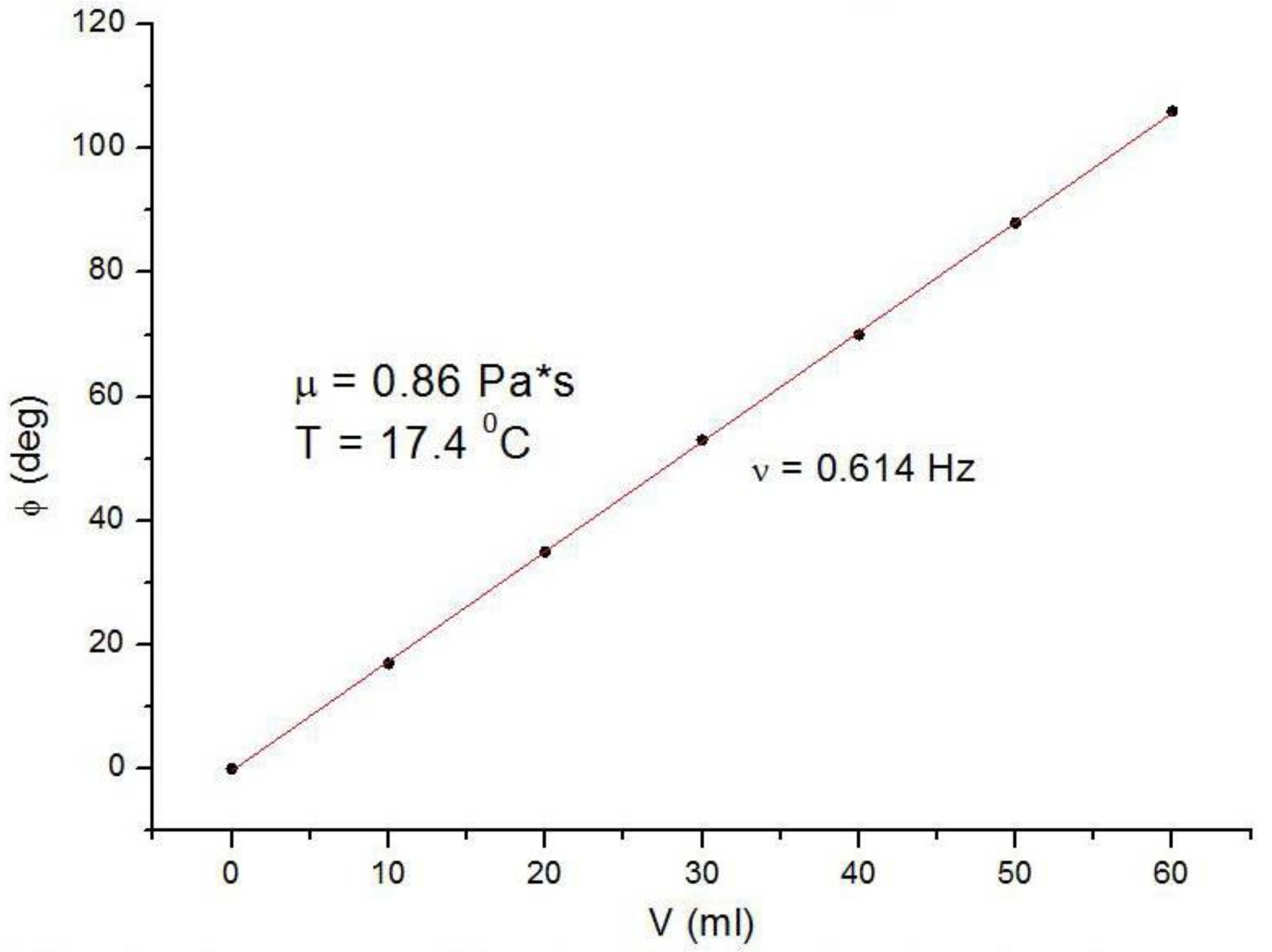
where $\Delta V = (r_2{}^2 - r_1{}^2)\pi h$

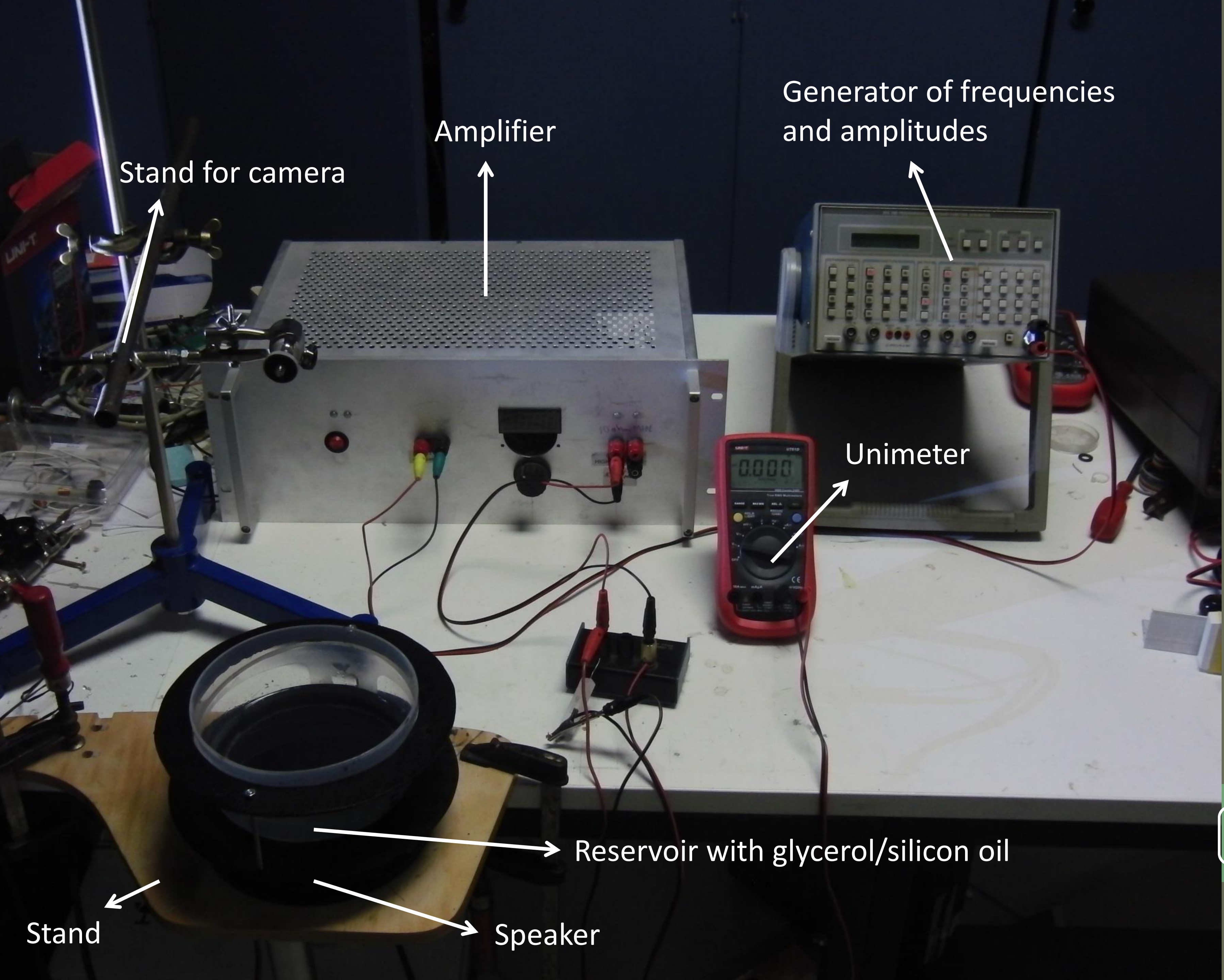
μ - viscosity
v – rotating frequency
$\Delta\varphi$ – change of angle
ΔV – change of volume
k - constant of the wire
h– height of fluuid in reservoir

Results - glycerol



Results – silicon oil




Generator of frequencies
and amplitudes
Amplifier
Stand for camera
Unimeter
Reservoir with glycerol/silicon oil
Stand
Speaker

Mathematical model

General equasion of standing wave:

$$\mathbf{y(x, t) = Acos(kx - \omega t)}$$

Velocity of particle situated on place x in moment t:

$$\mathbf{v_y(x, t) = A\omega sin(kx - \omega t)}$$

General equasion for two dimensional wave:

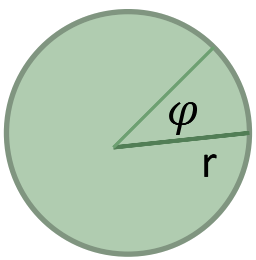
$$\frac{\partial^2 \mathbf{u}(\mathbf{x},\mathbf{y},\mathbf{t})}{\partial \mathbf{x}^2} + \frac{\partial^2 \mathbf{u}(\mathbf{x},\mathbf{y},\mathbf{t})}{\partial \mathbf{y}^2} - \frac{\mathbf{1}}{\mathbf{v}^2}\frac{\partial^2 \mathbf{u}(\mathbf{x},\mathbf{y},\mathbf{t})}{\partial \mathbf{t}^2} = \mathbf{0}$$

solution of this equasion is u(x,y,t) and gives us the deformation of point x,y in moment t

Applied equasion for two dimensional wave:

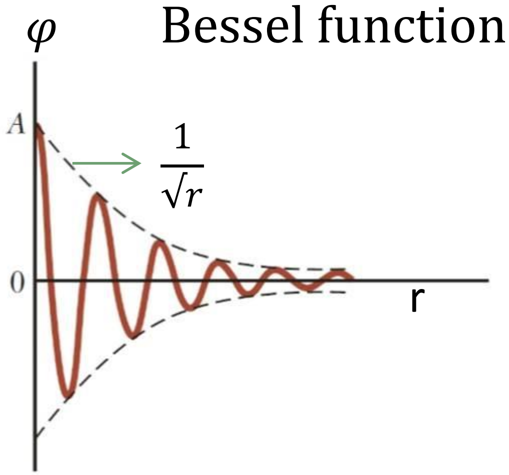
$$\Delta = \frac{\partial^2 \mathbf{u}(\mathbf{r}, \varphi, \mathbf{t})}{\partial \boldsymbol{r}^2} + \frac{\partial^2 \mathbf{u}(\mathbf{r}, \varphi, \mathbf{t})}{\partial \varphi^2} - \frac{\mathbf{1}}{\mathbf{v}^2} \frac{\partial^2 \mathbf{u}(\mathbf{r}, \varphi, \mathbf{t})}{\partial \mathbf{t}^2}$$

solution of this equasion is u(r, φ, t) and gives us the deformation of point r, φ in moment t



u ~ (radial part) * (angular part)

Bessel function



$$\phi(\varphi) = \begin{cases} \sin \lambda\varphi \\ \cos\lambda\varphi \end{cases}$$

$$\phi(\varphi + 2\pi) = \phi(\varphi) \rightarrow$$

λ = 1, 2, 3,... ϵN

Symmetries of third order

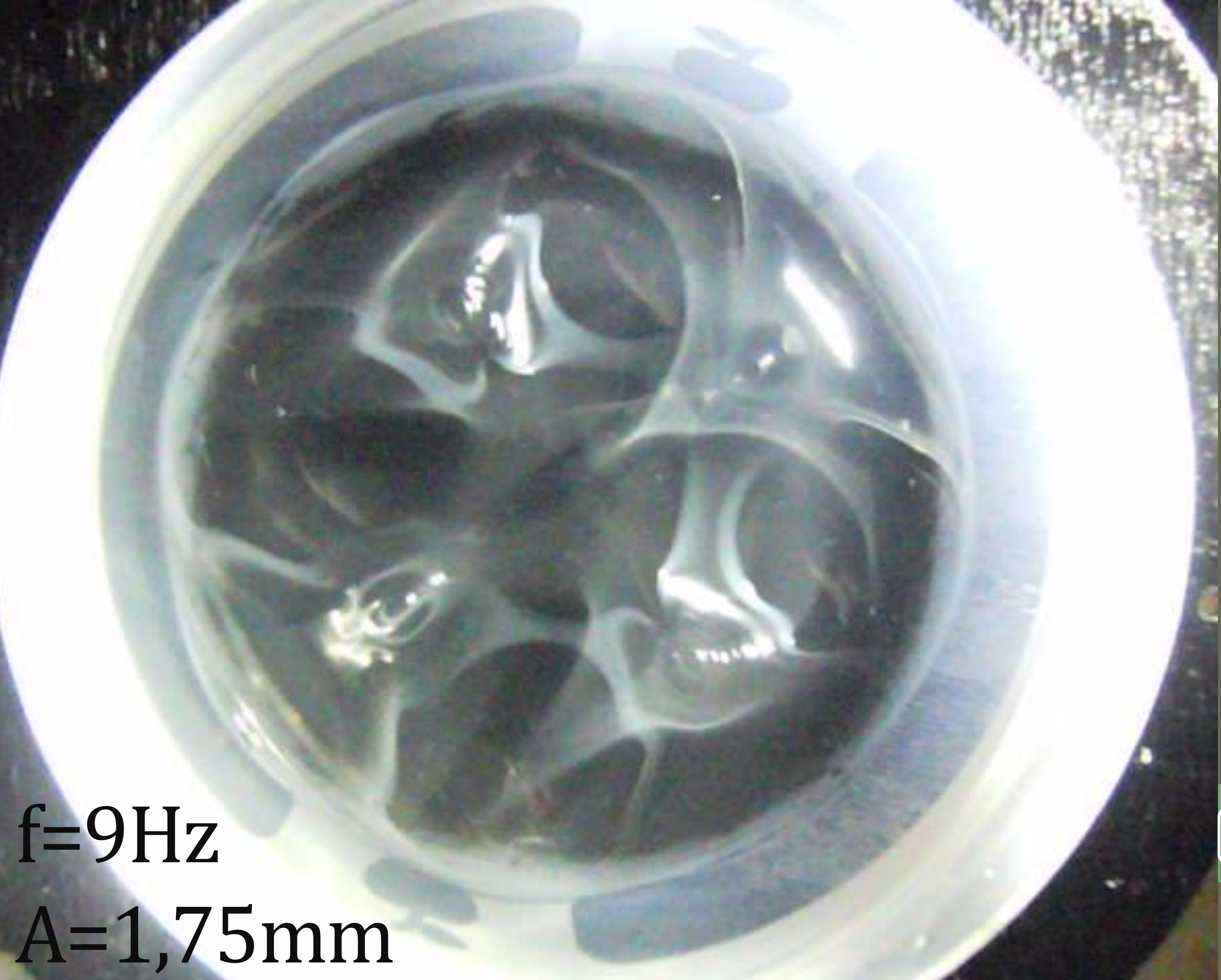
f= 8Hz

A=1,5 mm

Symmetries of fourth order

f=9Hz

A=1,75mm


f=9Hz
A=1,75mm

Symmetries of sixth order

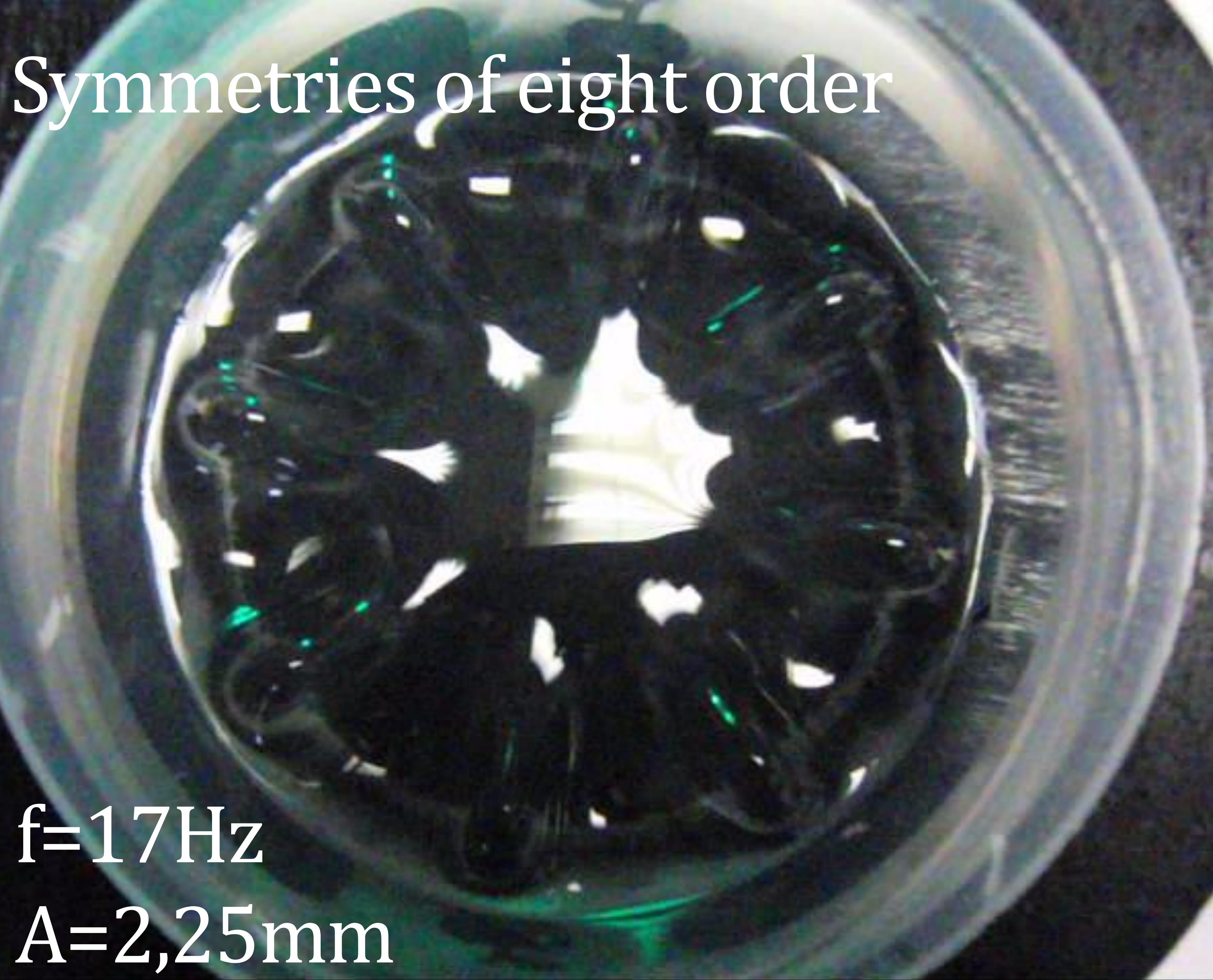
f=11Hz

A=2,1mm


f=11,2Hz
A=1,9mm

f=11Hz
A=2mm

Symmetries of eight order



f=17Hz

A=2,25mm

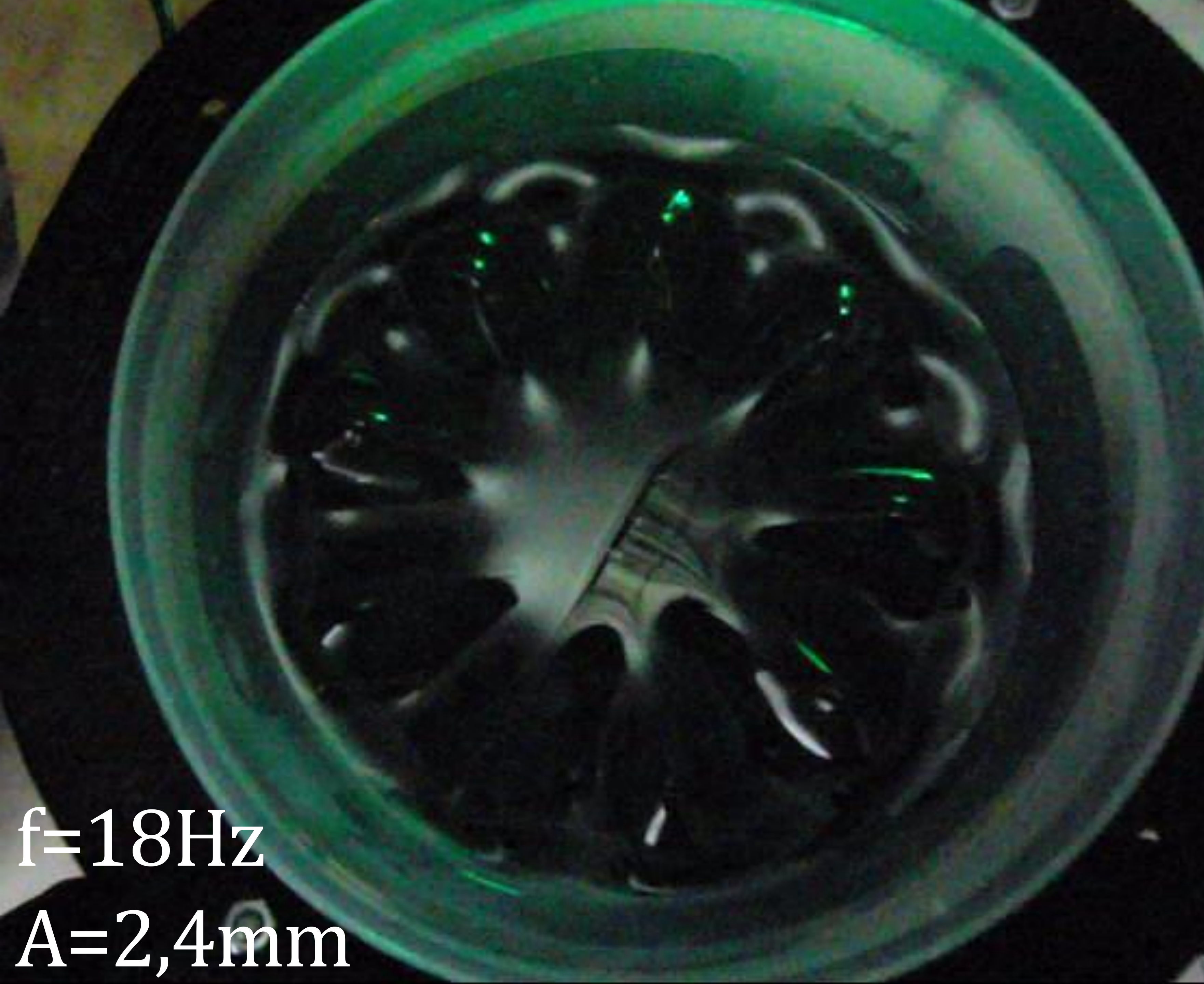
f=17,3Hz

A=2,2mm

Symmetries of ninth order

f=18Hz

A=2,4mm


f=18Hz
A=2,4mm

Hexagons

f=15Hz
A=3,3mm

Conclusion

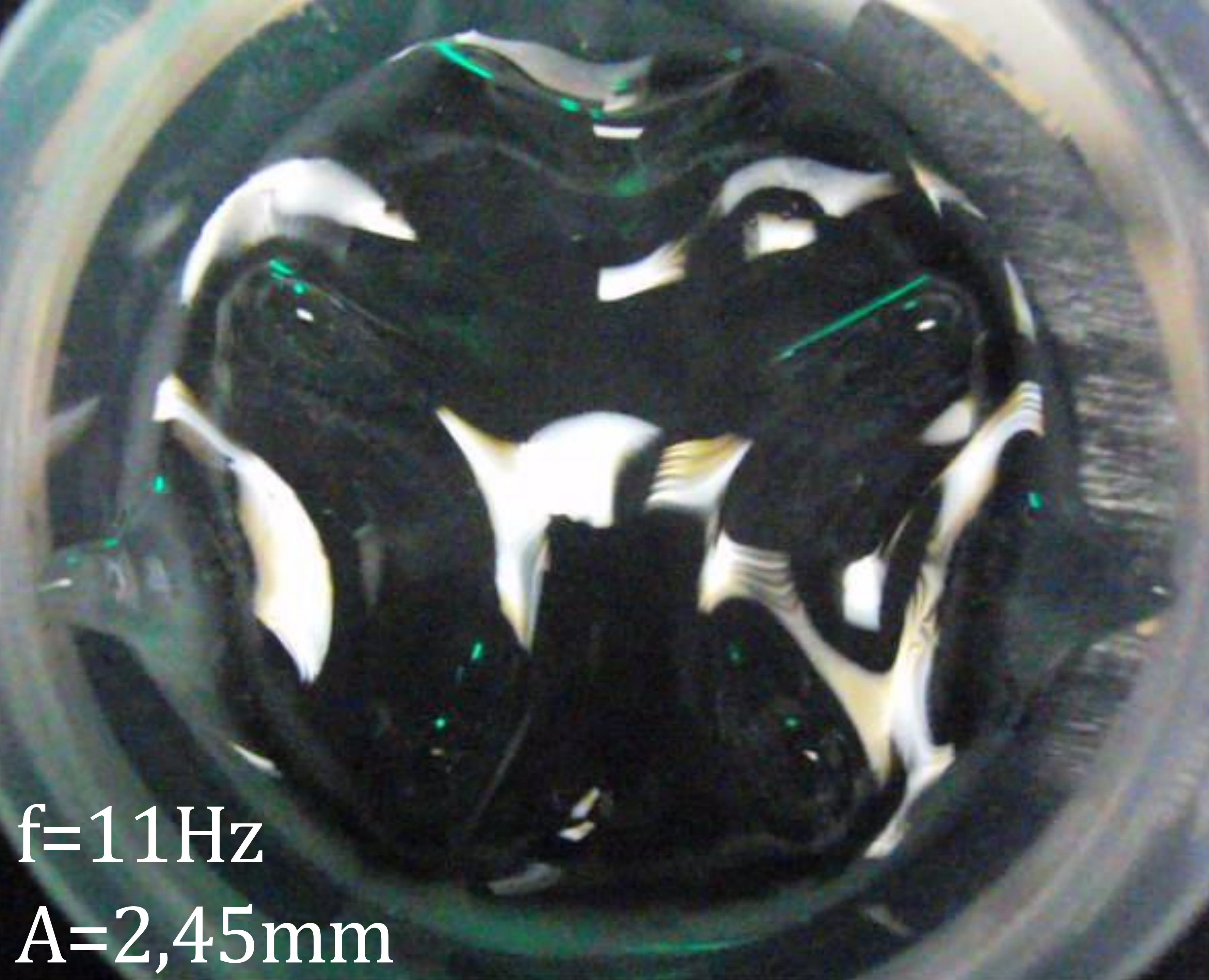
- existance, shape and behaviour of waves depends on:
  - viscosity of fluid
  - thickness of fluid in reservoir
  - frequency and amplitude of oscillations
- size and shape of reservoir don't have any influence (Faraday; 1831.)
- waves can be described with Bessel's functions
- stability of different patterns depends on relative value of energy which is necessary too keep them in that state (Skeldon, Guidoboni; 2005.)

Conclusion

- we have found six stabile systems with different number of symmetry lines - ‘starry’ shapes:
  - third order symmerty
  - fourth order symmerty
  - sixth order symmerty
  - eight order symmerty
  - ninth order symmerty
  - hexagons

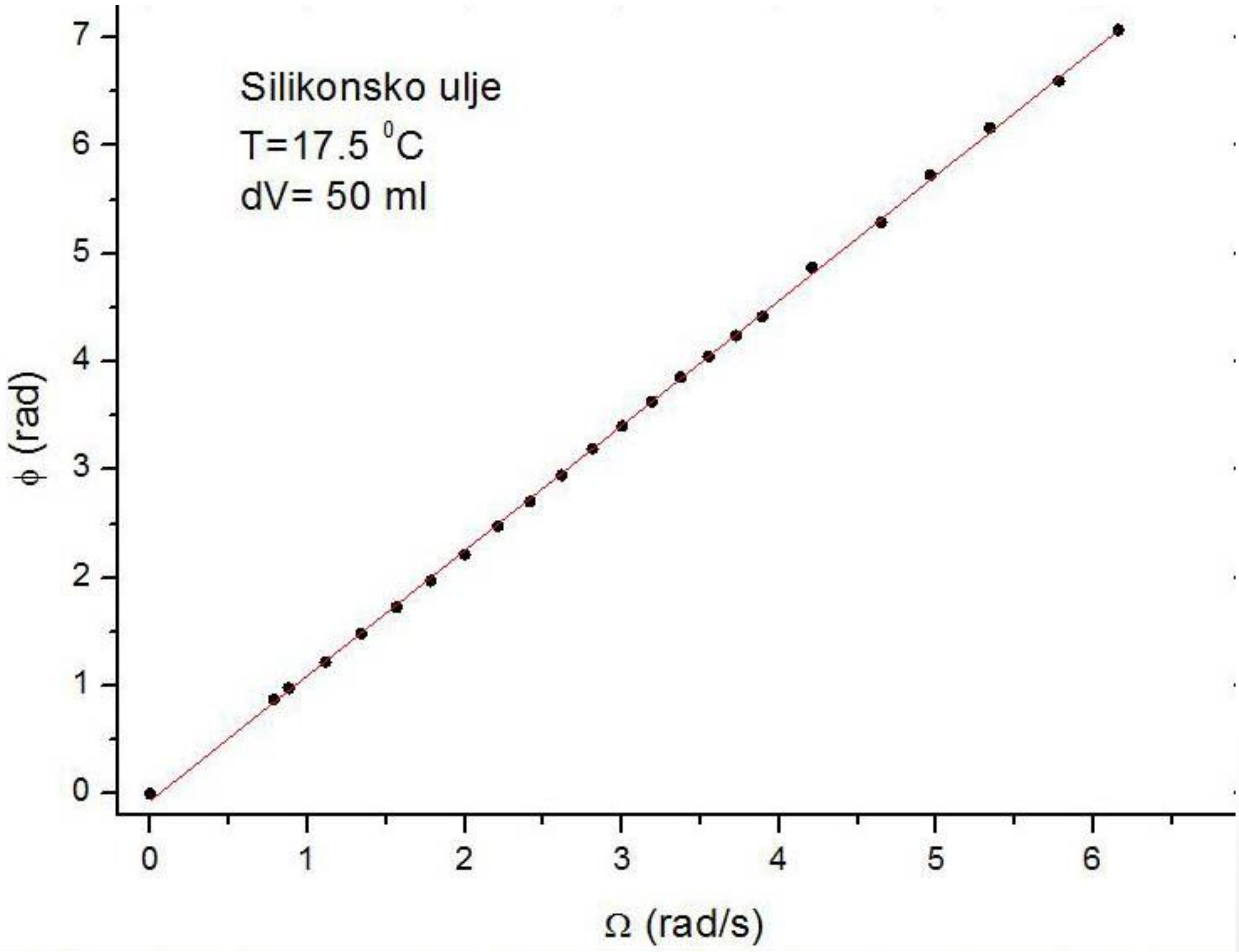
Thank you for your attention!

f=16Hz
A=2,2mm


f=11Hz
A=2,45mm


f=8Hz
A=1,7mm

Oil viscosity– low and high frequencies



Viscosity derivation

Velocity between walls

$$v(r) = A*r + \frac{B}{r}$$

$$A = \frac{r_2^{\,2}}{r_2^{\,2} - r_1^{\,2}} * \Omega\,,\quad B = -\,\frac{r_1^{\,2} * r_2^{\,2}}{r_2^{\,2} - r_1^{\,2}} * \Omega$$

$$\rho = \frac{4\pi * \mu}{k} * \frac{r_1^{\,2} * r_2^{\,2}}{r_2^{\,2} - r_1^{\,2}} * h * \Omega$$

$$h = \frac{V}{(r_2^{\,2} - r_1^{\,2})\pi},\quad k = \tau/\mu$$

$$\mu = \Delta\rho * \nu \qquad \mu = \frac{\Delta\rho}{\Delta V * k}$$