14 . Rubber Motor

‘A twisted rubber band **stores energy** and can be used to power a model aircraft for example. **Investigate the properties** of such an energy source and **how its power output changes with time**.’

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Team Croatia

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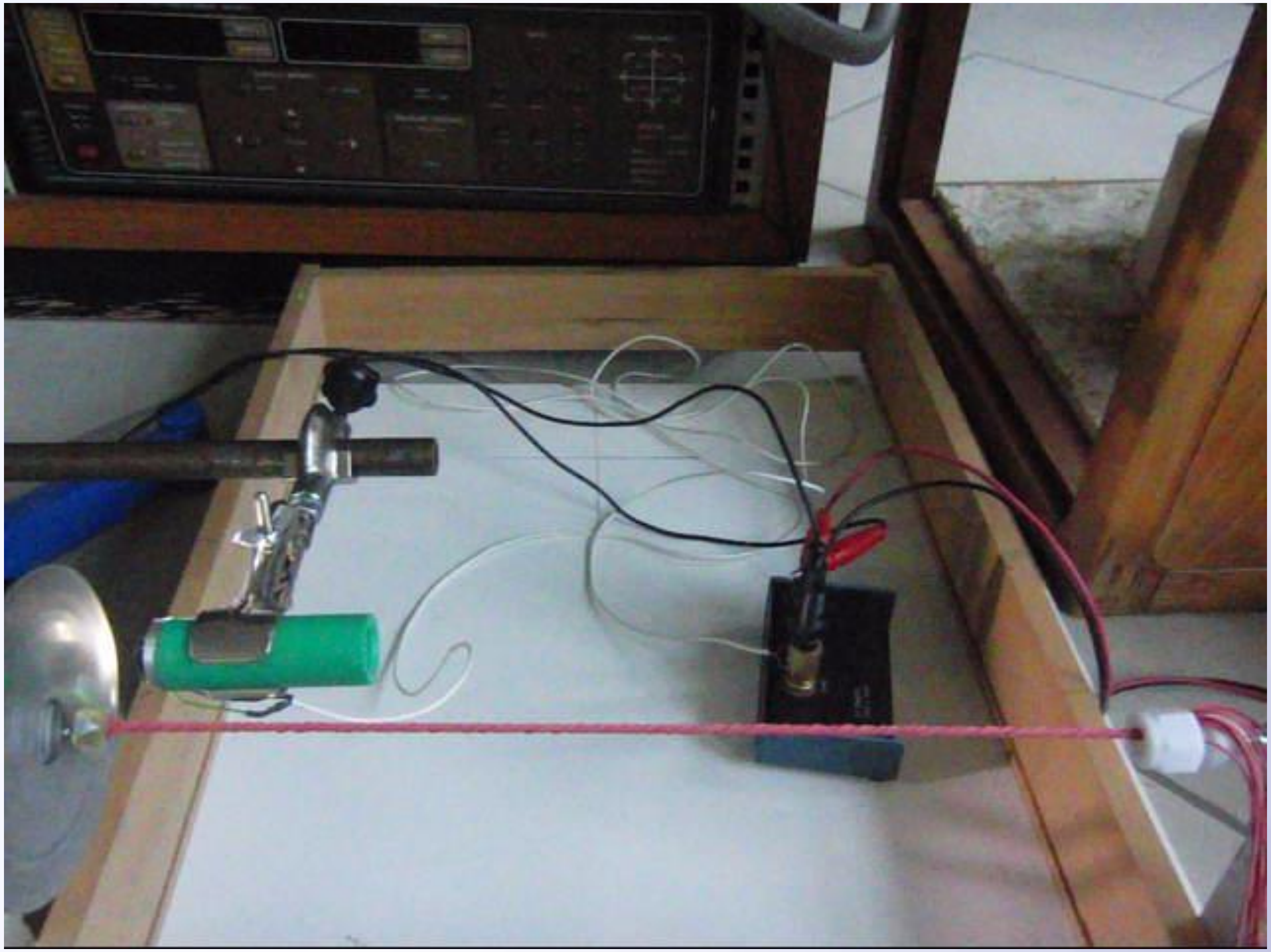
Properties of rubber materials

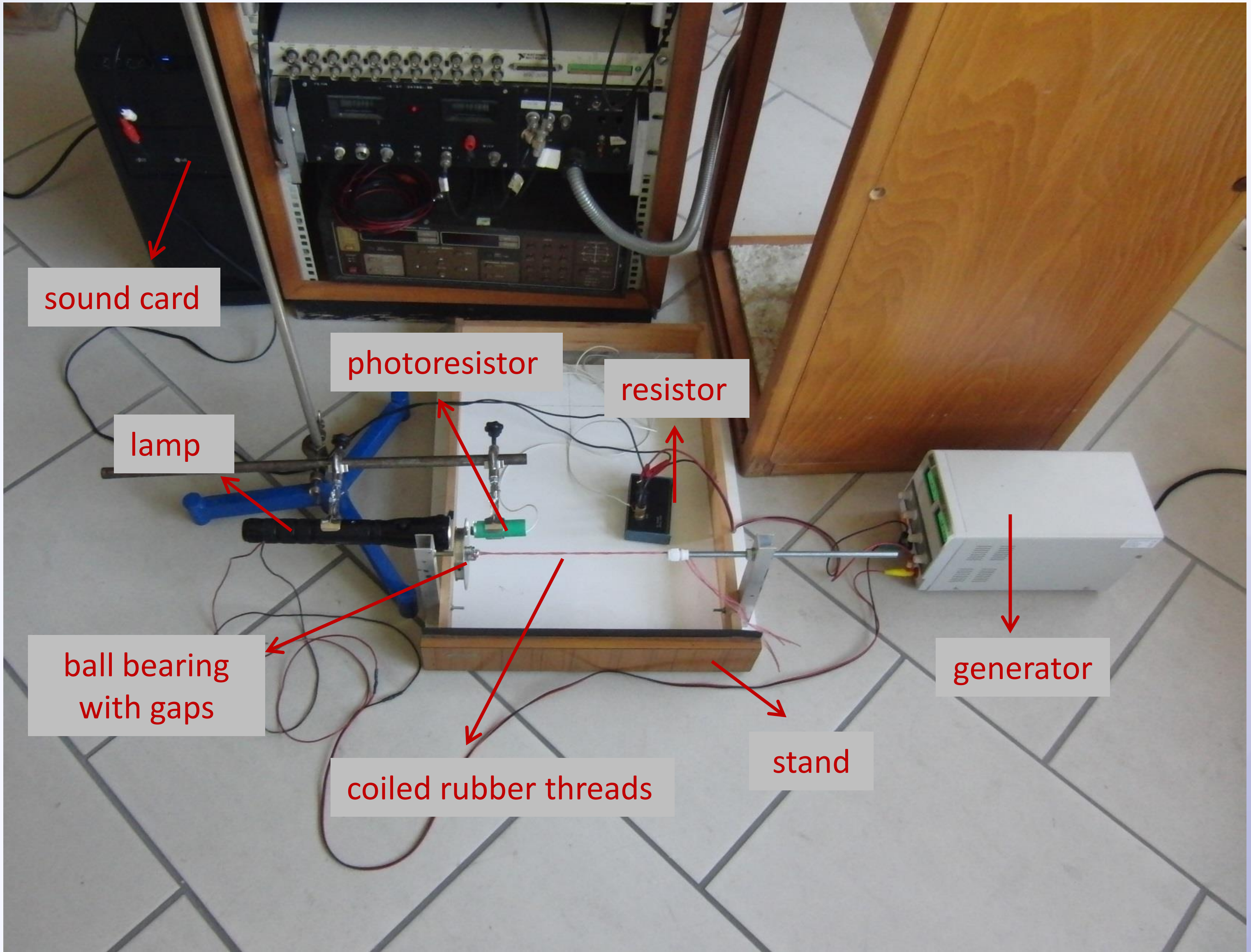
- elastomer → viscoelasticity
- Mullins' effect
  - hysteresis while stretching, entire energy isn't restored, waste in a form of thermal energy
- structure→ coiled molecular chains, polymers (covalent bonds) whose entropy decreases when rubber band is being stretched*
  - unstretched rubber band is in the state of maximum entropy

* I.M. Ward, J. Sweeney: *An Introduction to the Machanical Properties of Solid Polymers*

Observation of how does motor work

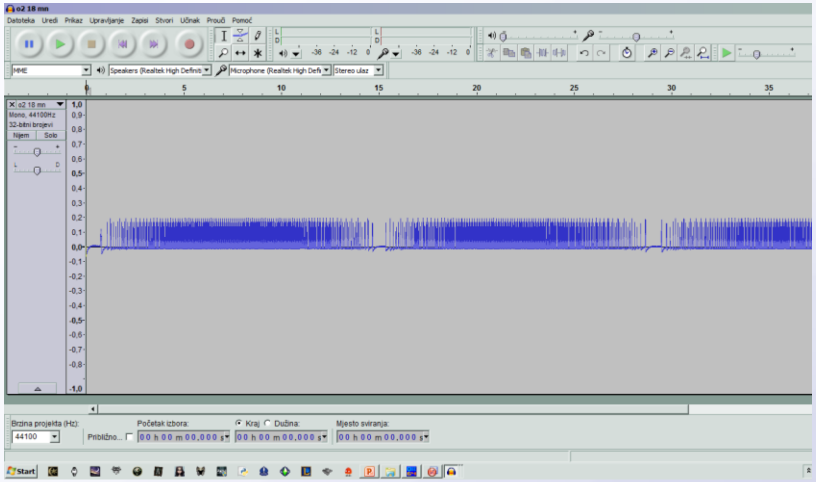


Experimental setup



Processing measurements

- coiling and uncoiling of rubber band recorded in *Audacity* (.wav)



- than reformatted into .txt files to continue further processing in our programs we made for analysis
- after processing in those programs, change of angle in time is known and angular velocity, kinetic energy, force and power output can be calculated

Mathematical model of osilaroty motion

$$\tau(\varphi) = -k\varphi$$

$$I\frac{d^2\varphi}{dt^2} = \sum \tau$$

Without friction:

$$I\frac{d^2\varphi}{dt^2} + k\varphi = 0$$

General solution of this equasion for mechanical oscillator is:

$$\varphi(t) = \varphi_0 \cos(\omega_0 t - \theta)$$

θ is starting phase, and $\omega_0 = \sqrt{\frac{k}{I}} = 2\pi f$

φ – angle [rad]

ω – angular velocity [rad/s]

f – frequency [Hz]

τ – torque [Nm]

k – force constant (for 5 threads = 5,35 $*10^{-5}$) [Nm/rad]

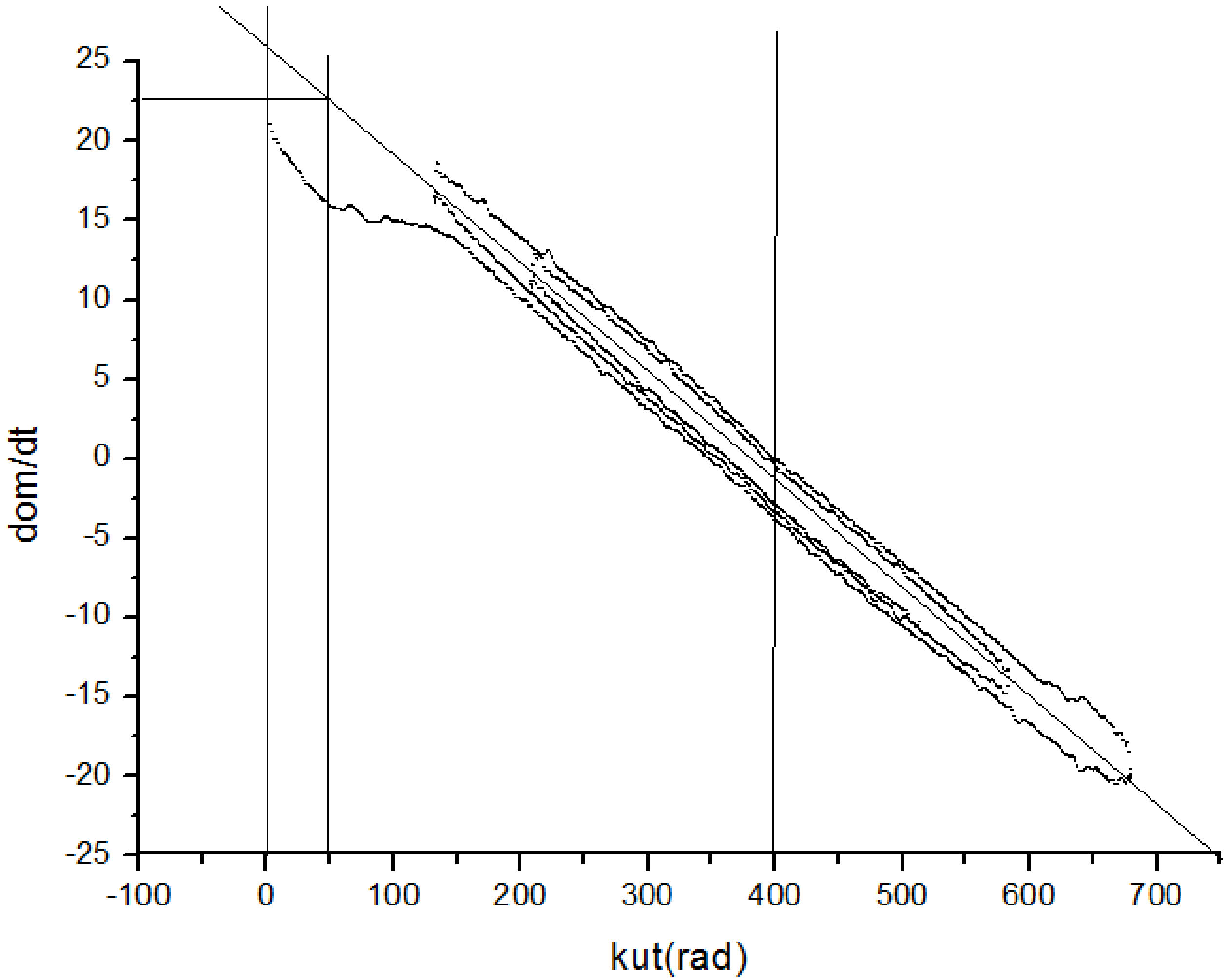
I - moment of inertia (7,8 $* 10^{-4}$)[kgm^2]

E_k - kinetic energy[J]

Ep – potential energy[J]

P - power[Watt]

Measuring ω_0



Graph: first derivation of angular velocity/time vs angle in radians

ω_0 - line coefficient of proportionality

Mathematical model of oscilaroty motion

Damped oscillations:

$$I\frac{d^2\varphi}{dt^2} + \gamma\frac{d\varphi}{dt} + k\varphi = 0$$

General solution of this equasion for mechanical oscillator is:

$$\varphi(t) = \varphi_0 e^{-\delta t}\cos(\omega_1 t - \theta)$$

θ is starting phase, $\omega_0 = \sqrt{\frac{k}{I}} = 2\pi f$,

$$\delta = \frac{\gamma}{2I},$$

$$\omega_1 = \sqrt{\omega_0^2 - \delta^2}$$

φ – angle[rad]

ω – angular velocity [rad/s]

f – frequency [Hz]

τ – torque [Nm]

k – force constant (for 5 threads = 5,35 $*10^{-5}$) [Nm/rad]

I - moment of inertia (7,8 $* 10^{-4}$)[kgm^2]

E_k - kinetic energy[J]

Ep – potential energy[J]

P - power[Watt]

δ – friction faktor

Mathematical model of osilaroty motion

Energy and power:

$$E_k = \frac{1}{2} \mathrm{I}\, \omega^2$$

$$Ep = \frac{1}{2} \mathrm{k}\, \varphi^2$$

$$P = \frac{dE}{dt}$$

φ – angle[rad]

ω – angular velocity [rad/s]

f – frequency [Hz]

τ – torque [Nm]

k – force constant (for 5 threads = 5,35 $*10^{-5}$) [Nm/rad]

I - moment of inertia (7,8 $* 10^{-4}$)[kgm^2]

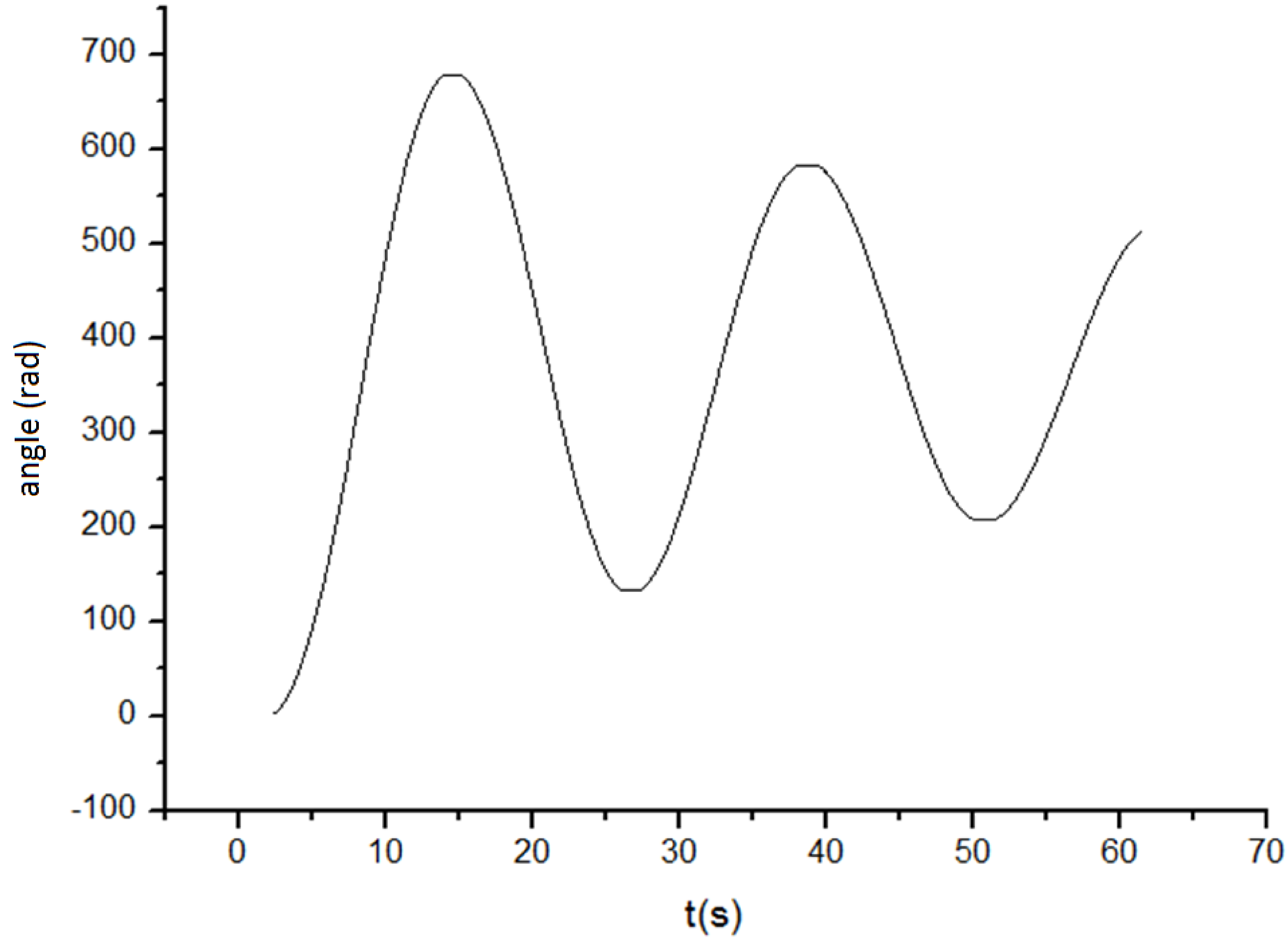
E_k - kinetic energy[J]

Ep – potential energy[J]

P - power[Watt]

Result analysis

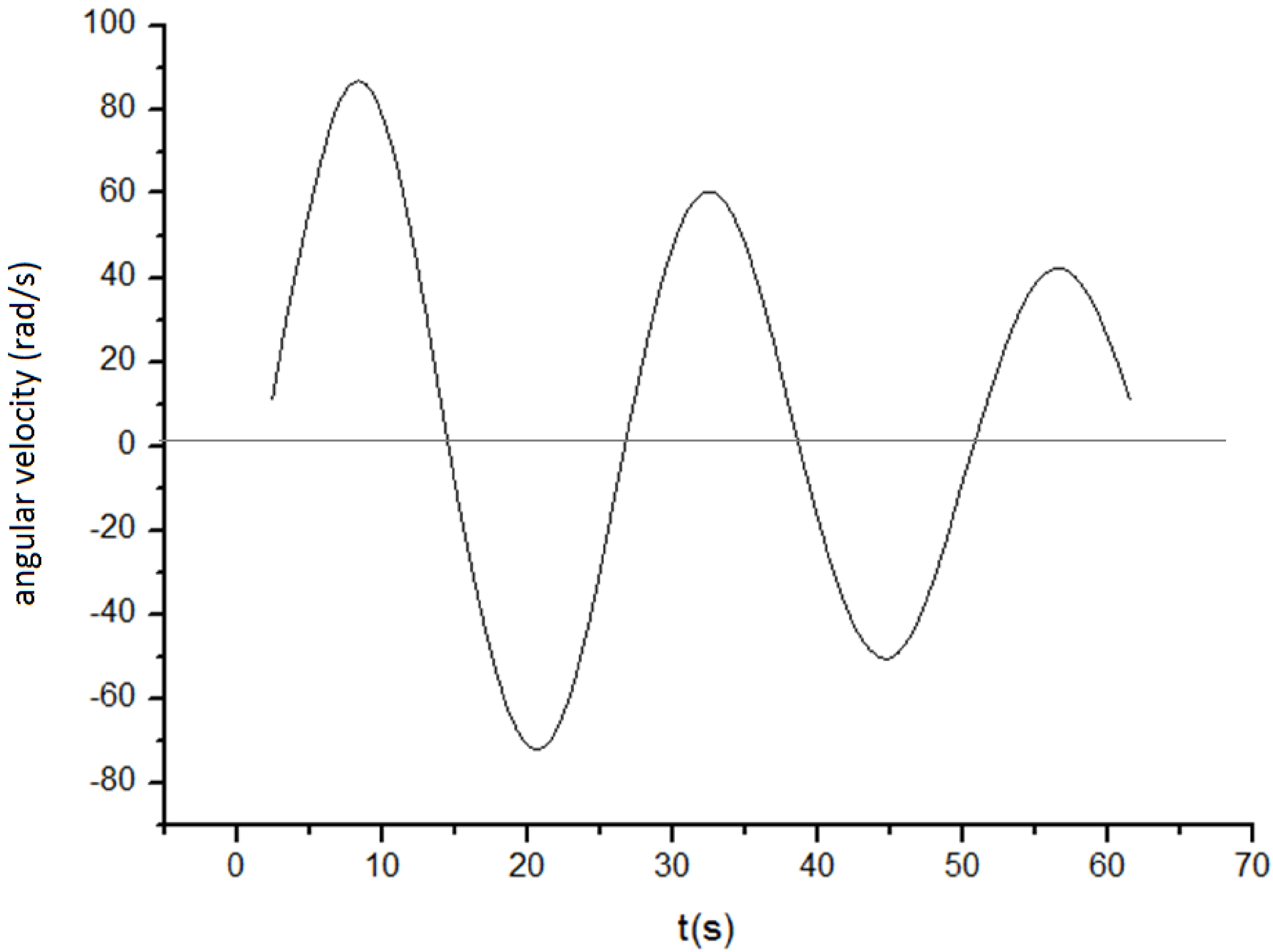
- bundle of 5 rubber threads
- bundle length: 16 cm
- nubmer of coils: 61,5



Graph 1: angle vs time

Result analysis

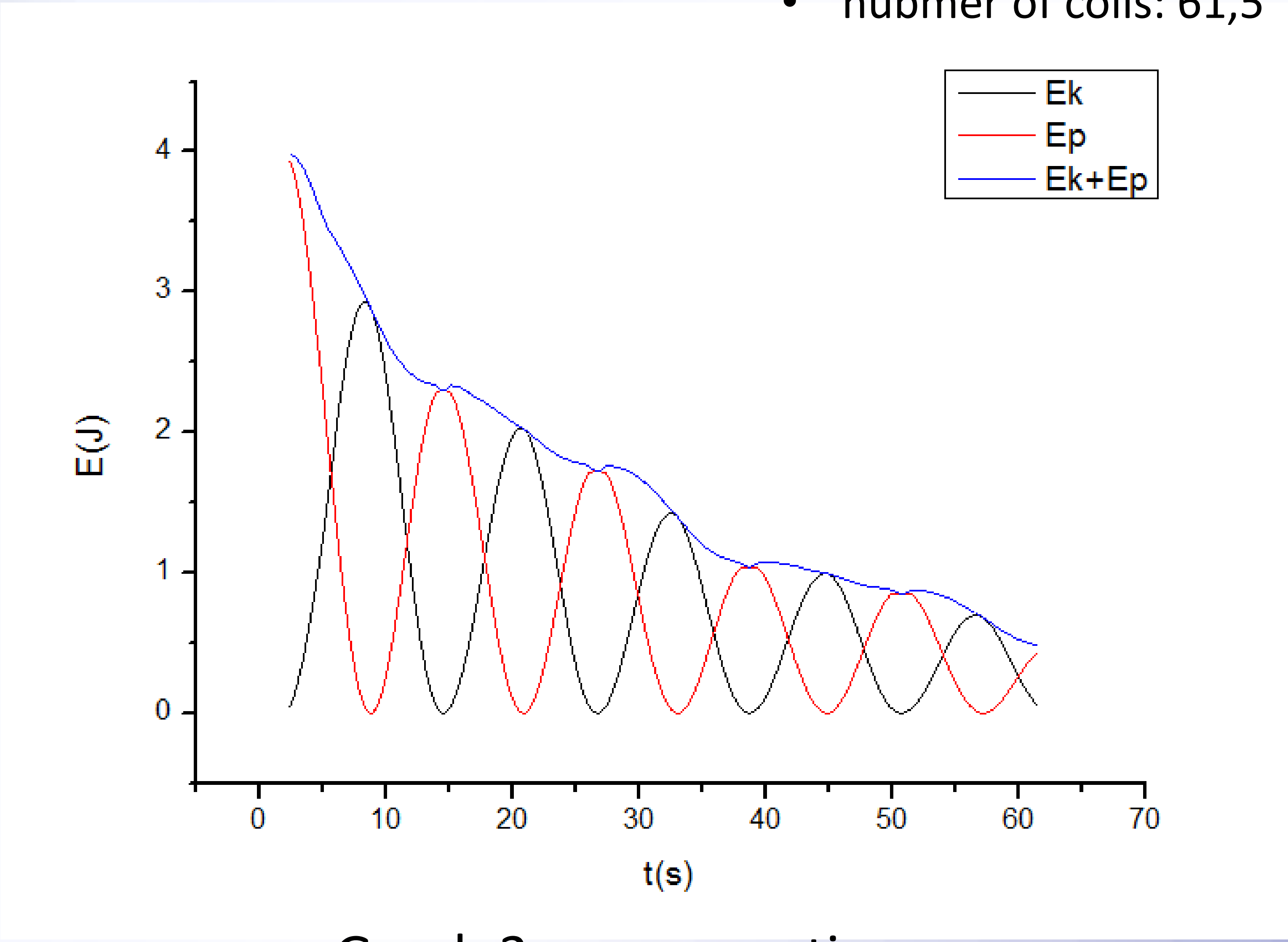
- bundle of 5 rubber threads
- bundle length: 16 cm
- nubmer of coils: 61,5



Graph 2: anglular velocity vs time

Result analysis

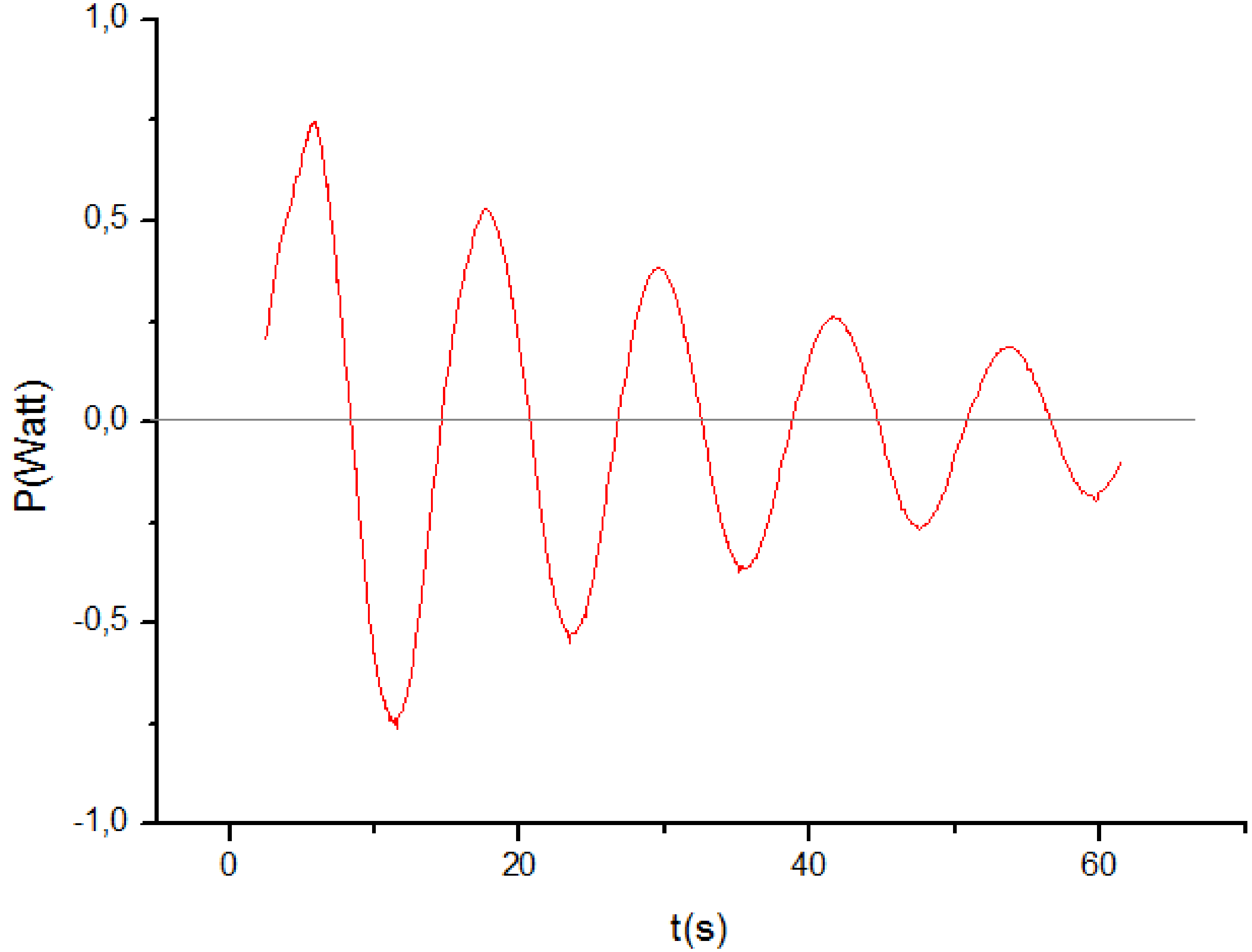
- bundle of 5 rubber threads
- bundle length: 16 cm
- nubmer of coils: 61,5



Graph 3: energy vs time

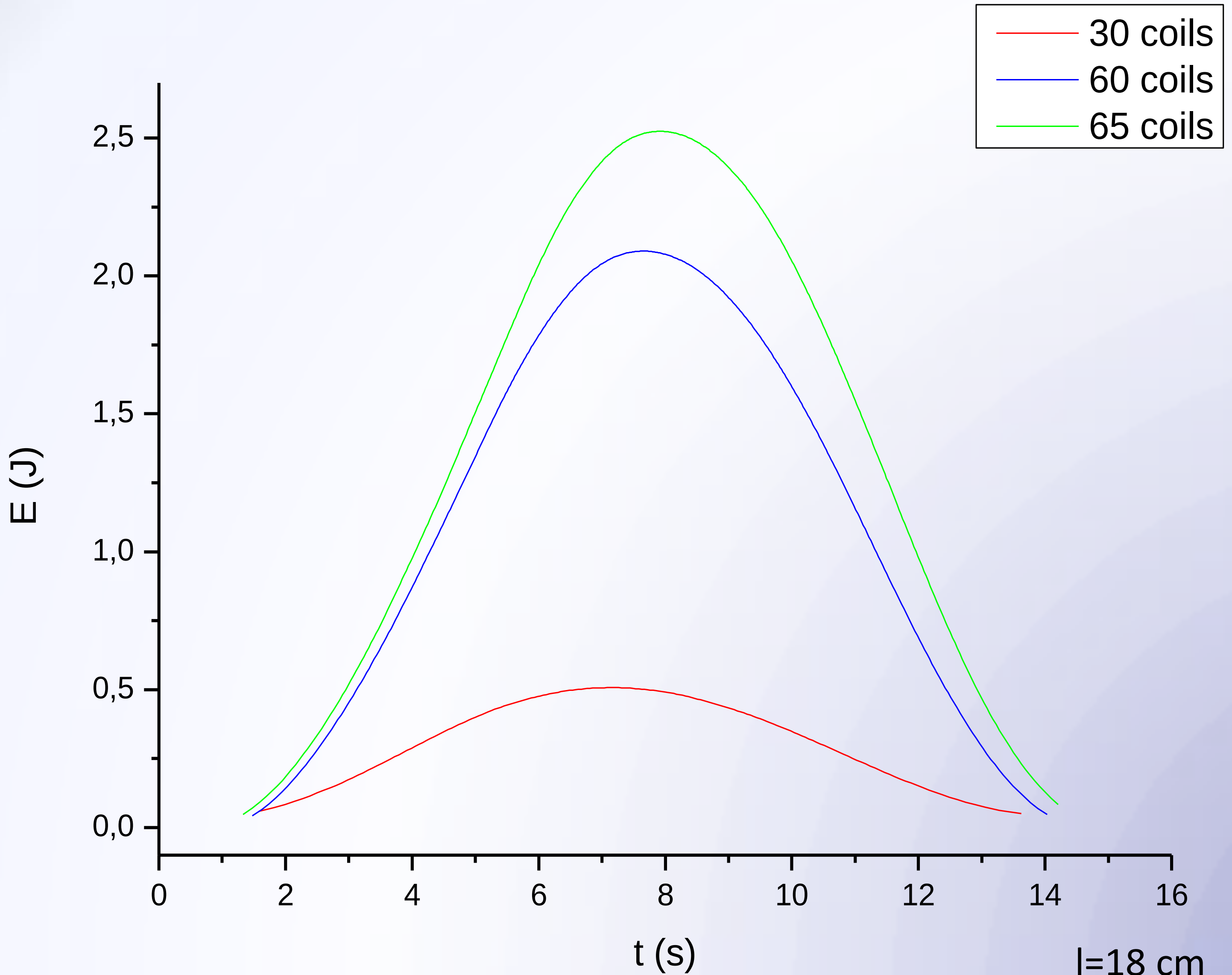
Result analysis

- bundle of 5 rubber threads
- bundle length: 16 cm
- nubmer of coils: 61,5



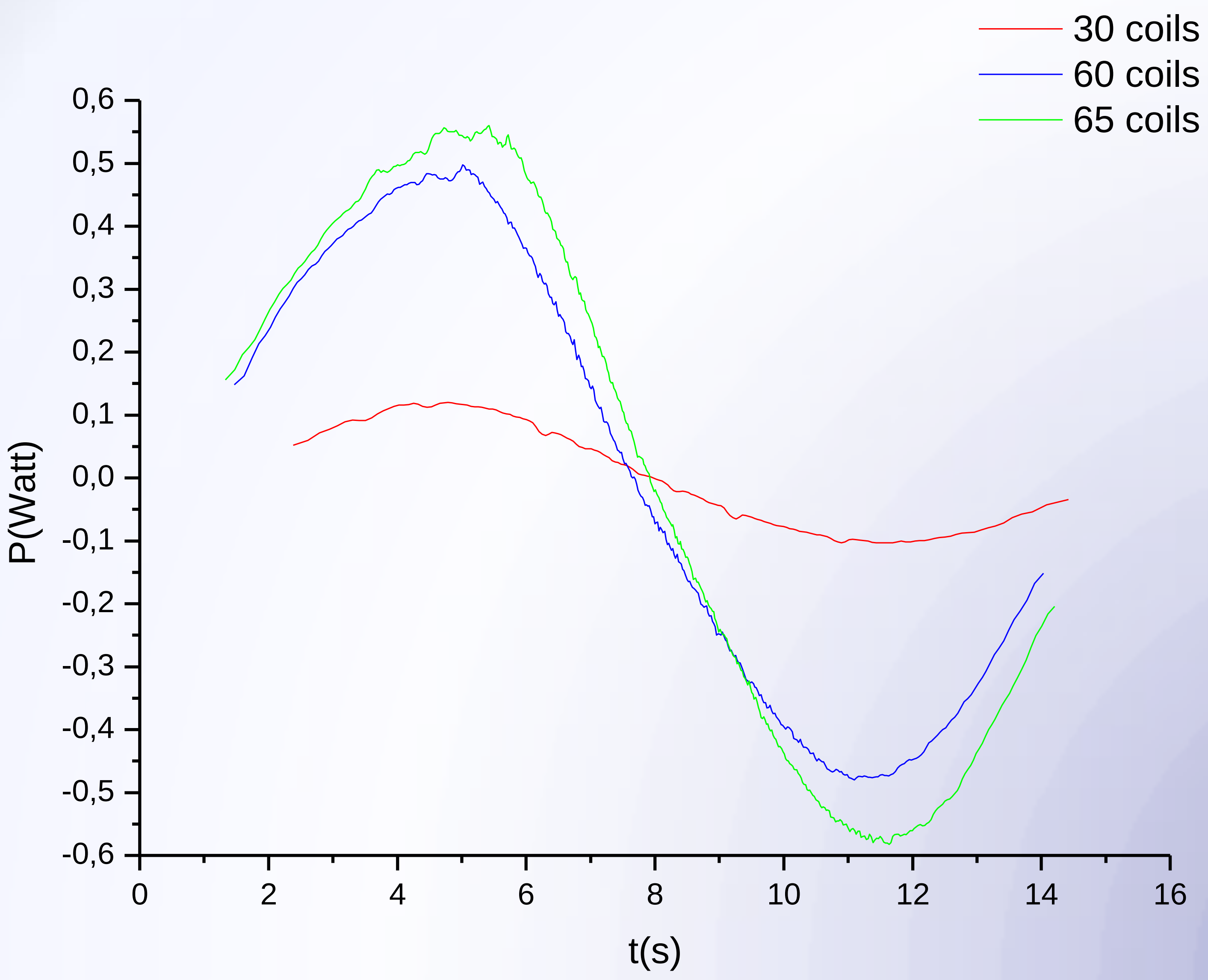
Graph 4: power vs time

Analysis of different number of coils



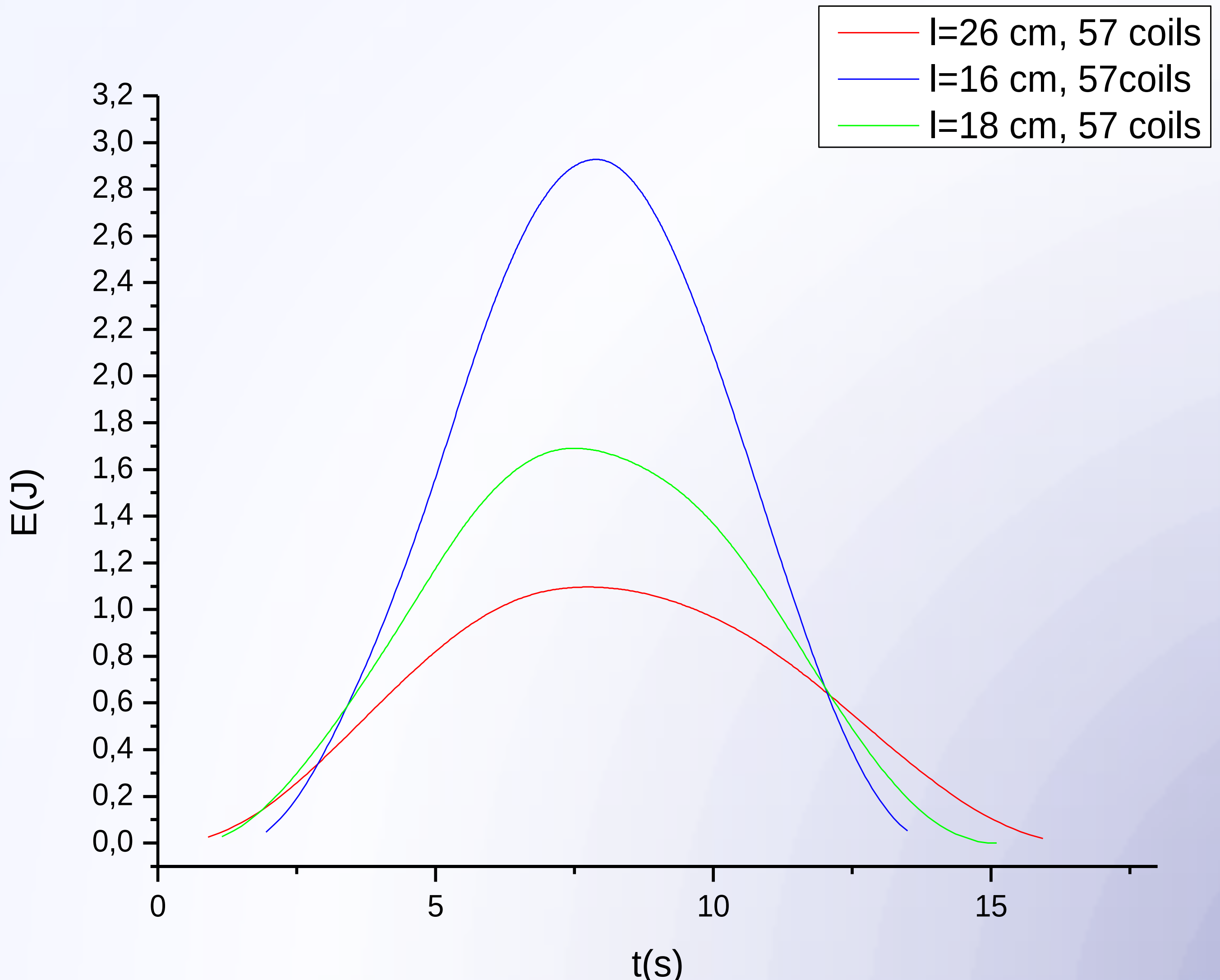
l=18 cm

Analysis of different number of coils

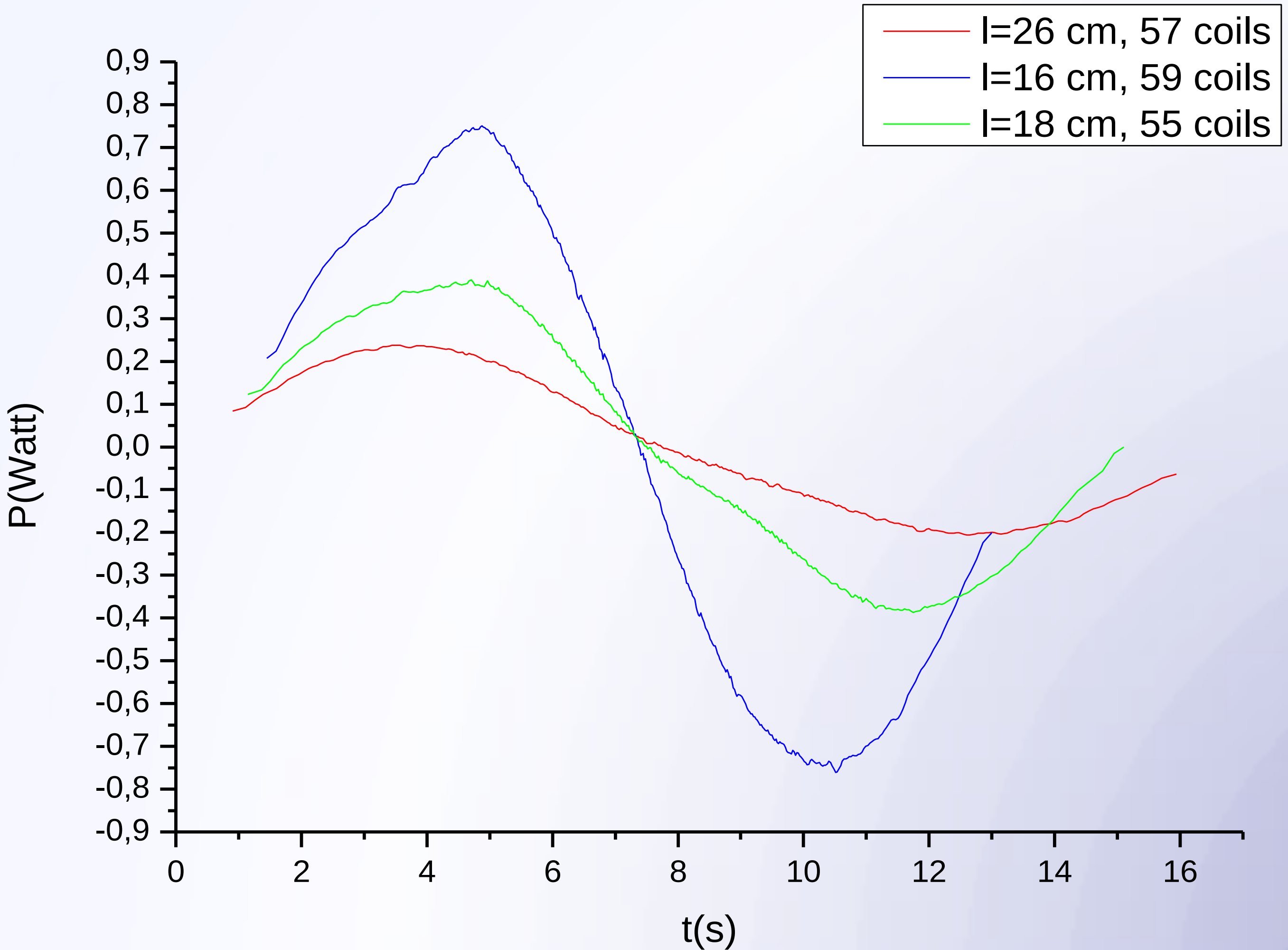


l=18 cm

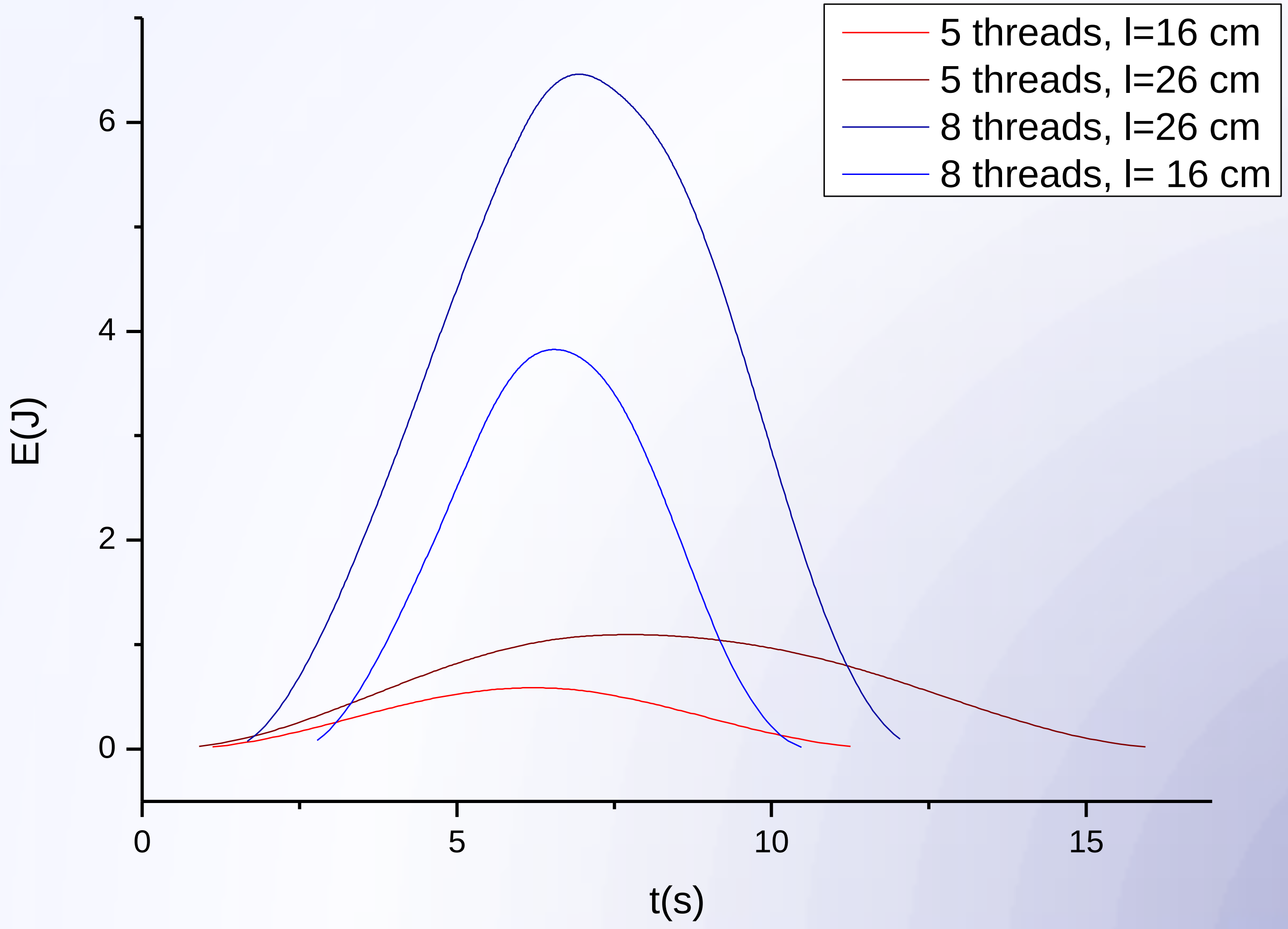
Analysis of different bundle lengths



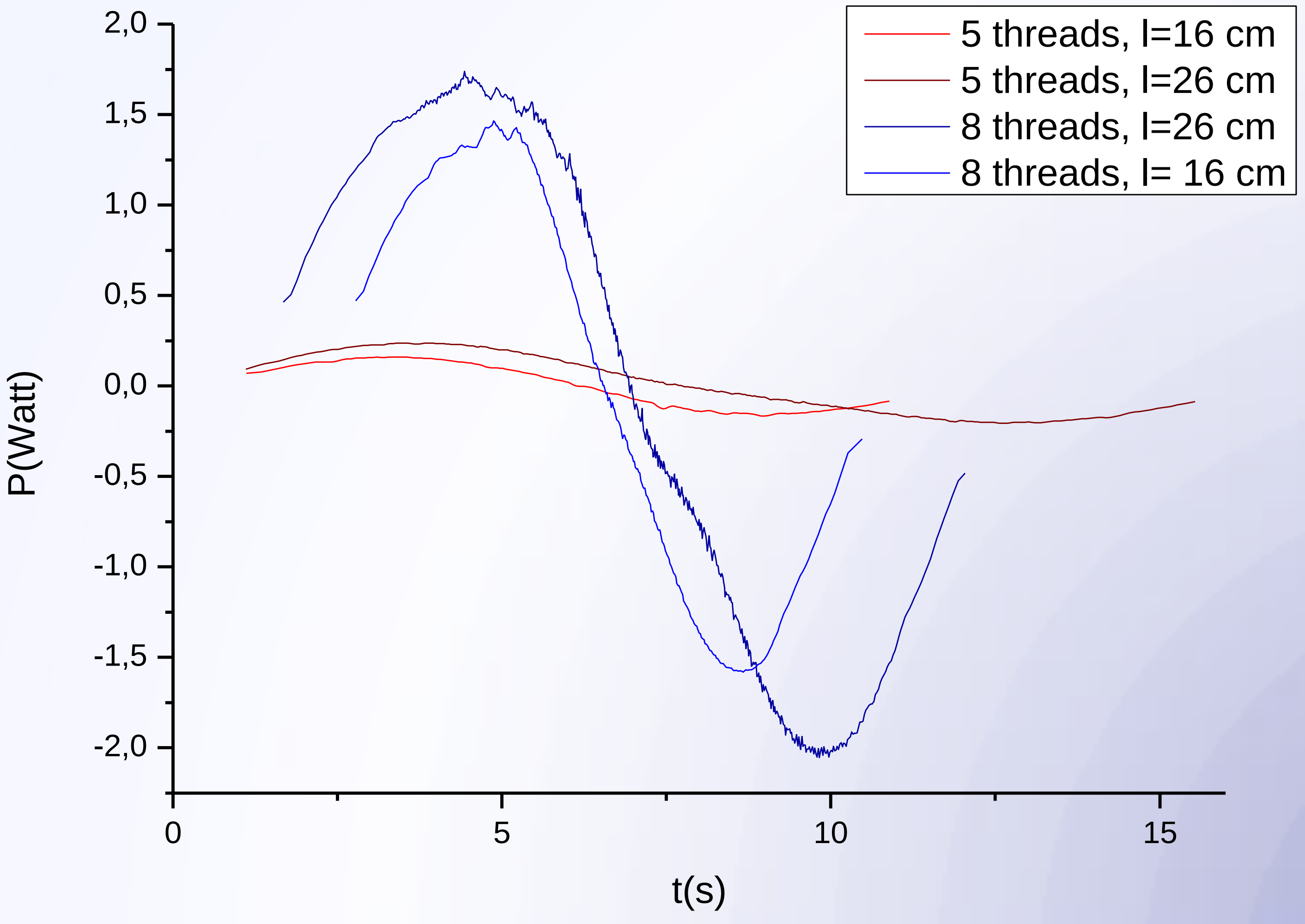
Analysis of different bundle lengths



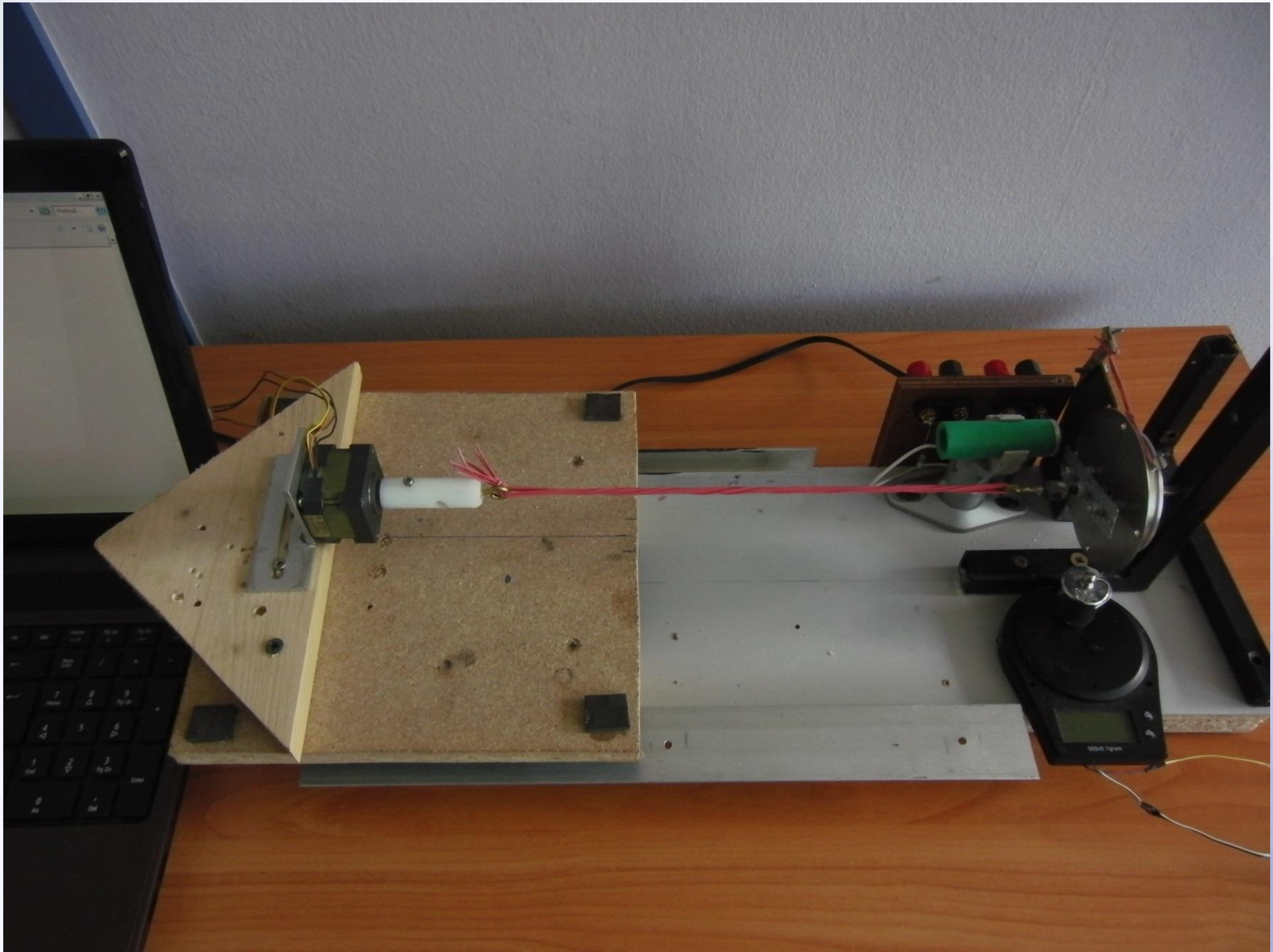
Analysis of different number of threads



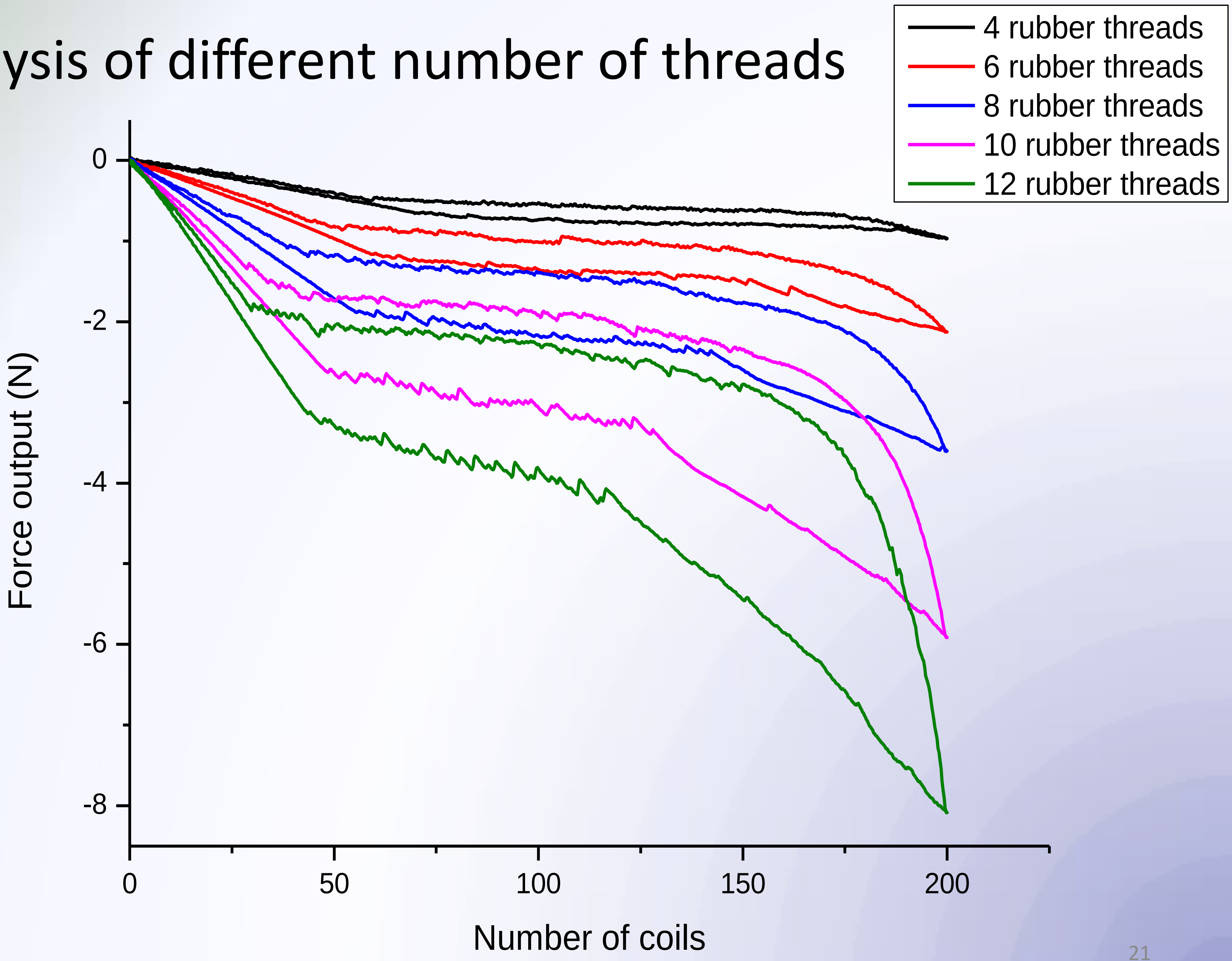
Analysis of different number of threads



New setup and measuring torque



Analysis of different number of threads

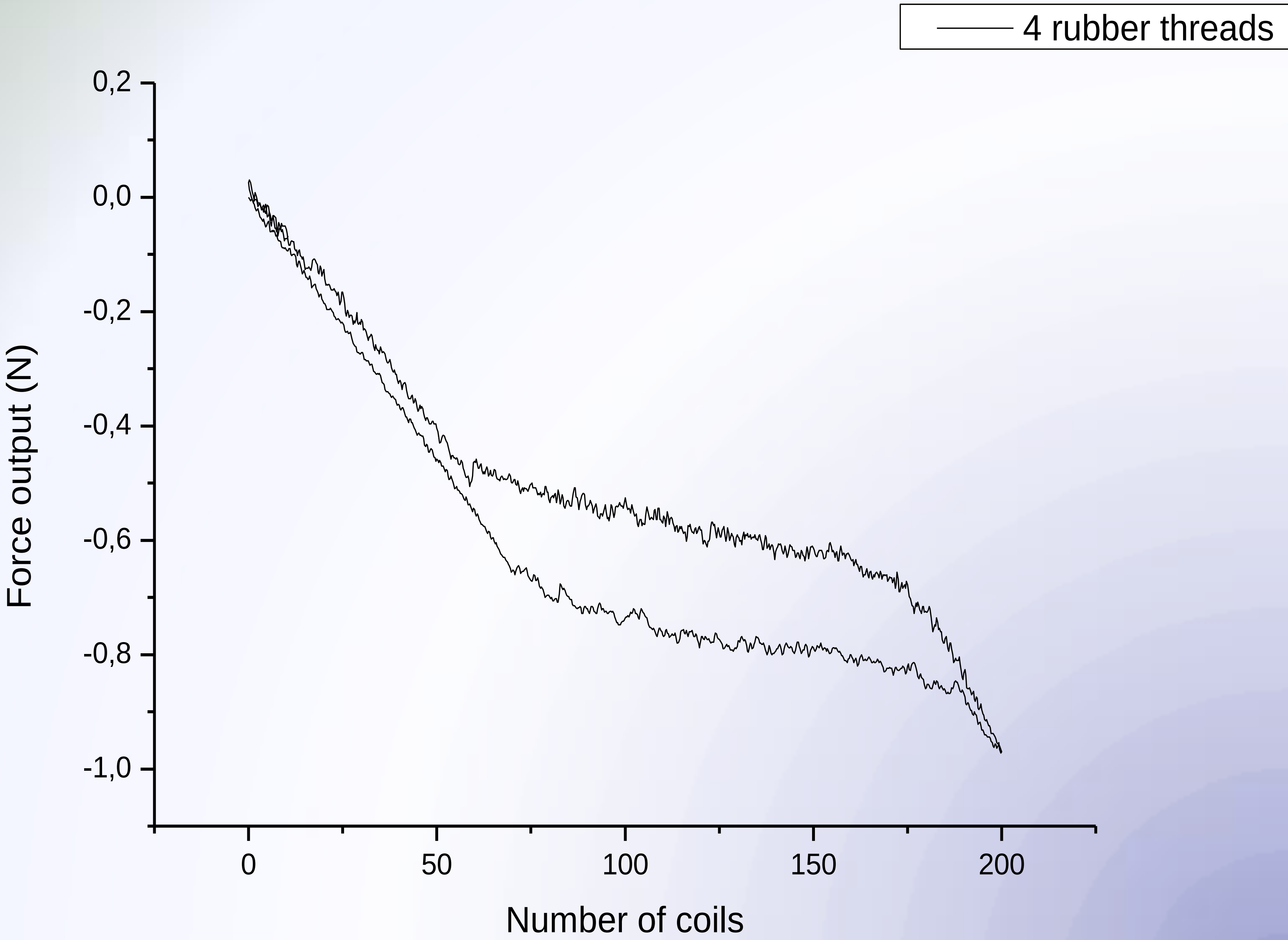


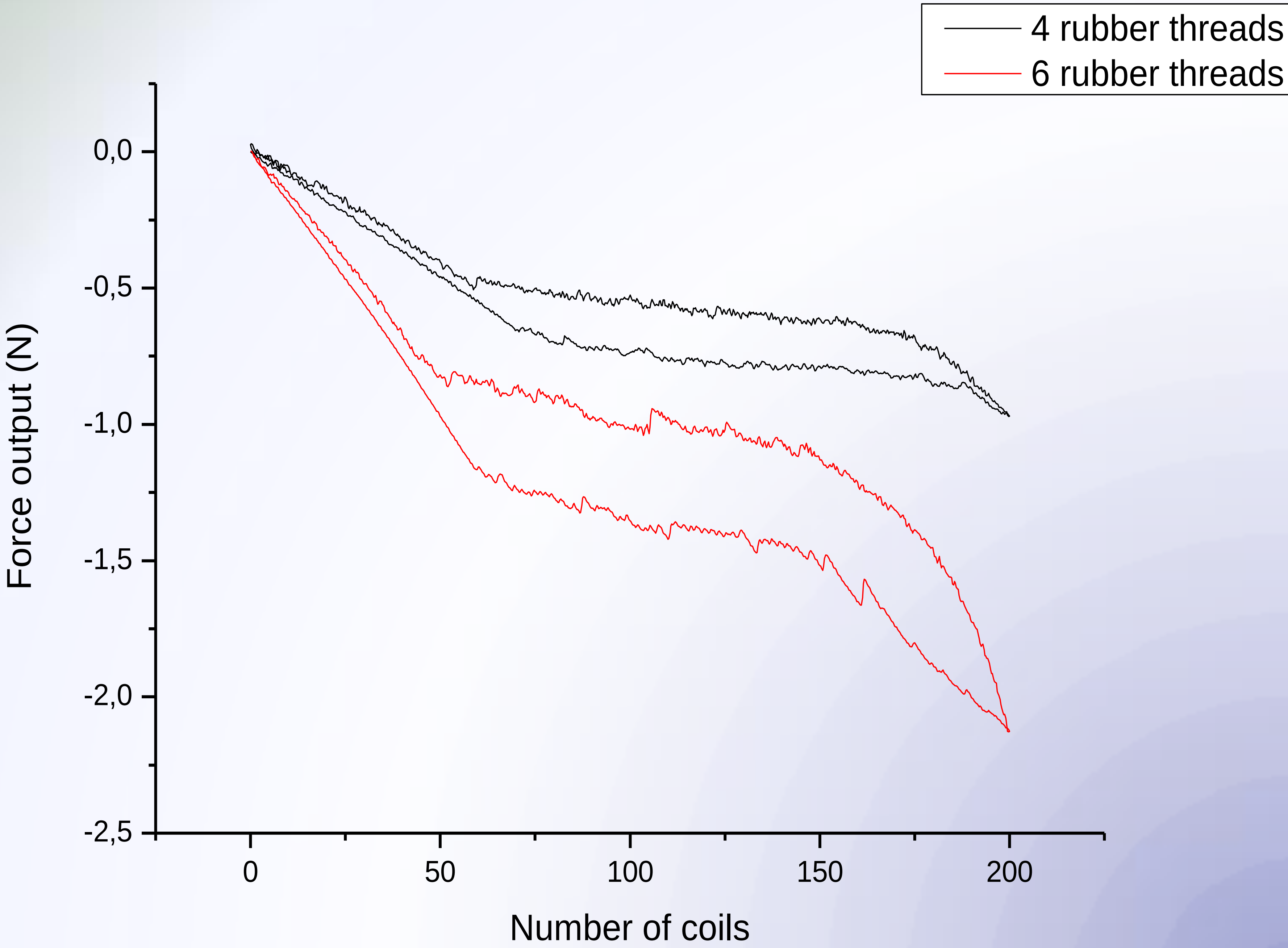
Conclusion

- energy which moves the motor comes from kinetic and potential energy stored in molecules of which rubber material is consisted
- while in motion, damping slows and finally stops motor
- hysteresis → energy loss happens because of heating and friction in oscillatory motion
- more power can be gained by:
  - using bundle made of maximum number of rubber threads
  - using longer bundle
  - increasing rubber tension or number of coils
- when tension is increased, more energy is gained in exact time interval
- hysteresis also rapidly increases as we gain more energy from motor

Literature list

- Hysteresis and Rubber Bands (madphysics.com)
- Elastomers and entropic springs (Aparna Baskaran, Syracuse University)
- Stretching a Rubber Band (schoolphysics.co.uk)
- I. M. Ward and J. Sweeney. An introduction to the mechanical properties of solid polymers (Wiley, 2005)
- A. N. Gent. Rubber Elasticity: Basic Concepts and Behavior; Science and Technology of Rubber (Elsevier, 2005)
- L. G. McAneny. Rubber band powered motor for model airplane
- G. Savarino and M. R. Fisch. A general physics laboratory investigation of the thermodynamics of a rubber band
- J. Pellicer, J. A. Manzanares, J. Zúniga, and P. Utrillas. Thermodynamics of rubber elasticity. J. Chem
- C. M. Roland. The Mullins effect in crosslinked rubber


4 rubber threads
Force output (N)
0,2
0,0
-0,2
-0,4
-0,6
-0,8
-1,0
0
50
100
150
200
Number of coils



4 rubber threads
6 rubber threads
0,0
-0,5
-1,0
-1,5
-2,0
-2,5
Force output (N)
0
50
100
150
200
Number of coils