1

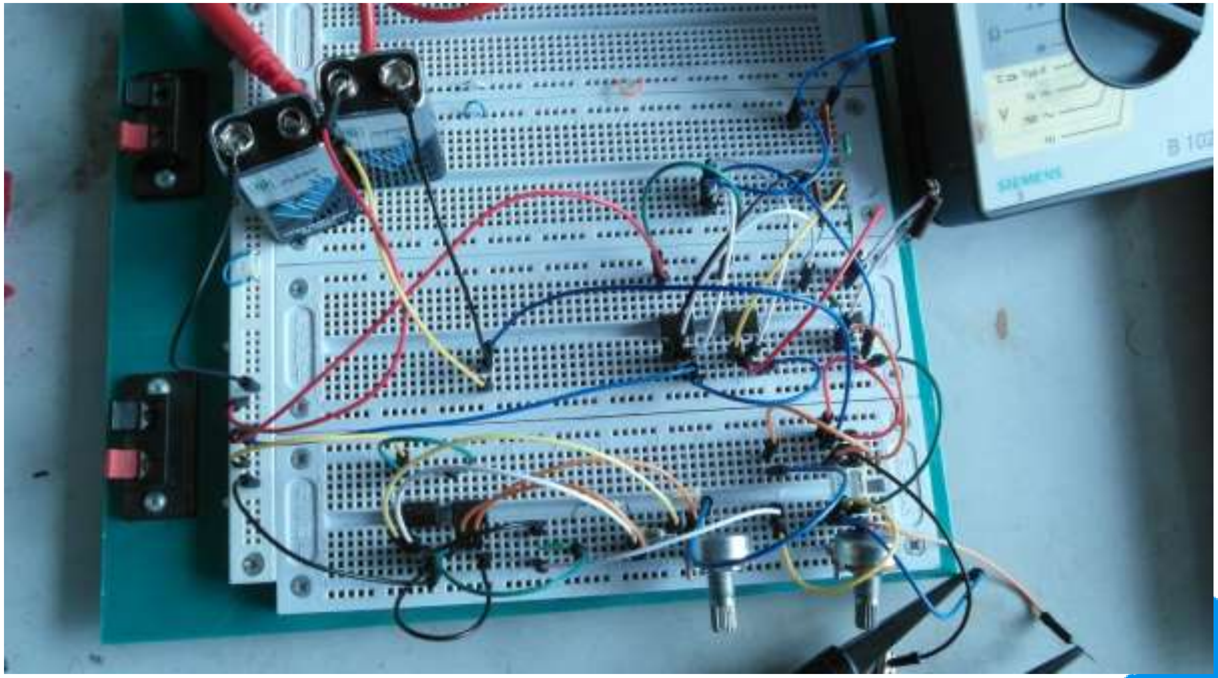
Invent Yourself

Matej Badin

Task

It is known that some electrical circuits exhibit chaotic behaviour.

Build a simple circuit with such a property, and investigate its behaviour.



Chaos?

Deterministic systems

"Chaotic"

Always the same output from the same initial conditions

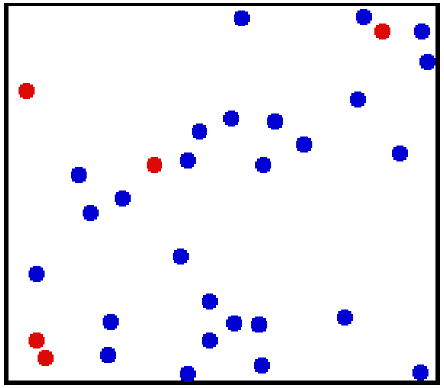
[http://en.wikipedia.org/wiki/File:Double-compound-pendulum.gif]

(Double pendulum)

Stochastic systems

"Random"

Future state cannot be determined from current (only probability)



[http://en.wikipedia.org/wiki/File:Translational_motion.gif]

(Thermal motion)

BUILDING A CIRCUIT

Requirements For Such a Circuit

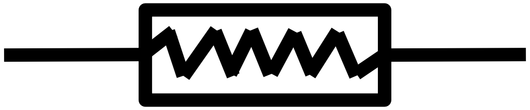
System of linear differential equations doesn't lead to chaos

Nonlinear element

System only with R,L,C will lead to damped oscillations

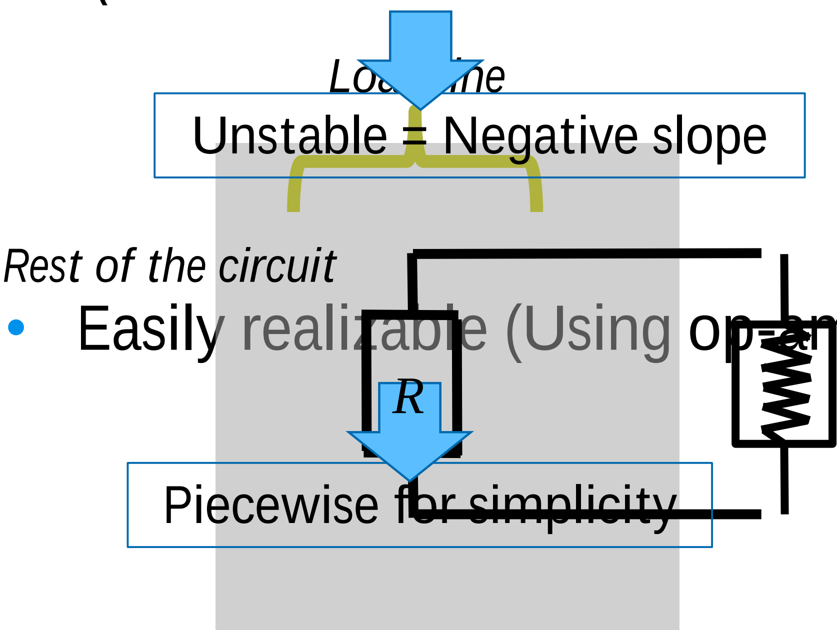
Active element

For simplicity – Nonlinear active resistor



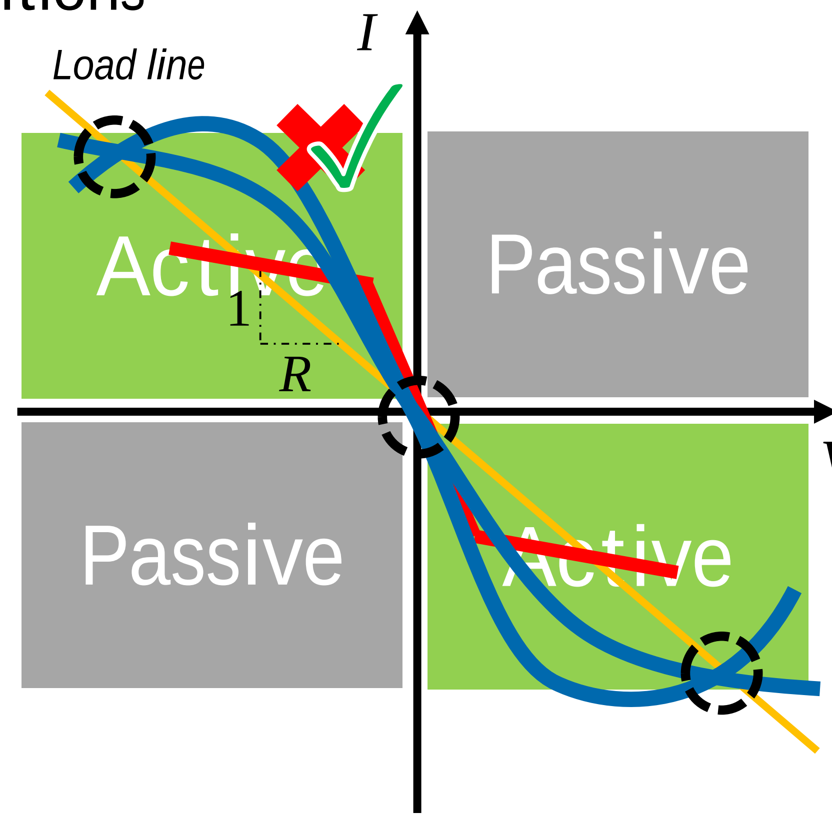
Requirements for Nonlinear Resistor

- Active resistor
- 2 or more Unstable equilibria positions
  (Intersection with circuit load line)
- Easily realizable (Using op-amps)

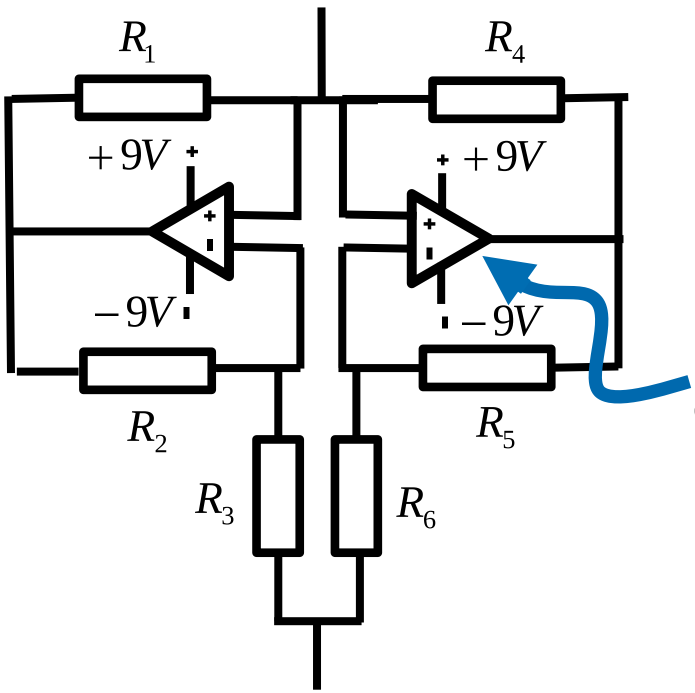


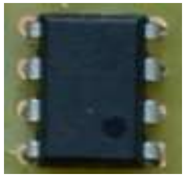
Only Resistance plays role
(In DC Equilibrium)

Let's look at I-V Characteristic:



Nonlinear Active Resistor - Chua's Diode





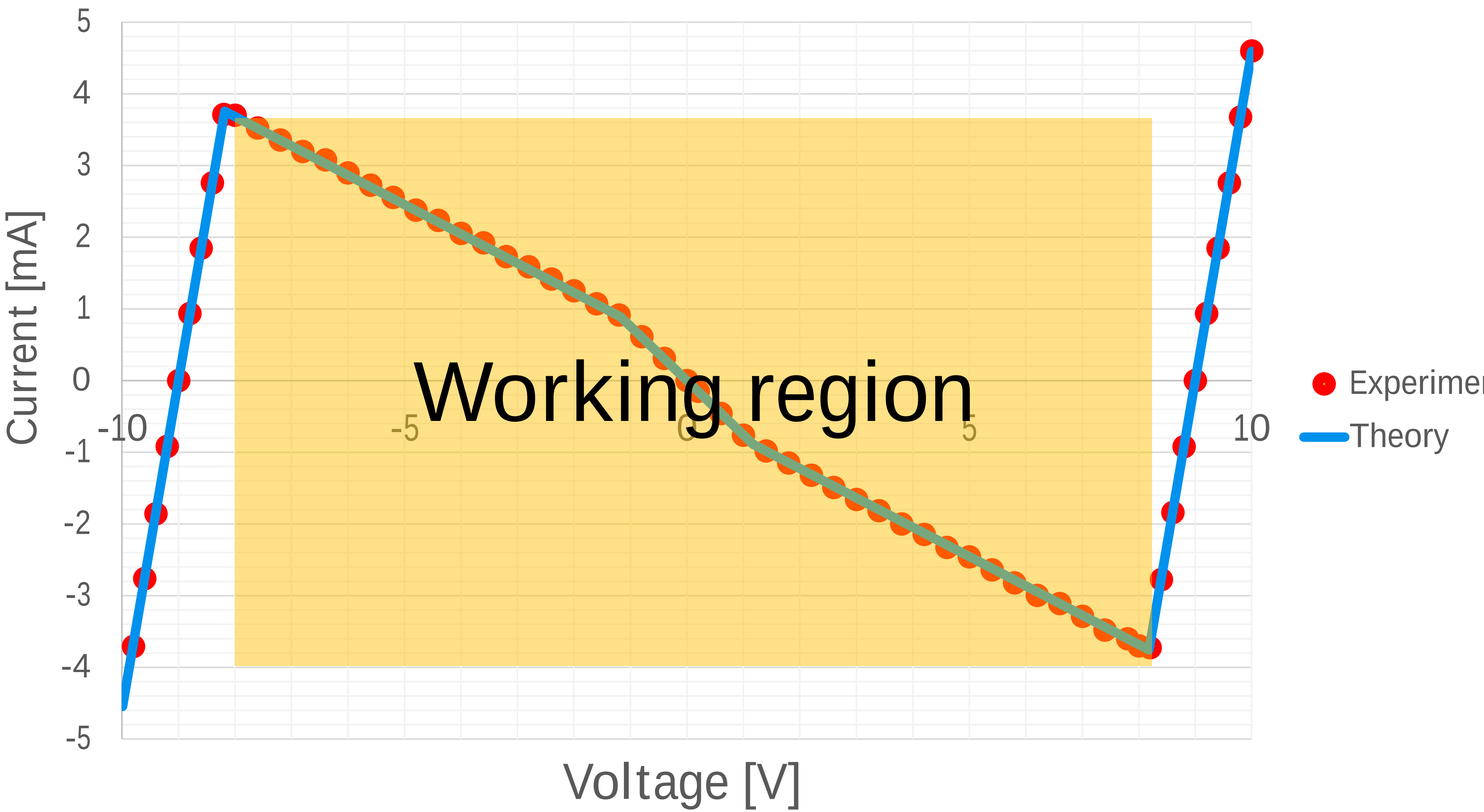
TL082CP
Operational
amplifier

Using parameters [V. Siderskiy – Chuacircuits.com]:

$$R_1 = 220\ \Omega$$
$$R_2 = 220\ \Omega$$
$$R_3 = 2.2\ k\Omega$$
$$R_4 = 22\ k\Omega$$
$$R_5 = 22\ k\Omega$$
$$R_6 = 3.3\ k\Omega$$
$$U_{\pm} = \pm 9\ V$$

Calculation of
I-V
Characteristic

I-V Characteristic of Chua's Diode



Requirements For Such a Circuit

System of linear differential equations doesn't lead to chaos

Nonlinear element

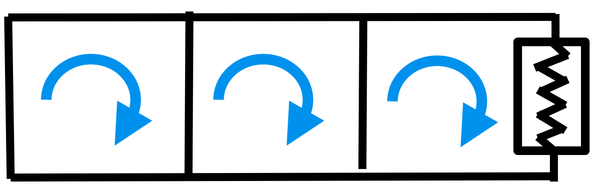
System only with R,L,C will lead to damped oscillations

Active element

Poincaré–Bendixson theorem[1]

Continuous system with less than 3 independent state variables cannot be chaotic

Minimum three loops



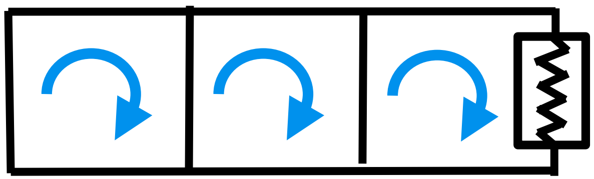
Minimum three energy storage (L,C) elements

[1] Explained in textbook - Teschl, Gerald (2012). Ordinary Differential Equations and Dynamical Systems

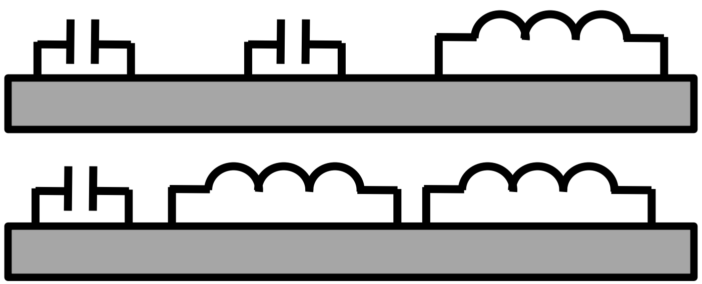
Requirements For Such a Circuit

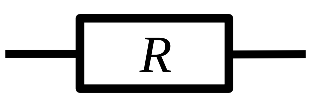
Known mathematical theorem[1]

System with less than 3 independent state variables cannot be chaotic



Only 2 Options





Minimum three loops

Minimum three energy storage (L,C) elements

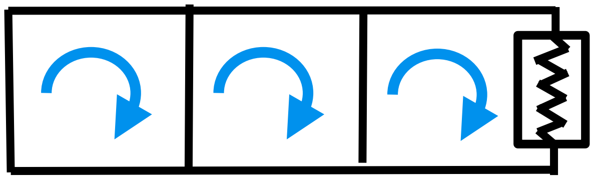
To oscillate it need both L & C

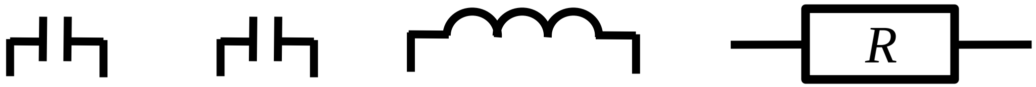
Don't forget we need another resistance (Unstable equilibrium)

[1] [Guckenheimer, Holmes, P.: Nonlinear oscillations,dynamical systems, and bifurcations of vector fields. Springer Verlag 1983

Topological Problem

How to put 5 elements into 3 loops ?

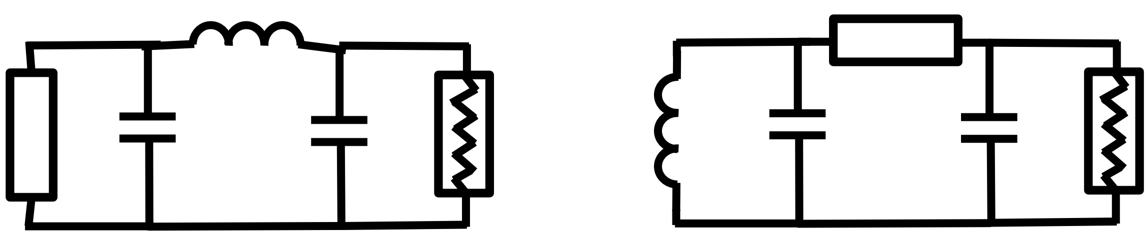




We don't want:

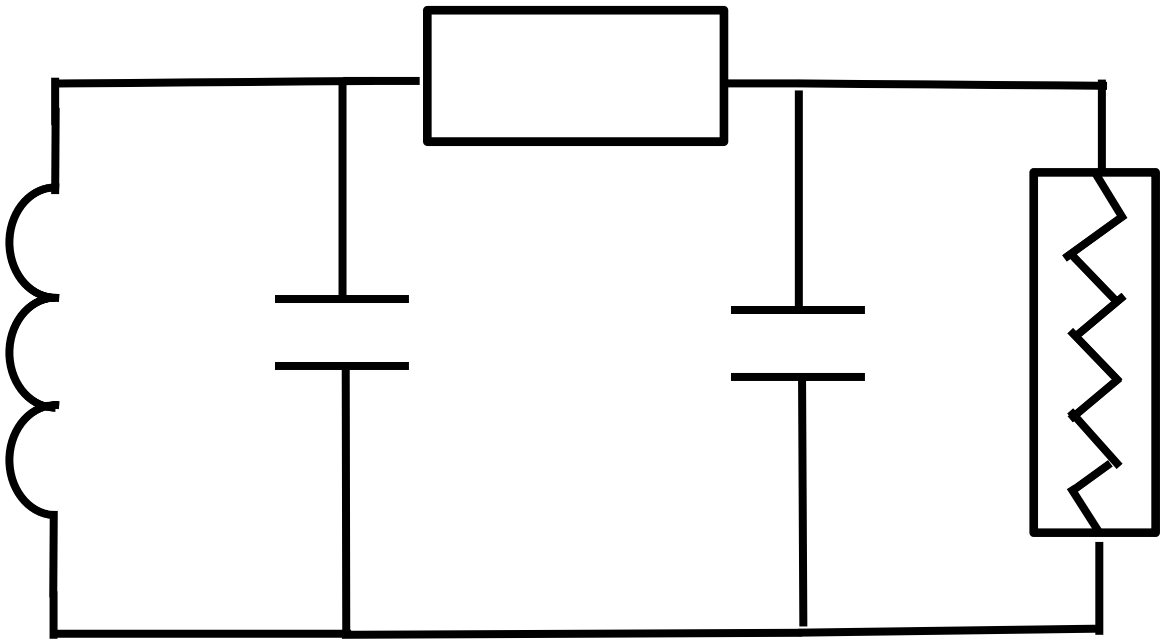
Open Circuit *(in DC Equilibrium)*	Short Circuit *(in DC Equilibrium)*	Parallel Capacitors or Resistors

Only two options:

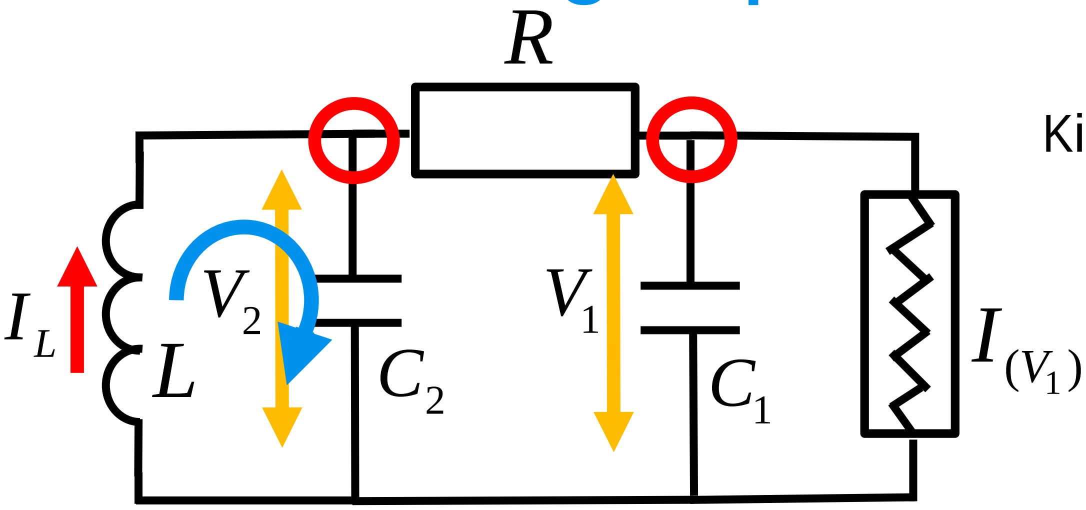


The Simplest Chaotic Circuit – Chua's

Chua, L. O. (1992) The Genesis of **Chua's** Circuit.



Governing Equations



Kirchhoff laws for junctions:

$$\dot{V}_1 C_1 + I_{(V_1)} = \frac{(V_2 - V_1)}{R}$$

$$I_L = \frac{(V_2 - V_1)}{R} + \dot{V}_2 C_2$$

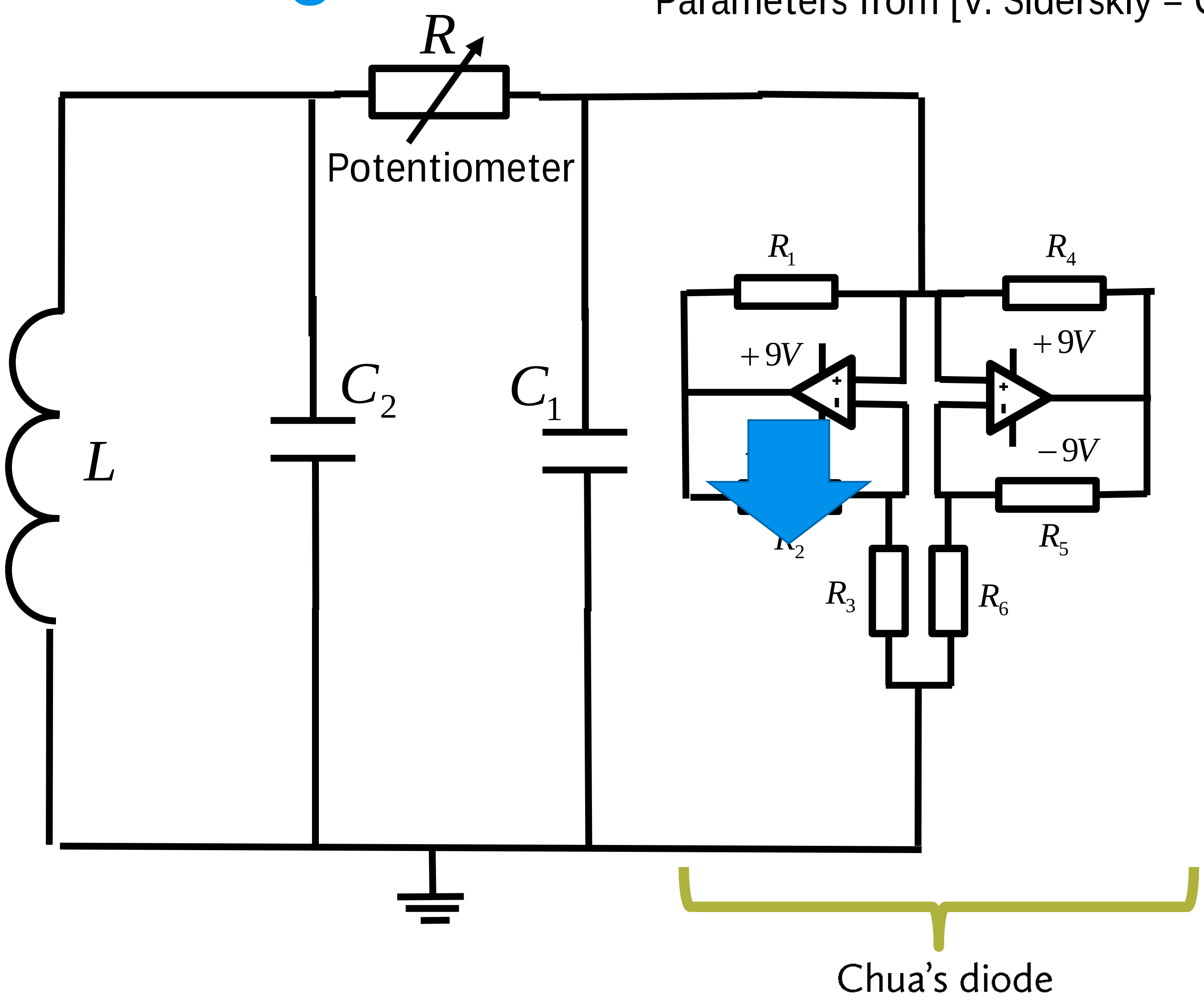
Kirchhoff law for loop :

$$L\dot{I}_L = -V_2$$

System of three differential equations

C_1, C_2	Capacitances
L	Inductance
R	Linear resistance
$I_{(V)}$	I-V Characteristic

Resulting Circuit



Parameters from [V. Siderskiy – Chuacircuits.com]:

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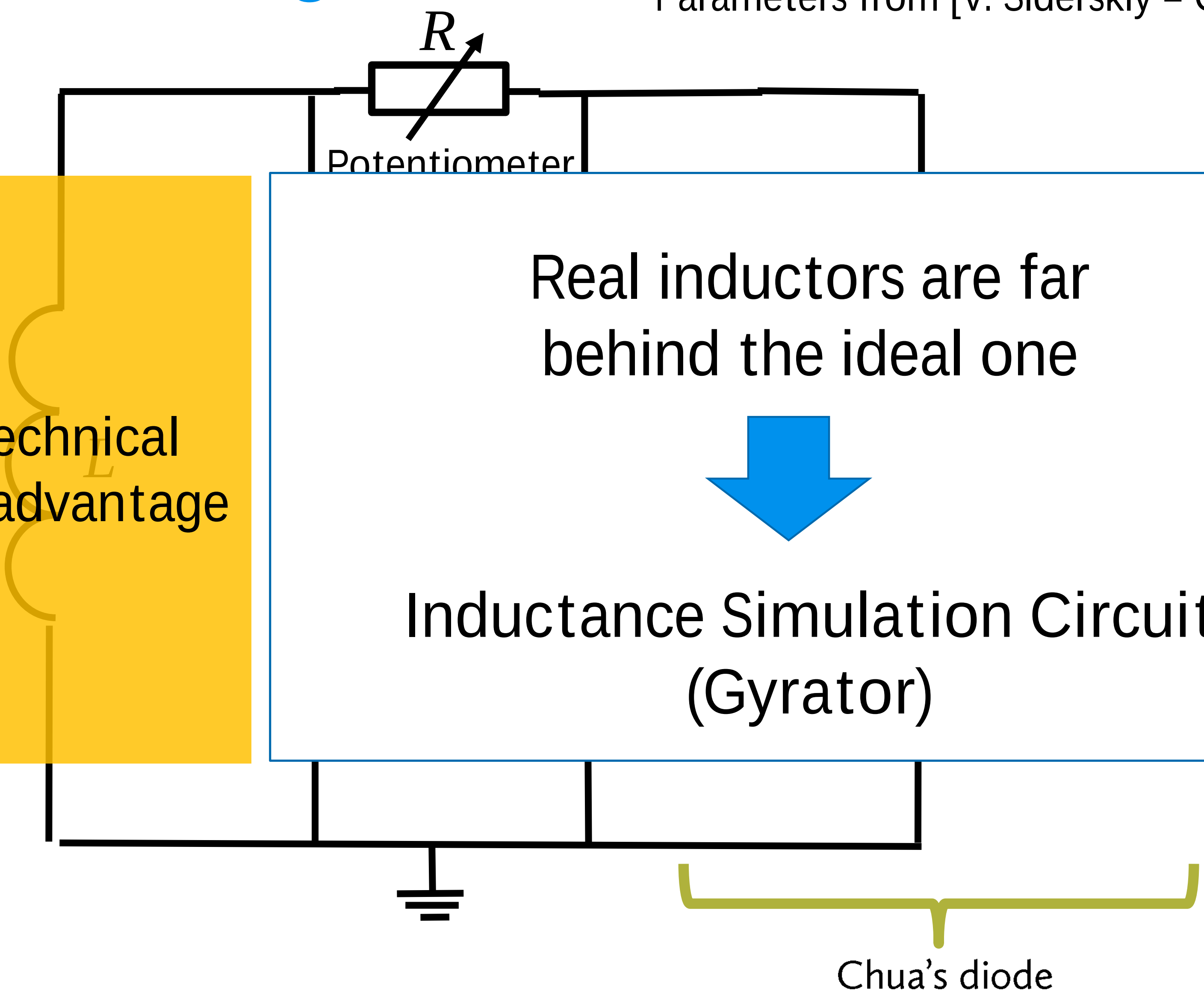
$R_2 = 220\ \Omega$
$R_3 = 2.2\ k\Omega$
$R_4 = 22\ k\Omega$
$R_5 = 22\ k\Omega$
$R_6 = 3.3\ k\Omega$

$U_\pm = \pm 9\ V$

$R_{g1} = 100\ \Omega$
$R_{g2} = 1\ k\Omega$
$R_{g3} = 1\ k\Omega$
$R_{g4} \leq 5\ k\Omega$
$C_g = 100\ nF$

$R \leq 2.5\ k\Omega$
$C_1 = 15\ nF$
$C_2 = 100\ nF$

Resulting Circuit



Technical Disadvantage

Real inductors are far behind the ideal one

Inductance Simulation Circuit (Gyrator)

Parameters from [V. Siderskiy – Chuacircuits.com]:

$R_1 = 220\ \Omega$

$R_2 = 220\ \Omega$

$R_3 = 2.2\ k\Omega$

$= 22\ k\Omega$

$= 22\ k\Omega$

$= 3.3\ k\Omega$

$= \pm 9\ V$

$= 100\ \Omega$

$= 1\ k\Omega$

$= 1\ k\Omega$

$\leq 5\ k\Omega$

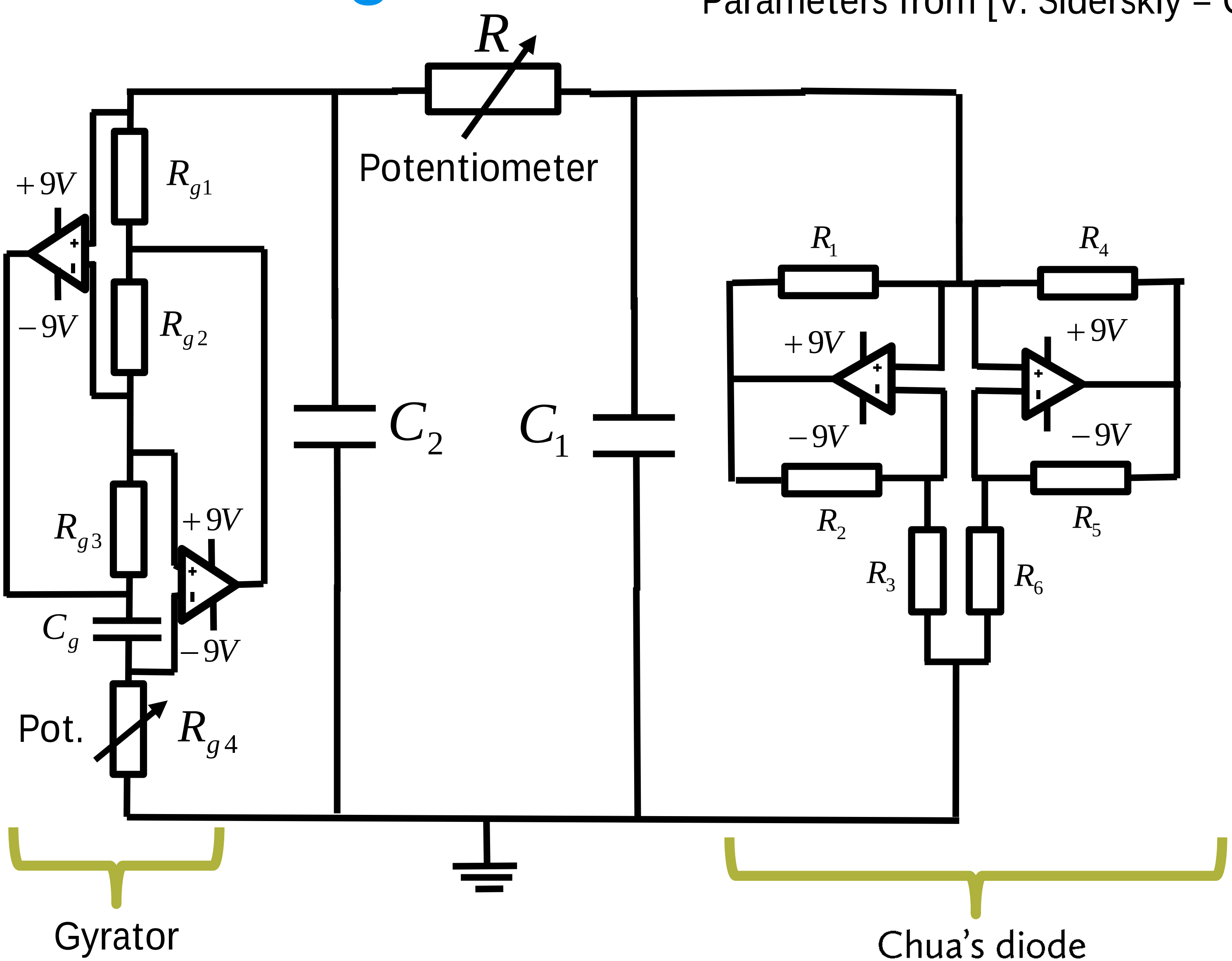
$C_g = 100\ nF$

$R \leq 2.5\ k\Omega$

$C_1 = 15\ nF$

$C_2 = 100\ nF$

Resulting Circuit



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$R_3 = 2.2\ k\Omega$
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$U_\pm = \pm 9\ V$

$R_{g1} = 100\ \Omega$
$R_{g2} = 1\ k\Omega$
$R_{g3} = 1\ k\Omega$
$R_{g4} \leq 5\ k\Omega$
$C_g = 100\ nF$

$R \leq 2.5\ k\Omega$
$C_1 = 15\ nF$
$C_2 = 100\ nF$

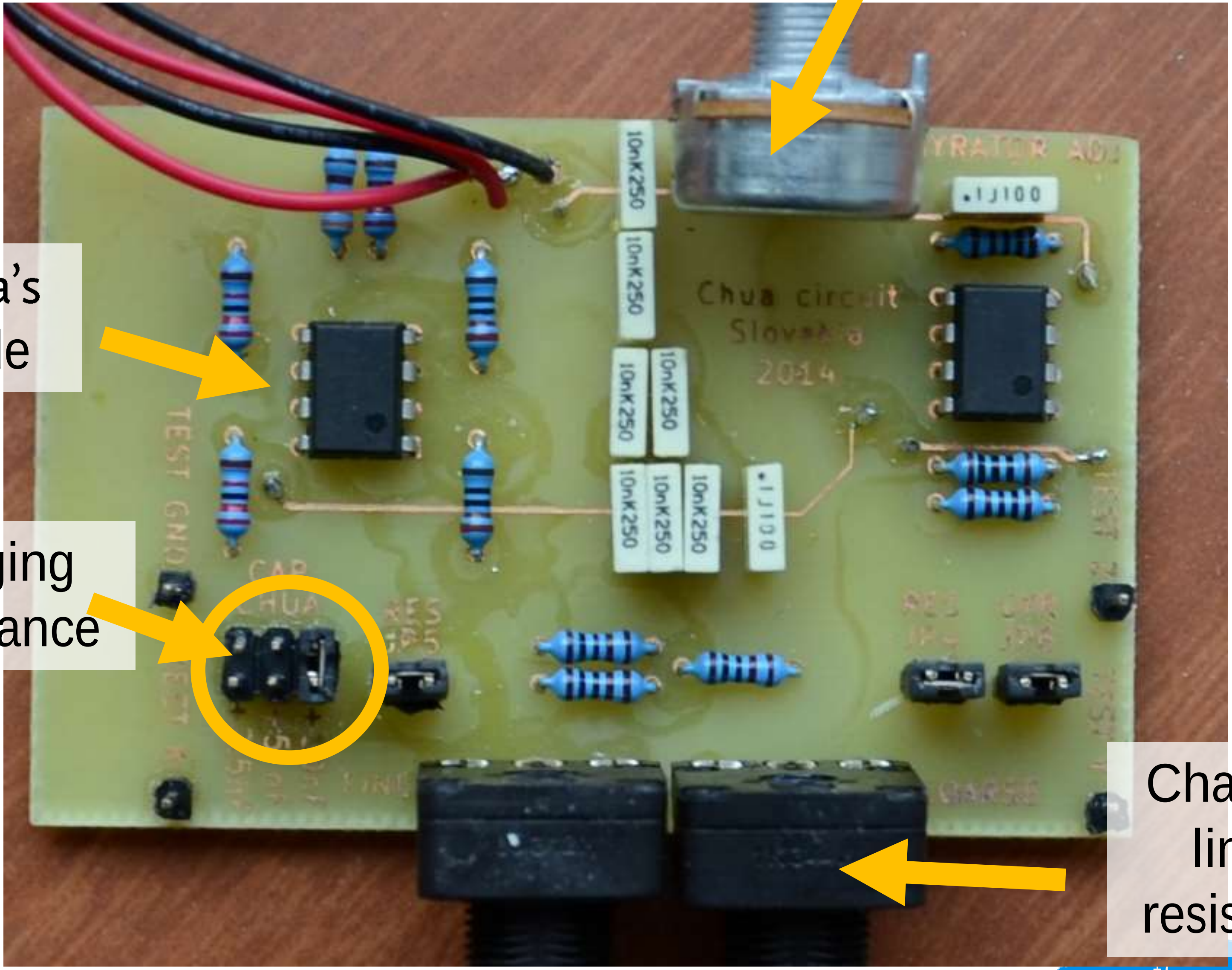
Resulting Circuit

Changing inductance

Chua's diode

Changing capacitance

Changing linear resistance



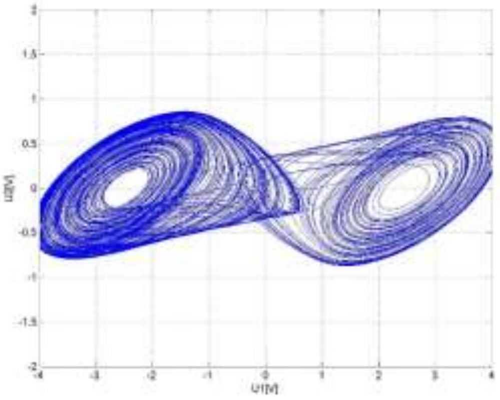
CHAOTIC BEHAVIOUR

How To Study Chaos?

Phase Space

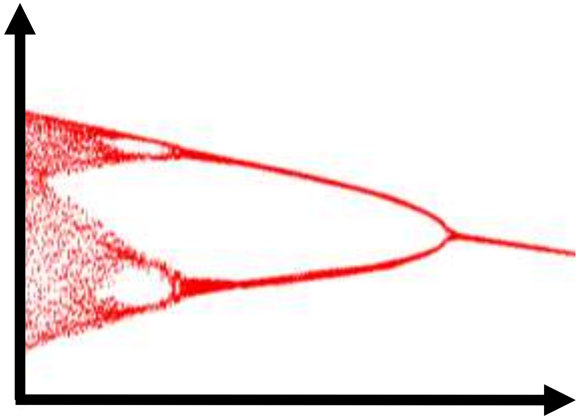
Look at possible states of a system for given parameters

Analog oscilloscope in XY mode

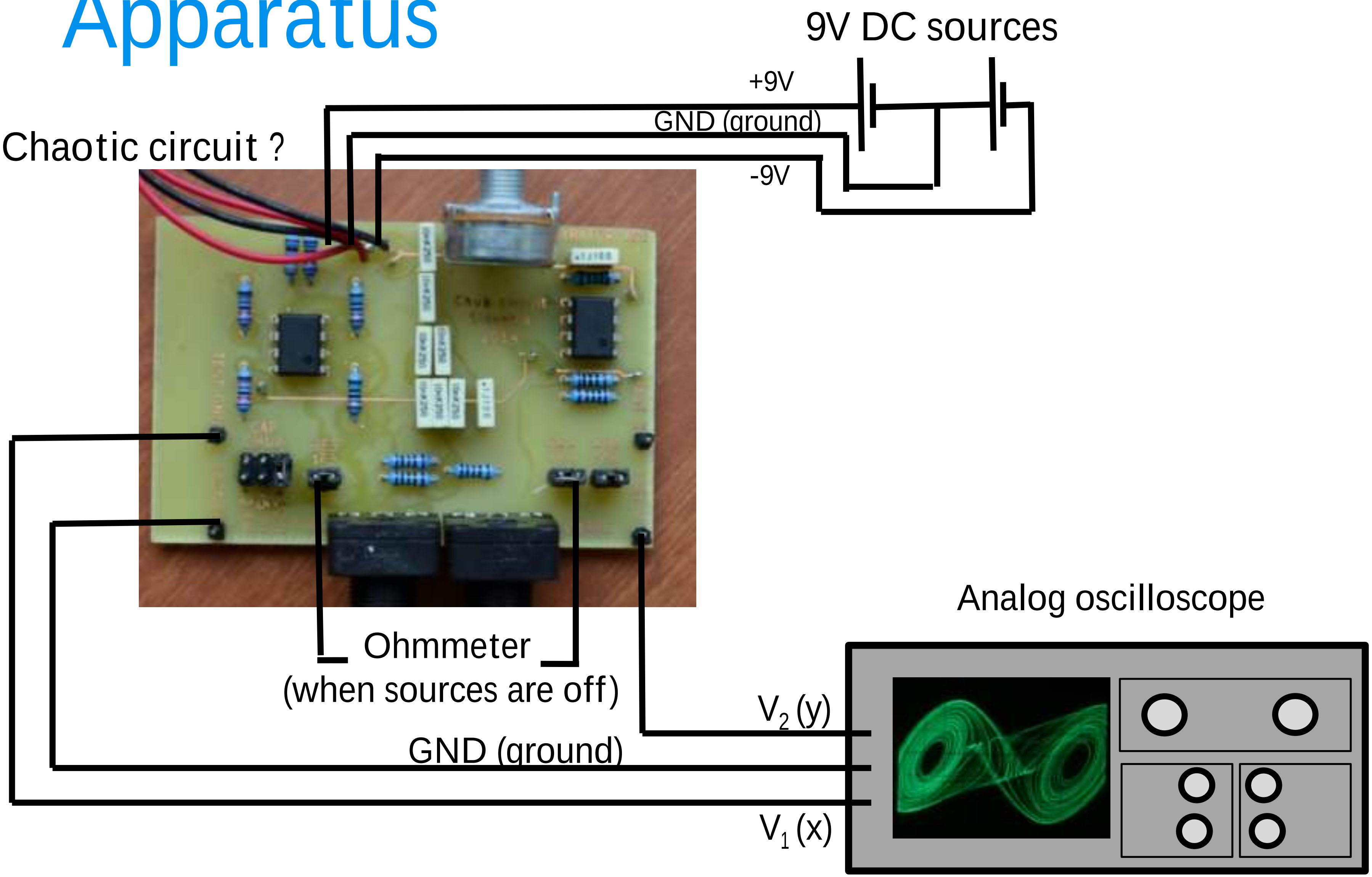


Bifurcation Diagrams

How many different states for different parameters



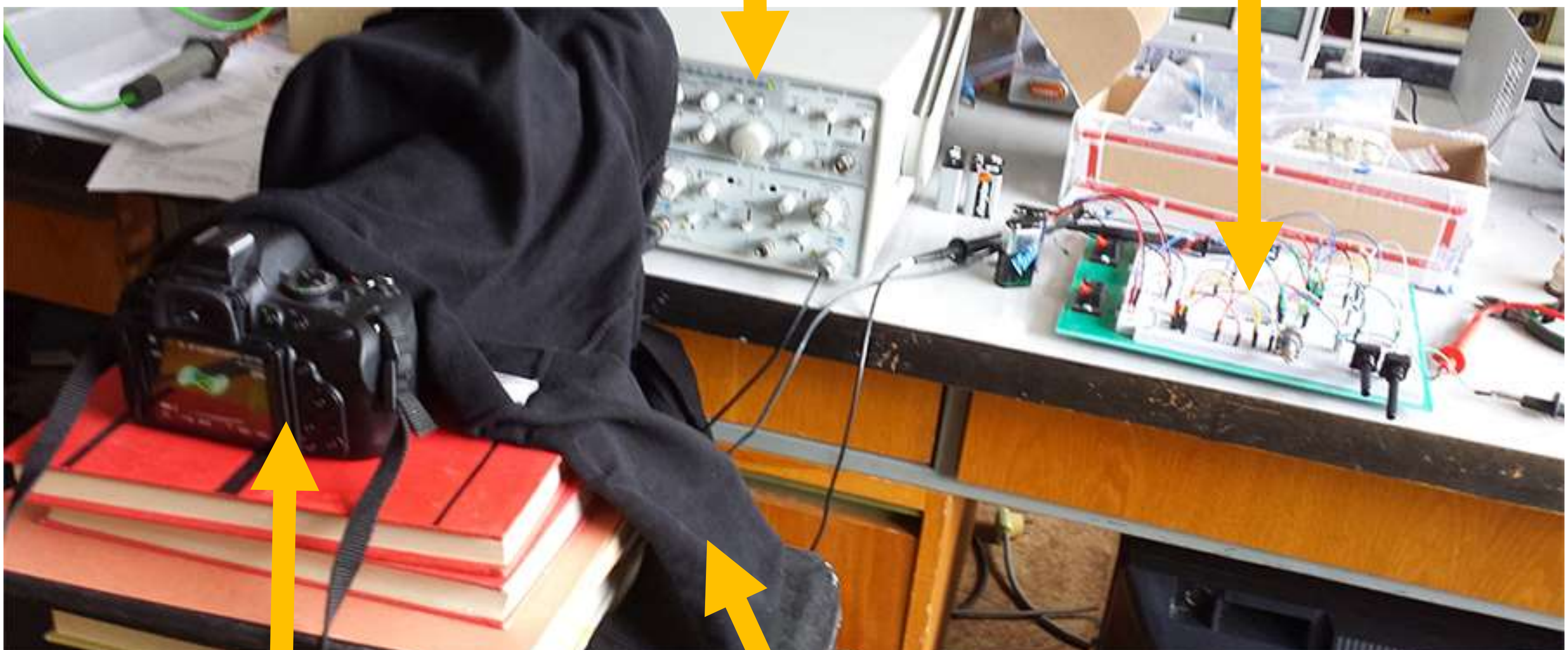
Apparatus


9V DC sources
+9V
GND (ground)
-9V
Chaotic circuit ?
Ohmmeter
(when sources are off)
Analog oscilloscope
V2 (y)
GND (ground)
V1 (x)

Apparatus

Analog oscilloscope
(XY mode)

Chua's circuit

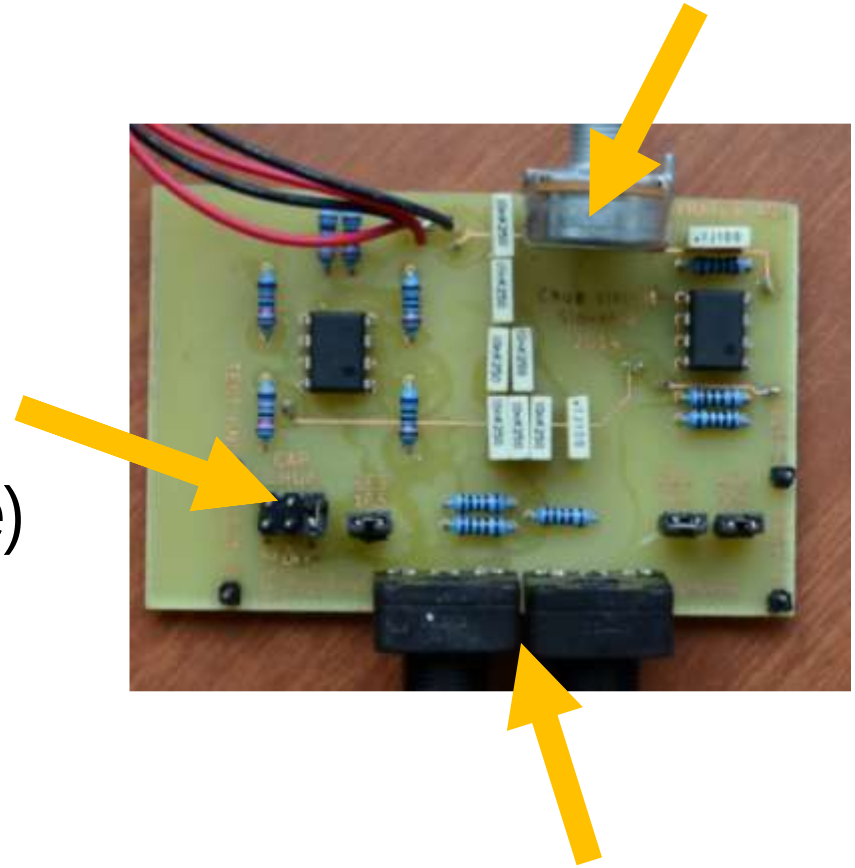


Camera
(Exposition 1/10 s)

Filtering light
from environment

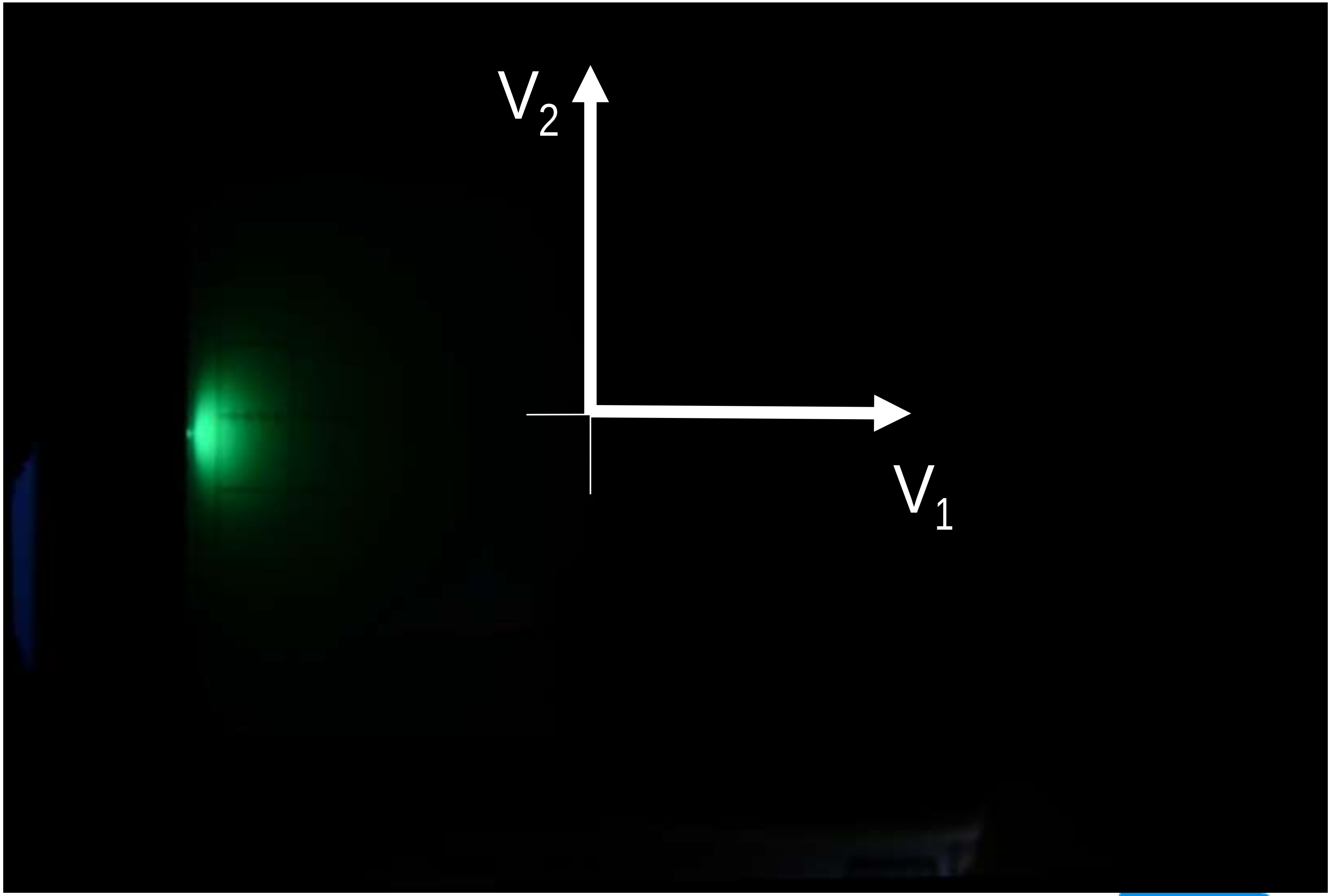
Experiments

- Varying:
  - Linear resistance
  - Capacitance
  - Inductance (Resistance)



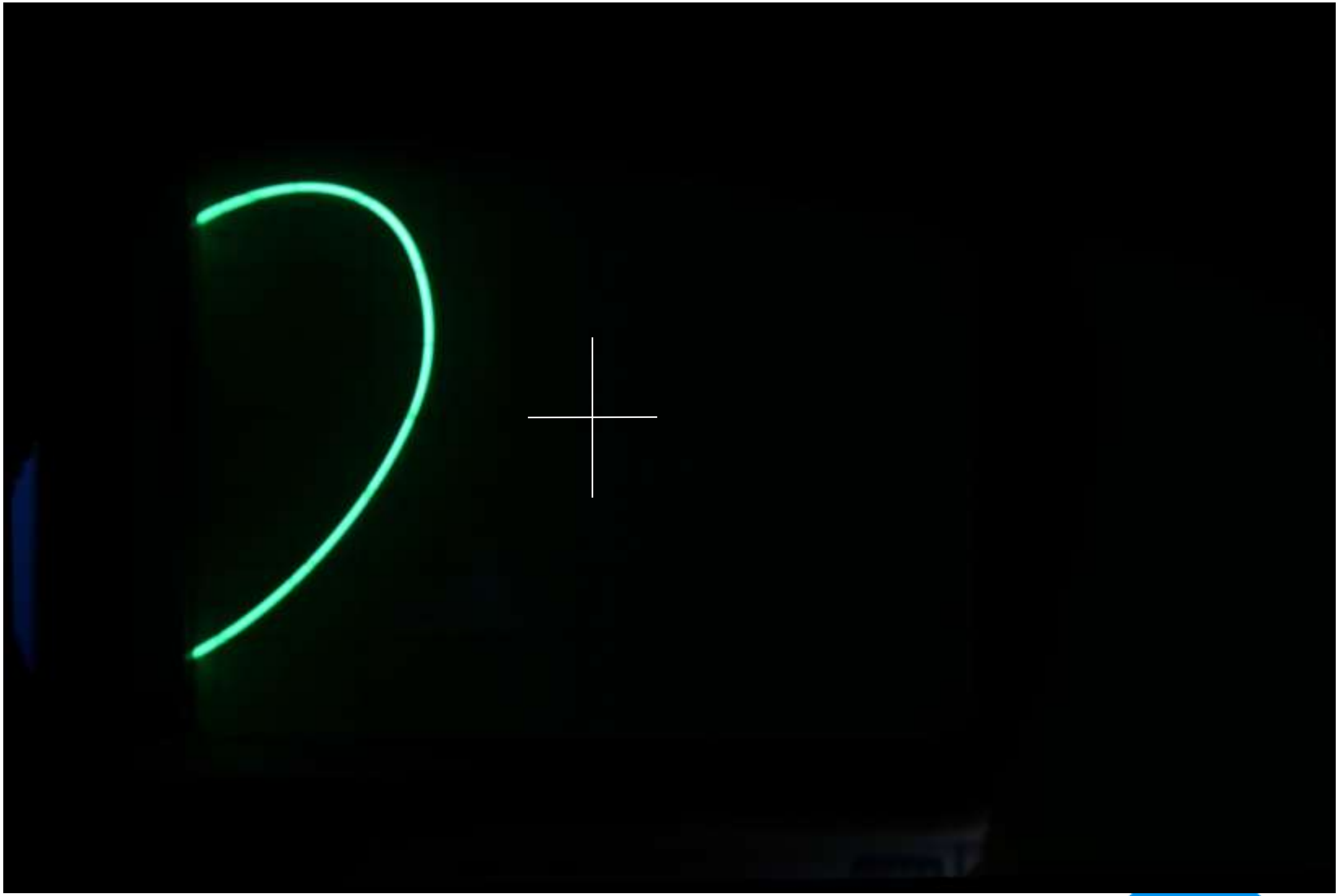
Phase Space

$R = 1866\,\Omega \quad L = 27{,}44\,mH$



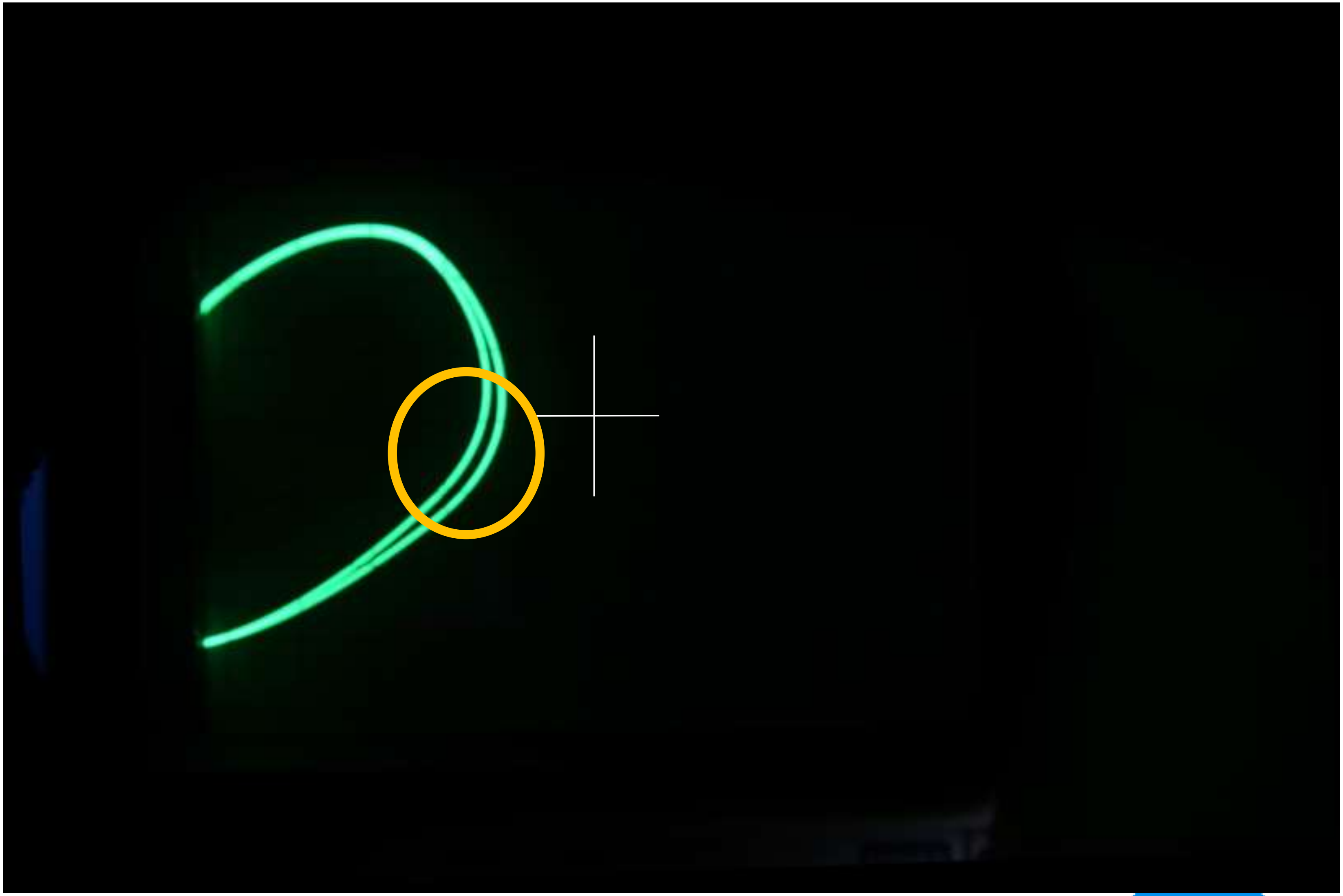
Phase Space

$R = 1833\,\Omega \quad L = 27{,}44\,mH$



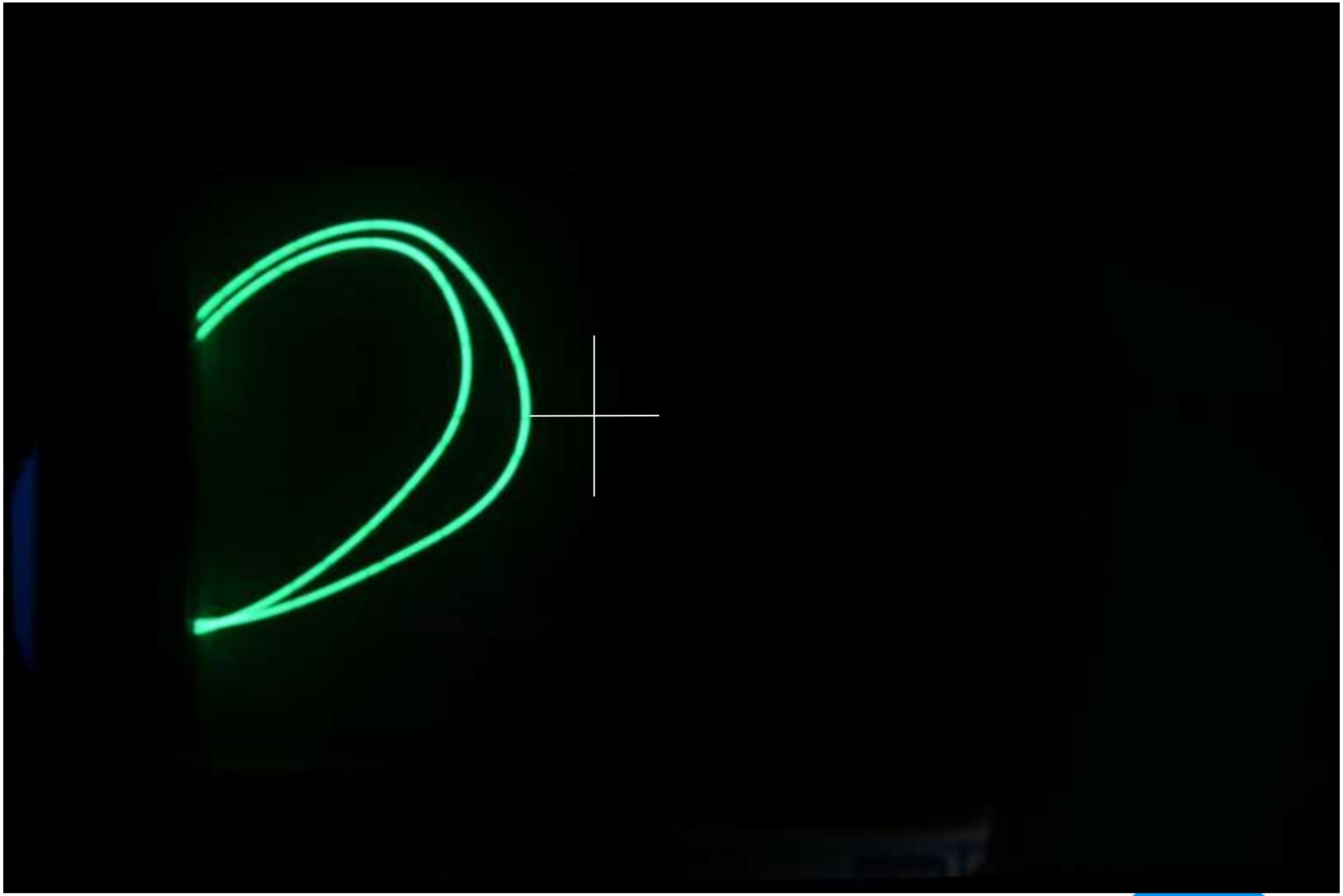
Biffurcation

$R = 1778\,\Omega \quad L = 27{,}44\,mH$



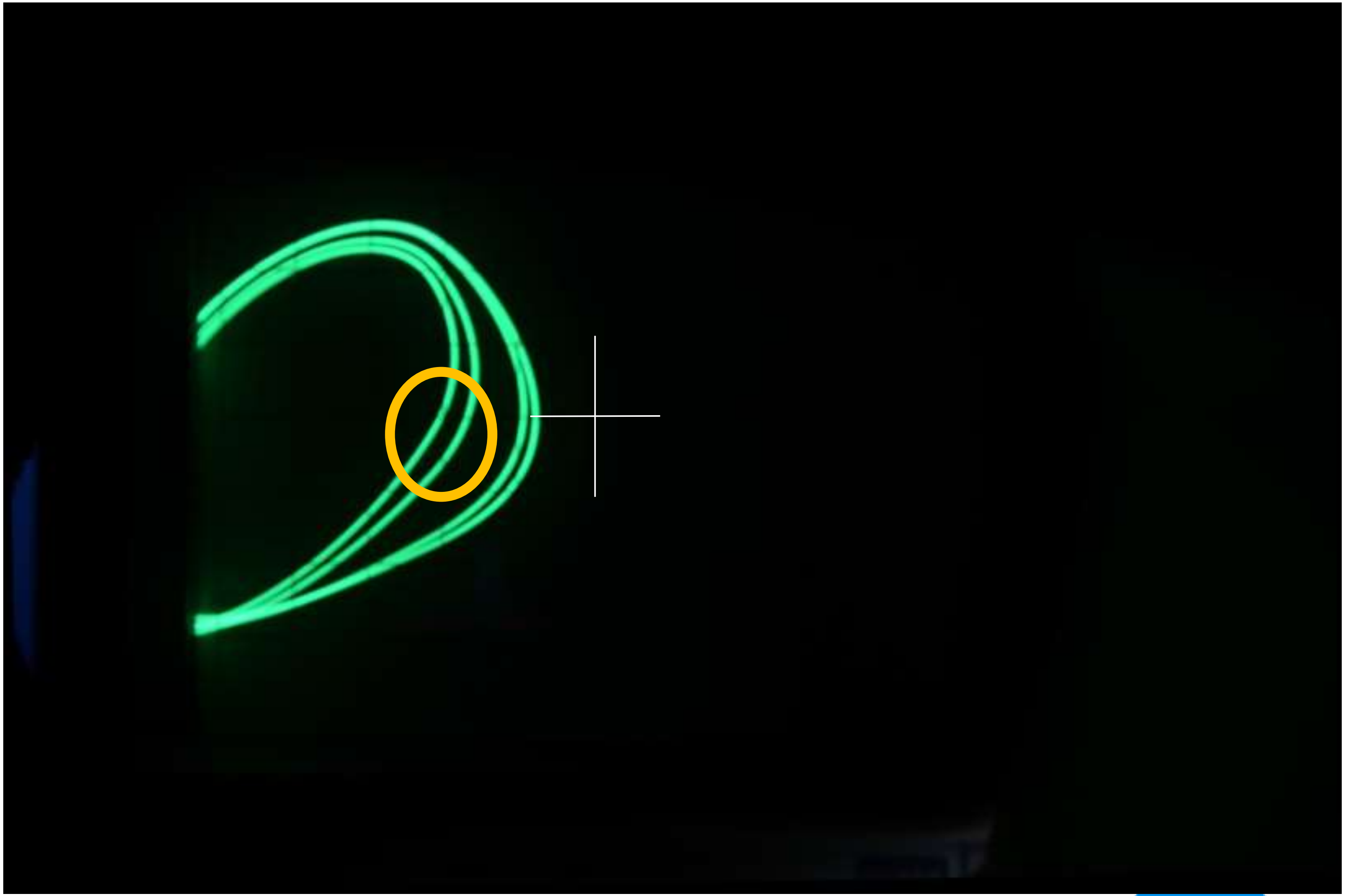
Phase Space

$R = 1763\,\Omega \quad L = 27{,}44\,mH$



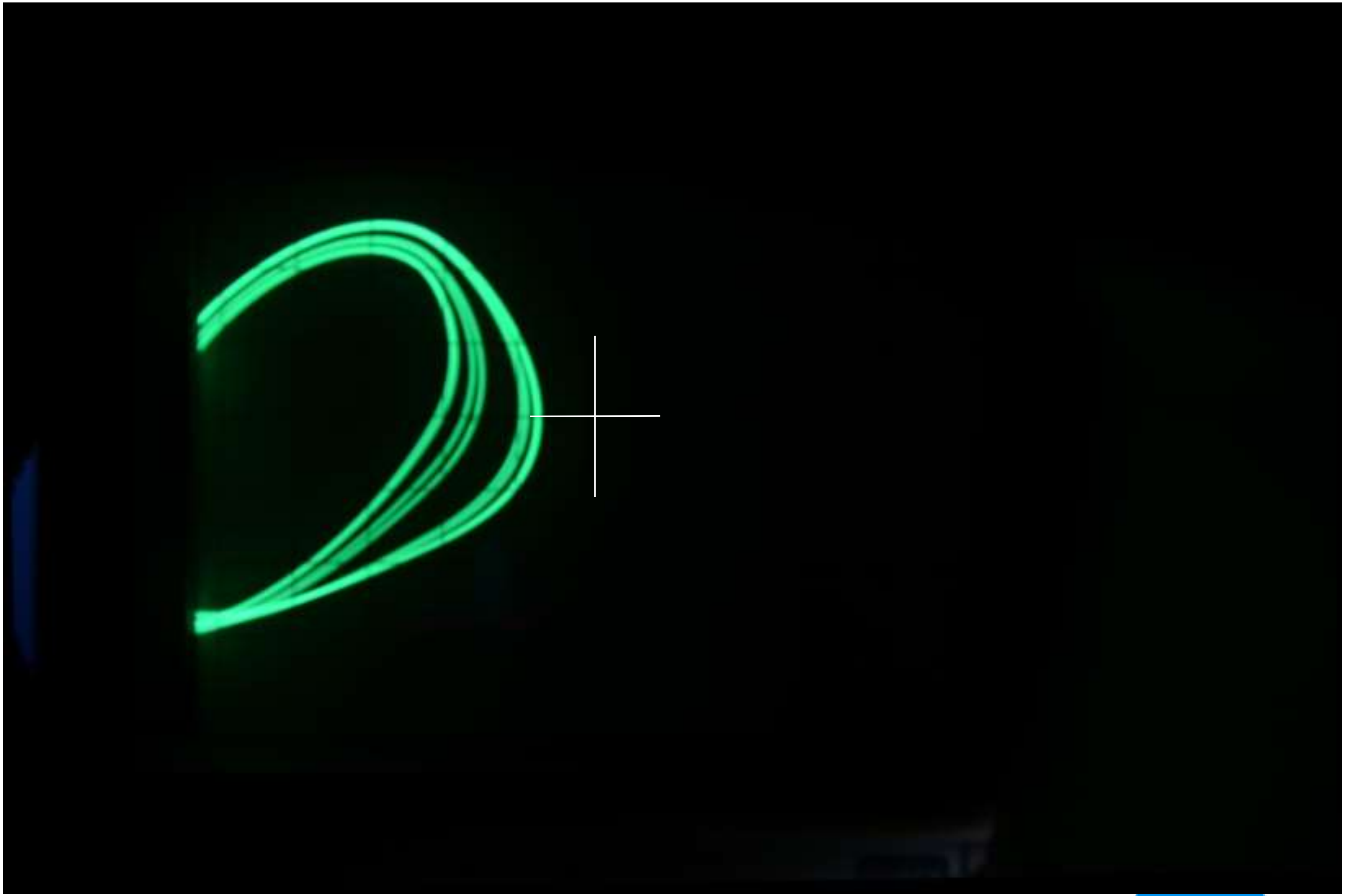
Biffurcations

$R = 1758\,\Omega \quad L = 27{,}44\,mH$



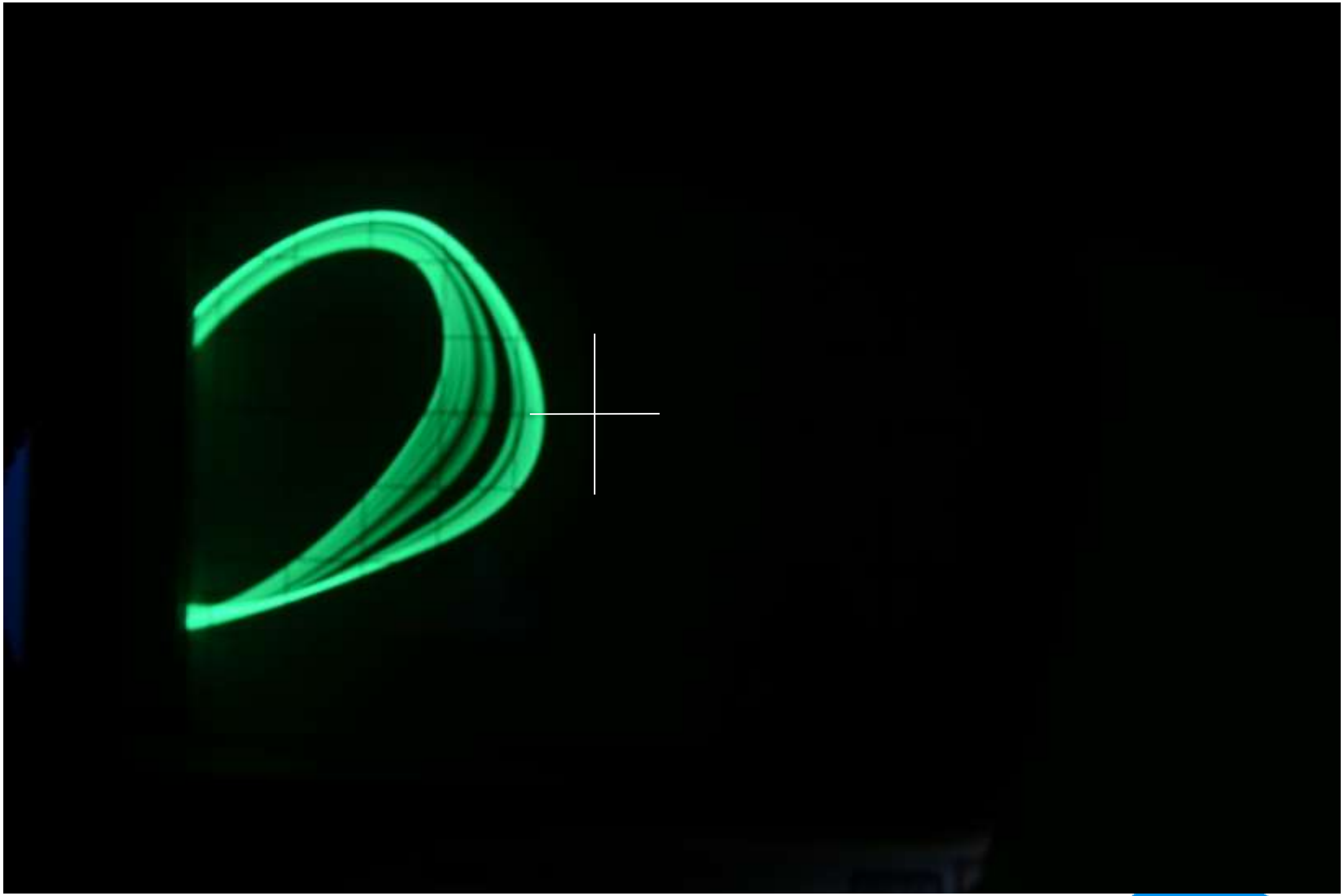
Biffurcations

$R = 1756\,\Omega \quad L = 27{,}44\,mH$



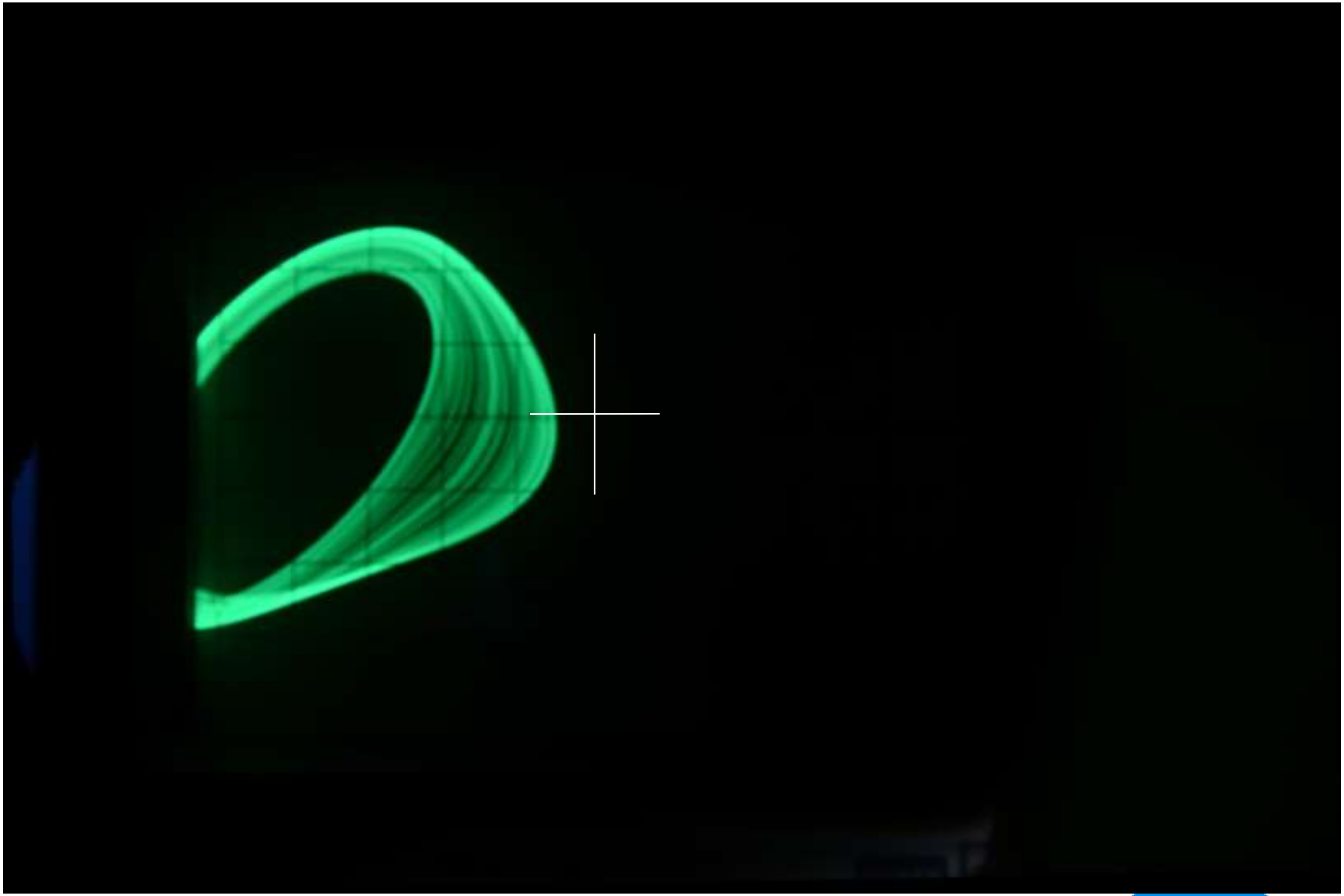
Biffurcations

$R = 1754\,\Omega \quad L = 27{,}44\,mH$



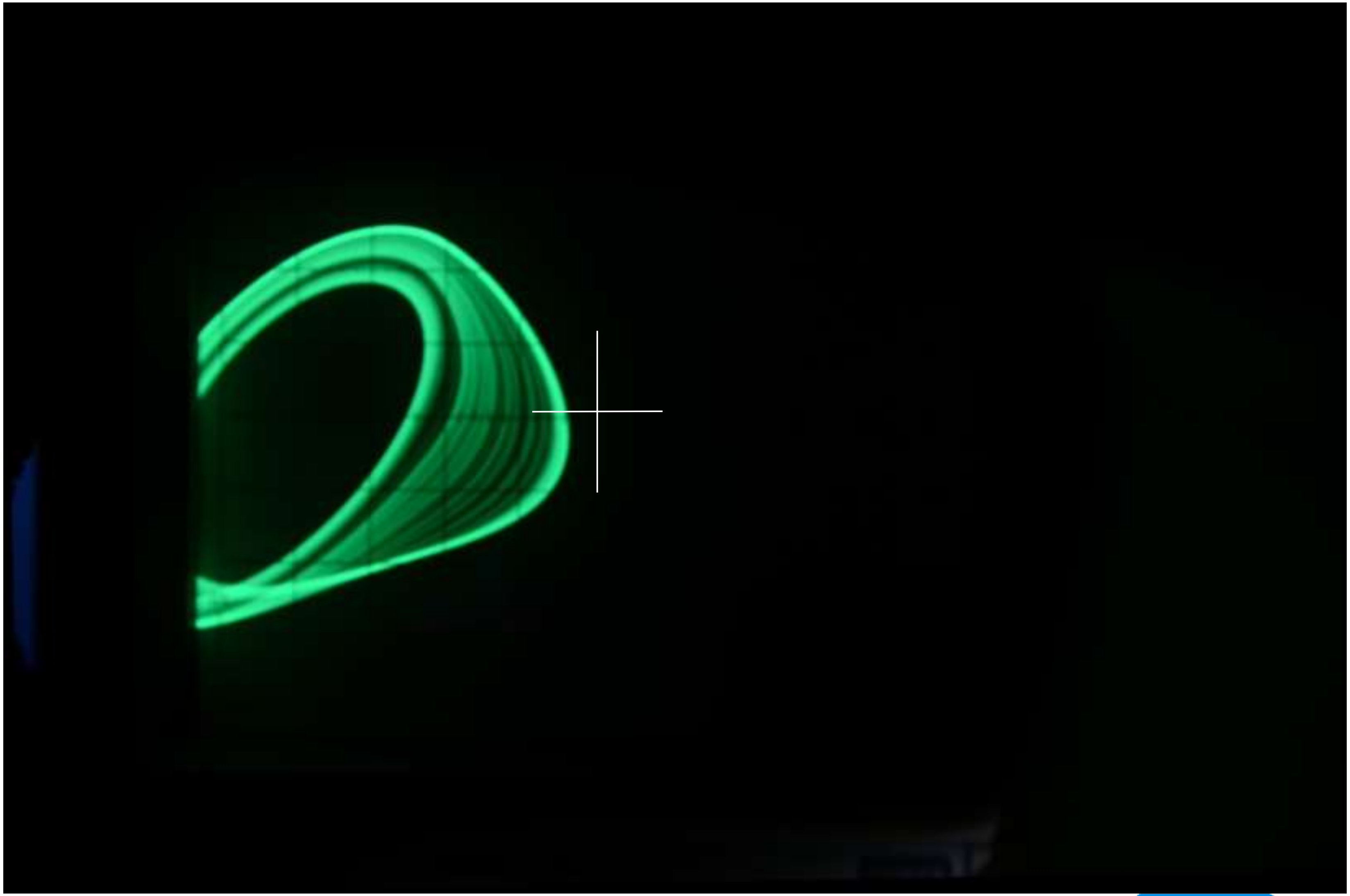
Biffurcations

$R = 1753\ \Omega \quad L = 27{,}44\ mH$



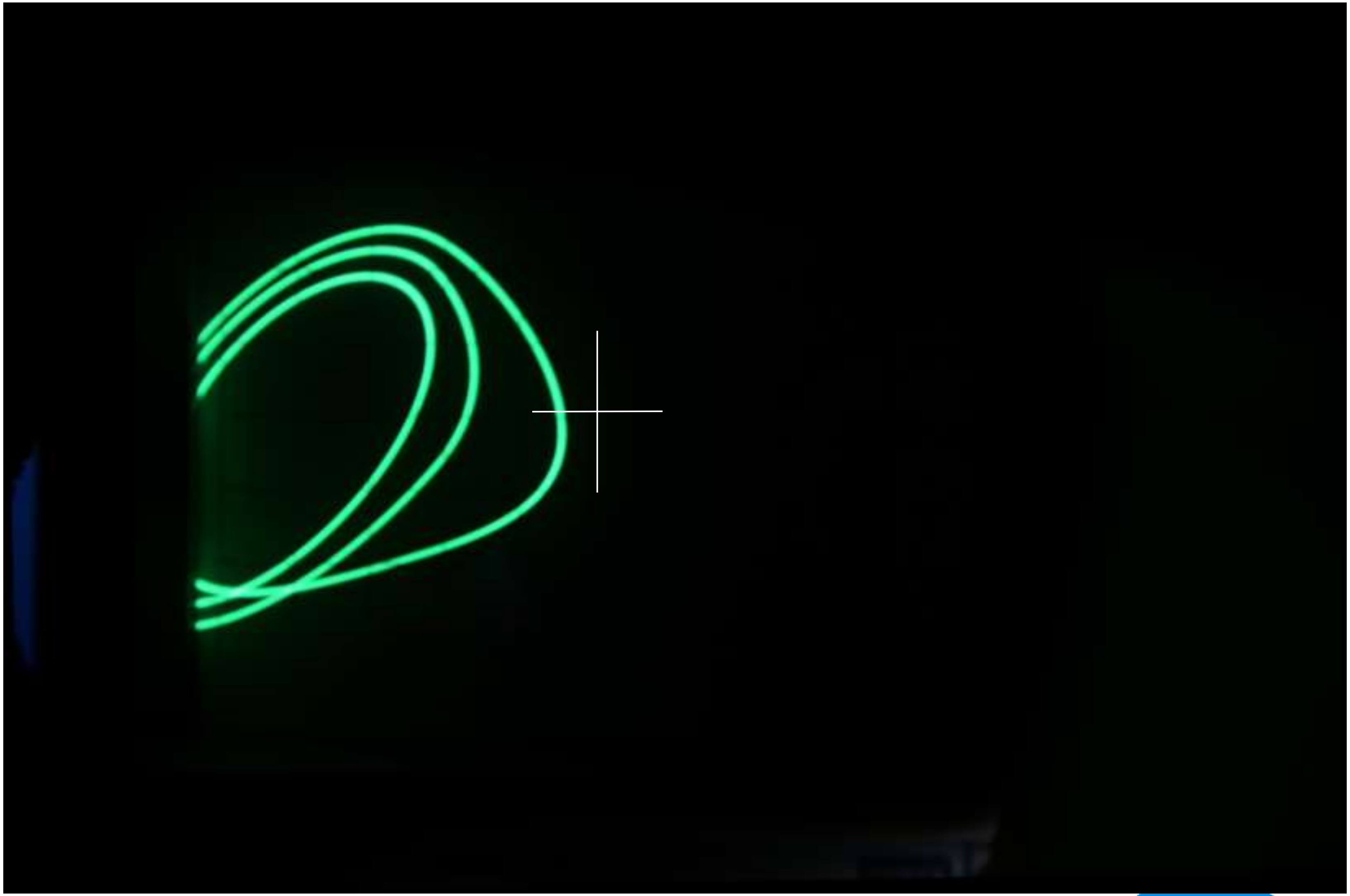
Rössler attractor

$R = 1750\,\Omega \quad L = 27{,}44\,mH$



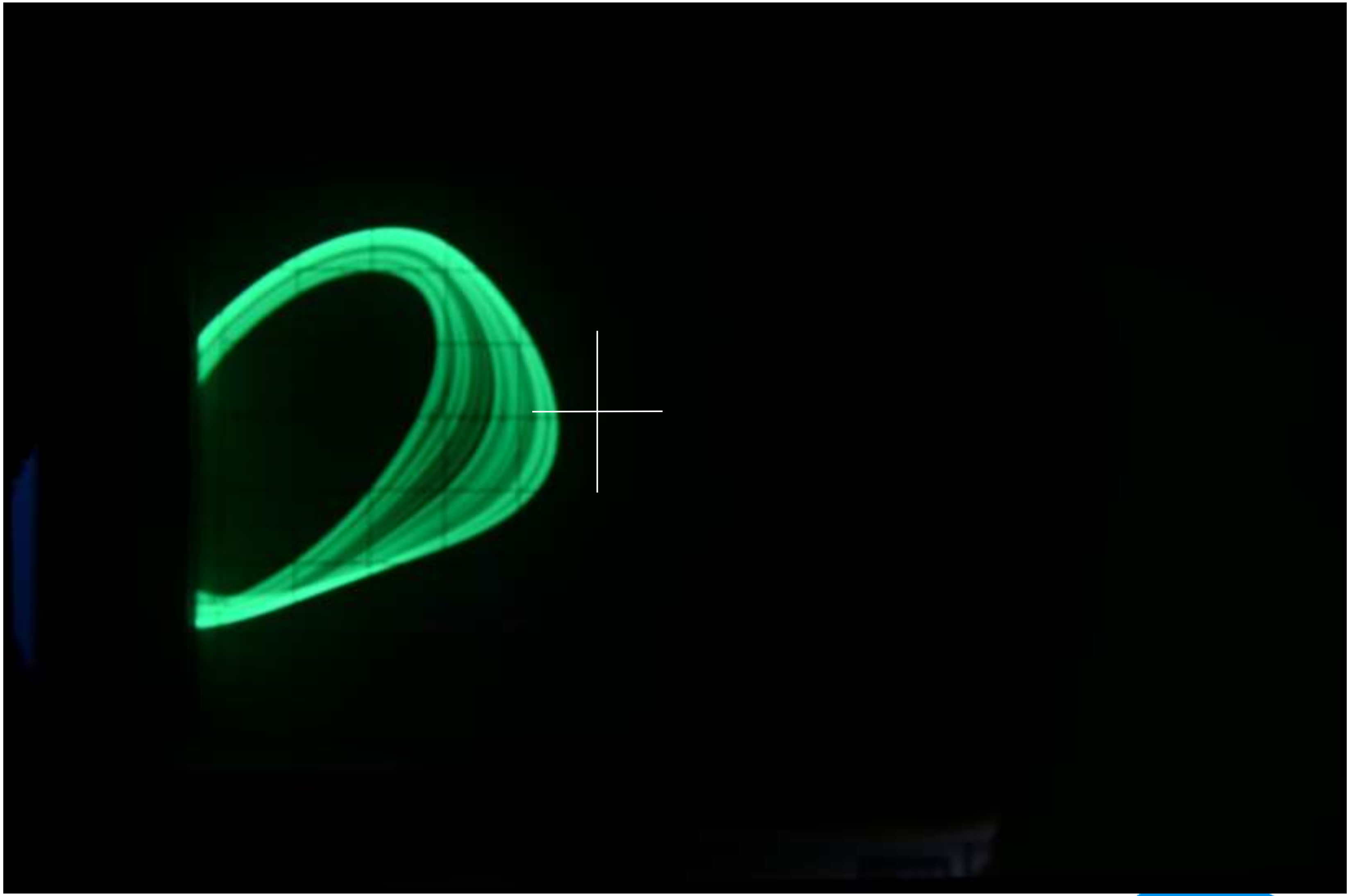
Phase Space

$R = 1750\ \Omega \quad L = 27{,}44\ mH$



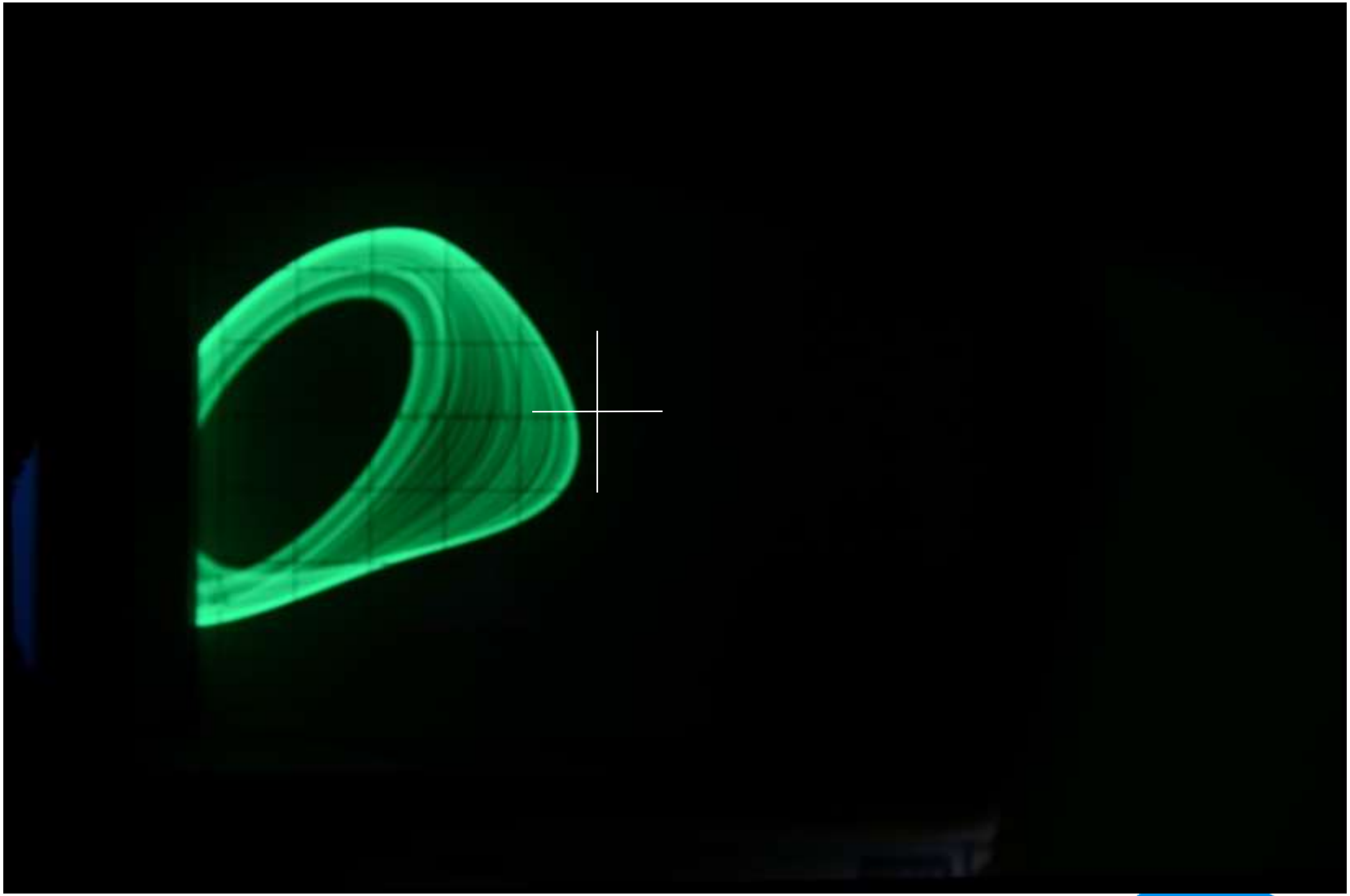
Phase Space

$R = 1746\,\Omega \quad L = 27{,}44\,mH$



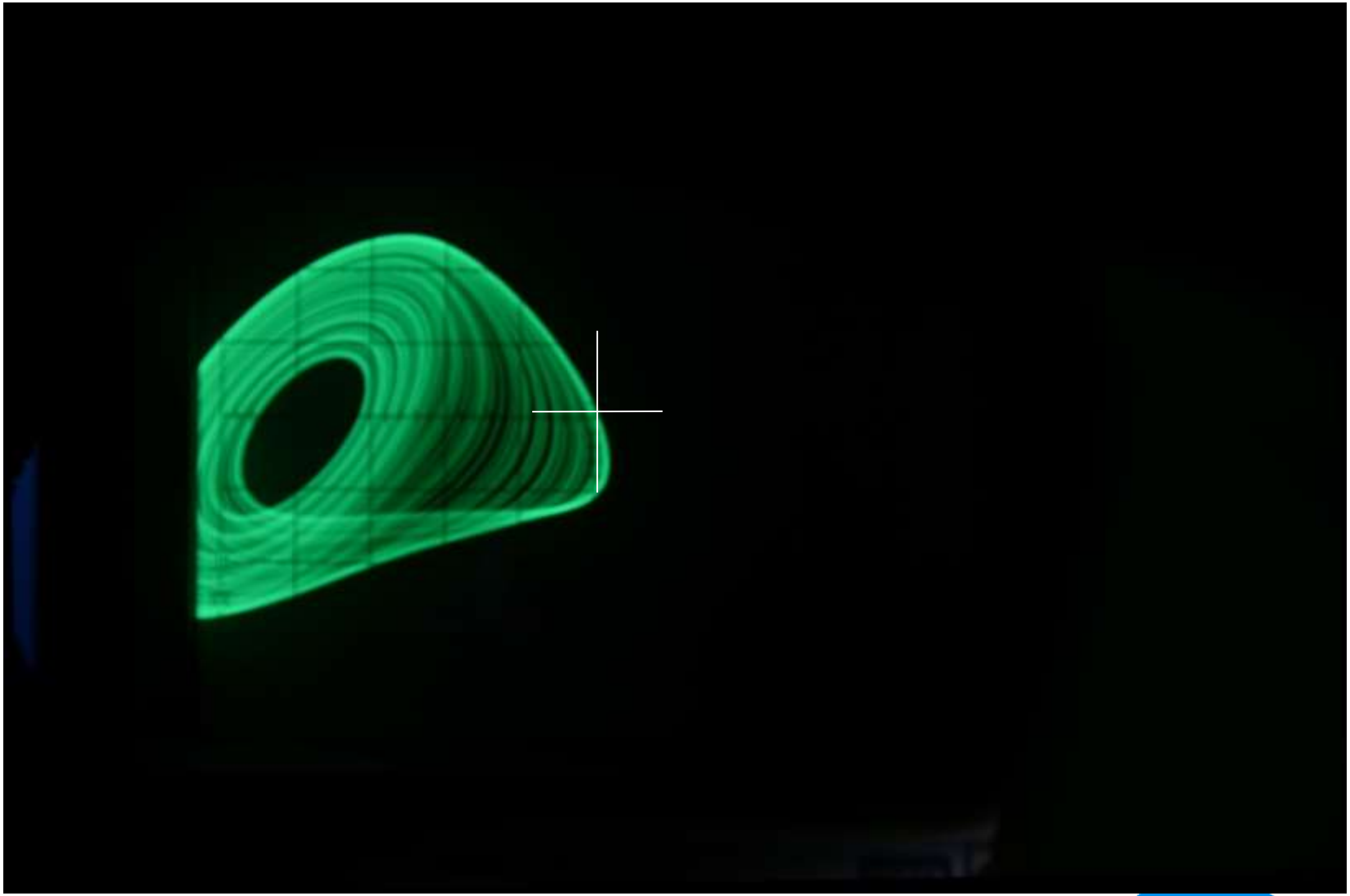
Phase Space

$R = 1742\ \Omega \quad L = 27{,}44\ mH$



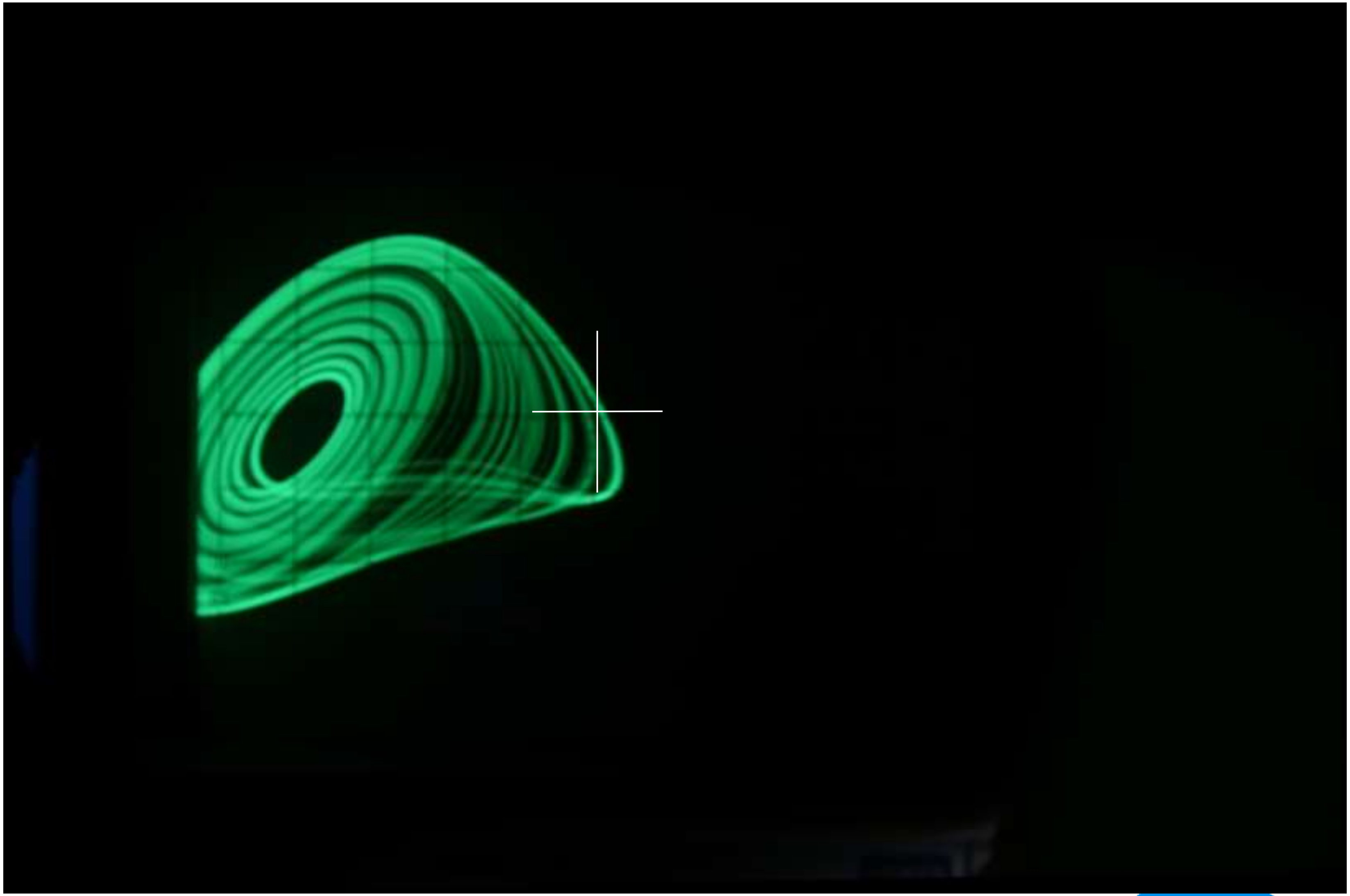
Phase Space

$R = 1728\,\Omega \quad L = 27{,}44\,mH$



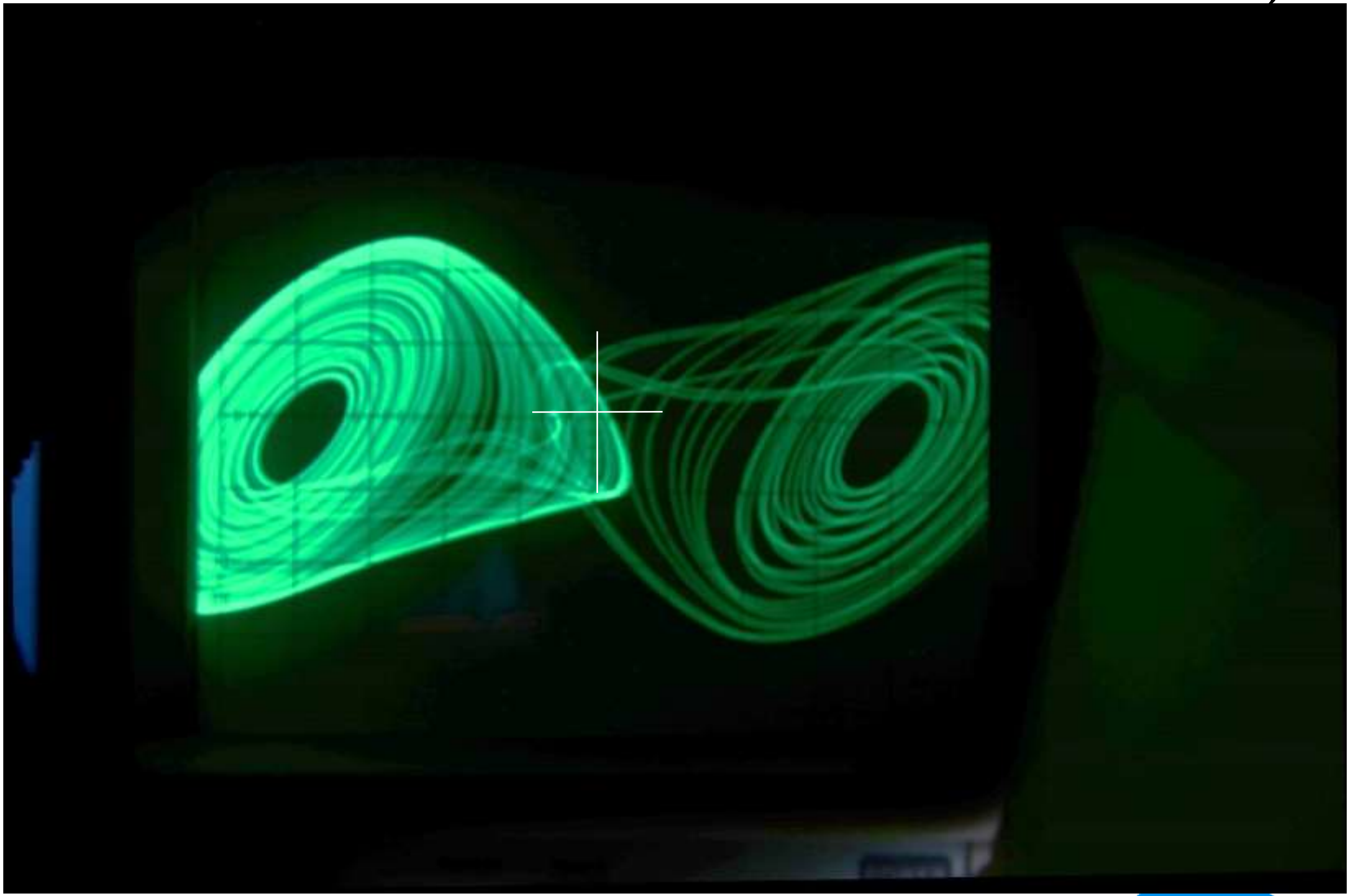
Phase Space

$R = 1723\ \Omega \quad L = 27{,}44\ mH$



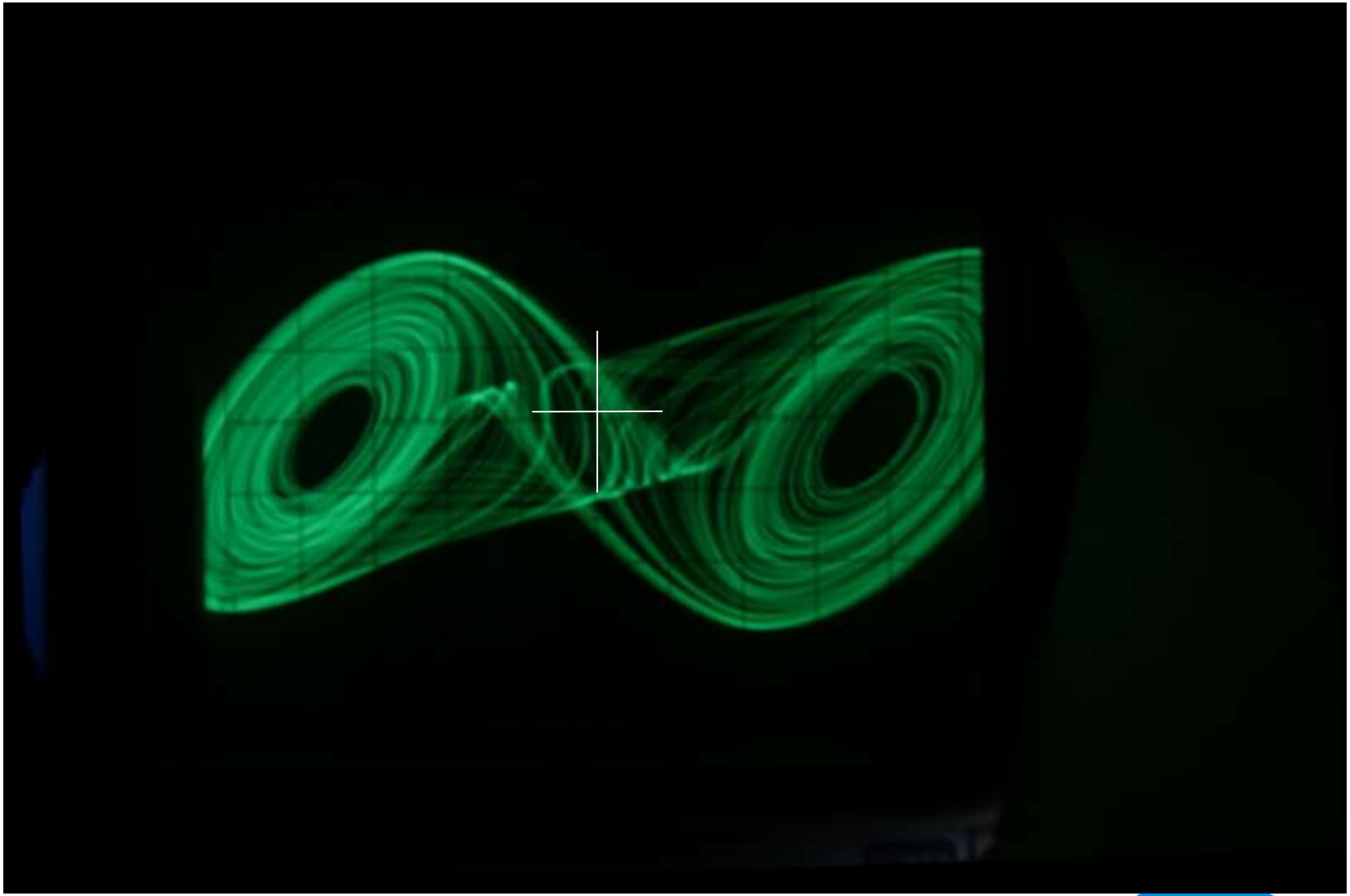
Transition to Double Scroll Attractor

$R = 1721\,\Omega \quad L = 27{,}44\ mH$



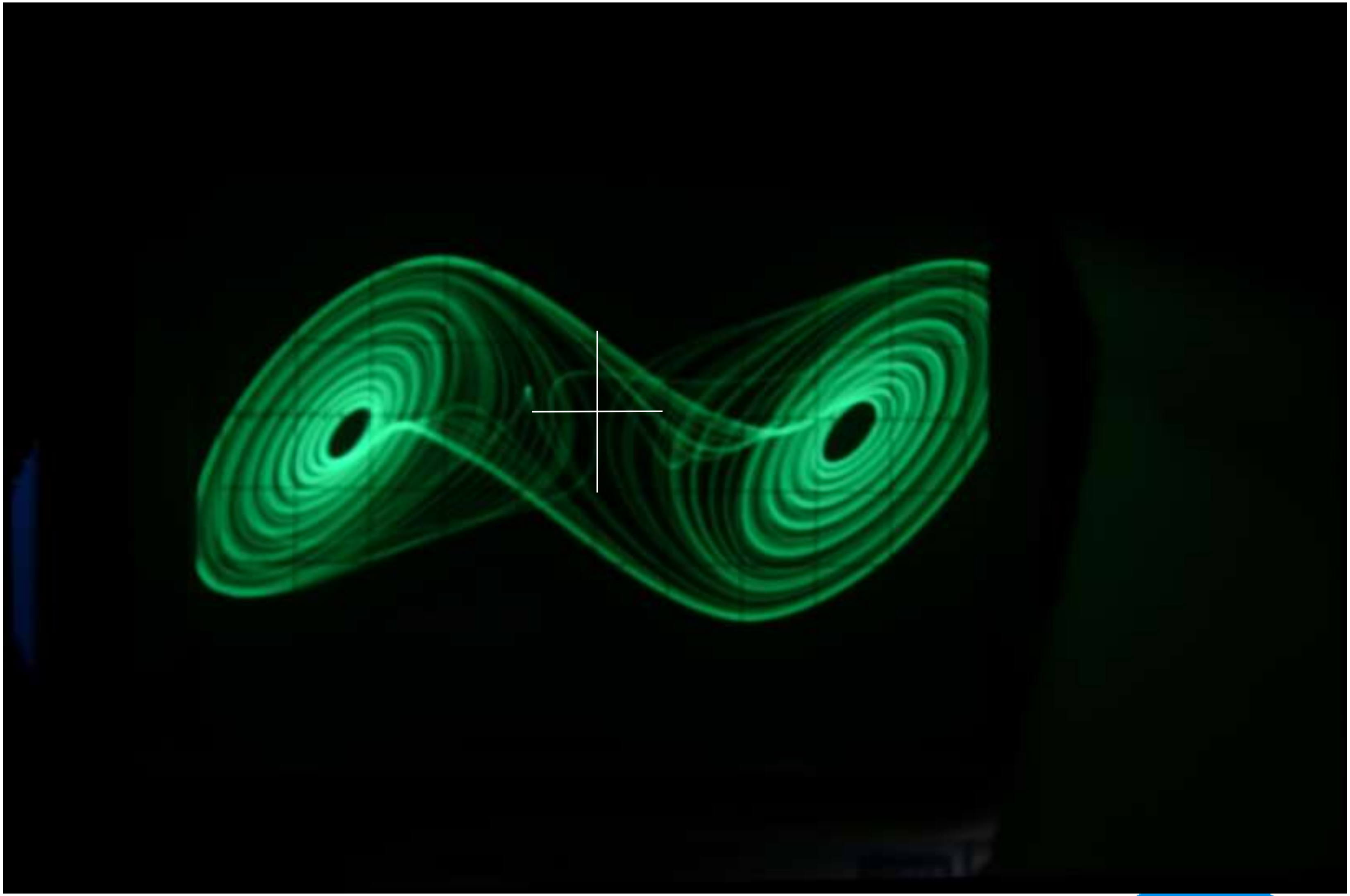
Double Scroll

$R = 1708\,\Omega \quad L = 27{,}44\,mH$



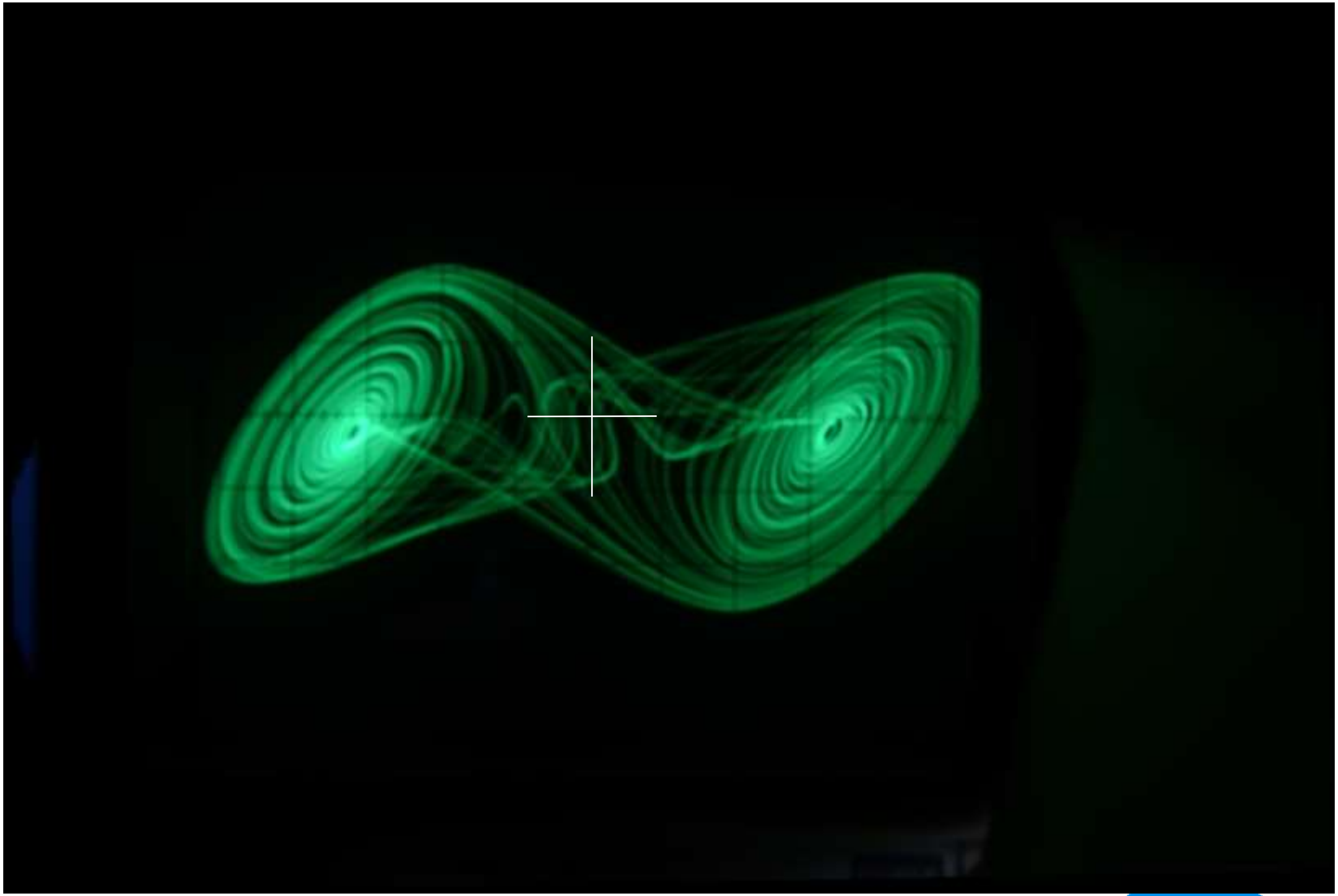
Double Scroll

$R = 1673\,\Omega \quad L = 27{,}44\,mH$



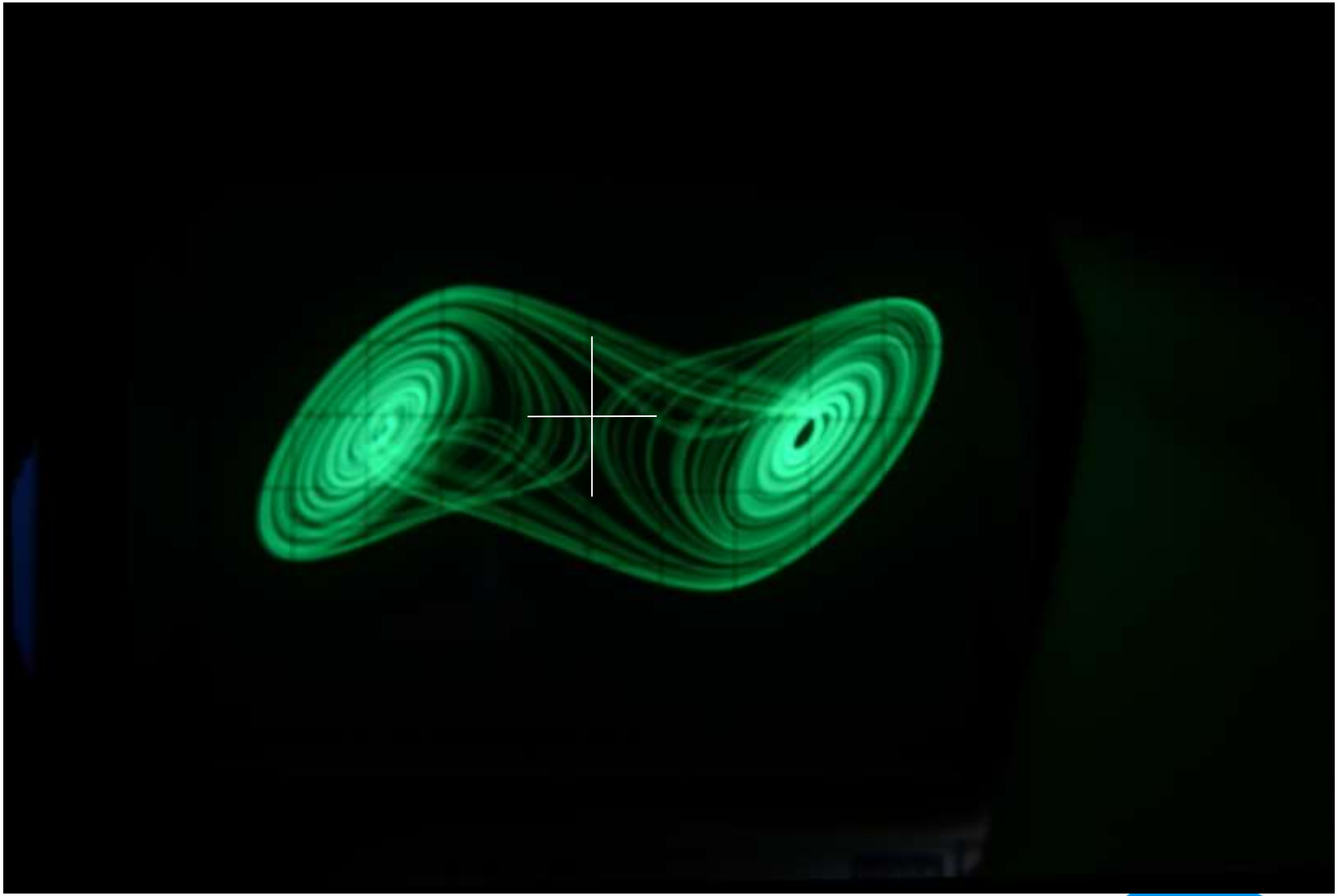
Double Scroll

$R = 1651\,\Omega \quad L = 27{,}44\,mH$



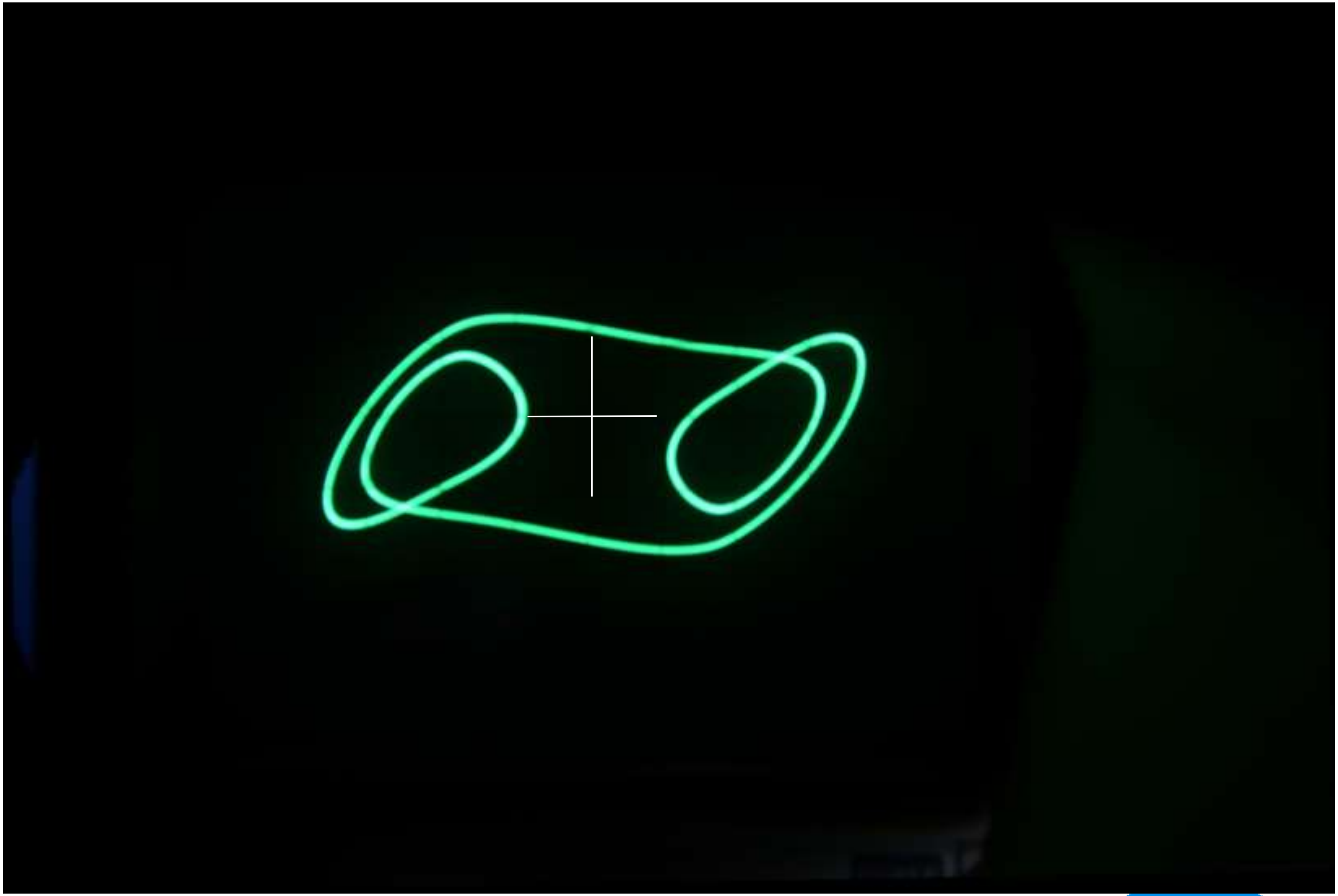
Double Scroll

$R = 1593\,\Omega \quad L = 27{,}44\,mH$



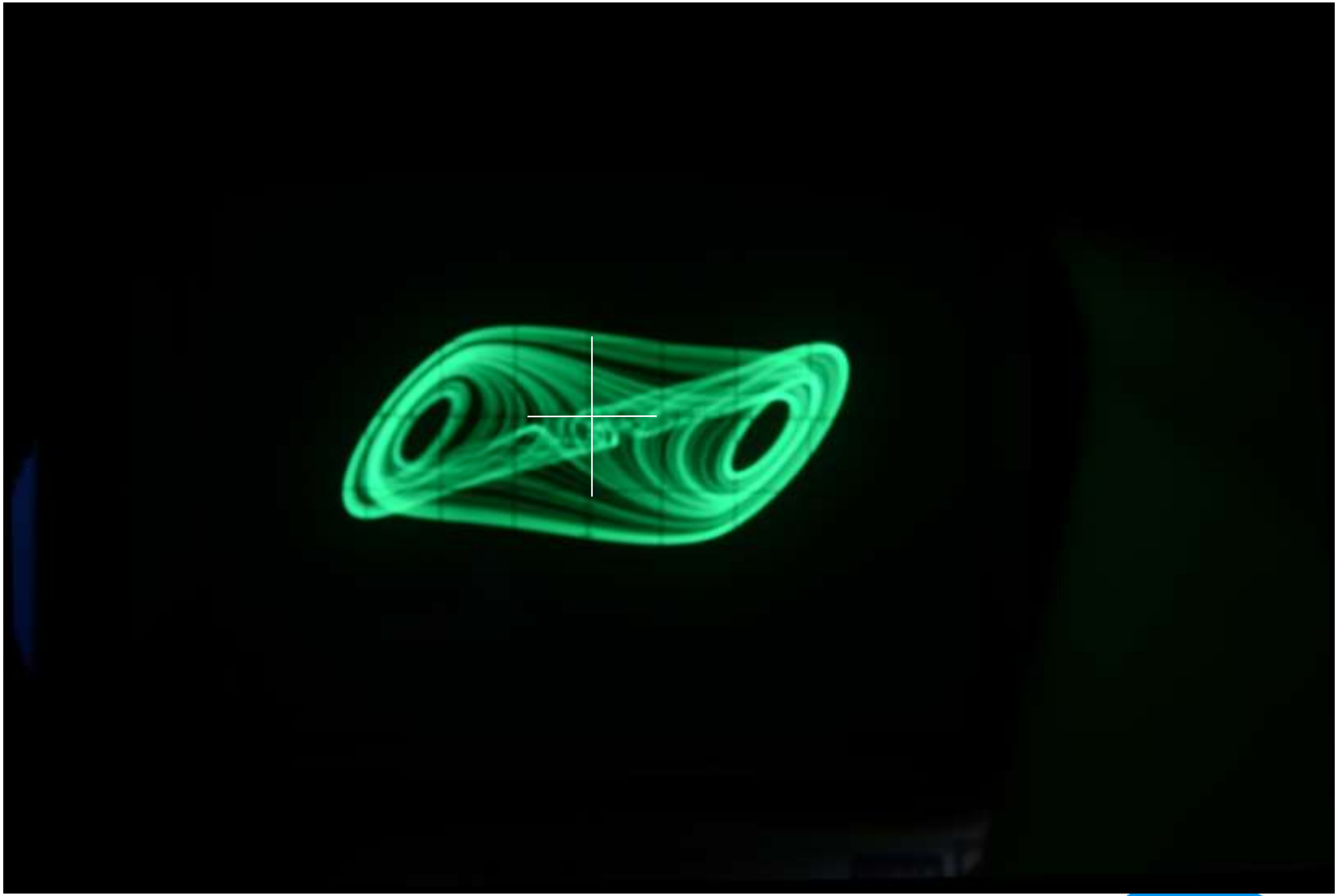
Phase Space

$R = 1500\,\Omega \quad L = 27{,}44\,mH$



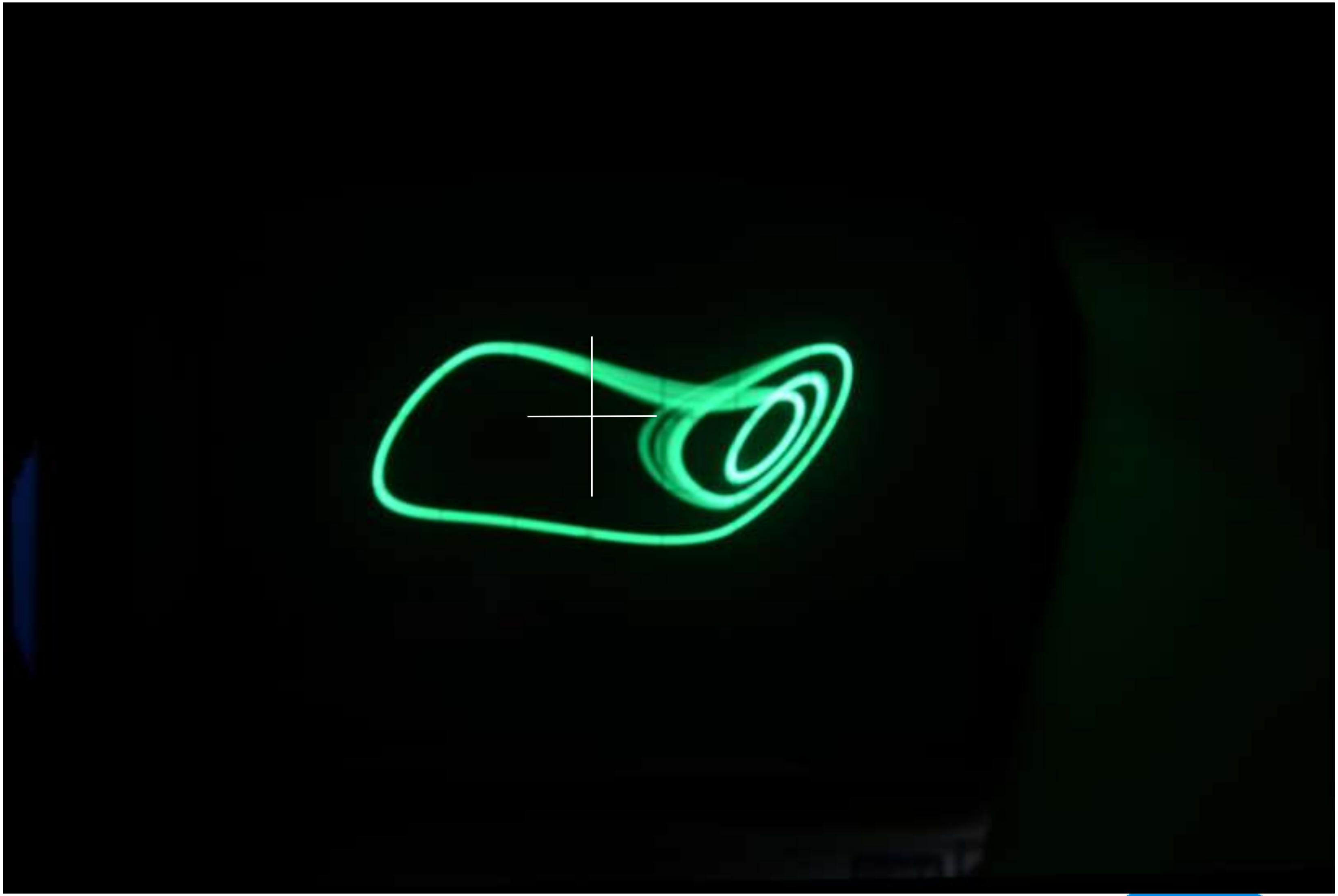
Phase Space

$R = 1476\,\Omega \quad L = 27{,}44\,mH$



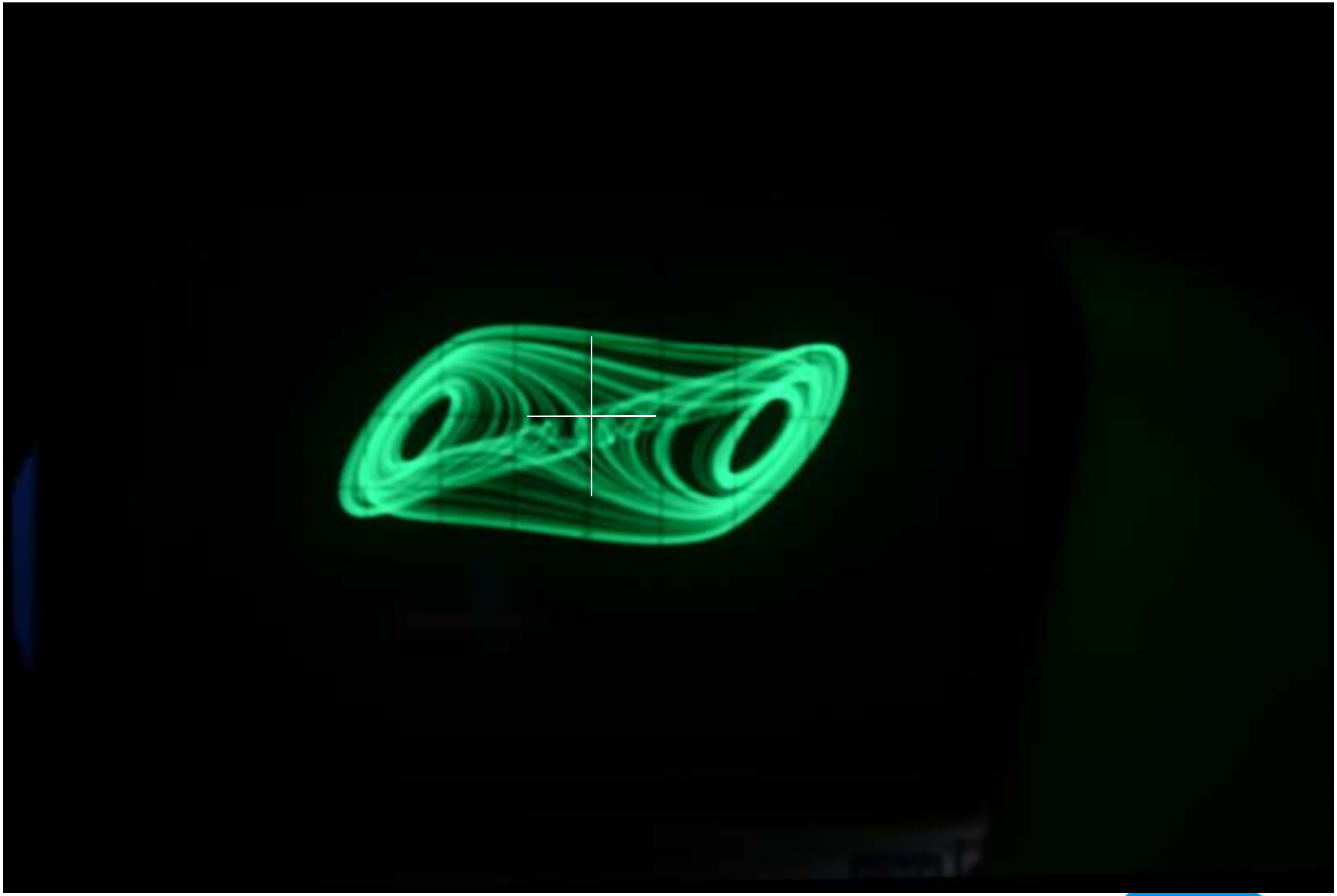
Phase Space

$R = 1472\,\Omega \quad L = 27{,}44\,mH$



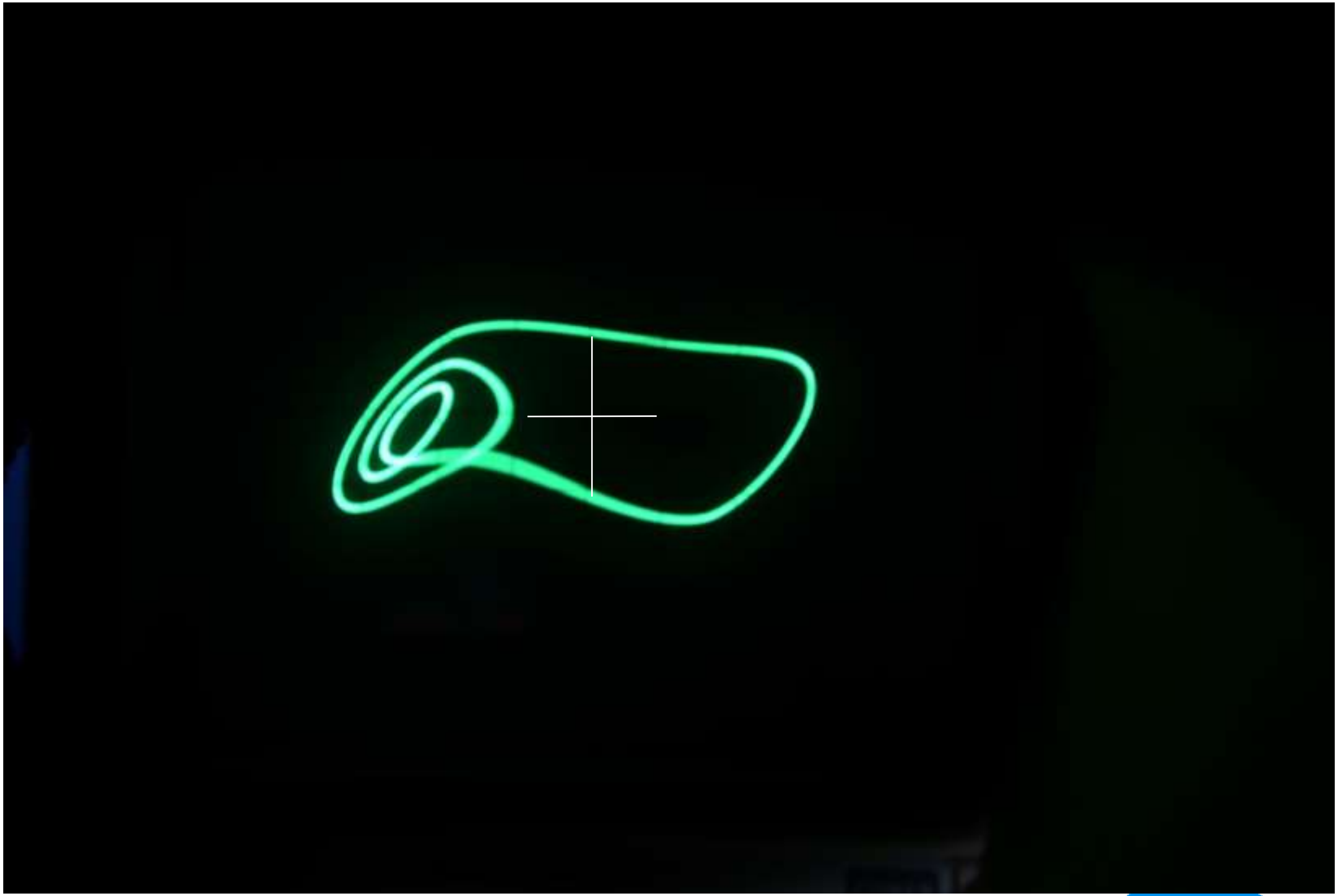
Phase Space

$R = 1469\,\Omega \quad L = 27{,}44\,mH$



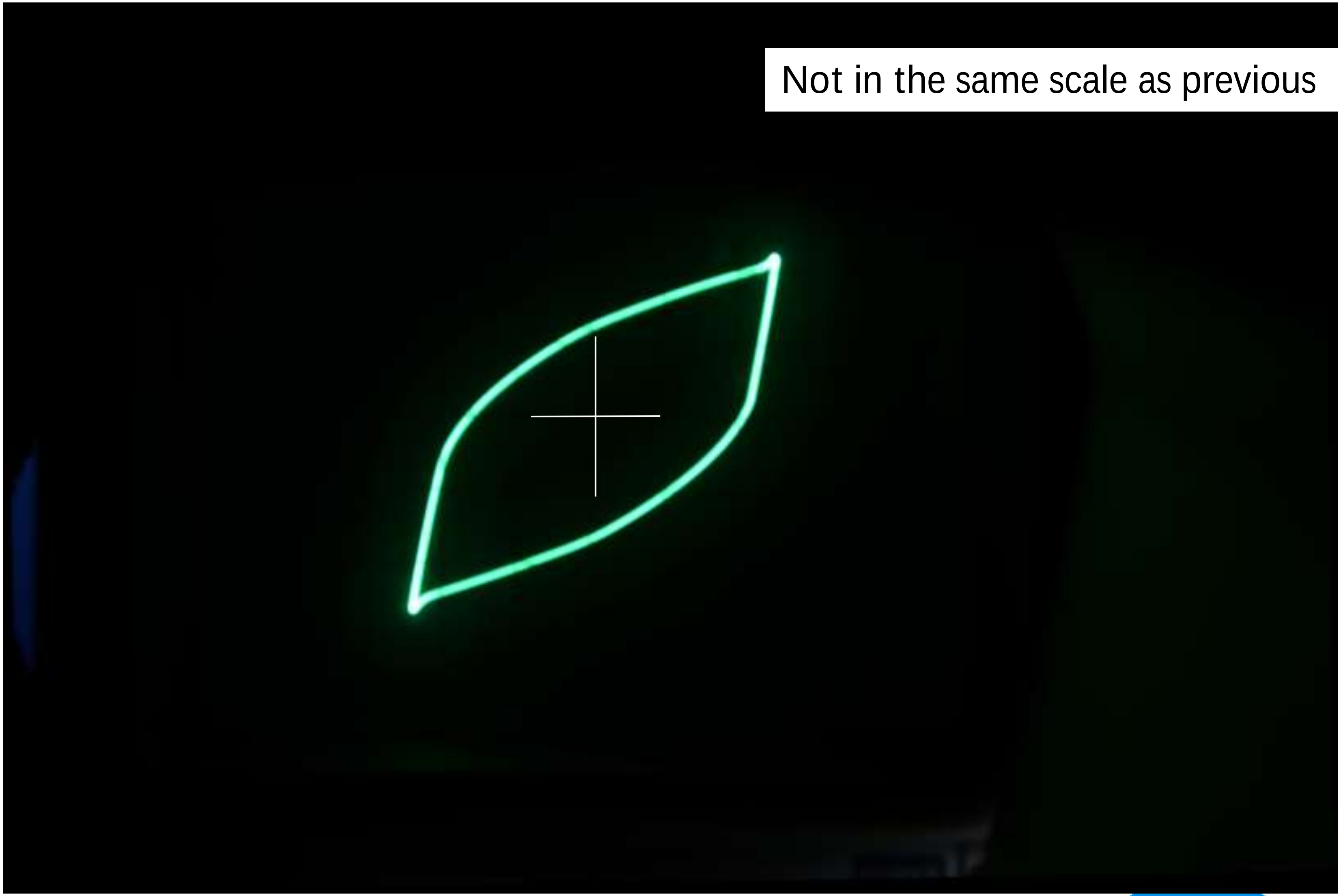
Phase Space

$R = 1464\,\Omega \quad L = 27{,}44\,mH$



Phase Space

$R = 1437\,\Omega \quad L = 27{,}44\,mH$

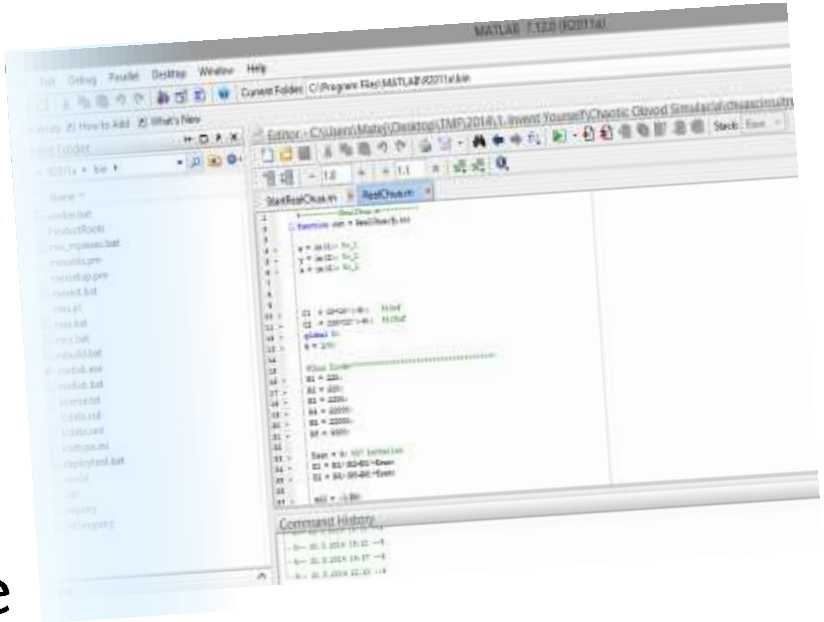


Not in the same scale as previous

Simulation

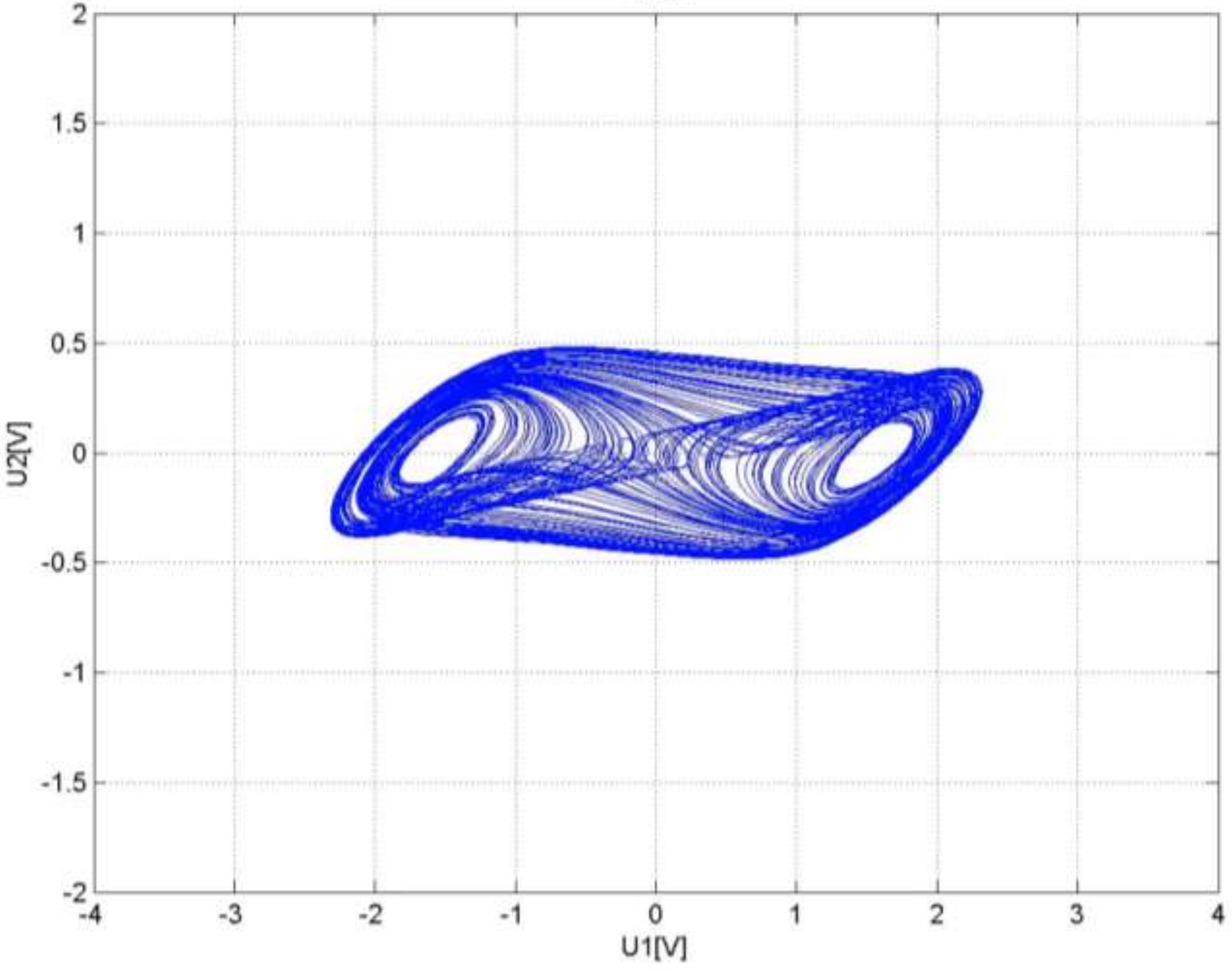
- Euler's method is insufficient
- Using Runge-Kutta 4th order ODE solver (in Matlab)
- Using calculated I-V characteristic of Chua's diode
- Arbitrary initial conditions (which lead to attractor)

We're interested in attractor



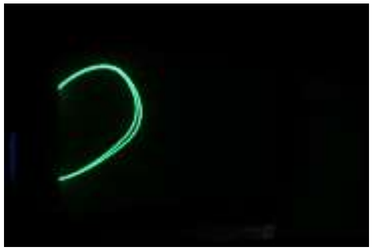
Theoretical result

Changing $R = 1500\ \Omega$ Other parameters constant

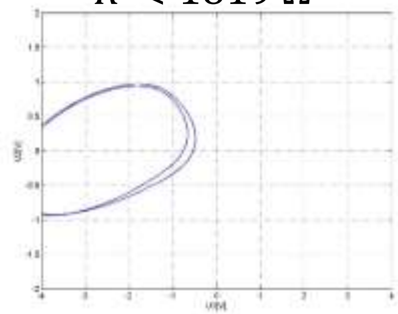


Comparison

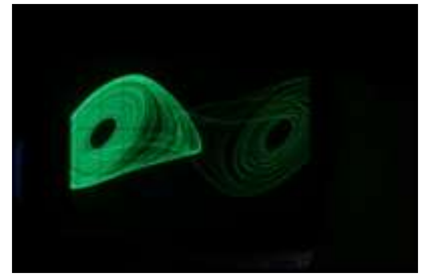
$R < 1778\ \Omega$



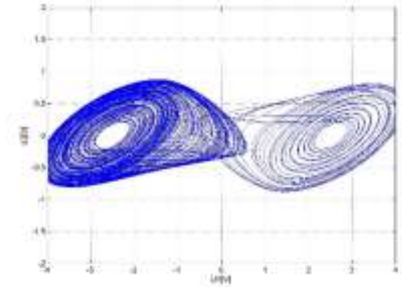
$R < 1819\ \Omega$



$R = 1721\ \Omega$

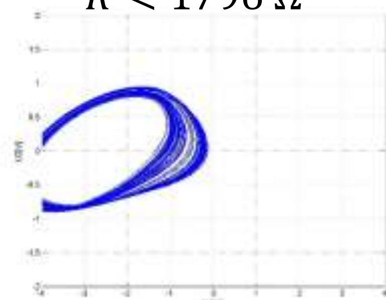


$R = 1757\ \Omega$

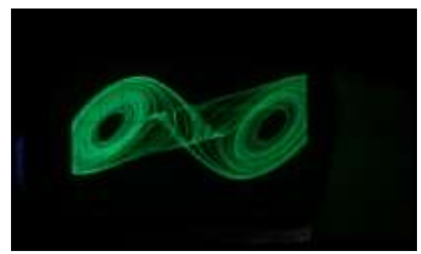


$R < 1753\ \Omega$

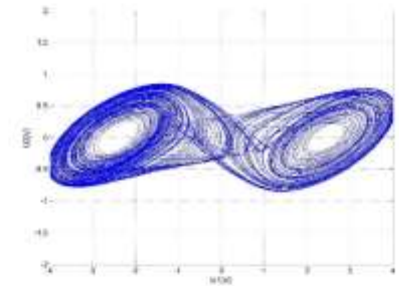
$R < 1796\ \Omega$



$R = 1708\ \Omega$

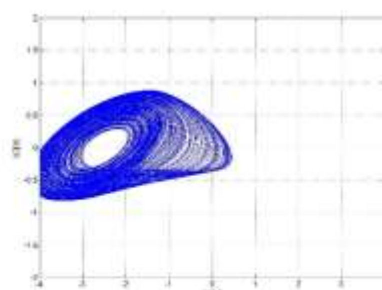


$R = 1742\ \Omega$

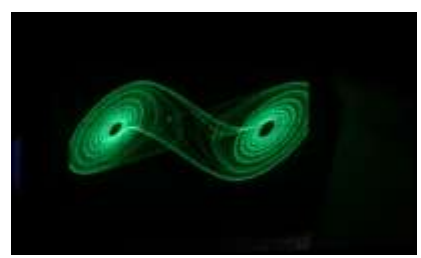


$R < 1728\ \Omega$

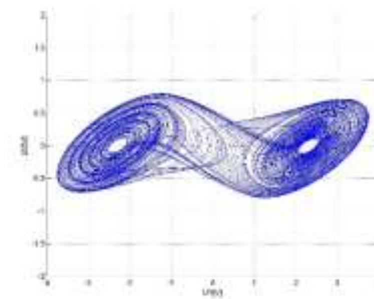
$R = 1766\ \Omega$



$1464\ \Omega < R < 1721\ \Omega$



$1540\ \Omega < R < 1757\ \Omega$

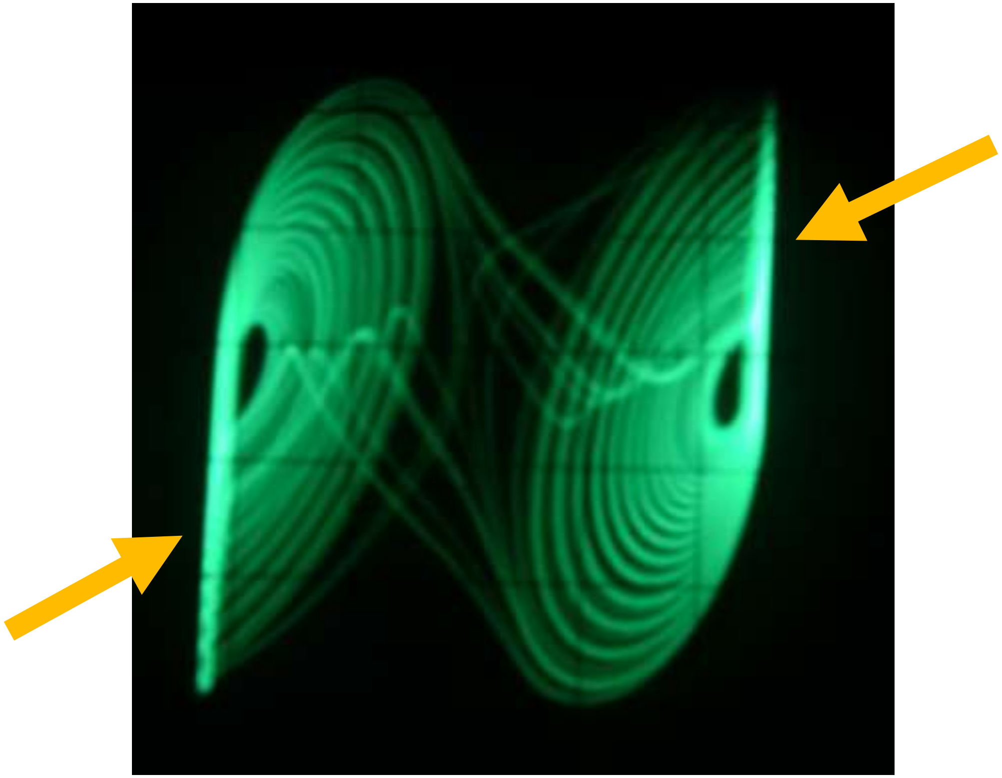


Varying Inductance

- Changes only width of range (for R,C) when chaos occurs
- Has no any other interesting effect

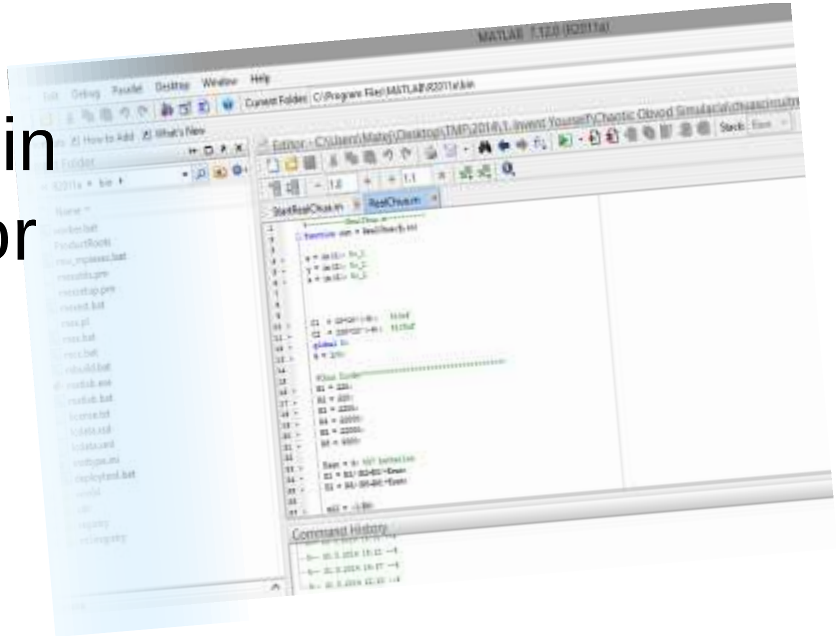
Varying Capacity

- Changes width of range (for R,L) when chaos occurs
- When using too small capacitance (e.g. 10 nF for our circuit) → saturation of capacitor



Bifurcation Diagrams

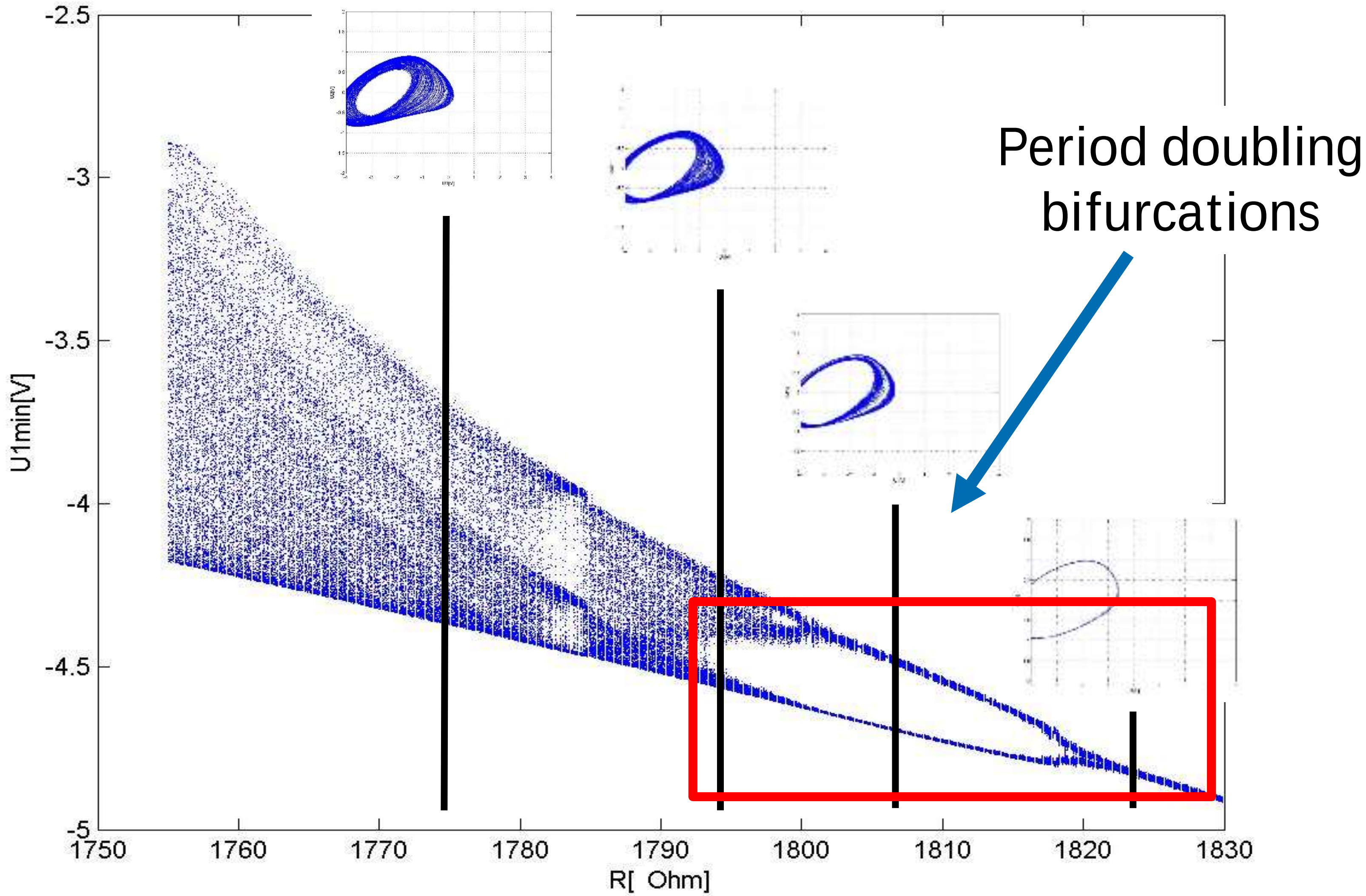
- Looking at local minima in voltage over 1st capacitor



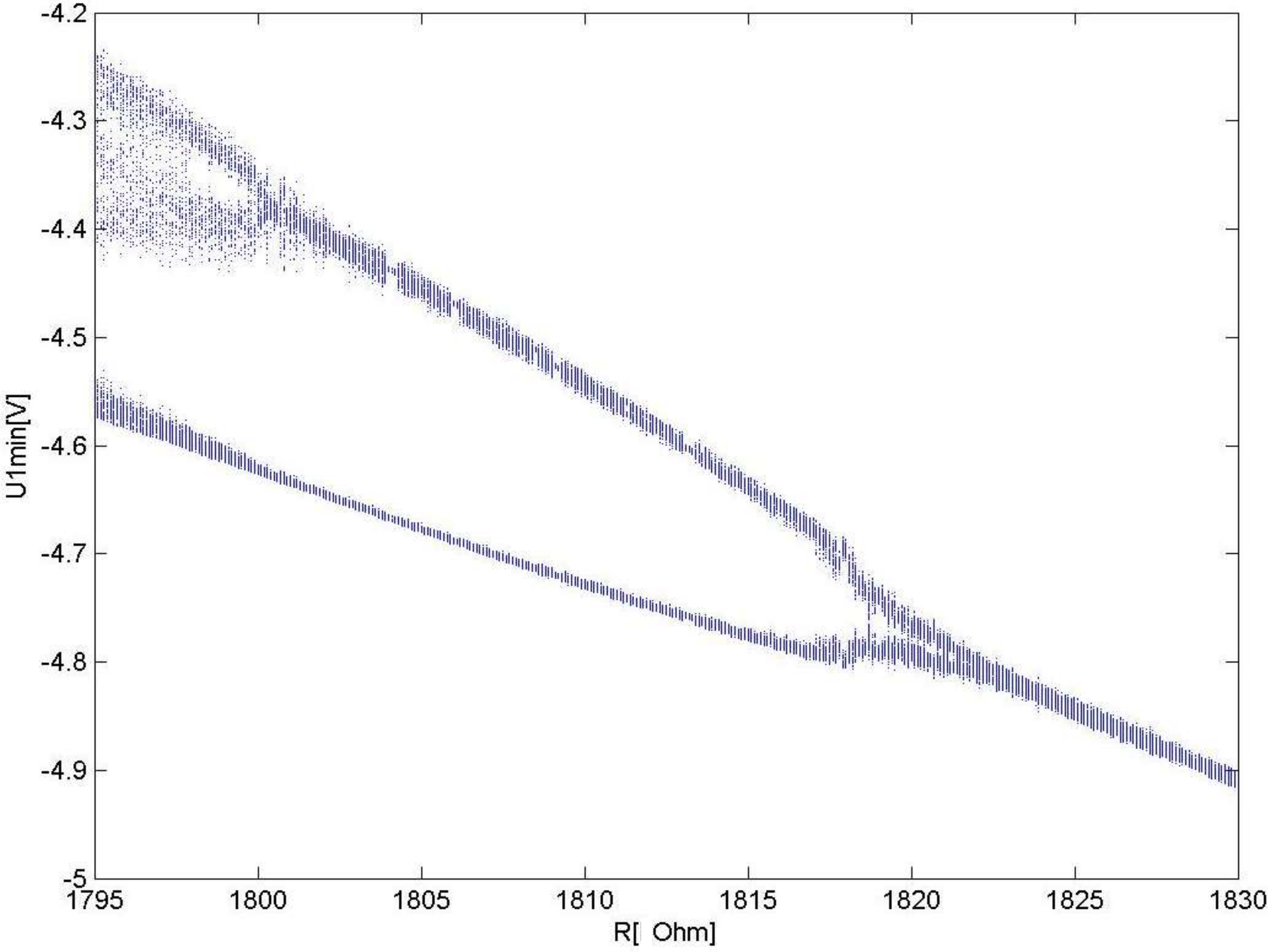
- Changing resistance, capacitance, inductance

- Using simulation to numerically solve for voltage over 1st capacitor

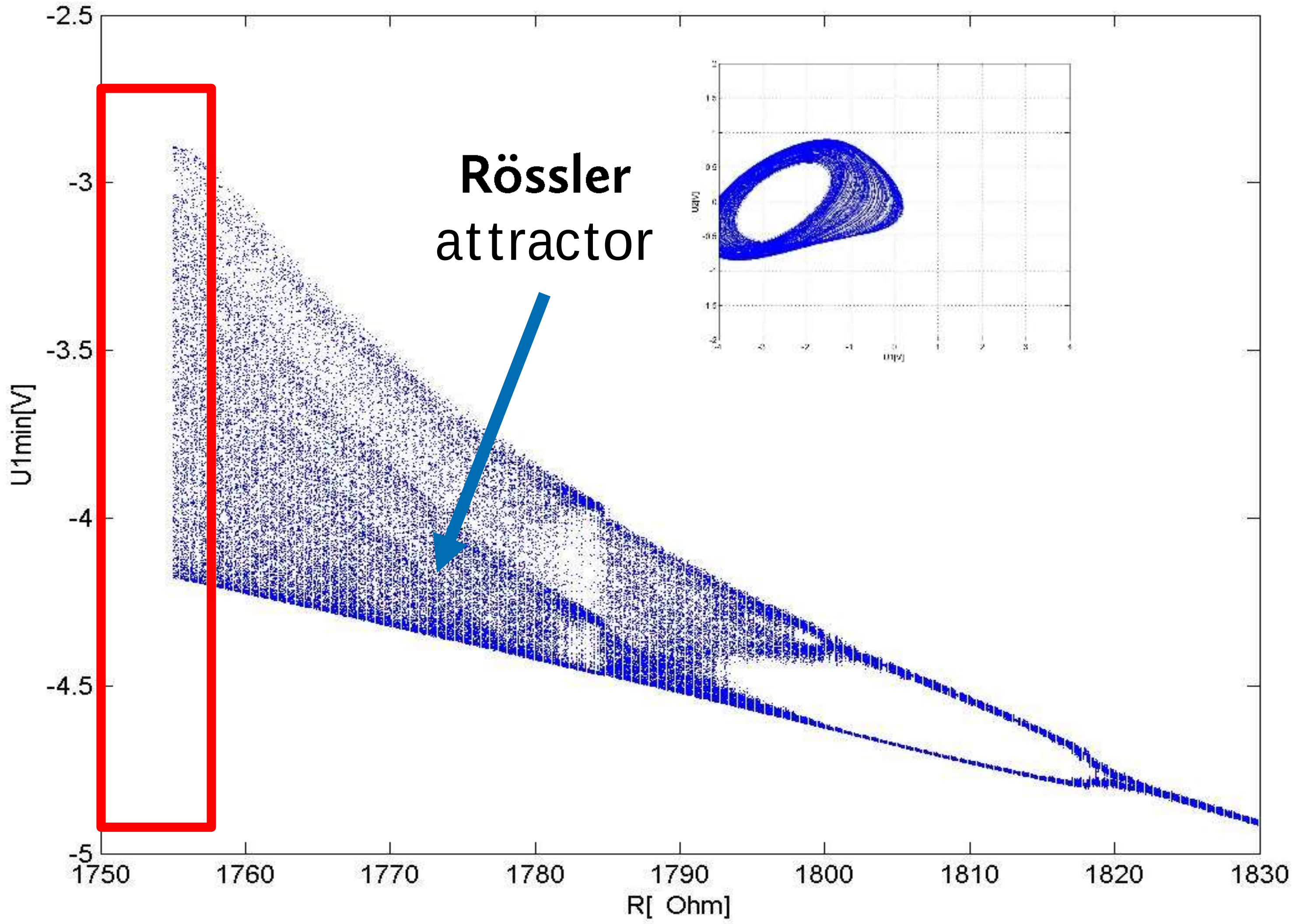
Bifurcation Diagram – Changing R



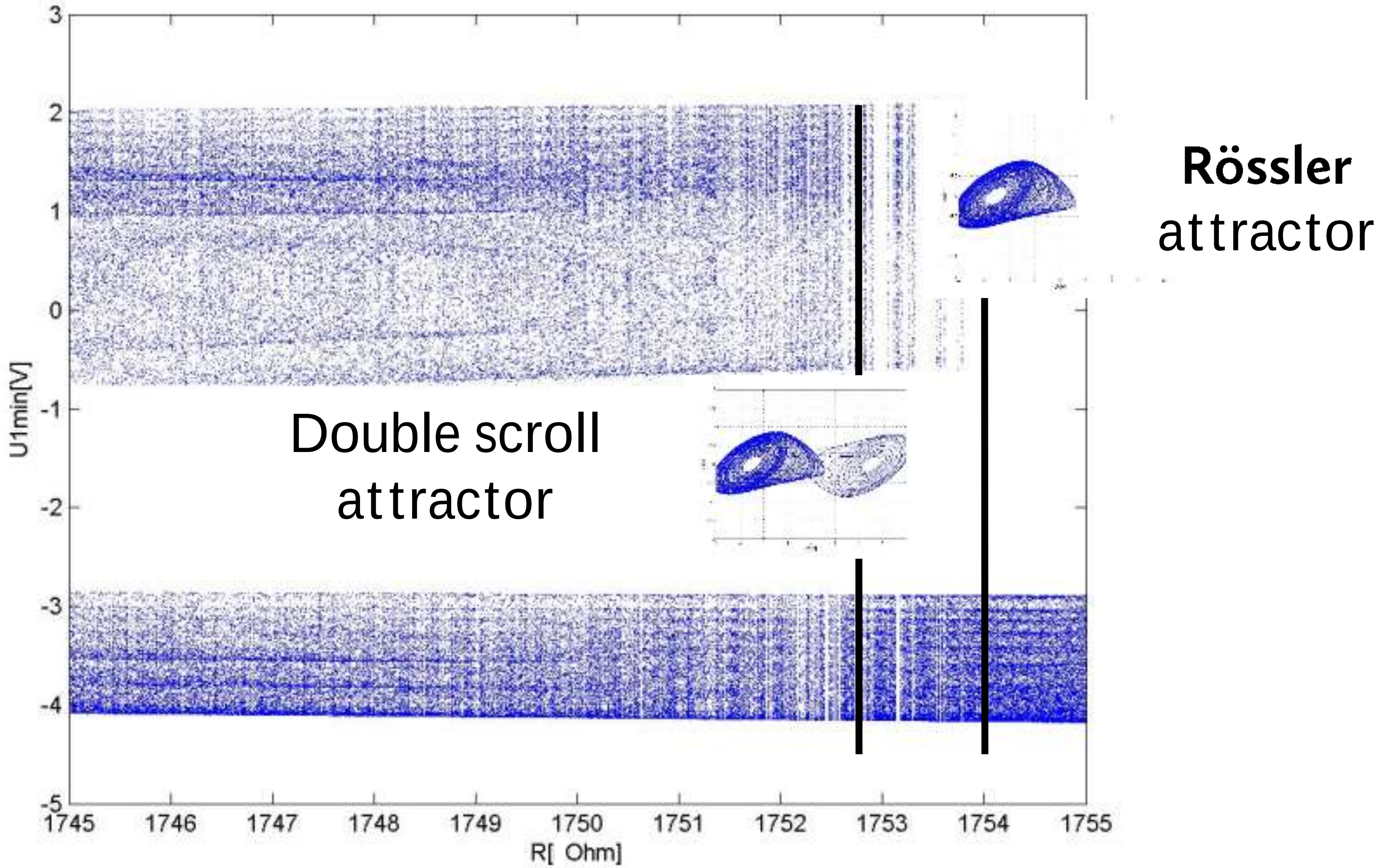
Period Doubling Bifurcation



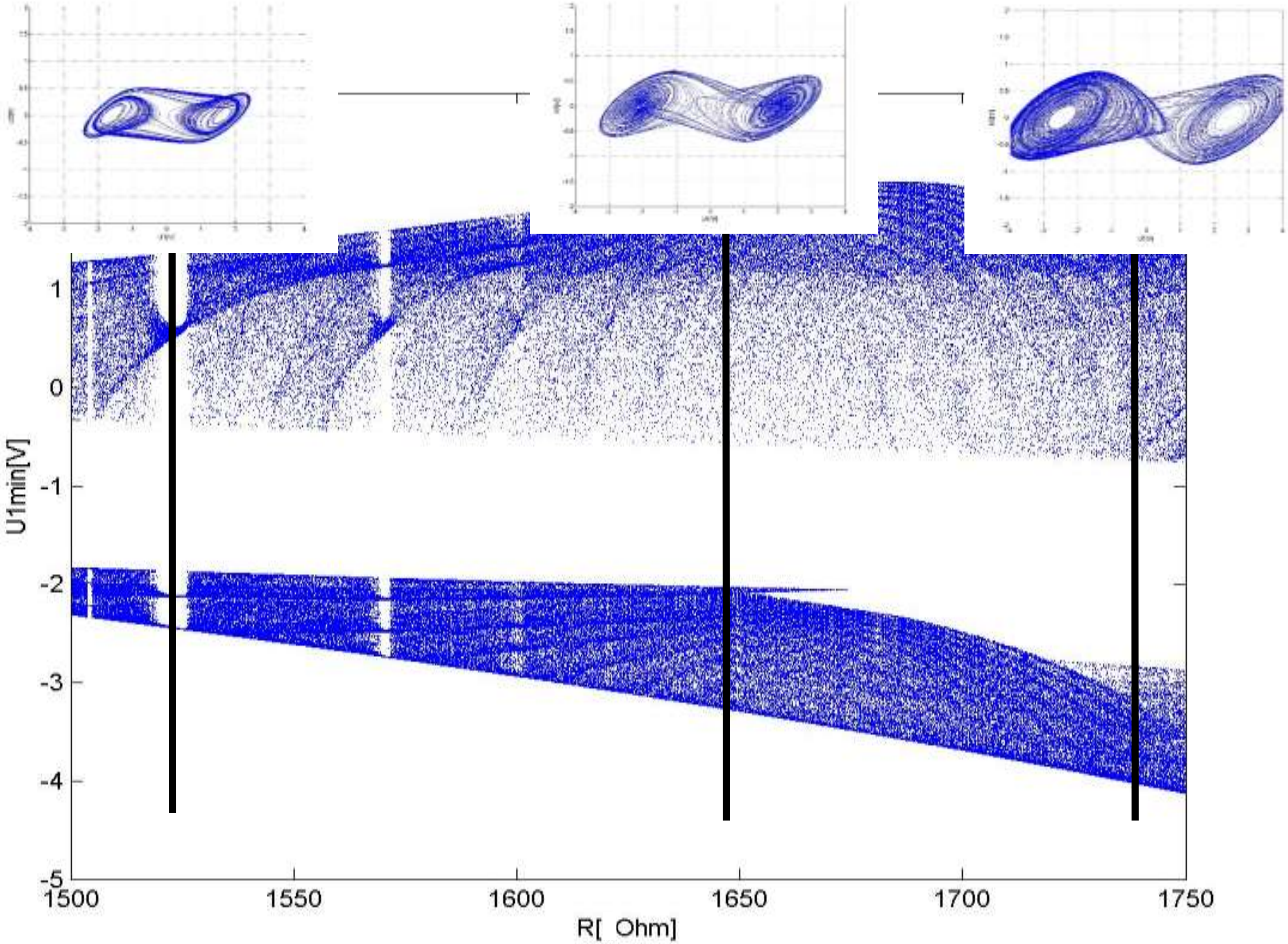
Bifurcation Diagram – Changing R



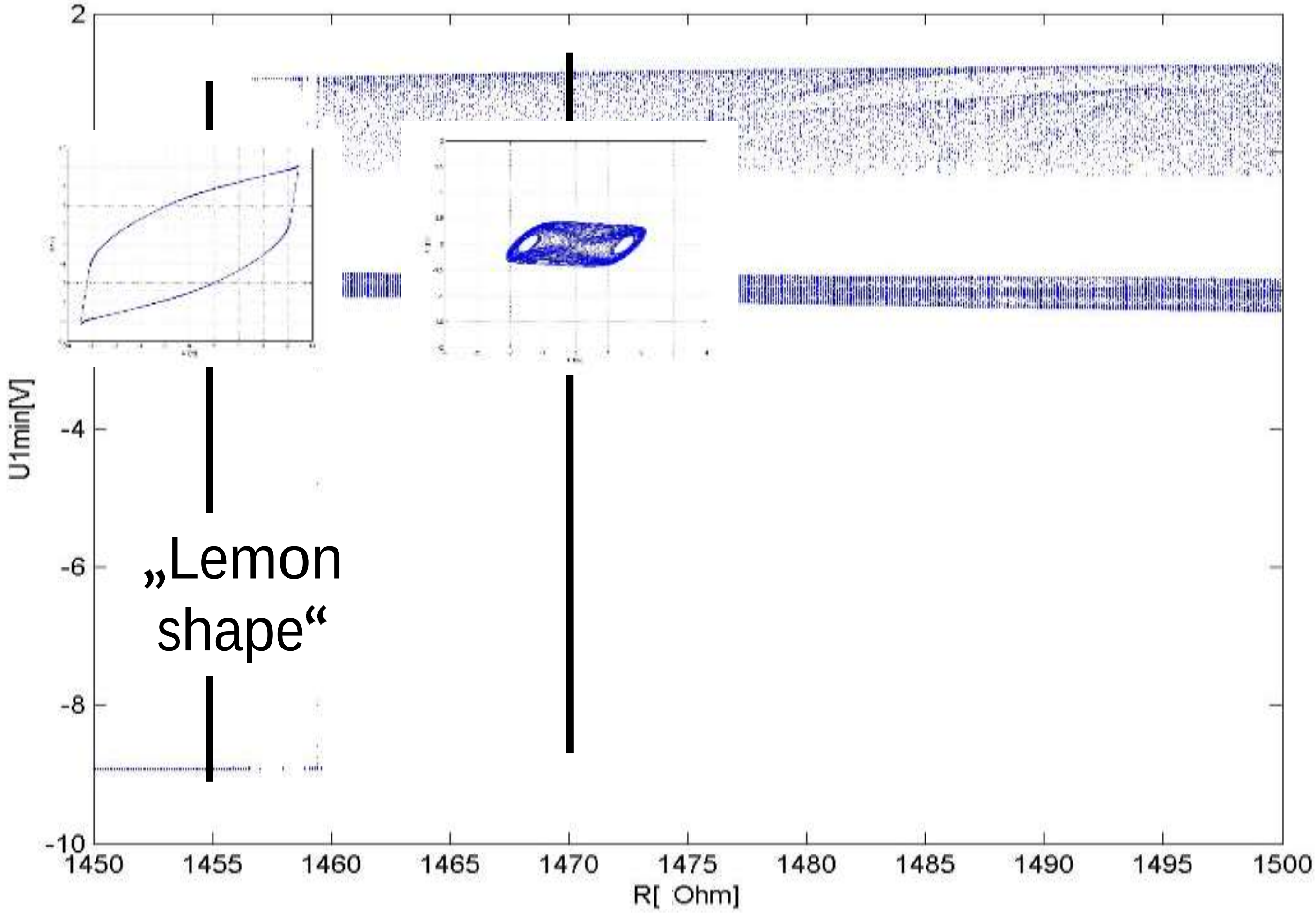
Start of the Chaos Region



Chaos



End of Chaos Region

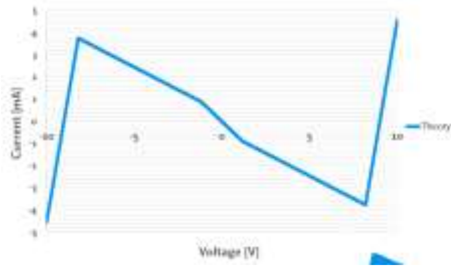


Conclusion

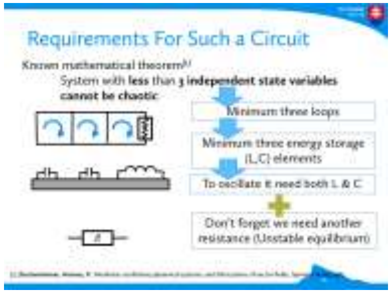
Thank you for your attention!

Deterministic Chaos

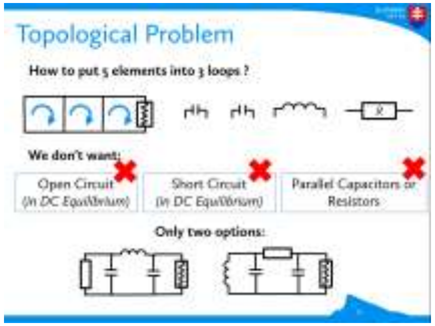
Nonlinearity is needed



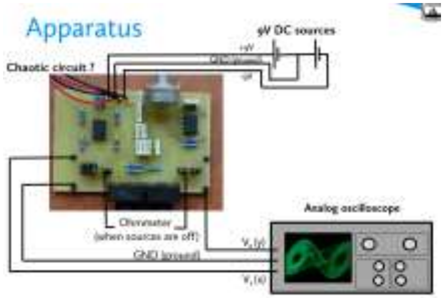
Requirements for a circuit



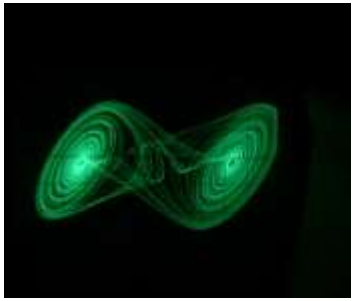
Found "chaotic" topology



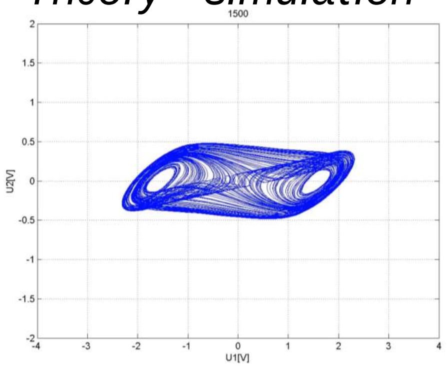
We've built one



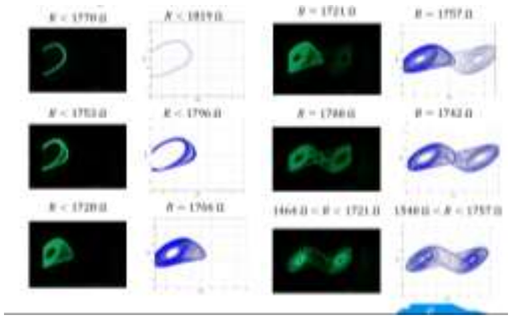
Phase space



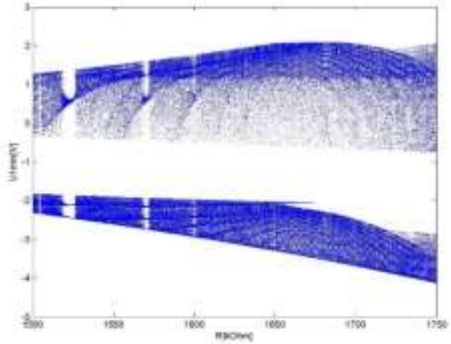
Theory - Simulation



Comparison



Bifurcation Diagrams



APPENDIX

Lyapunov exponents

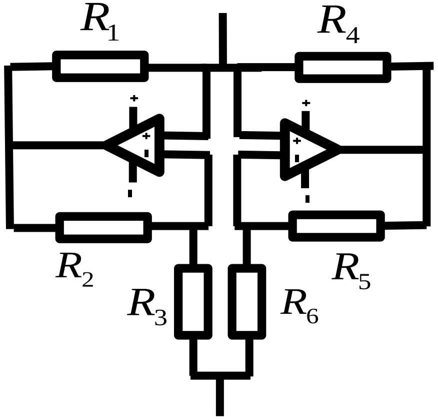
- [Parlitz, **Lyapunov exponents from the Chua's circuit**, Journal of Circuits, Systems and Computers, Vol. 3, No. 2, 1993]

- Quite hard to compute and get from exp. data

- Could be calculated from dimensionless diff. equations (Initial conditions – [0,0,0])

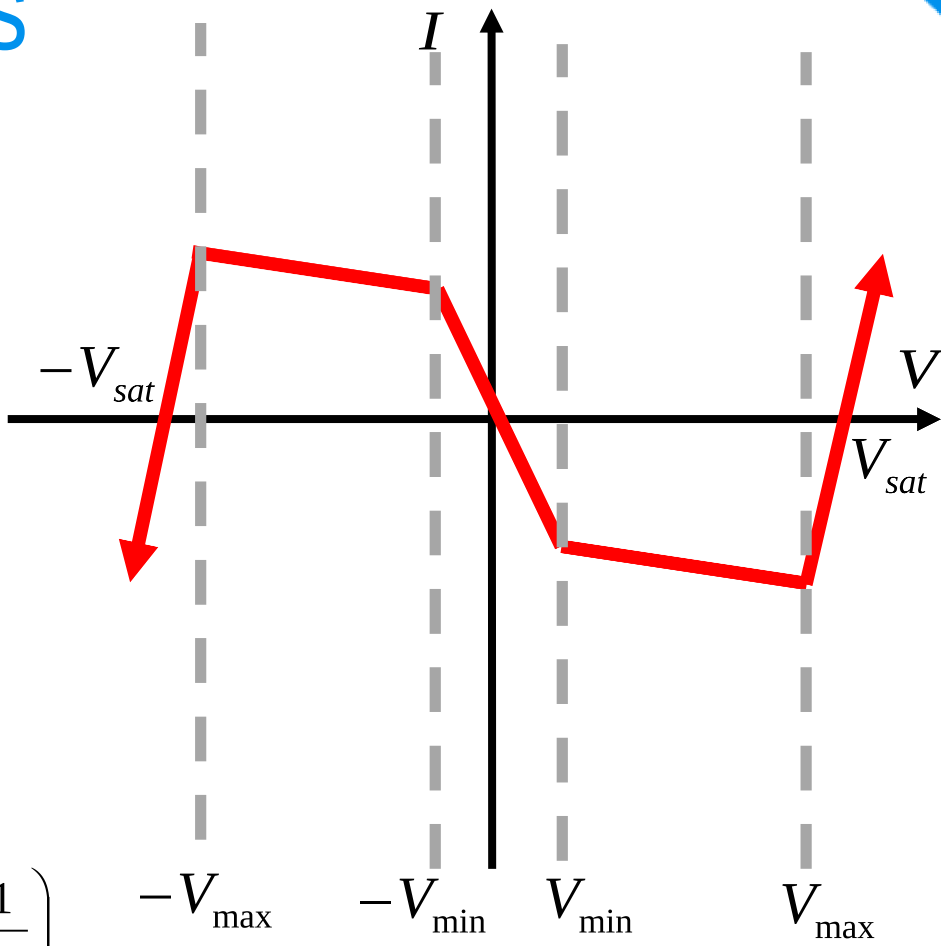
$$\lambda_{V_1} = 0.34 \frac{1}{RC_2} \qquad \lambda_{V_2} = 0 \qquad \lambda_{I_L} = -5.9 \frac{1}{RC_2}$$

I-V Characteristics



$$V_{\min} = \frac{R_6}{R_5 + R_6} V_{sat}$$

$$V_{\max} = \frac{R_3}{R_2 + R_3} V_{sat}$$



$$|V| < V_{\min} : I_{(V)} = -V\left(\frac{1}{R_6} + \frac{1}{R_3}\right)$$

$$|V| < V_{\max} : I_{(V)} = V\left(-\frac{1}{R_3} + \frac{1}{R_4}\right) - \operatorname{sgn}(V) V_{\min}\left(\frac{1}{R_6} + \frac{1}{R_4}\right)$$

$$|V| > V_{\max} : I_{(V)} = V\left(\frac{1}{R_1} + \frac{1}{R_4}\right) - \operatorname{sgn}(V) V_{\max}\left(\frac{1}{R_3} + \frac{1}{R_1}\right) - \operatorname{sgn}(V) V_{\min}\left(\frac{1}{R_6} + \frac{1}{R_4}\right)$$

Requirements for Nonlinear Resistor

- Active resistor
- 2 or more Unstable equilibria positions (intersections with load line)

↓ Negative slope

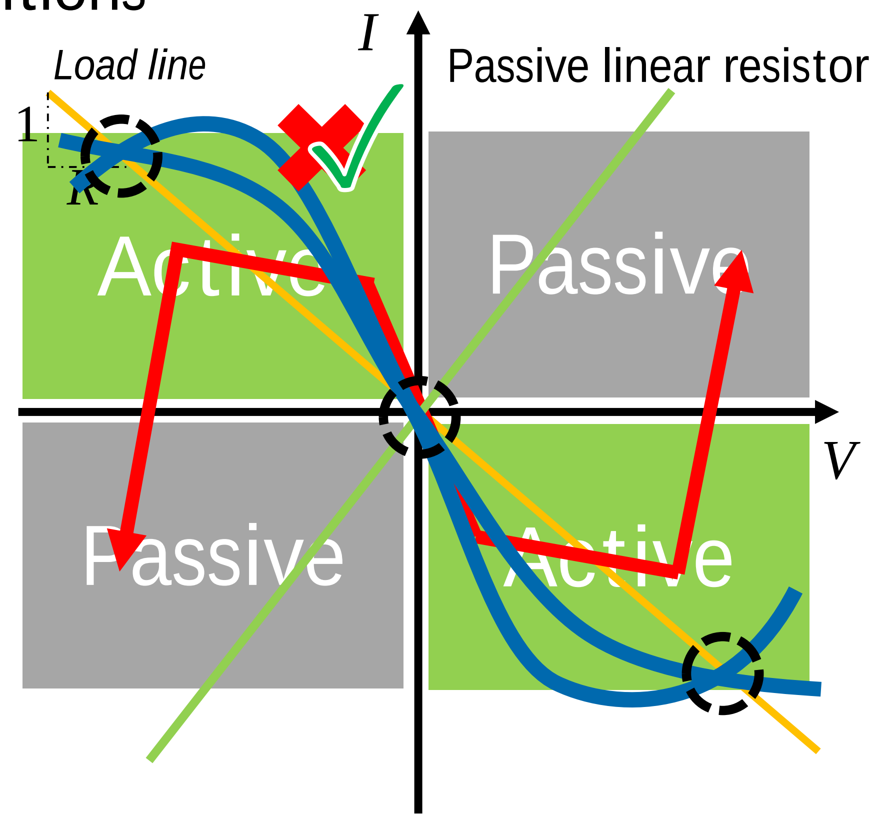
- Easily realizable (Using op-amps)

↓ Piecewise for simplicity

- Physically realizable

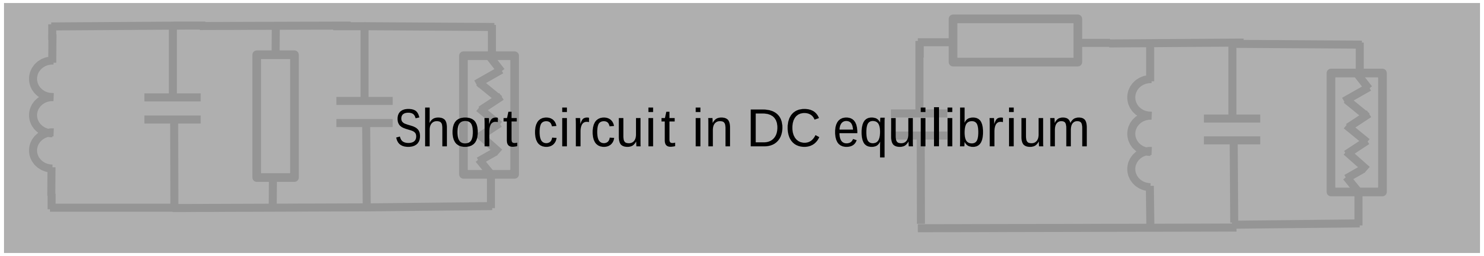
↓ Eventually passive (With positive slope)

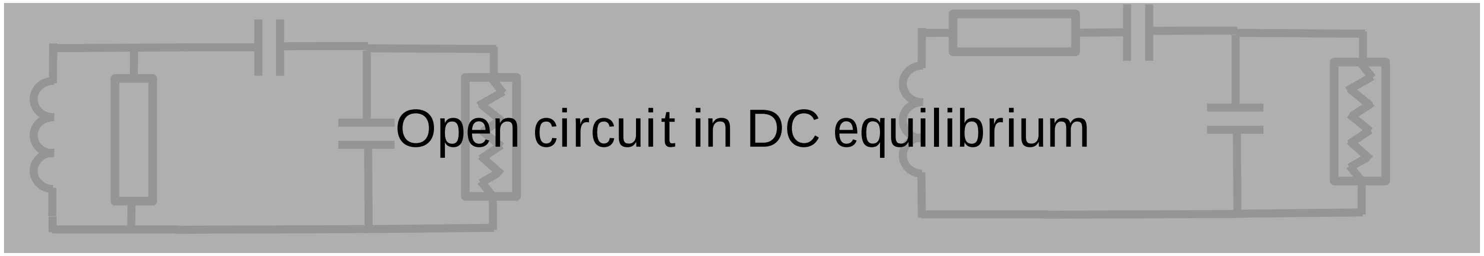
Let's look at I-V Characteristic:

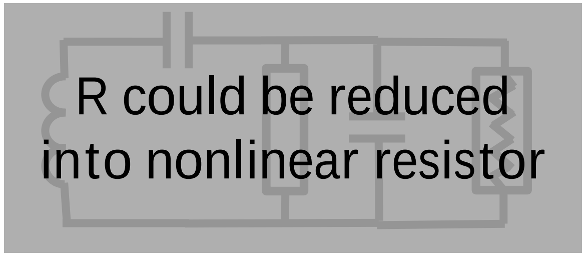


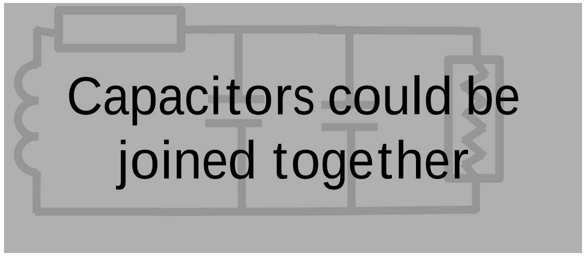
Possible candidates

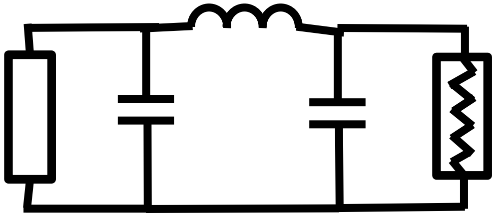
Totally 8 different possible topologies

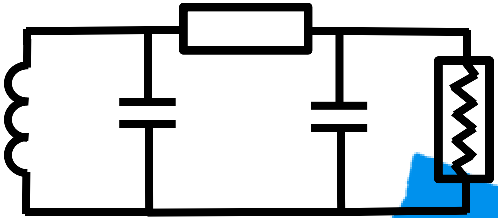






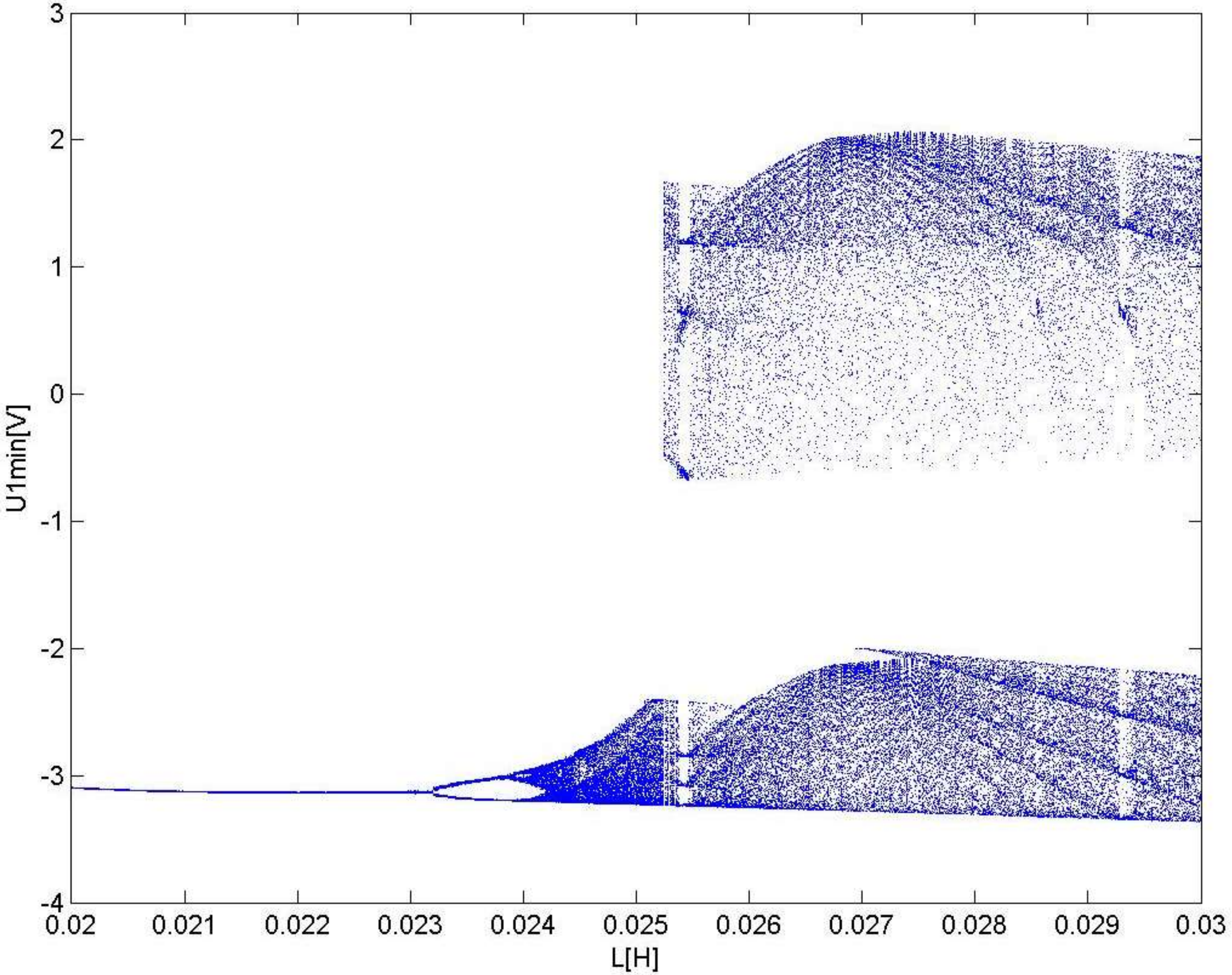






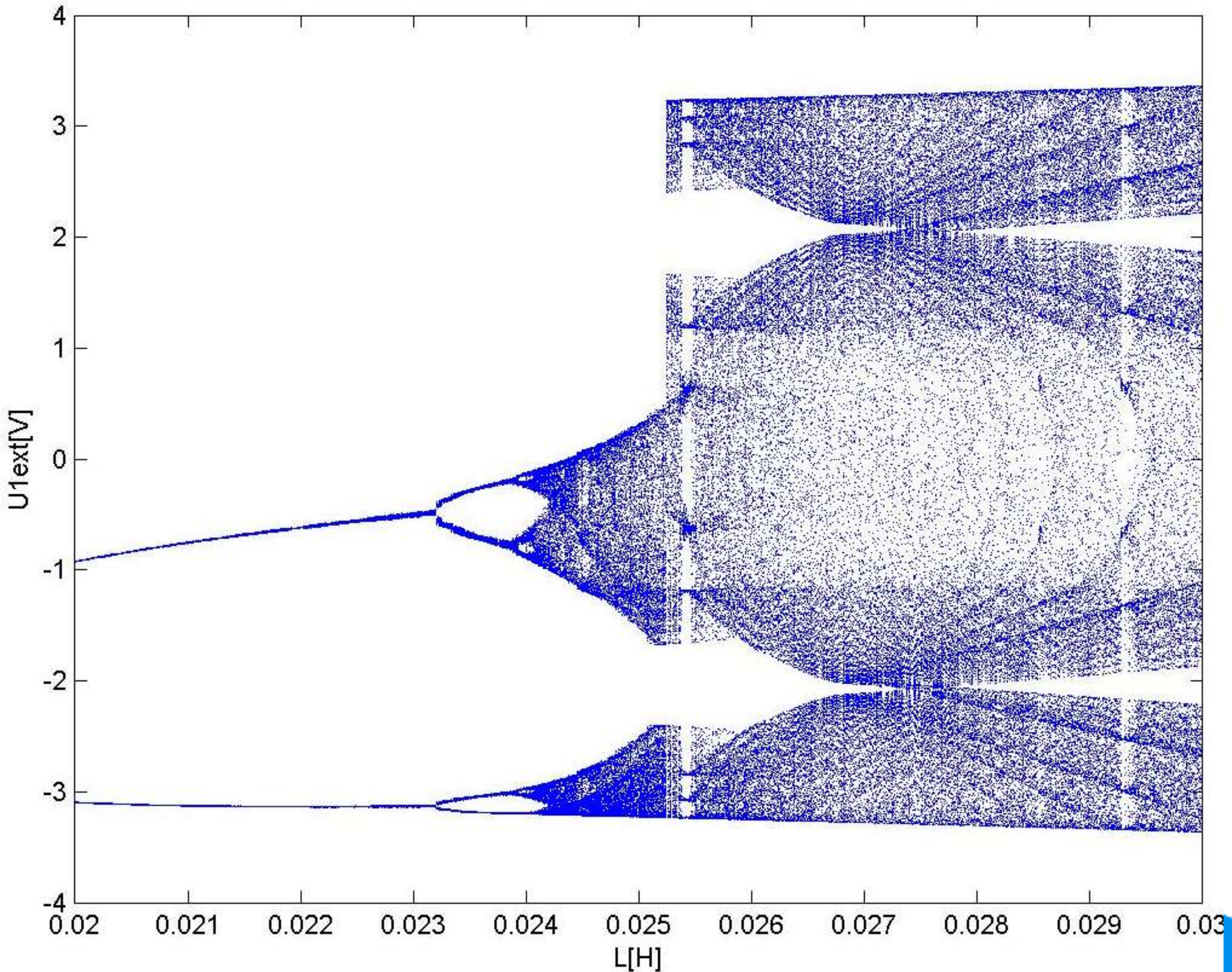
Bifurcation Diagram L

Local minima in voltage over 1st capacitor



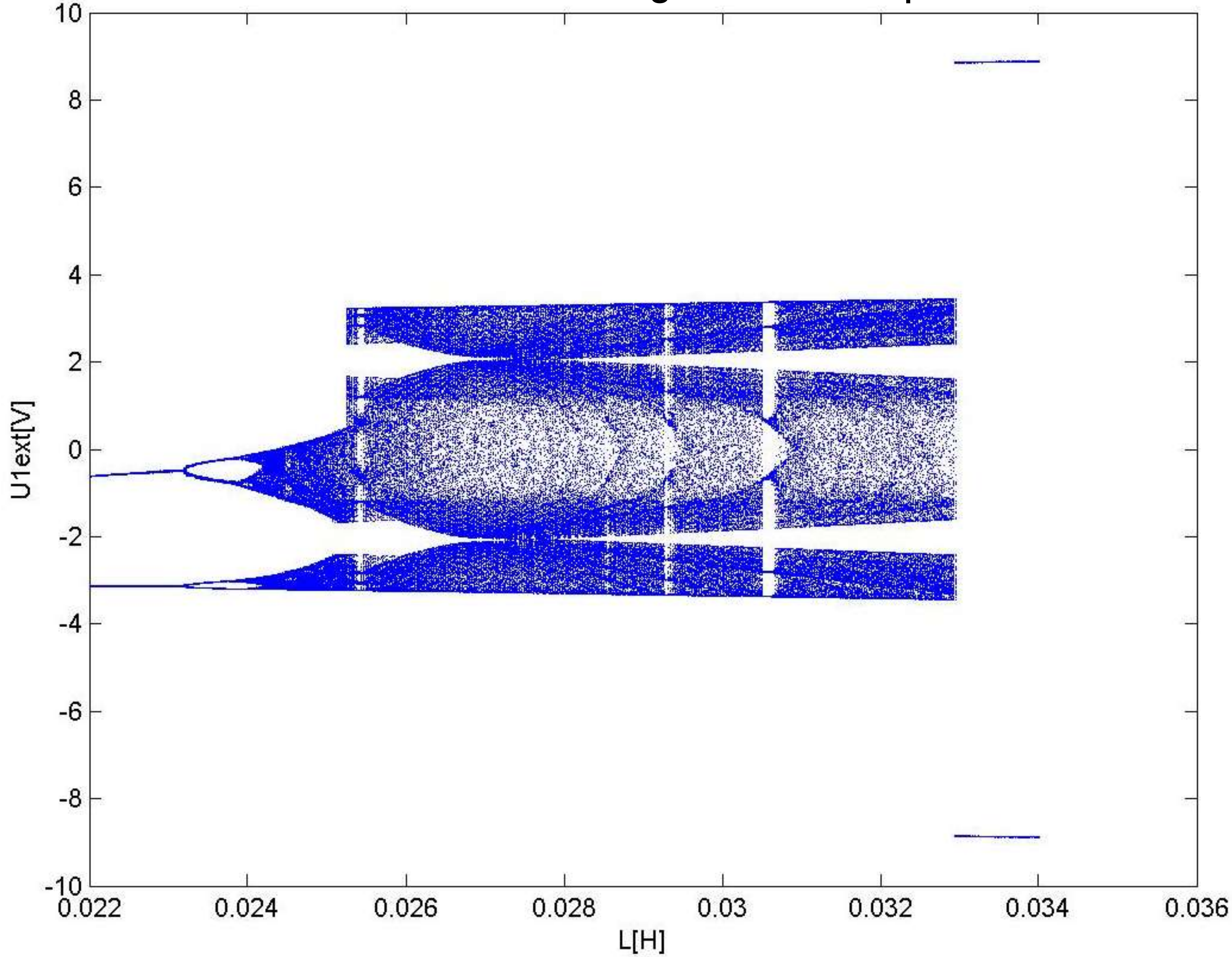
Bifurcation Diagram L

Local extremes in voltage over 1st capacitor



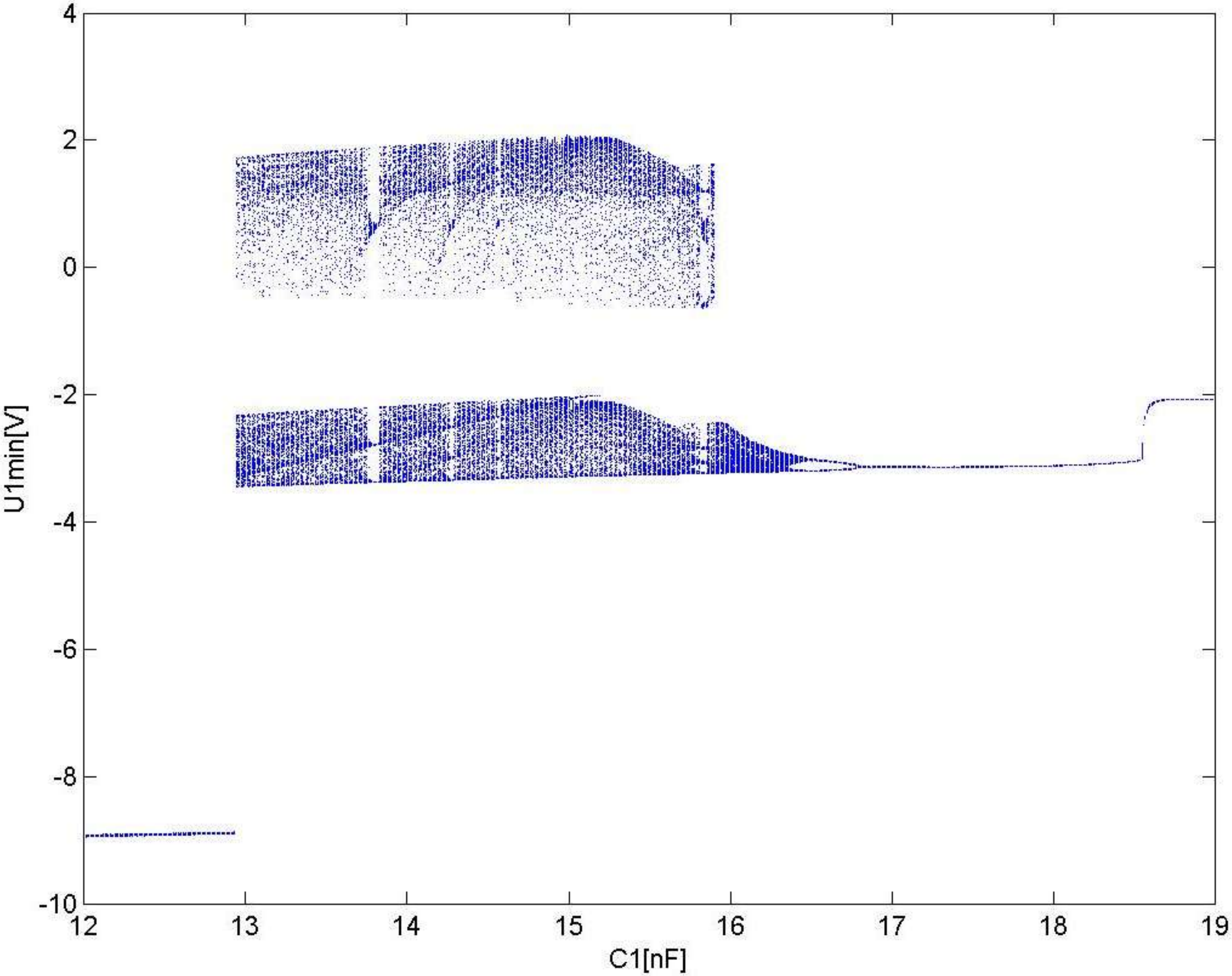
Bifurcation Diagram L

Local extremes in voltage over 1st capacitor



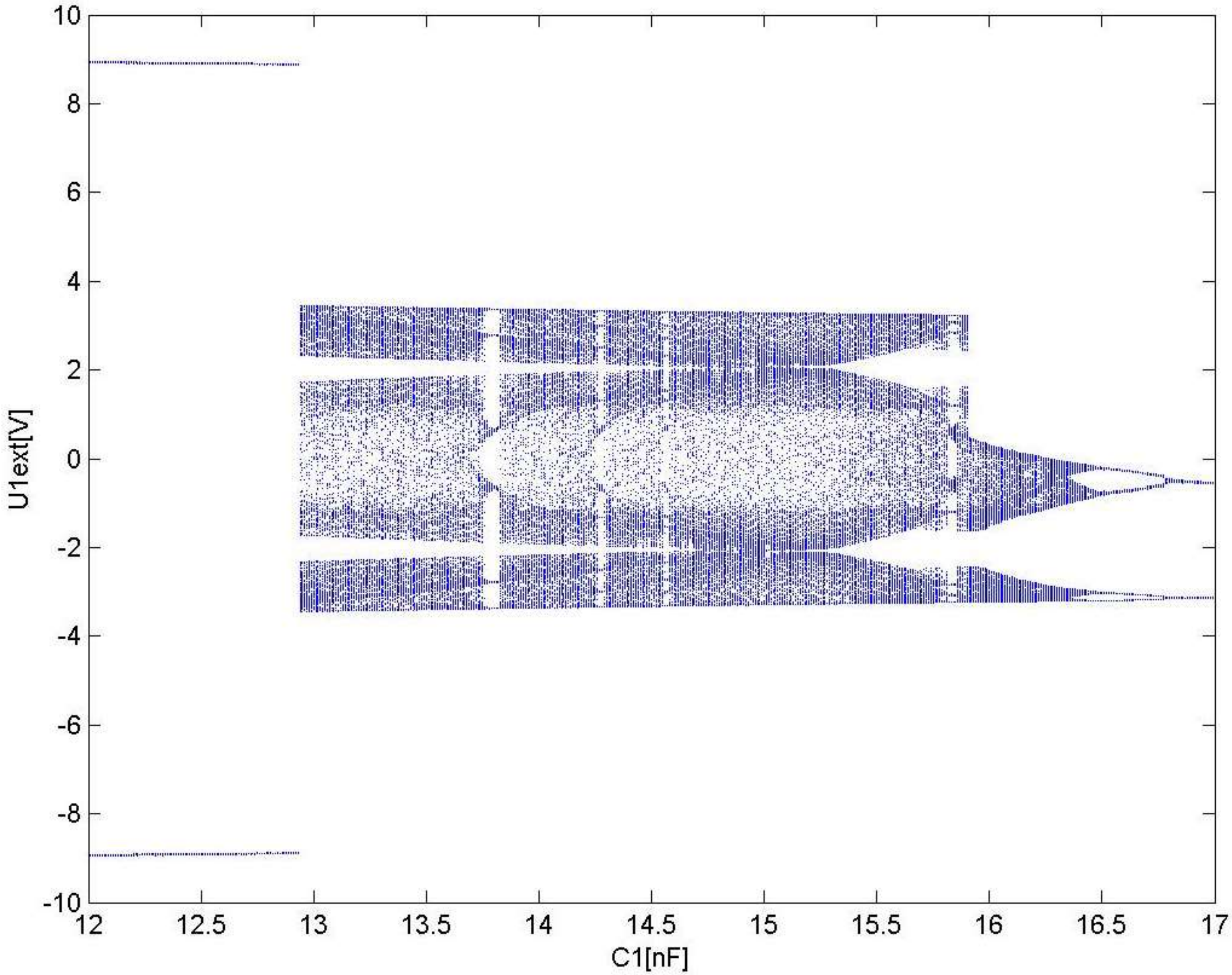
Bifurcation Diagram C

Local minima in voltage over 1st capacitor



Bifurcation Diagram C

Local extremes in voltage over 1st capacitor



Deterministic Chaos

[*Hasselblatt, Boris; Anatole Katok (2003).* A First Course in Dynamics: With a Panorama of Recent Developments. *Cambridge University Press]:*

- <u>No</u> universally accepted <u>mathematical</u> <u>definition</u> of chaos
- Commonly used definition
  Chaotic = have these properties:

  1. Sensitivity to the initial conditions

  2. “**Dense periodic orbits**”
     (Every point of phase space could be arbitrarily closed approached by periodical orbit)

  3. “**Topological mixing**” *(Any given region or open set of its phase space will eventually overlap with any other given region.)*