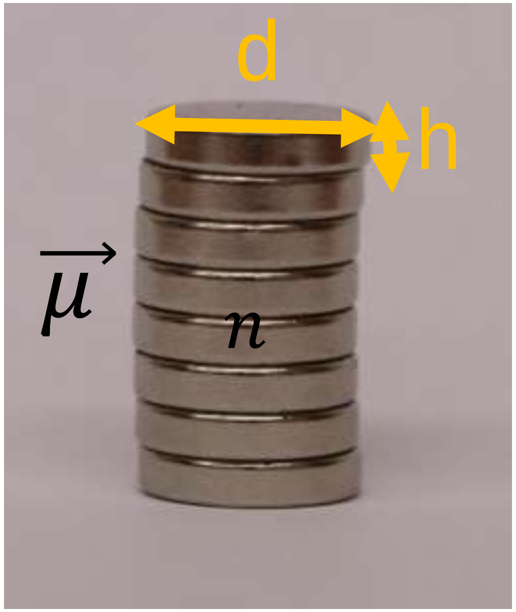
16

Magnetic Brakes

Matej Badin

Task

When a strong magnet falls down a non-ferromagnetic metal tube, it will experience a retarding force. Investigate the phenomenon.



$\vec{\mu}$ Magnetic moment

σ El. conductivity

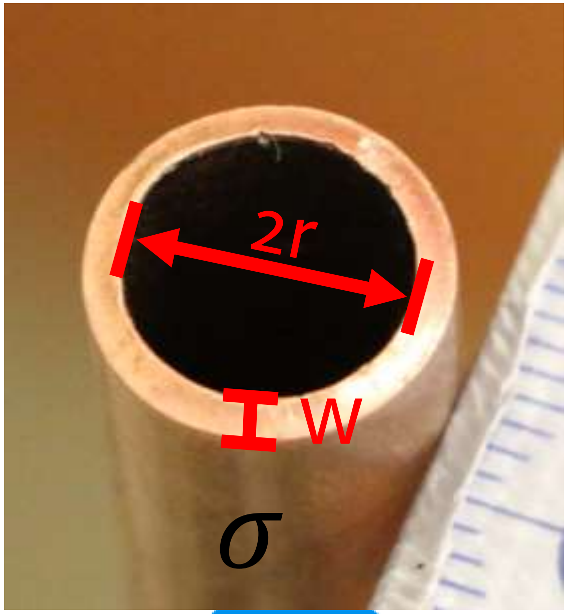
d Diameter

h Magnet height

n Number of magnets

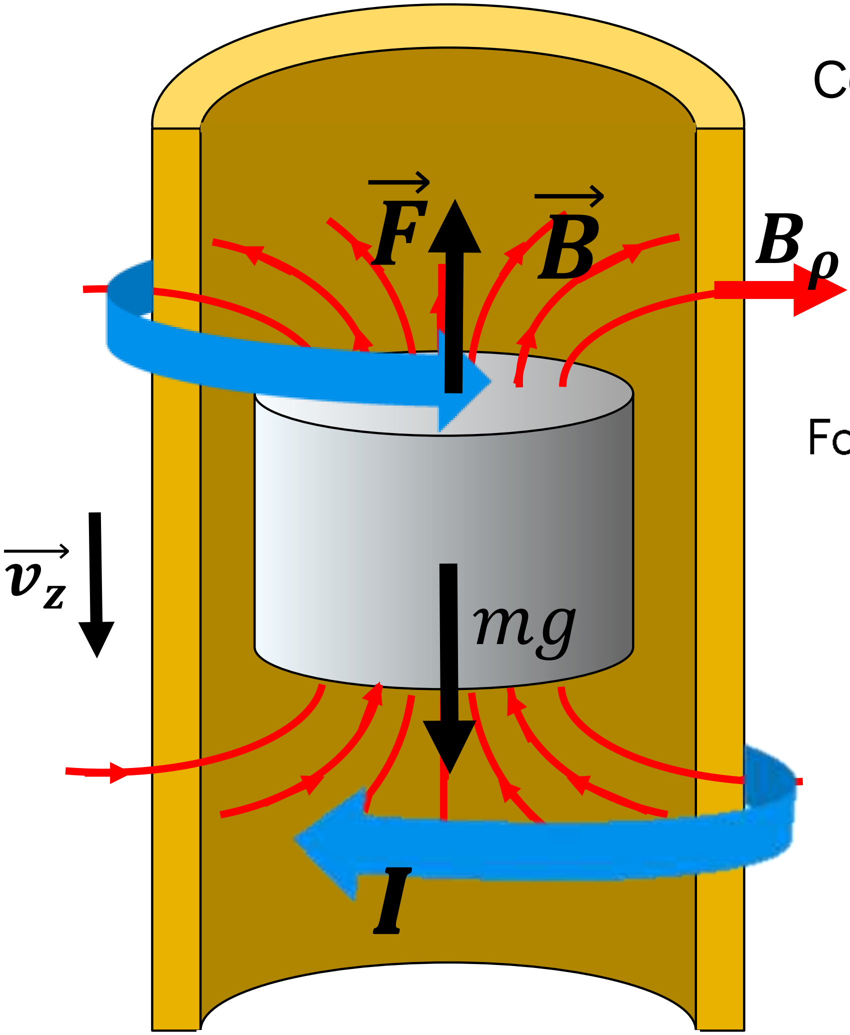
r Radius of tube

w Width of wall



What Happens?

Qualitative Explanation



Consequence of an electromagnetic induction

$$I \sim -\sigma \frac{d\emptyset_z}{dt} \sim \sigma v_z B_\rho$$

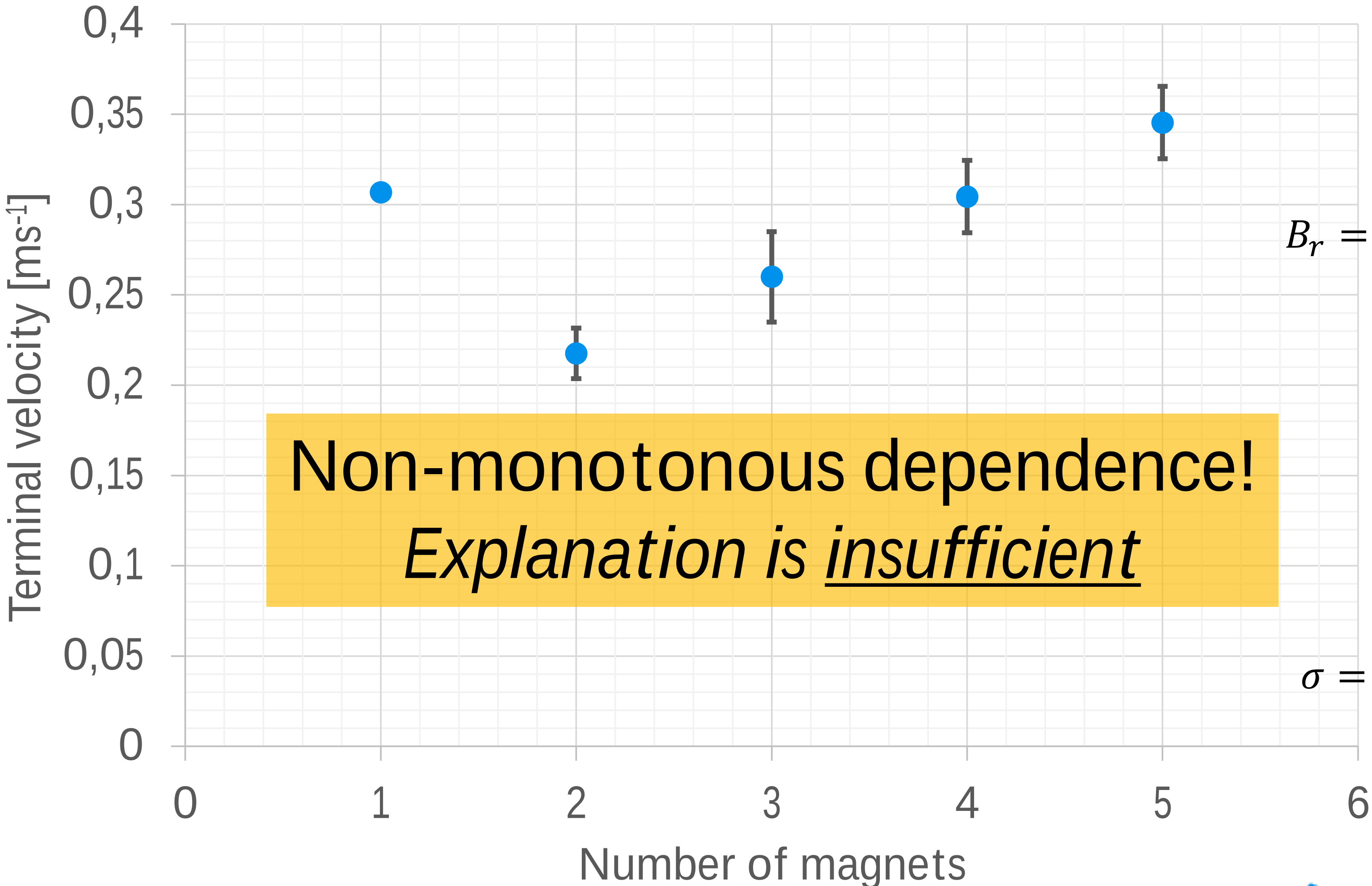
Force acting on a tube in magnet's ref. frame

$$F \sim B_\rho I 2\pi r \sim v_z \sigma B_\rho^2 2\pi r$$

Terminal velocity

$$v_z \sim \frac{mg}{\sigma B_\rho^2 r}$$

Is It Really so Simple?



Non-monotonous dependence!

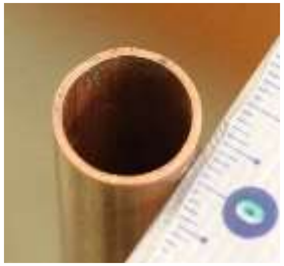
Explanation is *<u>insufficient</u>*

$B_r = 1{,}19 \pm 0{,}02\ T$

$m = 1{,}06\ g$

$h = 5\ mm$

$d = 6\ mm$



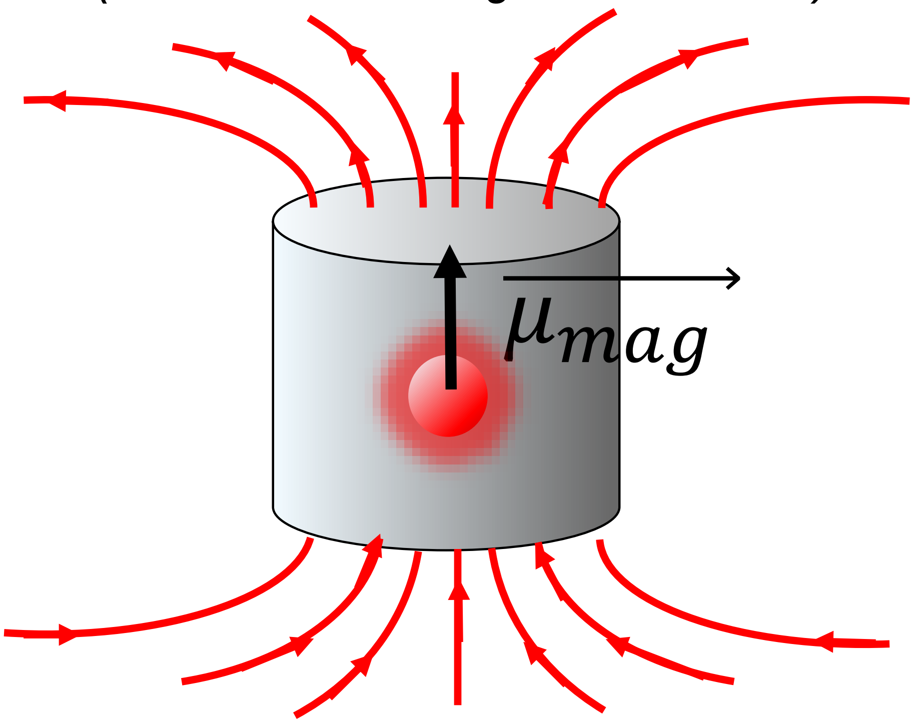
$\sigma = 42 \cdot 10^6\ Sm^{-1}$

$r = 6{,}5\ mm$

$w = 1{,}1\ mm$

Magnetic Dipole Model

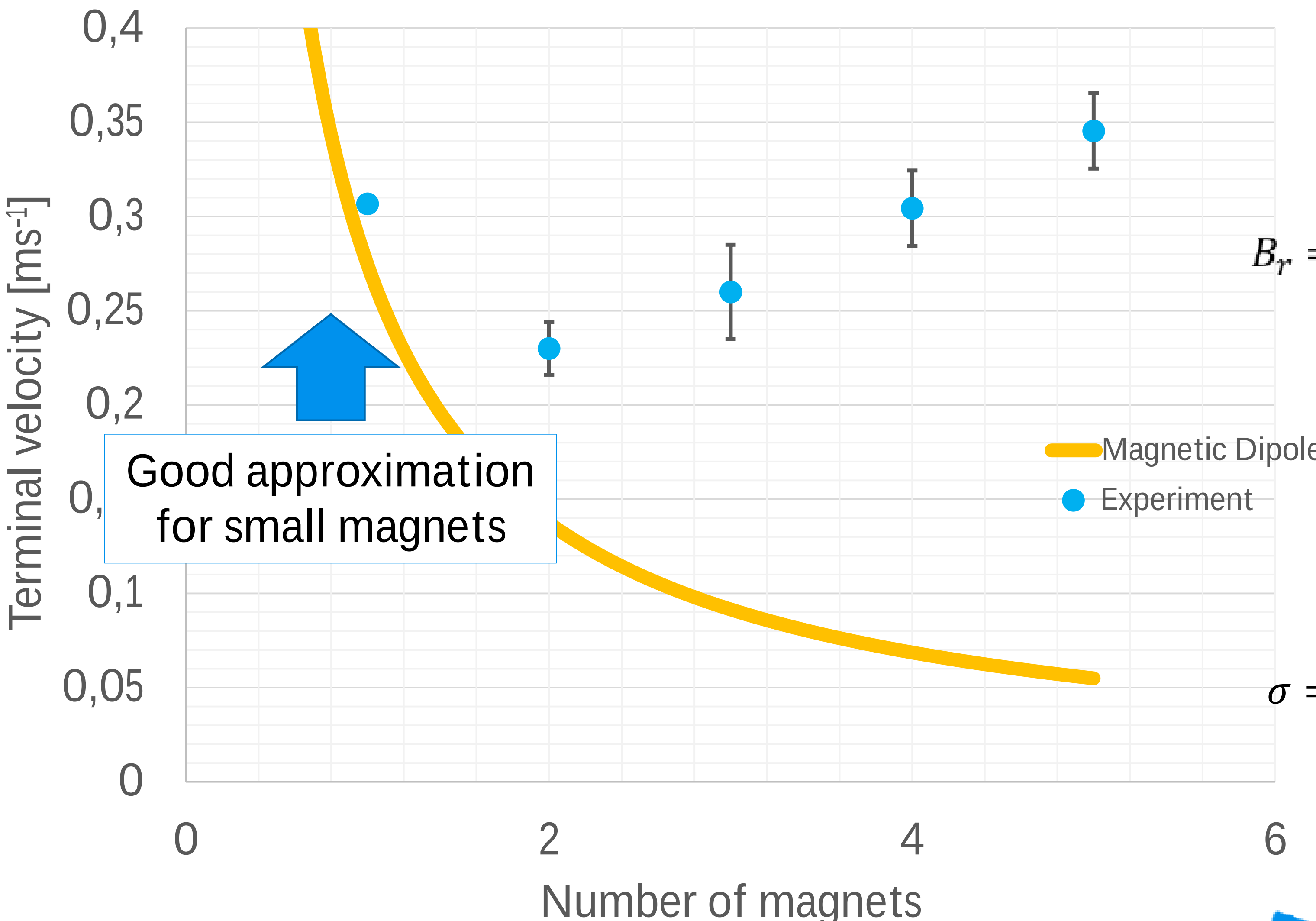
Threatening the magnetic field as the one created by magnetic dipole
(with the same magnetic moment)



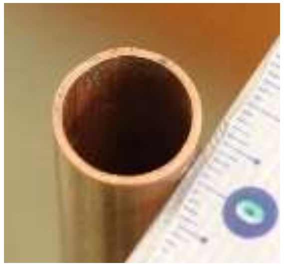
$$F = \frac{45\mu_0^2 \sigma \mu_{mag}{}^2 v}{1024} \frac{w}{r^4}$$

(Derivation in appendices)

Magnetic Dipole Model



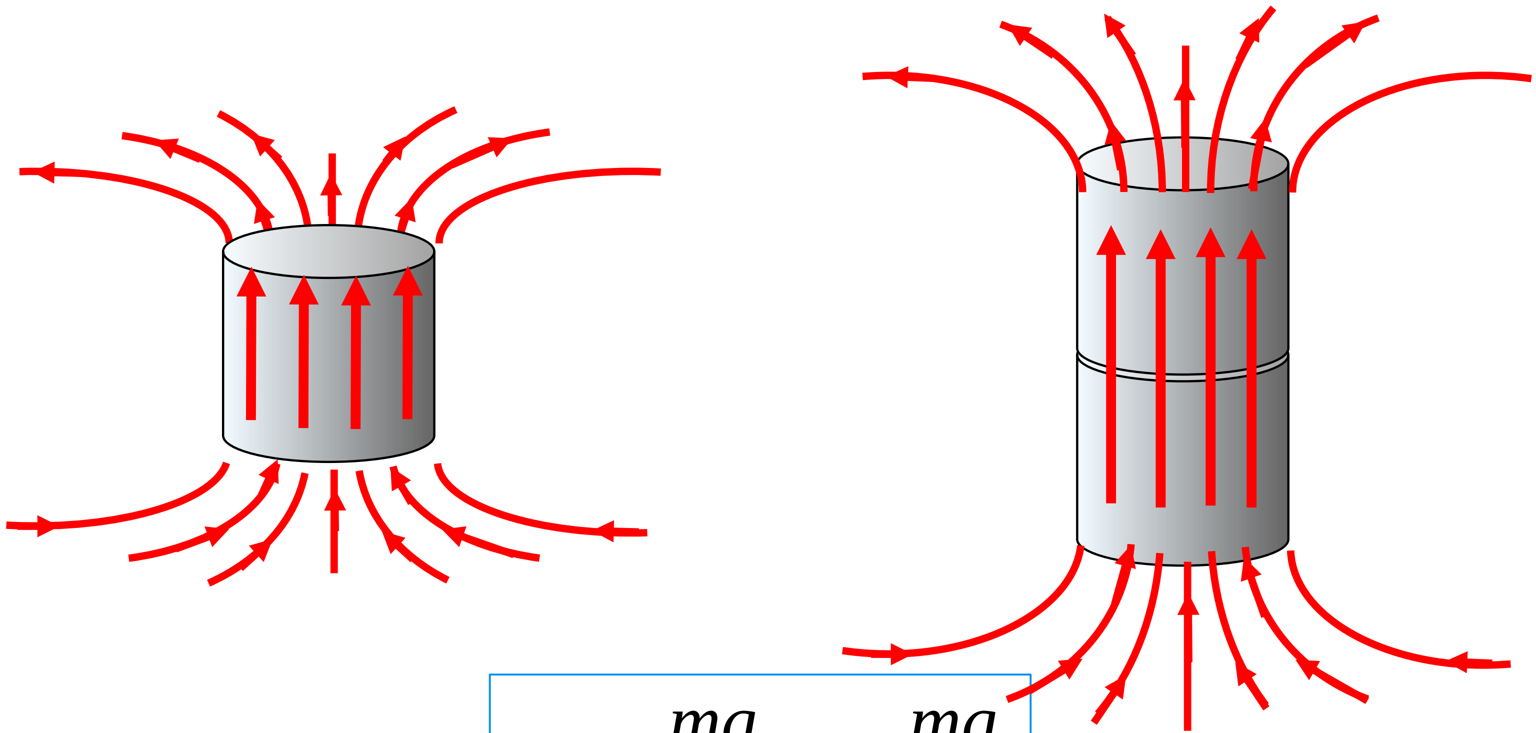
$$B_r = 1{,}19 \pm 0{,}02\ T$$
$$m = 1{,}06\ g$$
$$h = 5\ mm$$
$$d = 6\ mm$$



$$\sigma = 42 \cdot 10^6\ Sm^{-1}$$
$$r = 6{,}5\ mm$$
$$w = 1{,}1\ mm$$

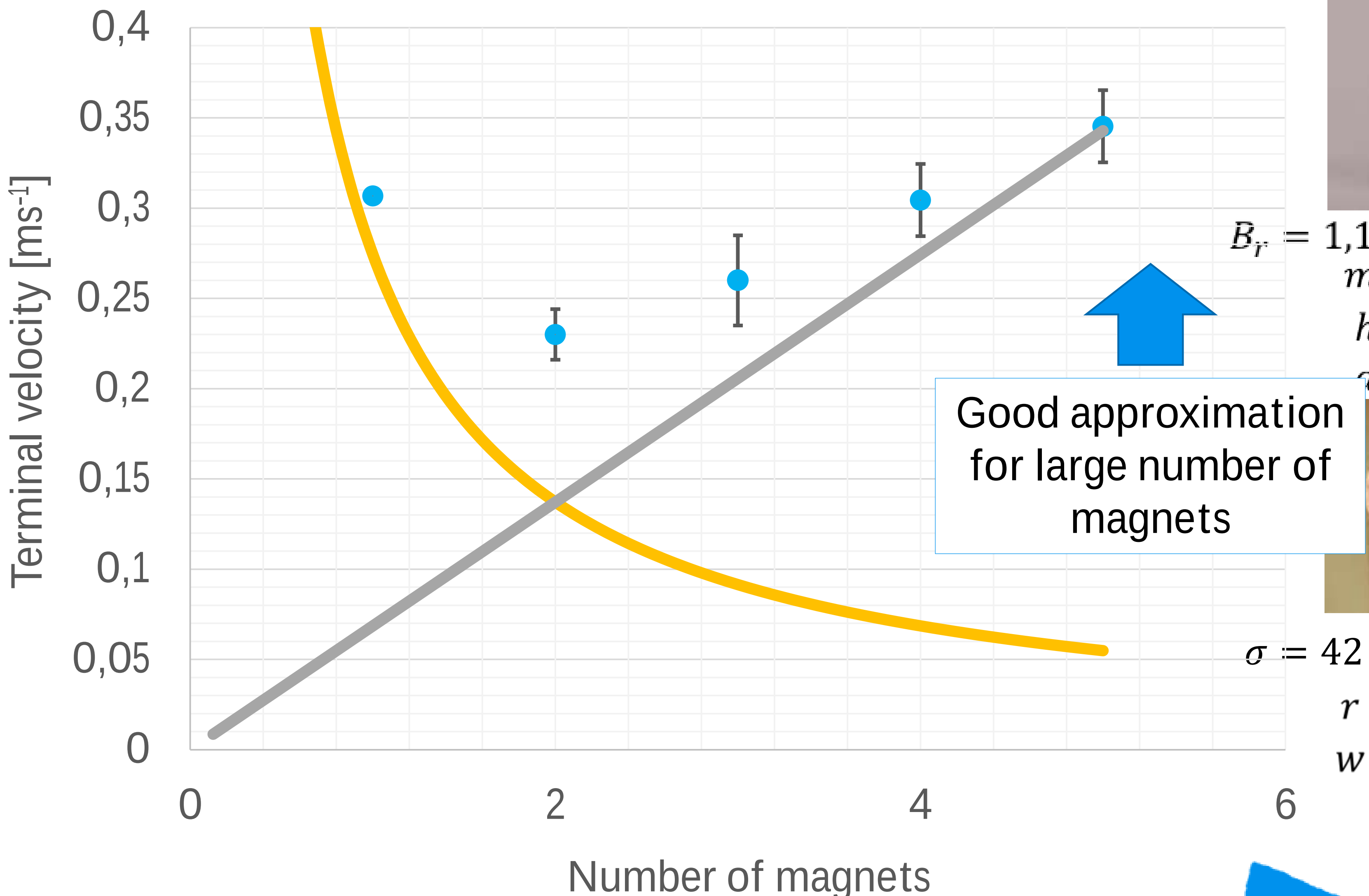
High-number of magnet limit

Flux of the magnetic field to the side effectively doesn't change
(for large number of magnets)



$$v \sim \mathrm{n}\frac{mg}{B_{eff}^{2}} \sim \mathrm{n}\frac{mg}{\mu_{eff}^{2}}$$

High-number of magnet limit



$B_r = 1{,}19 \pm 0{,}02\ T$

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$h = 5\ mm$

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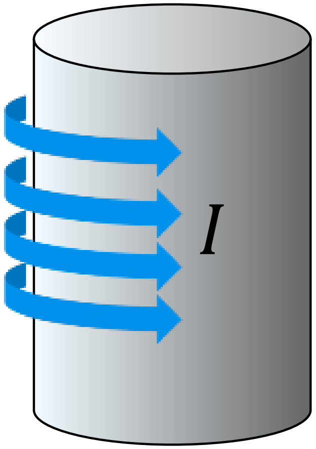
$\sigma = 42 \cdot 10^6\ Sm^{-1}$

$r = 6{,}5\ mm$

$w = 1{,}1\ mm$

Magnetic Dipole Model: Too rough



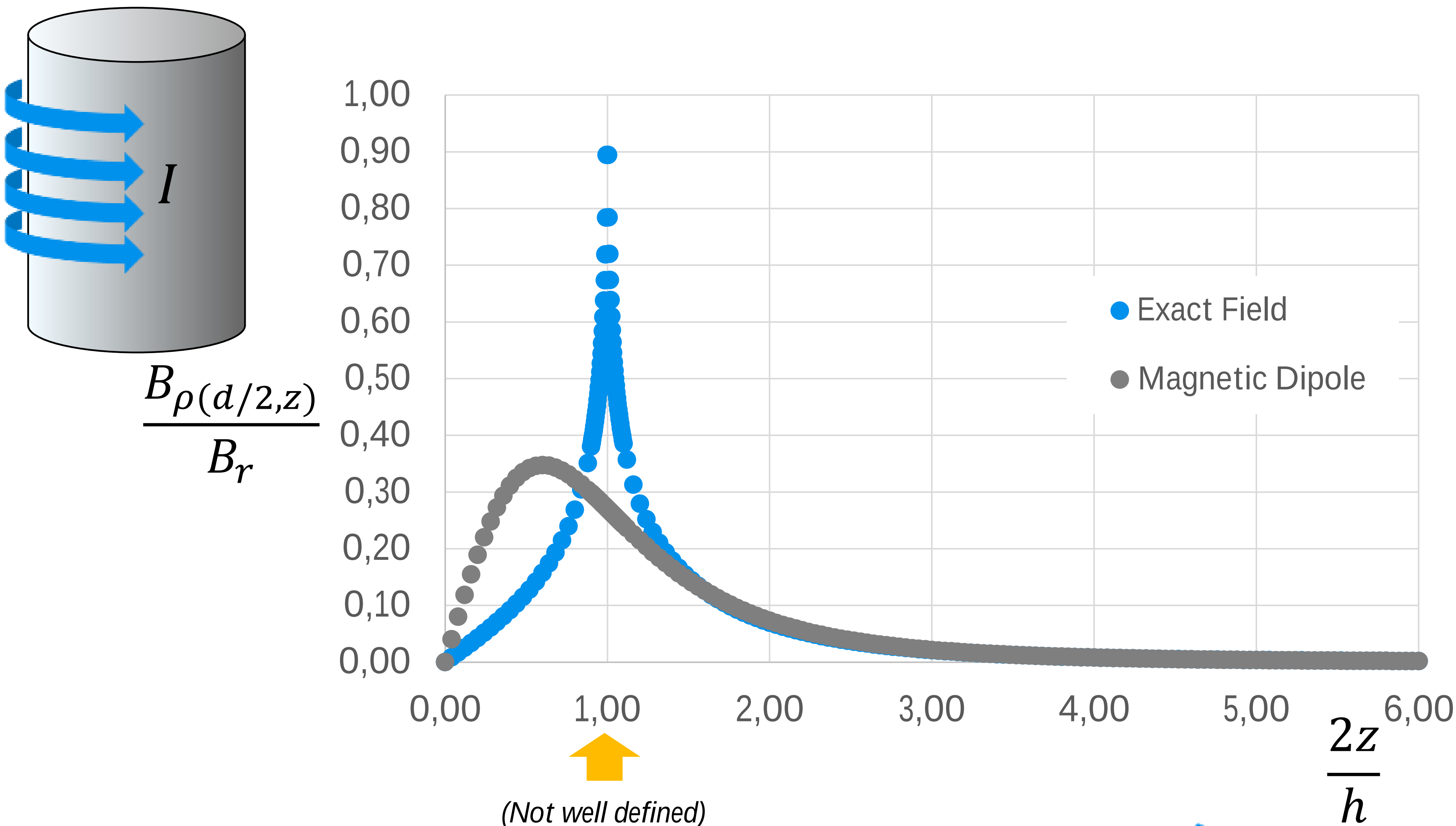
Instead we used expressions for solenoid field

(Equivalent to uniformly magnetized cylindrical magnets)

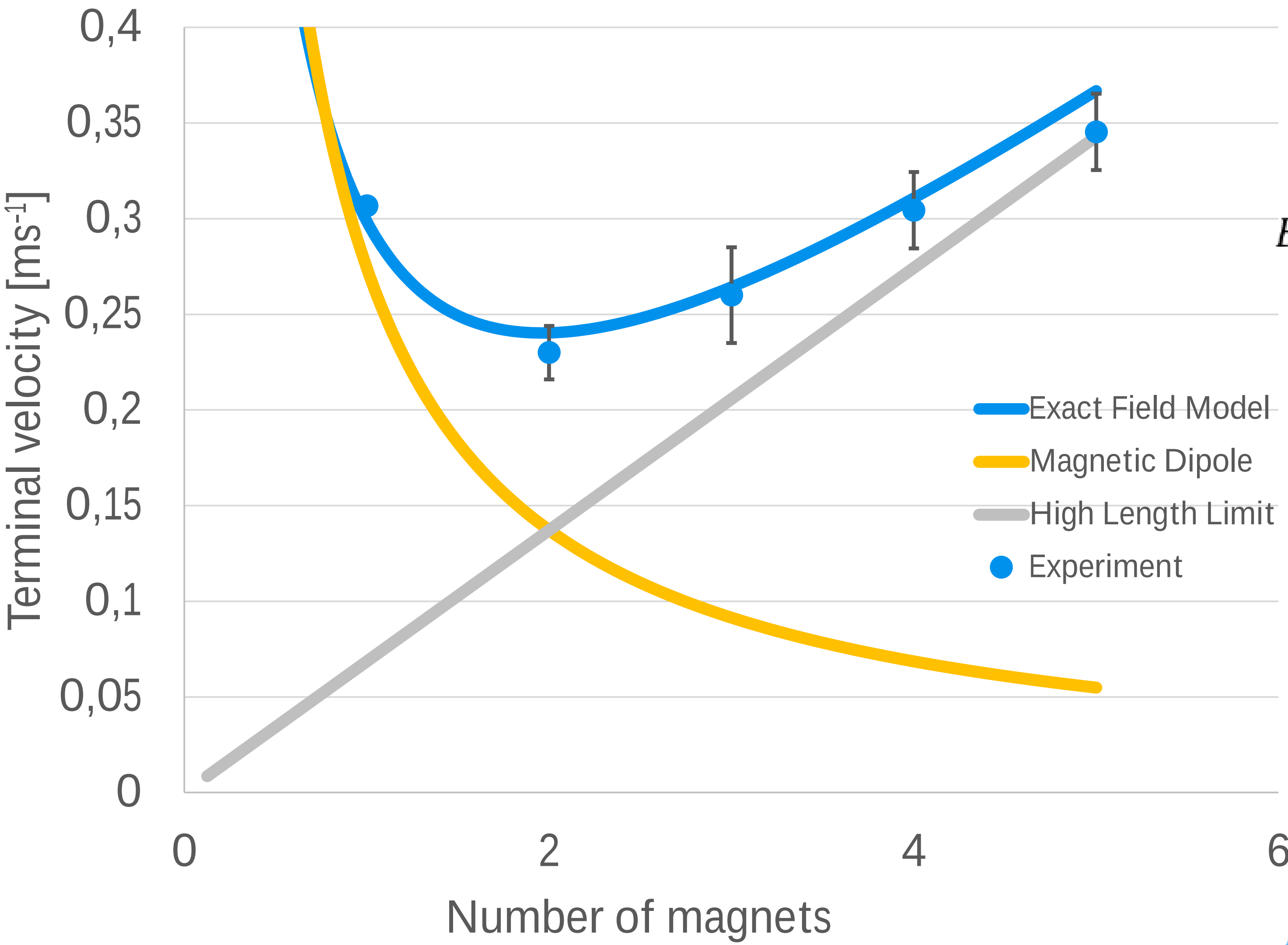
Polack, Stump "*Electromagnetism*" **Ch.** 9.2 pg. 319

(Proof in appendix)

Magnetic Dipole Model: Too rough



Exact Field Model



$B_r = 1{,}19 \pm 0{,}02\ T$

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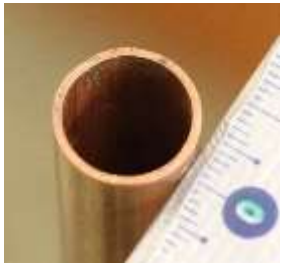


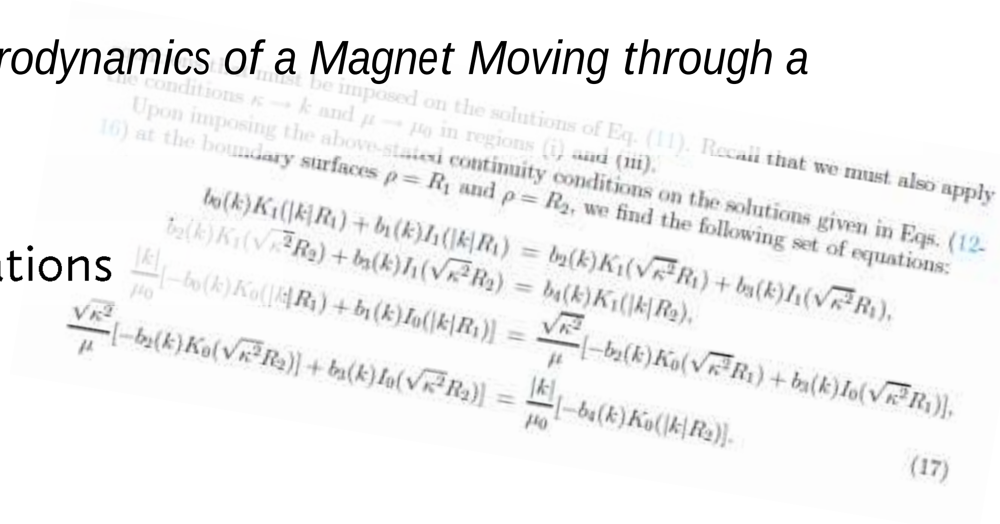
$\sigma = 42 \cdot 10^6\ Sm^{-1}$

$r = 6{,}5\ mm$

$w = 1{,}1\ mm$

Summary of (Good) Existing Work

M. Hossein Partovi, Eliza J. Morris, "*Electrodynamics of a Magnet Moving through a Conducting Pipe*," Can. J. Phys. 84, (2006)


... must be imposed on the solutions of Eq. (11). Recall that we must also apply the conditions $\kappa \to k$ and $\mu \to \mu_0$ in regions (i) and (iii).
Upon imposing the above-stated continuity conditions on the solutions given in Eqs. (12-16) at the boundary surfaces $\rho = R_1$ and $\rho = R_2$, we find the following set of equations:

$$\begin{aligned}
b_0(k)K_1(|k|R_1) + b_1(k)I_1(|k|R_1) &= b_2(k)K_1(\sqrt{\kappa^2}R_1) + b_3(k)I_1(\sqrt{\kappa^2}R_1),\\
b_2(k)K_1(\sqrt{\kappa^2}R_2) + b_3(k)I_1(\sqrt{\kappa^2}R_2) &= b_4(k)K_1(|k|R_2),\\
\frac{|k|}{\mu_0}[-b_0(k)K_0(|k|R_1) + b_1(k)I_0(|k|R_1)] &= \frac{\sqrt{\kappa^2}}{\mu}[-b_2(k)K_0(\sqrt{\kappa^2}R_1) + b_3(k)I_0(\sqrt{\kappa^2}R_1)],\\
\frac{\sqrt{\kappa^2}}{\mu}[-b_2(k)K_0(\sqrt{\kappa^2}R_2)] + b_3(k)I_0(\sqrt{\kappa^2}R_2)] &= \frac{|k|}{\mu_0}[-b_4(k)K_0(|k|R_2)].
\end{aligned} \quad (17)$$

- \+ Exact solution – Maxwell's equations
- – Cumbersome to handle

Norman Derby, Stanislaw Olbert, "*Cylindrical Magnets and Ideal Solenoids*," Am. J. Phys. 78, Issue 3, pp. 229-235 (2010)

- \+ Exact solution of the field of **cylindrical magnet's**
- – Experimental drawback

No.	b/a	m(g)	$\mu(A\cdot m^2)$	$v_{average}$ (m/s)	v_t-Partovi	v_t-Eq. (29)	$V_{average}$
1	1.0	12.1	1.76	0.0687	0.0669	0.0670	0.0674
2	1.5	17.9	2.36	0.1045	0.1050	0.1050	0.1058
3	2.0	23.8	3.23	0.1275	0.1243	0.1243	0.1254
4	3.0	36.4	5.00	0.1825	0.1710	0.1711	0.1731
5	4.0	48.2	6.37	0.2473	0.2451	0.2451	0.2486
6	1.0	12.9	1.17	0.1513	0.1615	0.1616	0.1622

Goals of Our Work

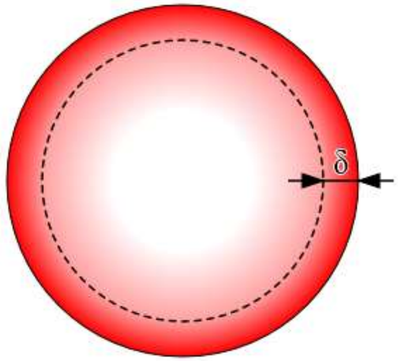
1. Simpler theory,
   numerical approach

2. Experimentally investigate
   large number of parameters

THEORETICAL MODEL

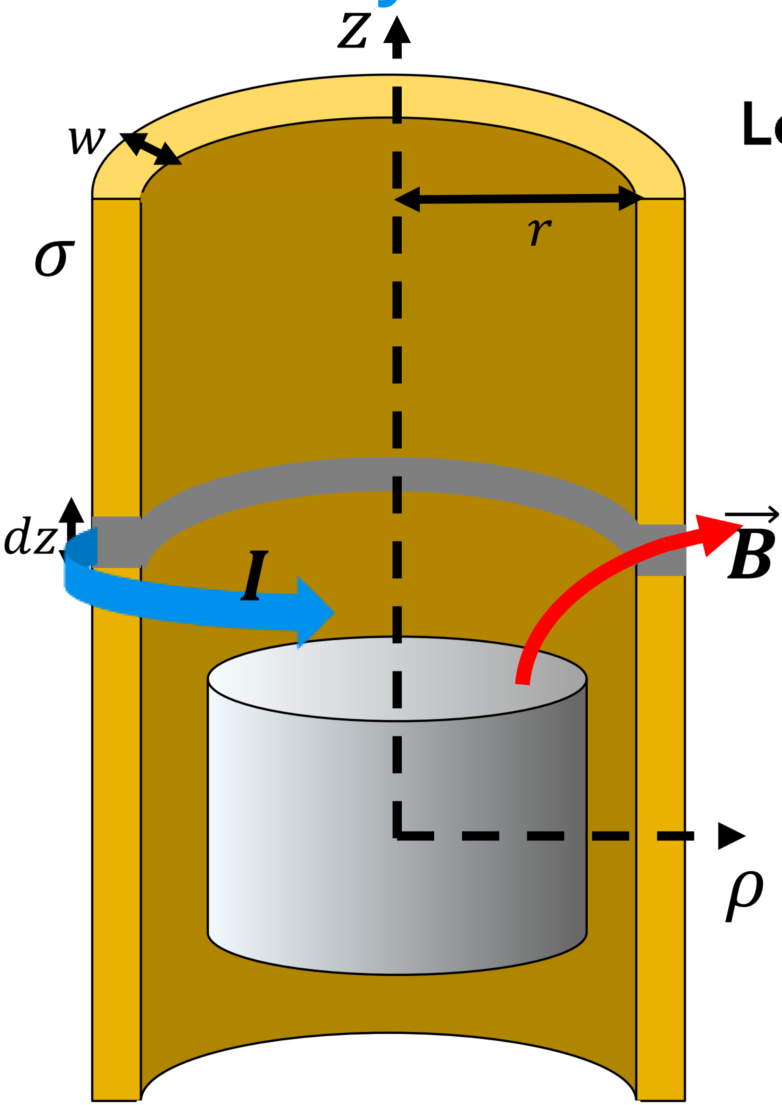
Our Approximations

- Quasi-static limit ($v \ll c$)
- Skin effect neglected
- Inducted currents in magnet neglected (Self-inductance)
- Cylindrical symmetry of the system
- Magnet falls down always in the center of tube & doesn’t rotate



[http://upload.wikimedia.org/wikipedia/commons/6/61/Skin_depth.svg]

Theory



Lenz's law & Gauss's law for magnetism

&

Calculating infinitesimal forces from each ring

$$F = -v_z \sigma 2\pi \left(r + \frac{w}{2}\right) w \int_{-\infty}^{\infty} B^2_{\rho(\rho,z)} dz$$

Numerical approach with Exact Magnetic Field

EXPERIMENTS

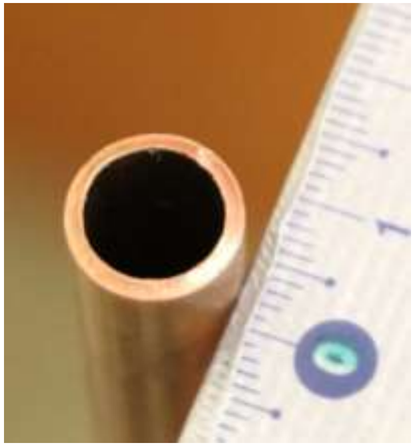
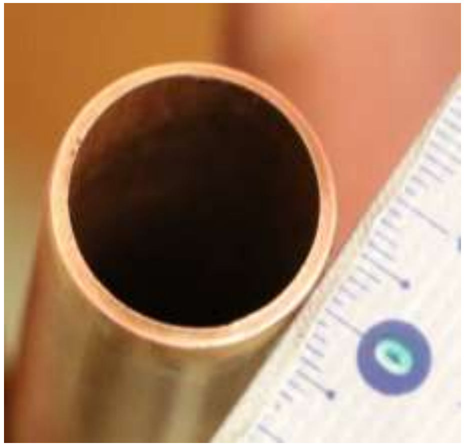
Uniformly Magnetized – NdFeB Magnets

(Photographs not in scale)

(Data for one magnet. Remanence given by the manufactor)

	N.1	N.2	N.3	N.4	N.5
Mass [g]	1,06	1,88	2,09	6,54	9,54
Remanence [T]	1,19±0,02	1,33	1,19±0,02	1,19±0,02	1,19±0,02
Magnetic moment [Am^2]	0,13±0,001	0,25±0,1	0,54±0,1	0,835±0,15	1,22±0,01
Diamater [mm]	6	8	12	15	18
Height of the magnet [mm]	5	5	5	5	5

Cu Tubes

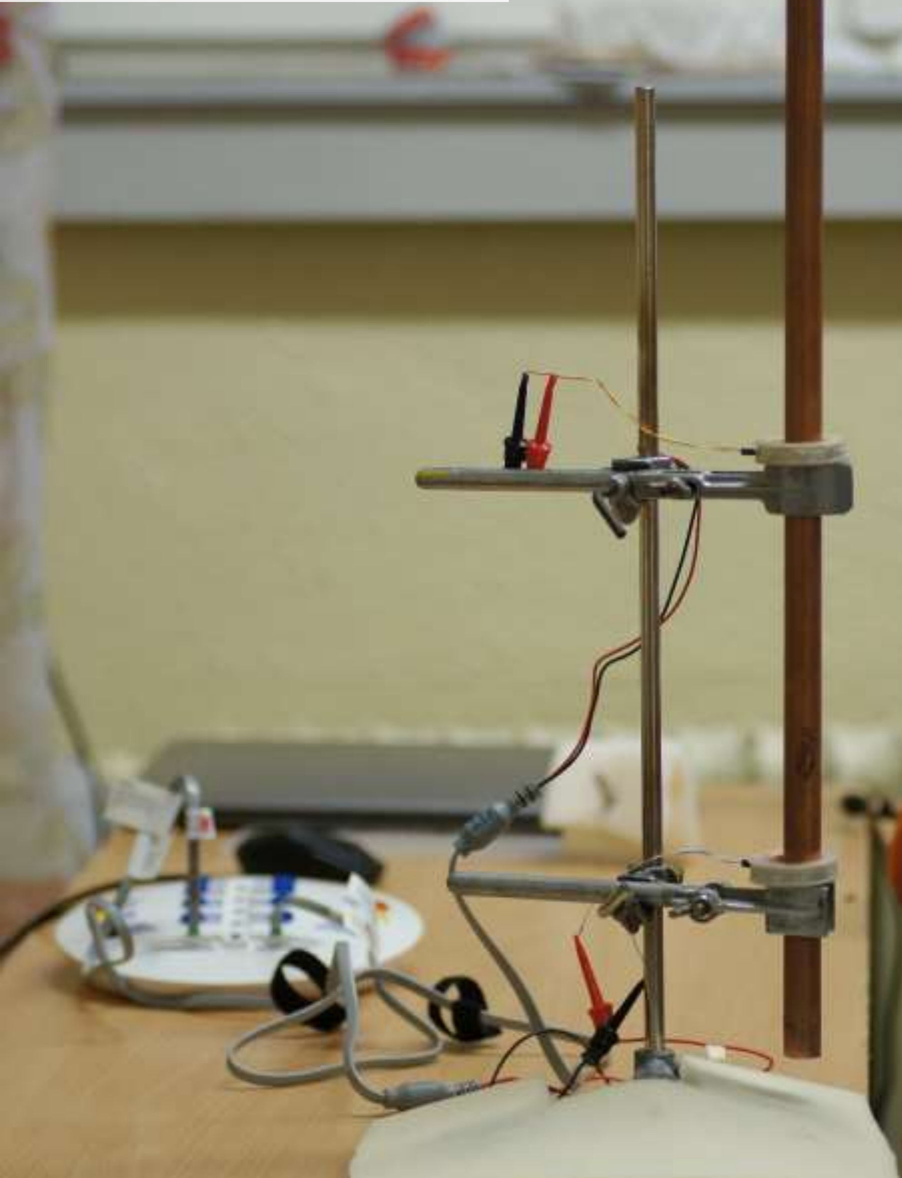
   

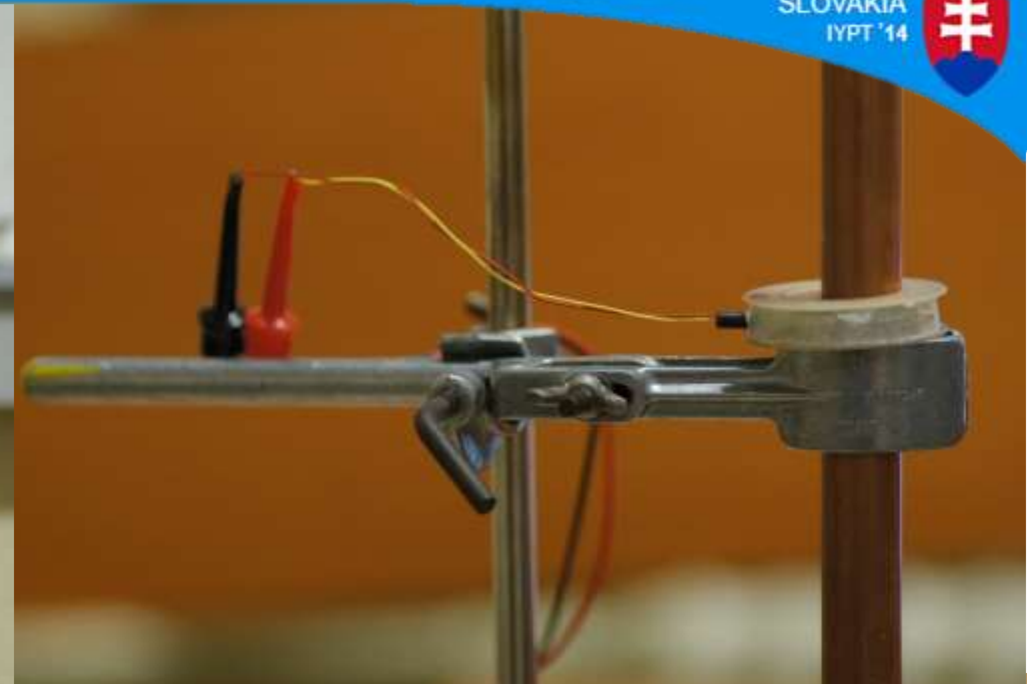
Tube	N.1	N.2	N.3	N.4
Inner radius [mm]	4,75	6,5	7,8	10,1
Width [mm]	1,25	1,1	1,2	1,02
Conductivity [10^6 Sm^{-1}]	42±2	42±2	42±2	42±2

Measured using Kelvin bridge

Cu Tubes

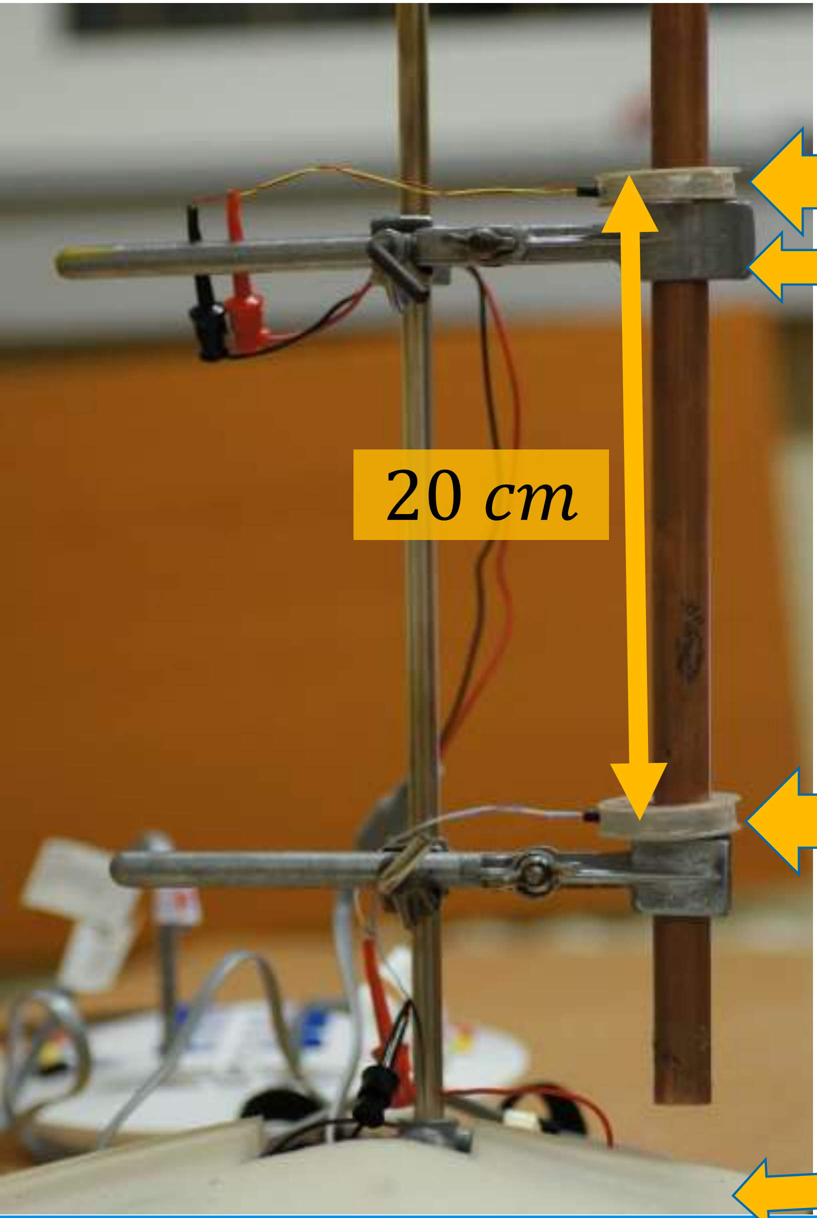
Apparatus





Measurement of Terminal Velocity

$0{,}8\ m$ Long tube → terminal velocity



Measurement coil

$R = 32\ \Omega$

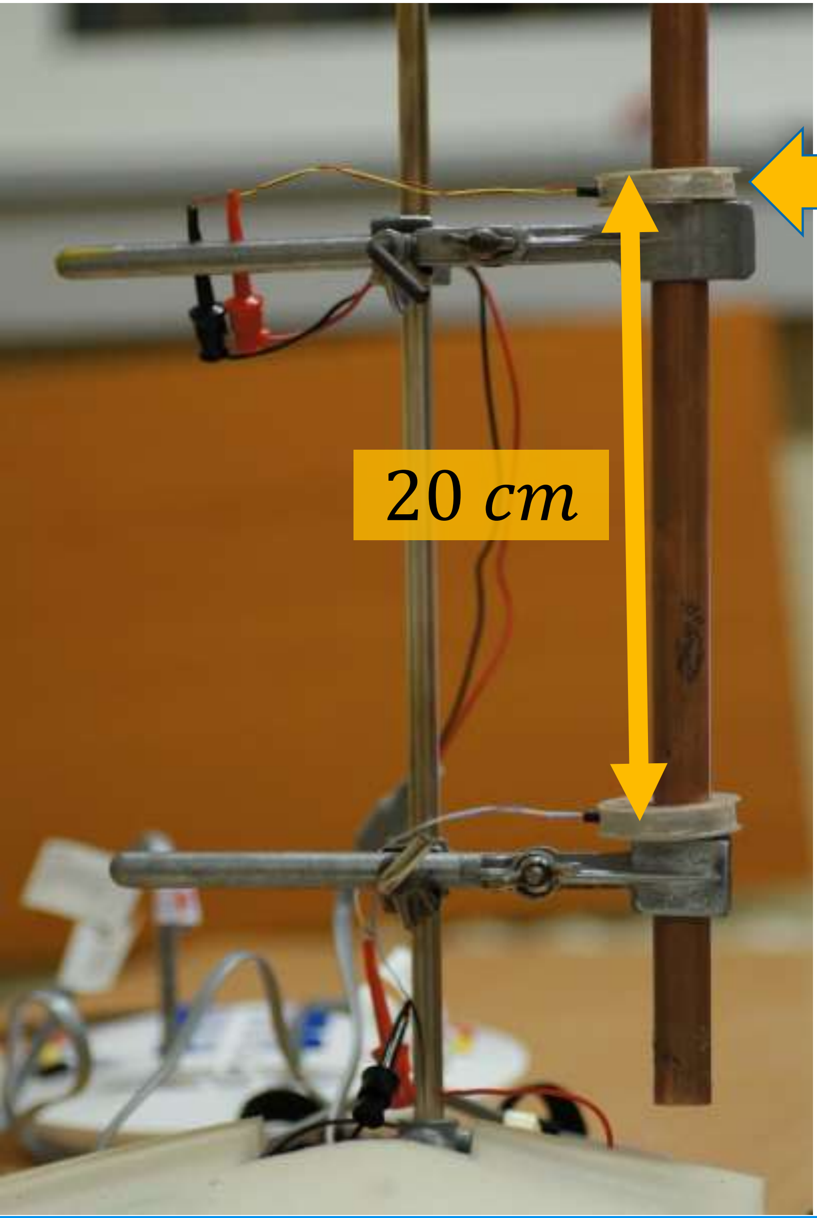
$L = 52\ mH$

Aluminum holder

Measurement coil

Foam ruber

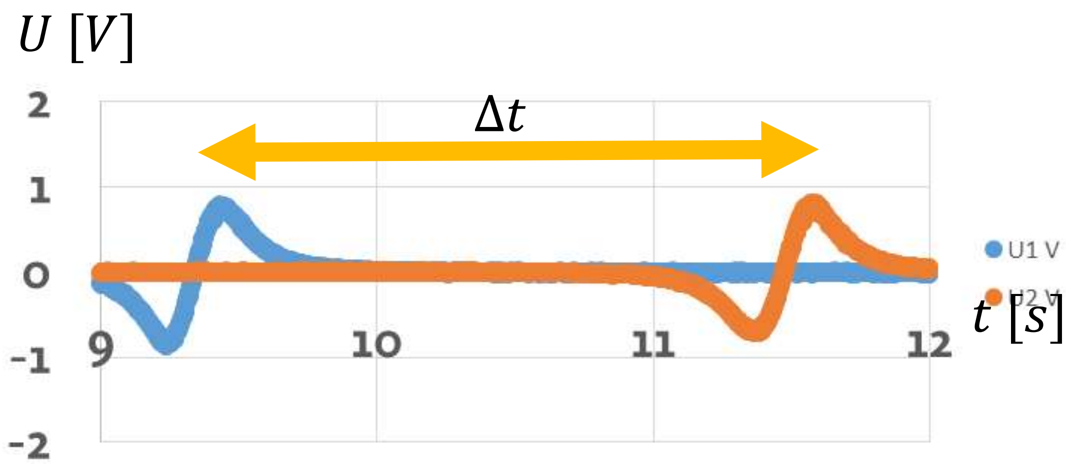
Measurement of Terminal Velocity



$0{,}8\ m$ Long tube → terminal velocity

Measurement coil $R = 32\ \Omega$ $L = 52\ mH$

Measuring the voltage course in time



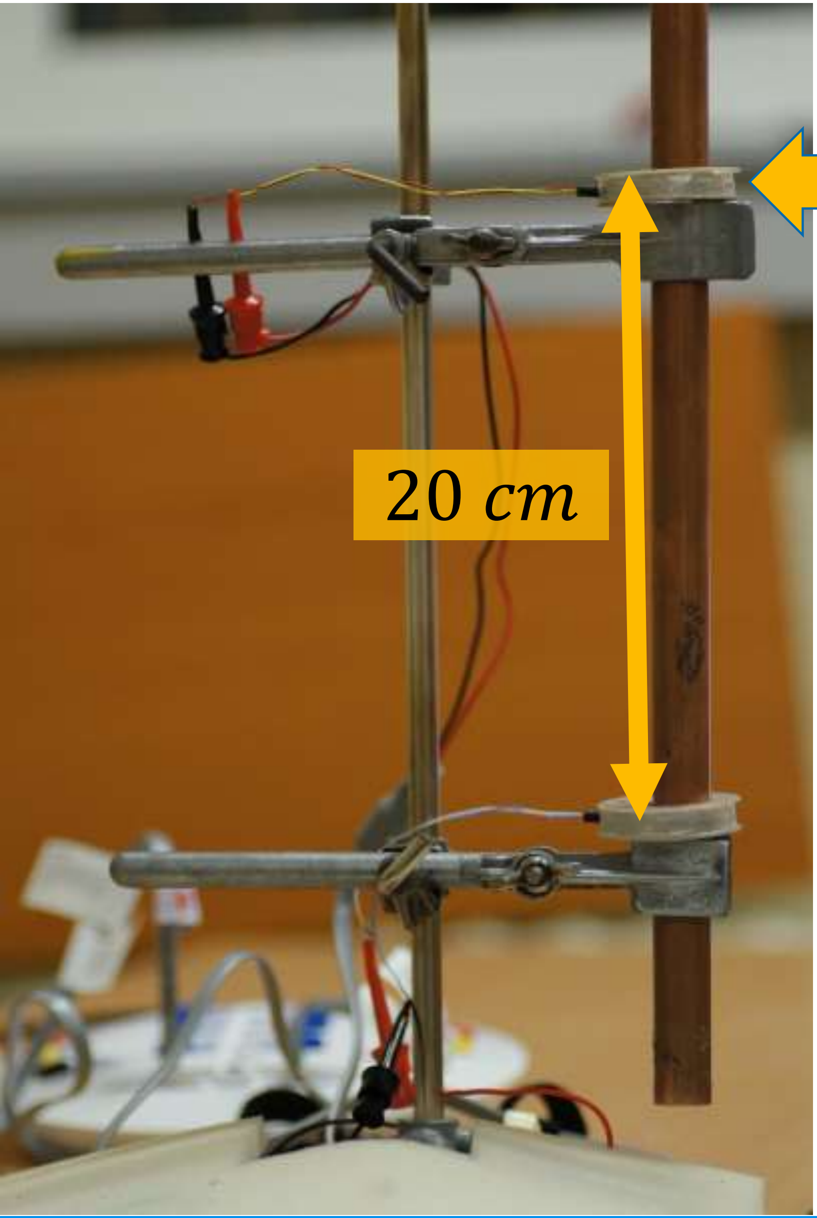
Error of the time measurement $\pm 2{,}5 \cdot 10^{-4}$ s

Typical velocity error < 5%

Force from coil << Gravity force

(≈ 100x smaller)

Measurement of Terminal Velocity

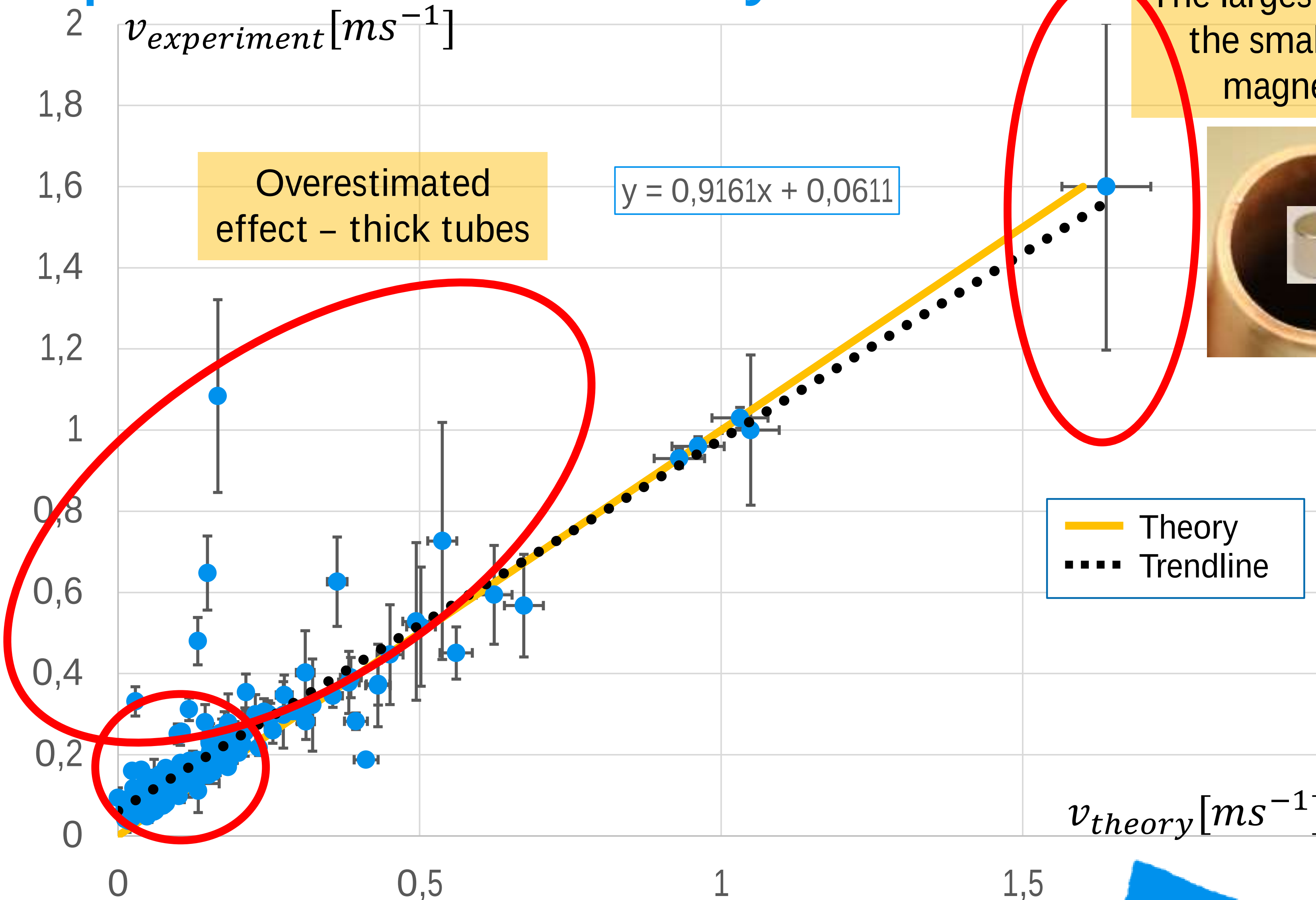


$0{,}8\ m$ Long tube → terminal velocity

Measurement coil $R = 32\ \Omega$, $L = 52\ mH$

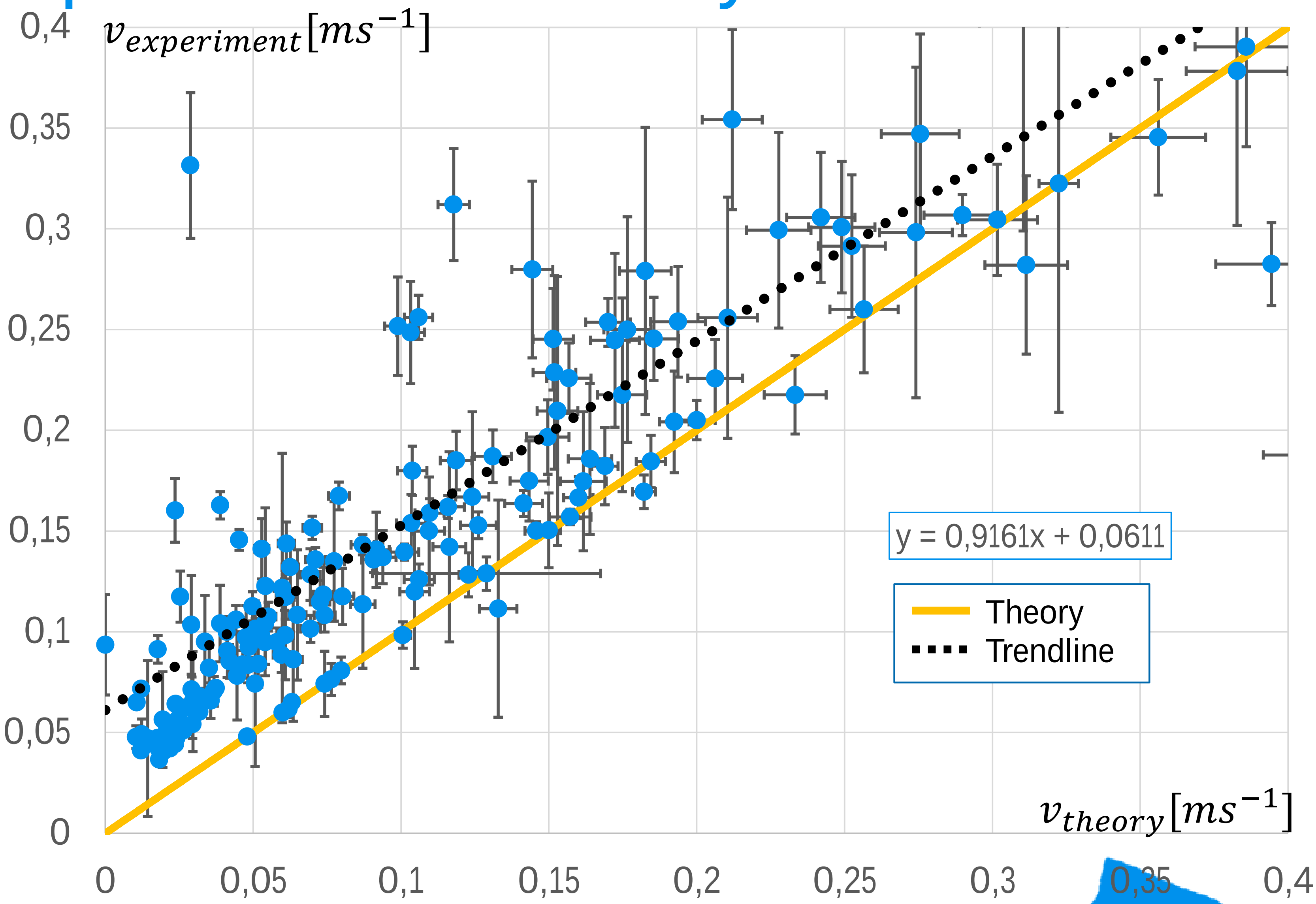
Measuring all the possible combinations ≈ 180 different set-**ups · min. 30** times repeated
≈ A lot of data!

Experiment vs. Theory

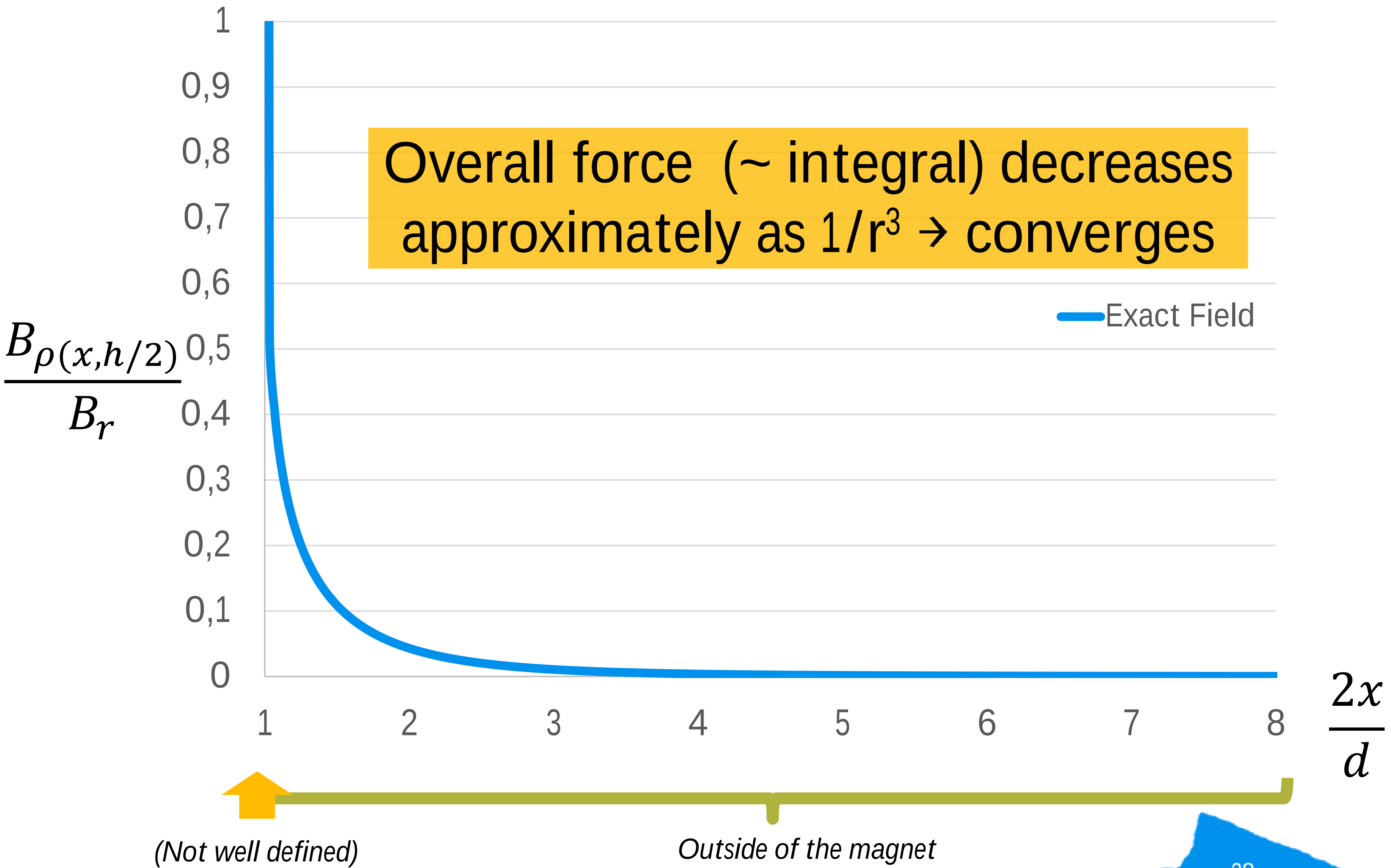


The largest tube, the smallest magnet

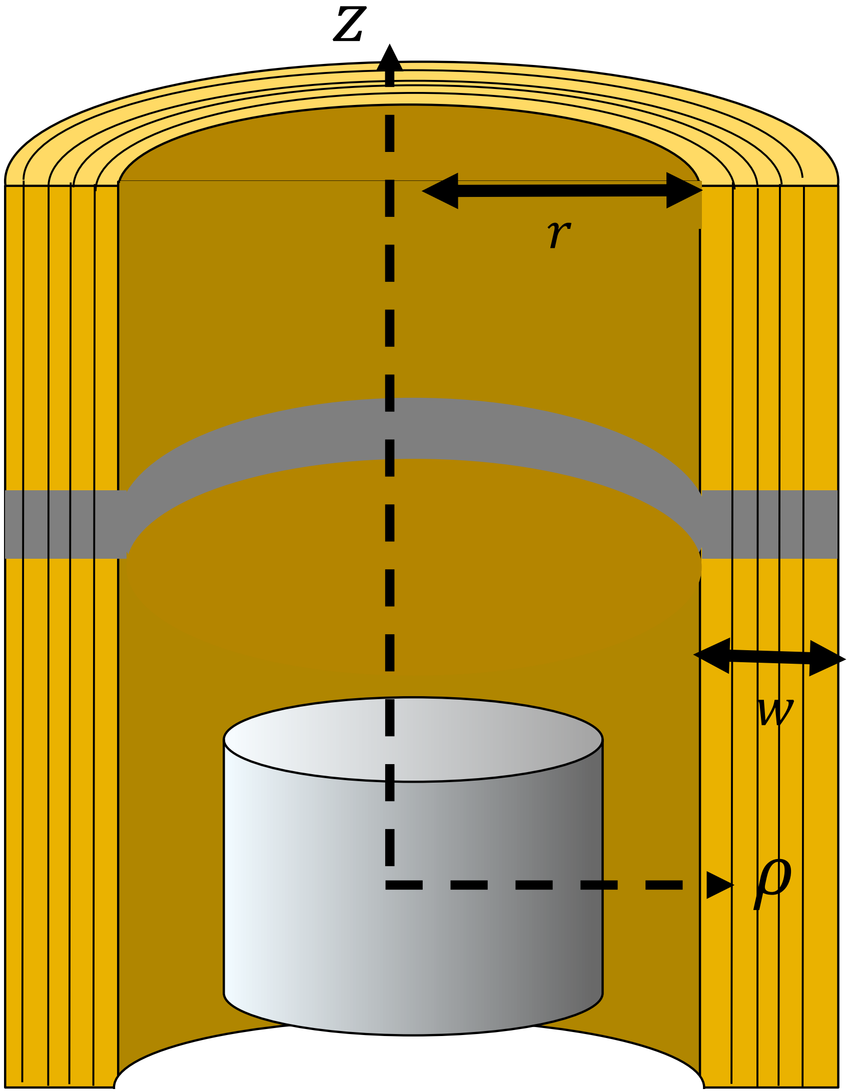
Experiment vs. Theory: Zoom In



Magnetic field: Change in radial direction



Correction of the Theory



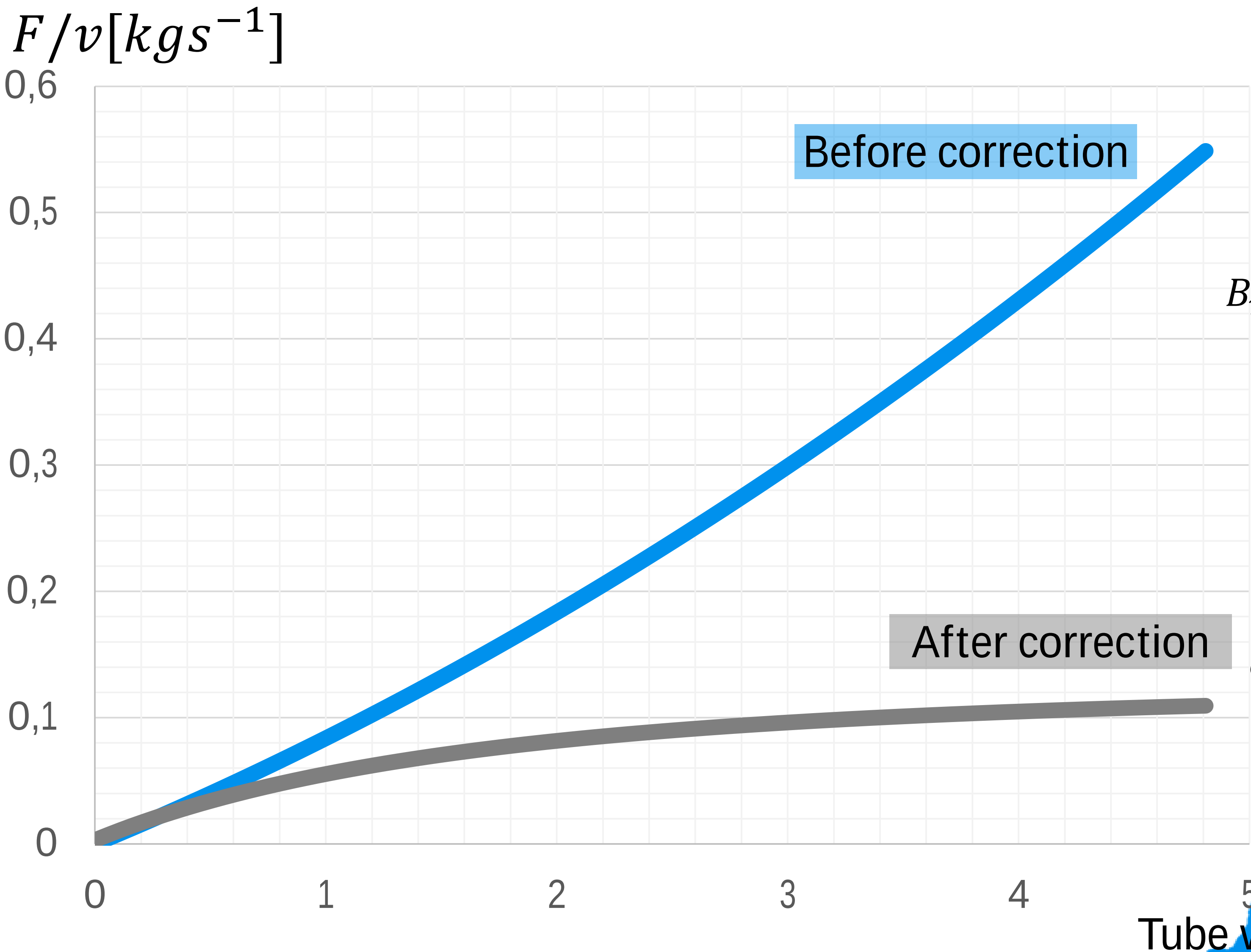
Force should converge

The tube is divided into large number of thin tubes

$$F = -v_z \sigma 2\pi \int_r^{r+w} \rho \int_{-\infty}^{\infty} B_{\rho(\rho,z)}^2 dz\, d\rho$$

Numerical calculation with exact magnetic field

Changing Width – Correction



$B_r = 1{,}19 \pm 0{,}02\ T$

$m = 1{,}06\ g$

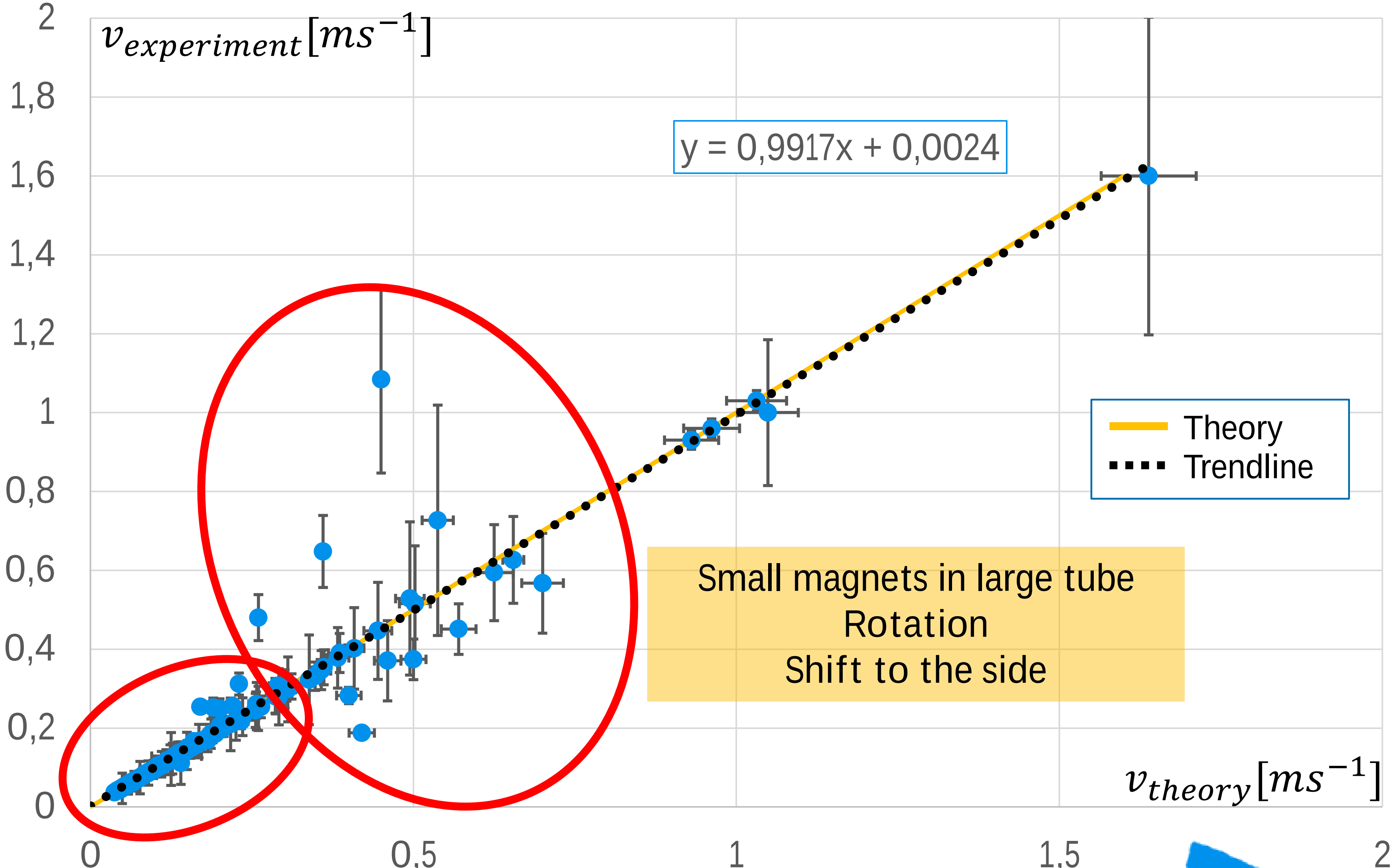
$h = 5\ mm$

$d = 6\ mm$

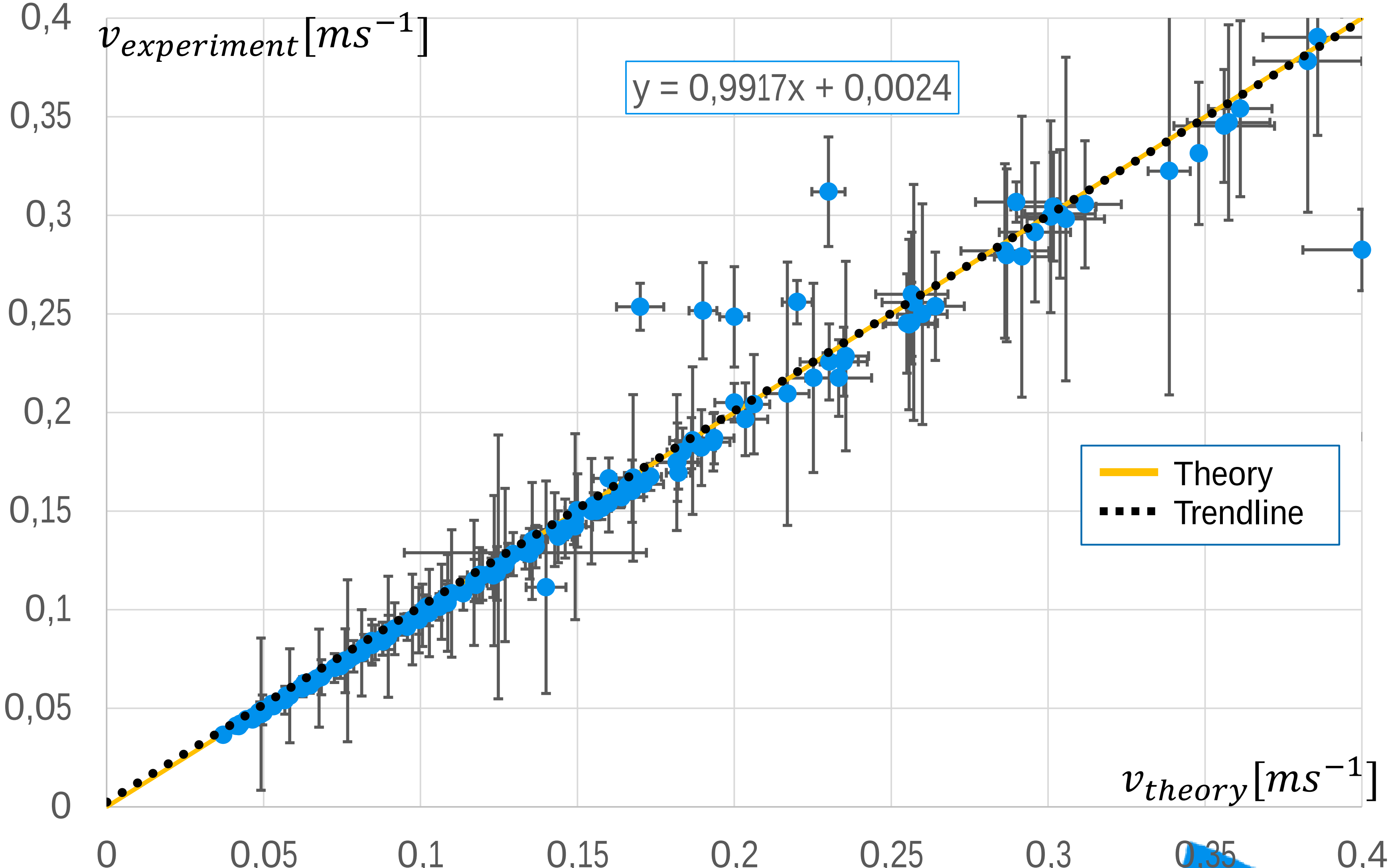
$\sigma = 42 \cdot 10^6\ Sm^{-1}$

$r = 4{,}75\ mm$

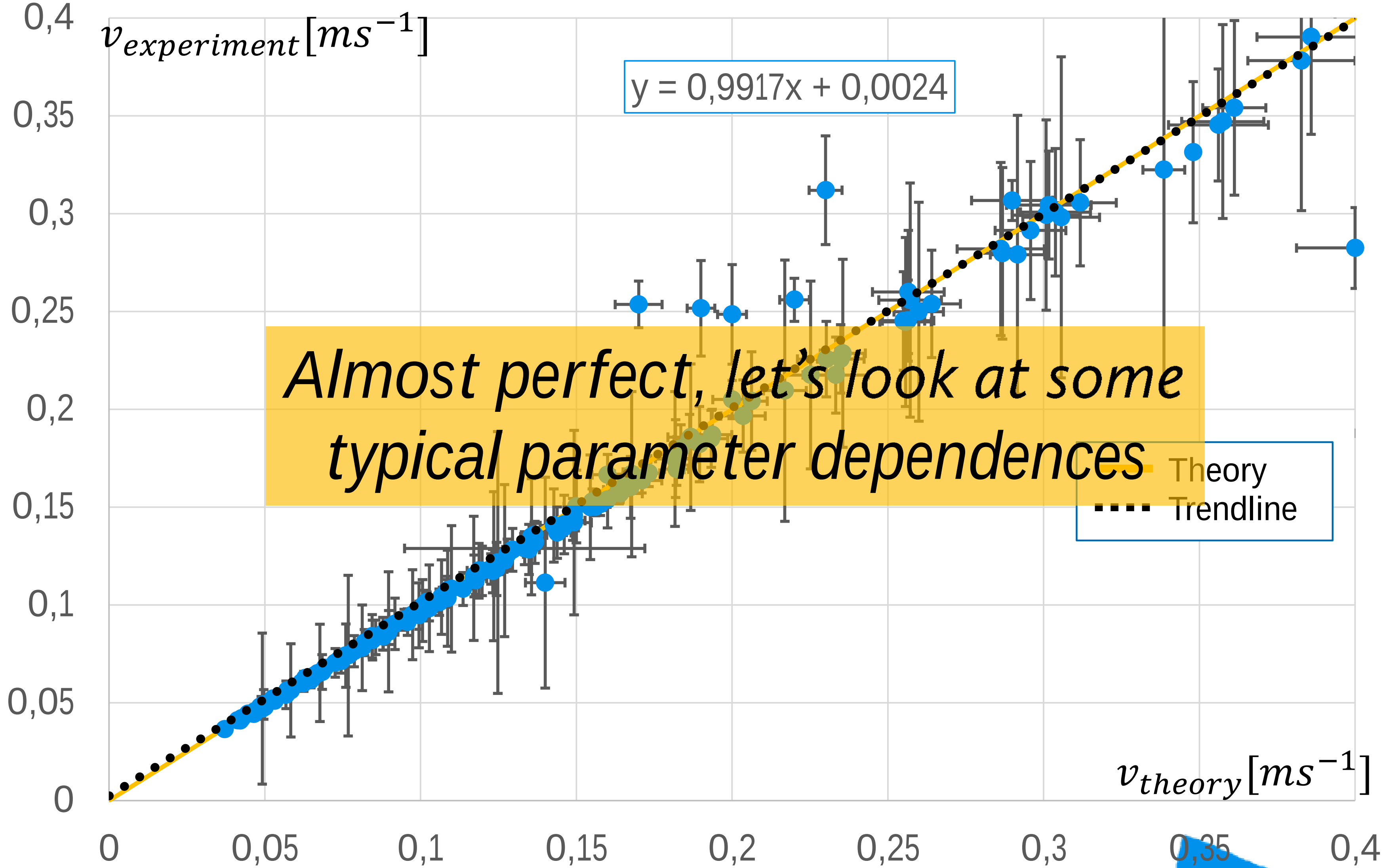
Experiment vs. Theory



Experiment vs. Theory: Zoom In



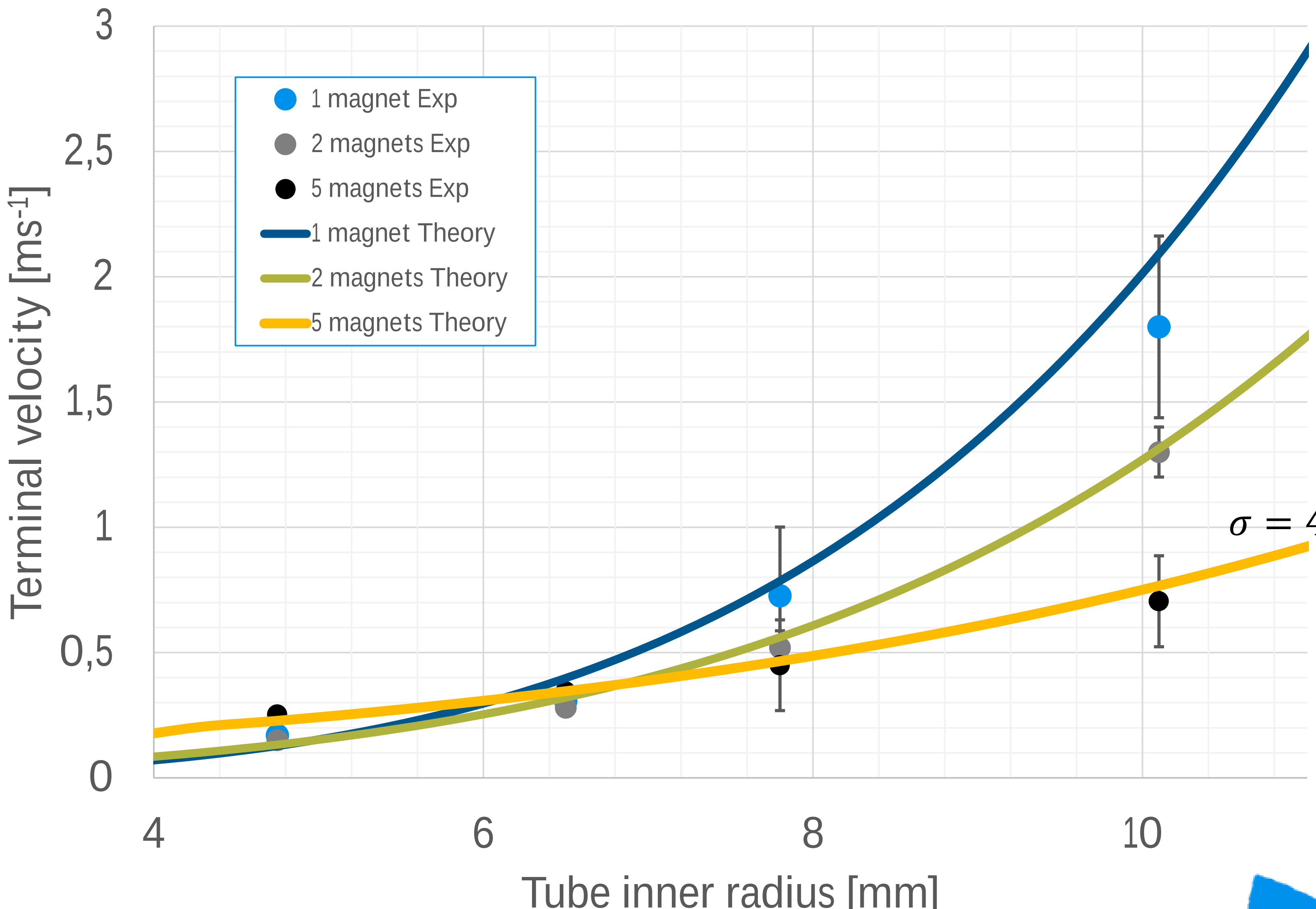
Experiment vs. Theory: Zoom In



Almost perfect, let's look at some typical parameter dependences

PARAMETER DEPENDENCIES

Tube radius

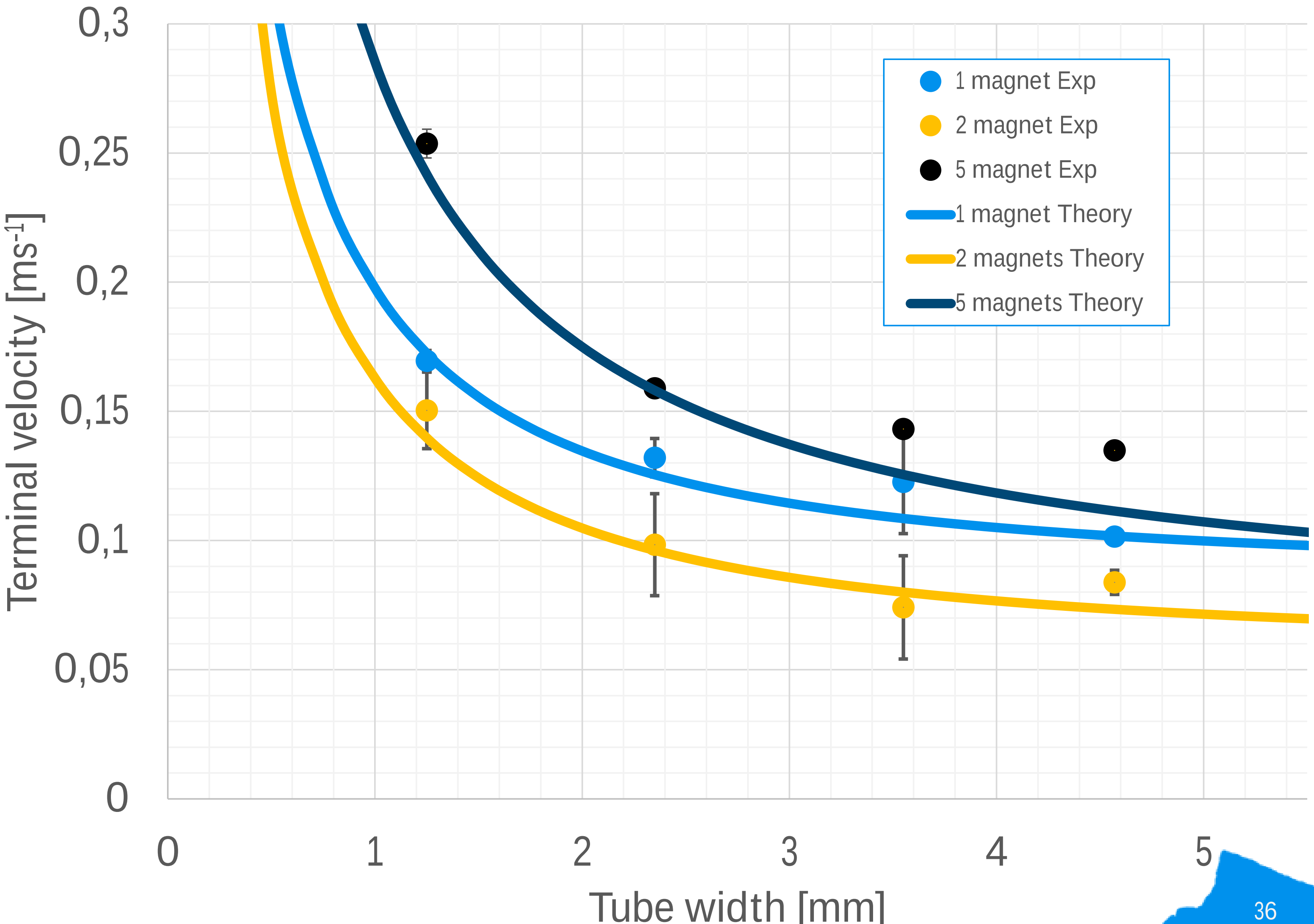


Magnet 1

(Fixed) tube width

$\sigma = 42 \cdot 10^6\ Sm^{-1}$

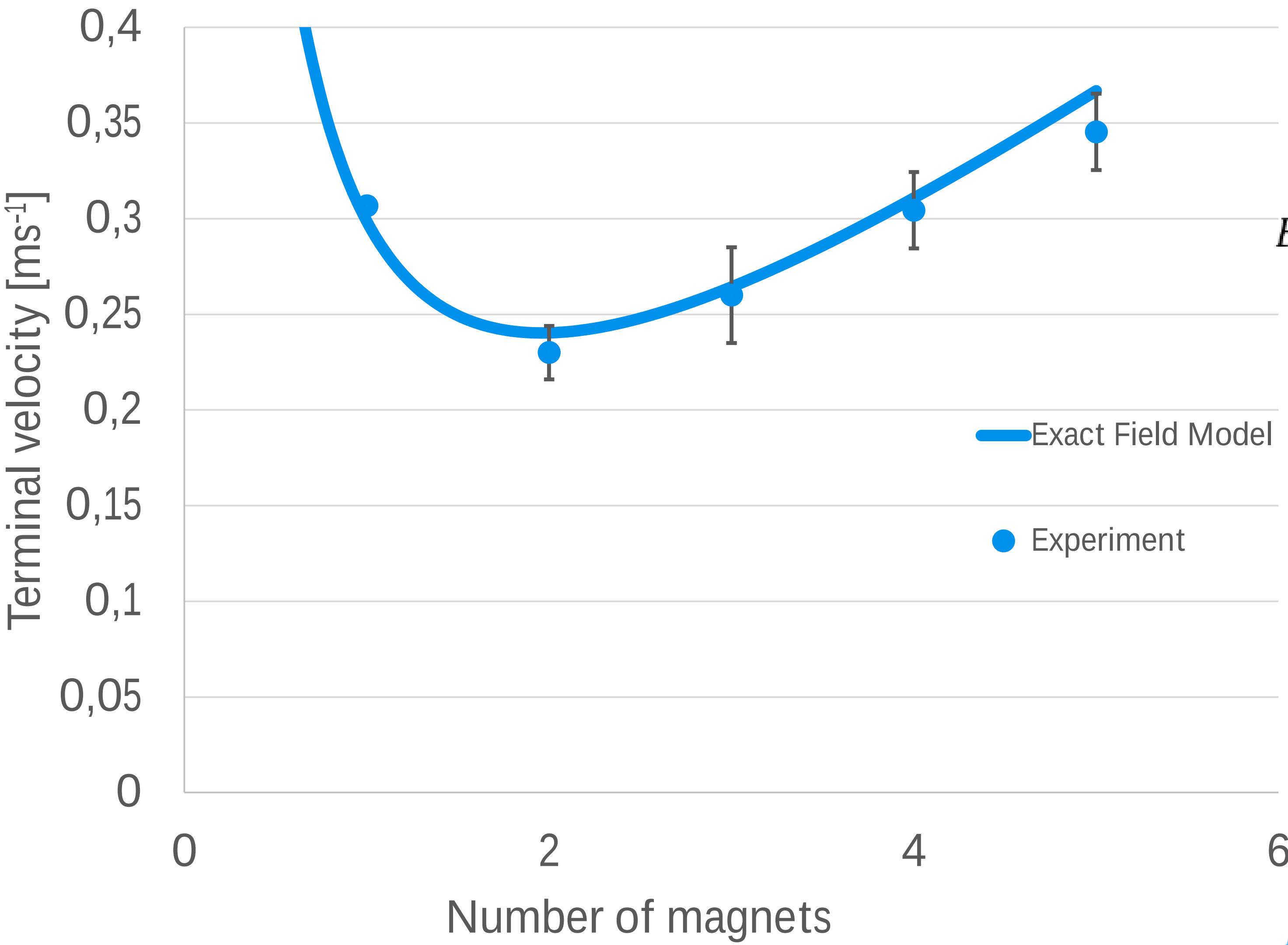
Tube width



Magnet 1

(Fixed) tube inner radius

Number of magnets in 1st tube



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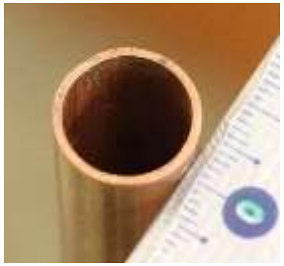
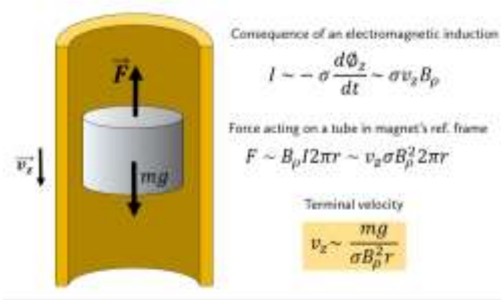


$$\sigma = 42 \cdot 10^6\ Sm^{-1}$$
$$r = 6{,}5\ mm$$
$$w = 1{,}1\ mm$$

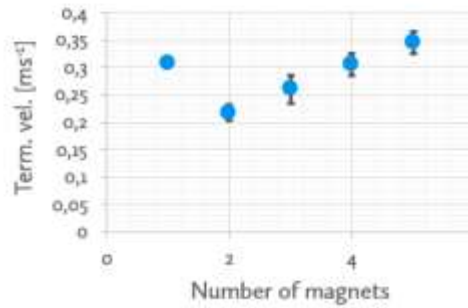
Conclusion

Thank you for your attention!

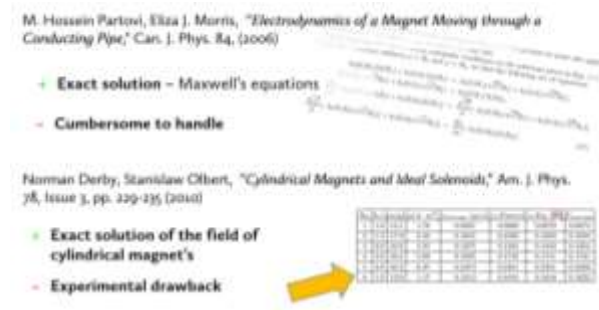
Qualitative explanation



Initial experiment



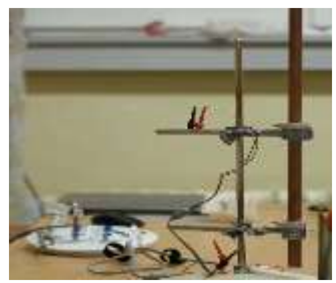
Challenging existing work



Theoretical Model using Solenoid field

$$F = -v_z \sigma 2\pi \left(r + \frac{w}{2}\right) w \int_{-\infty}^{\infty} B_{\rho(\rho,z)}^2 dz$$

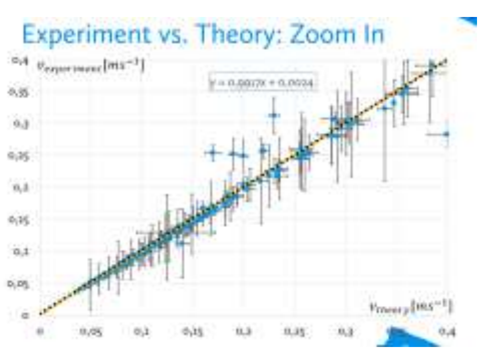
Challenged by experiments



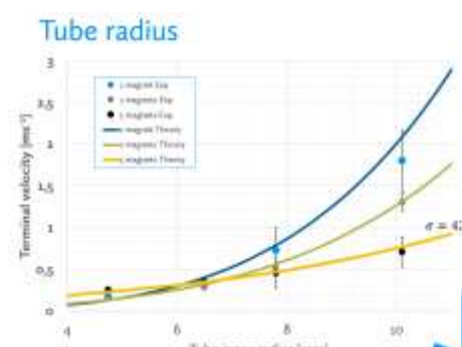
Correction for tube thickness

$$F = -v_z \sigma 2\pi \int_{r}^{r+w} \rho \int_{-\infty}^{\infty} B_{\rho(\rho,z)}^2 dz\, d\rho$$

Almost perfect correlation

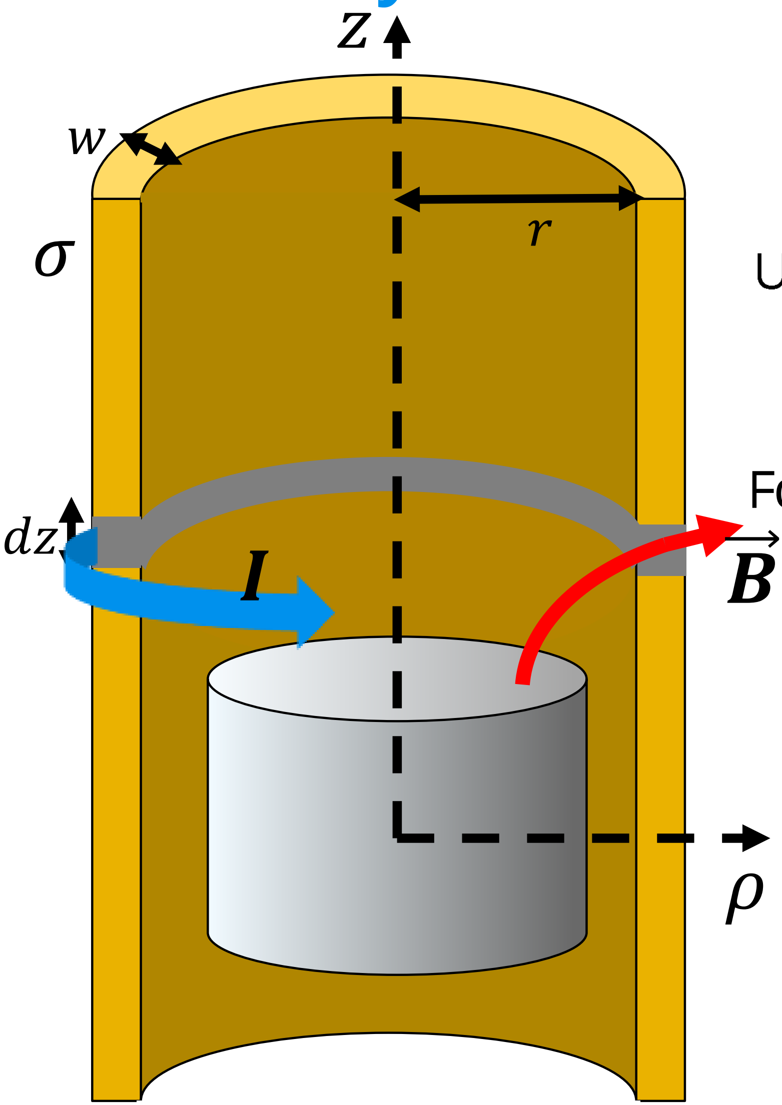


Parameter dependencies



Appendix

Theory



Lenz's law:

$$U_{ind} = -\frac{d\emptyset}{dt} = v_z \frac{d\emptyset}{dz} \qquad \emptyset = \oint \vec{B} \cdot \overrightarrow{dS}$$

Using Gauss's law for magnetism ($\nabla \cdot \vec{B} = 0$) :

$$\frac{d\emptyset}{dz} = -B_{\rho(\rho,z)} 2\pi r$$

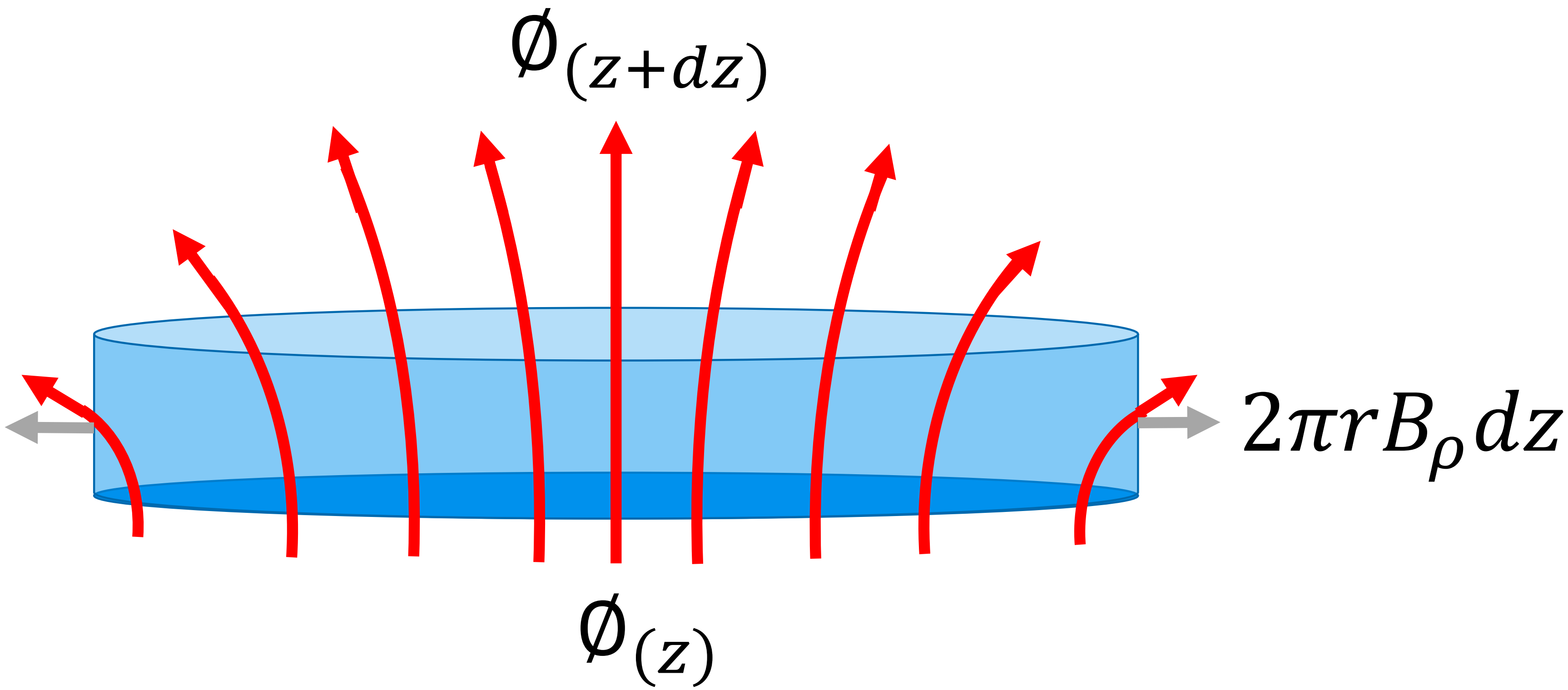
For the every ring in the height *z* above the magnet:

$$dI = \frac{U_{ind}}{dR} = -v_z \sigma w B_{\rho(\rho,z)} dz$$

$$dF = -2\pi r B_{\rho(\rho,z)} dI$$

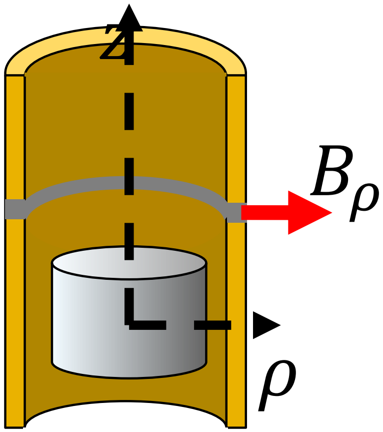
$$F = v_z \sigma 2\pi r w \int_x^{L-x} B^2_{\rho(\rho,z)} dz$$

Using Gauss's Law for Magnetism



$$\nabla \cdot \vec{B} = 0 \Rightarrow \emptyset_{(z+dz)} + 2\pi r B_\rho dz = \emptyset_{(z)} \Rightarrow \frac{d\emptyset}{dz} = -2\pi r B_\rho$$

Magnetic Dipole Model



Field of magnetic dipole

$$\vec{B}_{(r)} = \frac{\mu}{4\pi}\left(\frac{3\vec{r}(\vec{\mu}\cdot\vec{r})}{|r|^5} - \frac{\vec{\mu}}{|r|^3}\right)$$

Radial field

$$\vec{B}_\rho = \frac{\mu}{4\pi}\frac{3\rho z|\vec{\mu}|}{\sqrt{\rho^2+z^2}^5}$$

Resistance of small ring

$$dI = -2\pi\rho B_\rho v \frac{\sigma w dz}{2\pi\rho}$$

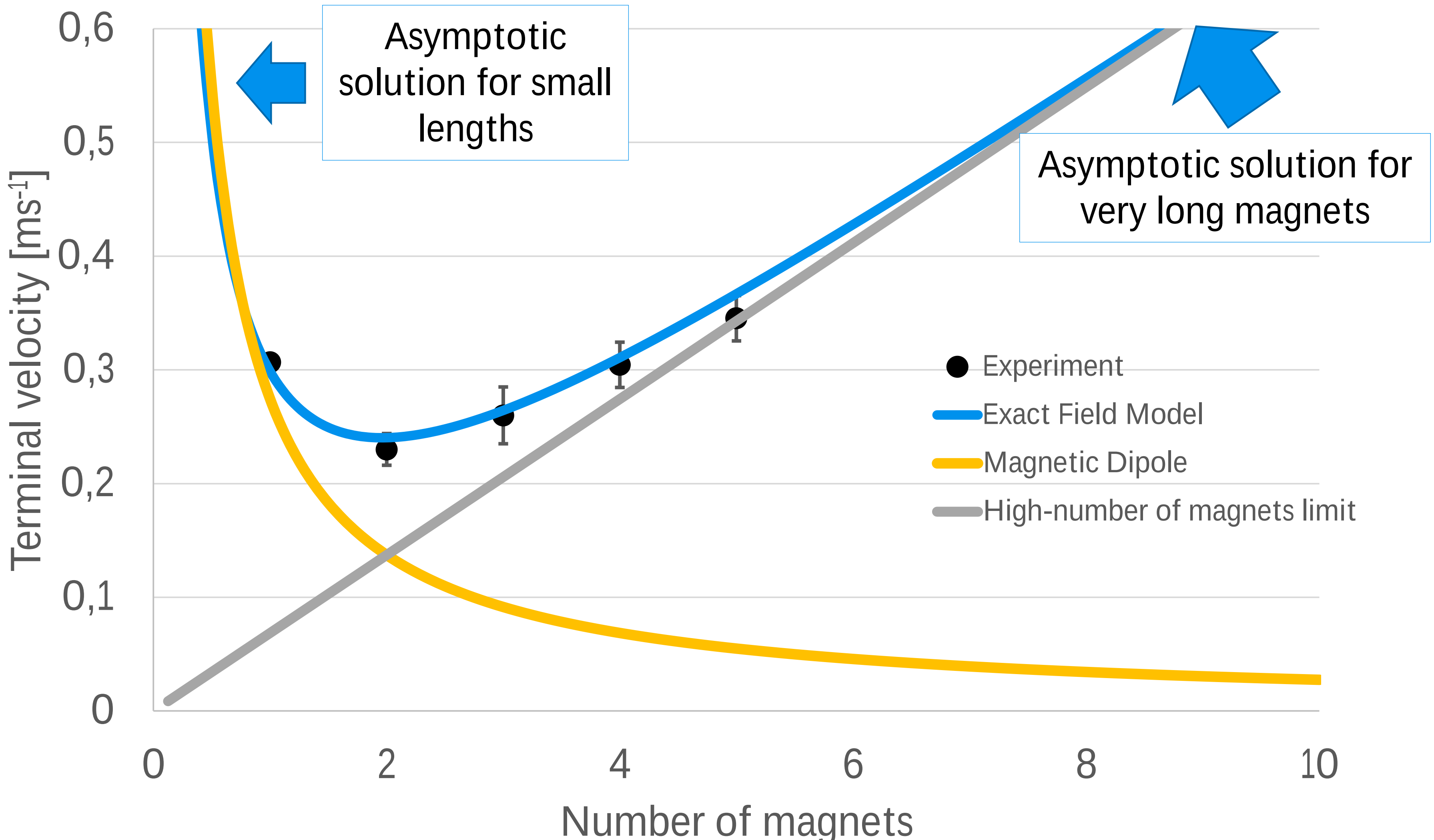
Inducted voltage

$$F = \int_{-L/2+x}^{L/2-x} 2\pi\rho B_\rho dI = -\int_{-L/2+x}^{L/2-x} 2\pi\rho B_\rho{}^2 v\sigma w dz$$

$$F \approx -\left(\frac{\mu_0}{4\pi}\right)^2 18\rho^3\mu_{mag}{}^2\pi v\sigma w\int_{-\infty}^{\infty}\frac{z^2}{\left(z^2+\rho^2\right)^5}dz = -\frac{45\mu_0^2\sigma\mu_{mag}{}^2 v}{1024}\frac{w}{r^4}$$

$$\frac{5\pi}{128\rho^7}$$

Models Comparison



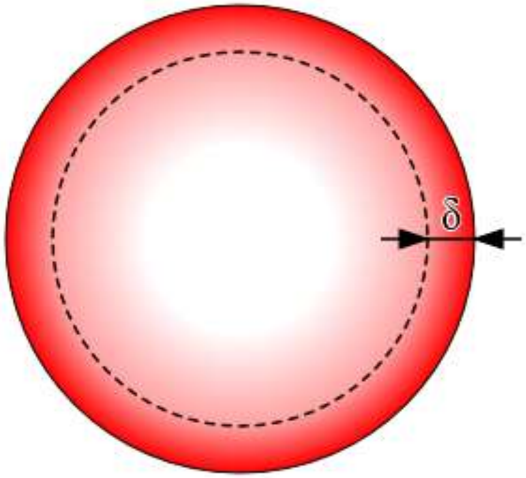
Justifications of Our Approximations

- Not considering:
  - Skin effect

$$\delta = \sqrt{\frac{2\rho}{\omega\mu_0}} \approx 10\,mm \gg r$$

ρ - Conductivity

ω - AC Frequency



[http://upload.wikimedia.org/wikipedia/commons/6/61/Skin_depth.svg]

Characteristic frequency

$$\omega \sim \frac{v}{r} \sim 10^2 s^{-1}$$

v – Magnet's velocity

r – Tube radius

  - Inducted currents in magnet (Self-inductance)

$$\frac{dz}{v} \gg \frac{L_{(dz)}}{R_{(dz)}}$$

$L_{(dz)}$ – Ring's self-inductance

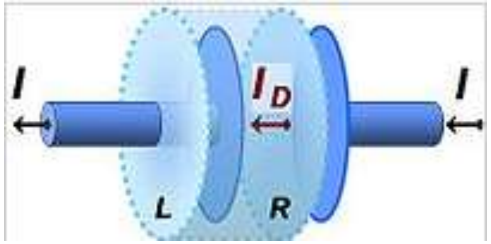
$R_{(dz)}$ – Ring's ohmic resistance

$\frac{L_{(dz)}}{R_{(dz)}}$ – Characteristic time of eddy currents decay

Another Assumptions

- Not considering:
  - Displacement currents $\frac{\partial E}{\partial t} \approx 0$

    (Consequence of quasi-static fields)



Displacement/Conduction currents $\frac{\epsilon_0 v}{\sigma r} \approx 10^{-16}$

  - Dipole radiation

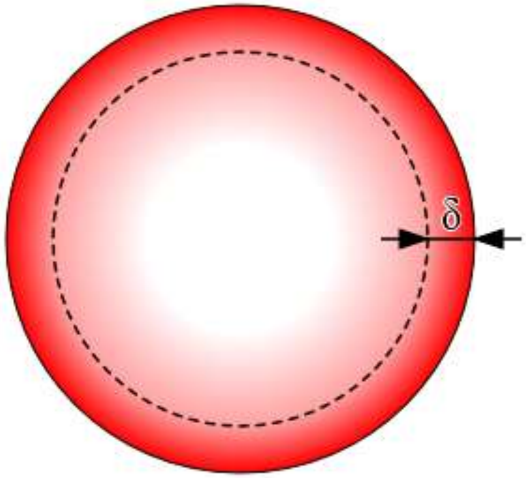
Dipole radiation vs. Ohmic dissipation

$$(\mu_0 m^2/6\pi c^7)(\dot{v}^2+v\ddot{v})^2 \ll \frac{45\mu_0^2\sigma\mu_{mag}{}^2 v^2}{1024}\frac{w}{r^4}$$

Skin effect

- Exact relationship for AC case:

$$\delta = \sqrt{\frac{2\rho}{\omega\mu_0}}\sqrt{\sqrt{1+(\rho\omega\varepsilon)^2}+\rho\omega\varepsilon}$$



Conductivity Measurement

- Using Kelvin bridge with

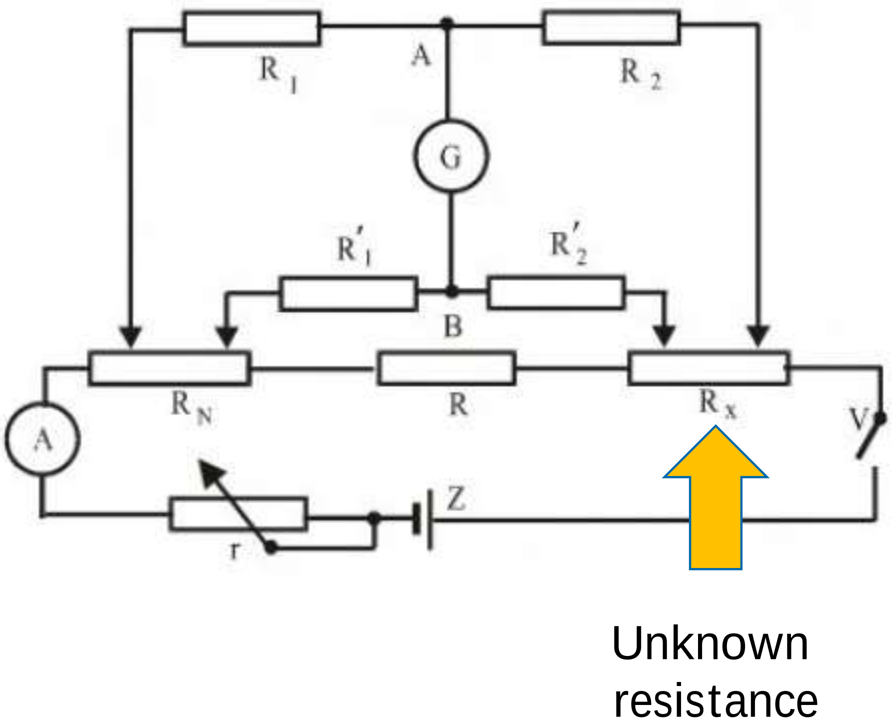
$$R_1 = R_1'$$
$$R_2 = R_2'$$

- In $I_G = 0$ state:

$$R_x = R_N \frac{R_1}{R_2}$$

- Used resistance:

$$R_N = 10^{-4}\ \Omega$$
$$R_{1,2} \approx 10 - 100\ \Omega$$



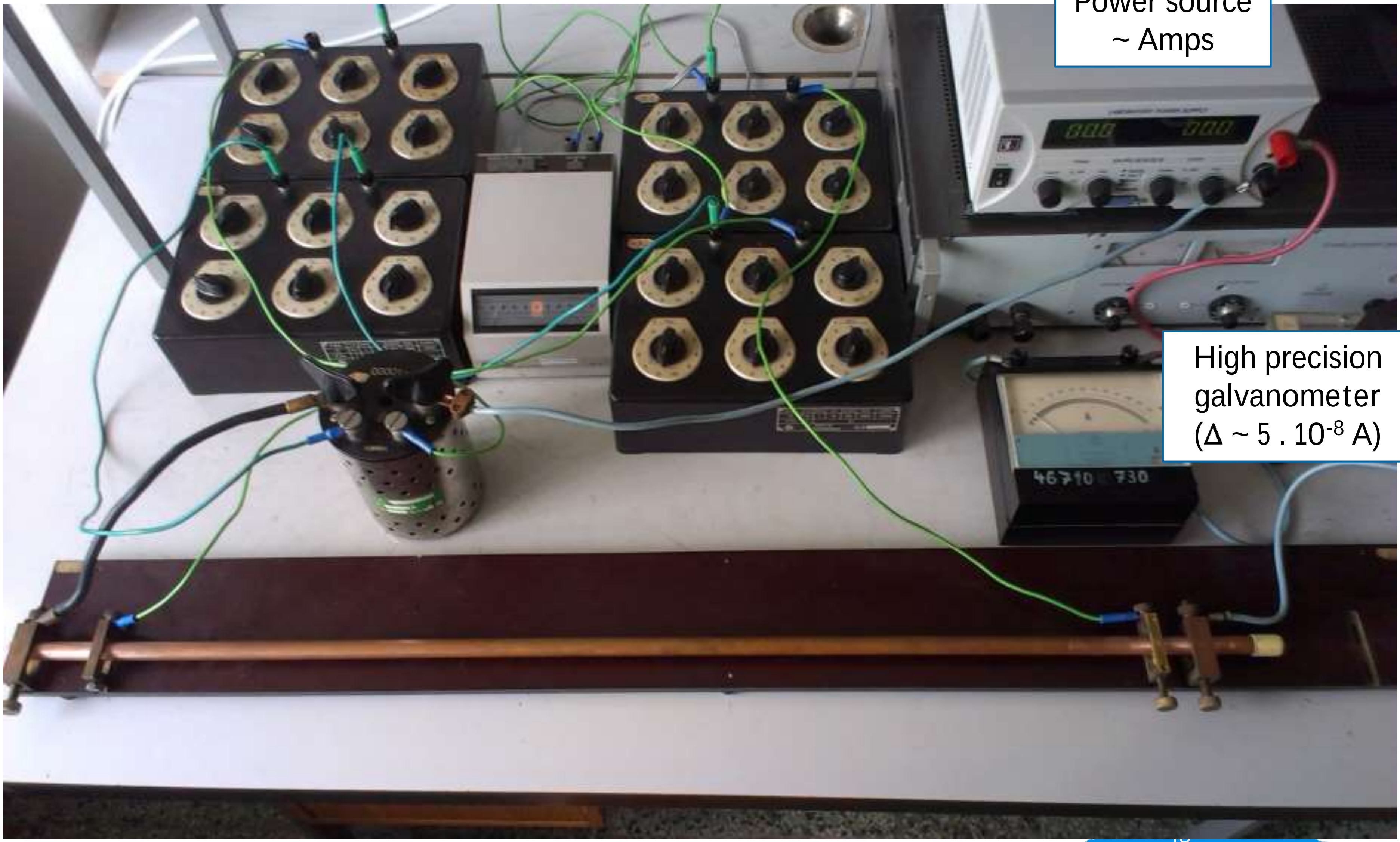
Conductivity of our copper pipes:

$$\sigma = (42 \pm 2)10^6\, Sm^{-1}$$

Pure copper conductivity

$$\sigma = 56 \cdot 10^6\, Sm^{-1}$$

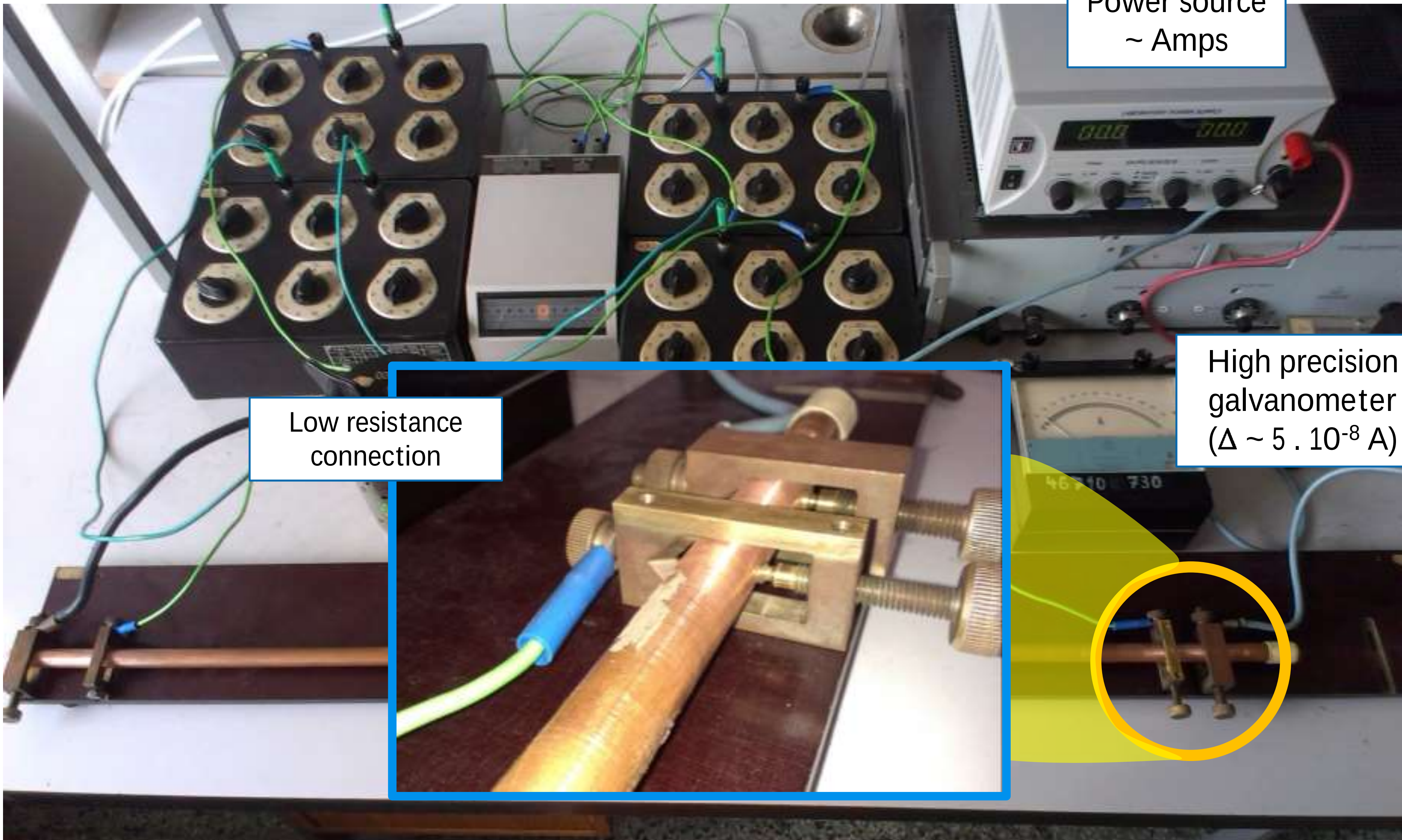
Conductivity Measurement



Power source
~ Amps

High precision
galvanometer
($\Delta \sim 5 \cdot 10^{-8}$ A)

Conductivity Measurement

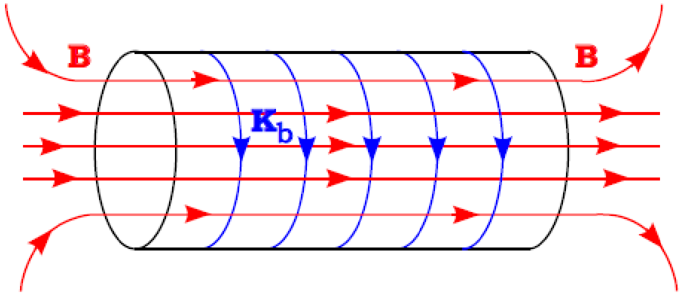


Equivalence of the Fields

- Simple proof from **Polack, Stump** “*Electromagnetism*” **Ch.** 9.2 pg. 319

Uniformly magnetized cylinder $M = M_0 \hat{k}$

$$A_{(x)} = \int \frac{\mu_0 M \times (x - x')}{4\pi |x - x'|^3} \mathrm{dx}'$$



By definition

The same B as if it was created by (bound) currents

$$B_{(x)} = \nabla \times A_{(x)}$$

$$J_b = \nabla \times M$$
$$K_b = M \times \hat{n}$$

In our case

$$J_b = \nabla \times M = 0$$
$$K_b = M \times \hat{n} = M_0 \hat{\varphi}$$

Magnetic field is caused by azimuthal bond surface currents

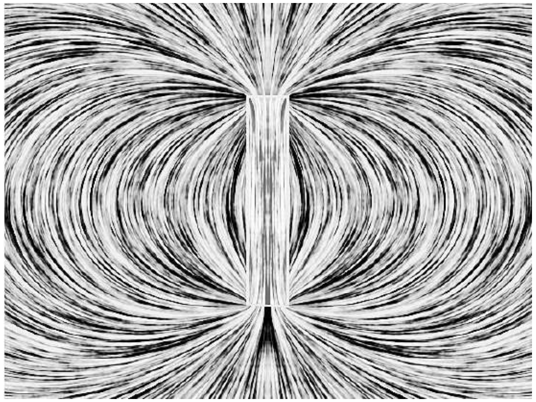
Ttherefore the same field as Coil

Magnetic Field of Cylindrical Magnet

[Cylindrical Magnets and Ideal Solenoids]

$$B_\rho = B_0\left(\alpha_+ C_{(k_+,1,1,-1)} - \alpha_- C_{(k_-,1,1,-1)}\right)$$

$$B_z = \frac{B_0 d}{d+2\rho}\left(\beta_+ C_{(k_+,\gamma^2,1,\gamma)} - \beta_- C_{(k_-,\gamma^2,1,\gamma)}\right)$$



Where $C_{(k,p,c,s)} = \int_0^{\pi/2} \frac{c\cos^2\varphi + s\sin^2\varphi}{(\cos^2\varphi + p\sin^2\varphi)\sqrt{\cos^2\varphi + k\sin^2\varphi}} d\varphi$ Generalized complete elliptic integral

$$z_\pm = z \pm \frac{h}{2}$$

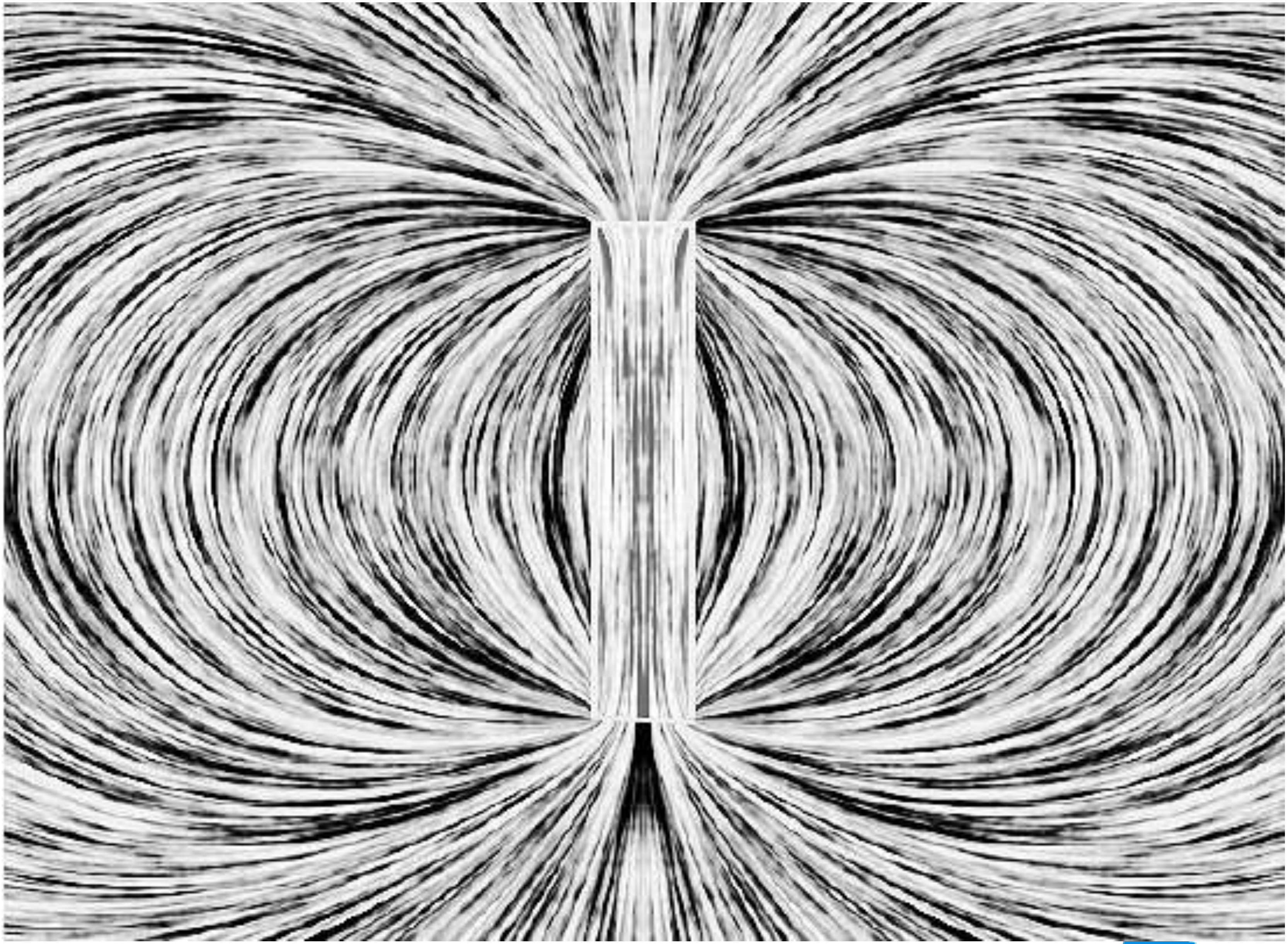
$$\gamma = \frac{d-2\rho}{d+2\rho}$$

$$\alpha_\pm = \frac{d}{2\sqrt{z_\pm{}^2 + \left(\rho + \frac{d}{2}\right)^2}}$$

$$B_0 = \mu_0 \frac{4\vec{\mu}}{\pi^2 h d^2}$$

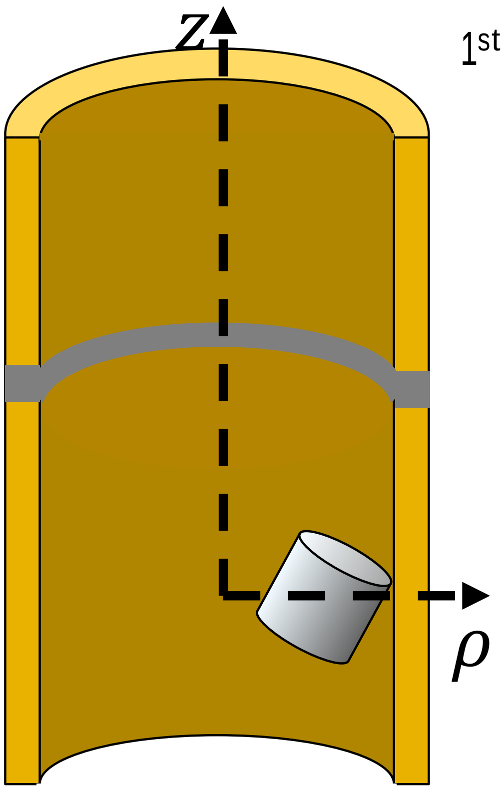
$$k_\pm = \sqrt{\frac{z_\pm^2 + \left(\frac{d}{2} - \rho\right)^2}{z_\pm^2 + \left(\frac{d}{2} + \rho\right)^2}}$$

$$\beta_\pm = \frac{z_\pm}{\sqrt{z_\pm{}^2 + \left(\rho + \frac{d}{2}\right)^2}}$$



[Cylindrical Magnets and Ideal Solenoids]

Modificated theory – Shift & Rotation



1st step - Transformations of the coordinate system – Shift & Rotation

$$dI_{(z)} = -2\pi\rho v \frac{\sigma d\rho dz}{2\pi\rho} \int_0^{2\pi} (\vec{B}_{\rho(\beta)} \cdot \vec{n}_{(\beta)}) d\beta$$

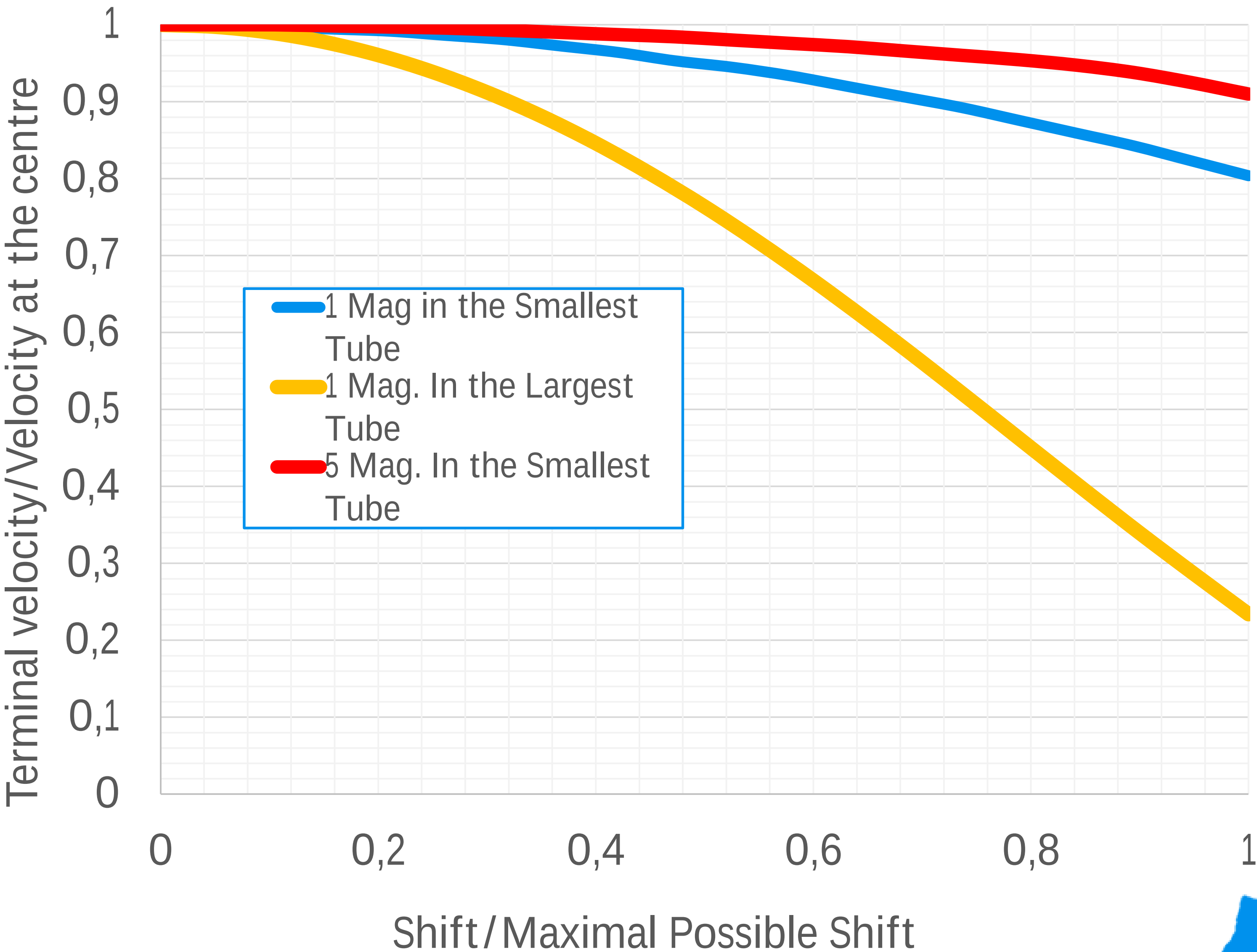
Magnetic field flowing outside the tube in radial direction

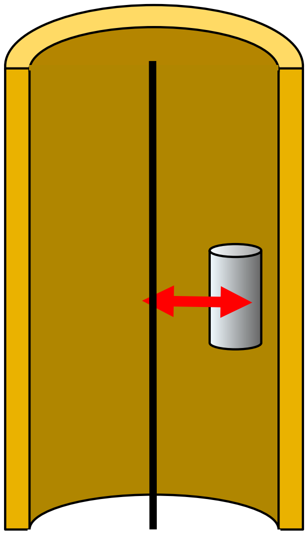
$$dF = -\frac{2\pi}{4\pi^2} \rho \left(\int_0^{2\pi} (\vec{B}_{\rho(\beta)} \cdot \vec{n}_{(\beta)}) d\beta \right)^2 v$$

$$F = \int_{-\infty}^{\infty} \int_r^{r+w} dF_{(z,\rho)} d\rho dz$$

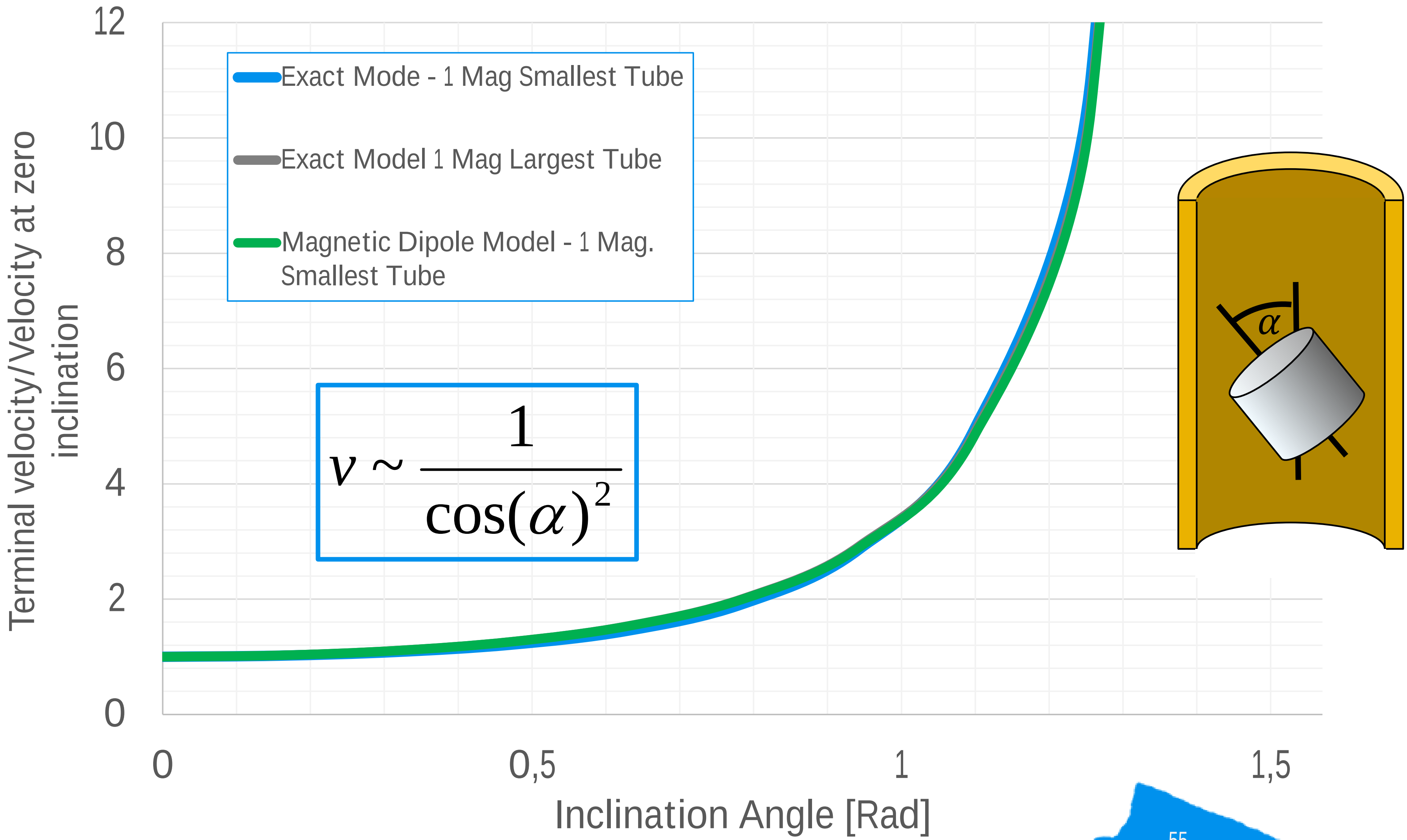
$$F = -\int_{-\infty}^{\infty} \int_r^{r+w} \frac{1}{2\pi} \rho v \left(\int_0^{2\pi} (\vec{B}_{\rho(\beta)} \cdot \vec{n}_{(\beta)}) d\beta \right)^2 d\rho dz$$

Shift to the side – zero inclination





Inclination of the magnet – zero shift

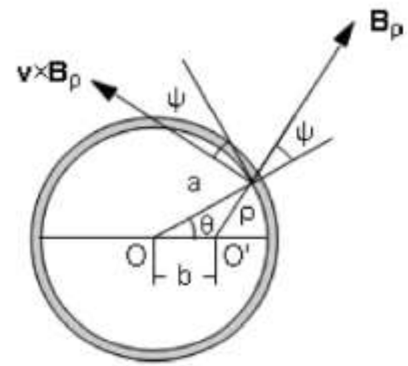


$$v \sim \frac{1}{\cos(\alpha)^2}$$

Shift – Magnetic Dipole Model

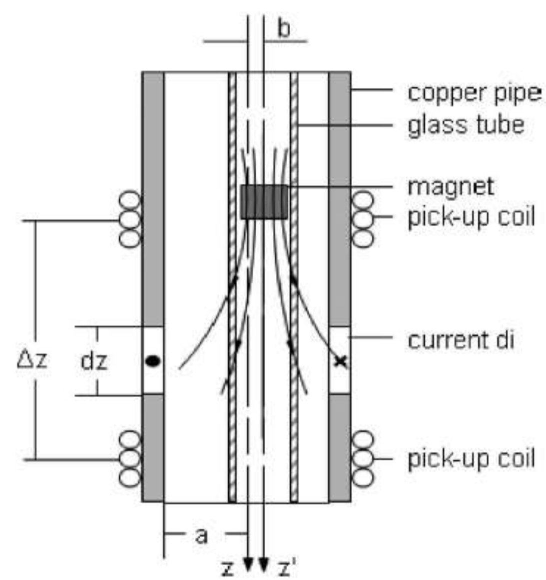
Analytical solution doesn't exist – Elliptic integrals – No advantage against our model

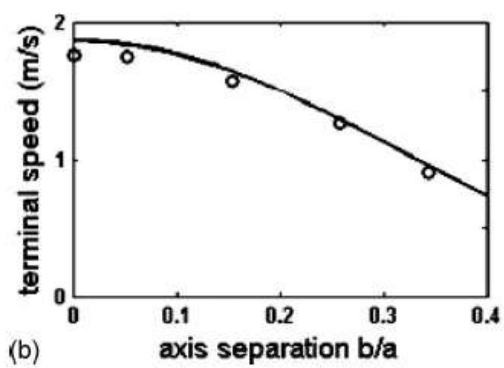
- Solved in G. Donoso, C. L. Ladera, and P. **Martín** "*Damped fall of magnets inside a conducting pipe*" Am. J. Phys. 79, 193 (2011)



$$F_z = \frac{36\pi\sigma\tau v\tilde{\mu}^2}{\alpha a^4}\int_0^\infty [G(u,b)]^2 du \equiv \frac{36\pi\sigma\tau v\tilde{\mu}^2}{\alpha a^4}\tilde{f}(b/a)$$

$$G(u,b) = \int_0^{2\pi} \frac{u[1-(b/a)\cos\theta]}{2\pi\left[1+\left(\frac{b}{a}\right)^2 - 2\left(\frac{b}{a}\right)\cos\theta + u^2\right]^{5/2}} d\theta.$$

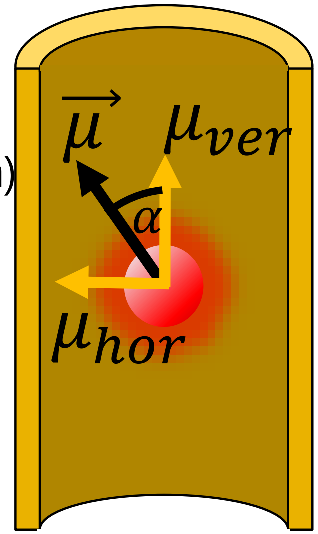




Rotation – Magnetic Dipole

Analytical solution possible only for magnetic dipole model

Magnetic moment could be divided into two (with the same position)



$$\mu_{ver} = |\vec{\mu}|\cos(\alpha)$$
$$\mu_{hor} = |\vec{\mu}|\sin(\alpha)$$

Resulting magnetic field is the superposition of the two partial

Thanks to: $(\vec{a} + \vec{b}) \cdot \vec{c} = (\vec{a} \cdot \vec{c}) + (\vec{b} \cdot \vec{c})$

Associated radial fields (Flowing perpendicular to tube)

Induced voltages (Over infinitesimal ring)

μ_{ver} $\quad \vec{B}_\rho = \frac{\mu}{\ } \frac{3\rho z|\mu_{ver}|}{\ }$

$$U = \int_0^{2\pi} B_\rho \rho v d\beta \neq 0$$

Horizontal magnetic moment doesn't contribute to braking force

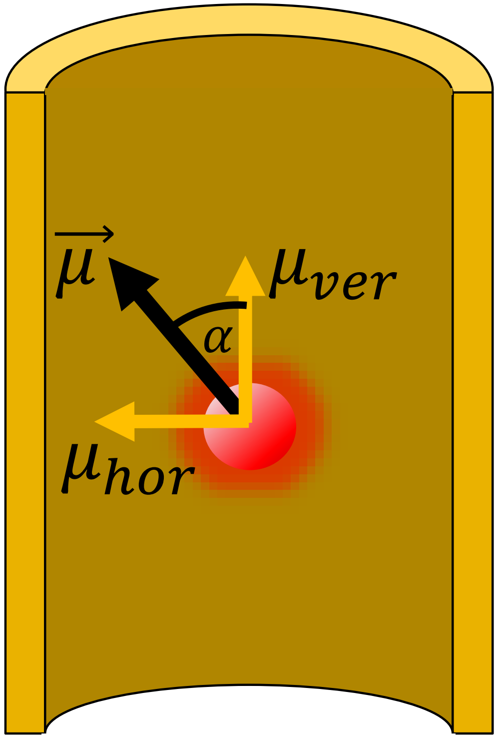
μ_{hor} $\quad \vec{B}_\rho = \frac{\mu}{4\pi} \frac{3\rho^2|\mu_{hor}|}{\sqrt{\rho^2+z^2}^5}\left(\cos(\beta)^3 + \sin(\beta)^2\cos(\beta) - \frac{\rho^2+z^2}{3\rho^2}\cos(\beta)\right)$

$$U = \int_0^{2\pi} B_\rho \rho v d\beta = 0$$

Rotation – Magnetic Dipole

Analytical solution possible only for magnetic dipole model

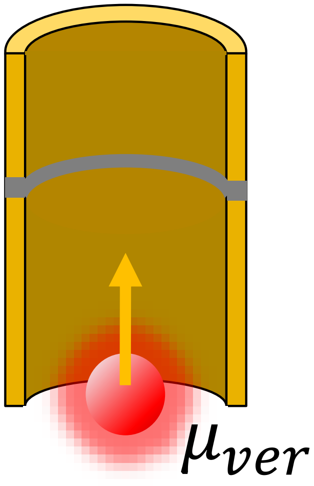
$$F = \frac{45\mu_0^2 \sigma \mu_{mag}{}^2 v}{1024} \frac{w}{r^4} \cos(\alpha)^2$$



Torque on Magnet

Induced voltage (In infinitesimal ring) due to vertical component of the magnetic moment:

$$U = -2\pi\rho\,\frac{\mu}{4\pi}\,\frac{3\rho z\left|\mu_{ver}\right|}{\sqrt{\rho^2+z^2}^{\,5}}\,v \;\Rightarrow\; dI_{(z)} = -\frac{\mu}{4\pi}\,\frac{3\rho z\left|\mu_{ver}\right|}{\sqrt{\rho^2+z^2}^{\,5}}\,v\sigma w dz$$



Inducted current induces magnetic field in the position of the dipole:

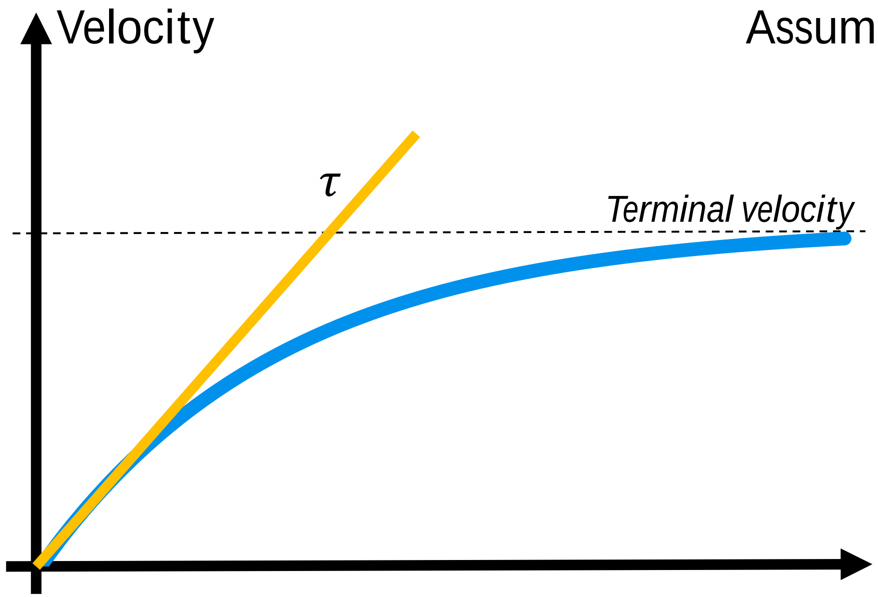
$$dB_{(z)} = \frac{\mu}{4\pi}\,\frac{2\pi\rho^2 dI_{(z)}}{\sqrt{\rho^2+z^2}^{\,3}} \;\Rightarrow\; B = \int_{-L/2+x}^{L/2+x} dB_{(z)} = \left(\frac{\mu_0}{4\pi}\right)^2 6\rho^3\pi\mu_{ver} w\sigma v\left[\frac{1}{6}\left(z^2+\rho^2\right)^{-3}\right]_{-L/2+x}^{L/2+x}$$

(Biot-Savart law for current loop at distance z)

Which exerts torque on magnetic dipole:

$$\vec{\tau} = \vec{\mu}\times\vec{B} = \mu B\sin(\alpha) = \left(\frac{\mu_0}{4\pi}\right)^2 6\rho^3\pi\mu^2 w\sigma v\left[\frac{1}{6}\left(z^2+\rho^2\right)^{-3}\right]_{-L/2+x}^{L/2+x} \cos(\alpha)\sin(\alpha)$$

Motion of the magnet



Assuming braking force from infinite tube all time:

$$v_{(t)} = \frac{mg}{C}\left(1 - e^{-\frac{C}{m}t}\right)$$

Distance travelled ($v_{(t=0)} = 0$):

$$s_{(t)} = \frac{mg}{C}t + \left(\frac{m}{C}\right)^2 g\left(e^{-\frac{C}{m}t} - 1\right)$$

Worst case scenario

$$v_{terminal} = \frac{mg}{C} = 1{,}6 ms^{-1}$$

Characteristic time

$$\tau = \frac{m}{C} = 0{,}16\ s$$

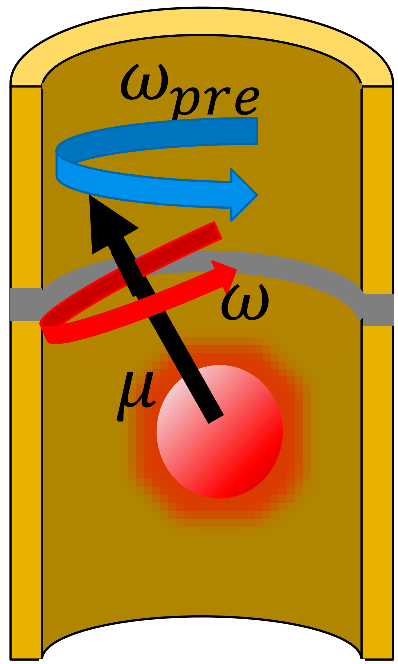
95 % of terminal velocity

$$3\tau = 0{,}48\ s$$

Distance

$$s_{(3\tau)} = 52\ cm$$

Precession of the magnet



Inclination

Position in the tube

$$\omega_{pre} = \frac{\mu B}{L} = \frac{2\mu B}{mR^2\omega} = \frac{2\mu}{mR^2\omega}\left(\frac{\mu_0}{4\pi}\right)^2 6\rho^3\pi\mu^2 w\sigma v\left[\frac{1}{6}\left(z^2+\rho^2\right)^{-3}\right]_{-L/2+x}^{L/2+x}\cos(\alpha)$$

Rotation about magnet axis

Magnet's velocity

[Partovi & Morris] Result

- Obtained the general solution for uniformly magnetized cylinder via solution of Maxwell's equations in magnet-pipe system:

$$\mathbf{F}^{uni} = -\hat{\mathbf{v}}\frac{\mu_0 m^2}{2\pi^2}\int_0^{+\infty} dk k^3 \left[\frac{\sin(kL/2)}{(kL/2)}\right]^2 \left[\frac{I_1(ka)}{(ka/2)}\right]^2 \mathrm{Im}[Q(k)]$$

Where Q(k) is ratio of $b_1(k)/b_0(k)$

$$\begin{aligned} b_1(k) = & \{[K_0(|k|R_1)K_0(|k|R_2)T_{11} + \beta K_0(|k|R_1)K_1(|k|R_2)T_{10} \\ & -\beta K_1(|k|R_1)K_0(|k|R_2)T_{01} - \beta^2 K_1(|k|R_1)K_1(|k|R_2)T_{00}] \\ & \div[I_0(|k|R_1)K_0(|k|R_2)T_{11} + \beta I_0(|k|R_1)K_1(|k|R_2)T_{10} \\ & +\beta I_1(|k|R_1)K_0(|k|R_2)T_{01} + \beta^2 I_1(|k|R_1)K_1(|k|R_2)T_{00}]\} b_0(k), \end{aligned}$$

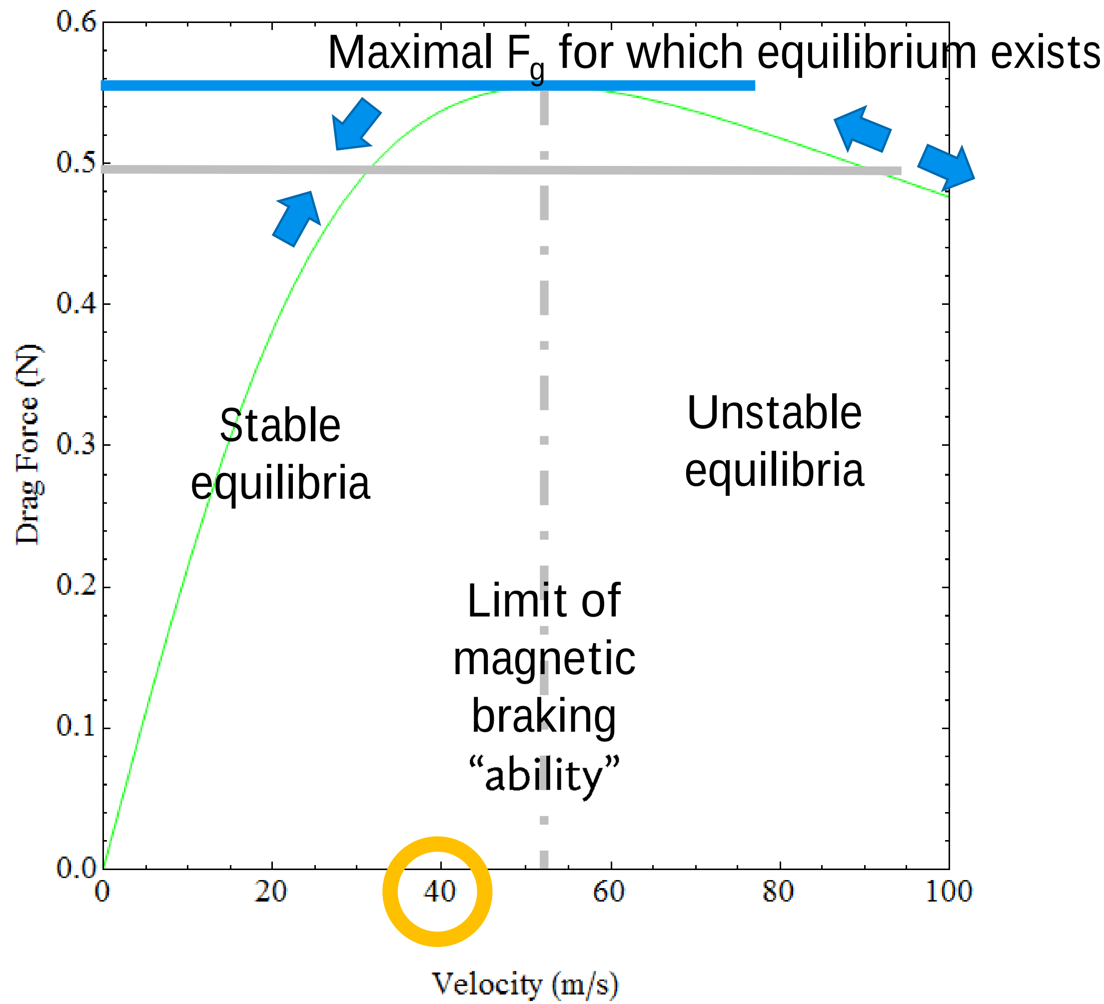
Exact, but <u>very cumbersome</u> to handle

$$\begin{aligned} T_{00} &= K_0(\alpha|k|R_1)I_0(\alpha|k|R_2) - I_0(\alpha|k|R_1)K_0(\alpha|k|R_2) \\ T_{01} &= K_0(\alpha|k|R_1)I_1(\alpha|k|R_2) + I_0(\alpha|k|R_1)K_1(\alpha|k|R_2) \\ T_{10} &= K_1(\alpha|k|R_1)I_0(\alpha|k|R_2) + I_1(\alpha|k|R_1)K_0(\alpha|k|R_2) \\ T_{11} &= K_1(\alpha|k|R_1)I_1(\alpha|k|R_2) - I_1(\alpha|k|R_1)K_1(\alpha|k|R_2), \end{aligned}$$

$$\begin{aligned} \alpha &= \sqrt{\kappa^2}/|k| = \sqrt{1 - i\frac{\mu_0\mu_{rel}\sigma v}{k}}, \\ \beta &= \frac{\alpha}{\mu_{rel}} = \frac{1}{\mu_{rel}}\sqrt{1 - i\frac{\mu_0\mu_{rel}\sigma v}{k}}. \end{aligned}$$

Large velocities – Skin effect

- Using results from [Partovi & Morris] for those parameters:

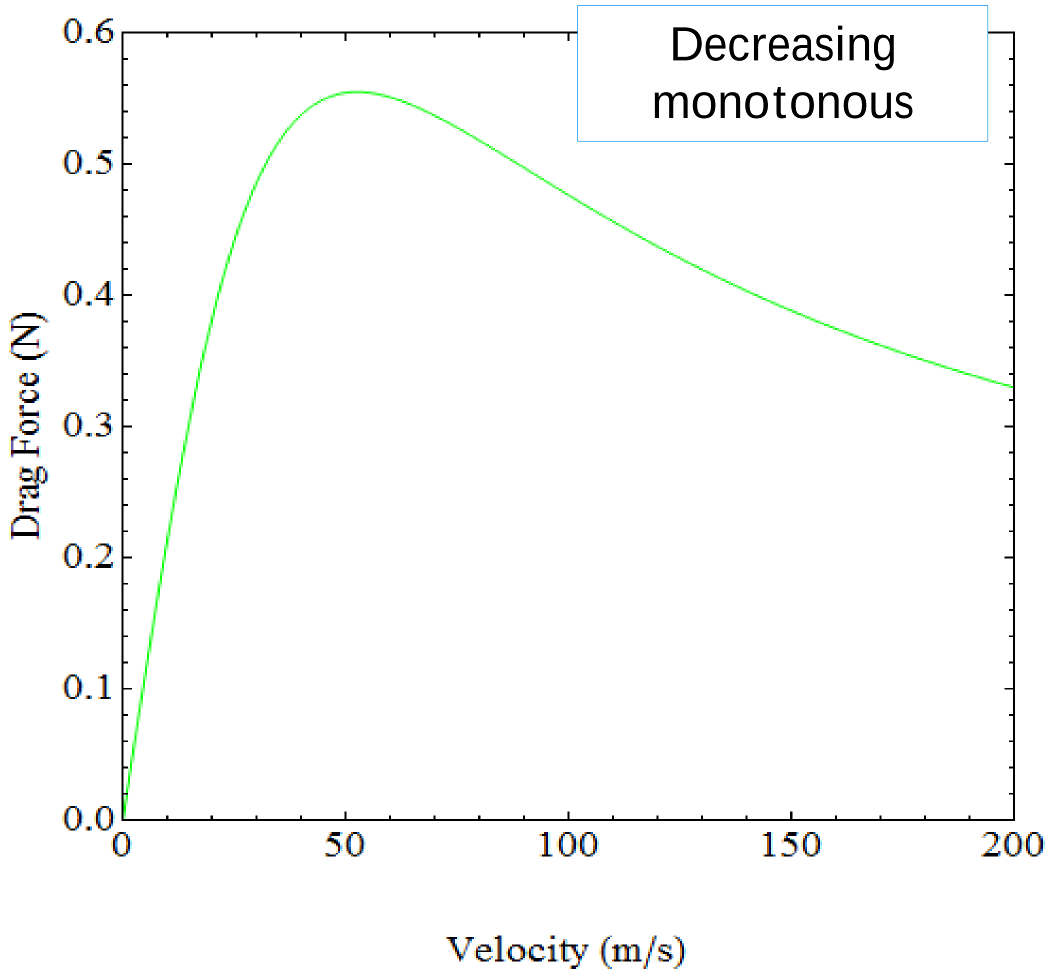


$$B_r = 1{,}19 \pm 0{,}02\ T$$
$$m = 1{,}06\ g$$
$$h = 5\ mm$$
$$d = 6\ mm$$

$$\sigma = 42 \cdot 10^6\ Sm^{-1}$$
$$r = 6{,}5\ mm$$

Large velocities – Skin effect

- Using results from [Partovi & Morris] for those parameters:



$m = 1{,}06\ g$

$h = 5\ mm$

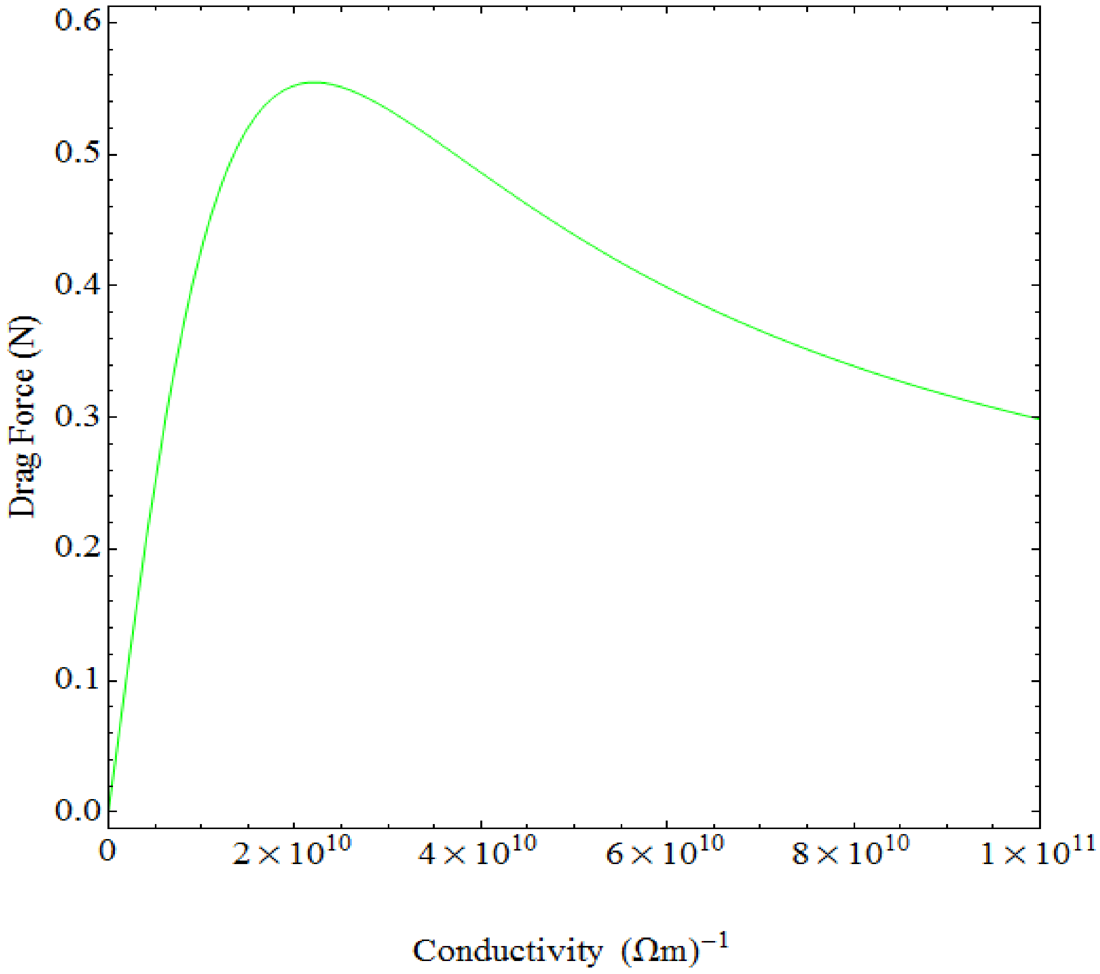
$d = 6\ mm$

$\sigma = 42 \cdot 10^6\ Sm^{-1}$

$r = 6{,}5\ mm$

Conductivity dependence – Skin effect

- Drag force (*at v = 10 cms*$^{-1}$) for different conductivities



$B_r = 1{,}19 \pm 0{,}02\ T$

$m = 1{,}06\ g$

$h = 5\ mm$

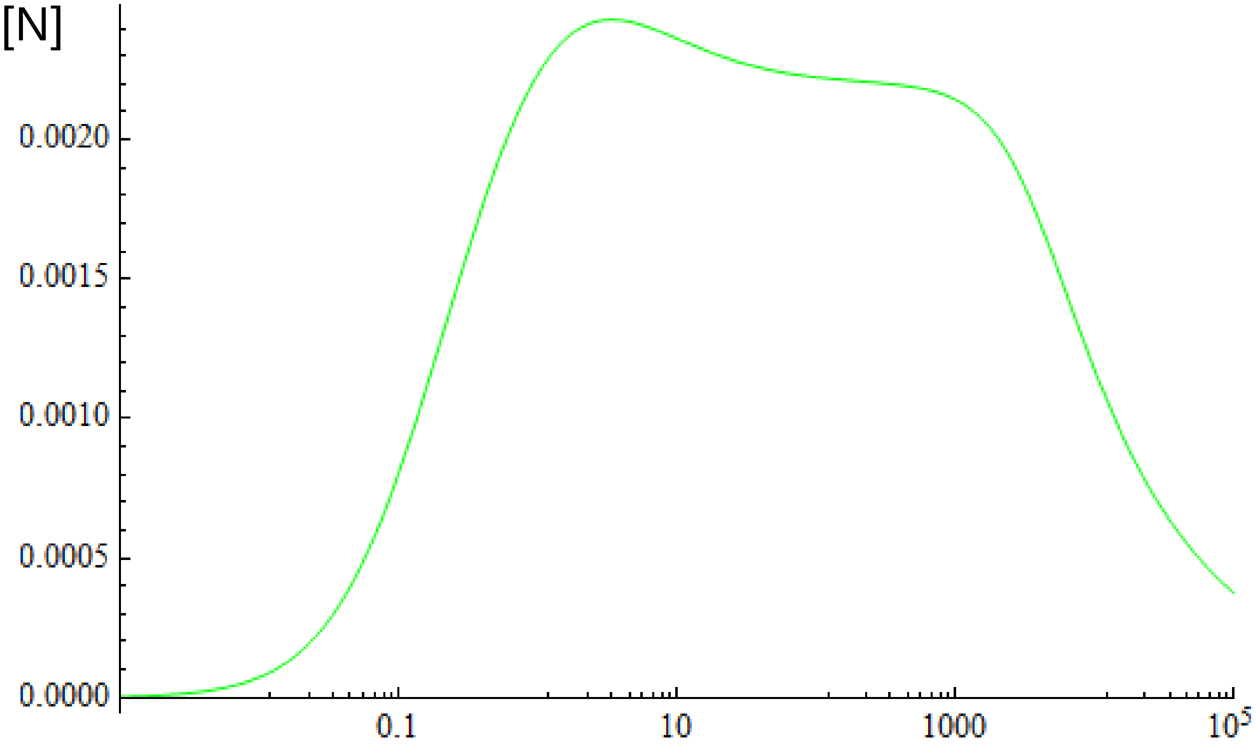
$d = 6\ mm$

$r = 6{,}5\ mm$

Magnetic permeability

- Drag force ($v = 10\ cms^{-1}$) for different permeabilities

Drag force [N]



Relative Permeability

$B_r = 1{,}19 \pm 0{,}02\ T$
$m = 1{,}06\ g$
$h = 5\ mm$
$d = 6\ mm$

$\sigma = 42 \cdot 10^6\ Sm^{-1}$
$r = 6{,}5\ mm$

Cuted Slit

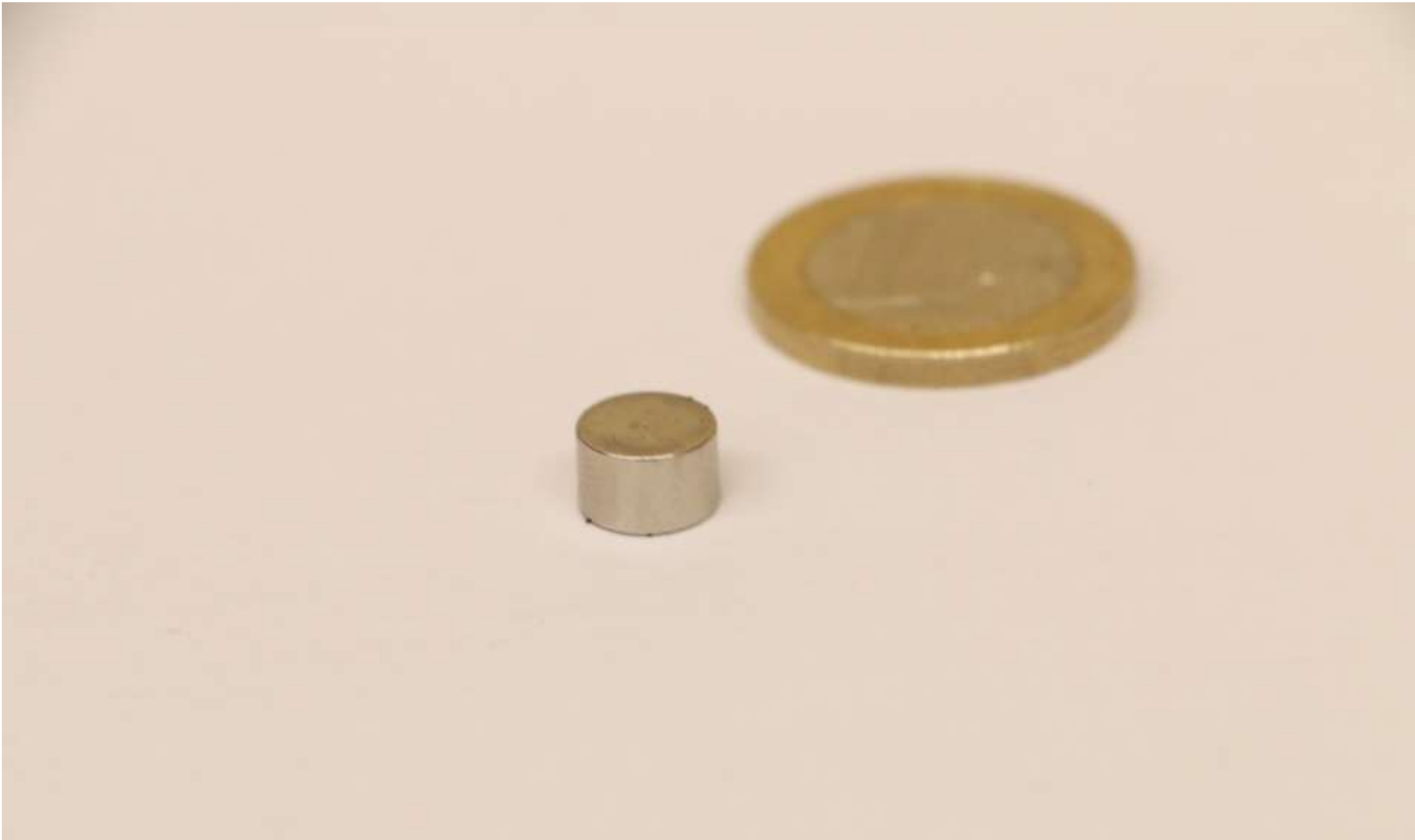
Tube without slit

Tube with slit

Cuted Slit Comparison

Magnet n.1

Magnet n.2

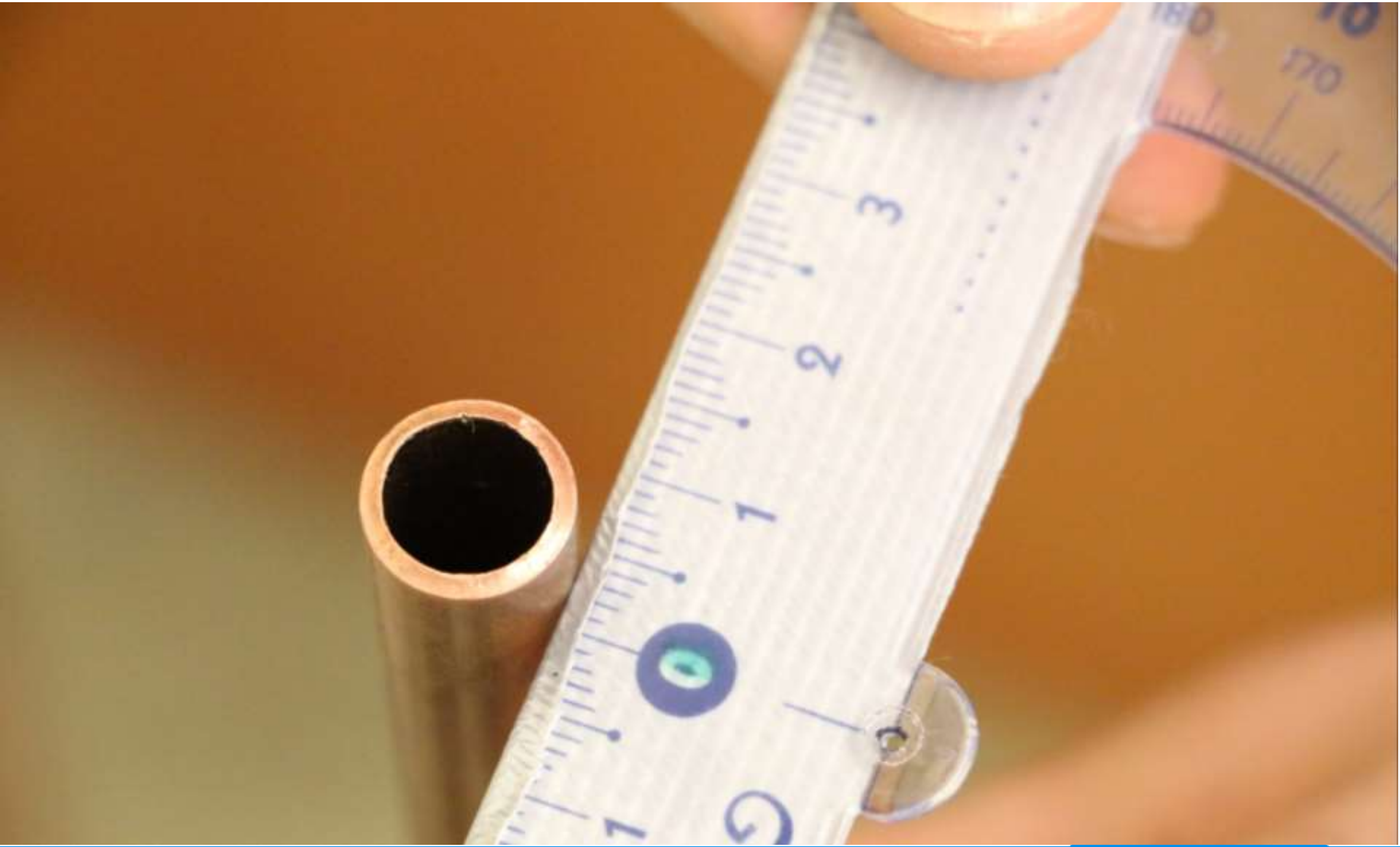


Magnet n.3

Magnet n.4

Magnet n.5

Tube n.1



Tube n.2

Tube n.3

Tube n.4

Tube n.5