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# Water bombs

Mário Lipovský



# Task

Some students are ineffective in water balloon fights as the balloons they throw rebound without bursting.

Investigate the motion, deformation, and rebound of a balloon filled with fluid.

Under what circumstances does the balloon burst?



# Content

- Motion and deformation
  - Two types
- Rebound
  - Angle of rebound
- Burst
  - Energies in balloon
    - Elastic
    - Kinetic
    - Potential
- Summary

# How it looks



# How it looks





# Motion and

# deformation

$$E_k = 0$$

$$E_p = mgh$$

$$E_e = A$$



$E_k$  *Kinetic energy*

$E_p$  *Potential energy*

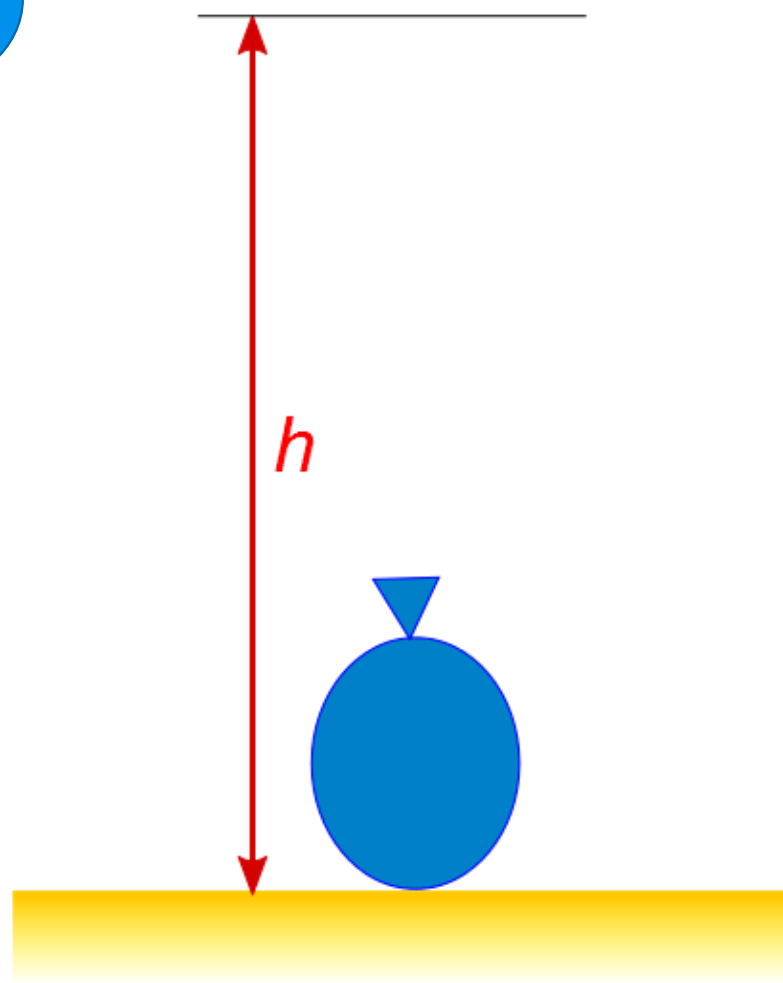
$E_e$  *Elastic energy*

# Motion and deformation

$$E_k = mgh$$

$$E_p = 0$$

$$E_e = A$$



# Motion and deformation

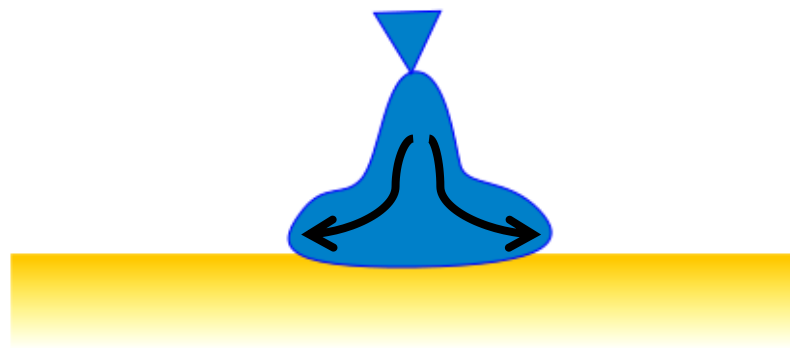
$$E_k = mgh - \Delta E$$

$$E_p = 0$$

$$E_e \approx A + \Delta E$$



*Water flows to the sides*







# Motion and deformation

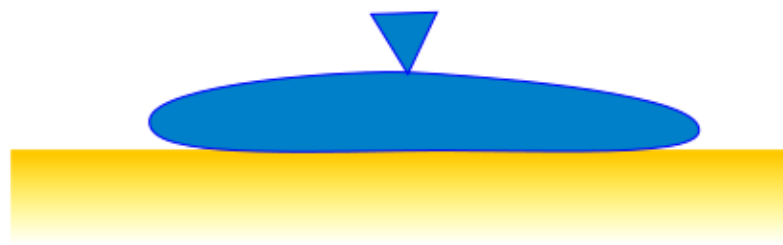
$$E_k = 0$$

$$E_p = 0$$

$$E_e \approx A + mgh$$

*Maximal radius of balloon  
Water stopped by rubber*

*Some energy lost  
- water flows*



# Motion and deformation

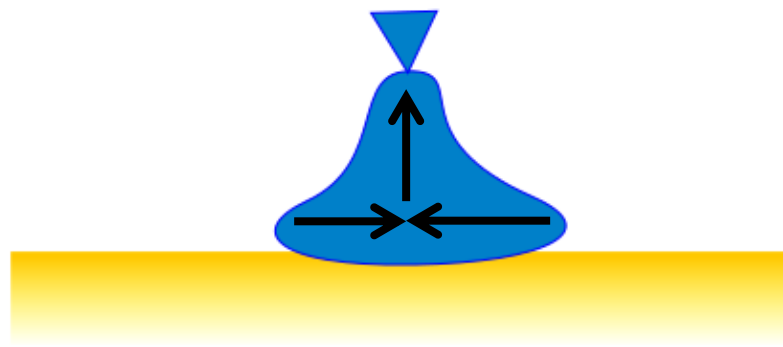
$$E_k = \Delta E$$

$$E_p = 0$$

$$E_e \approx A + mgh - \Delta E$$



*Rubber is contracting*





# Motion and deformation





# Motion and deformation



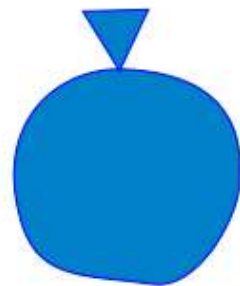


# Motion and deformation

$$E_k = 0$$

$$E_p < mgh$$

$$E_e = A$$



*Energy losses*  
*Water convection*  
*Heat*

# Different type of deformation



The view is from the bottom of the aquarium 14

# Different type of deformation





# Different type of deformation





# Different type of deformation



# Different type of deformation



# Different type of deformation



# Different type of deformation

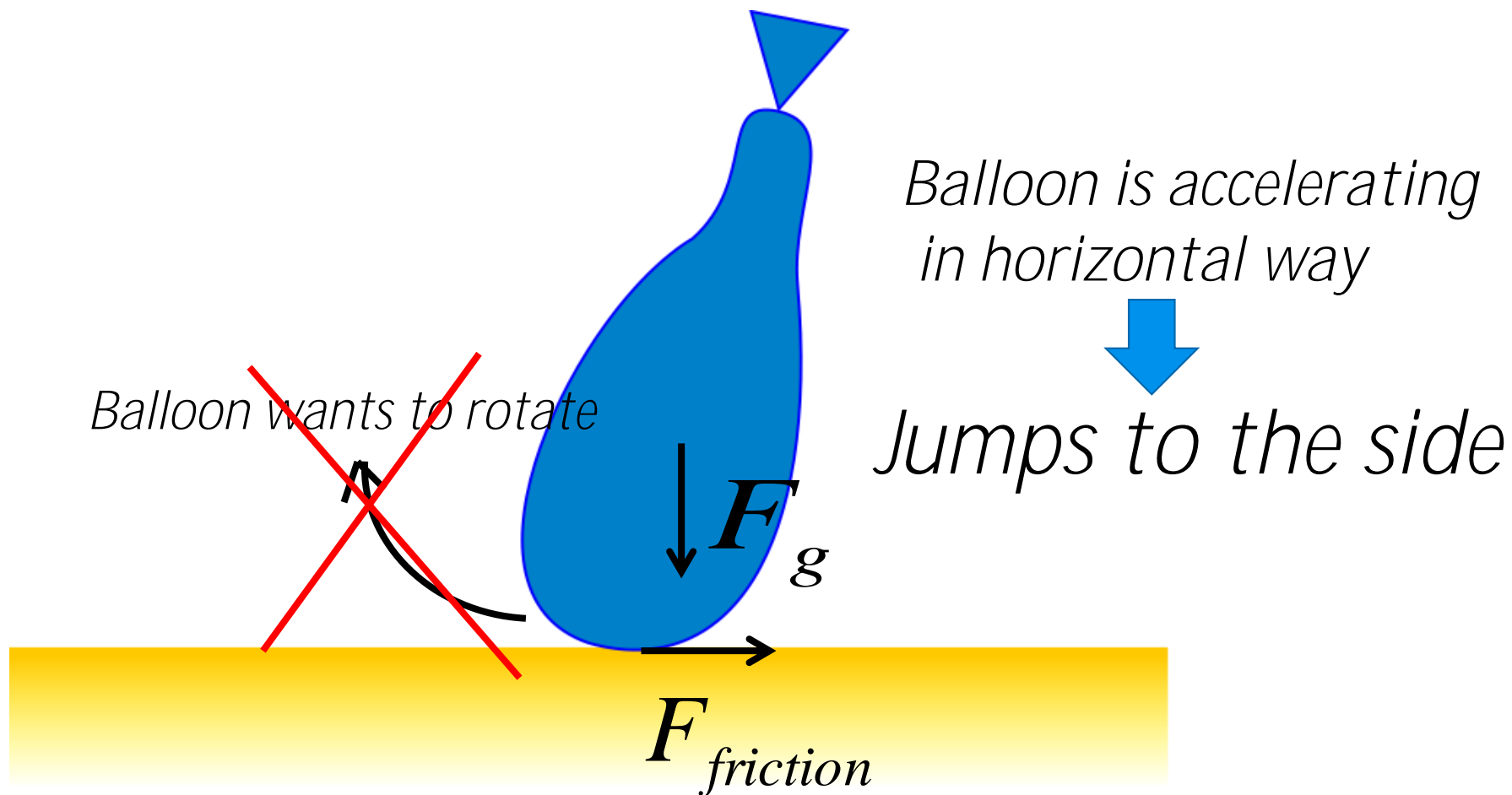
*Balloon jumps*



“Investigate the motion, deformation and rebound...”



# Rebound



# Jumping balloon



# Jumping balloon





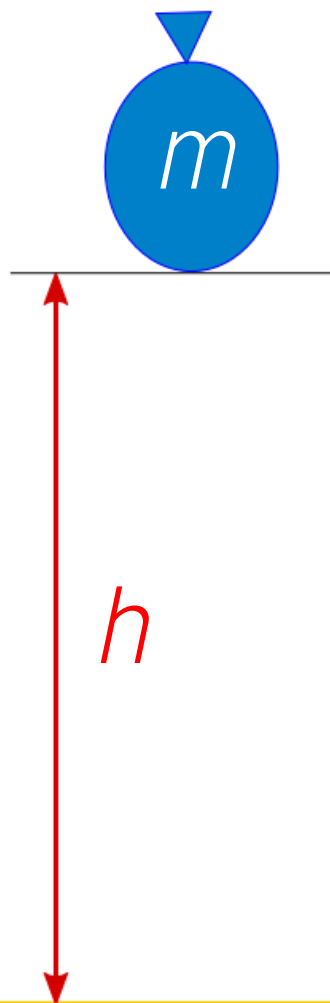
# Jumping balloon



*“Investigate the motion, deformation, and rebound of a balloon filled with fluid. Under what circumstances does the balloon burst?”*



# Apparatus



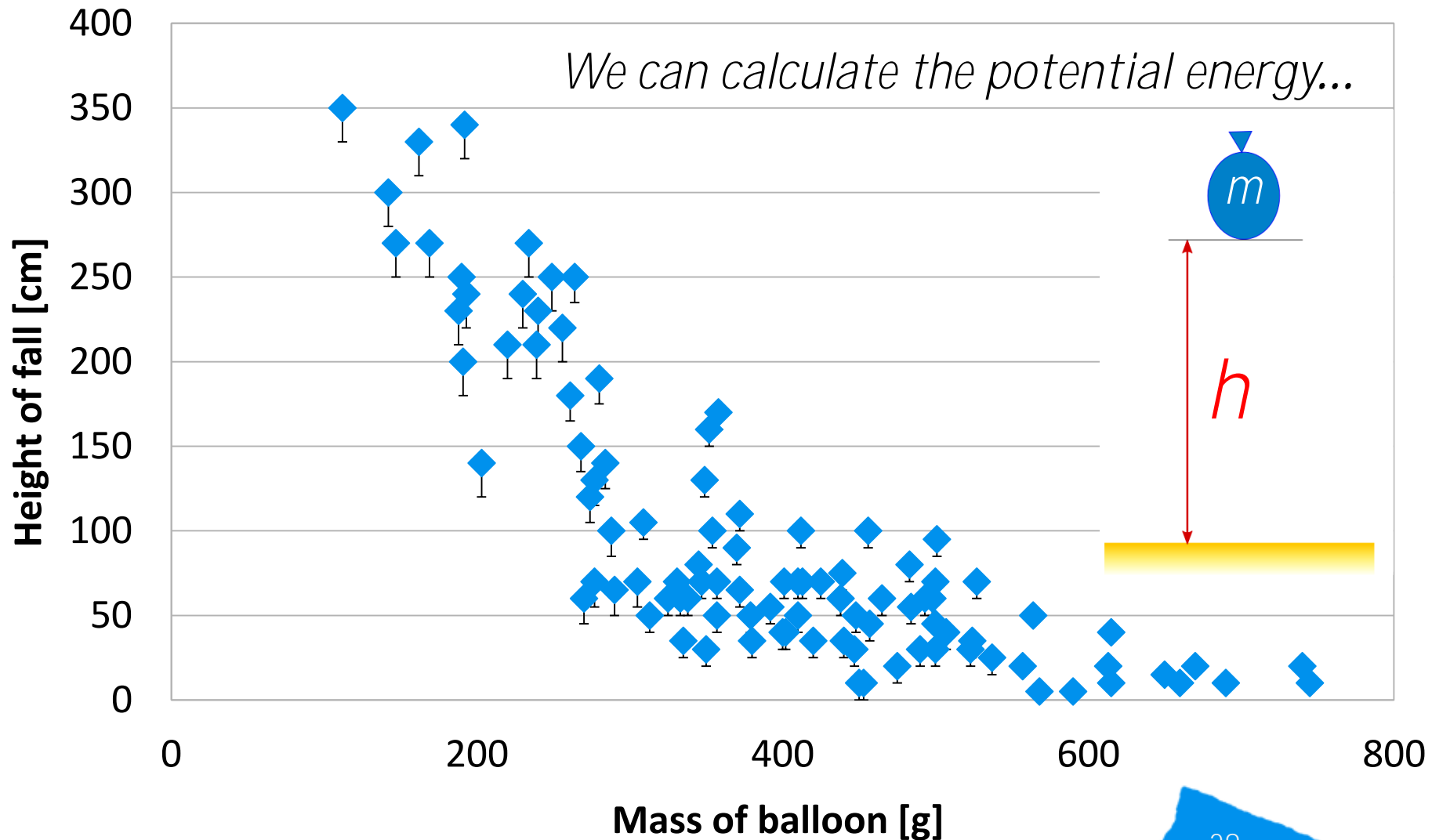
Balloon with mass  $m$

Changing height  $h$  from  $h=0\text{cm}$

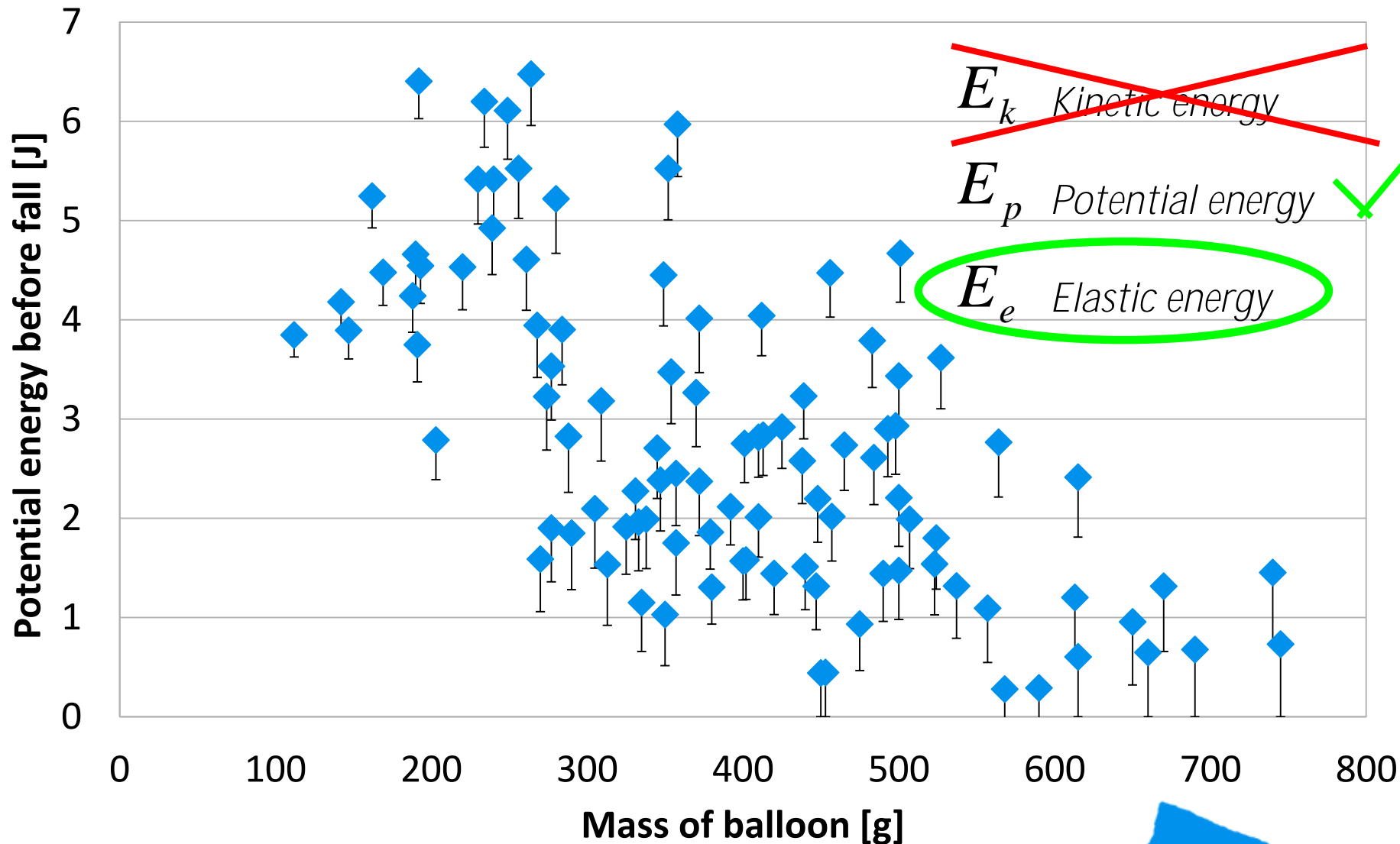
Find the **smallest**  $h$ , when it bursts

Do it with many balloons

# Height, which causes burst

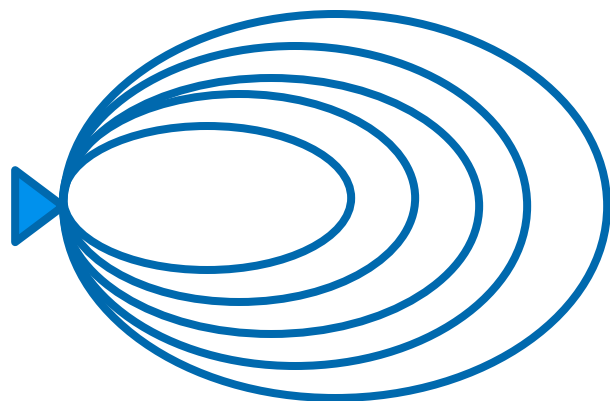


# Potential energy needed to burst



# Elastic energy of balloon

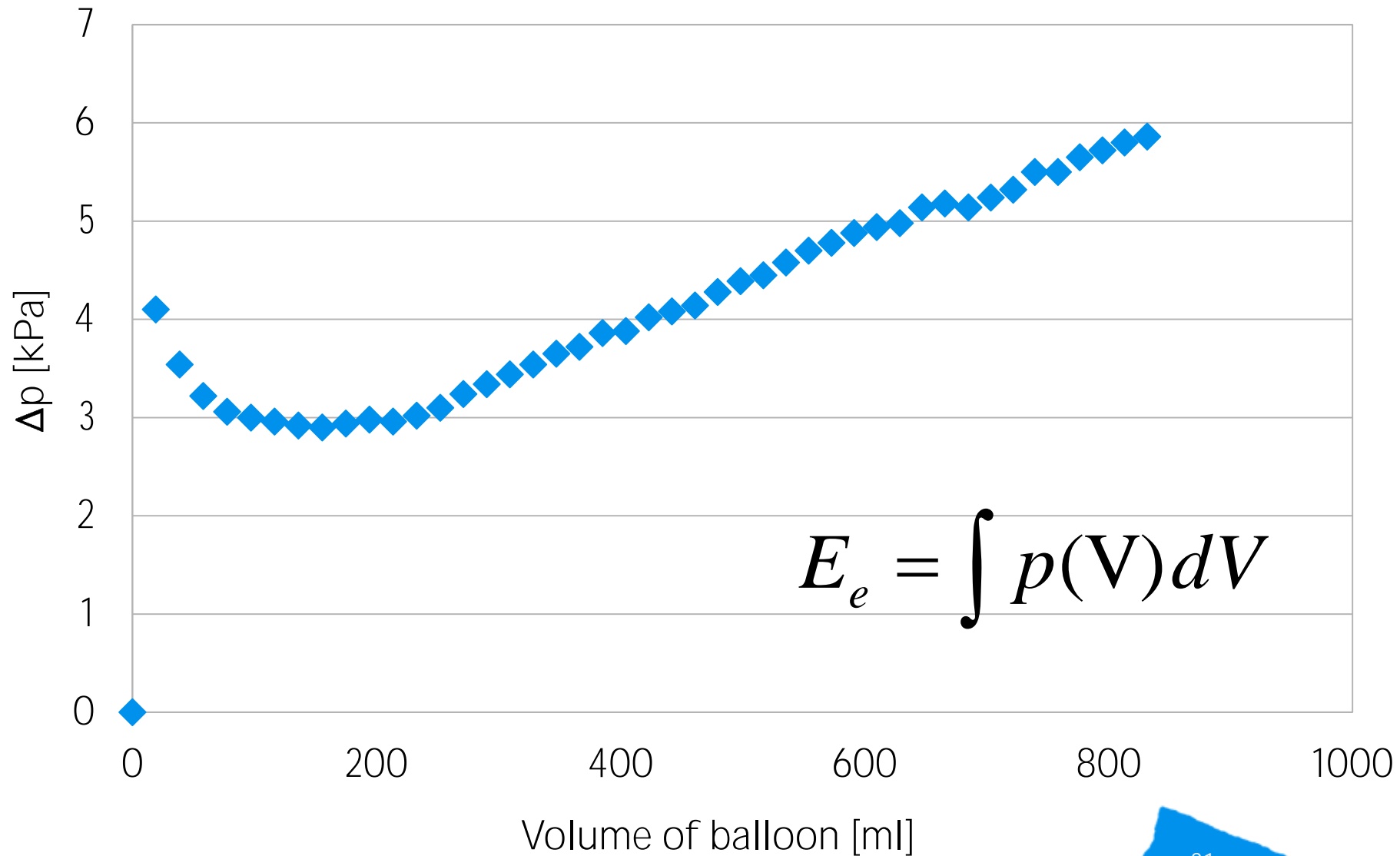
Can be calculated from  
pressure change during inflation



$$E_e = \int p(V) dV$$

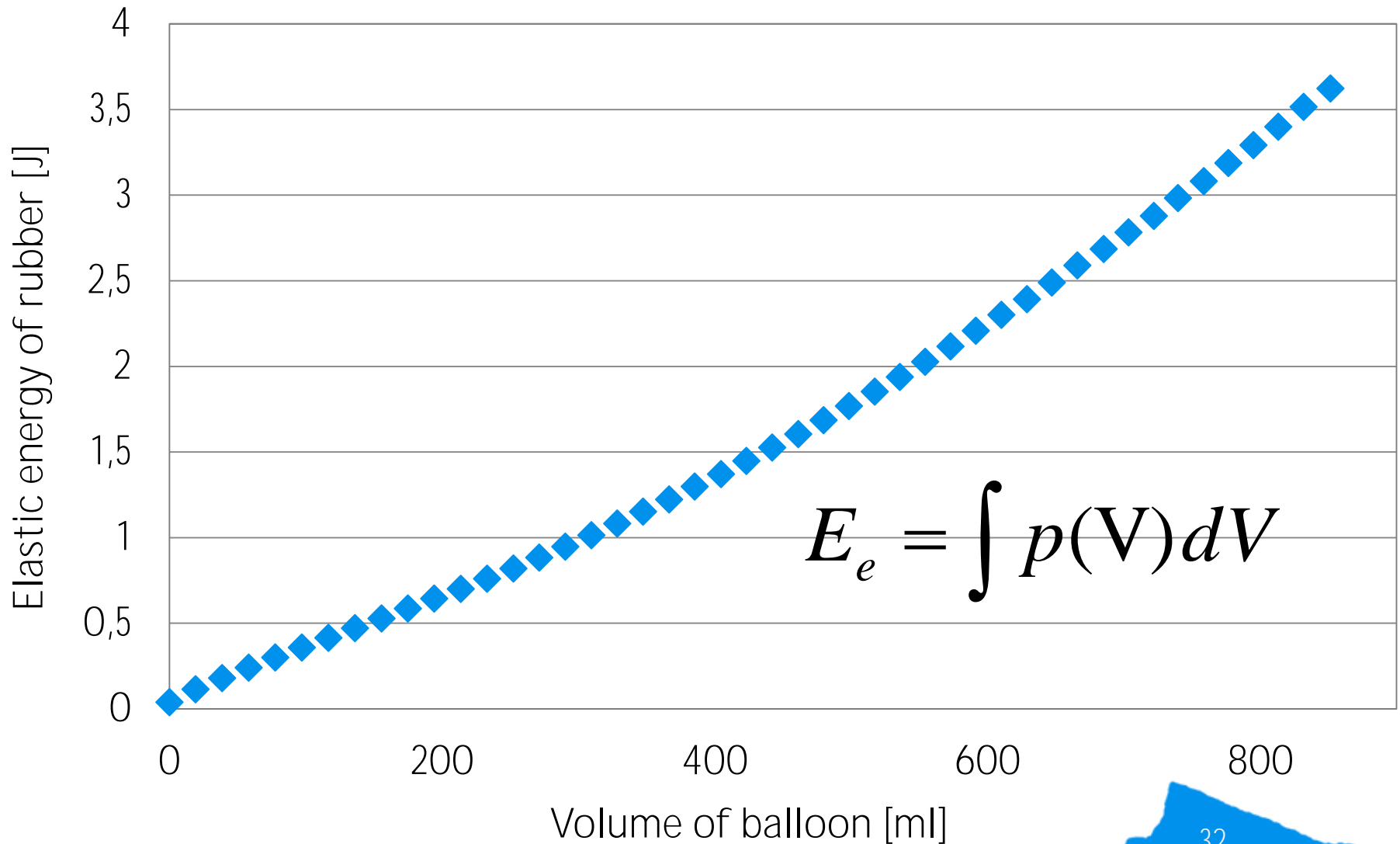


# Pressure measurement



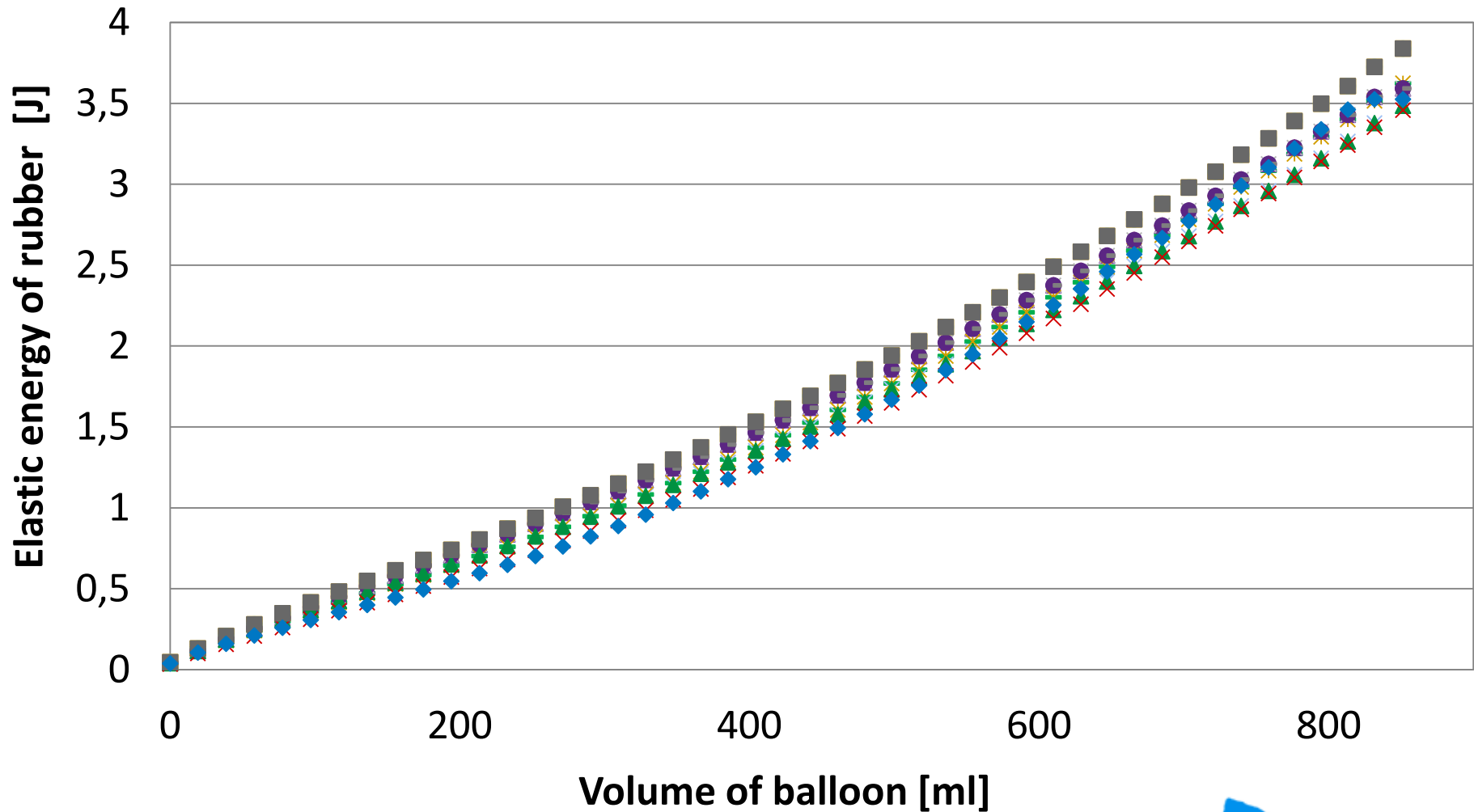


# Elastic energy of balloon



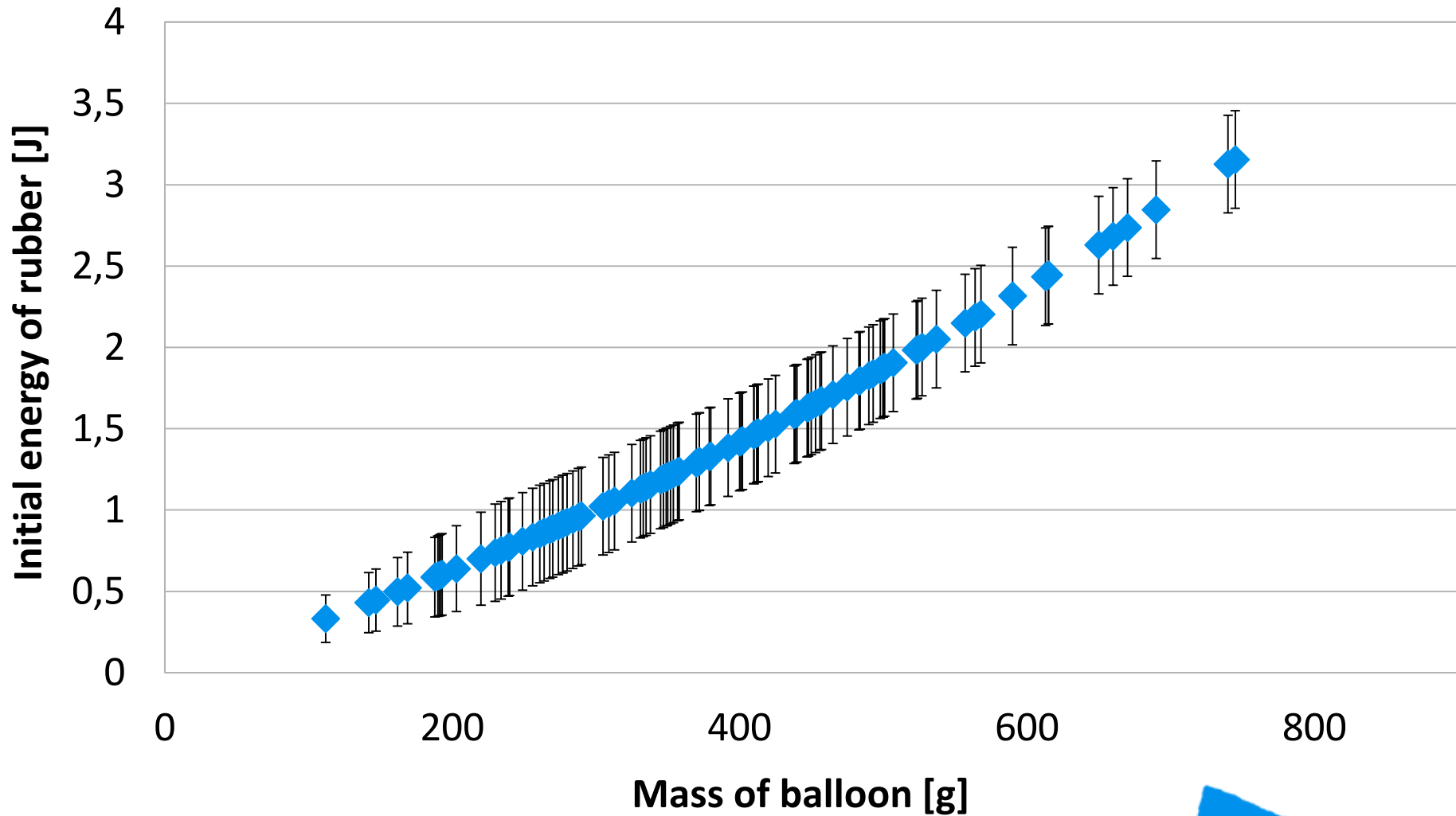


# More measurements of elastic energy



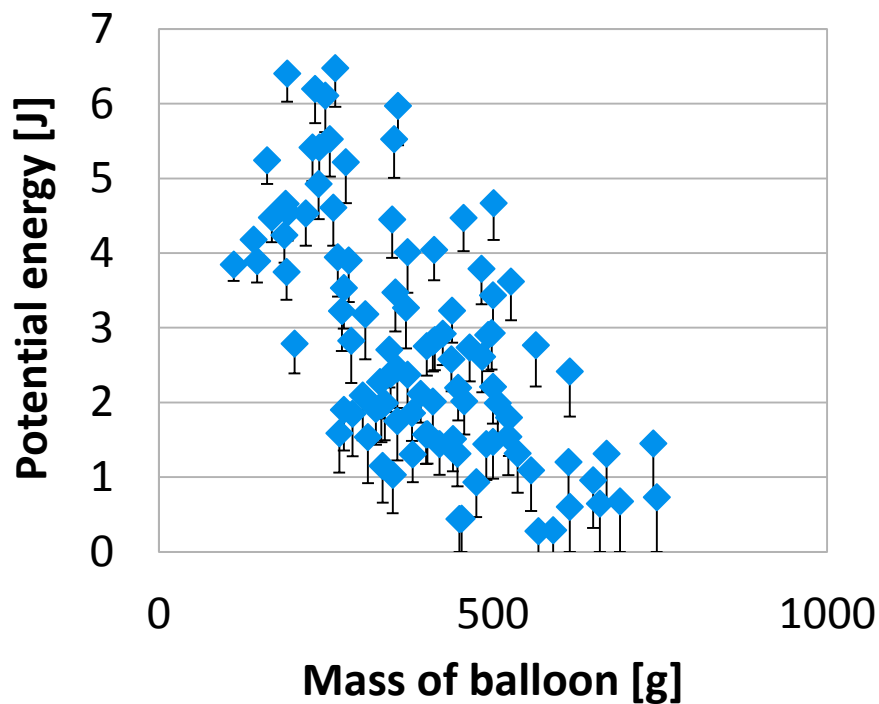


# Elastic energy of balloon with water

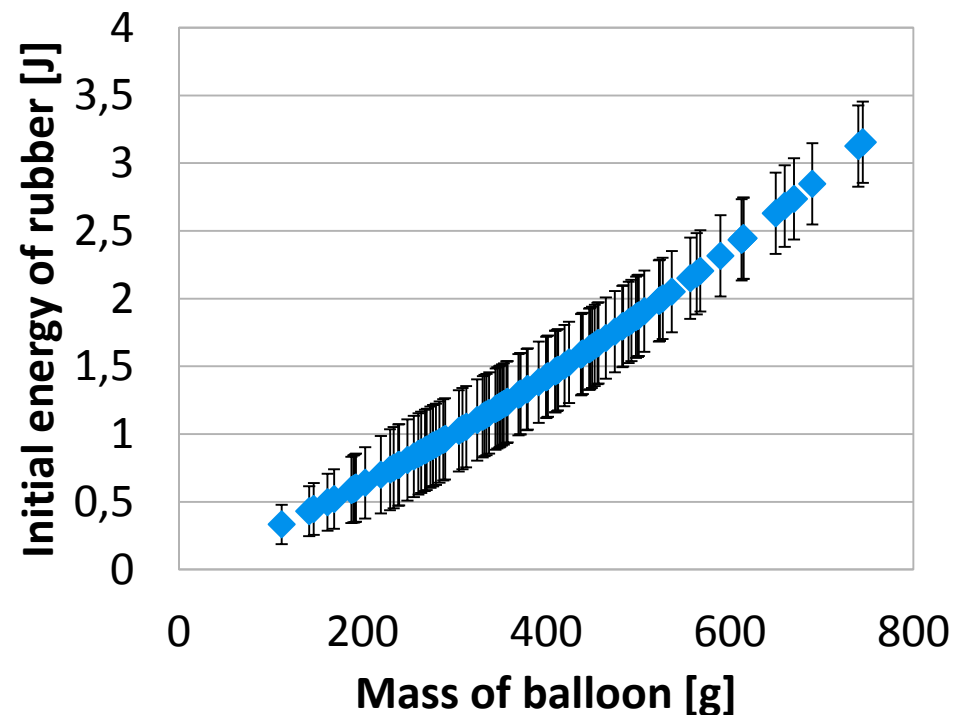


# Total energy given to balloon

## Potential energy of fall



## Elastic energy

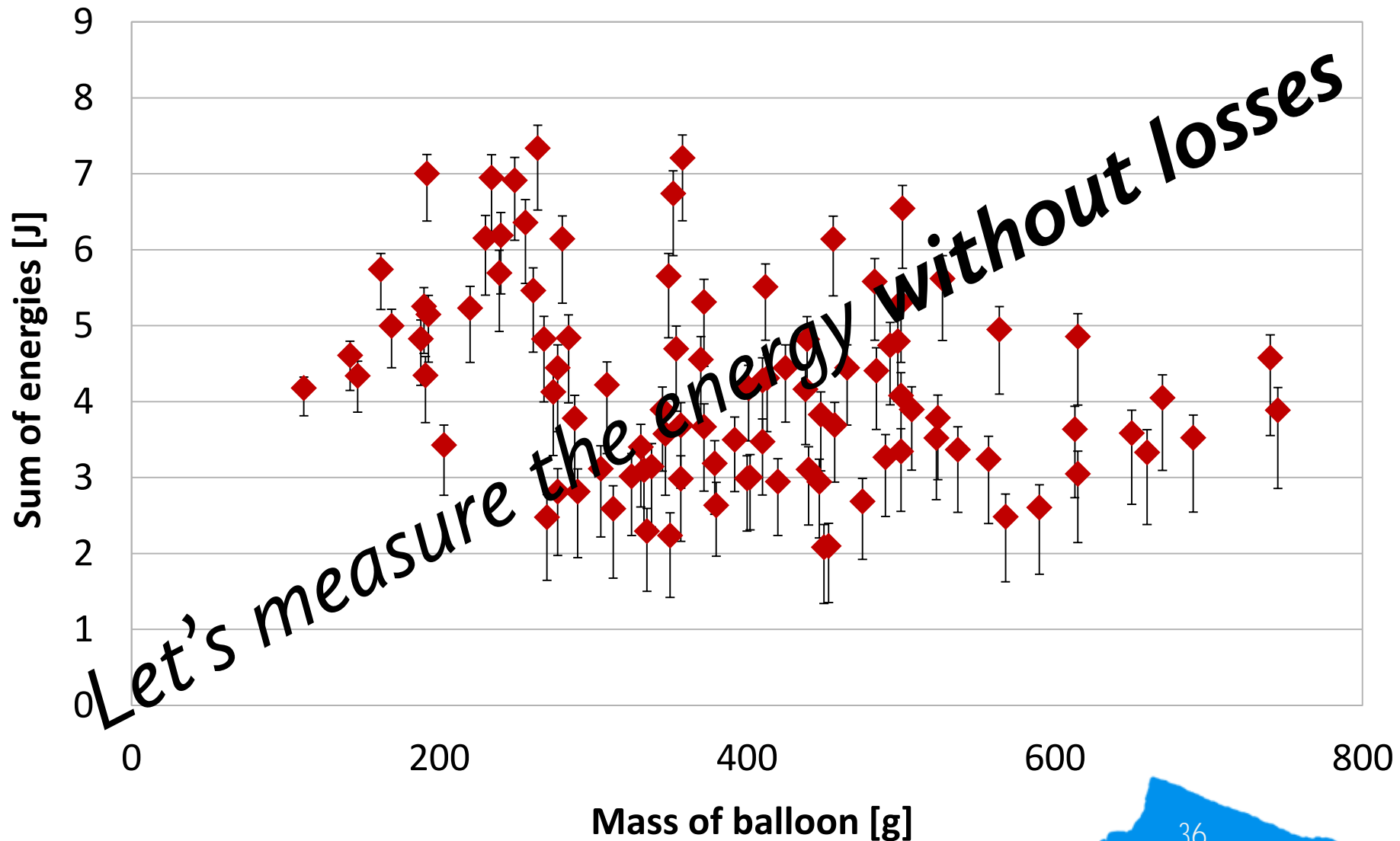


$$E_{total} = E_p + E_e$$

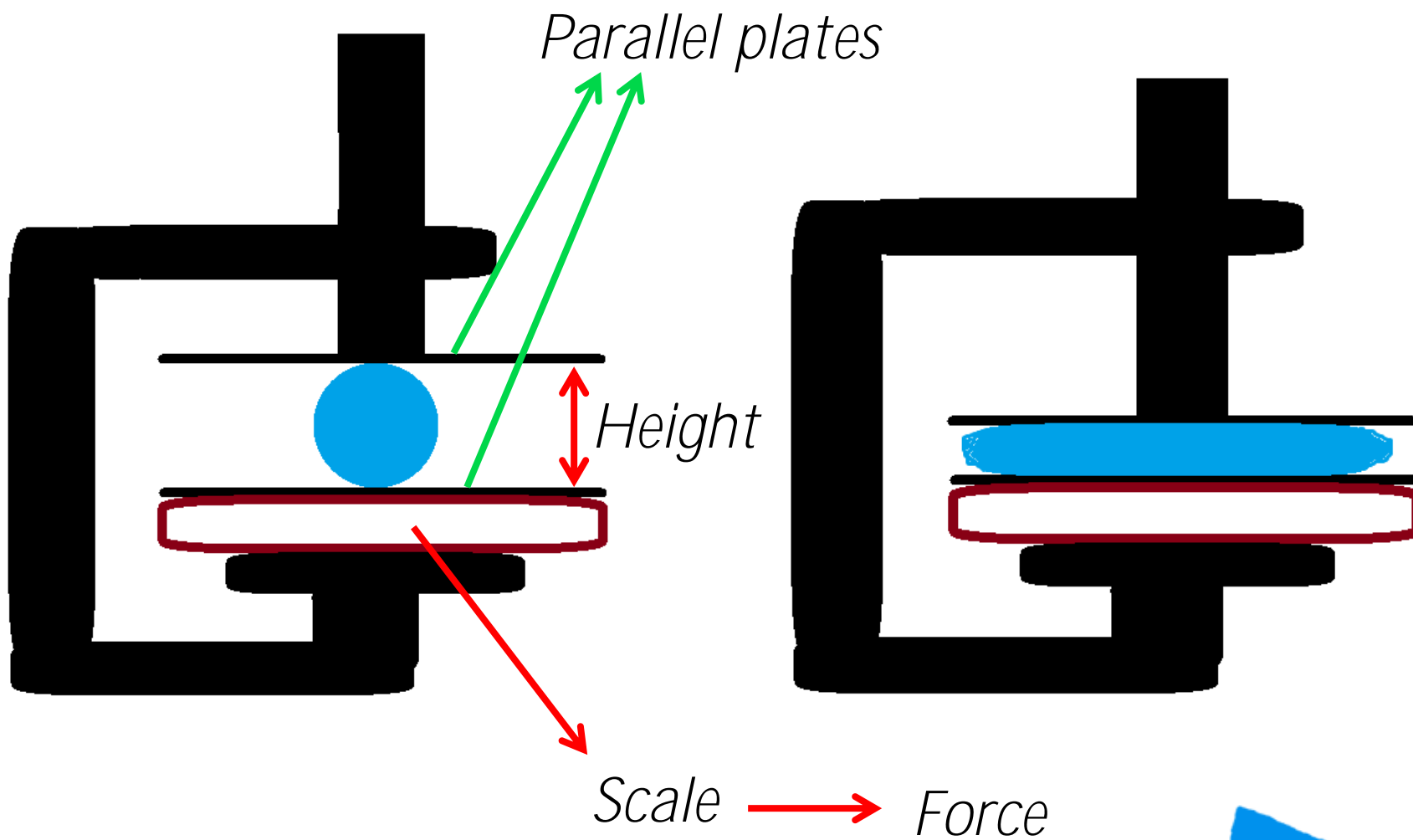


# Total energy needed to burst

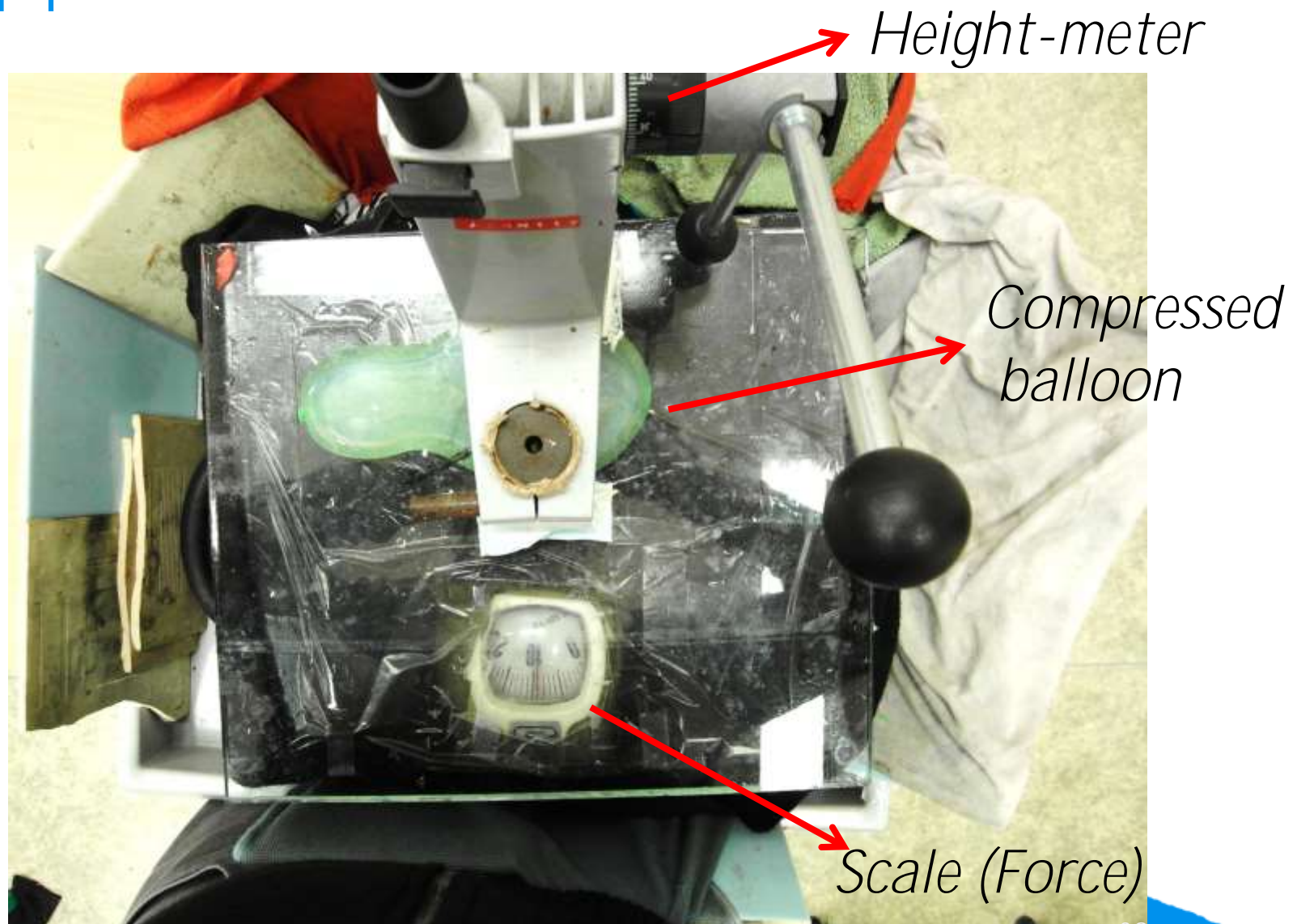
(Potential + Elastic)



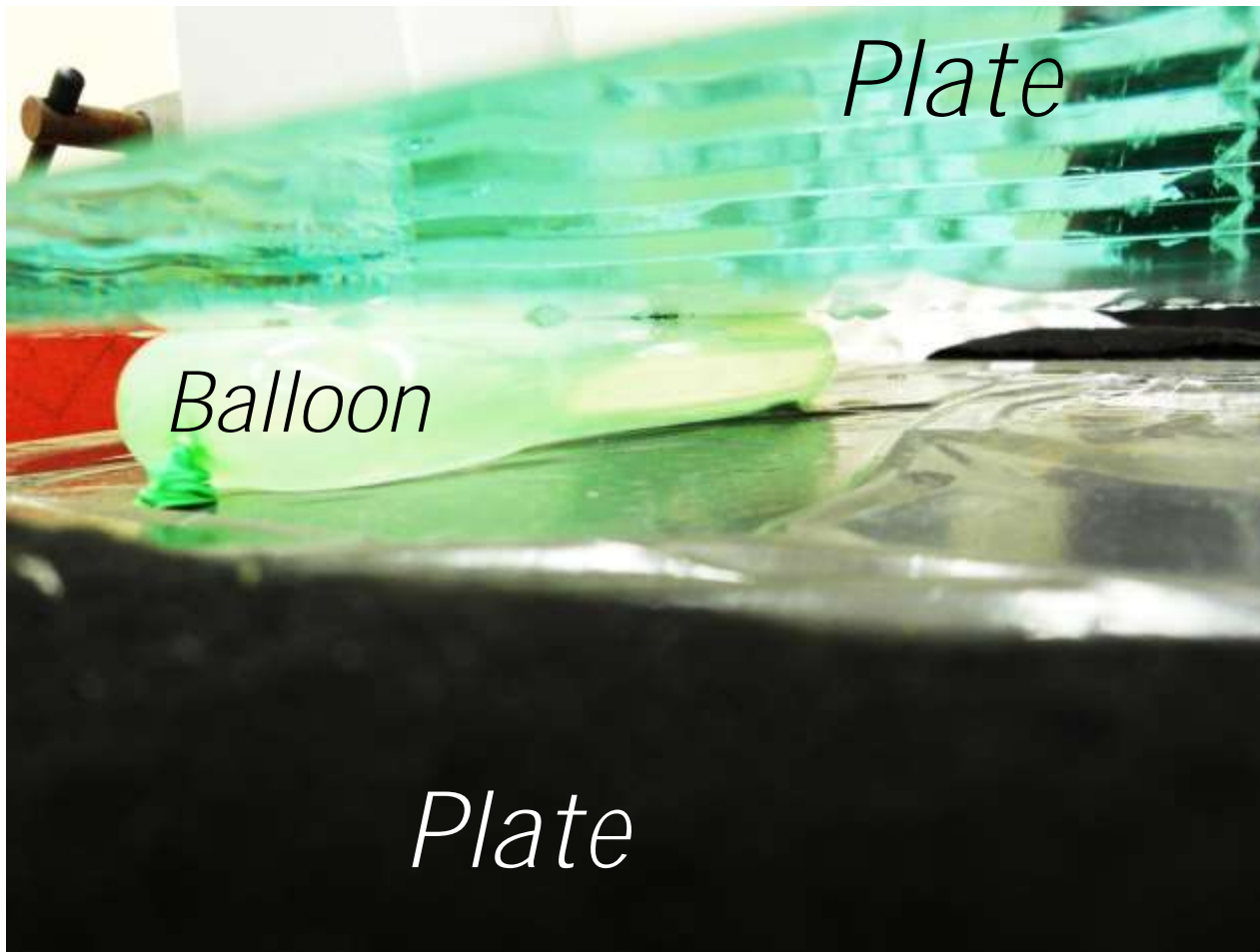
# Compressing



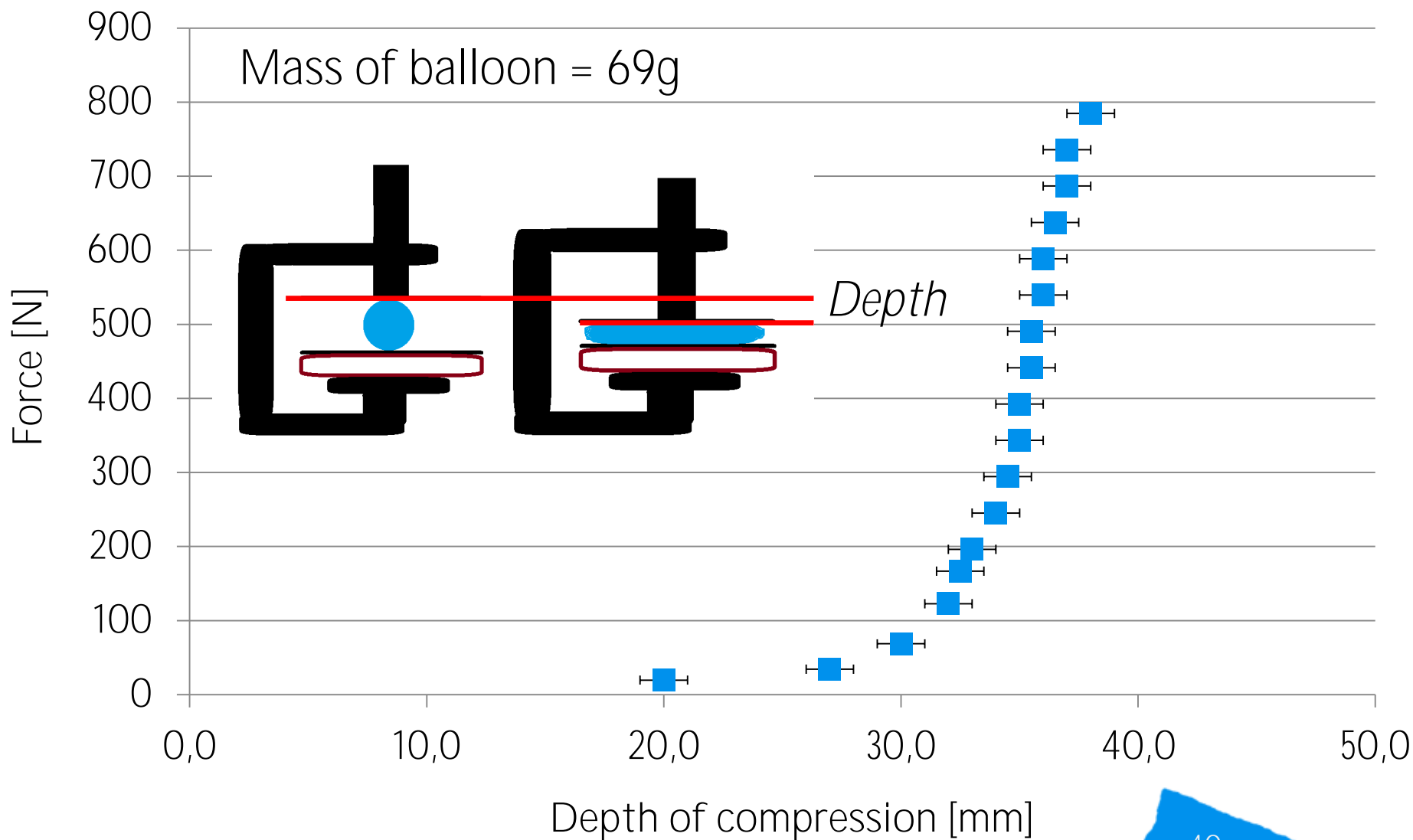
# Apparatus



# Compressed balloon



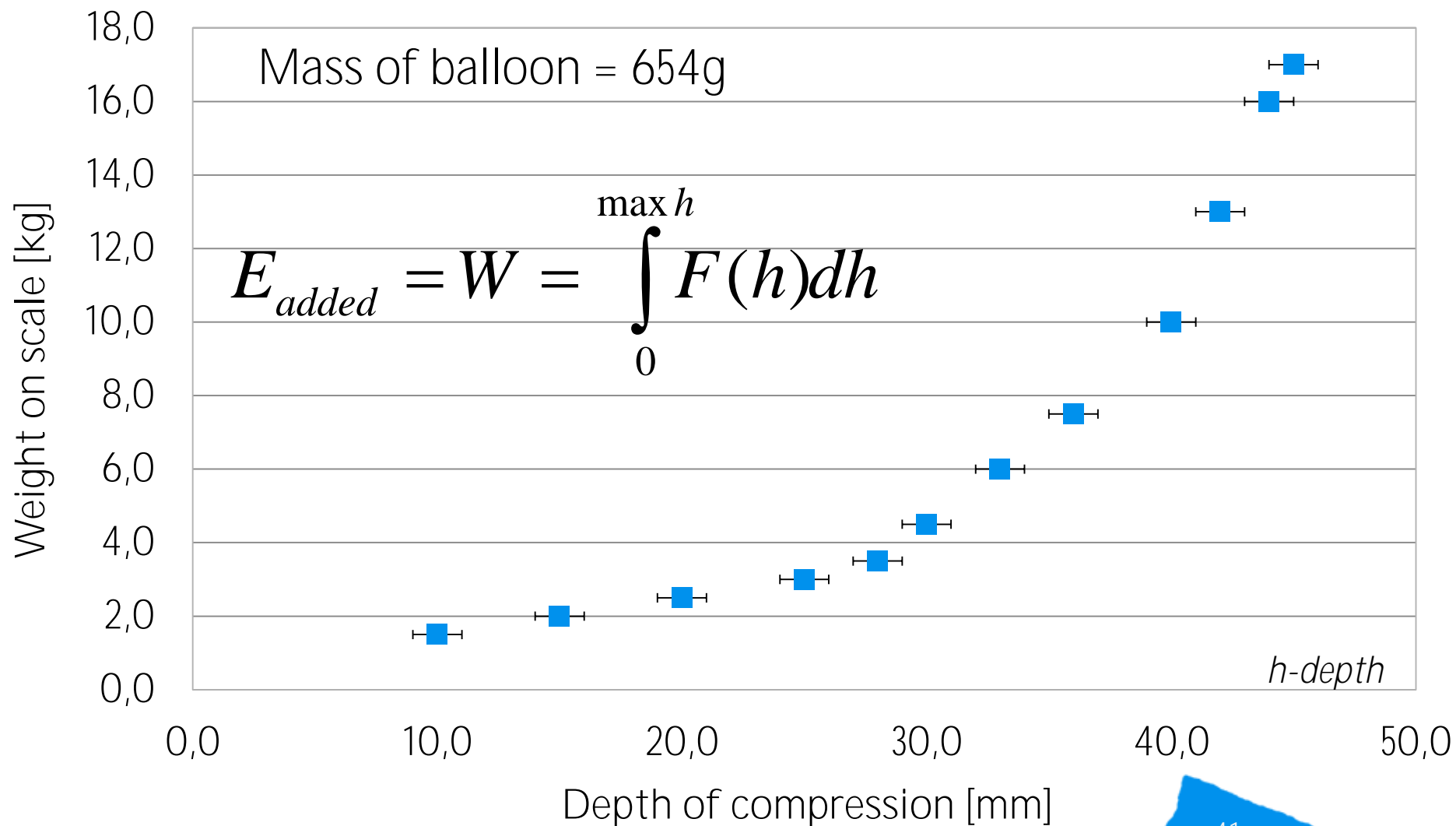
# Force vs. depth of compression







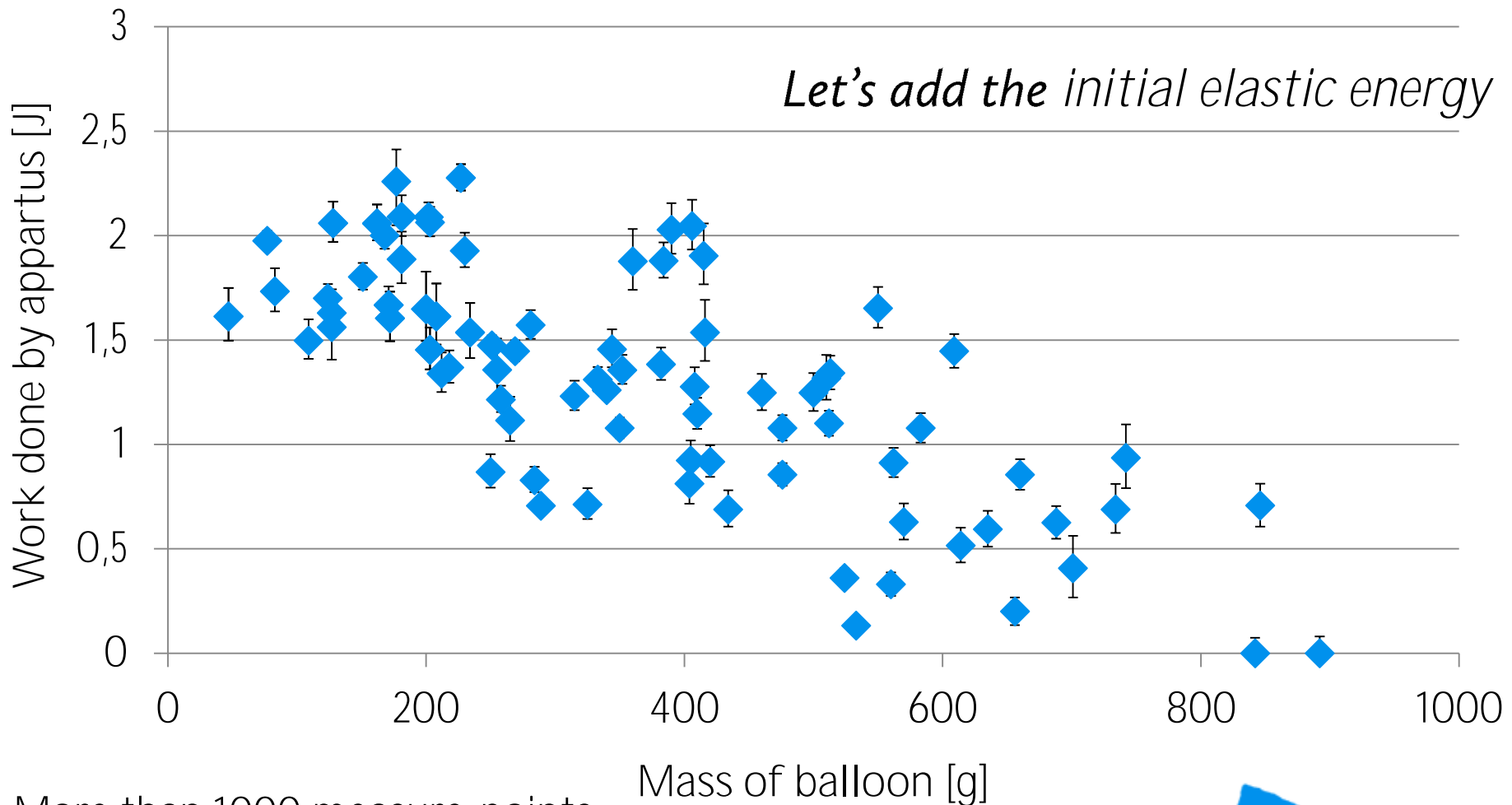
# Force vs. depth of compression





Work done by apparatus  
for many balloon volumes

$$W = \int_0^{\max h} F(h) dh$$

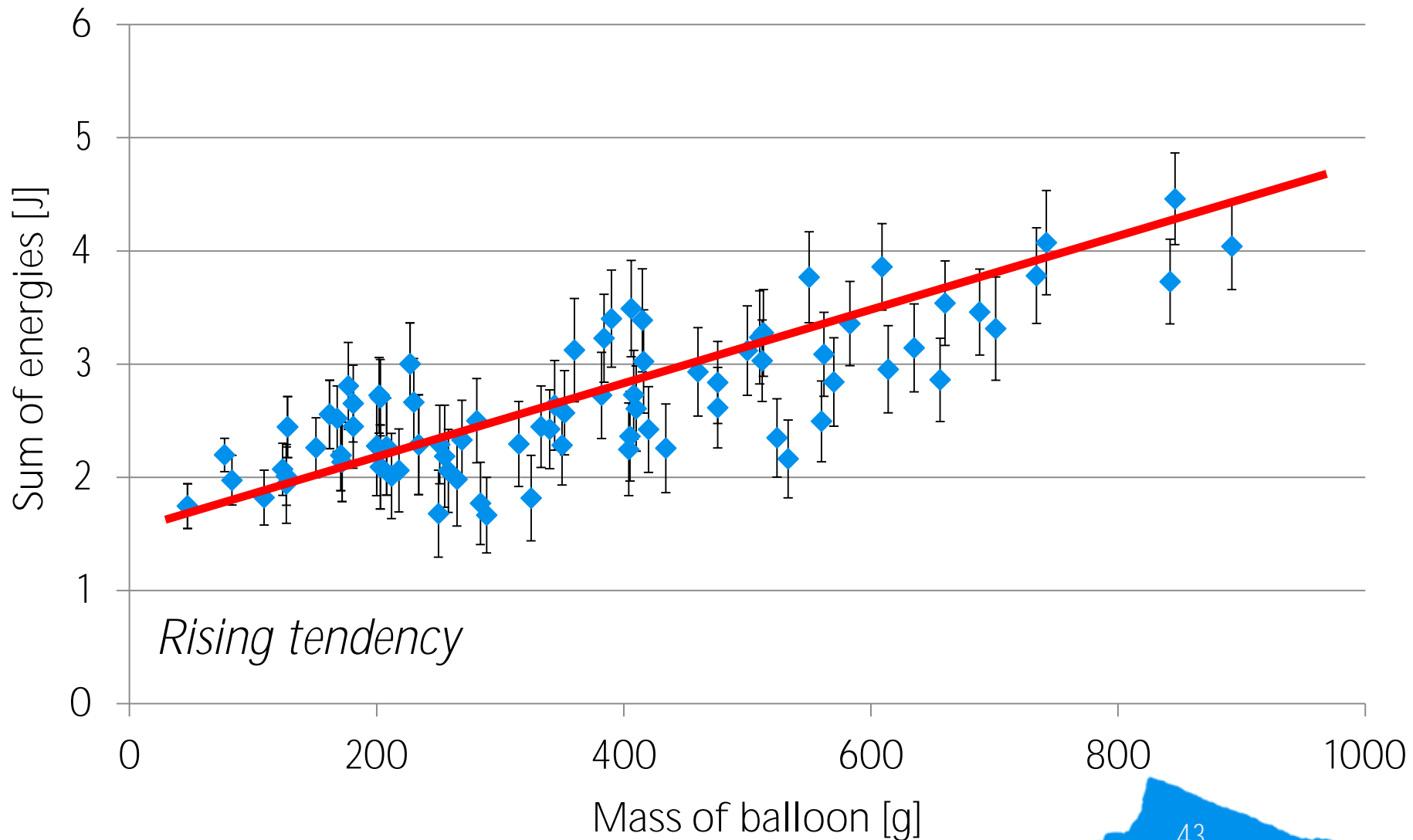


More than 1000 measure-points



# Total energy needed for burst

(Work + Elastic)





# Big balloon

*Almost maximal tension*

*Almost maximal tension*

*Almost maximal tension*

*Almost maximal tension*

*Almost maximal tension*

*Almost maximal tension*

*Almost maximal tension*

*Almost maximal tension*

*Almost maximal tension*

*Almost maximal tension*

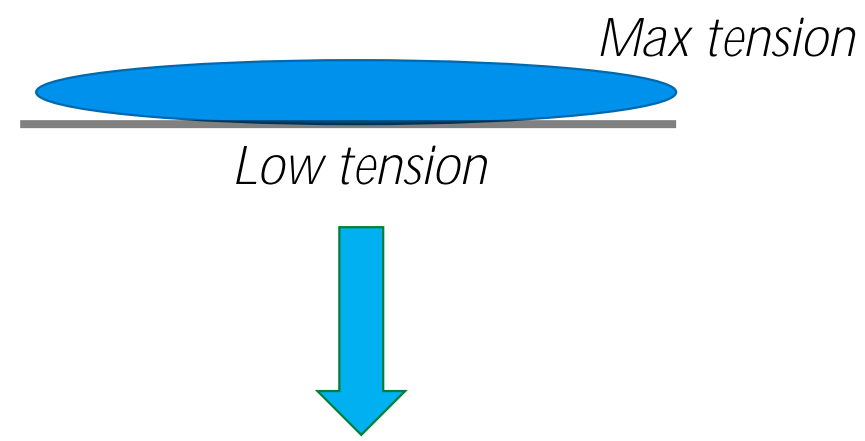
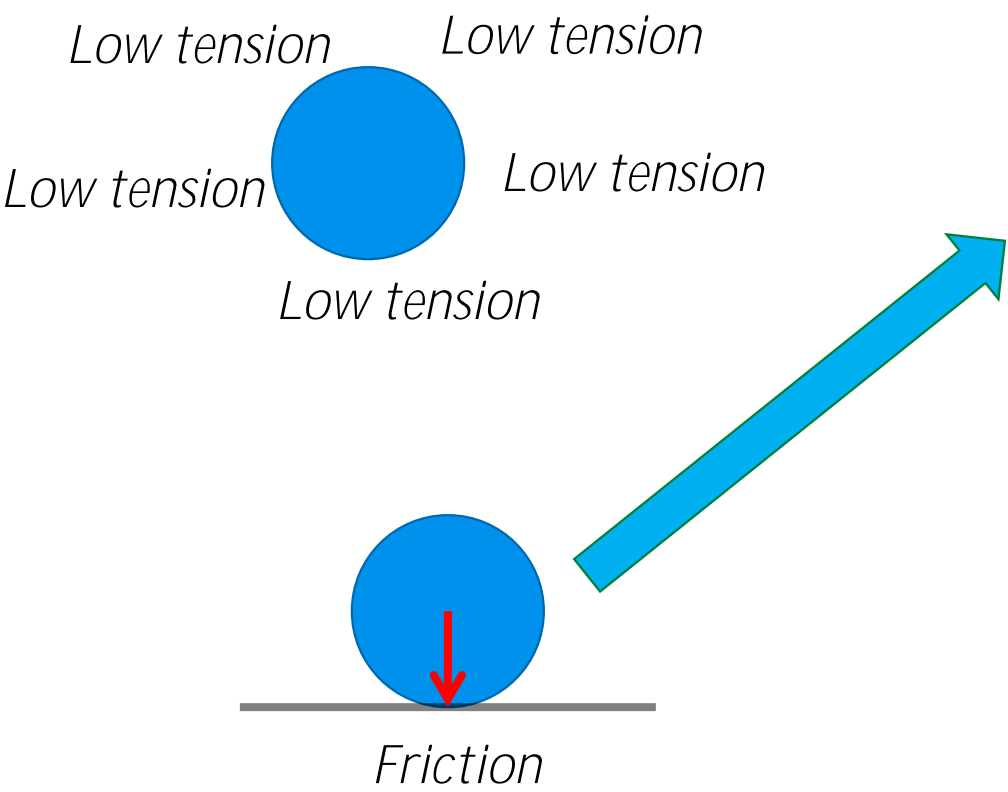
*Almost maximal tension*

*Almost maximal tension*

*Elastic energy is almost maximal*



# Small balloon

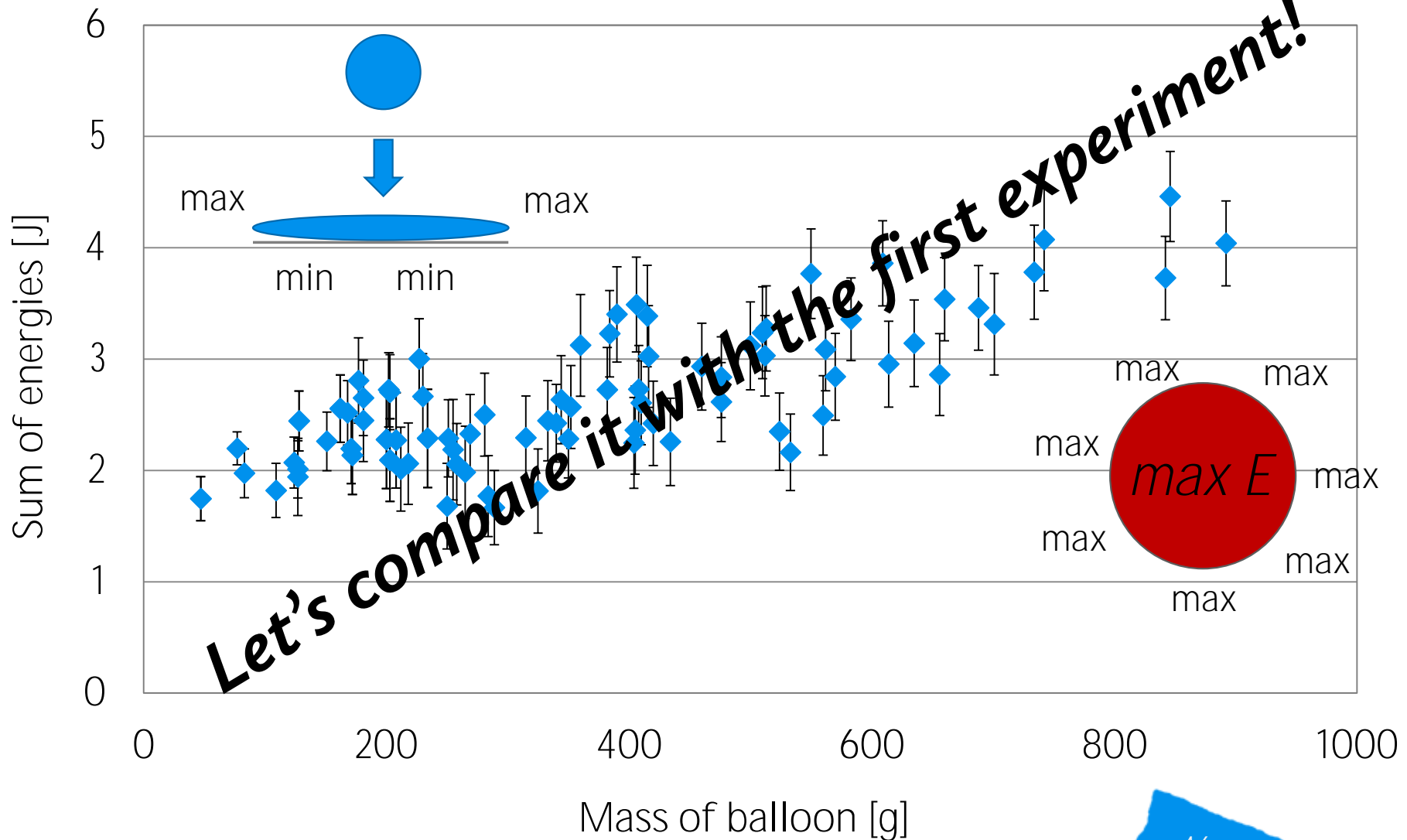


*Elastic energy is less than maximal*

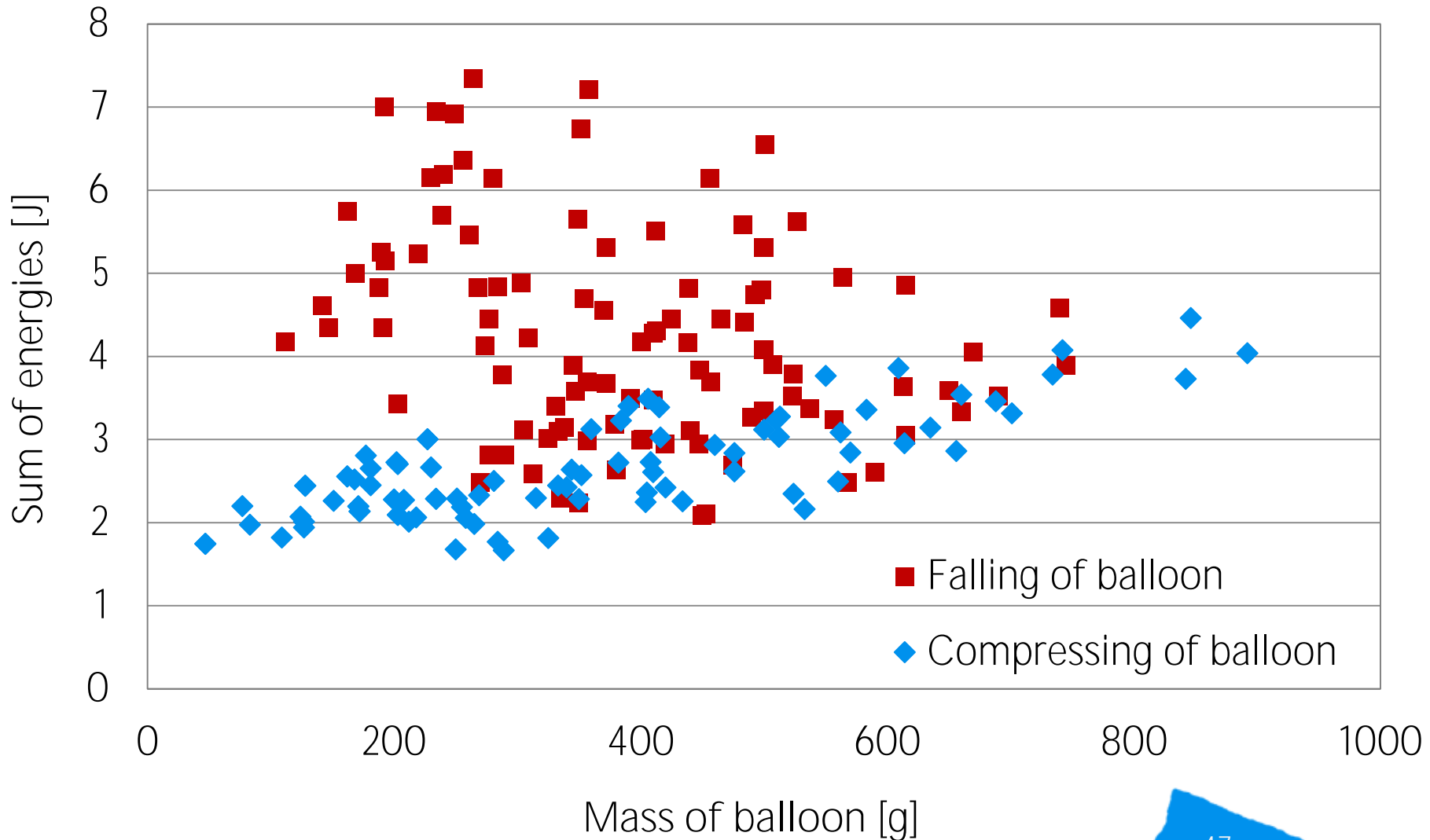
*Tension at the bottom remains the same*

# Total energy needed for burst

(Work + Elastic)



# Energy needed to burst through **fall** & through **compression**





# Falling balloon

In graph

$$E = E_p + E_e$$

$$E = mgh + E_e$$

In real life

There are losses  
Water convection  
Heat

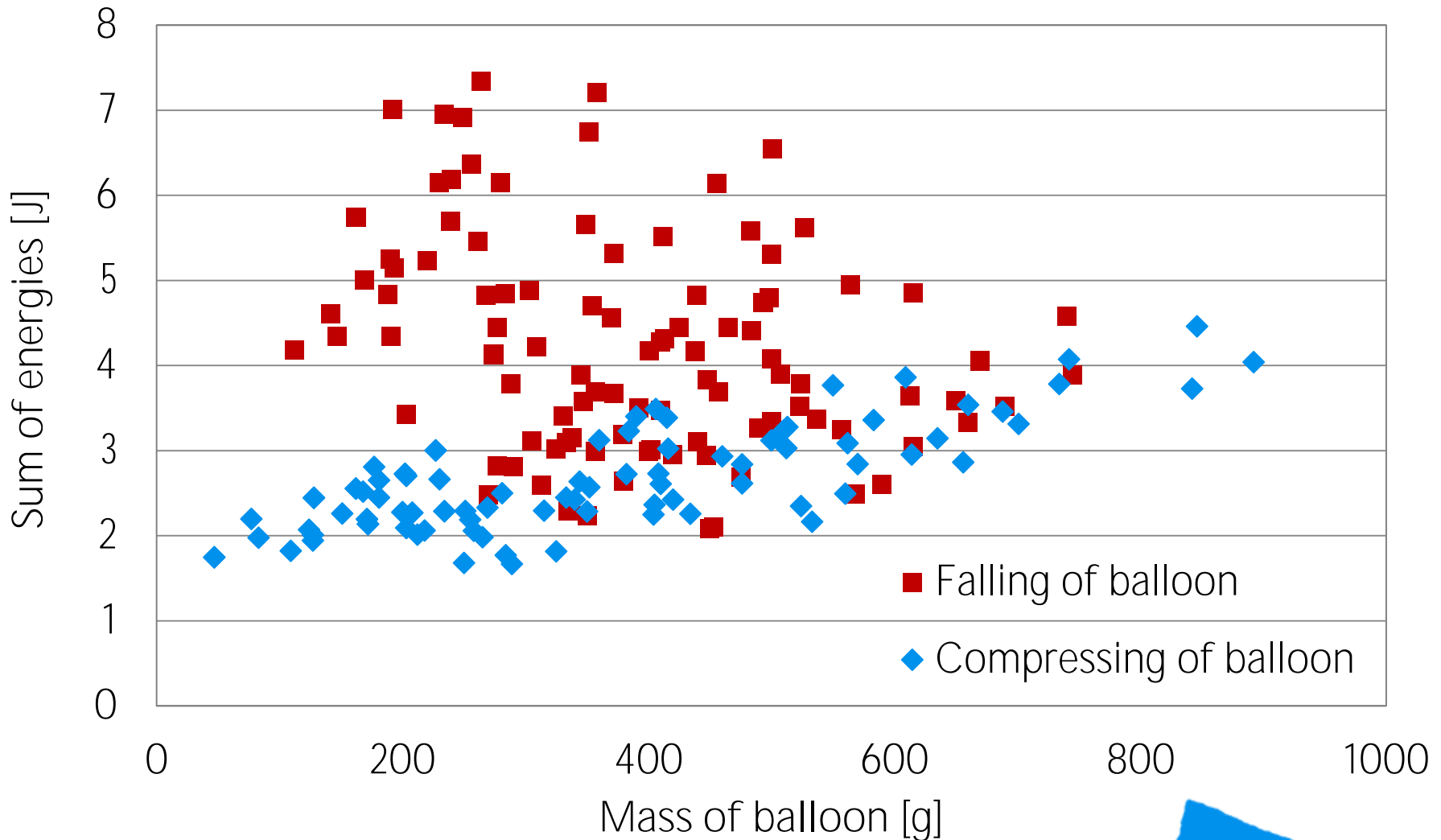
Let's **assume** that  
constant ratio of energy  
is preserved

$$E = k \cdot mgh + E_e$$



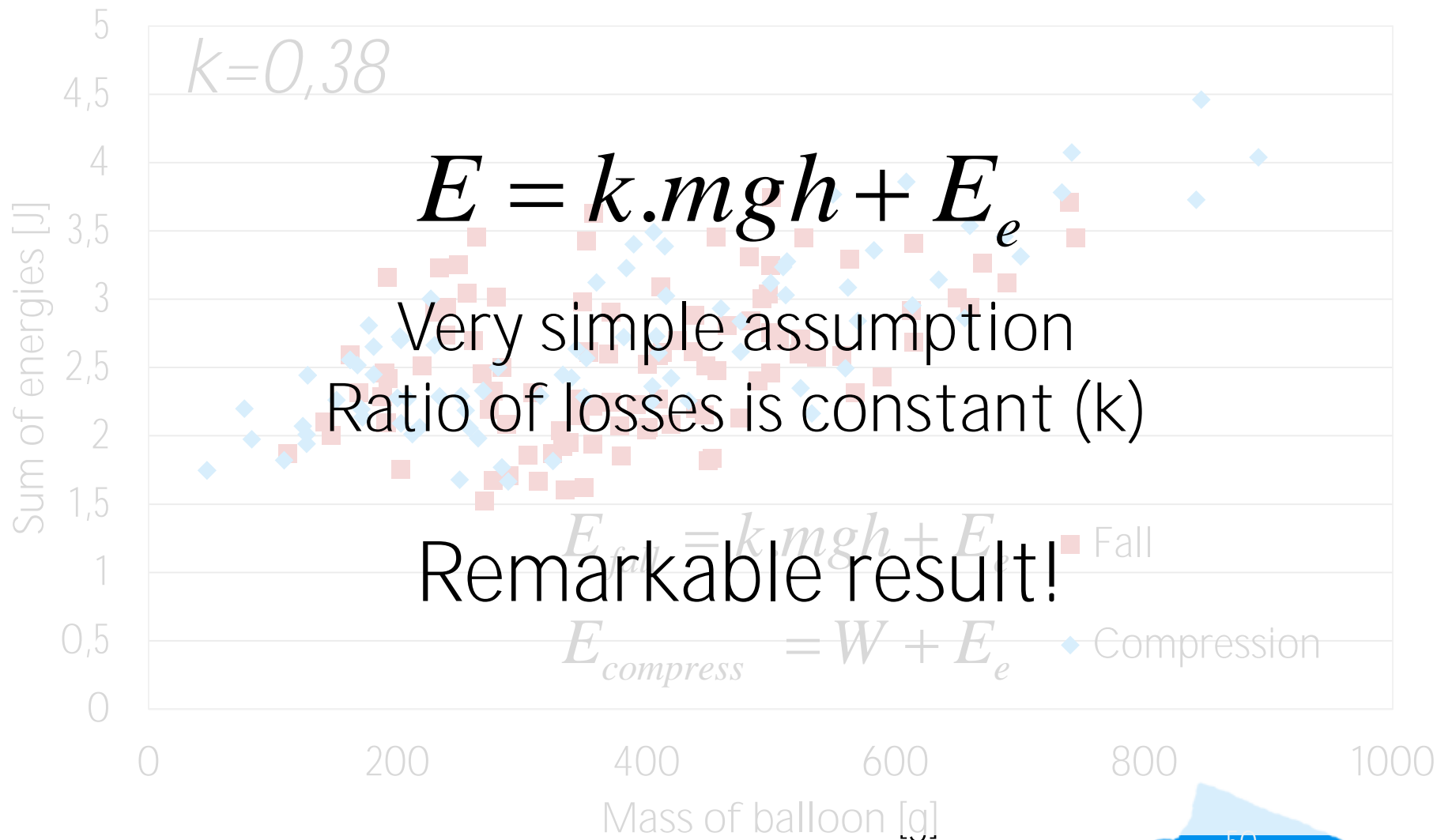


# Energy needed to burst through **fall** & through **compression**

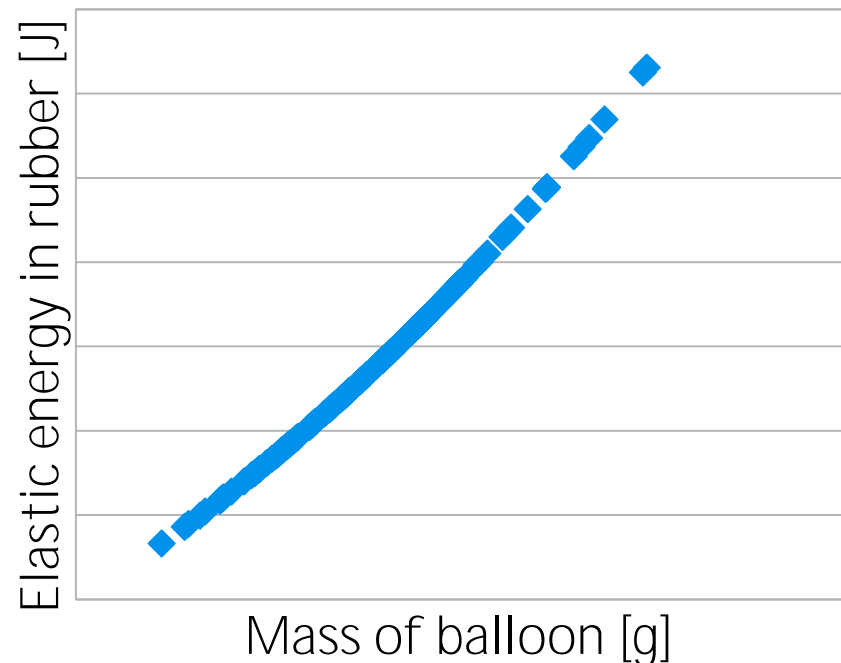
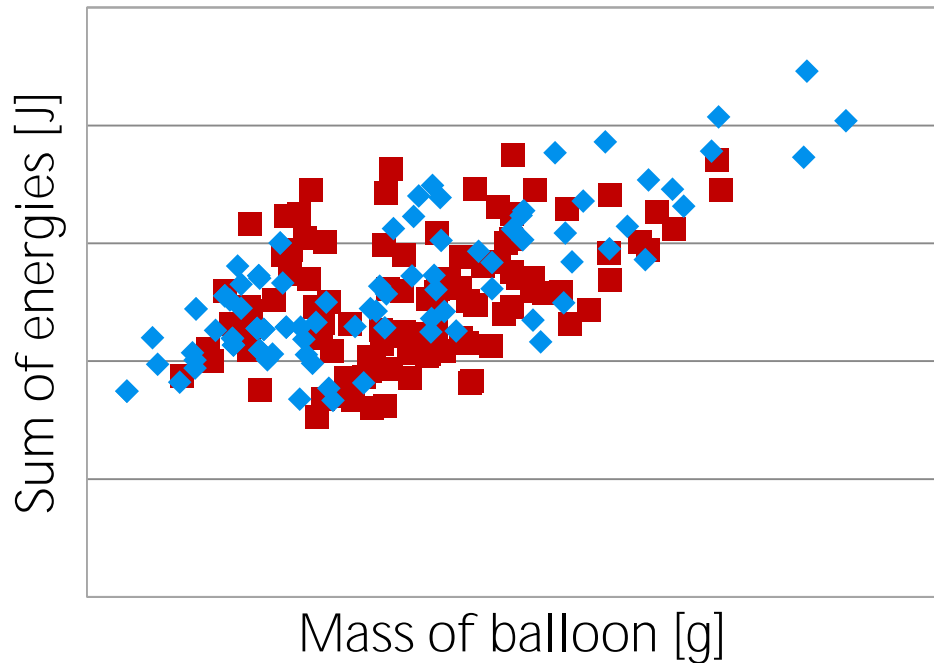




# Energy needed to burst through **fall** & through **compression**



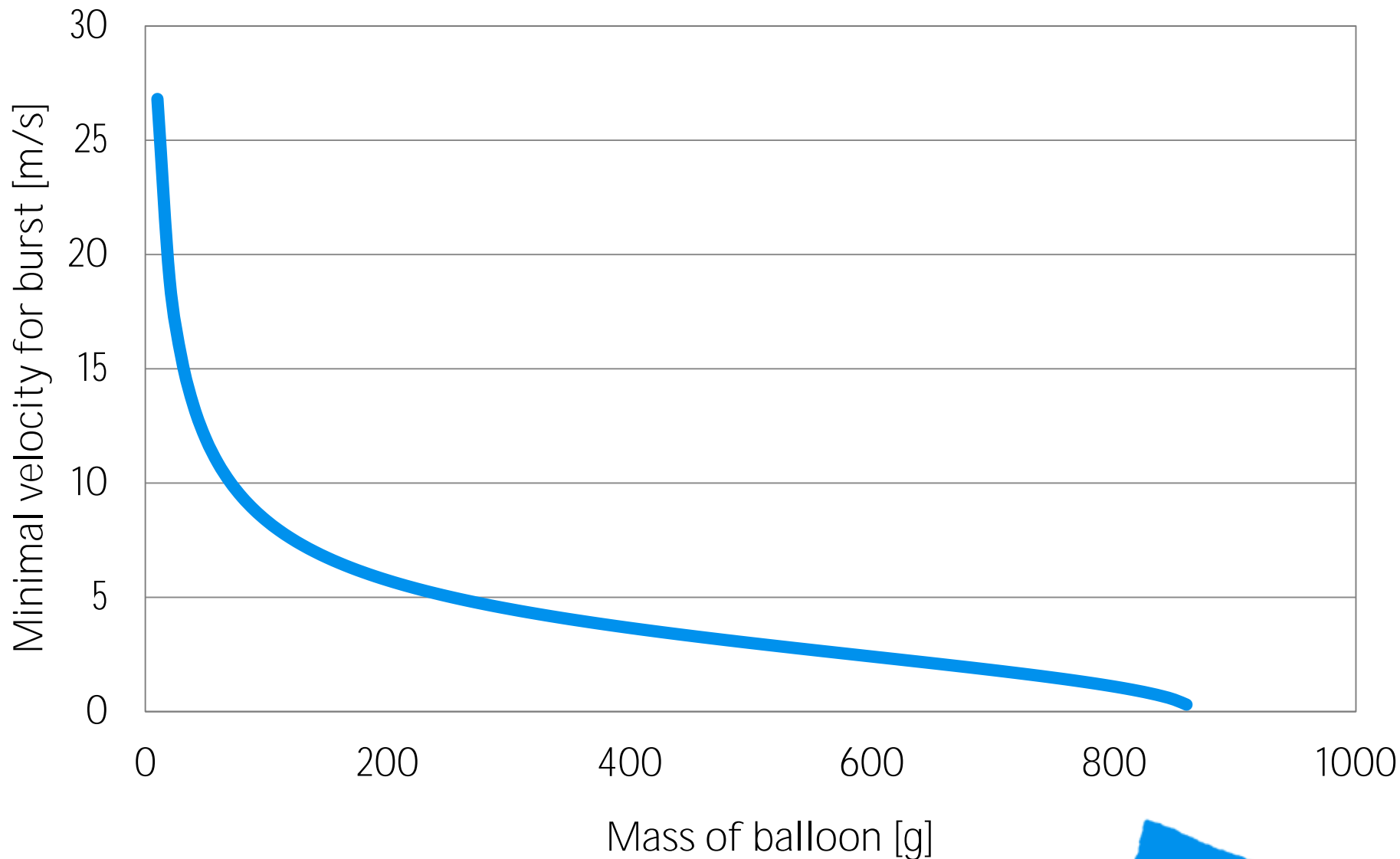
# What we have



*We can derive approximately the velocity of throw needed to burst a balloon*

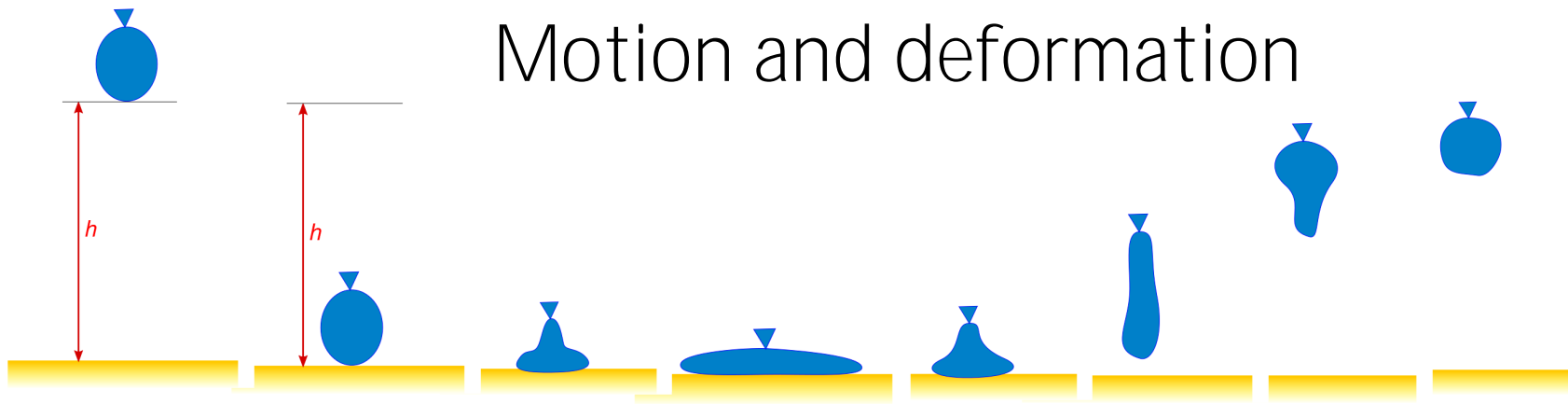


# Approximate velocity needed for burst

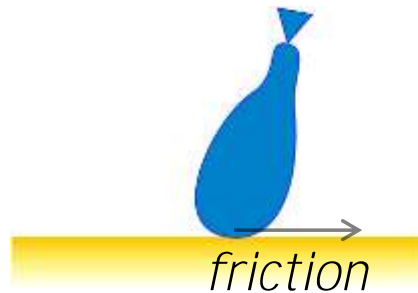


# Summary

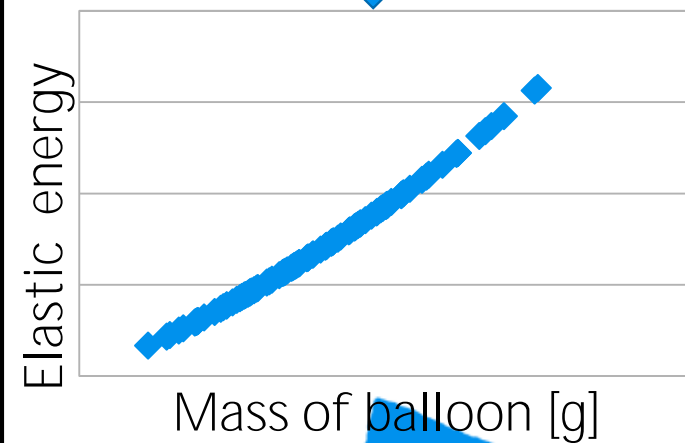
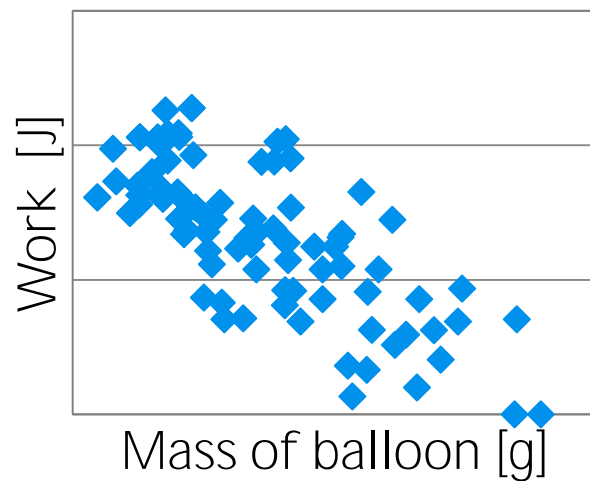
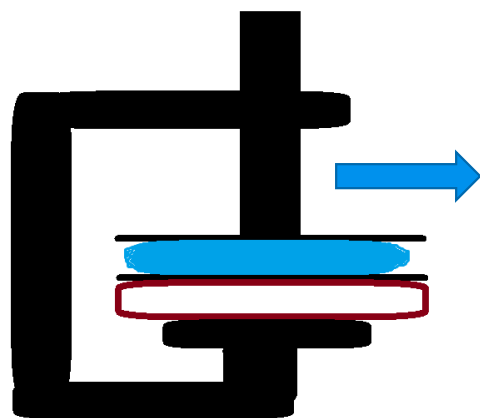
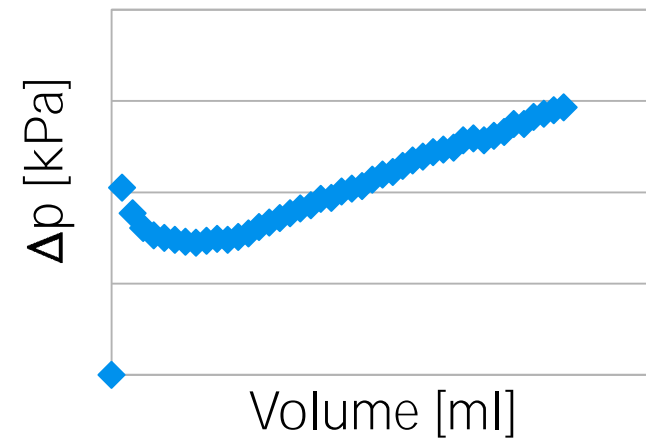
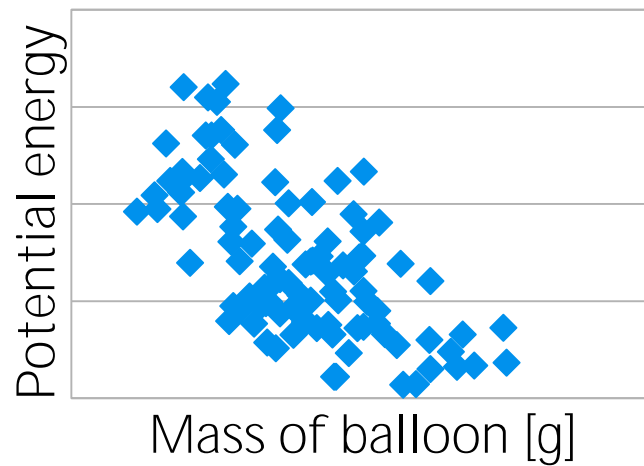
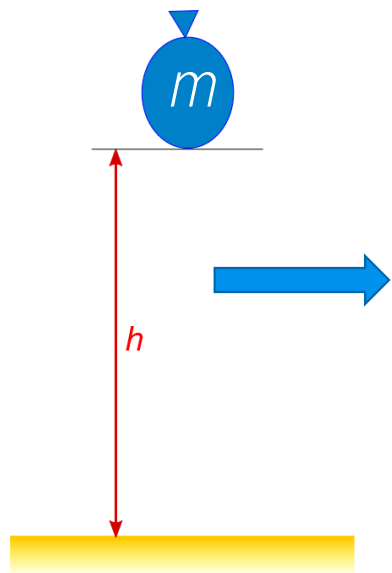
## Motion and deformation



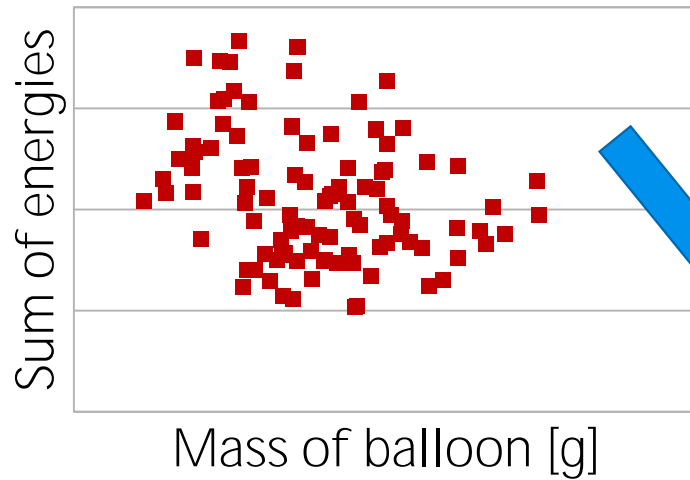
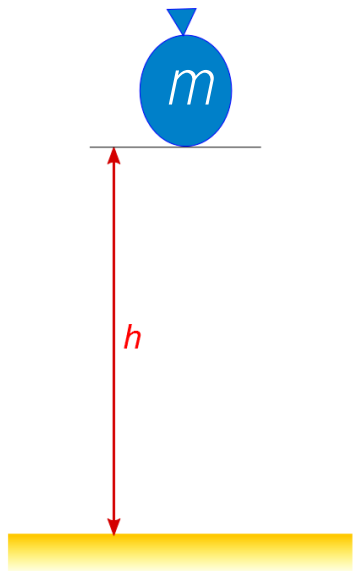
## Rebound



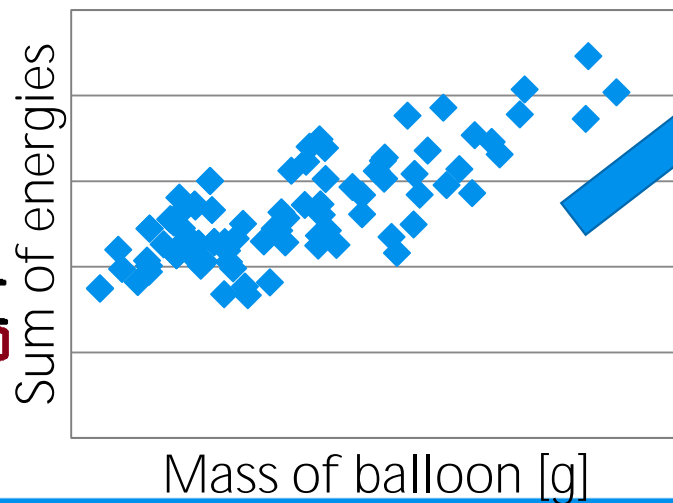
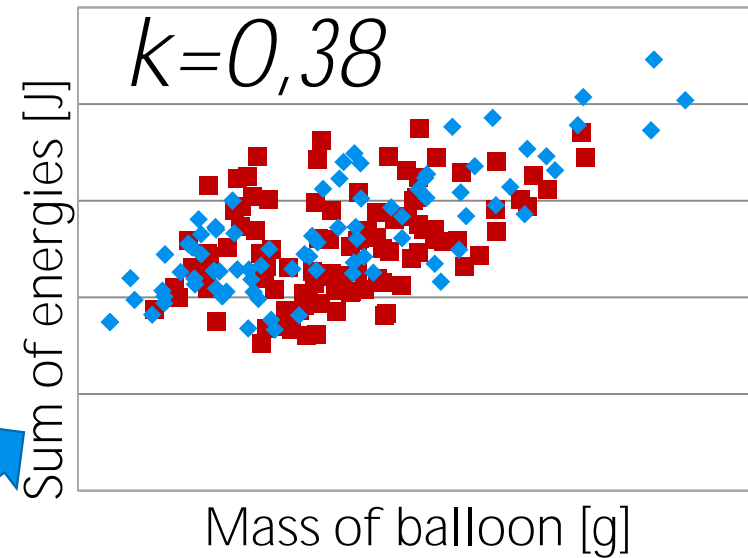
# Summary - Burst



# Summary - Burst

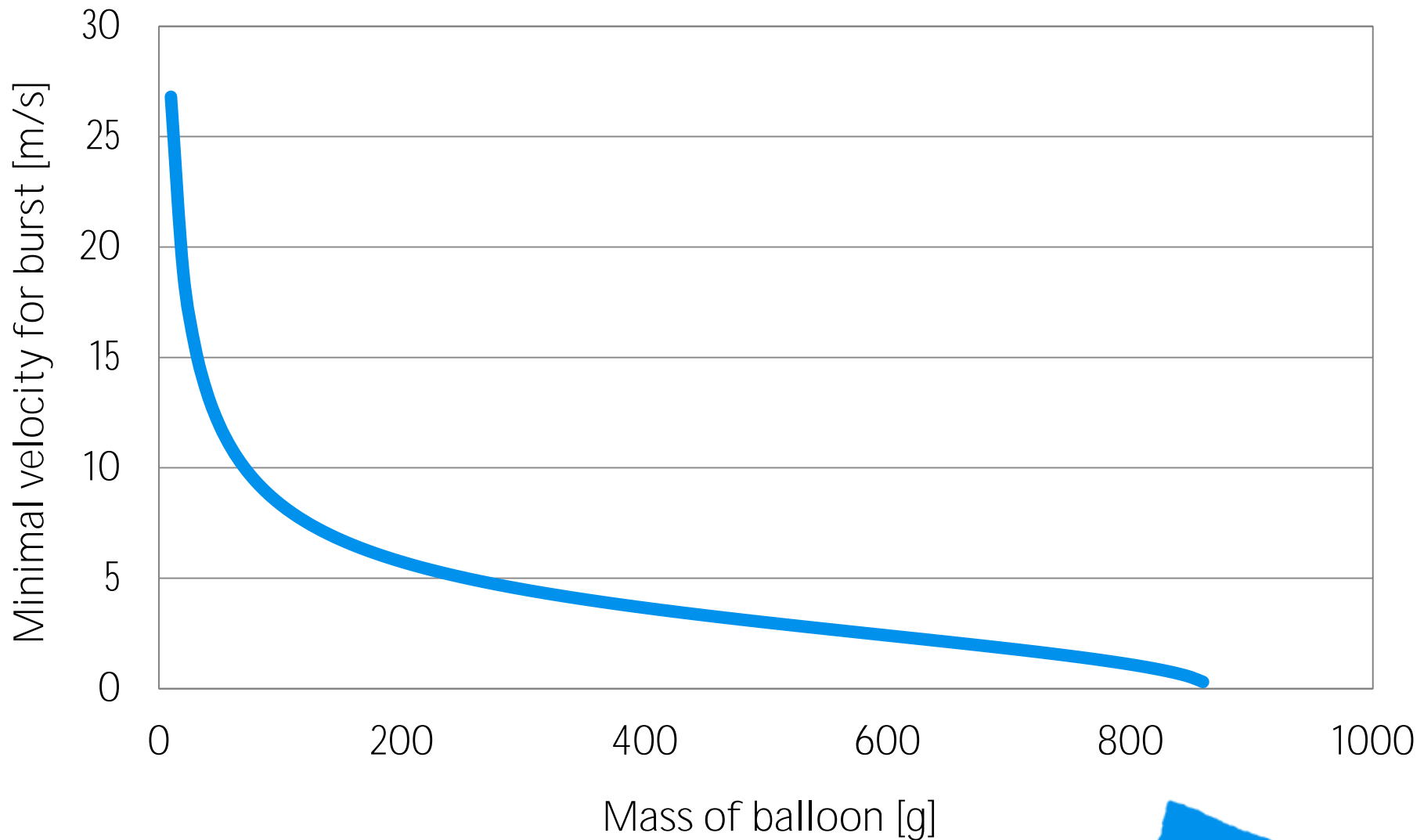


$$E_{total} = kE_p + E_e$$





# Thank you for your attention!

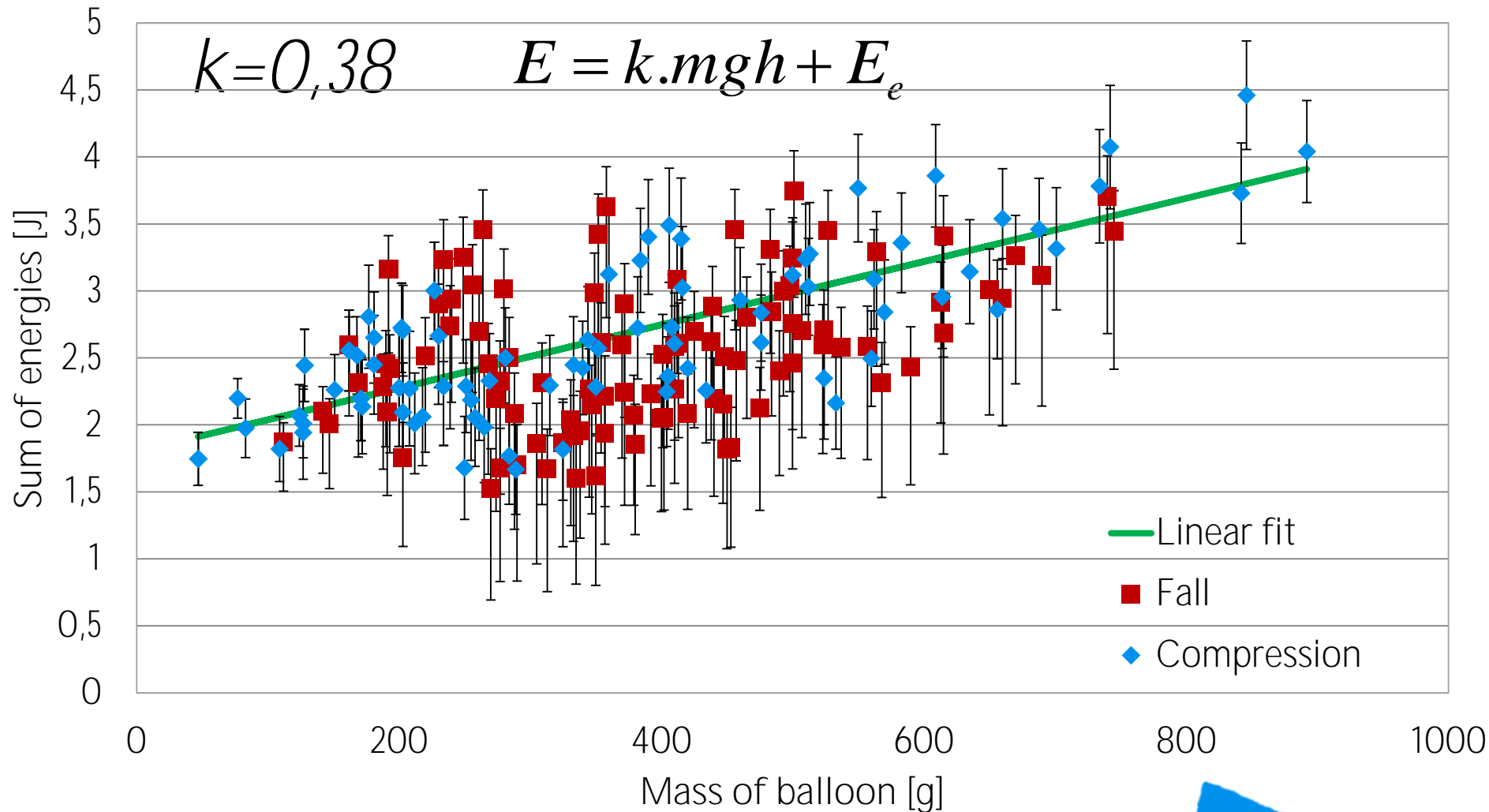






# APPENDICES

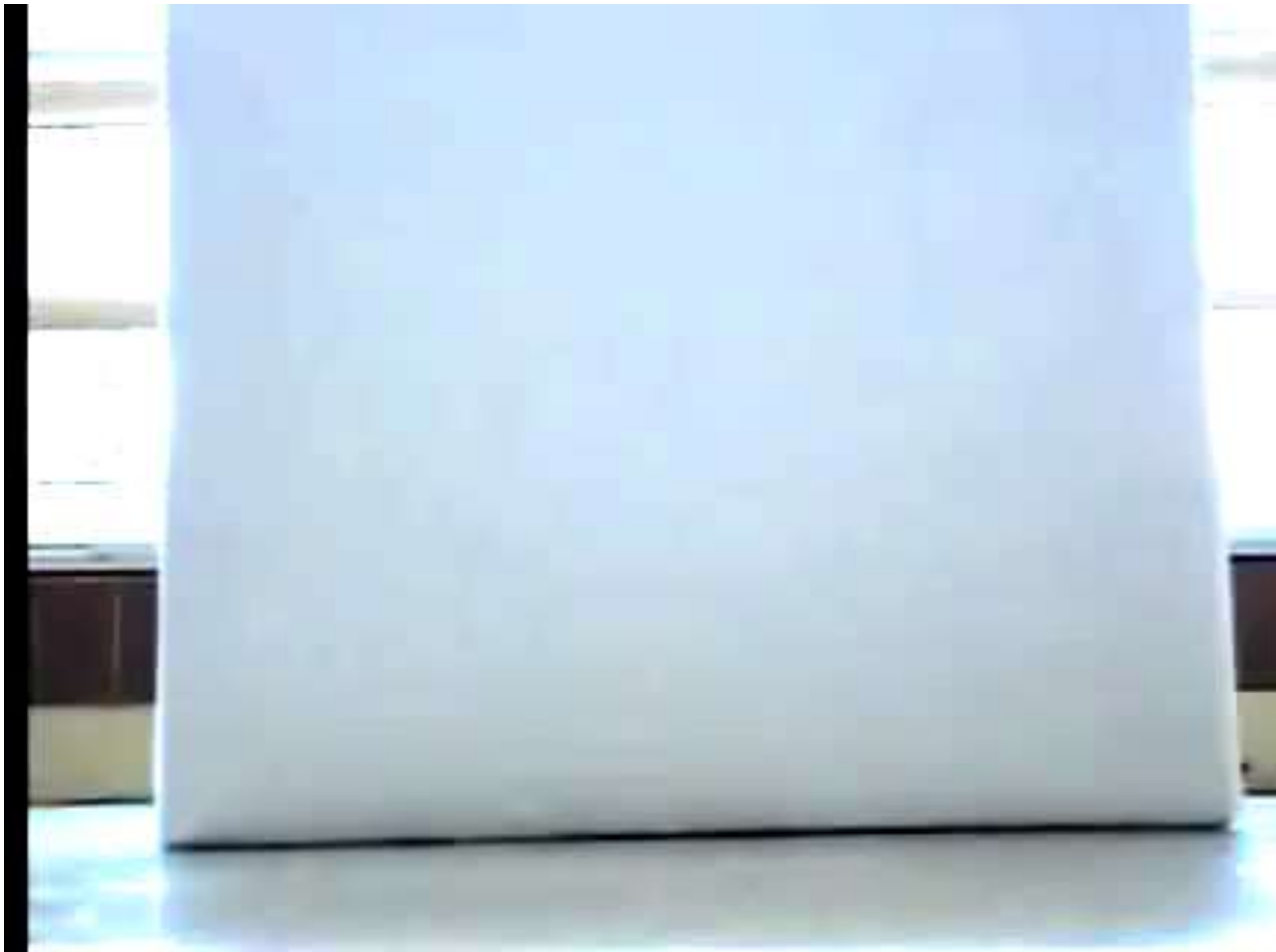
# Energy needed to burst through **fall** & through **compression**



# Rubber stretch at rebound

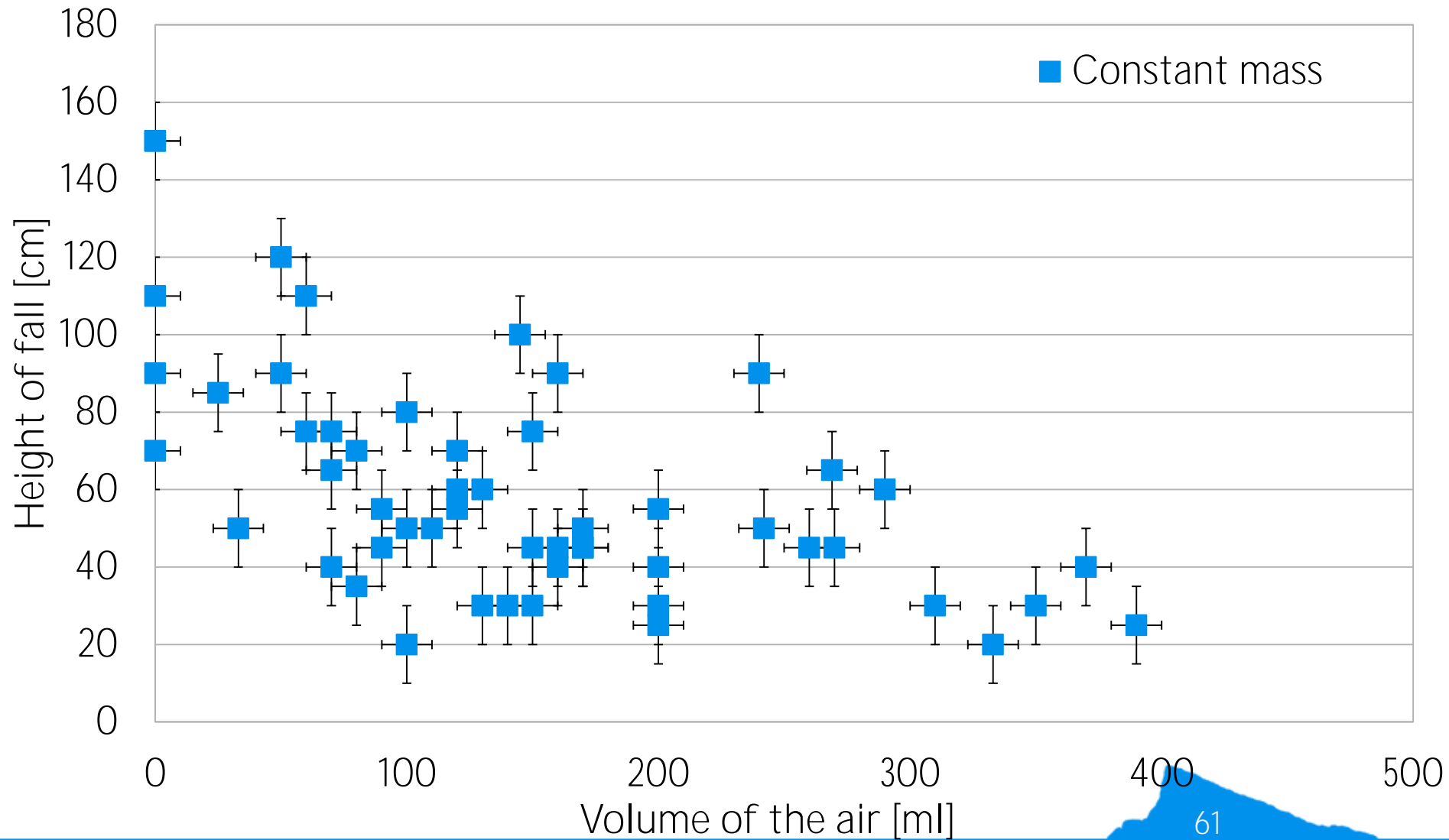


# Rubber stretch at rebound

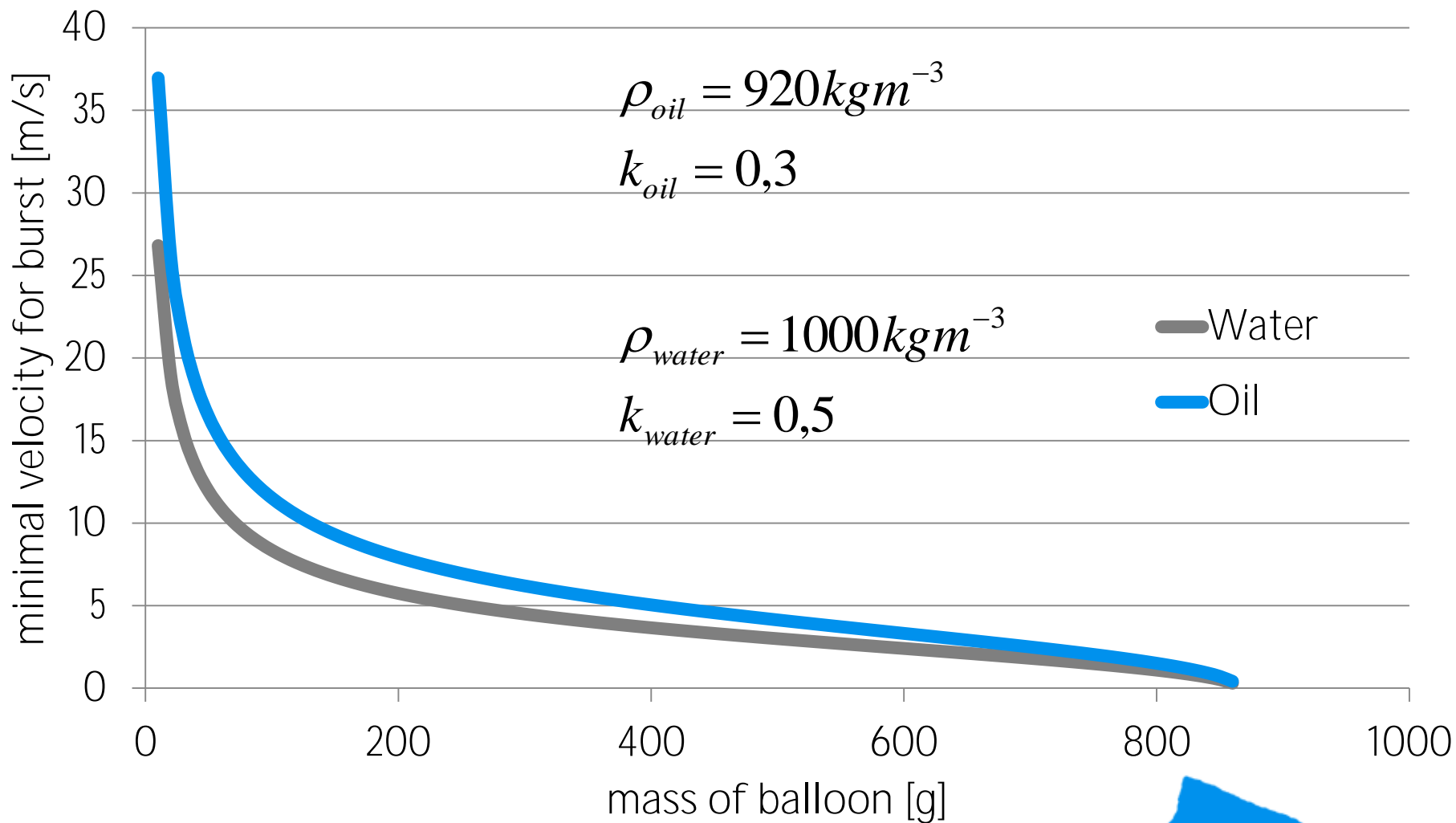




# Adding air



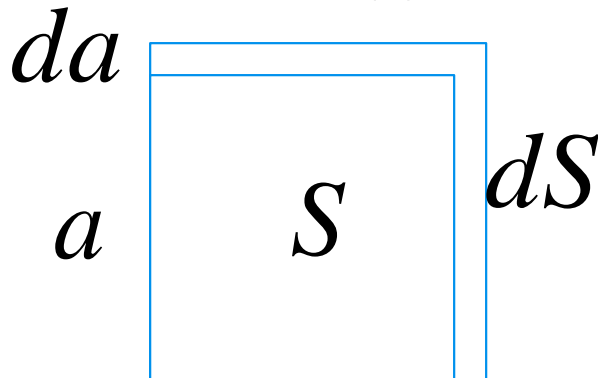
# Other liquid



# Elastic energy of the balloon

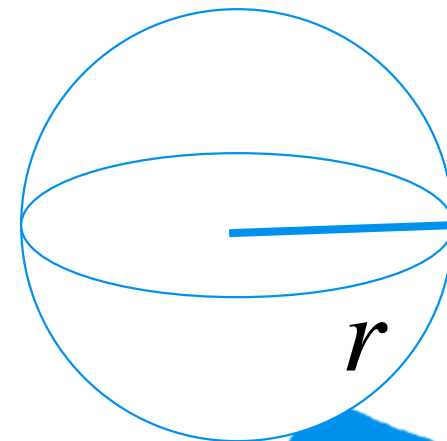
- Stretching a piece of rubber, surface tension  $\sigma$

$$dW = 2a\sigma da = \sigma dS$$



- In terms of pressure inside:

$$p = \frac{2\pi r \sigma}{\pi r^2} \quad \sigma = \frac{rp}{2}$$





- Spherical shape:  $S = 4\pi r^2$      $dS = 8\pi r dr$   
 $V = \frac{4}{3}\pi r^3$      $dV = 4\pi r^2 dr$

$$dW = \frac{pr}{2} dS = p4\pi r^2 dr = pdV$$

- Elastic energy  $= \int pdV$