



**Russia IYPT**

# Coffee cup

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Physicists like drinking coffee, however walking between laboratories with a cup of coffee can be problematic.

Main task

Investigate how the shape of the cup, speed of walking and other parameters affect the likelihood of coffee being spilt while walking.



# First observations

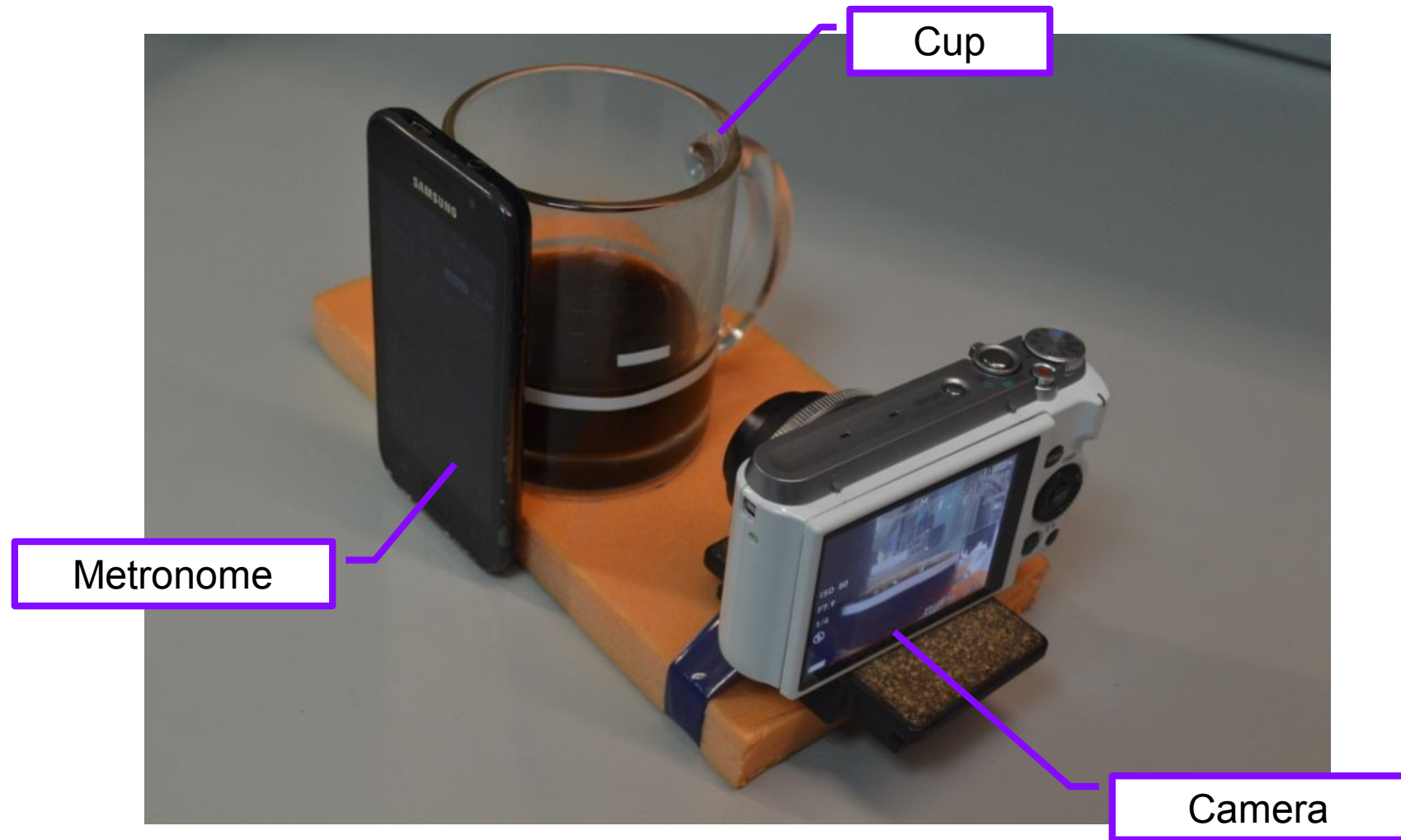
# Cylindrical cup

4



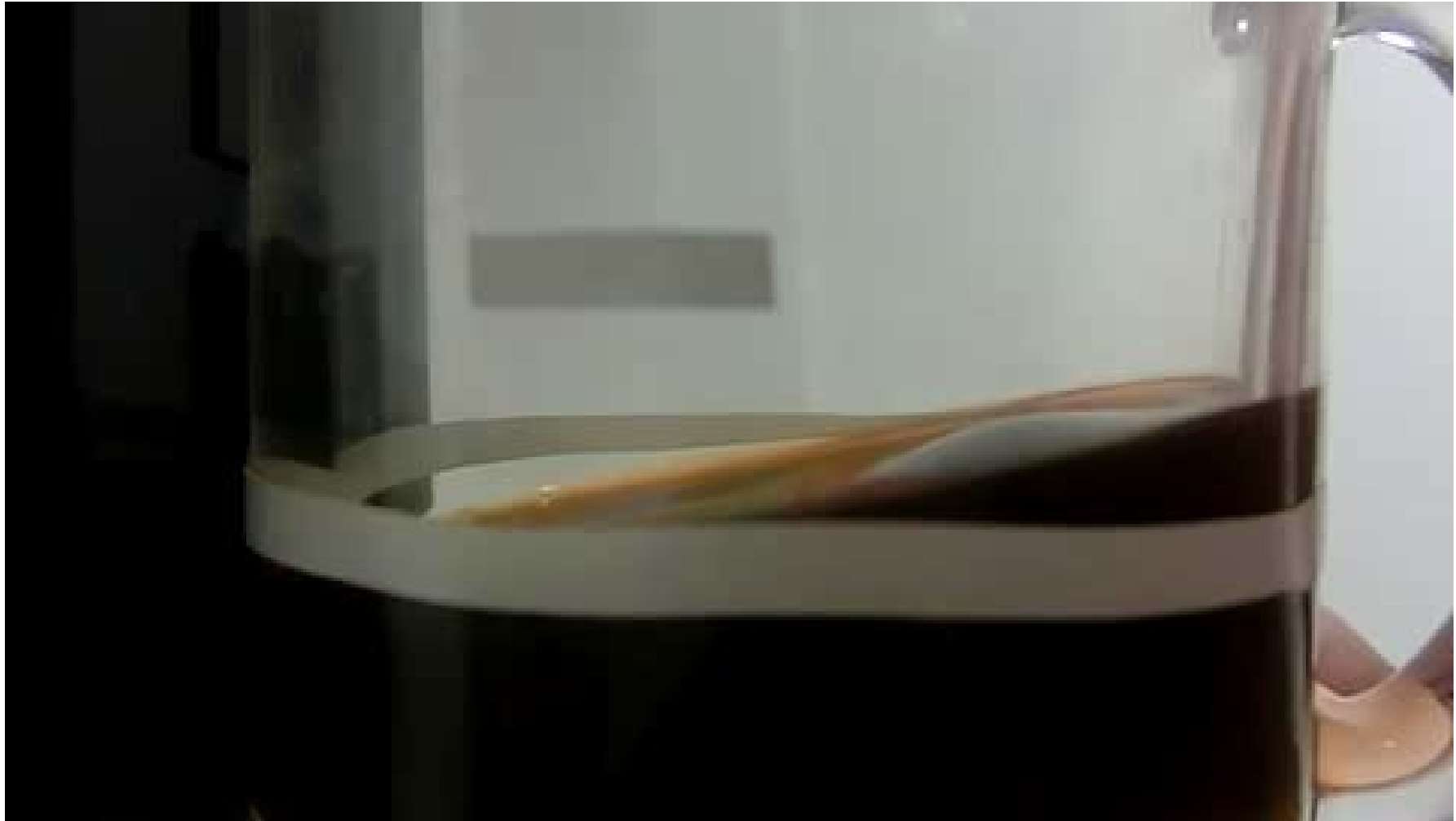
# Mobile measurements

5



# Natural step (2.5 Hz)

6



## Quick step (3.5 Hz)

7



Even faster (4 Hz)

8





Excitation of coffee sloshing has  
a resonance character.



Resonant excitation of sloshing modes



Excitation during walking



Human hand as a dumper

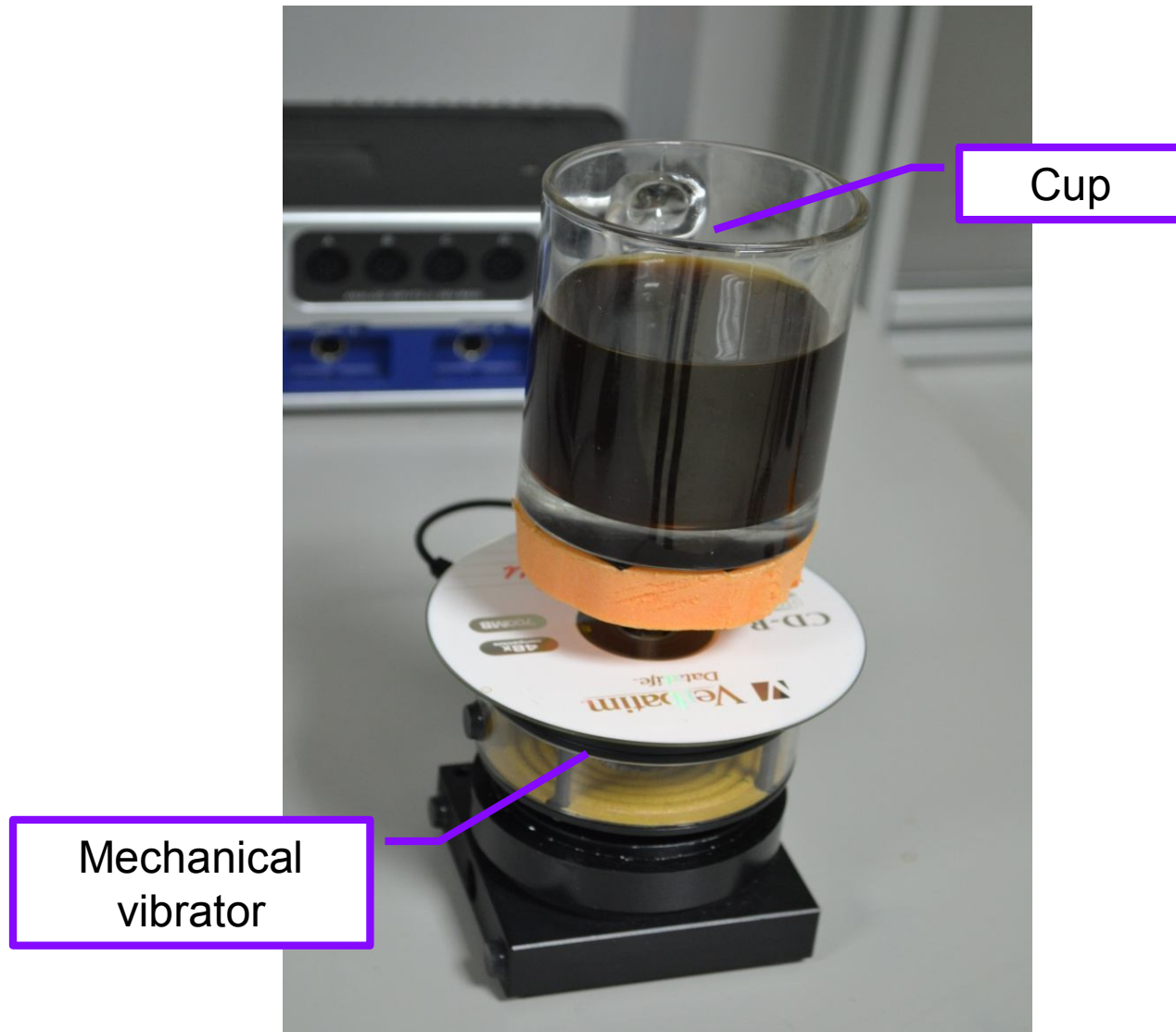


The shape of a cup

# Forced oscillations

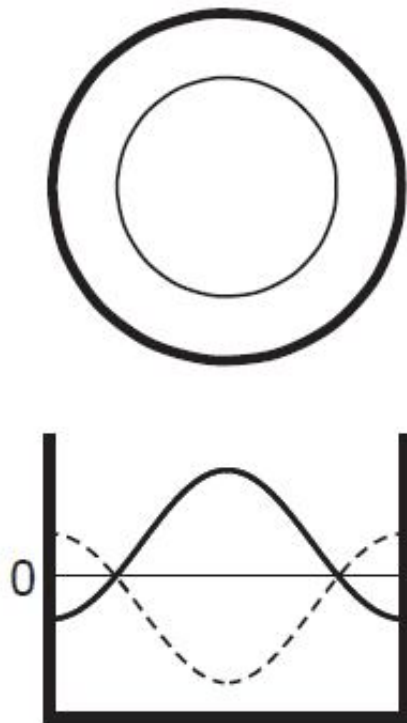
# Setup for vertical excitations

12



# Radial oscillation mode (240 fps)

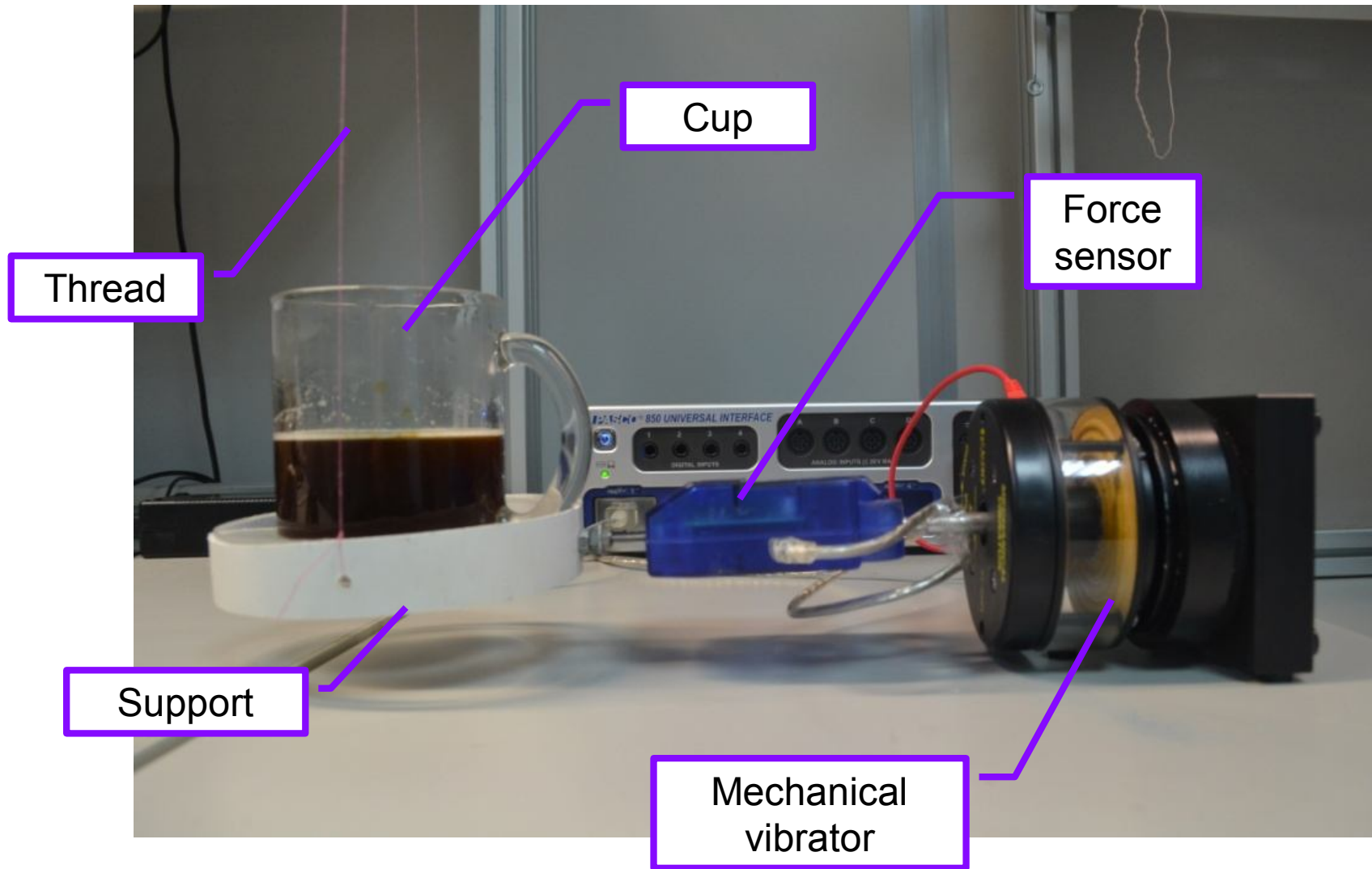
13



$$f = 3.8 \text{ Hz}$$

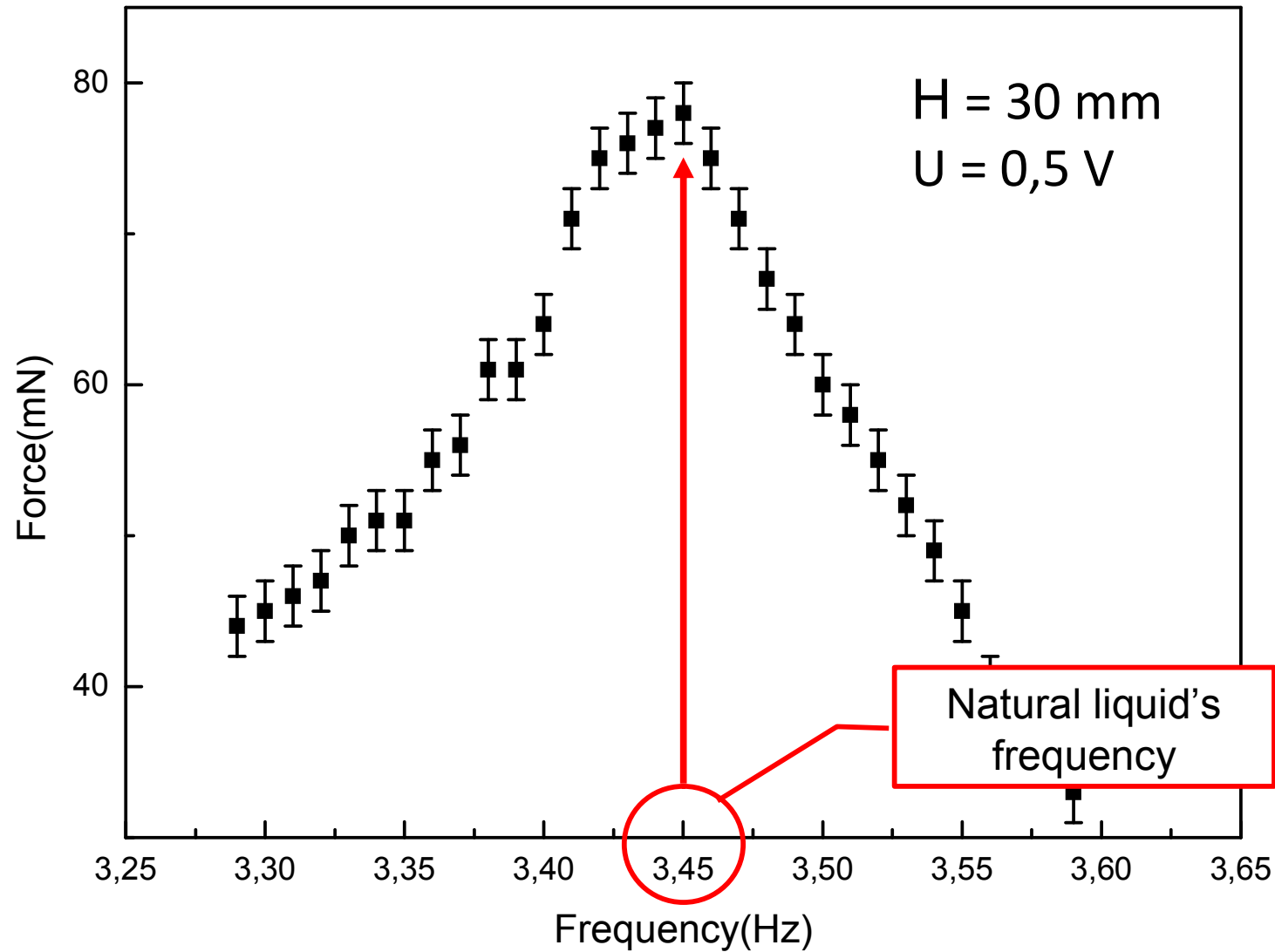
# Setup for horizontal excitations

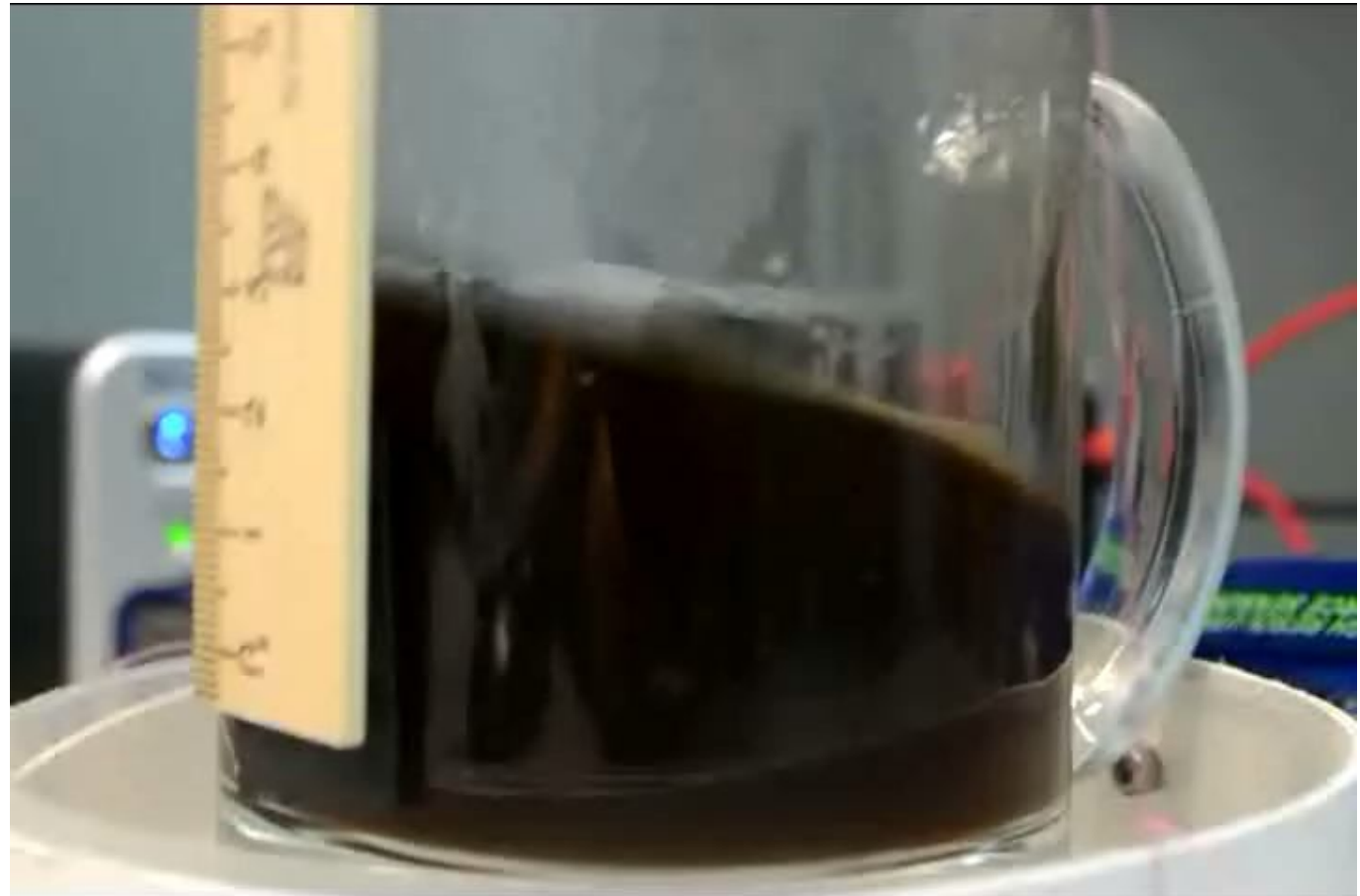
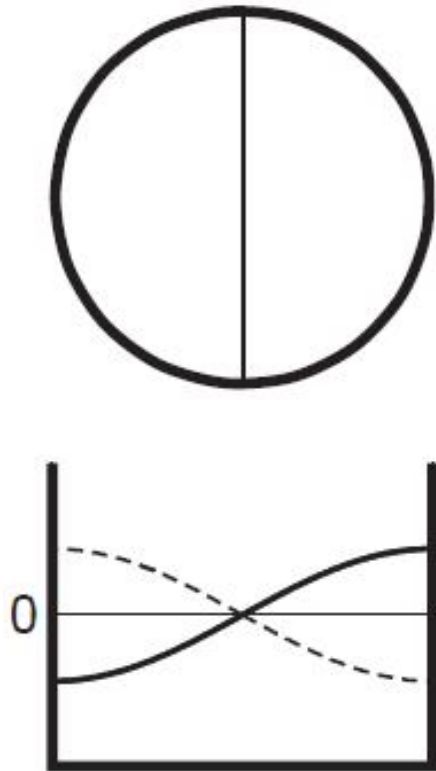
14



# Force vs. frequency

15





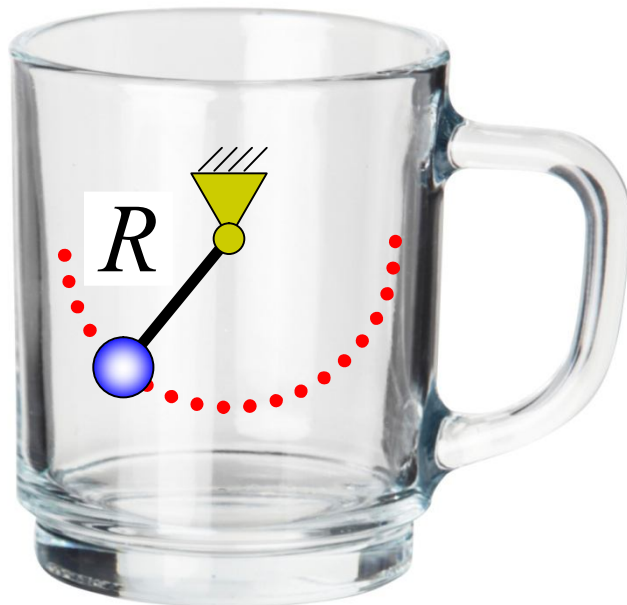
Resonance  $f = 3.5$  Hz



# Theoretical model. Pendulum hypothesis.

# Frequency estimation

18



$$\omega^2 \propto \frac{g}{R}$$

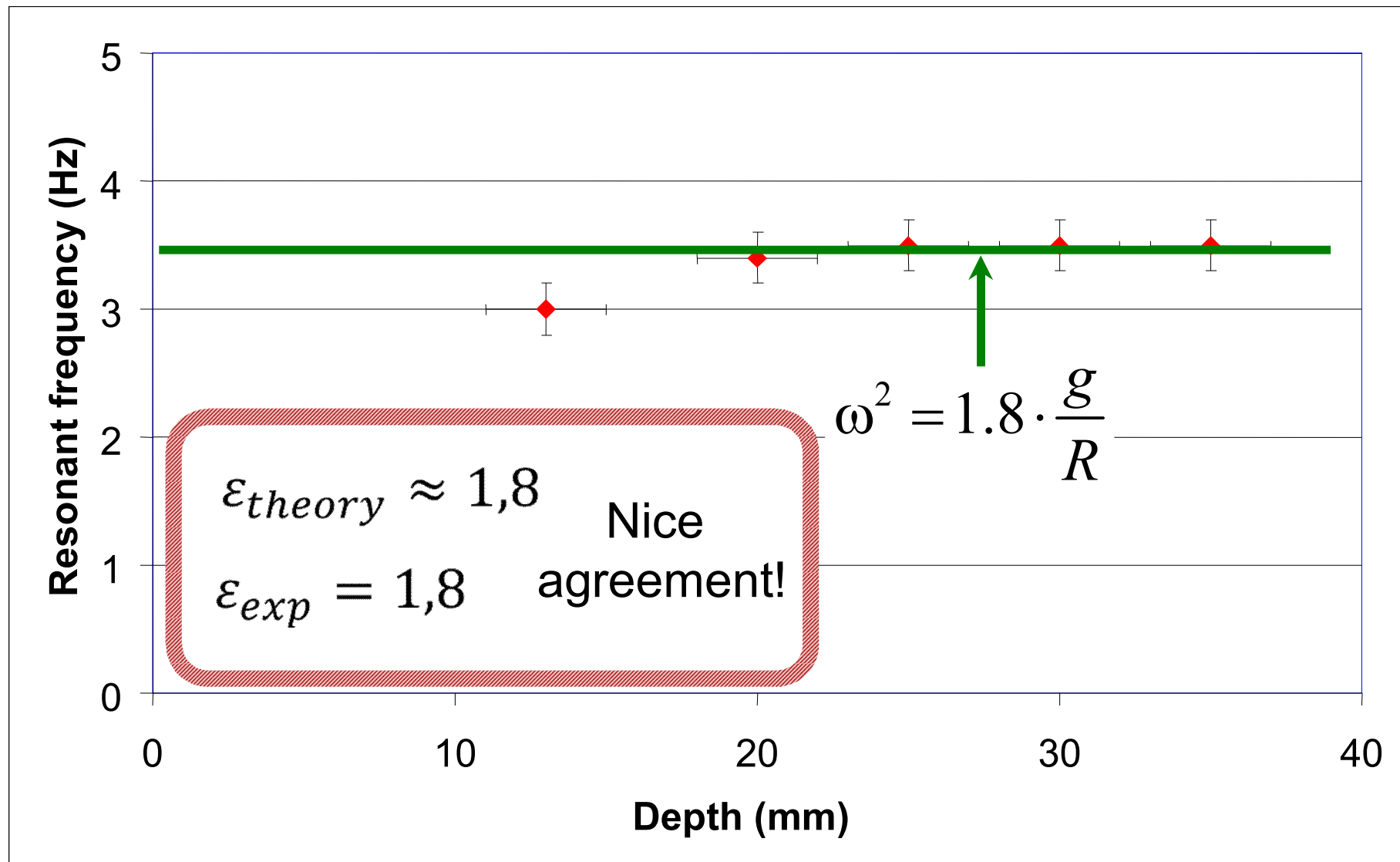
$$\omega^2 = \underbrace{\varepsilon \frac{g}{R} \tanh\left(\varepsilon \frac{H}{R}\right)}_{\approx 1} \cdot \underbrace{\left(1 + \varepsilon^2 \frac{\sigma g}{\rho R^2}\right)}_{\approx 0,02} \rightarrow \omega^2 = \varepsilon \frac{g}{R}$$

Theory

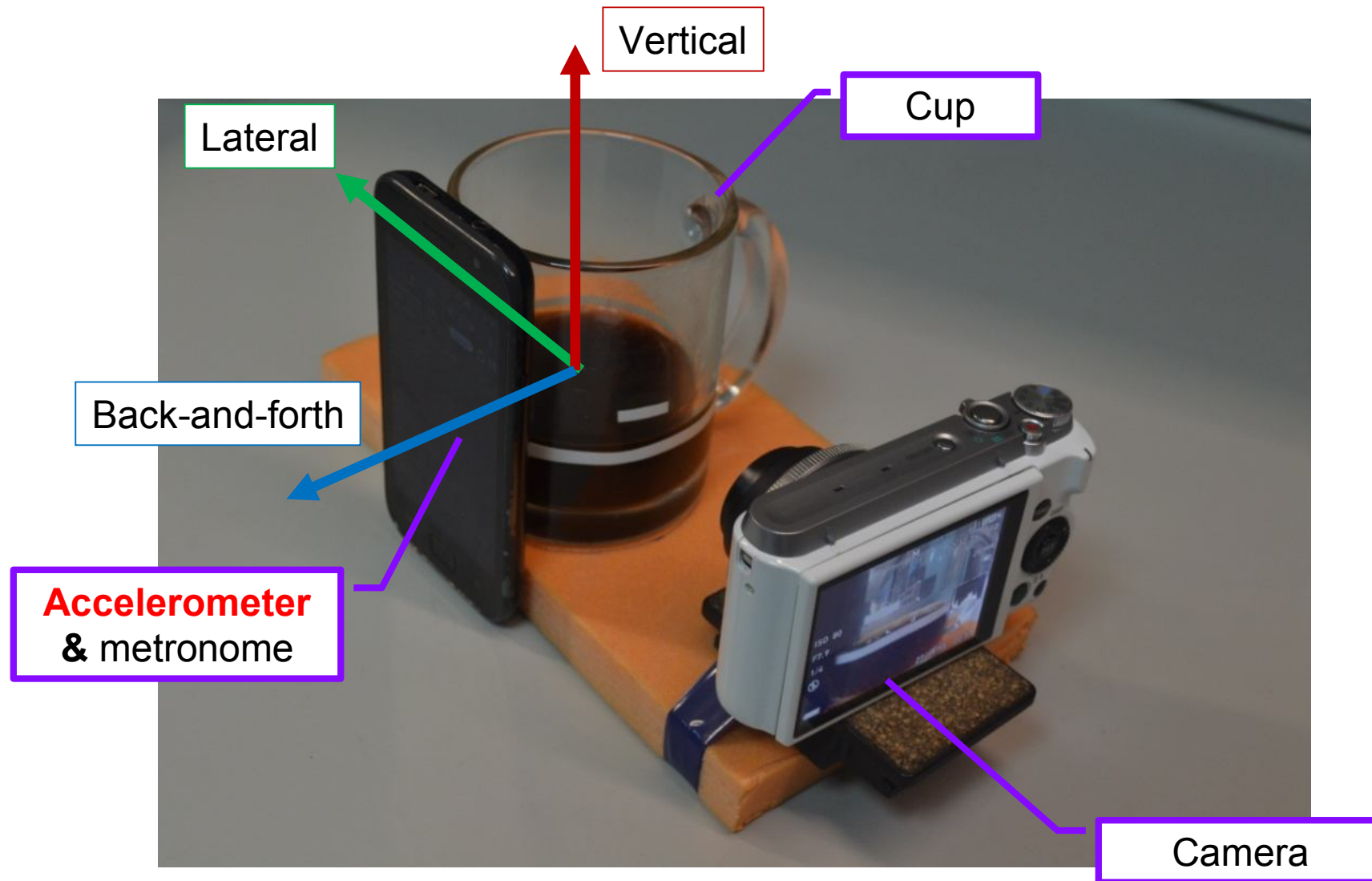
$$\varepsilon \approx 1,8$$

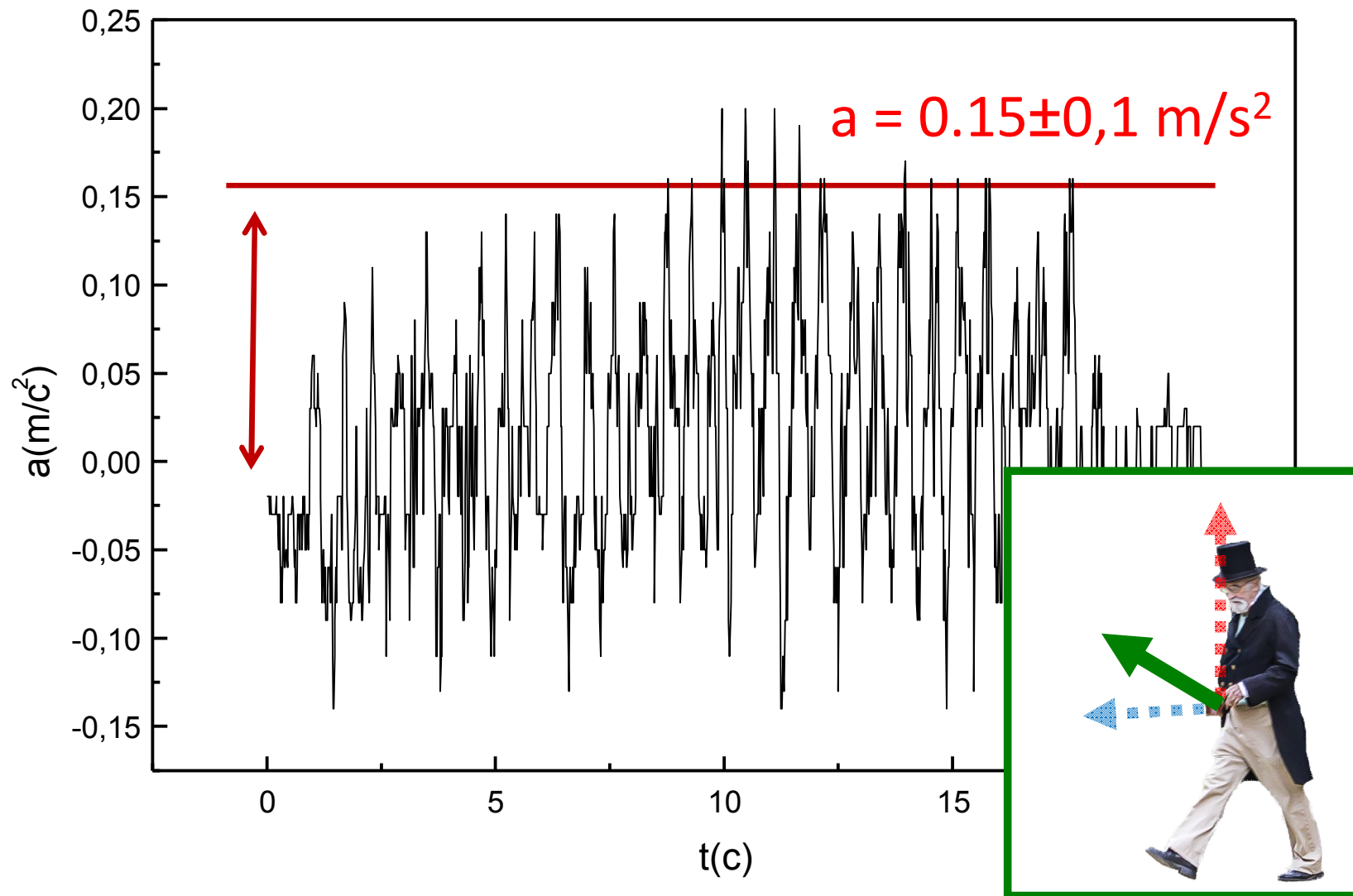
# Resonant frequency vs. depth

20



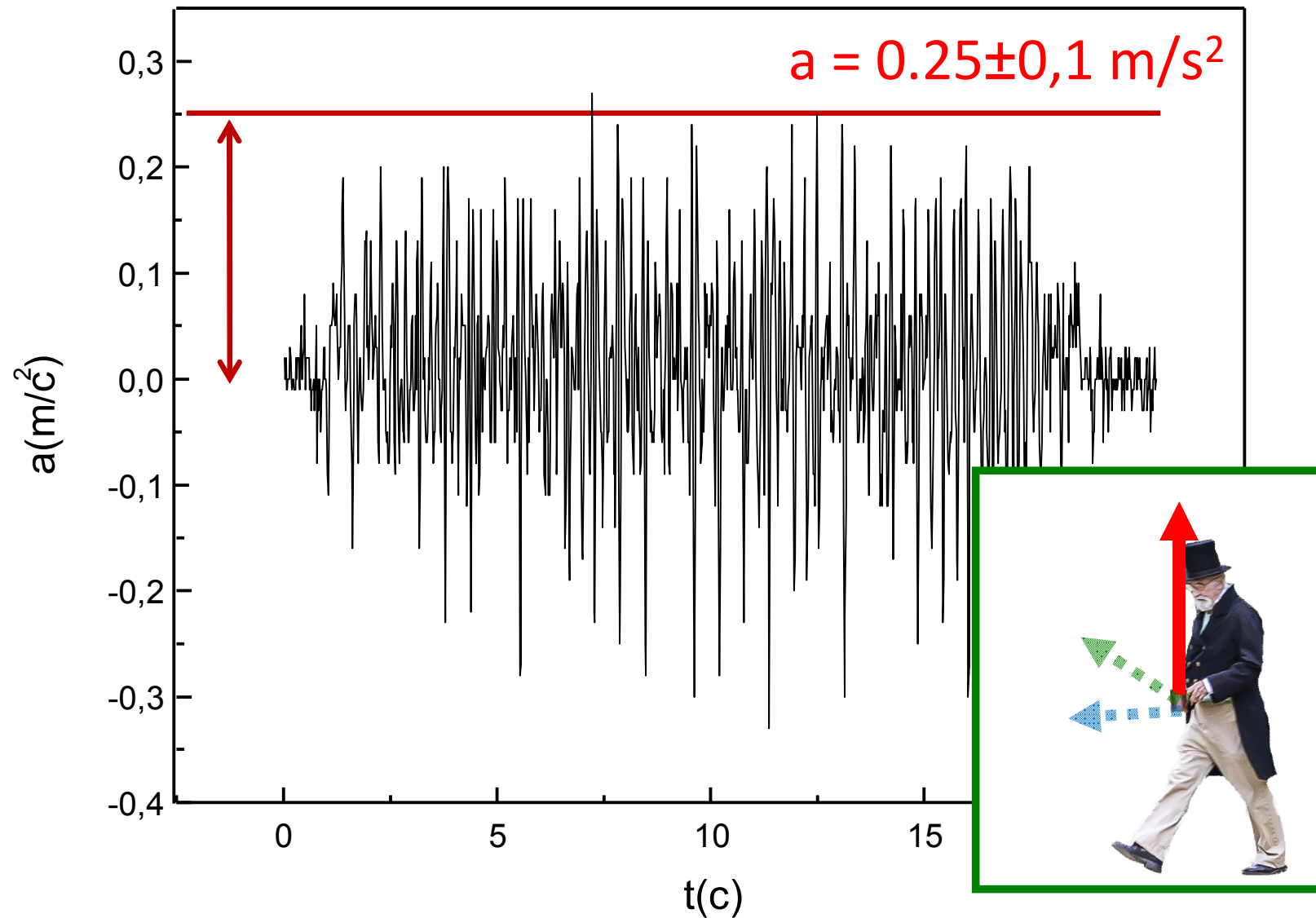
# Acceleration of a cup during walking



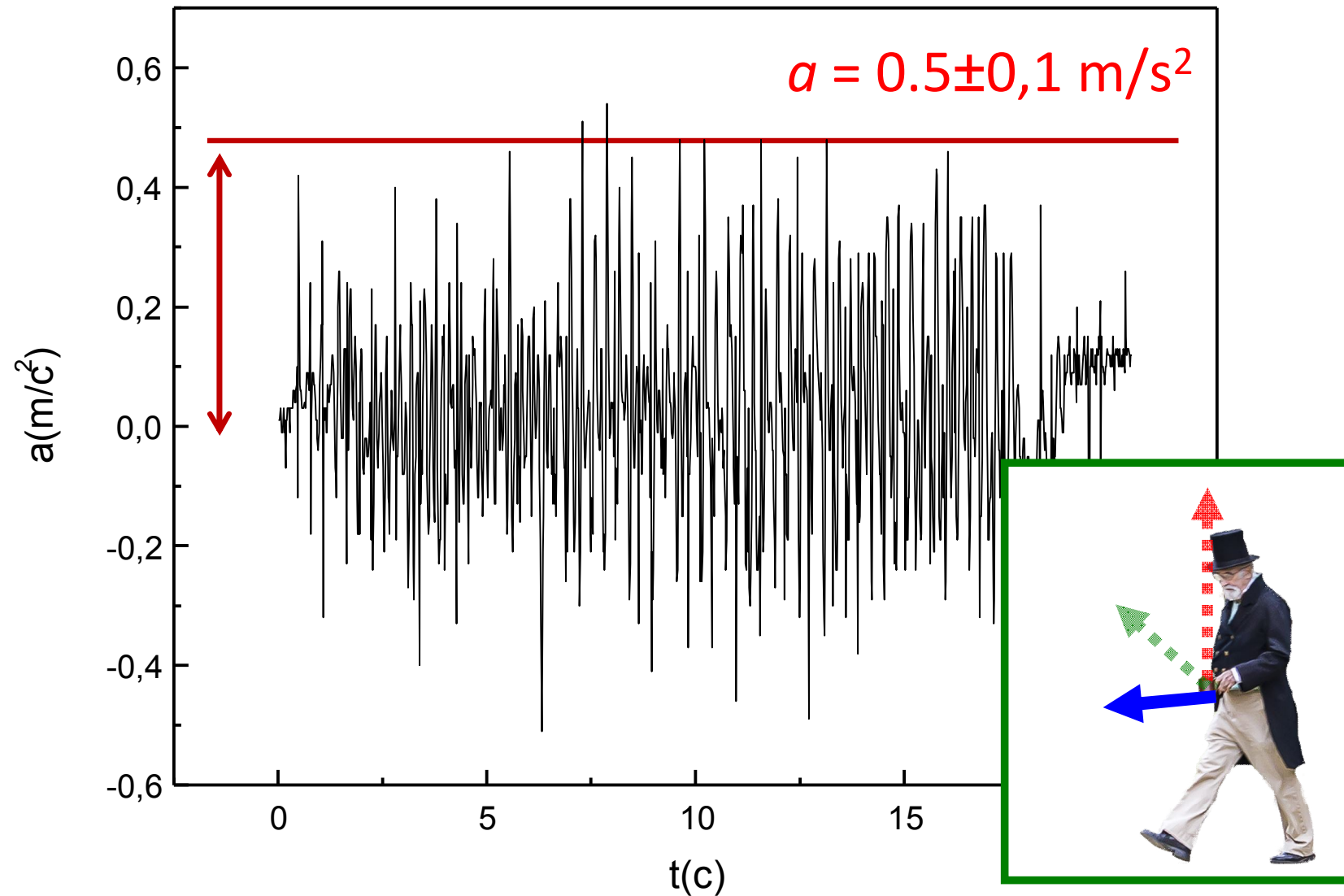


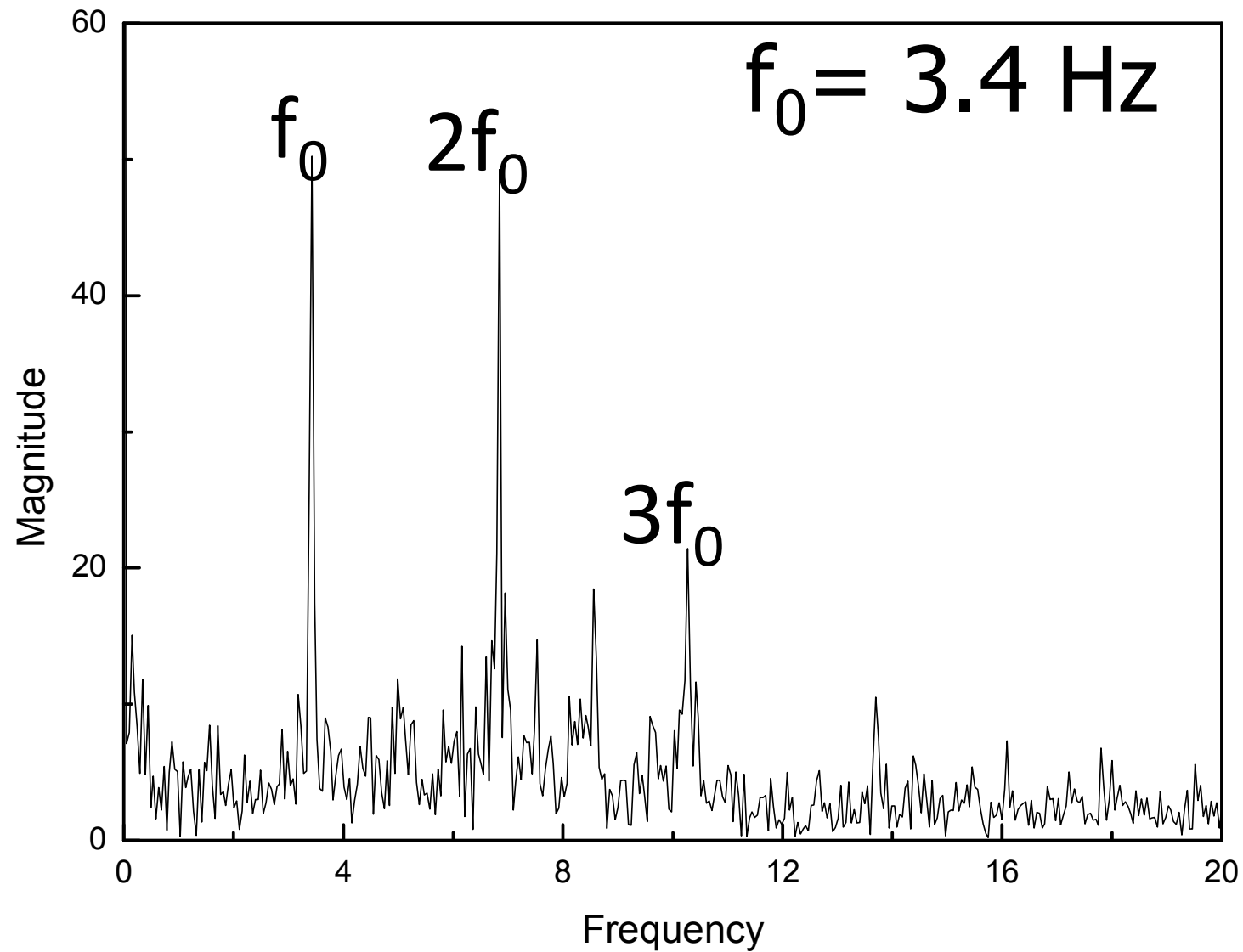
# Vertical resonance acceleration

24



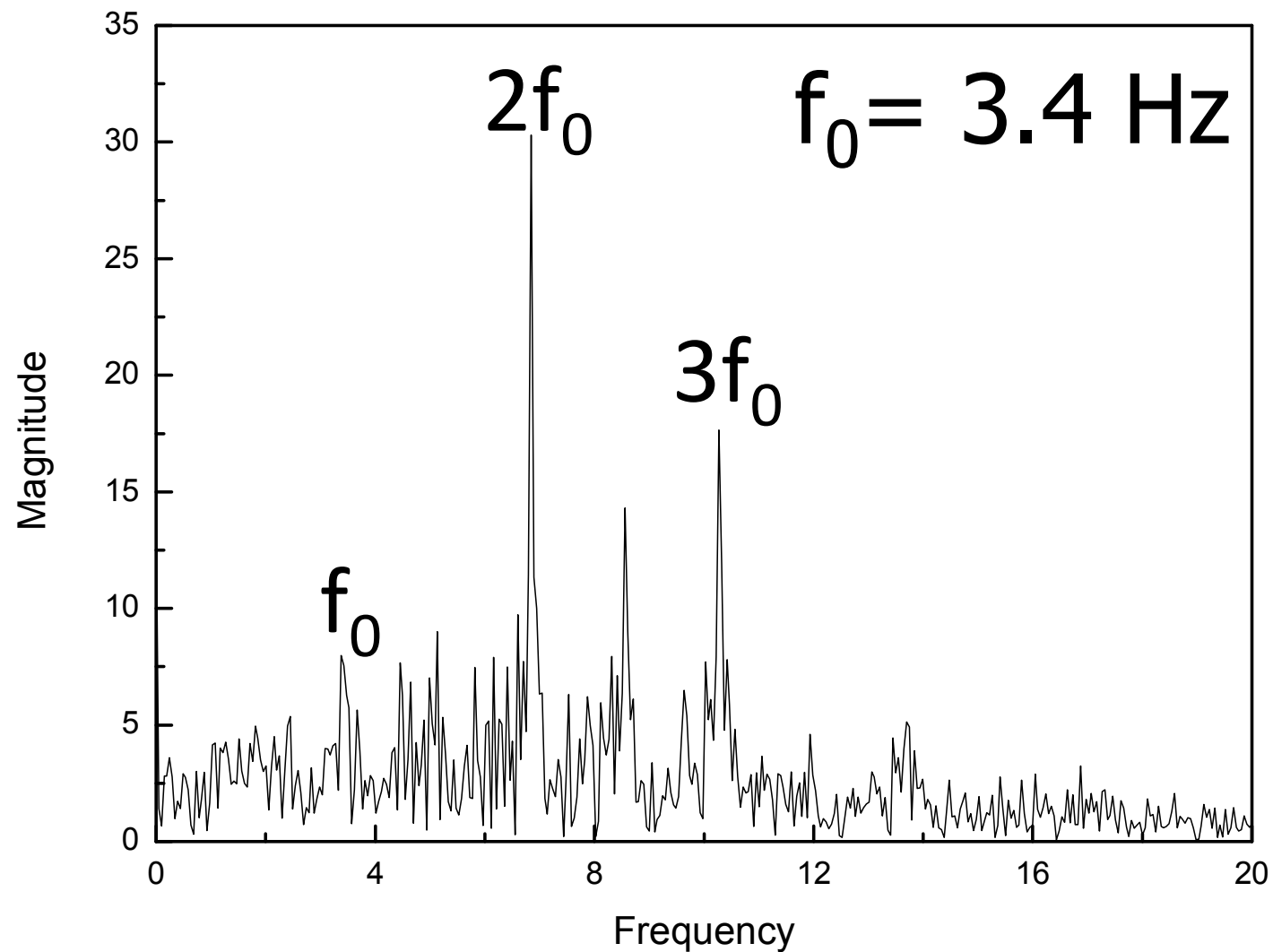


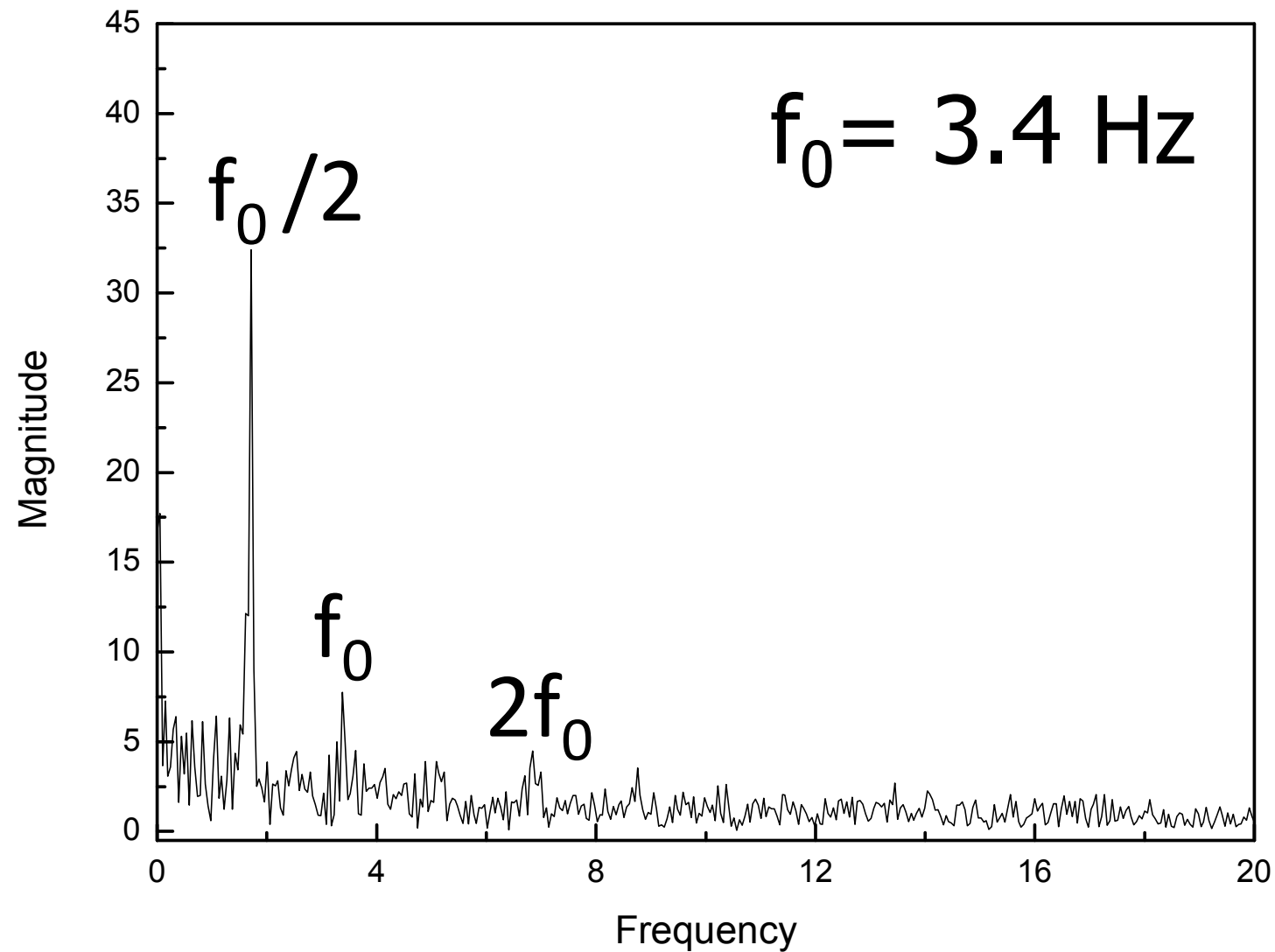




# Vertical acceleration spectrum

27



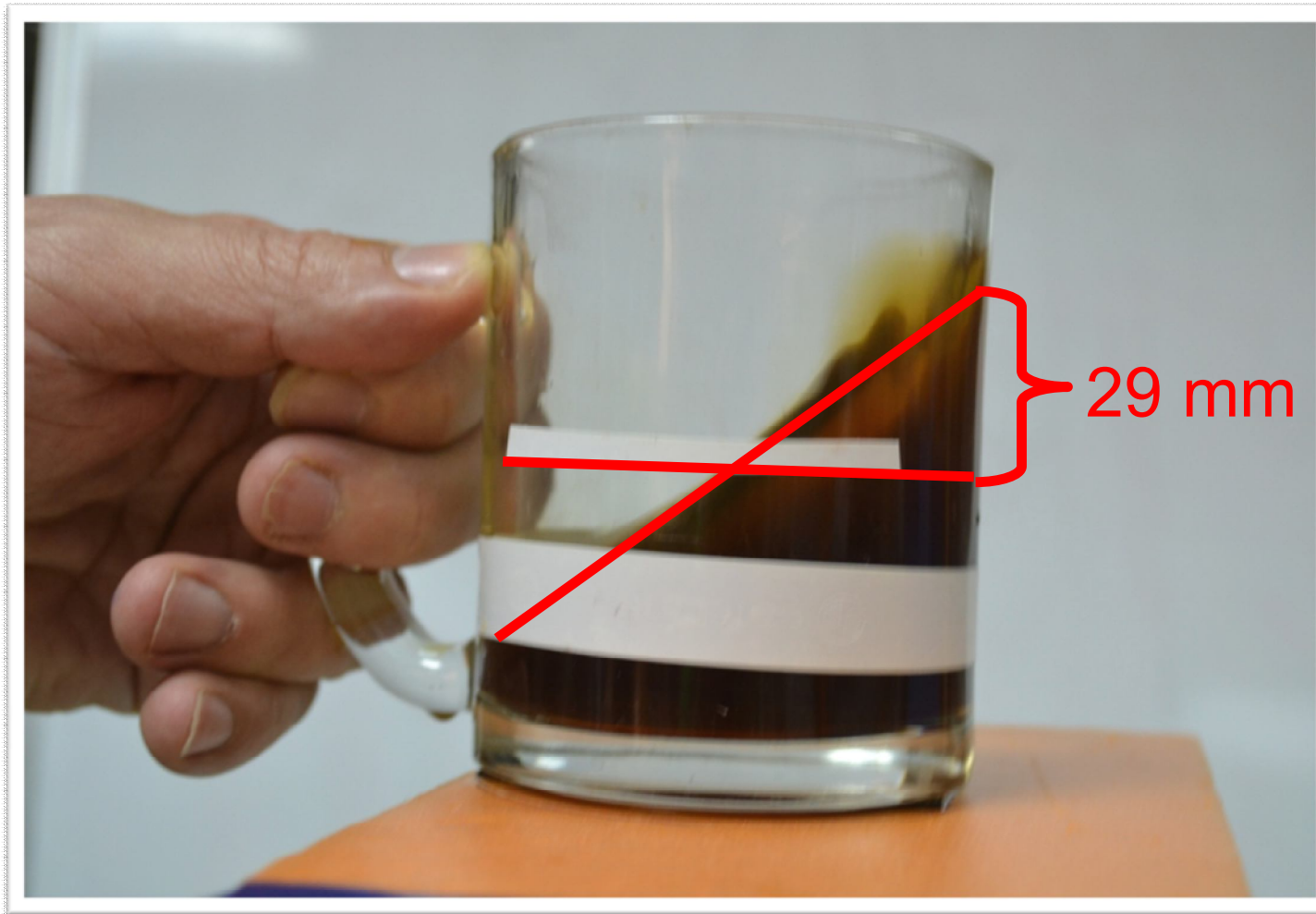


Back-and-forth acceleration  
makes a dominant contribution  
in coffee sloshing.



# Amplitude during walking

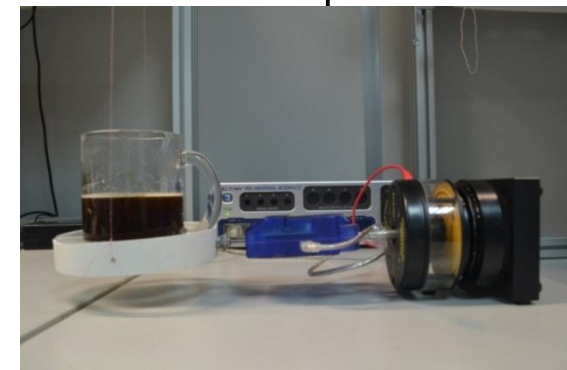
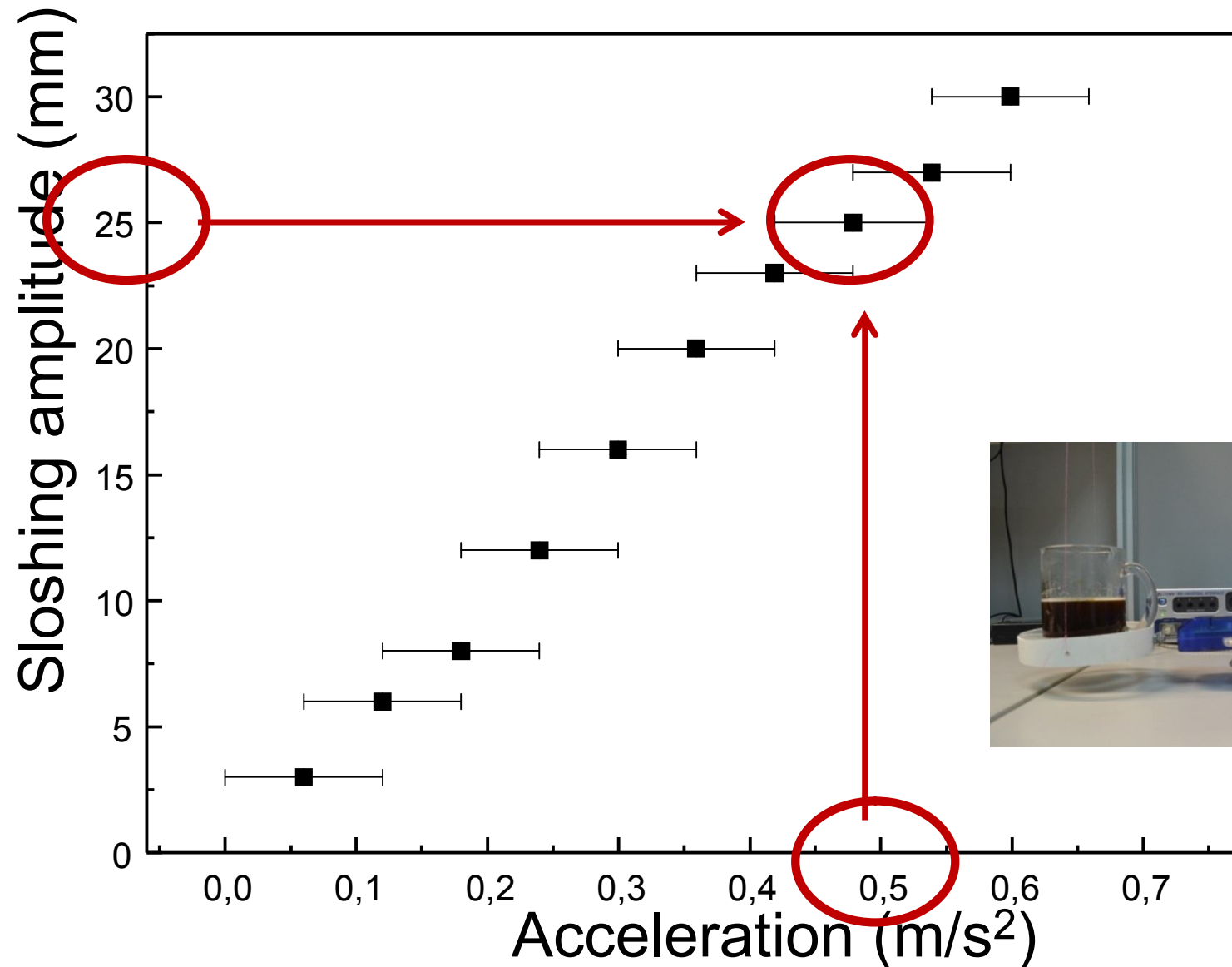
30



Back-and-forth acceleration  $0.5 \text{ m/s}^2$

# Horizontal excitation experiment

31



Human hand  
as a dumper



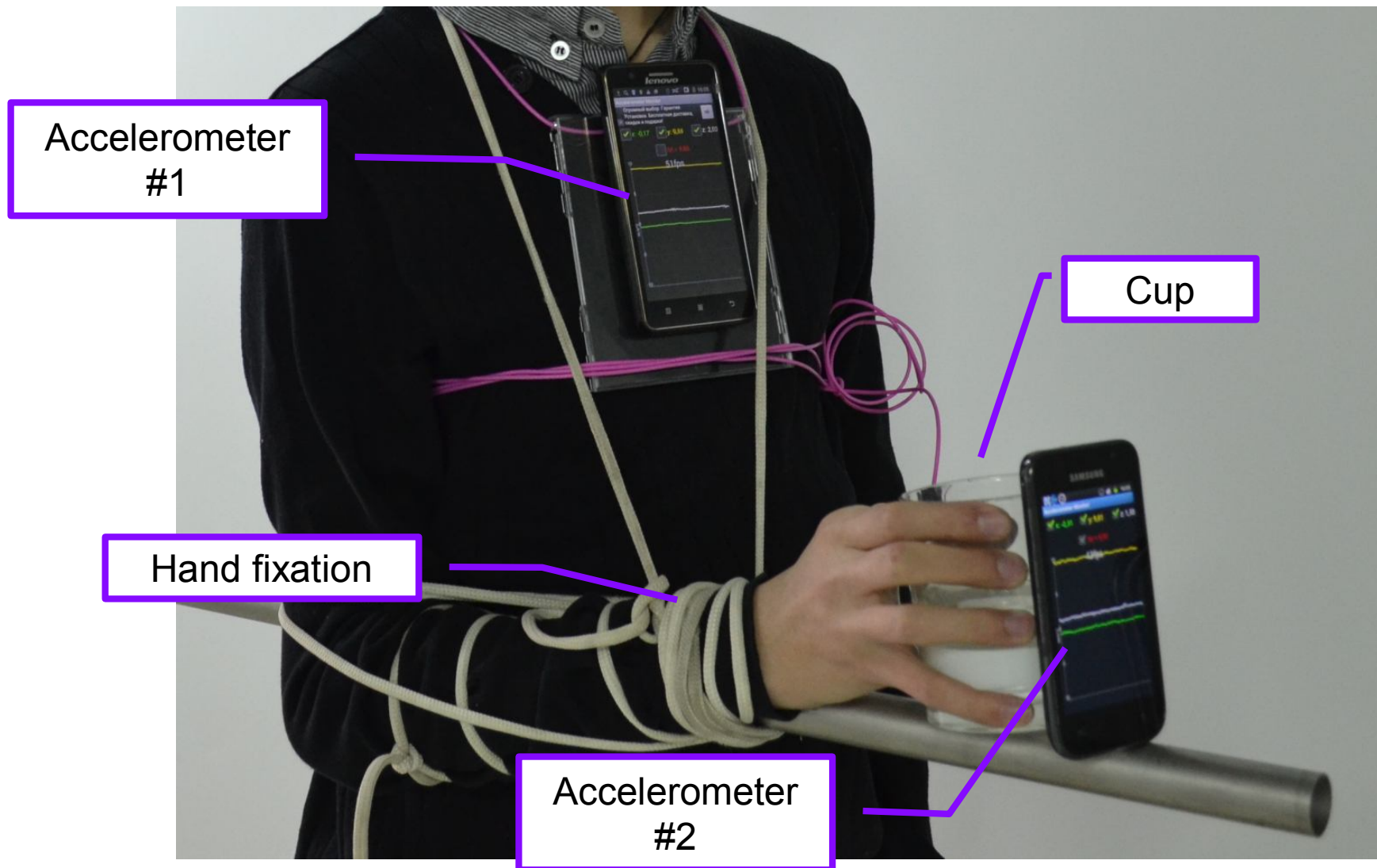
Frequencies of human walking

Due to its own natural frequencies  
human hand damps this excitation

Natural frequencies of  
liquid oscillations in the cup

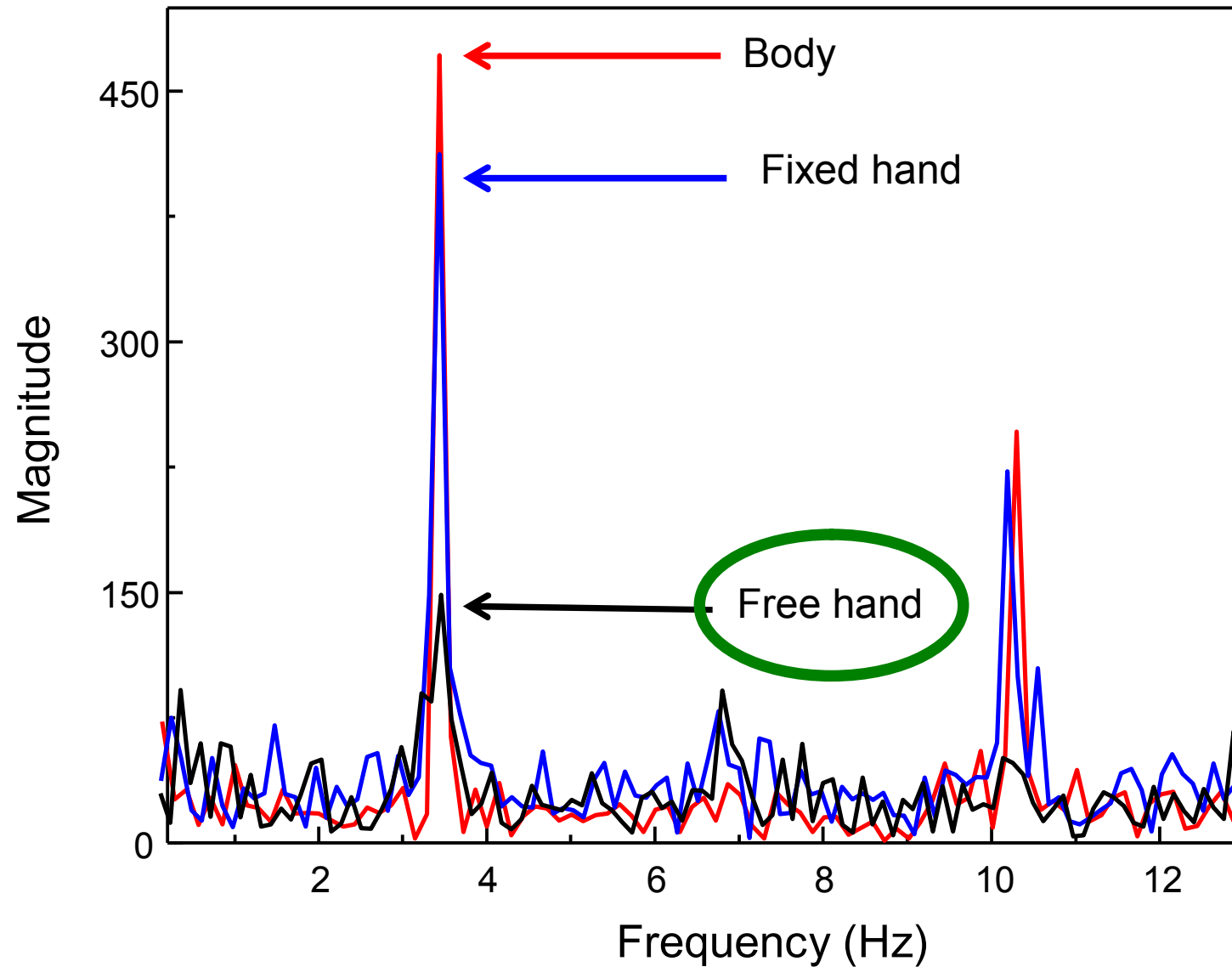
# Experiment with fixed hand

34



# Acceleration spectrum

35



# Cup geometry

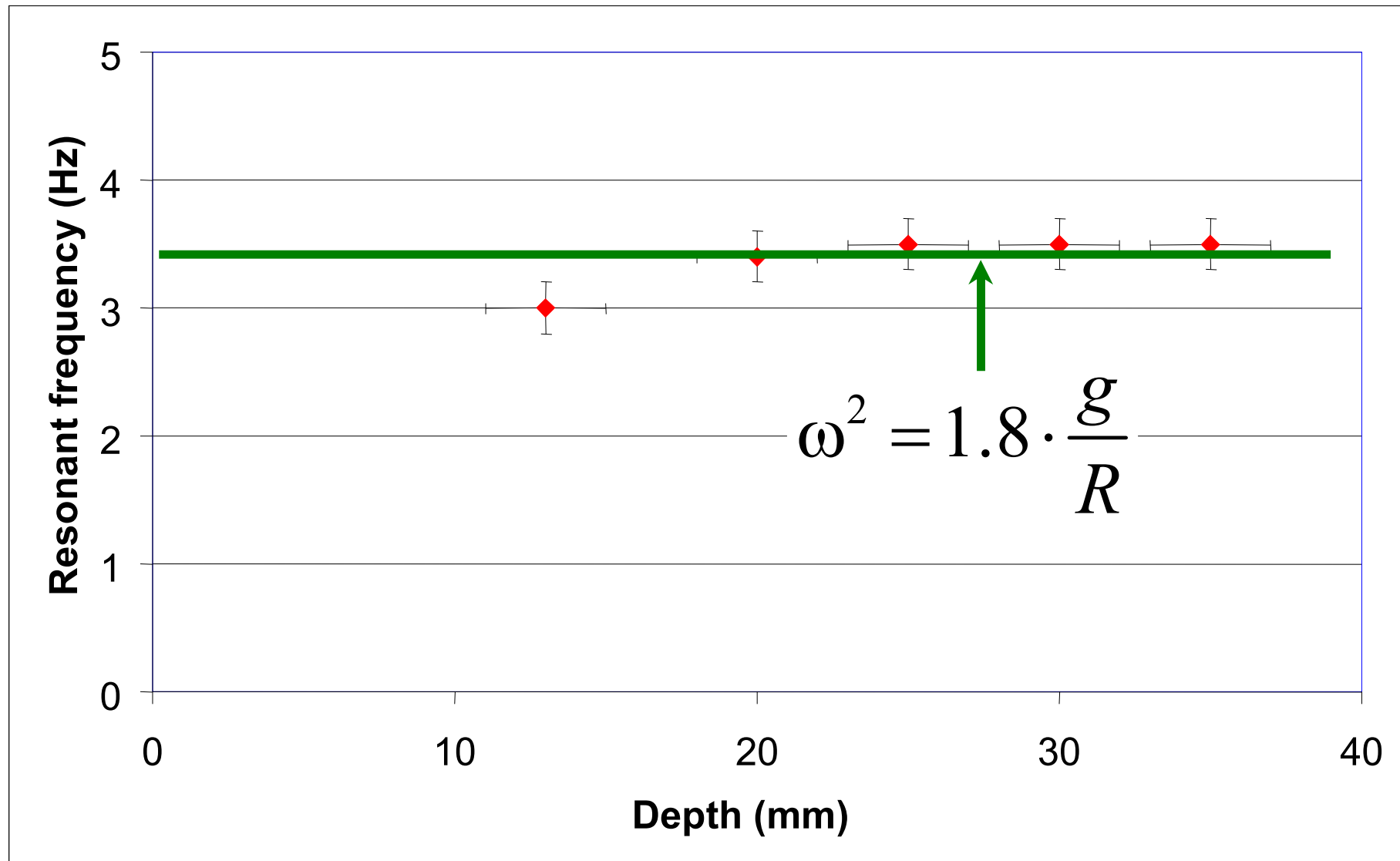
# Cylindrical cup

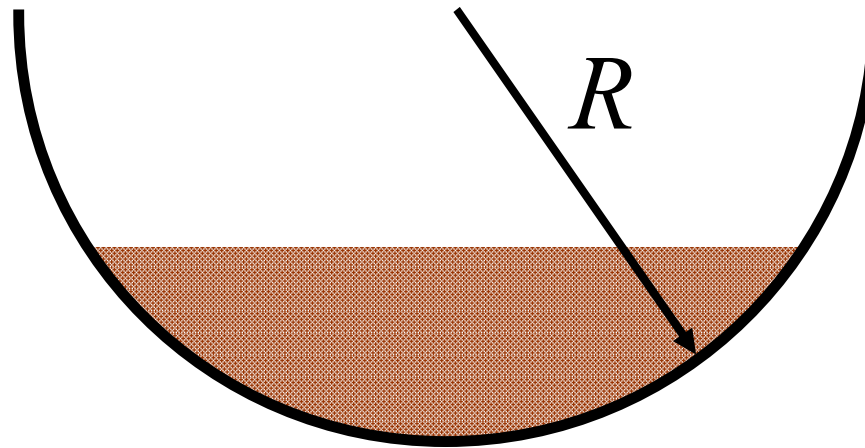
37



# Resonant frequency vs. depth

38



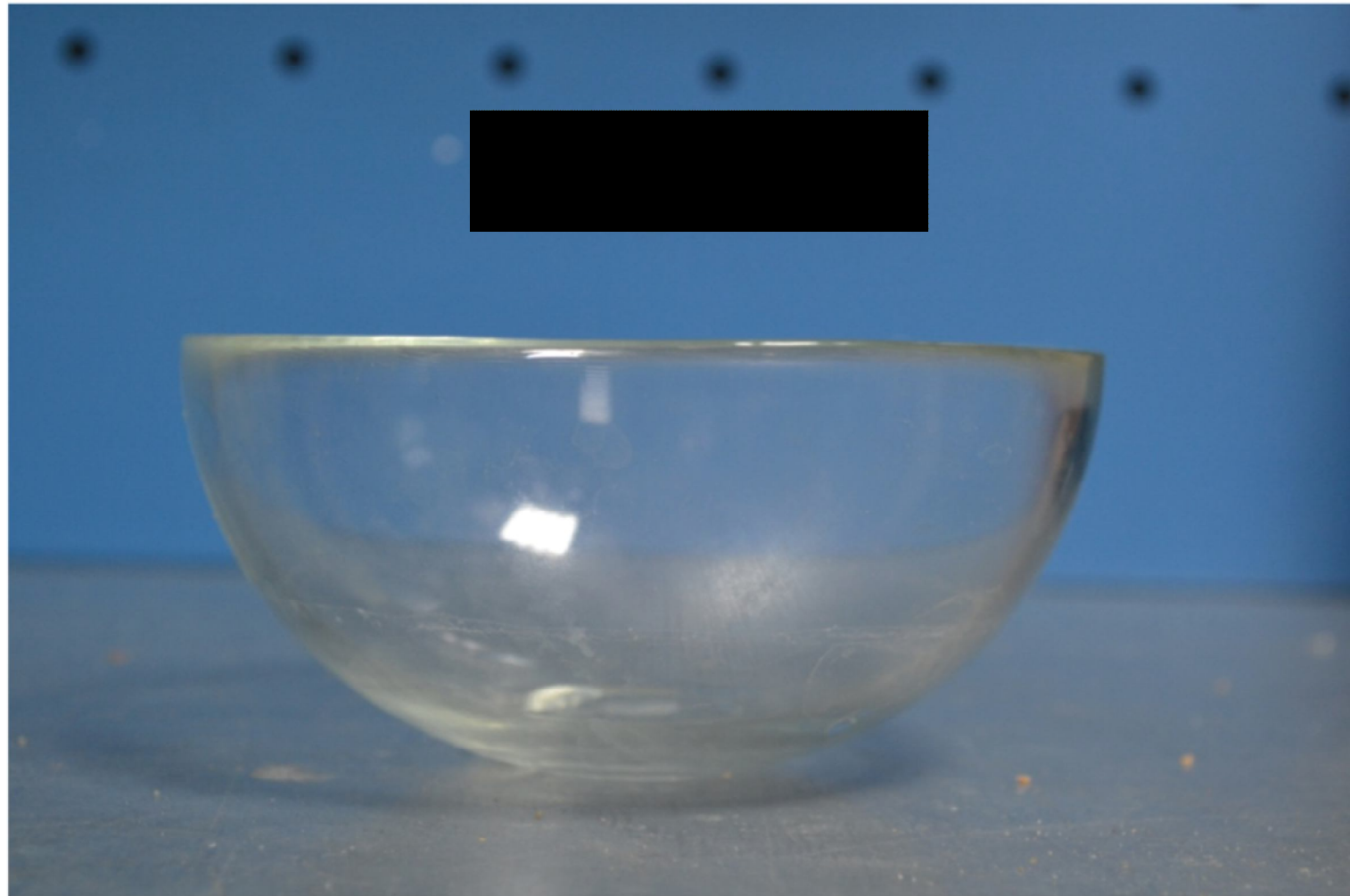


$$\omega^2 \approx \frac{g}{R}$$

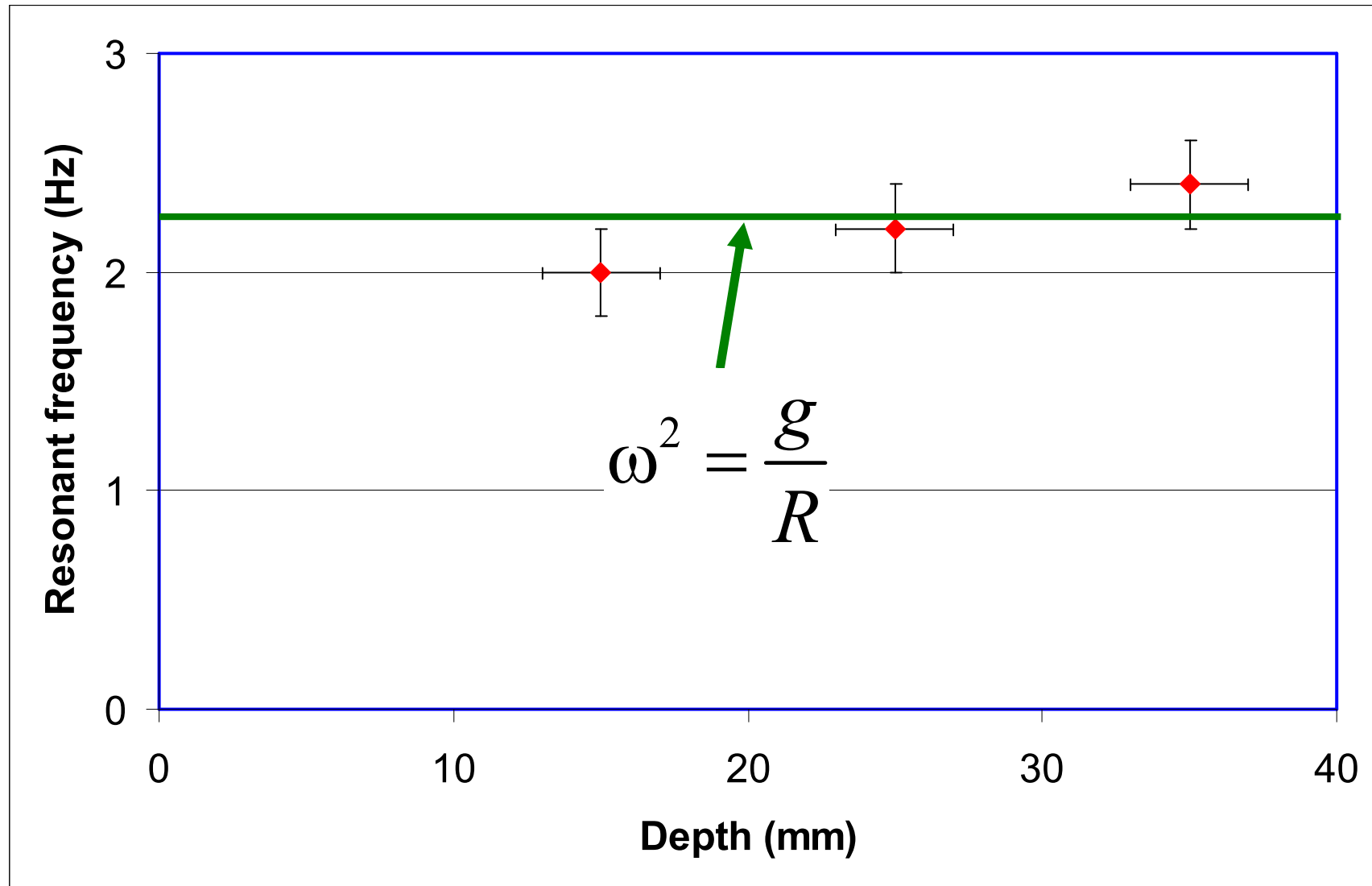
**Resonant frequency weakly depends on the depth until the depth becomes comparable with the radius**

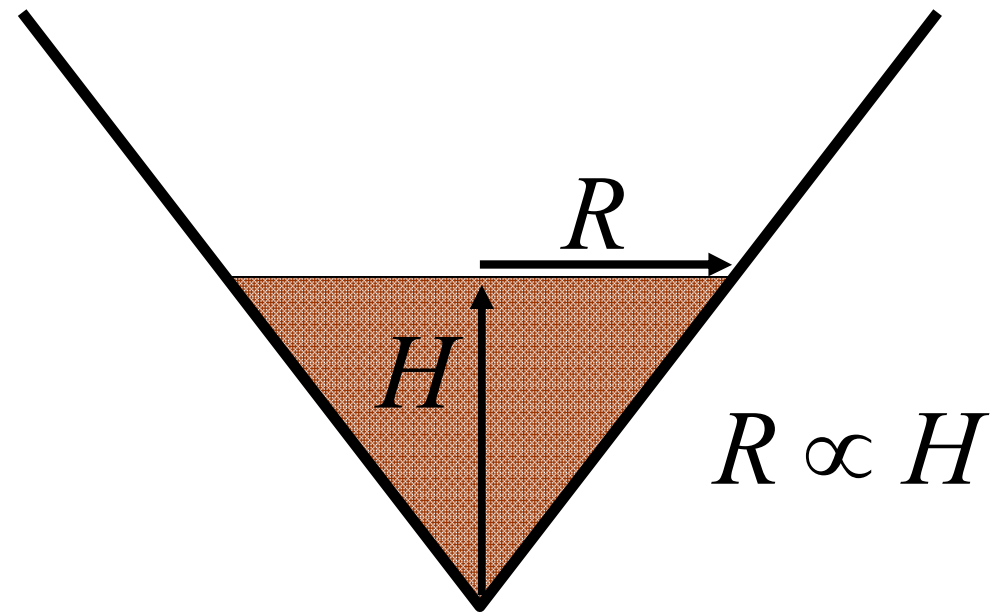
# Hemispherical cup

40

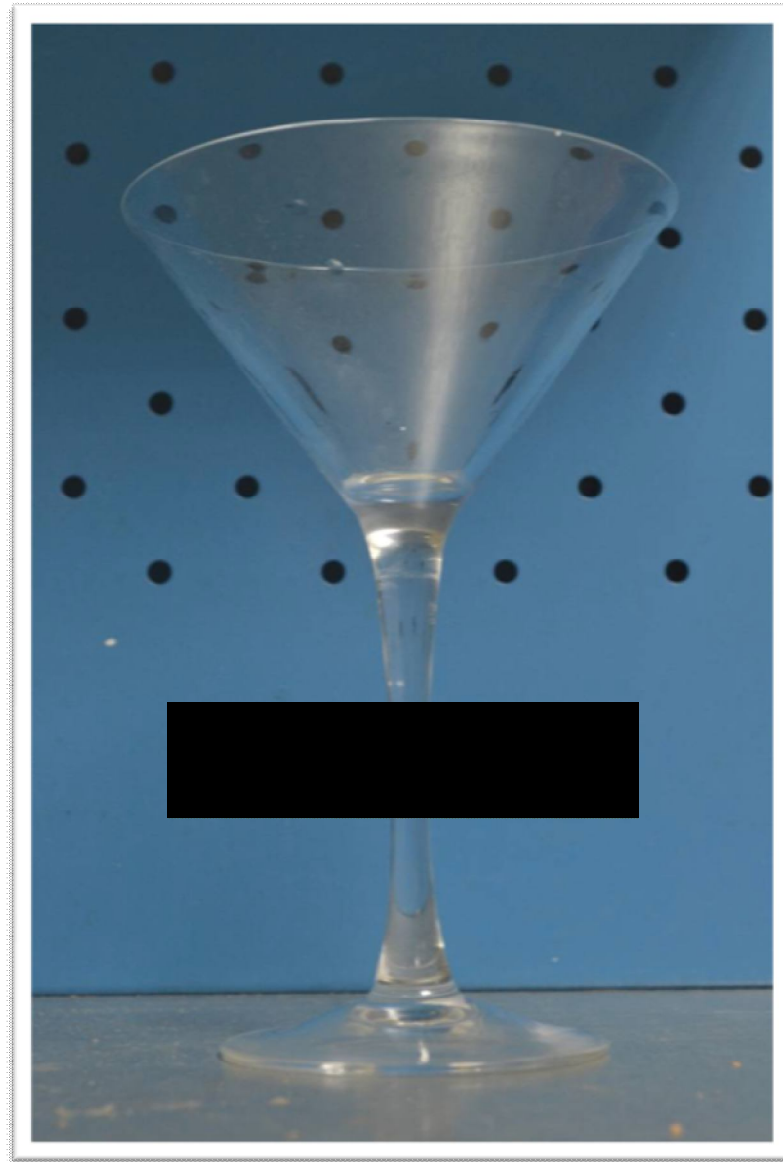






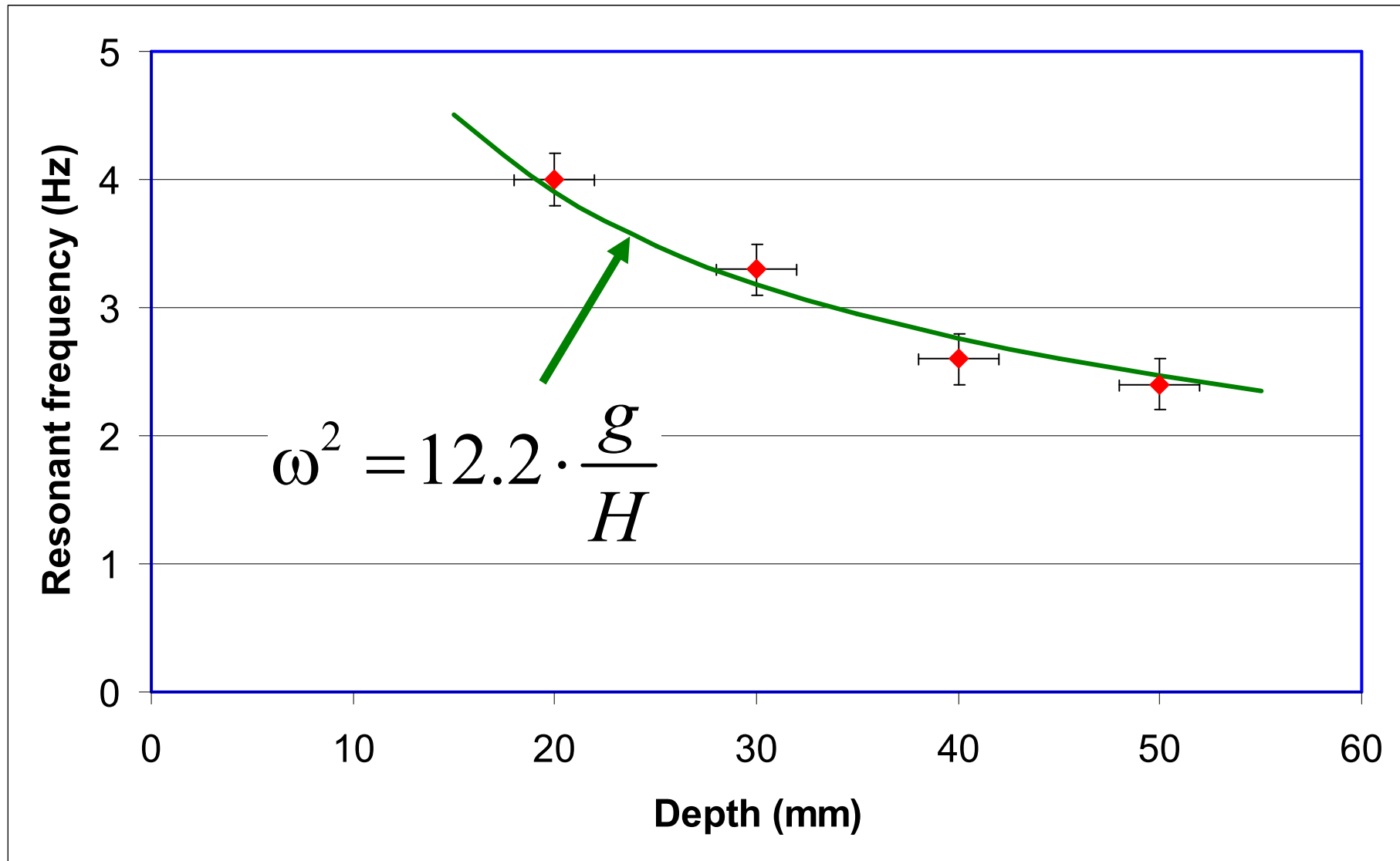


$$\omega^2 \propto \frac{g}{R} \propto \frac{g}{H}$$



# Resonant frequency vs. depth

44



# Summary

# Conclusions

46

Quick step (3.5 Hz)

7

Setup for vertical excitations

12

Force vs. frequency

15

Theory for cylindrical cup

19

Resonant frequency vs. depth

20

$$\omega^2 = \varepsilon \frac{g}{R} \tan^2 \alpha$$

Back-and-forth resonance acceleration

25

Acceleration spectrum

35

Model

42

Force(mN)

80  
60  
40  
3.25

Mech  
vib

Resonant frequency (Hz)

5  
4  
3  
2  
1  
0

$\varepsilon_{the}$   
 $\varepsilon_{exp}$

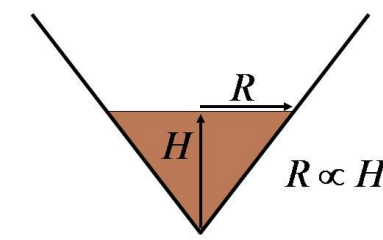
H. Lamb, Hydrody

$a(m/s^2)$

0.6  
0.4  
0.2  
0.0  
-0.2  
-0.4  
-0.6

Magnitude

450  
300  
150  
0



$$\omega^2 \propto \frac{g}{R} \propto \frac{g}{H}$$

- Ikeda T., Murakami S. (2005) “Autoparametric resonances in a structure/fluid interaction system carrying a cylindrical liquid tank.” *J. Sound Vibr.* **285**, 517–546.
- Mayer H.C., Krechetnikov R. (2012) “Walking with coffee: Why does it spill?” *Phys. Rev. E* **85**, 046117



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**Thank you for  
your attention!**



- The frequency of your steps should not be equal to the natural frequency of coffee oscillations.
- Relax your hand and it will act as a damper for coffee sloshing.
- Small cups are better than large ones, for their natural frequencies are higher.

