Russia IYPT

# Hovercraft

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# The problem



A simple model hovercraft can be built using a CD and a balloon filled with air attached via a tube. Exiting air can lift the device making it float over a surface with low friction. Investigate how the relevant parameters influence the time of the 'low-friction' state.

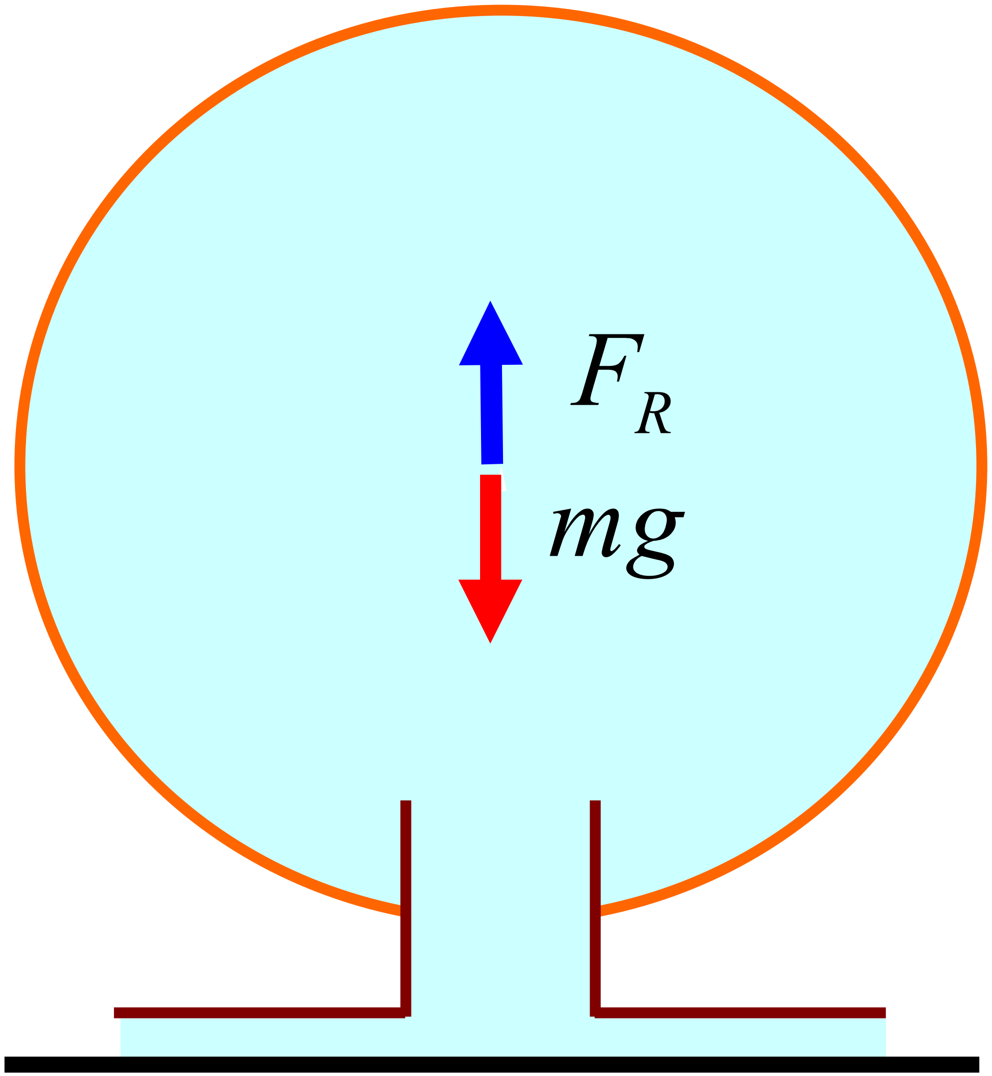
# First observations

# Floating over the table

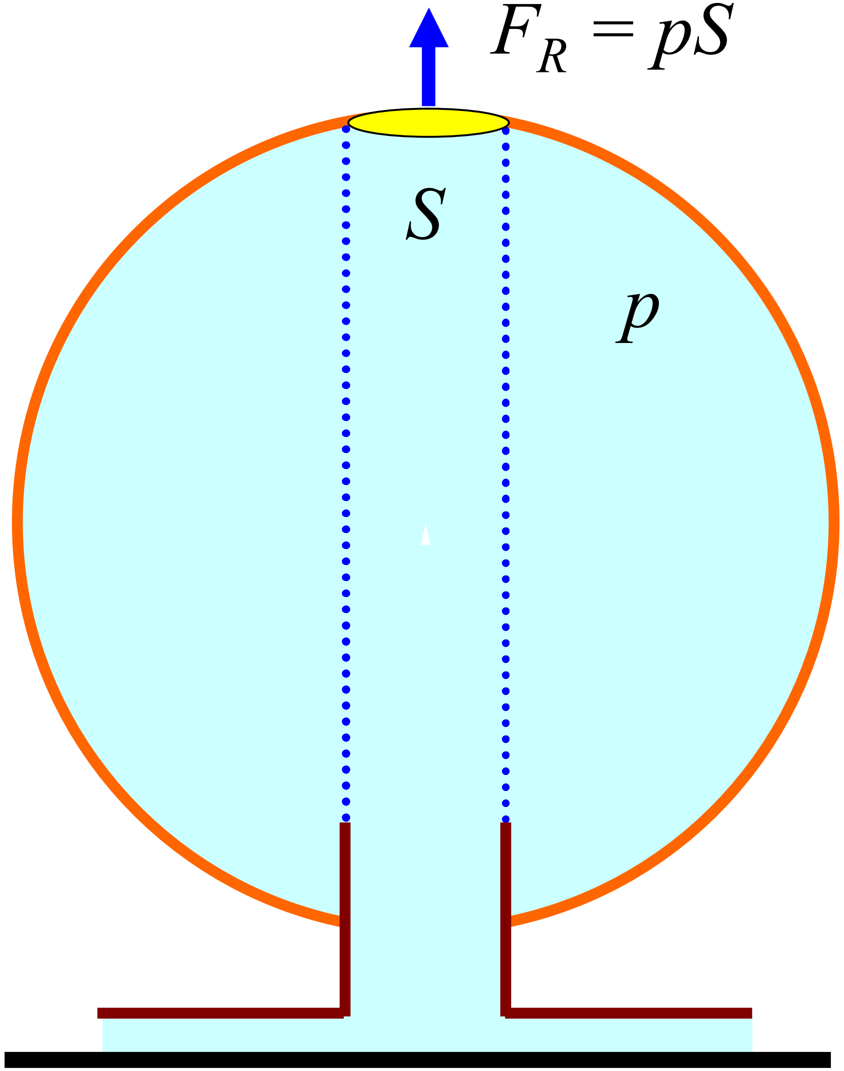


# Force balance





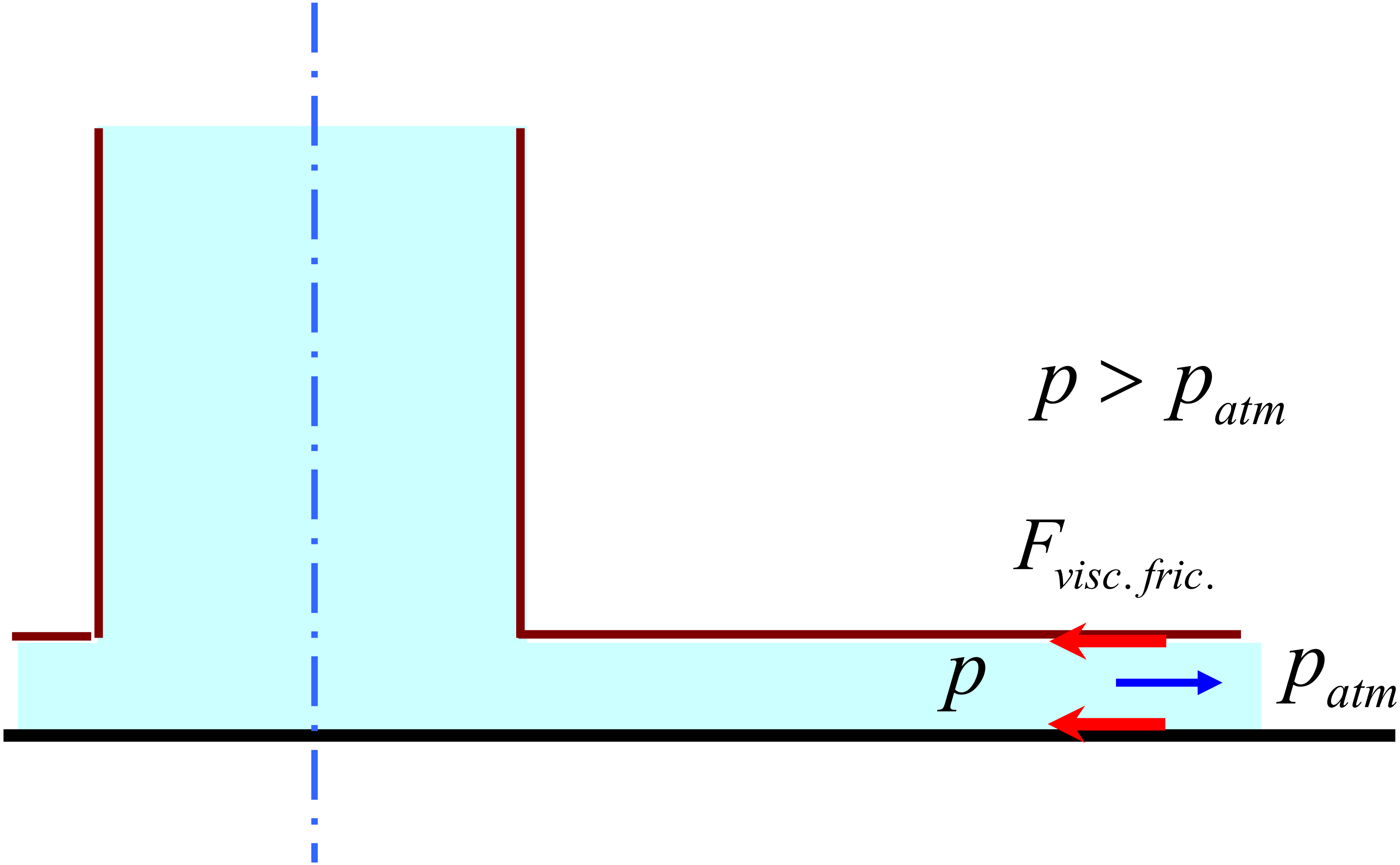
# The reactive force




$$d = 13{,}8\ \mathrm{mm}$$
$$S = 1{,}5\ \mathrm{cm}^2$$
$$F_R = 0{,}15\ \mathrm{N}$$

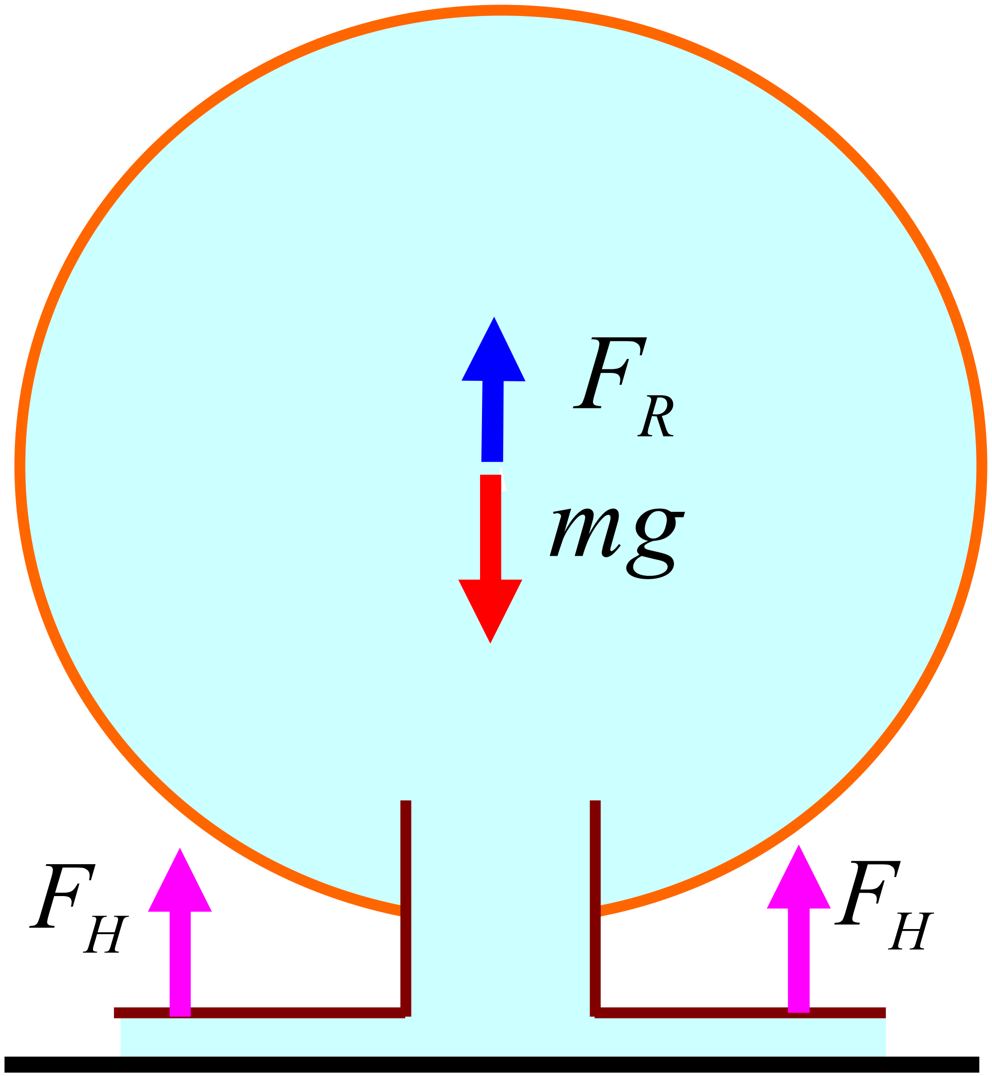
$$mg = 0{,}28\ \mathrm{N}$$

# Viscous friction



# Force balance over the table



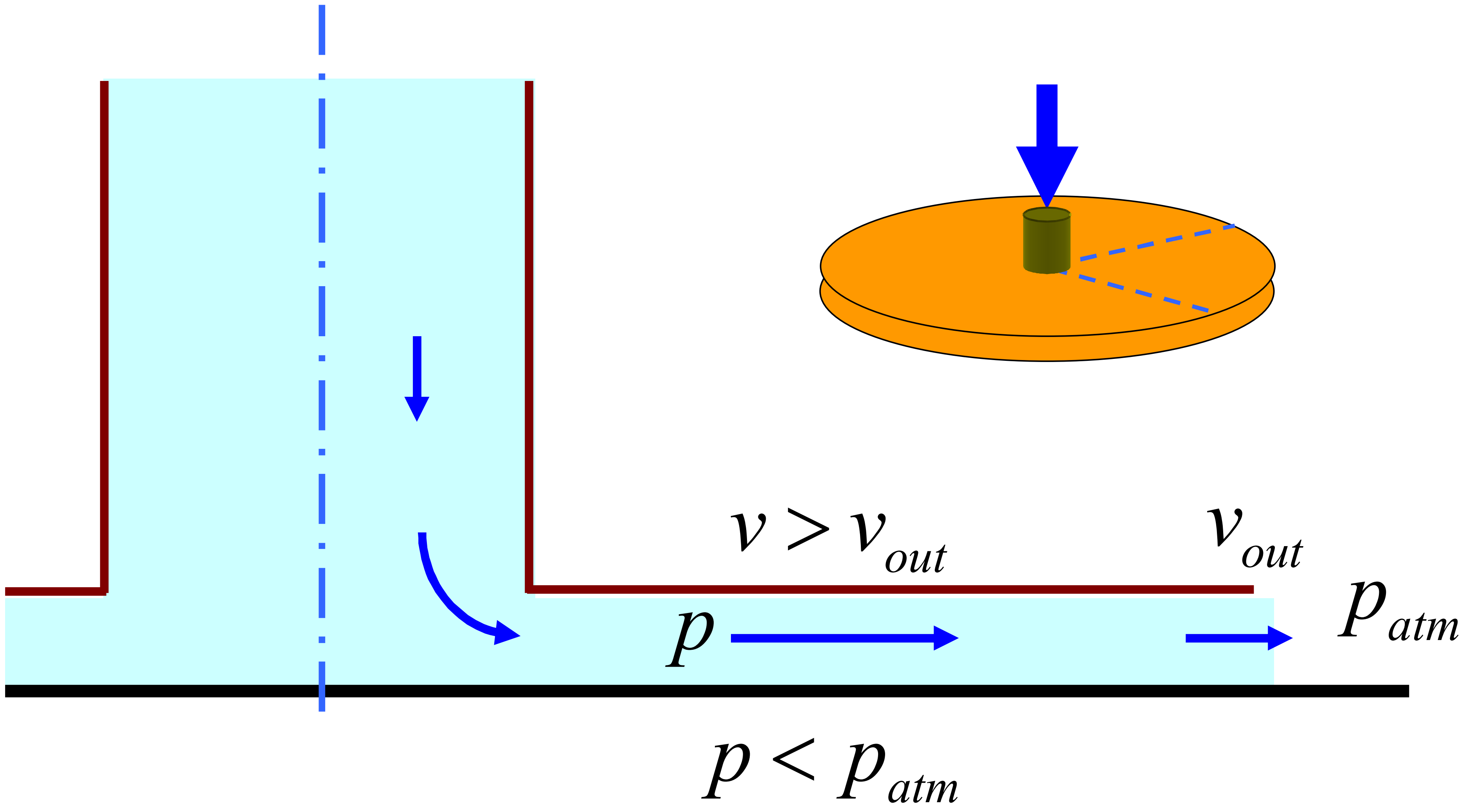


# Floating under the ceiling



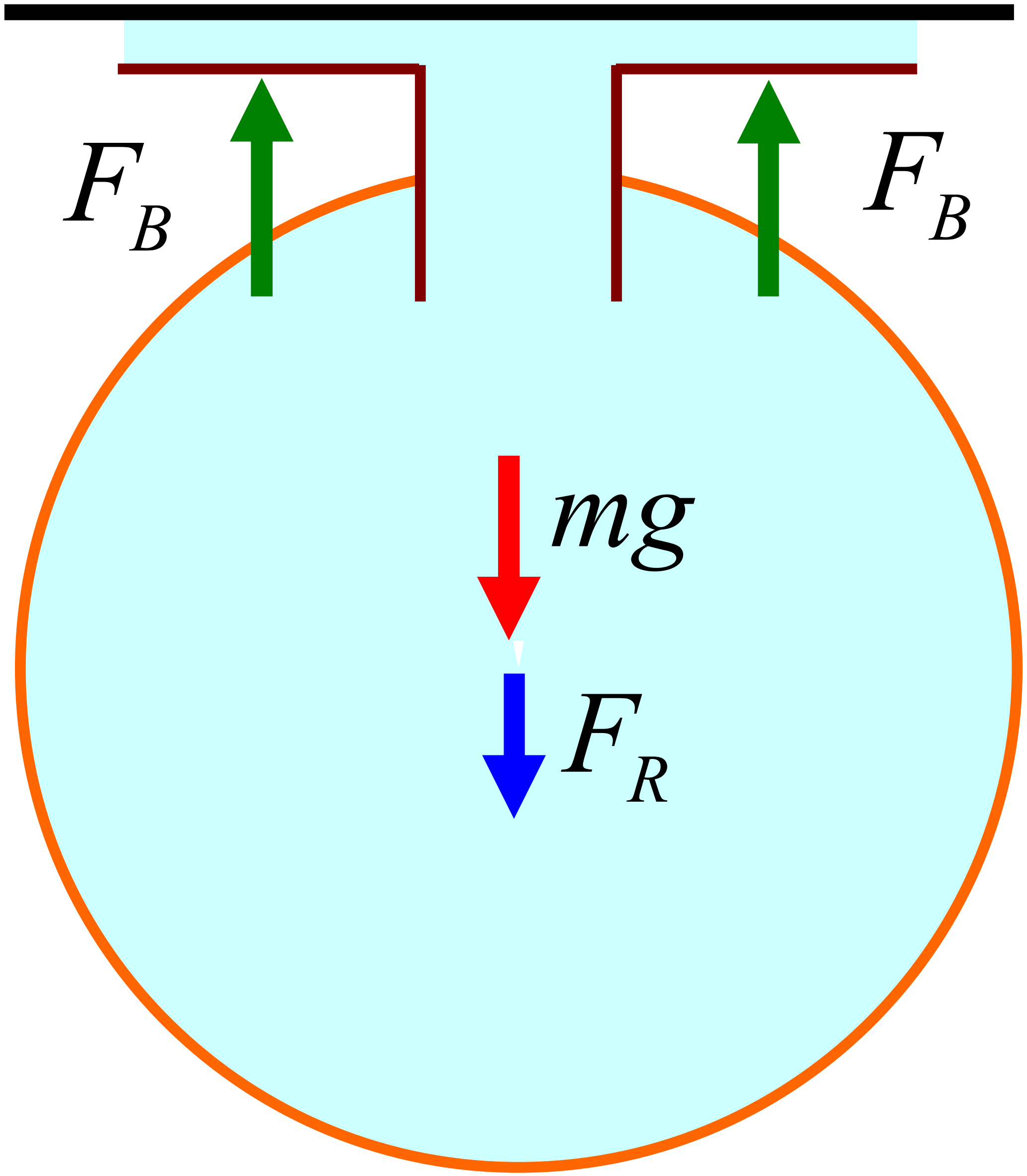
# Bernoulli's principle





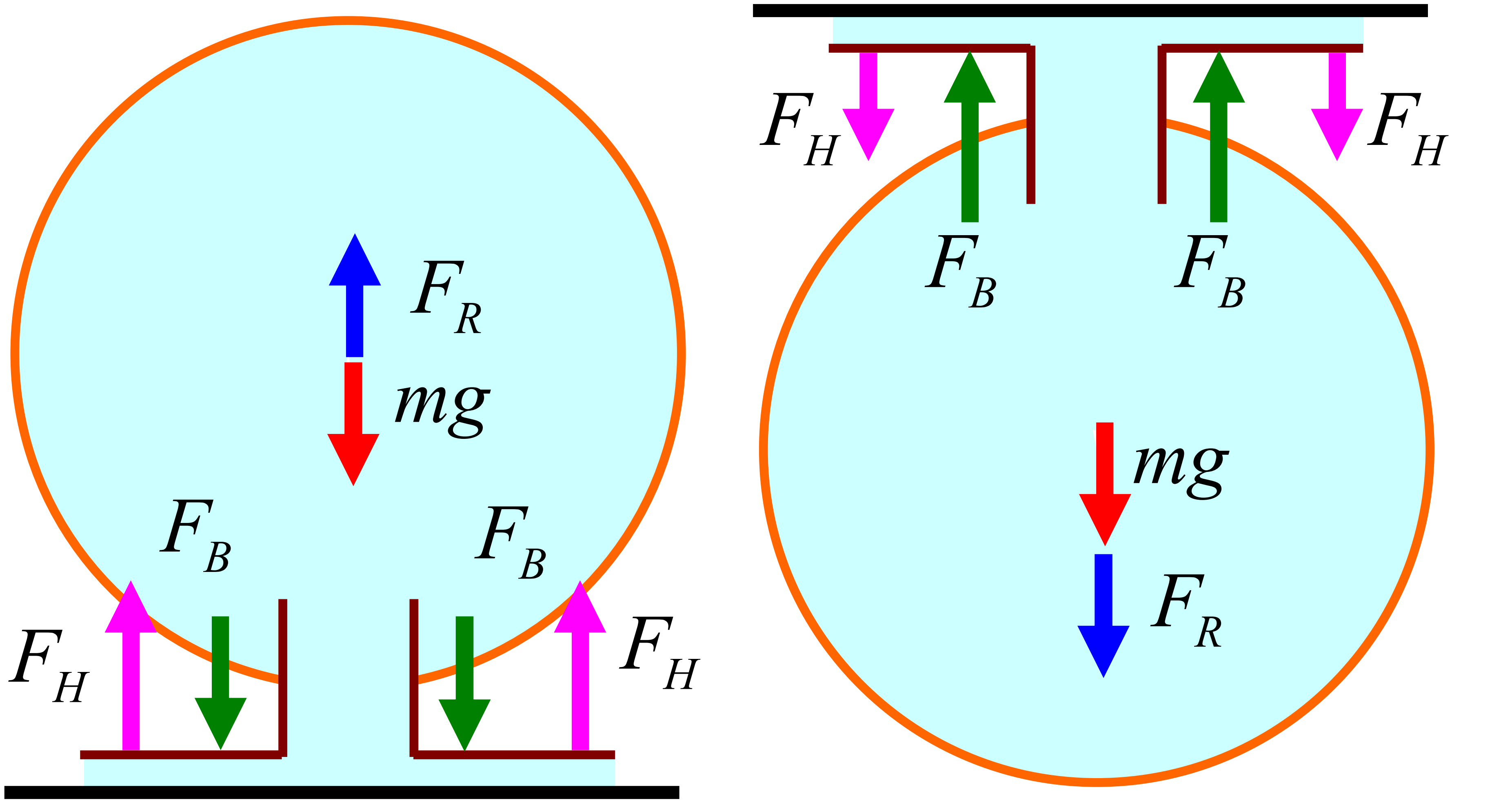
# Force balance under the celling





# Final force balance





# Outline of the report



Pressure under the disk

↓

Hovering experiments

↓

Theoretical model

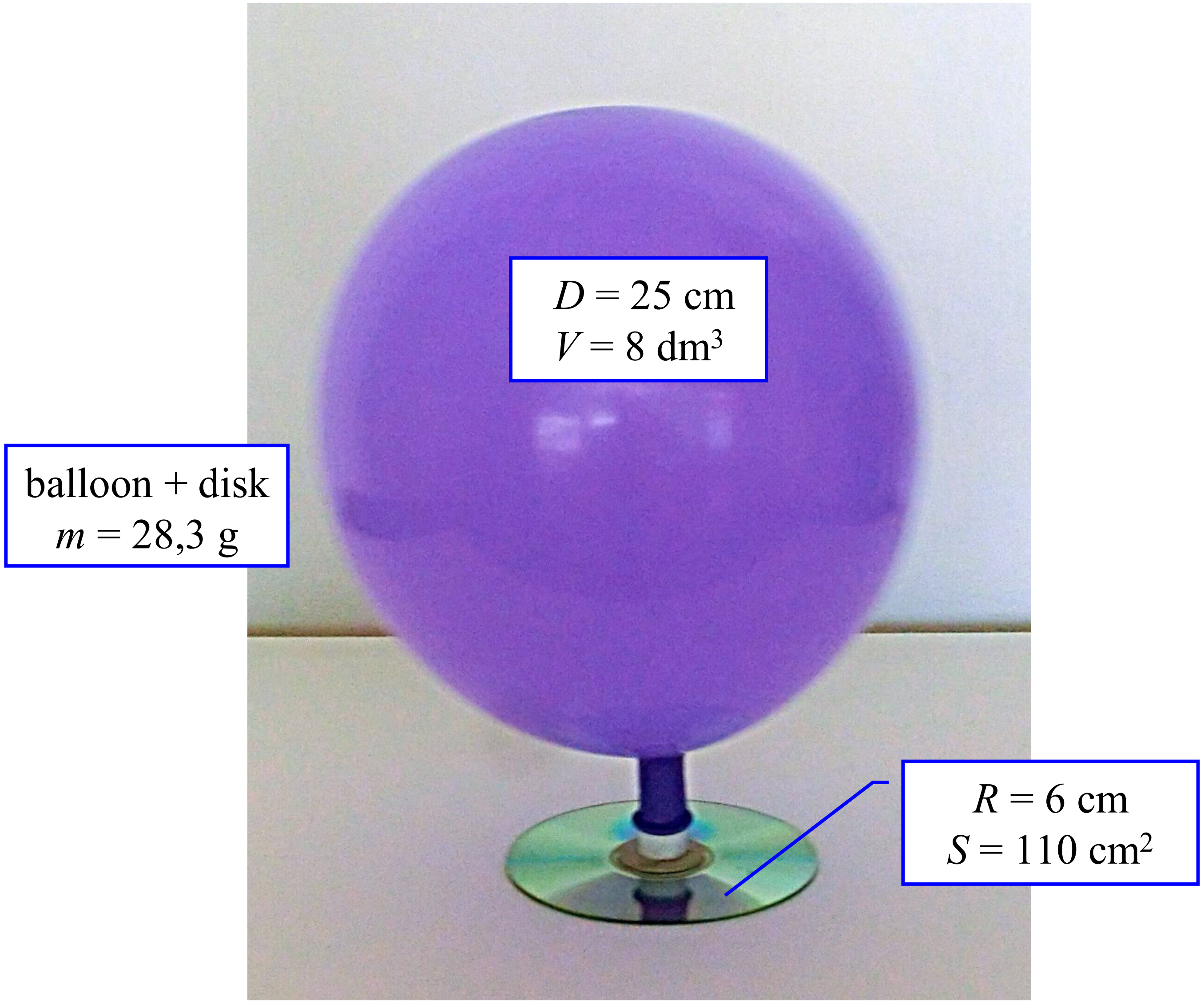
↓

Hovering time vs. craft's weight

…how the relevant parameters influence the time of the 'low-friction' state…

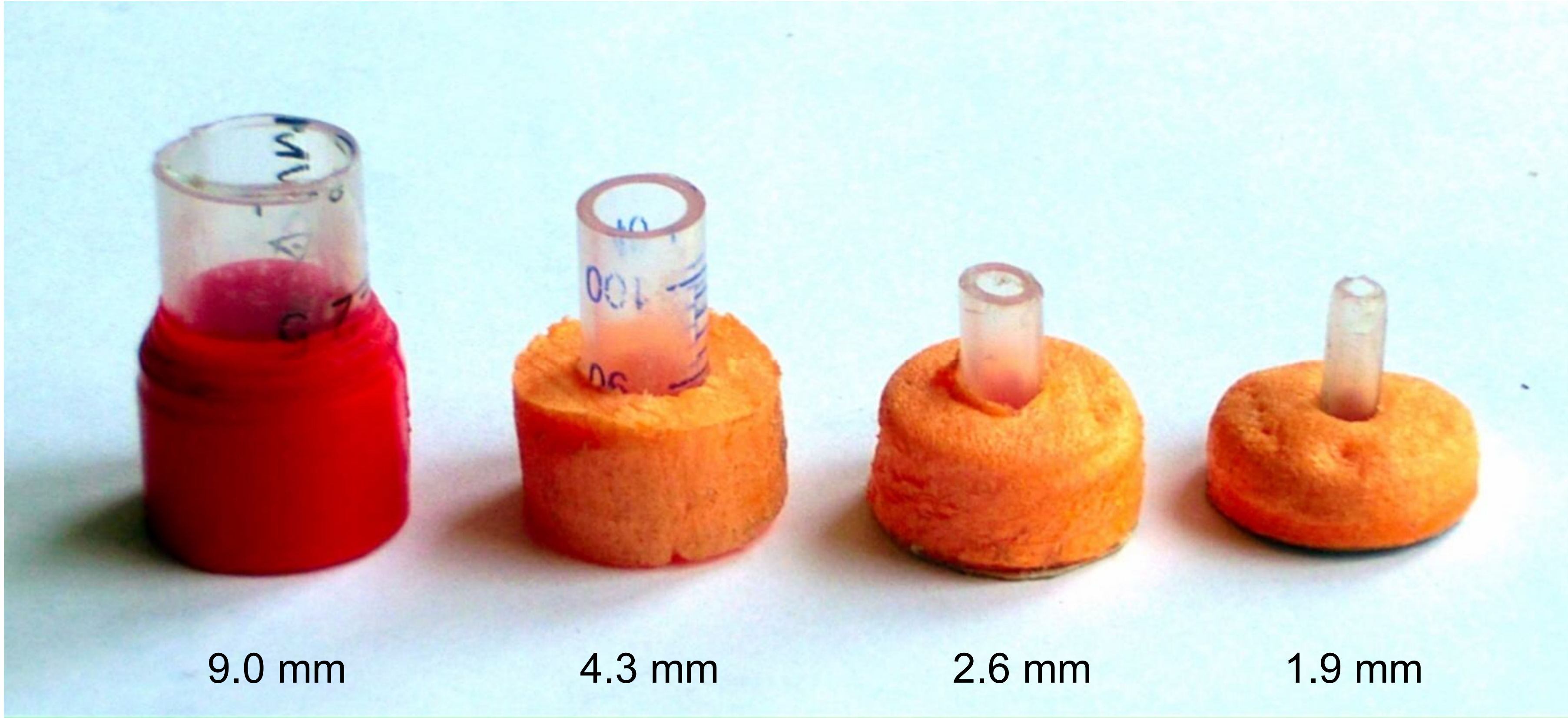
# Air pressure under the disk

# Parameters of the hovercraft

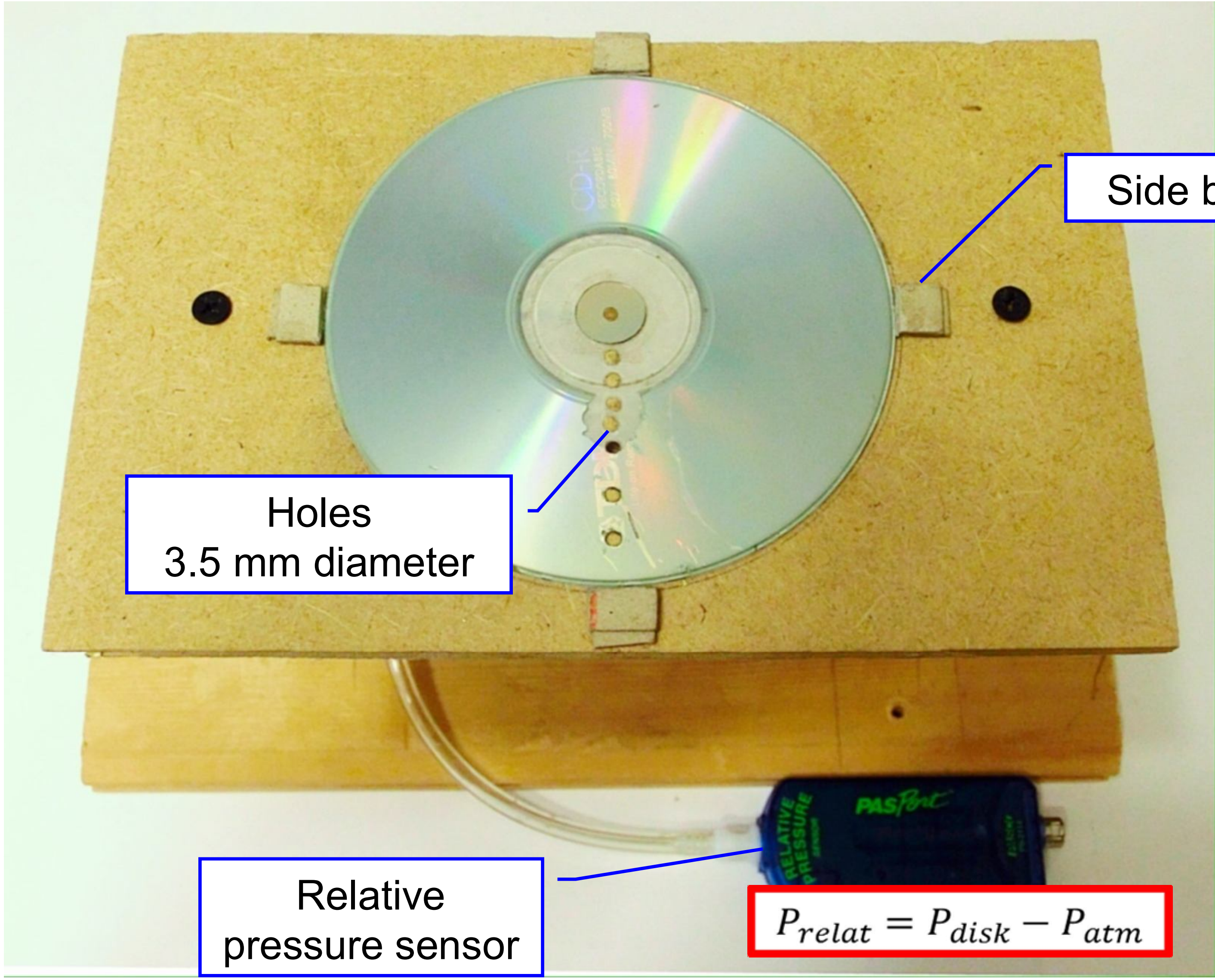


# Plug-in nozzles



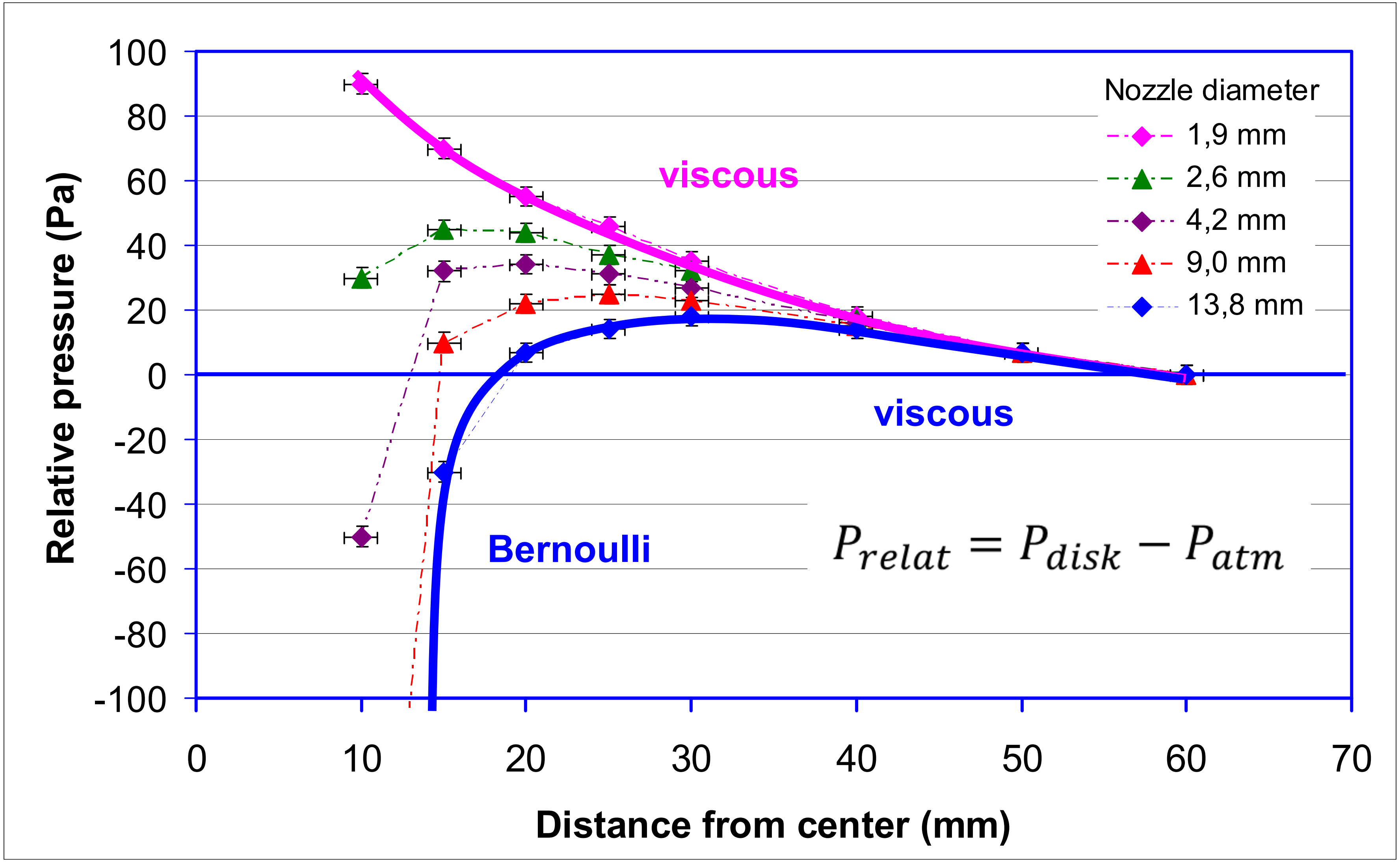


# Setup for pressure measurement 




$$P_{relat} = P_{disk} - P_{atm}$$

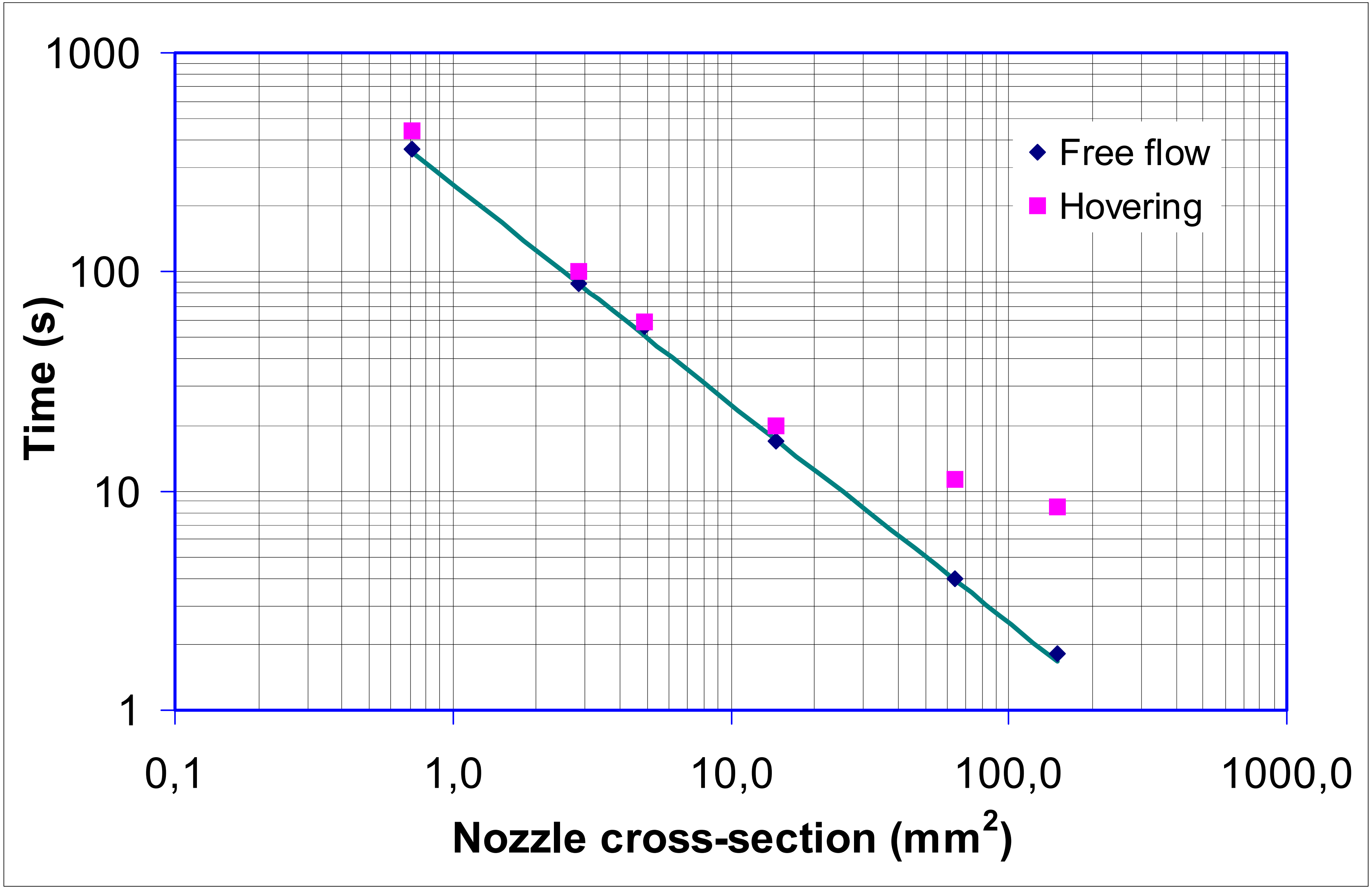
# Pressure distribution under the disk



# Hovering experiments

"…how the relevant parameters influence
the time of the 'low-friction' state…"

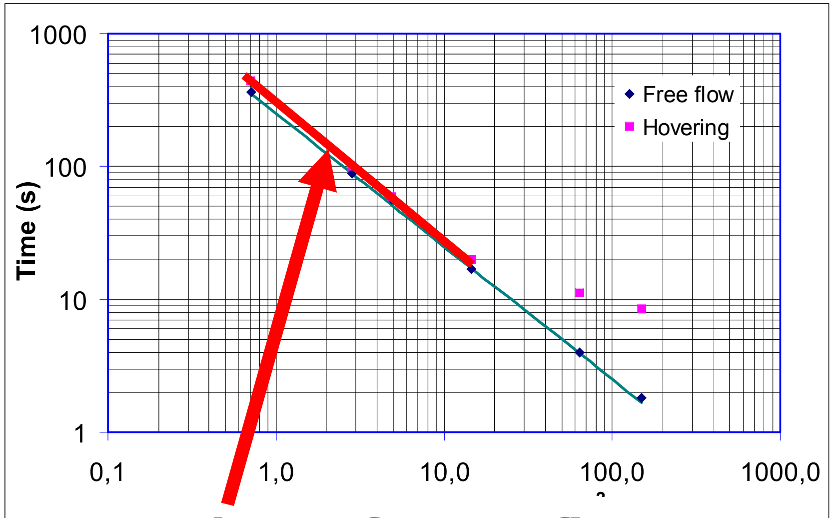
# Time vs. nozzle cross-section



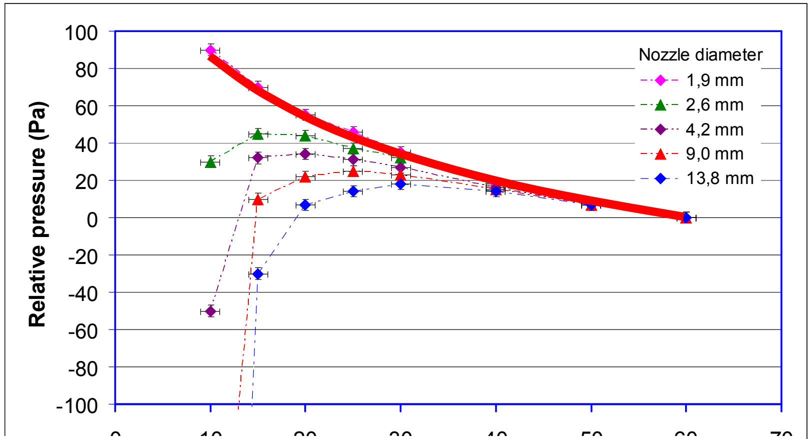
# Narrow nozzles
# Viscosity dominates

# What do we already know?

Small nozzle diameter

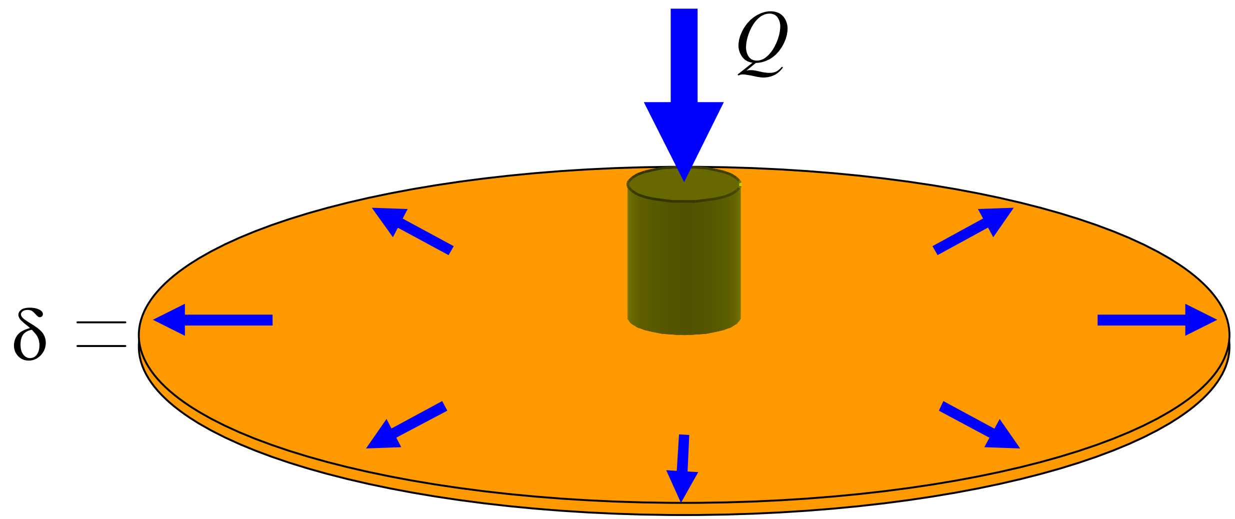



Almost free outflow




Pressure drops along the radius

# Viscous regime: Hele-Shaw cell



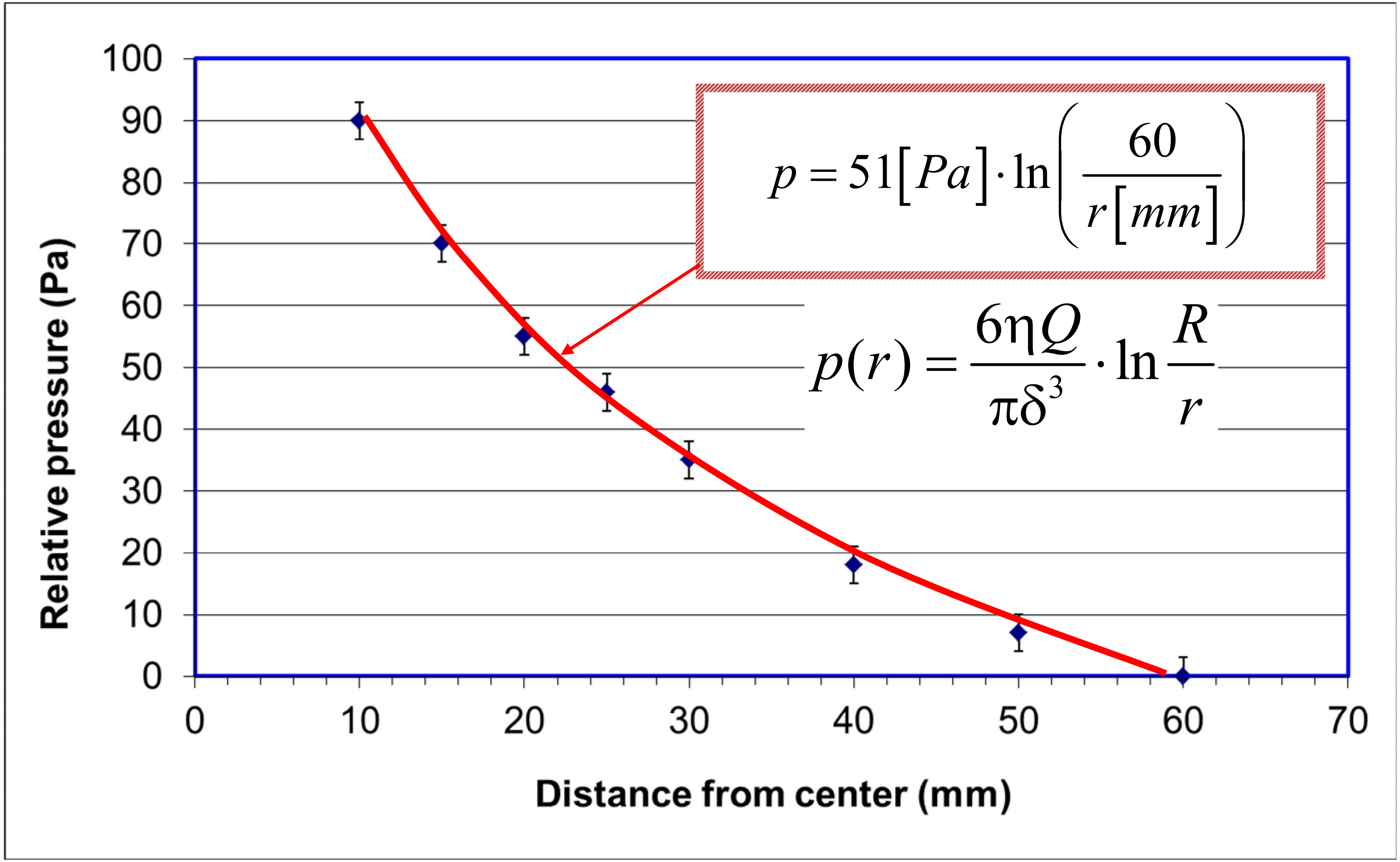

Continuity condition: $$v(r) = \frac{Q}{2\pi r\delta}$$

Darcy's law: $$v(r) = -\frac{\delta^2}{12\eta}\frac{dp}{dr}$$

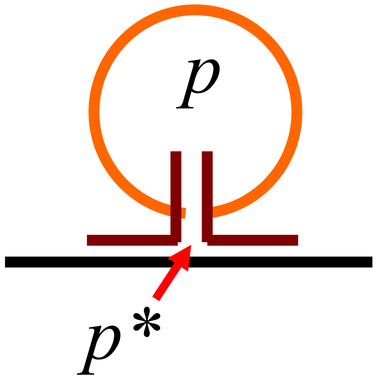
$$\left.\begin{array}{l} v(r) = \dfrac{Q}{2\pi r\delta} \\ v(r) = -\dfrac{\delta^2}{12\eta}\dfrac{dp}{dr} \end{array}\right\} \quad p(r) = \frac{6\eta Q}{\pi\delta^3}\cdot\ln\frac{R}{r}$$

# Narrow nozzle (d = 1.9 mm)





# Hovering time with narrow nozzle




Flow under the disk: $$p^* = \frac{mg}{\pi R^2} \cdot 2\ln\frac{R}{a}$$

Flow through the nozzle: $$p^* = p - \rho u^2$$

Velocity in the nozzle:

$$u^2 = \frac{p}{\rho} \cdot \left\{ 1 - \frac{mg}{p \cdot \pi R^2} \cdot 2\ln\left(\frac{R}{a}\right) \right\}$$

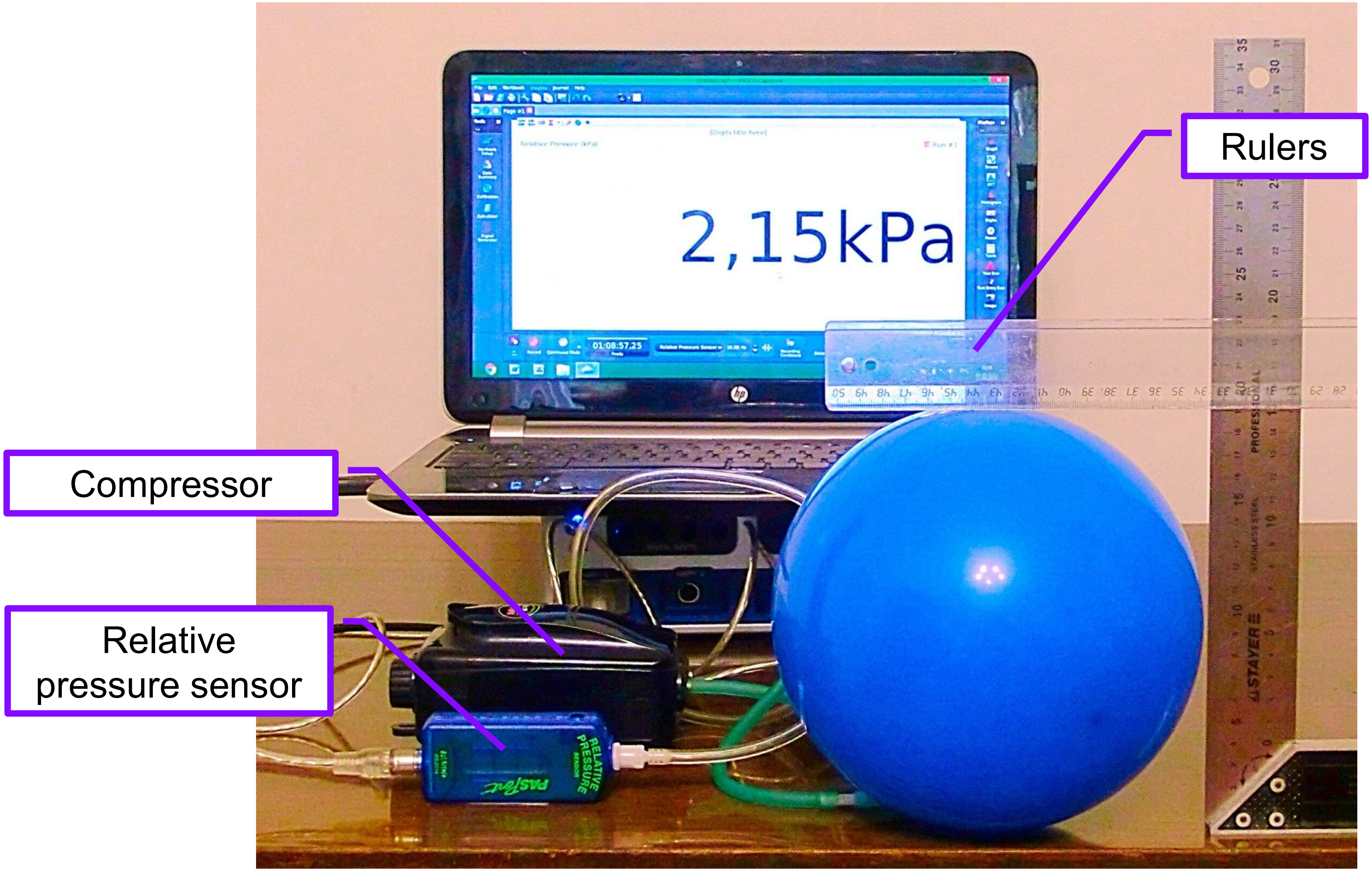
# Hovering time with narrow nozzle



$$\tau = \underbrace{\boxed{\frac{V}{S}\sqrt{\frac{\rho}{p}}}}_{\tau_0} \cdot \left(1 - \frac{mg}{\boxed{p} \cdot \pi R^2} \cdot 2\ln\frac{R}{a}\right)^{-1/2}$$

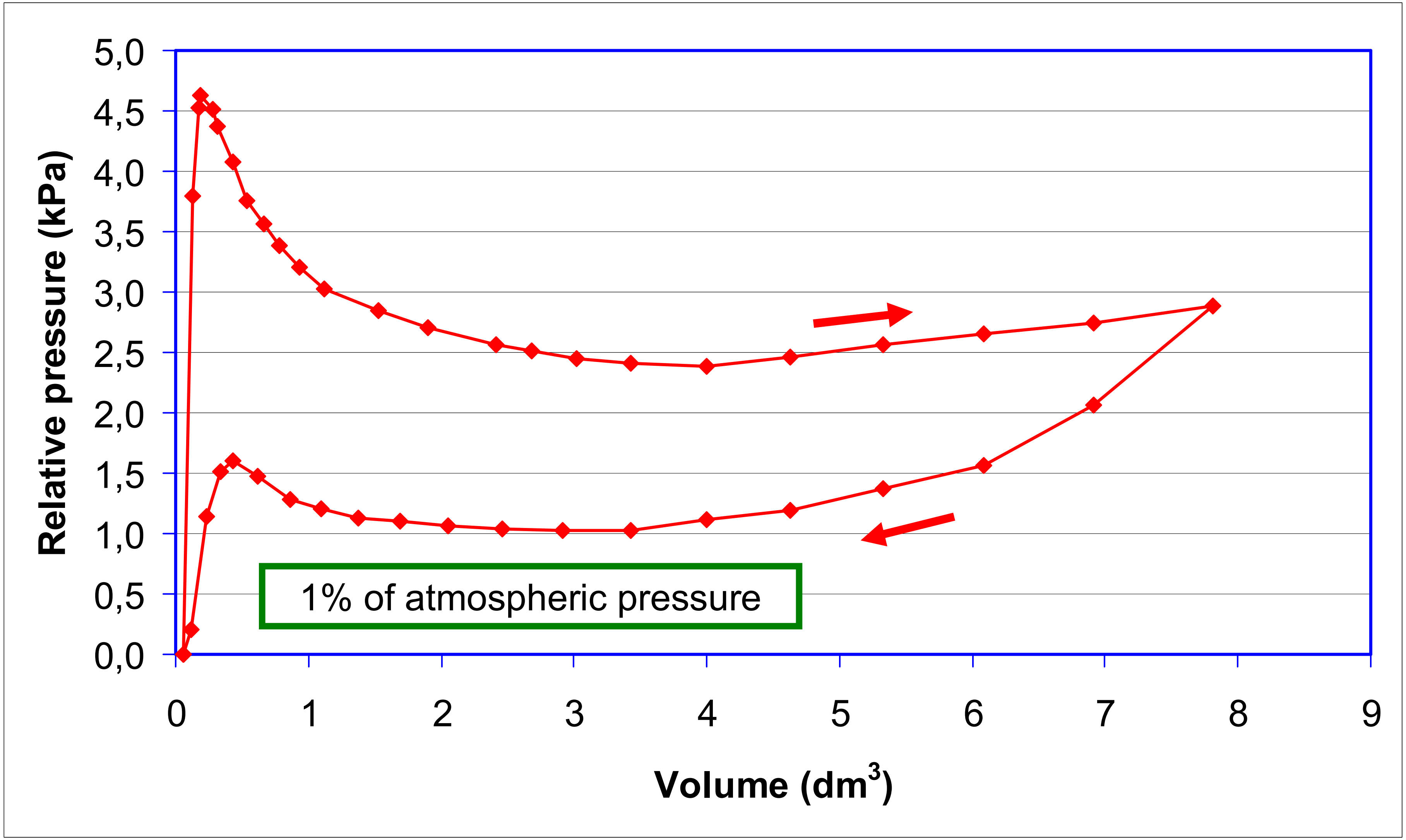
We should know it!

# Measurement of relative pressure



# Pressure vs. volume





# Hovering time with narrow nozzle



$$\tau = \tau_0 \cdot \left( 1 - \frac{mg}{p \cdot \pi R^2} \cdot 2 \ln \frac{R}{a} \right)^{-1/2}$$

For our parameters of the balloon and the disk

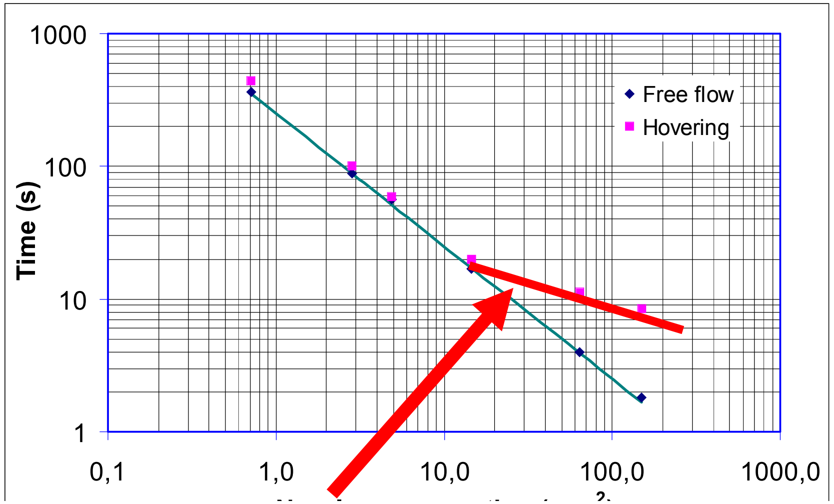
$$\tau \approx \tau_0 \left( 1 + 0{,}02 \cdot \ln \frac{R}{a} \right)$$

If R/a = 60, $\tau = 1{,}08 \cdot \tau_0$

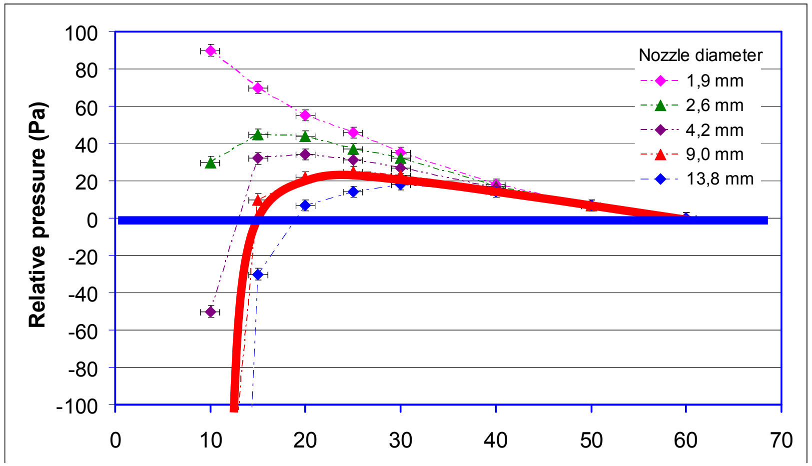
# Wide nozzles
# Viscosity + Bernoulli

# What do we already know?


**Big nozzle diameters**

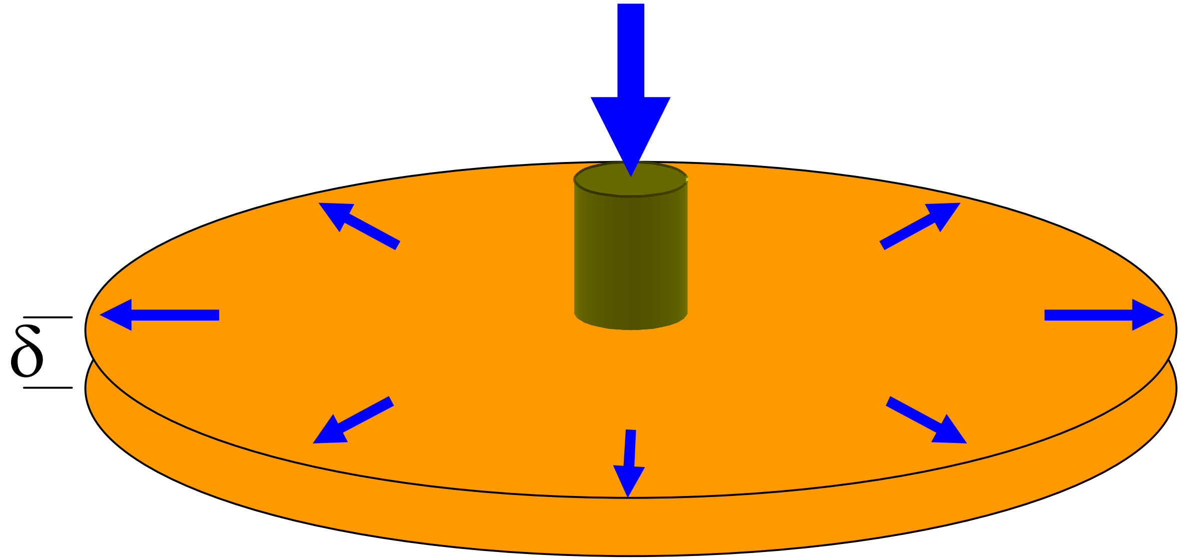



**Outflow differs from the free outflow**




**Pressure under the central area is less than atmospheric**

# Wide gap: Bernoulli's regime




Relative Pressure is negative

Continuity condition: $v(r) = \dfrac{Q}{2\pi r\delta}$

Bernoulli's principle: $p + \dfrac{\rho v^2}{2} = \mathrm{const}$

$$p(r) = \frac{1}{4\pi^2} \cdot \frac{\rho Q^2}{\delta^2}\left(\frac{1}{R^2} - \frac{1}{r^2}\right)$$

# Combined regime

$$p(r) = \underbrace{\frac{6\eta Q}{\pi\delta^3} \cdot \ln\frac{R}{r}}_{\textbf{Viscous}} + \underbrace{\frac{1}{4\pi^2} \cdot \frac{\rho Q^2}{\delta^2}\left(\frac{1}{R^2} - \frac{1}{r^2}\right)}_{\textbf{Bernoulli}}$$

# Parabolic velocity profile

$$p(r) = \underbrace{\frac{6\eta Q}{\pi\delta^3}\cdot\ln\frac{R}{r}}_{\text{viscous}} + \underbrace{\frac{27}{140\pi^2}\cdot\frac{\rho Q^2}{\delta^2}\left(\frac{1}{R^2}-\frac{1}{r^2}\right)}_{\text{Bernoulli with a parabolic profile}}$$

Armengoll J., Calbó J., Pujol T., Roura P. (2011) "Bernoulli correction to viscous losses: Radial flow between two parallel discs".

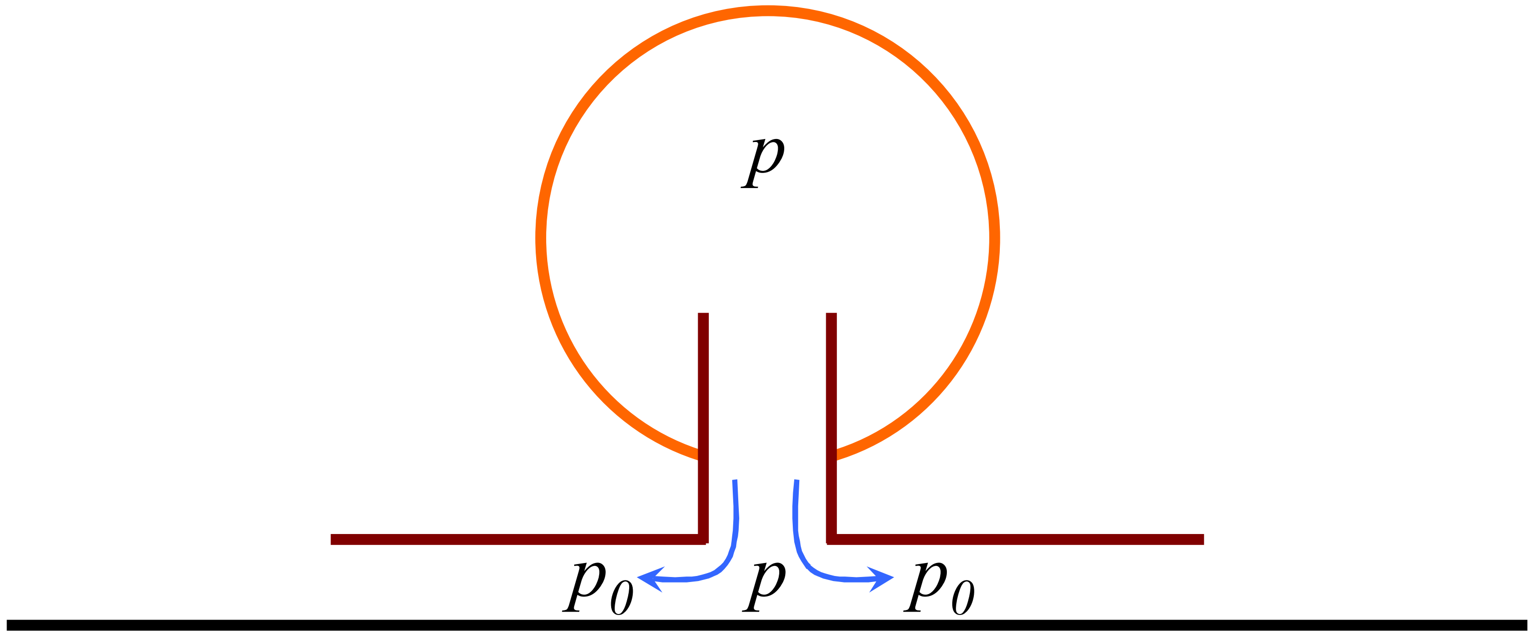
# Force balance

$$mg - F_R \approx \underbrace{\frac{3\eta Q R^2}{\delta^3}}_{\text{Viscous}} - \underbrace{\frac{27\rho Q^2}{70\pi\delta^2}\ln\left(\frac{R}{a}\right)}_{\text{Bernoulli}}$$

**Both terms are large compared with the weight, so they are approximately equal to each other**

$$\boxed{Q\cdot\delta \approx \frac{70}{9}\cdot\frac{\eta\cdot\pi R^2}{\rho\ln\left(R/a\right)}}$$

# Outflow from the nozzle under the disk




**As outflow from the nozzle into the atmosphere**

$$\boxed{Q = \underbrace{2\pi a \delta}_{\text{Cylindrical entry}} \cdot v_0} \qquad v_0 = \sqrt{\frac{p}{\rho}}$$

# Hovering time with wide nozzle

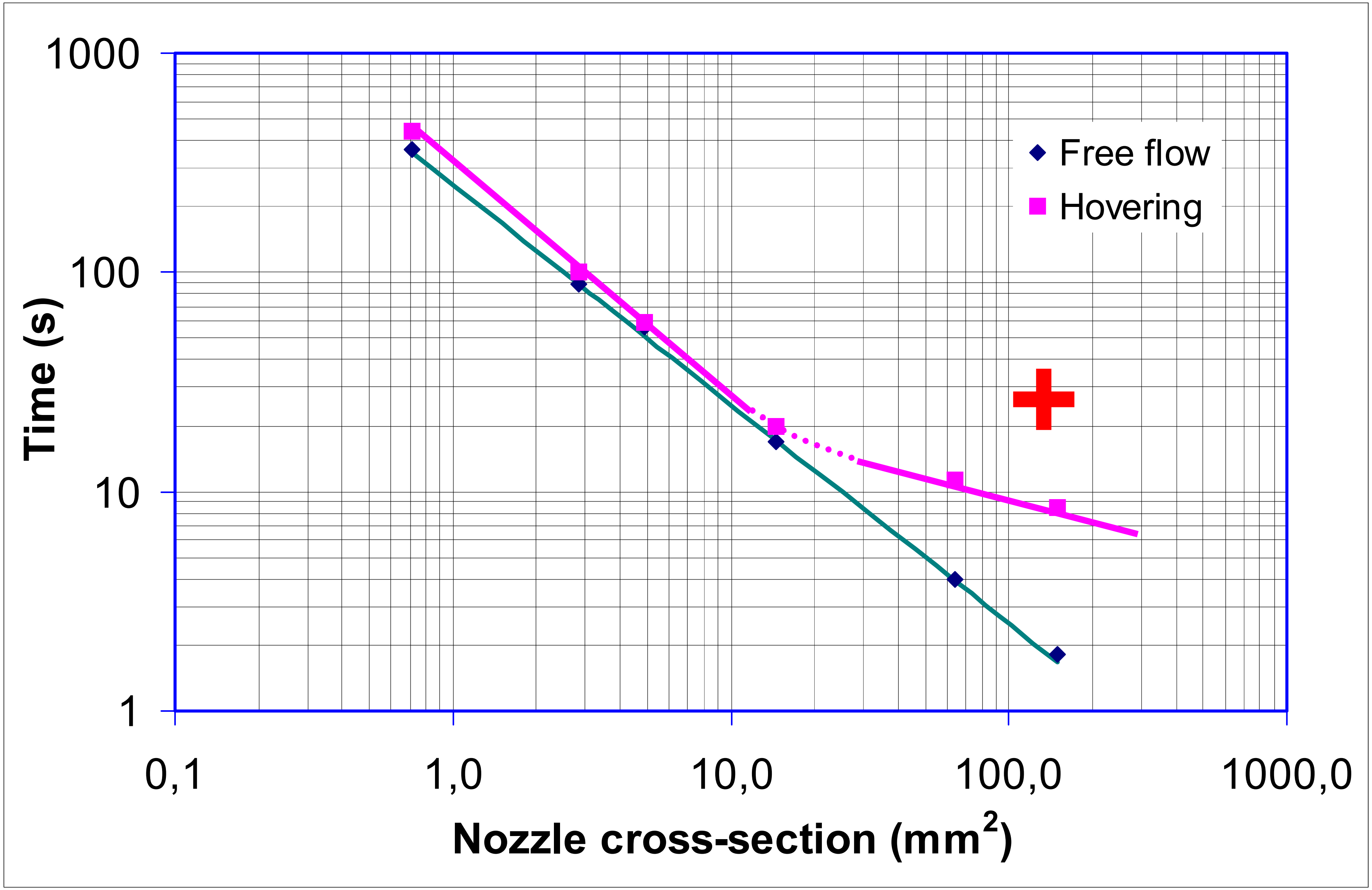


$$Q \cdot \delta \approx \frac{70}{9} \cdot \frac{\eta \cdot \pi R^2}{\rho \ln\left(R / a\right)}$$

$$Q = 2\pi a \delta \cdot v_0$$

$$\boxed{\tau = V \cdot \sqrt{\frac{9}{70} \cdot \frac{\rho \ln\left(R / a\right)}{2\pi a \cdot \pi R^2 \cdot \eta v_0}}}$$

# Theory-experiment comparison



# Hovering time vs. weight

# The hovering time



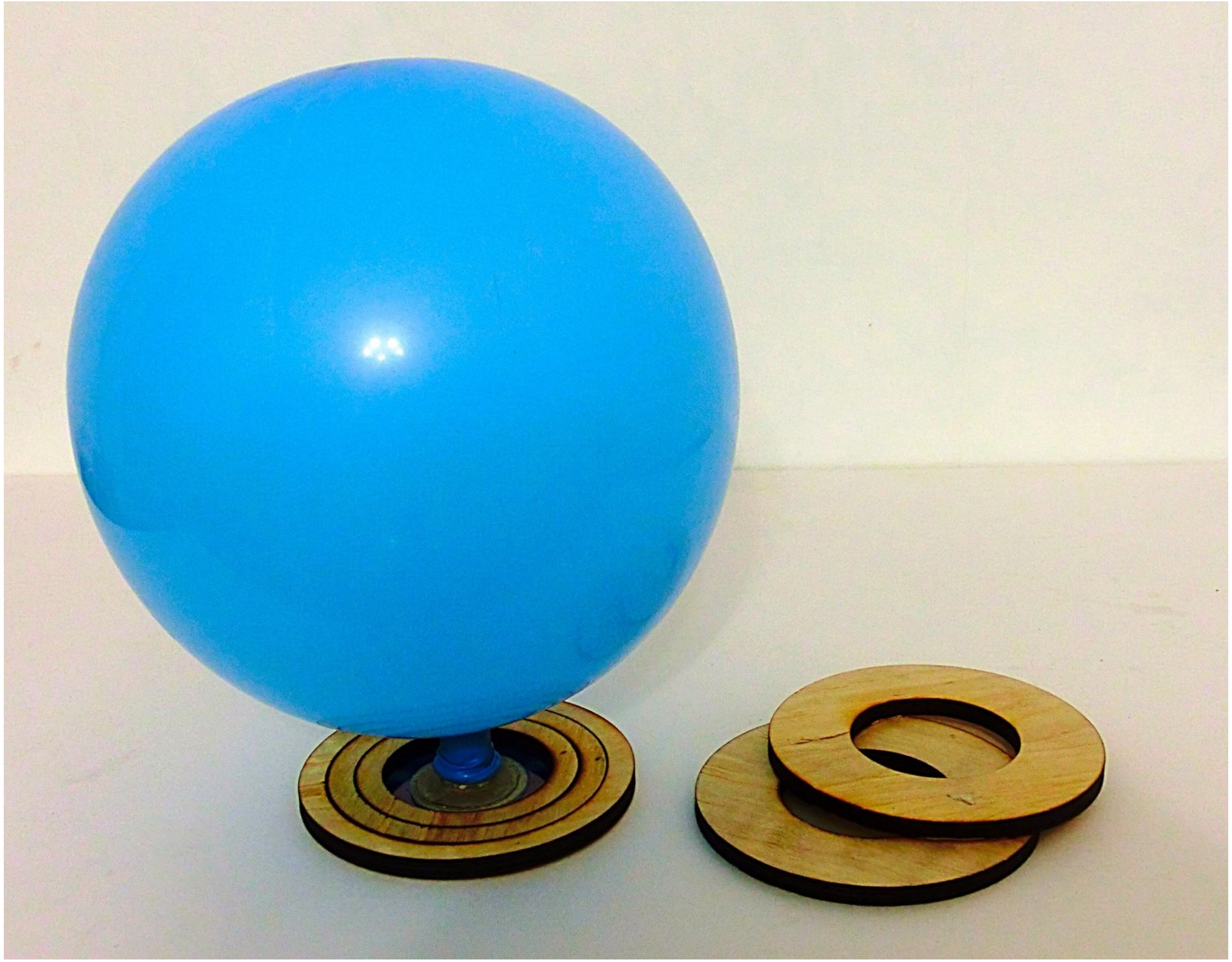
$$\tau = \frac{\tau_0}{\sqrt{1 - \frac{mg}{p \cdot \pi R^2} \cdot 2\ln\frac{R}{a}}}$$

$$\tau = V \cdot \sqrt{\frac{9}{70} \cdot \frac{\rho \ln\left(R/a\right)}{2\pi a \cdot \pi R^2 \cdot \eta v_0}}$$

The hovering time almost does not depend on the weight of the vessel until this weight is not very large.

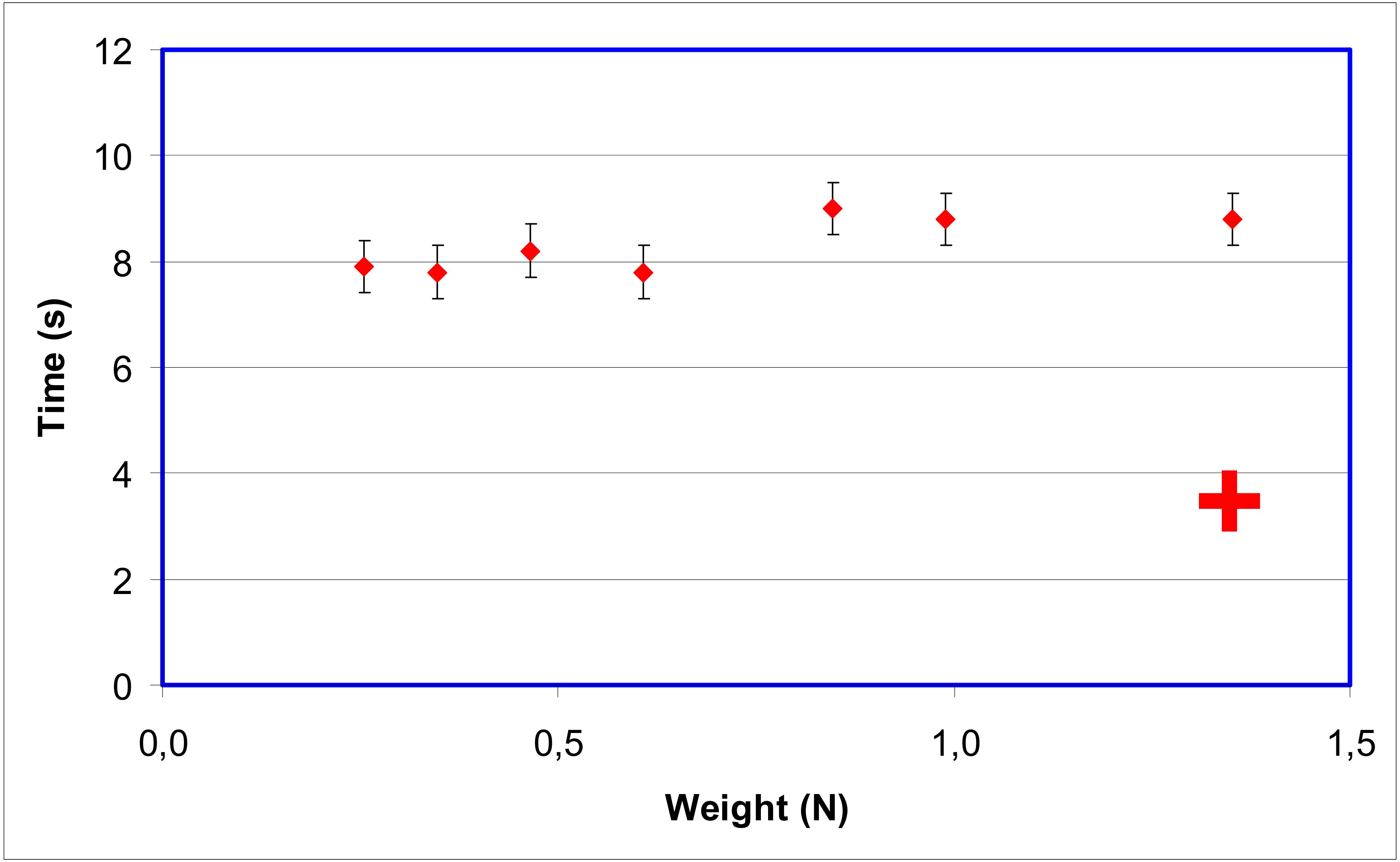
# Experiment





# Time vs. weight (nozzle 13.8 mm)

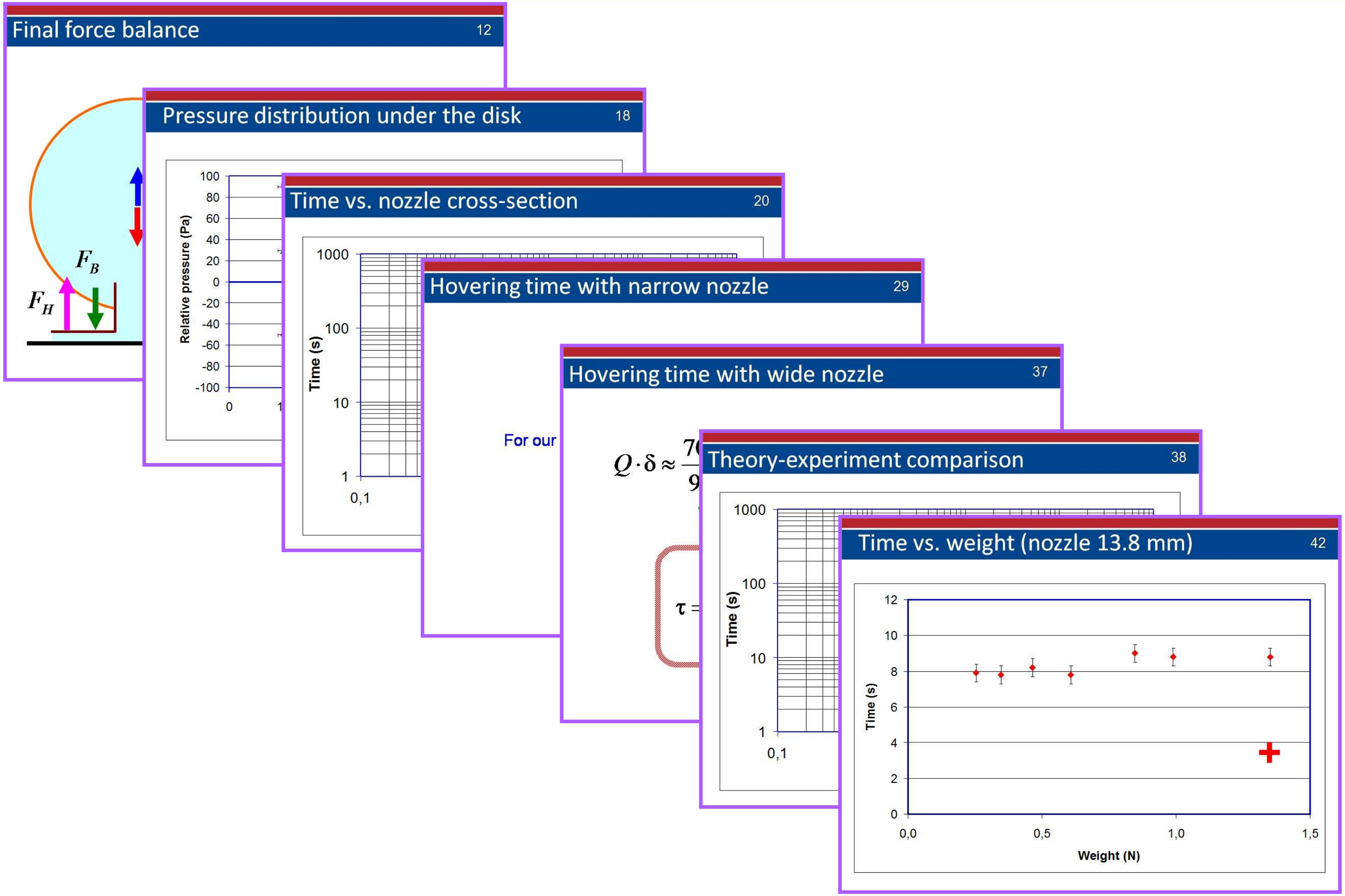




# Summary

# Conclusions





# References




- Jackson J.D., Symmons G.R. (1965) “An investigation of a laminar flow between two parallel disks”. *Appl. Sci. Res.* **15**, 59–75.
- Armengoll J., Calbó J., Pujol T., Roura P. (2011) “Bernoulli correction to viscous losses: Radial flow between two parallel discs”. *Am. J. Phys.* **76**, 730–737.
- Izarra Ch., Izarra G. (2014) “Stokes equation in a toy CD hovercraft”. *Eur. J. Phys*. **32**, 89–99.

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# Thank you for your attention!