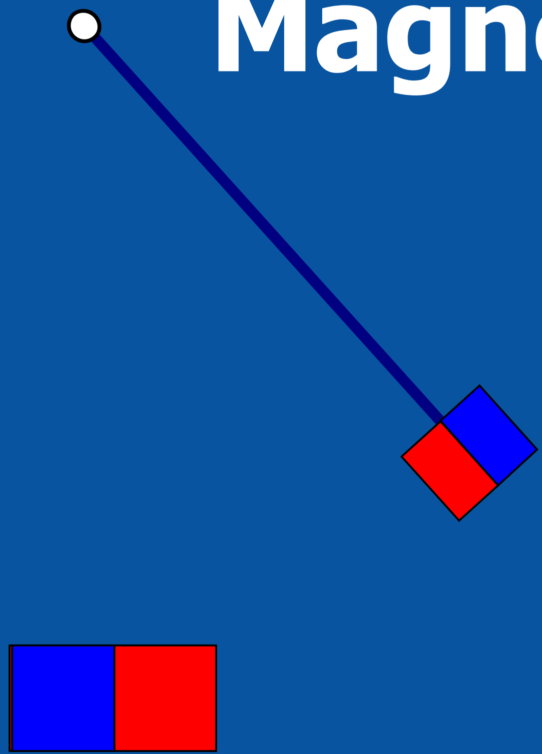
Russia IYPT

Magnetic pendulum



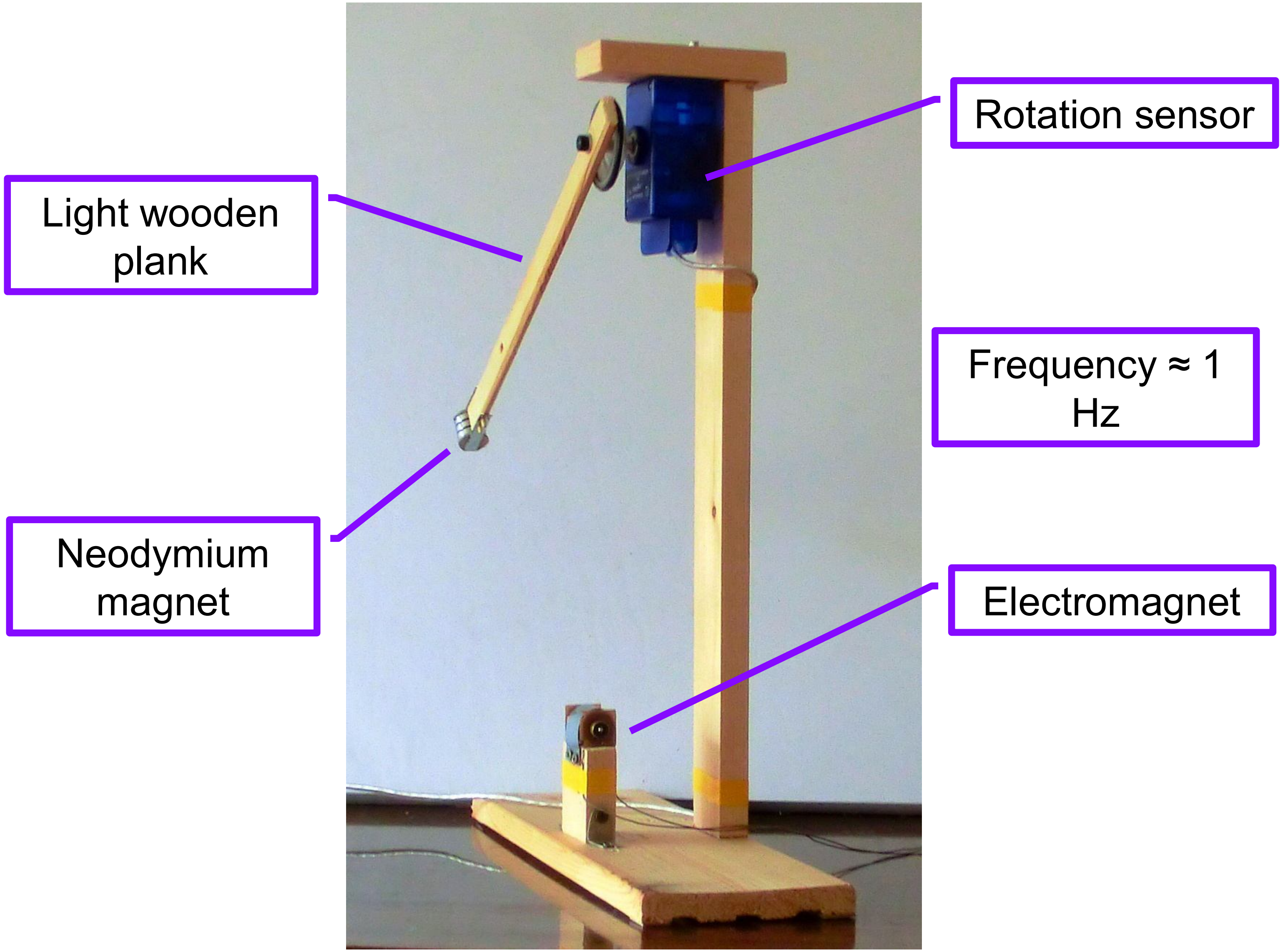
Vitaly Matiunin

The problem

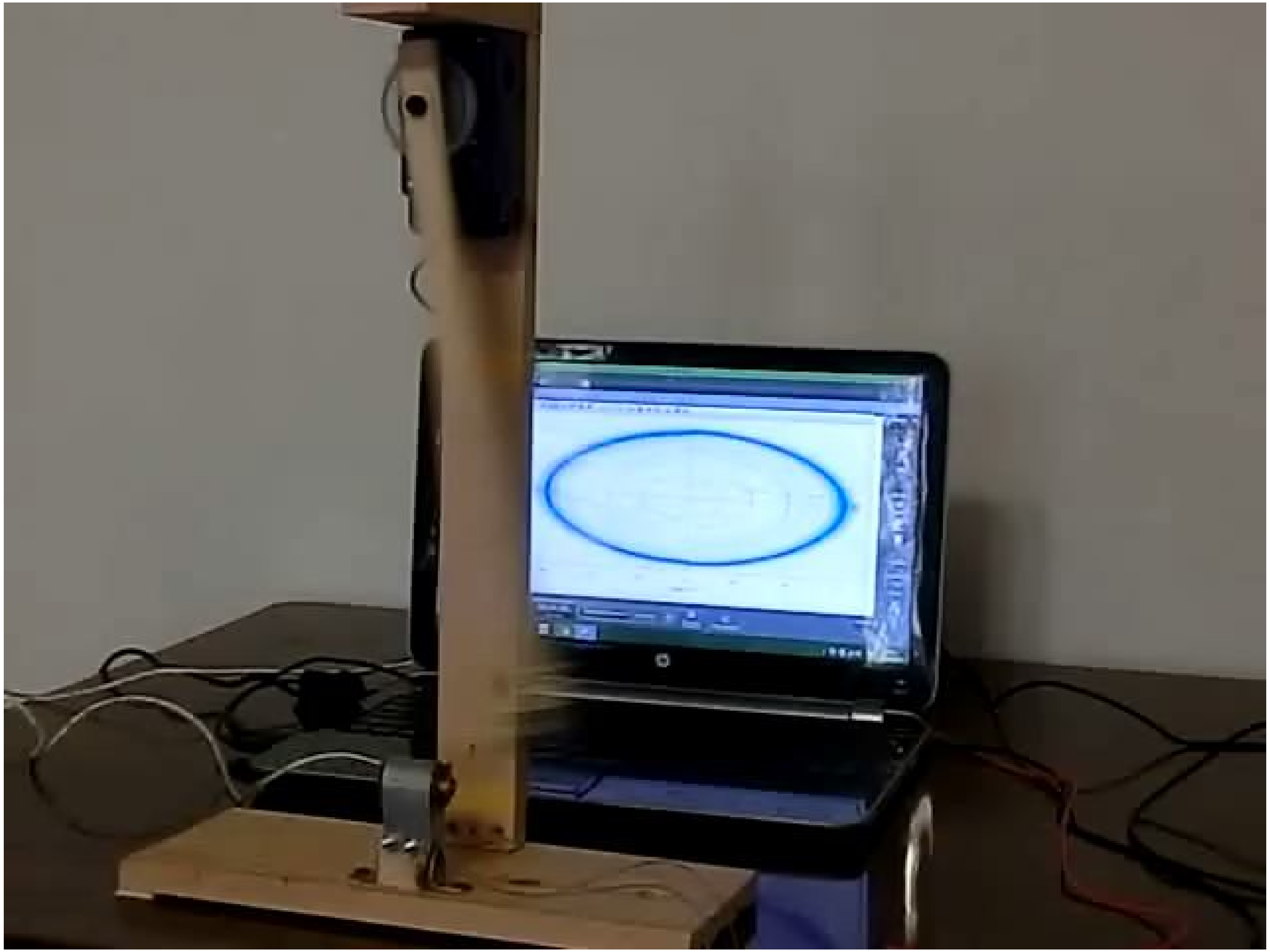
Make a light pendulum with a small magnet at the free end. An adjacent electromagnet connected to an AC power source of a much higher frequency than the natural frequency of the pendulum can lead to undamped oscillations with various amplitudes. Study and explain the phenomenon.

First observations

Experimental setup



Undamped oscillations



Plan of the report

Magnets interactions

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Conditions for undamped oscillations

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Experimental detection of undamped oscillations

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Orders of resonance

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Cycles of undamped oscillations

Main goal of our investigation

Prediction of undamped oscillations

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Computer simulation

↓

Theoretical model

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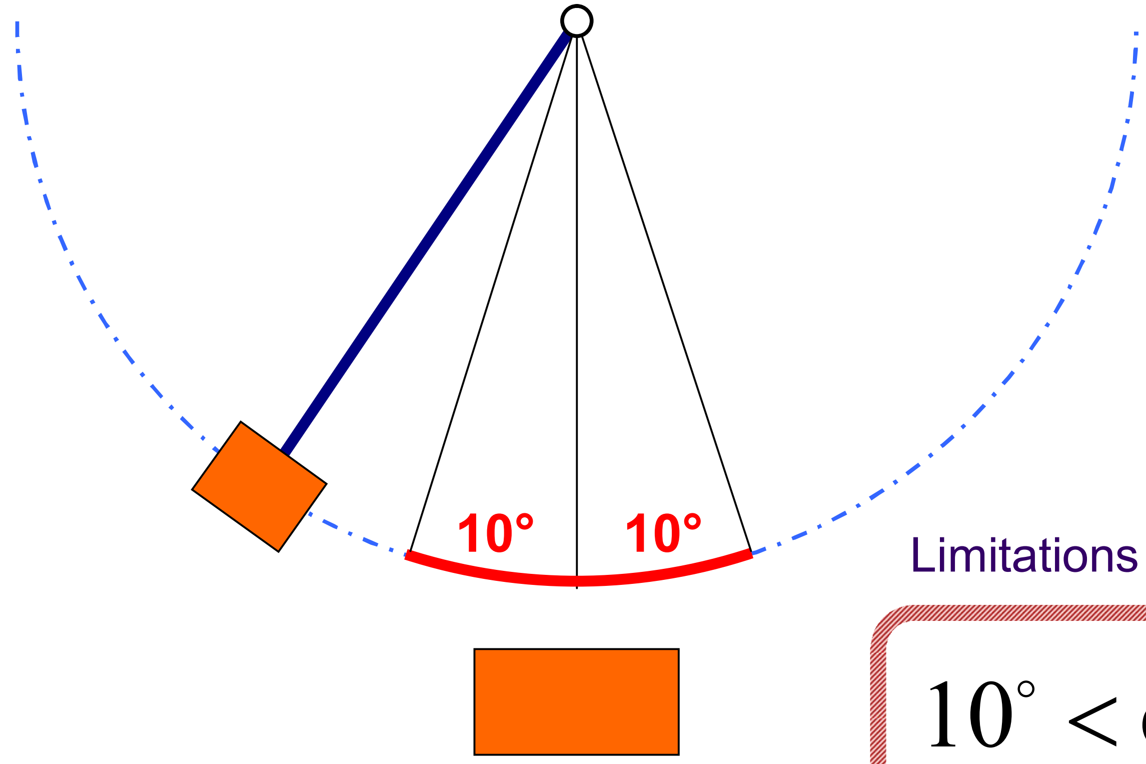
Poincare map

Qualitative explanation

Region of magnets' interaction

A magnet on the pendulum interacts with
the electromagnet only flying over it.
The span happens with almost constant velocity.



Limitations of amplitudes

$$10^{\circ} < \alpha < 180^{\circ}$$

Characteristic times and frequencies

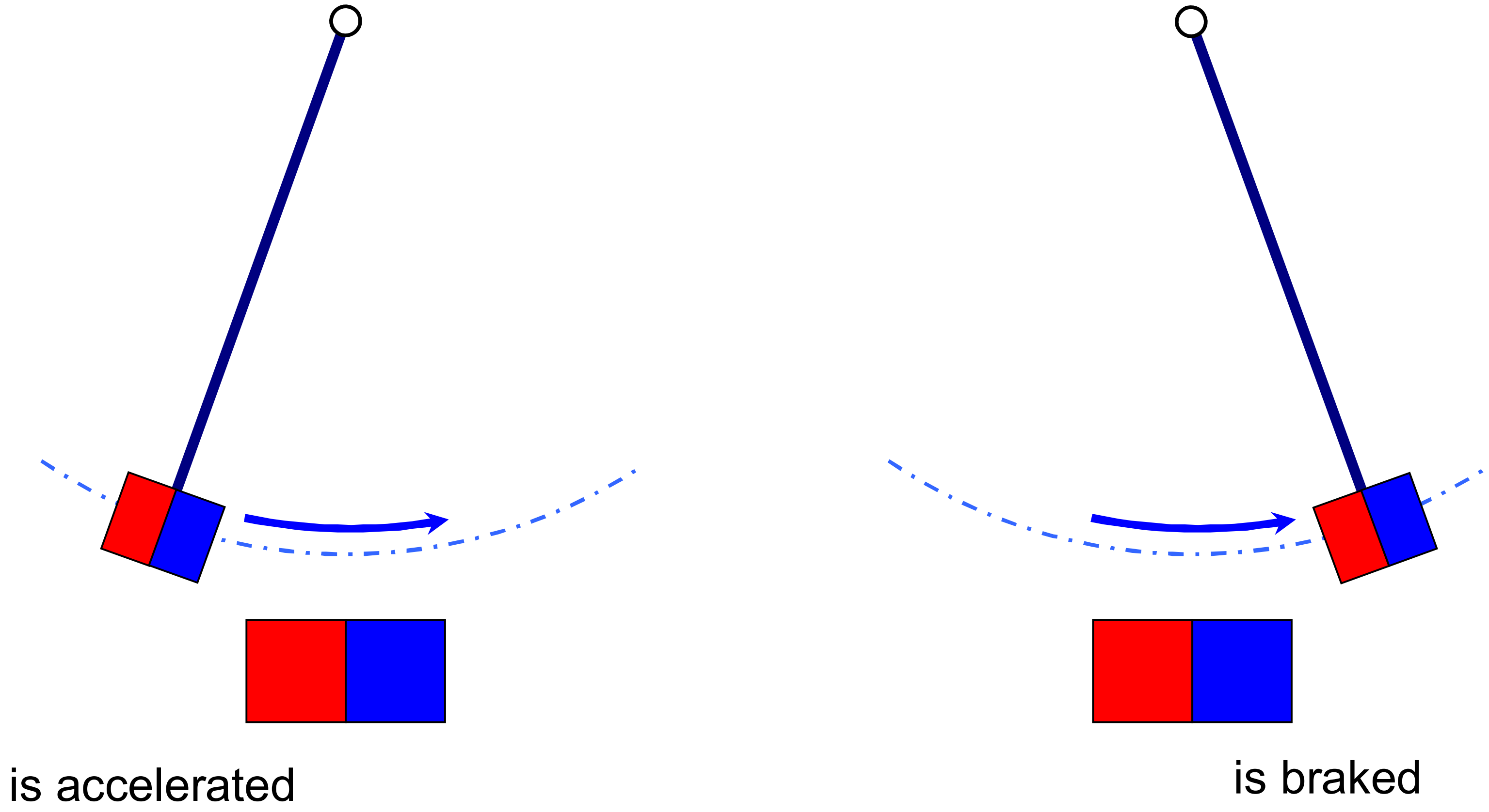
$$v = \sqrt{2gl} \approx 2.5 \text{ m/s}$$

$$\tau = \frac{2d}{v} \approx 0.04 \text{ s}$$

$$f = \frac{1}{\tau} \approx 25 \text{ Hz}$$

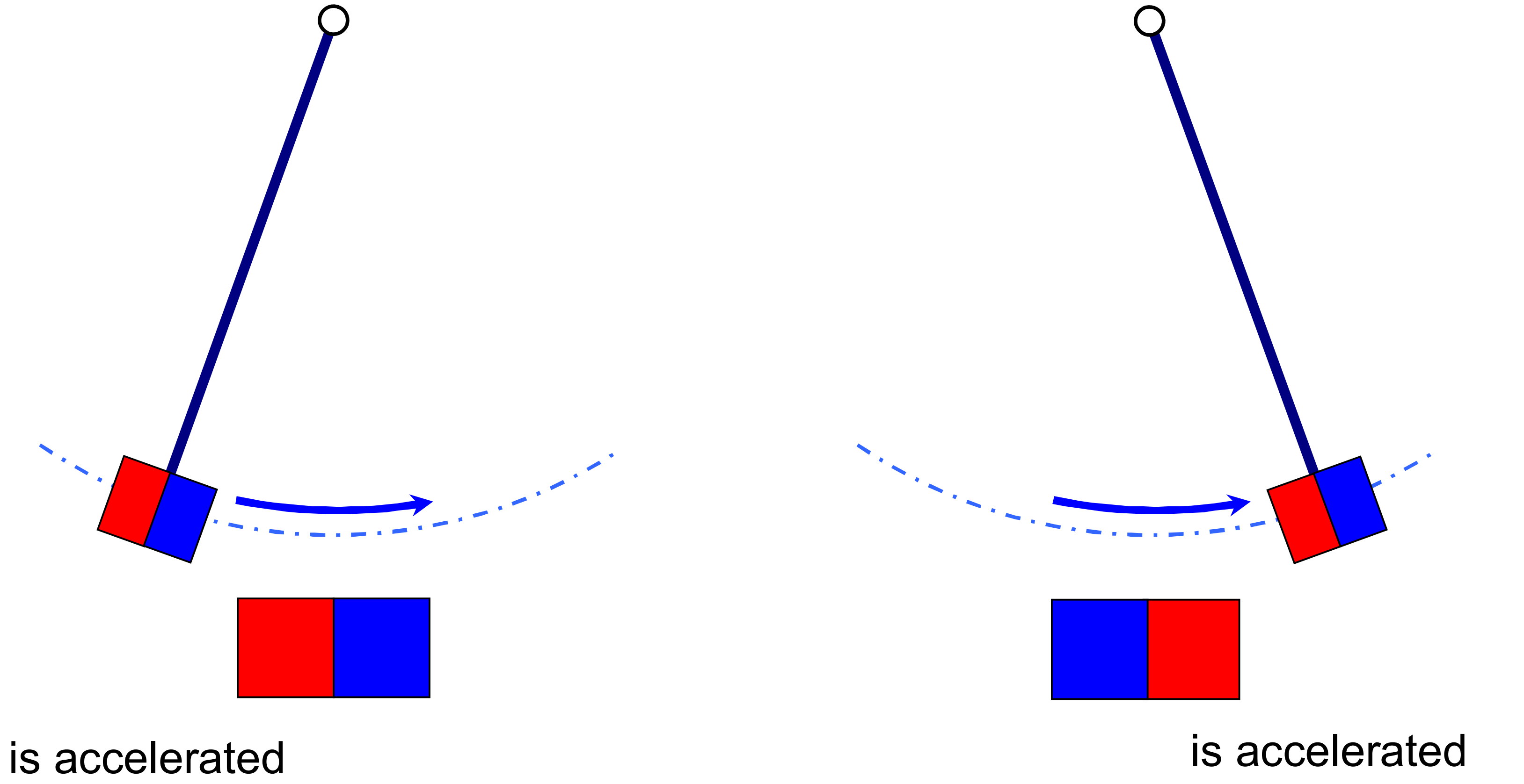
DC solenoid

If the current in solenoid is direct, the change in energy of the pendulum is equal to zero.



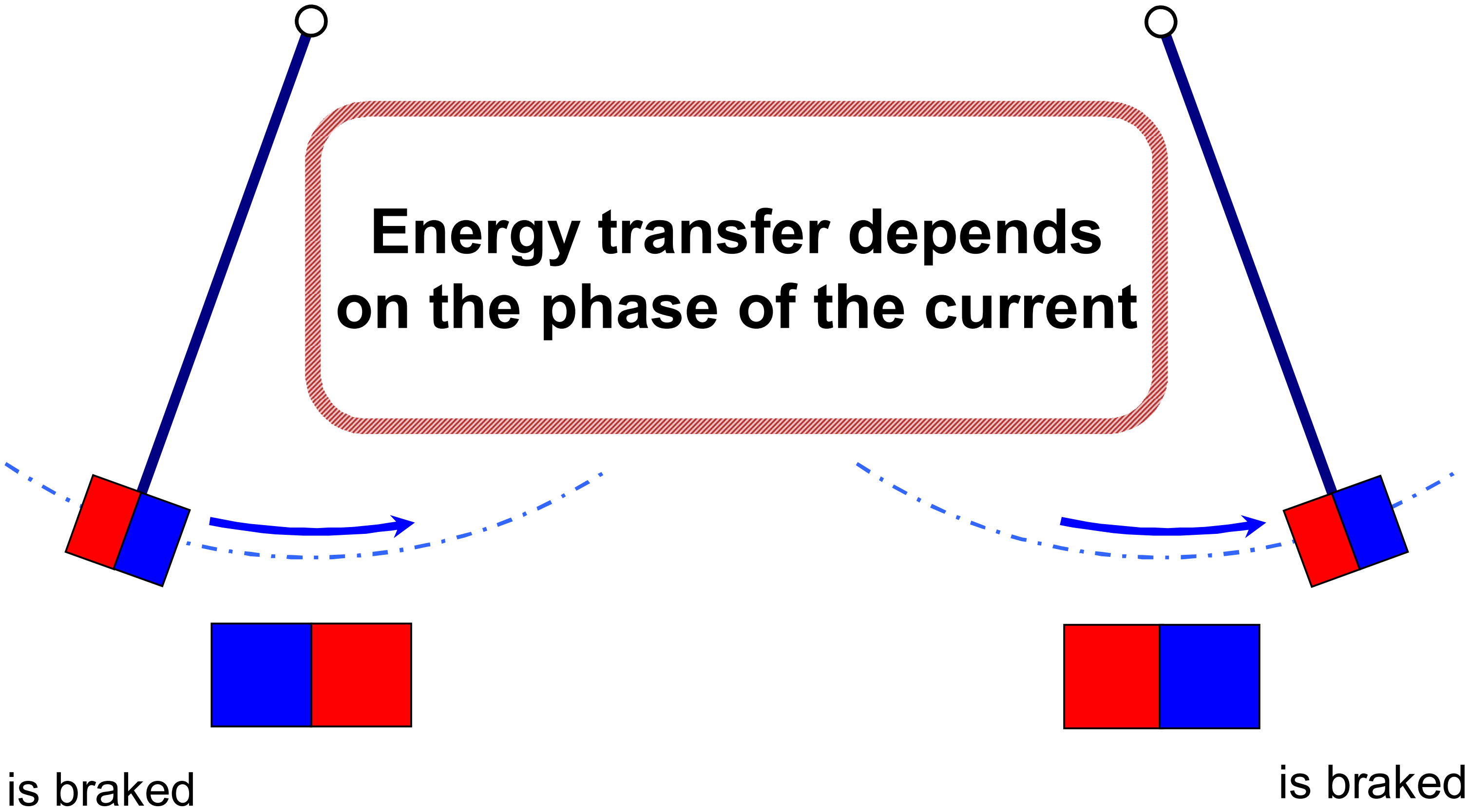
AC solenoid

The energy of the pendulum can either increase…



AC solenoid

…and decrease.



Dependence on the phase

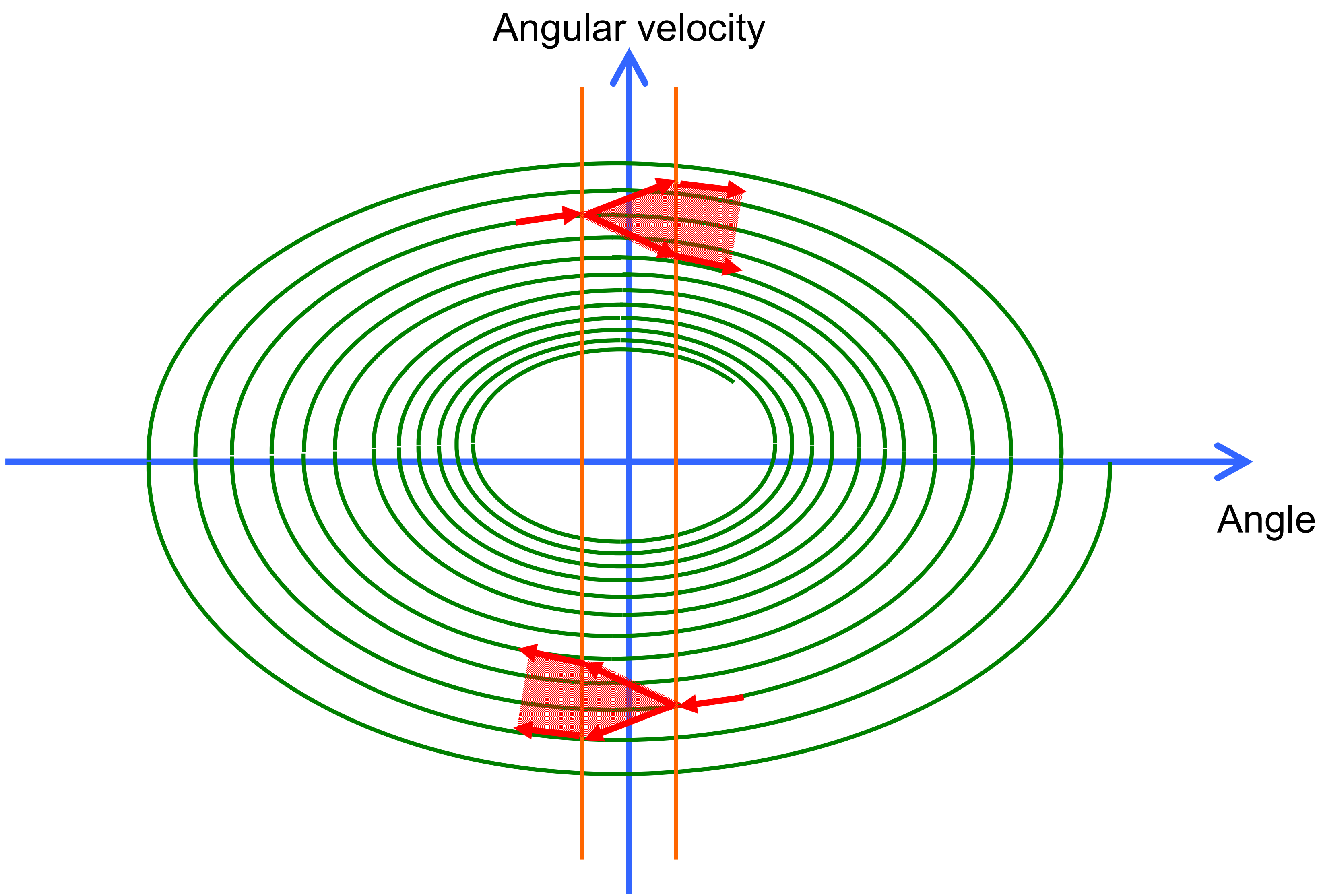
We denote the time when the pendulum passes its lowest point as $t = 0$.

Current oscillations in the coil:

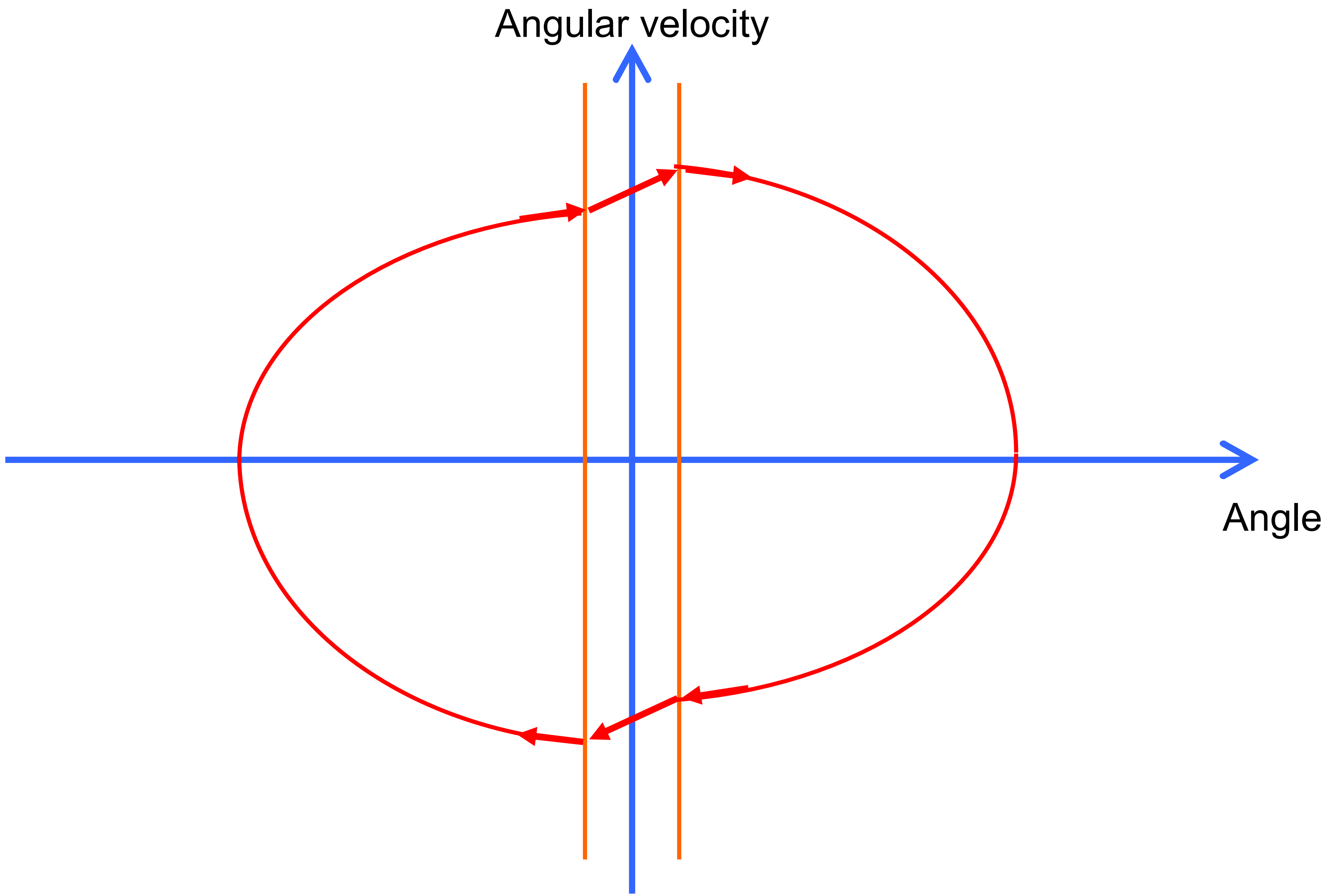
$$I(t) = I_0 \sin(\omega t + \varphi)$$

$$I(t) = \underbrace{\left(I_0 \cos\varphi\right) \cdot \sin\omega t}_{\text{Energy transfer}} + \underbrace{\left(I_0 \sin\varphi\right) \cdot \cos\omega t}_{\text{No energy transfer}}$$

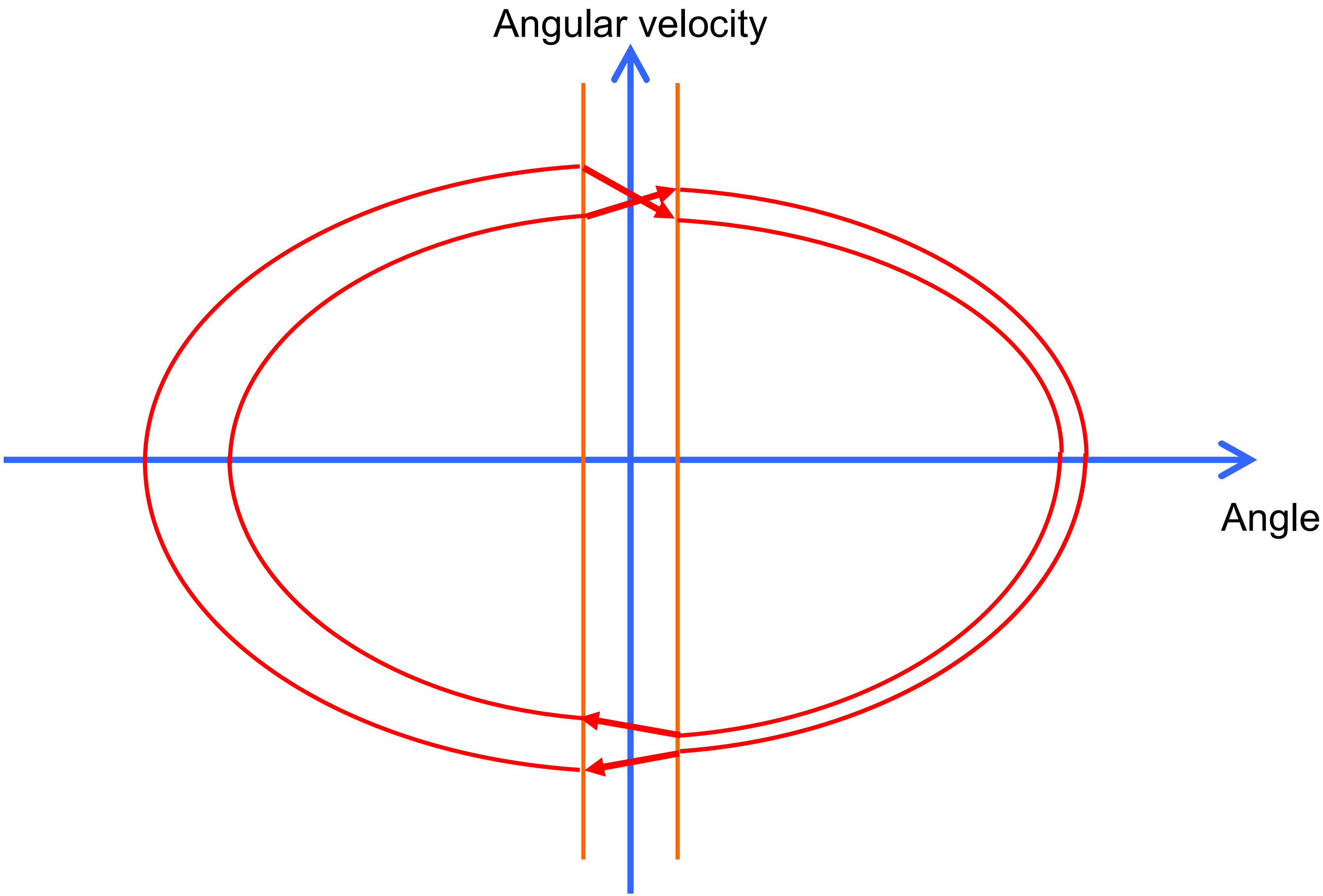
Impact of the electromagnet



Undamped oscillations (1 period)

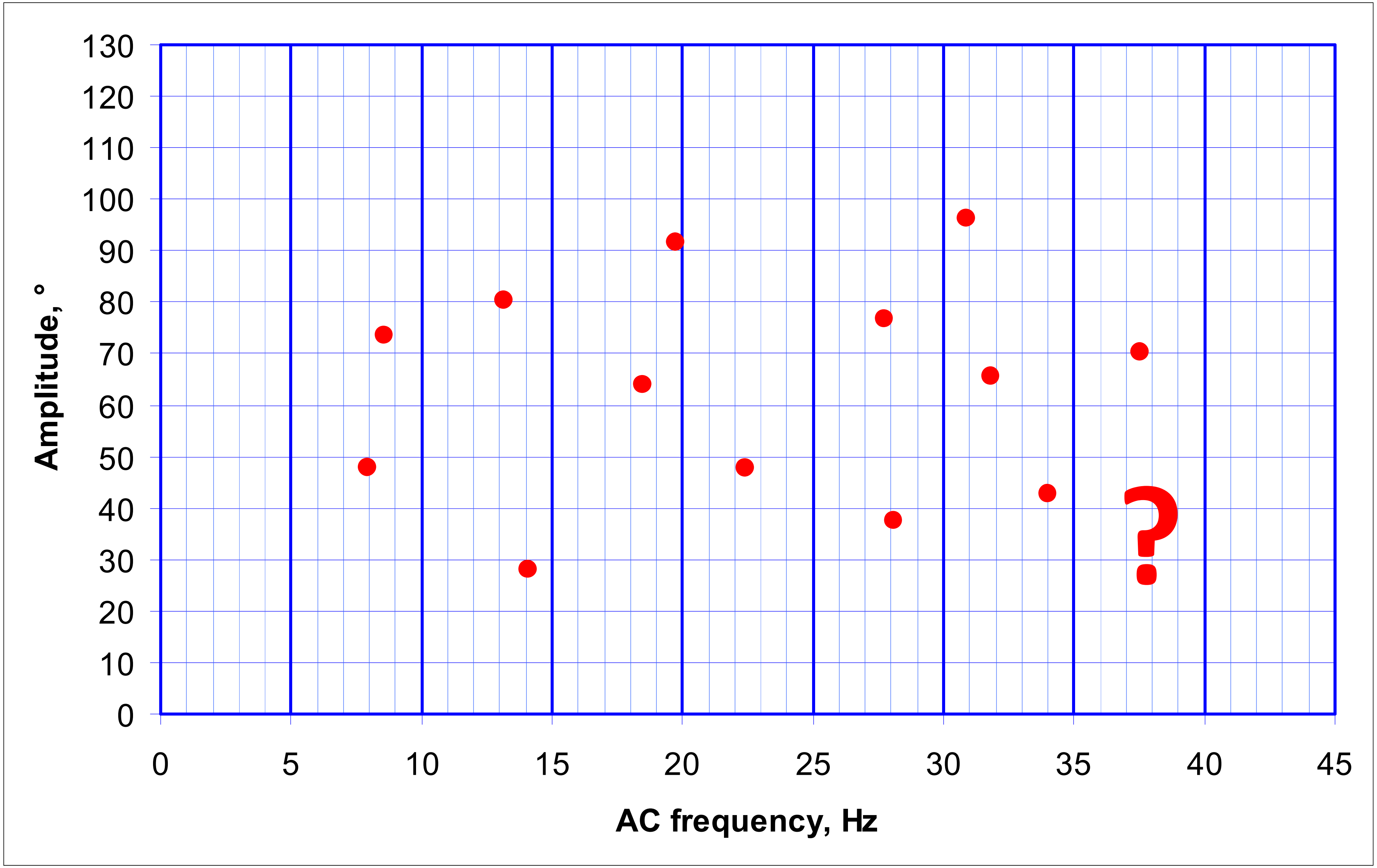


Undamped oscillations (2 periods)

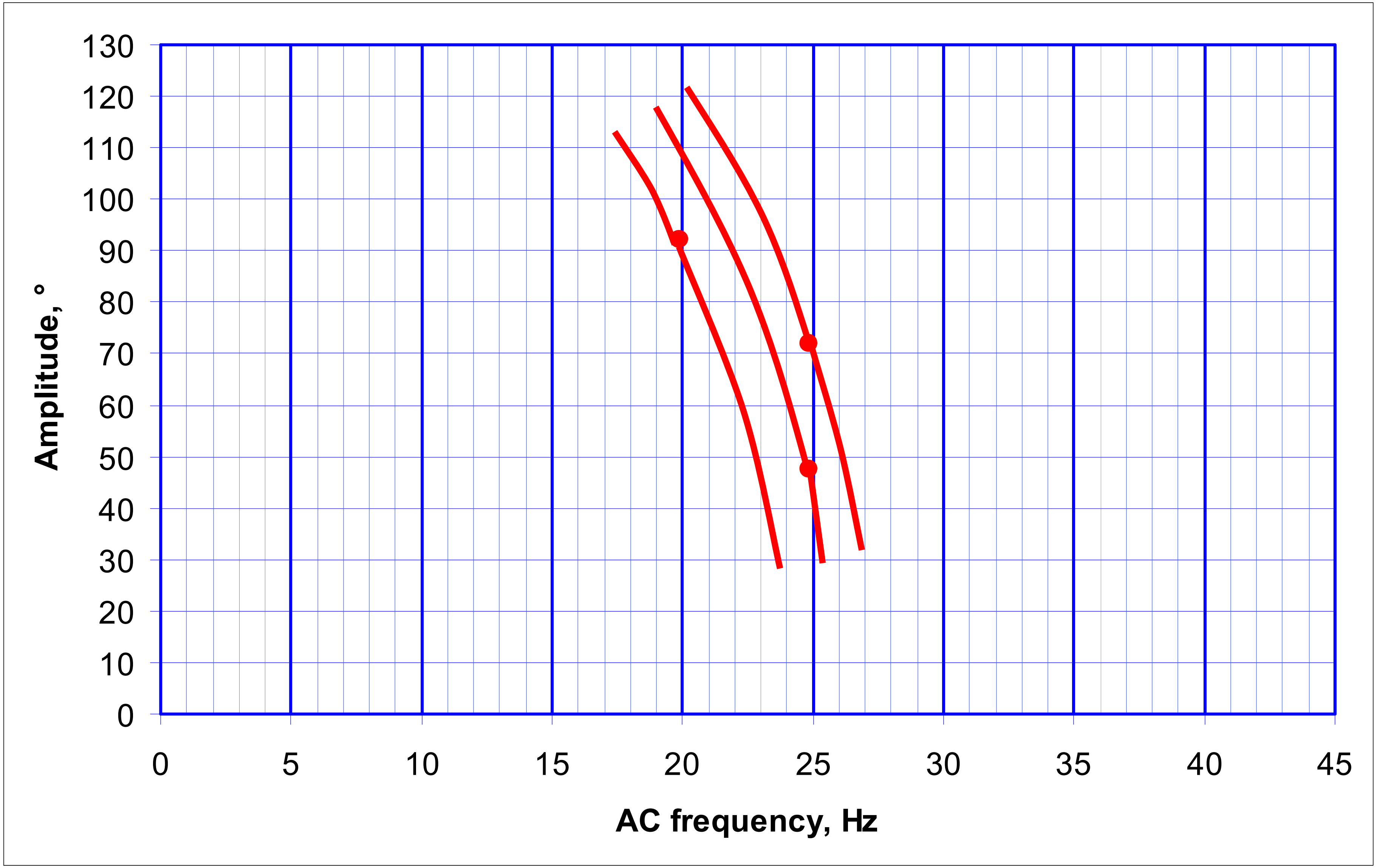


Experiments

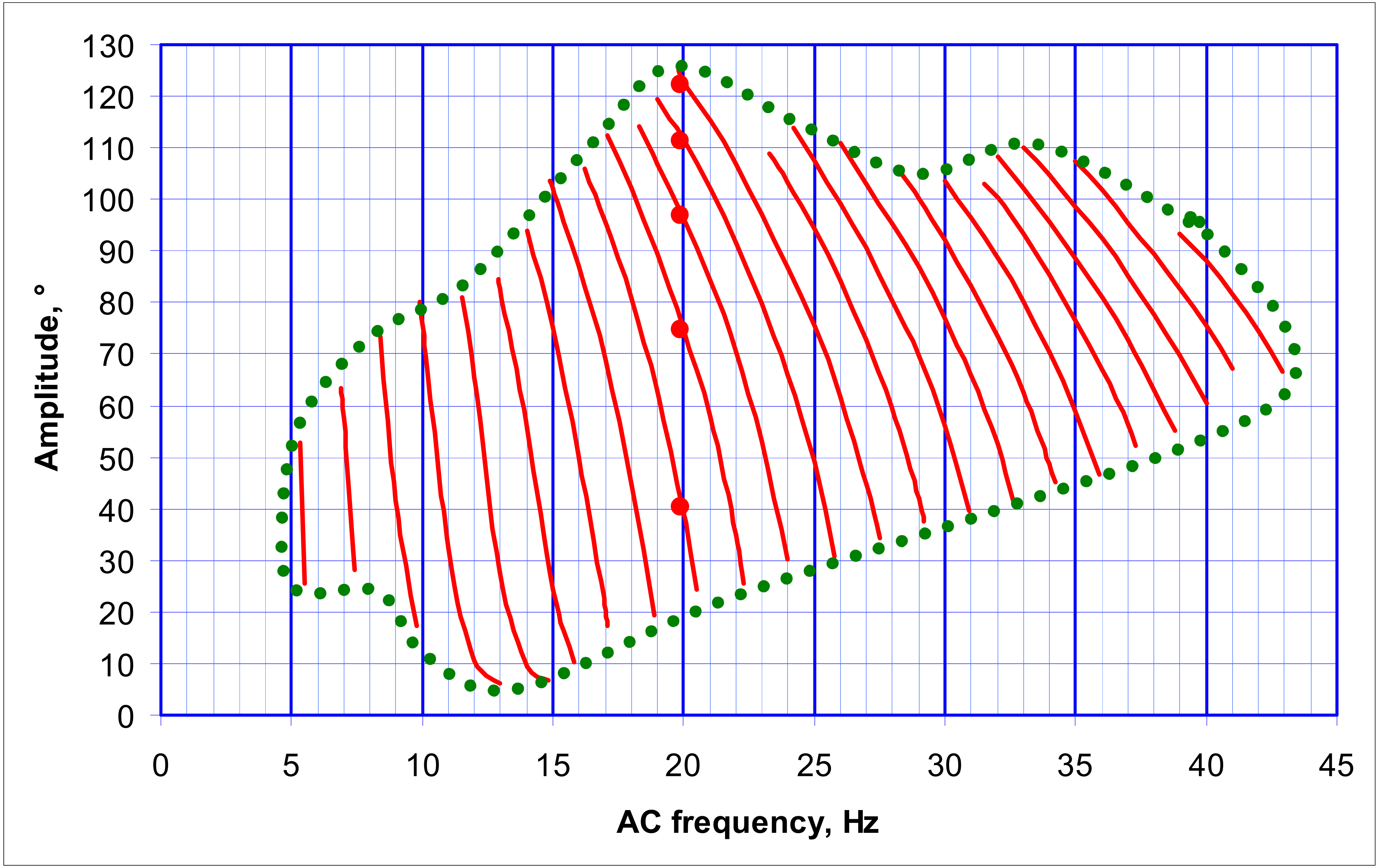
First attempt



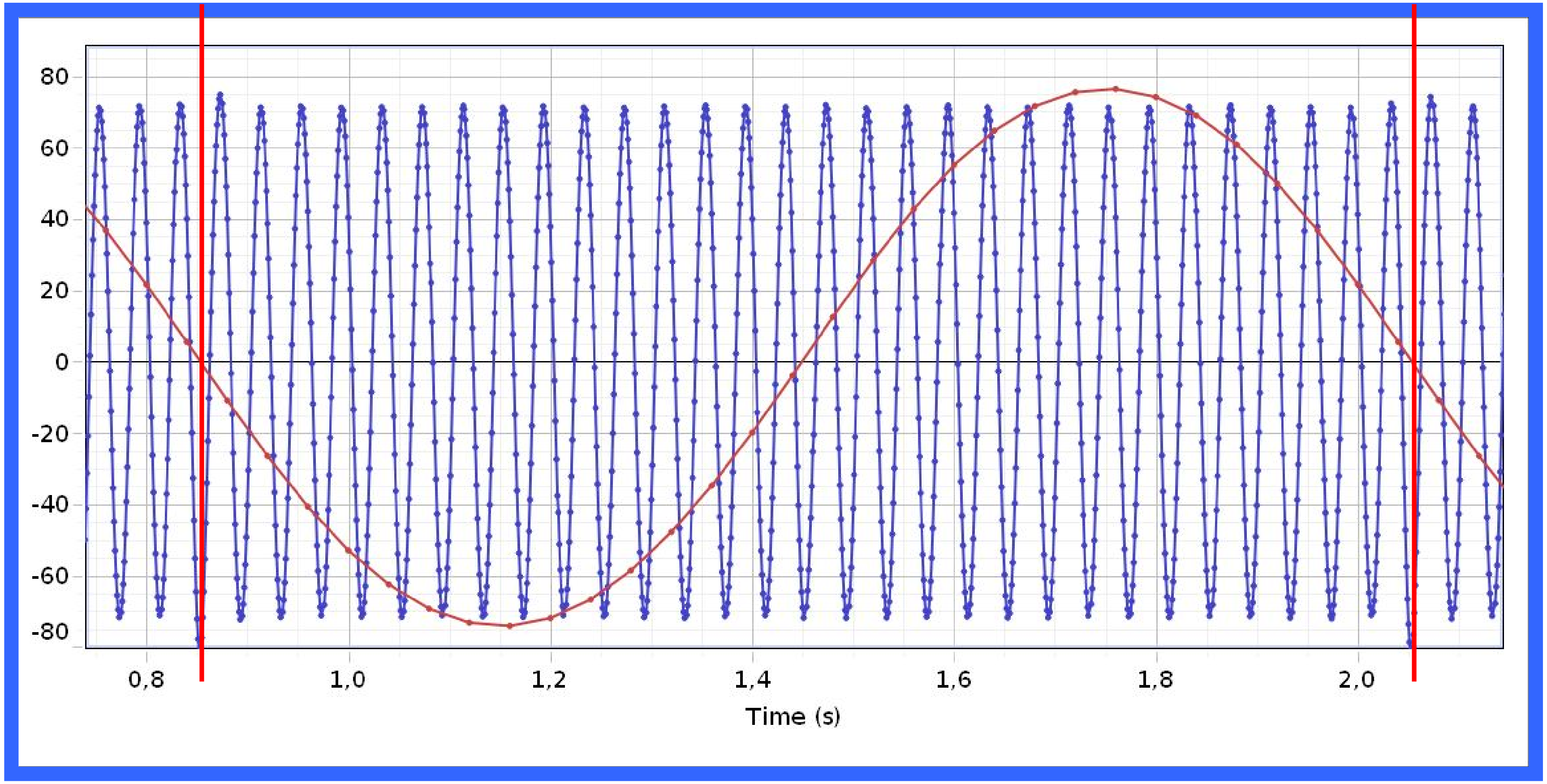
Experiment #1



Undamped oscillations

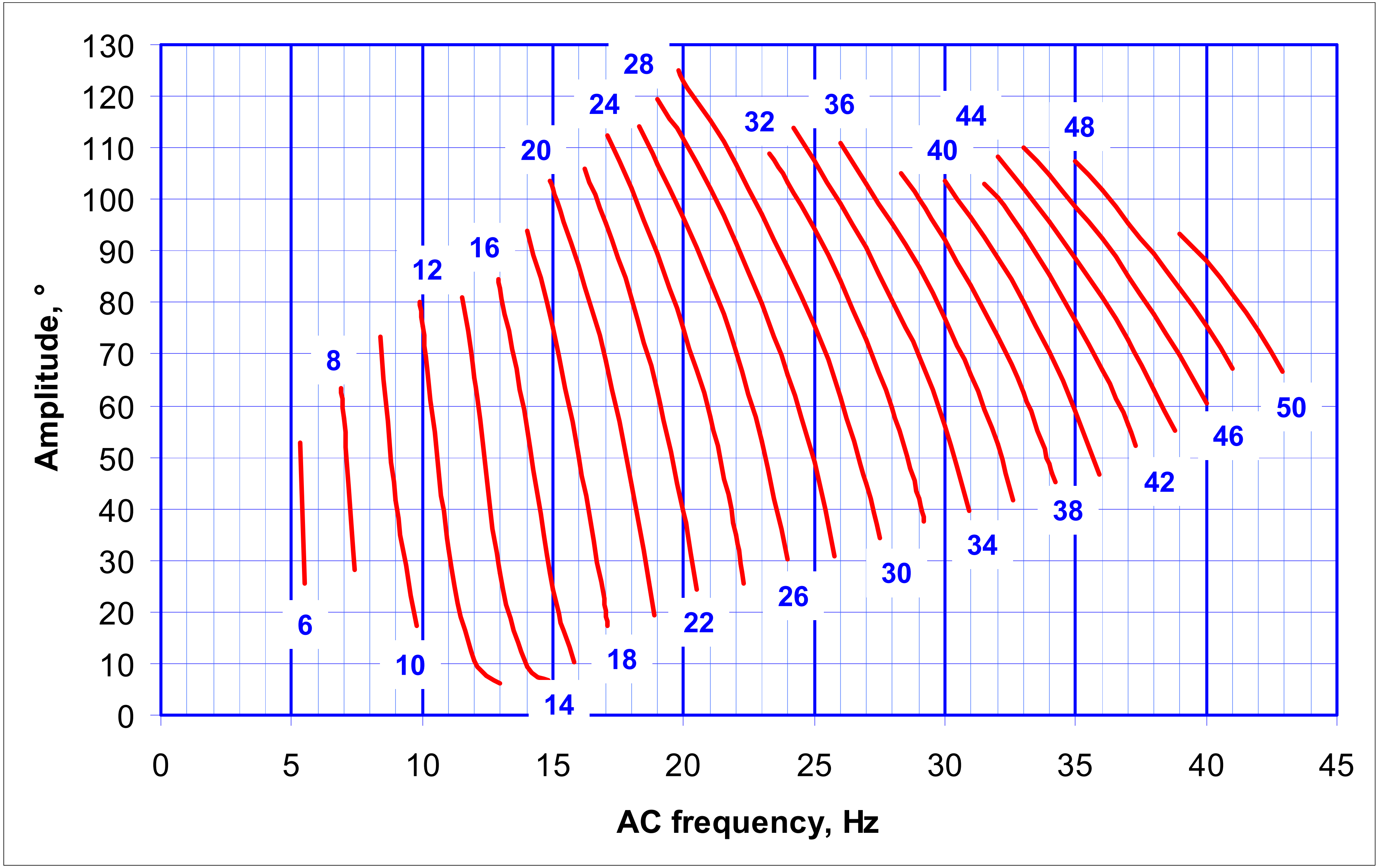


Order of resonance

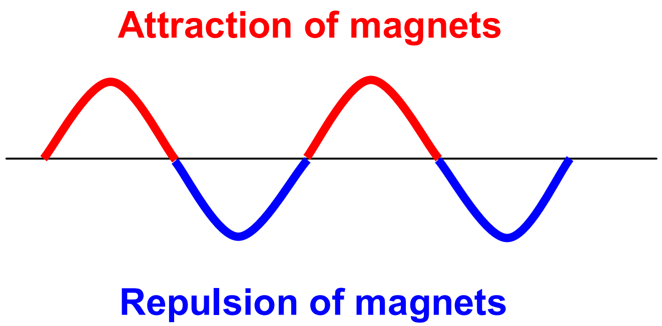


In this test 30 current oscillations occur during 1 pendulum oscillation. **Order of resonance = 30.**

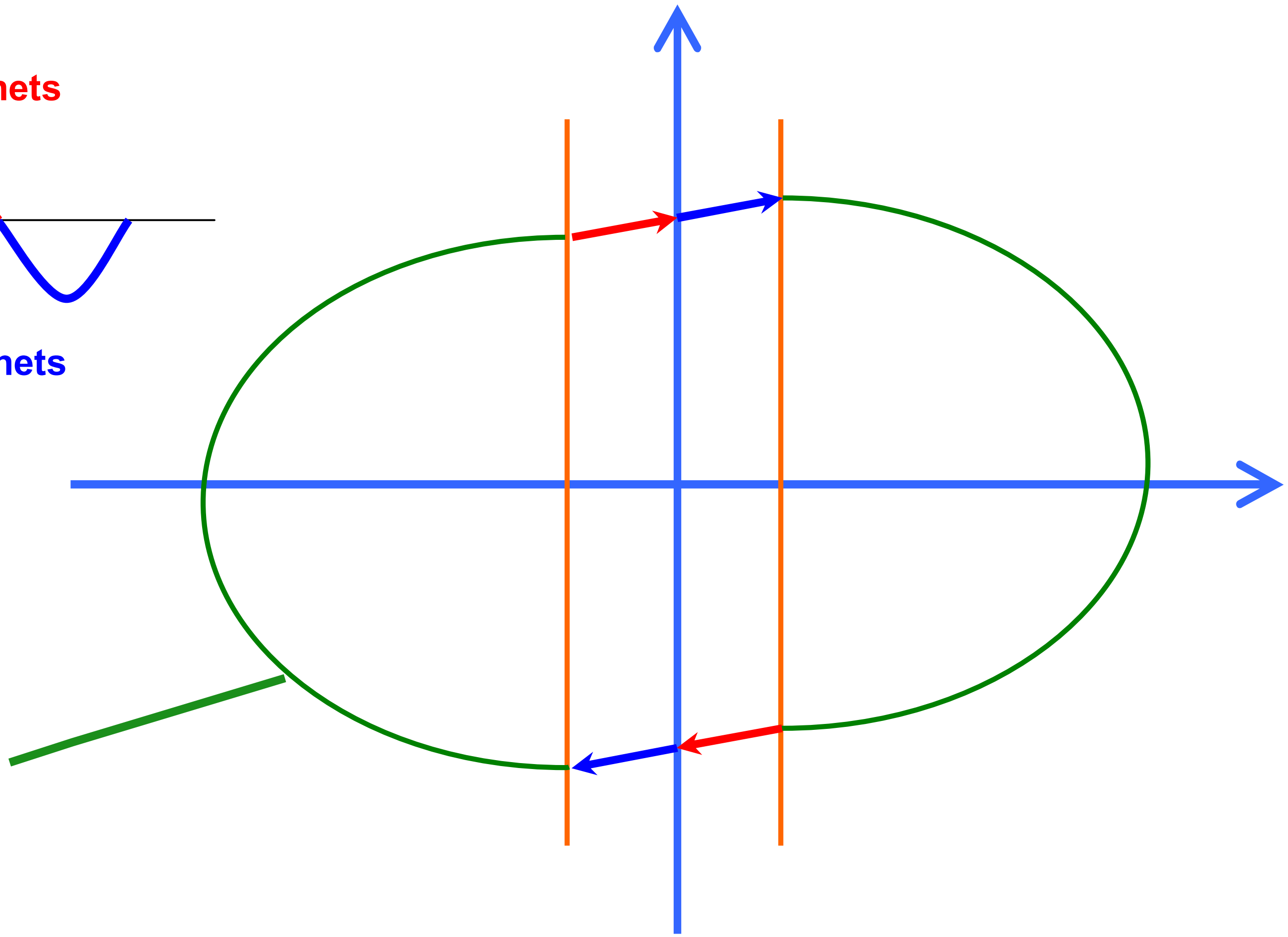
Orders of resonance



Why all orders are even?



Half of a cycle



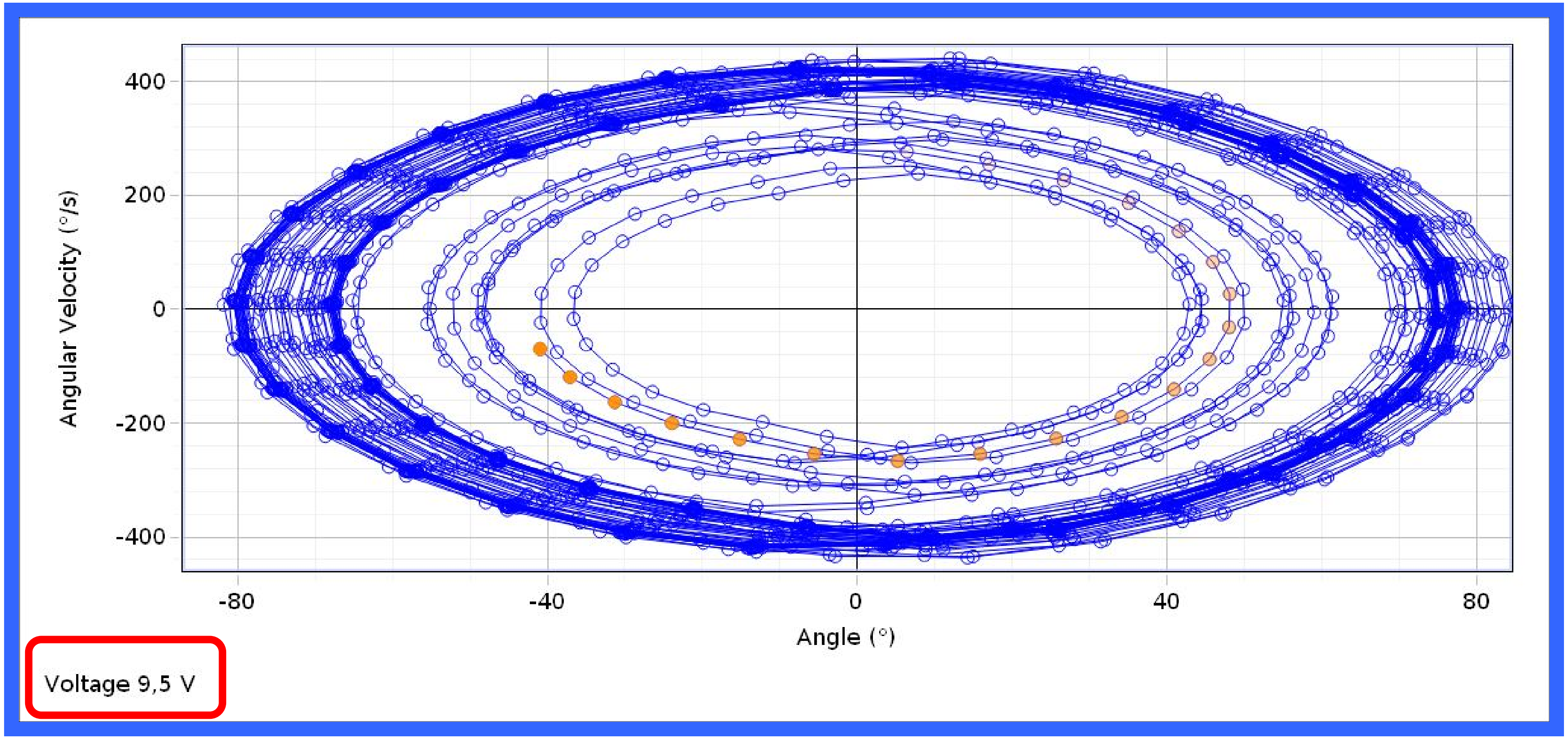
$$N = 2 + 2 \cdot k$$

Cycles of undamped oscillations

Experiment #2

- AC frequency is fixed (25 Hz in the main series).
- Some undamped oscillation of kicked pendulum is found (resonance order = 30, amplitude ≈ 80° in the main series).
- Slowly shifting the voltage we monitor changes of the fine structure of the phase portrait.

Period doubling

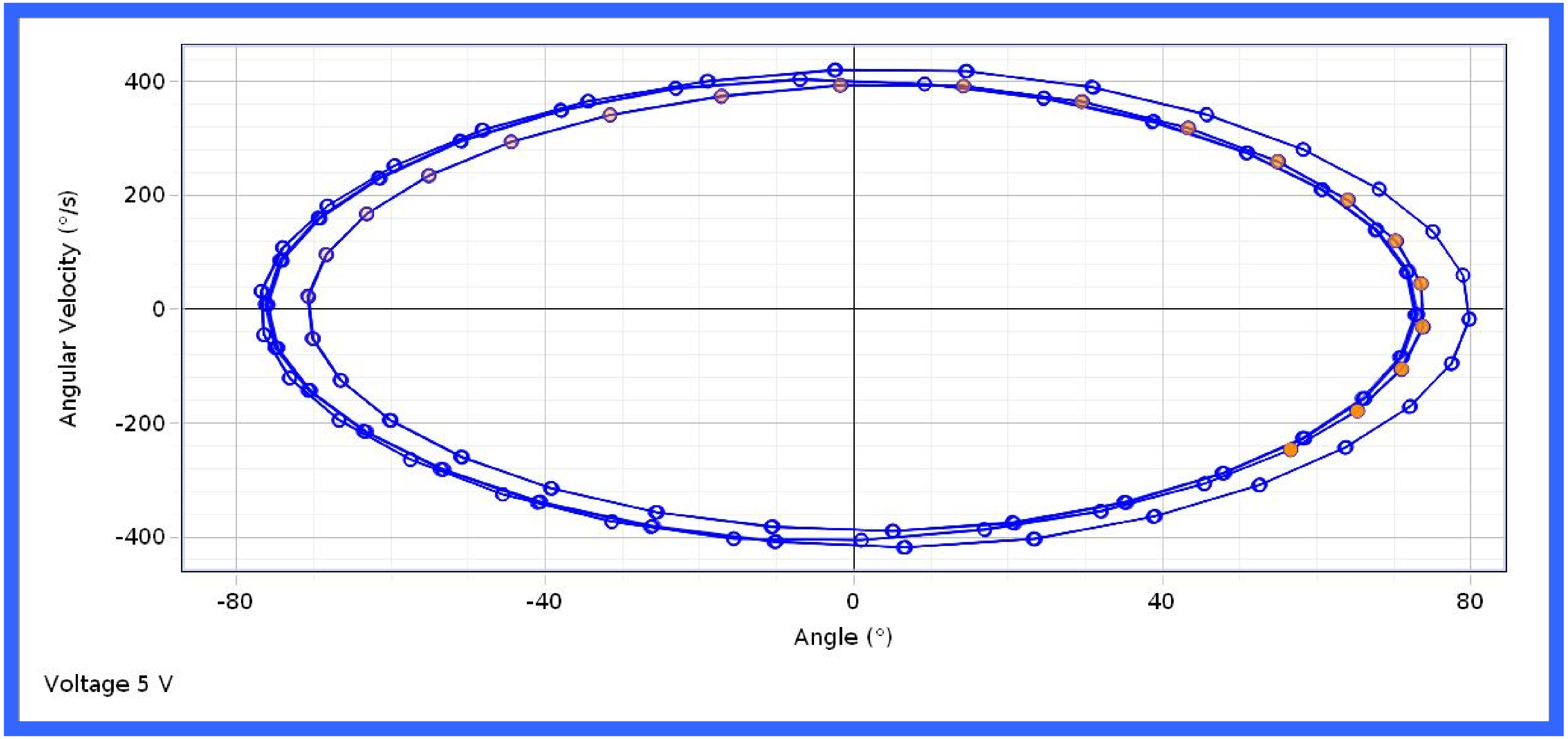


Appearance of the cycle = **1.7 V**

Period doubling = **6.9 V**

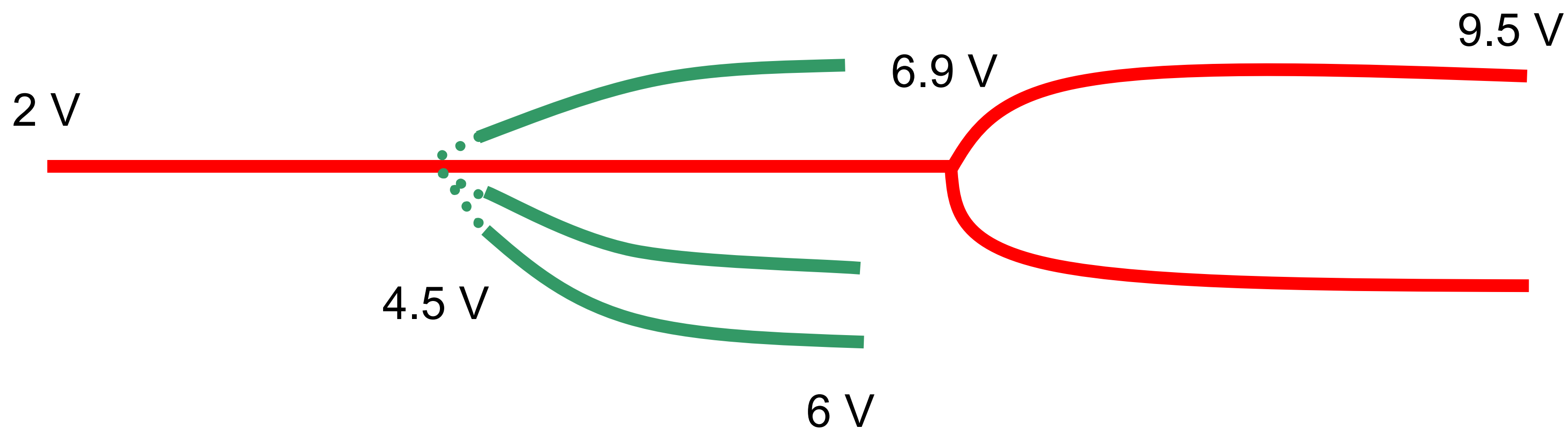
Destruction of the cycle= **9.5 V**

3-periodic cycle



This 3-periodic cycle exists
in the range from **4.5 V** to **6.0 V**

The fine structure of the resonance



Why the cycles are destroyed?

- **At low currents:** kicks are too weak to compensate the energy dissipation.
- **At high currents:** phase adjustment can’t exactly compensate too strong too strong kicks. This eventually leads to destruction of the cycle.

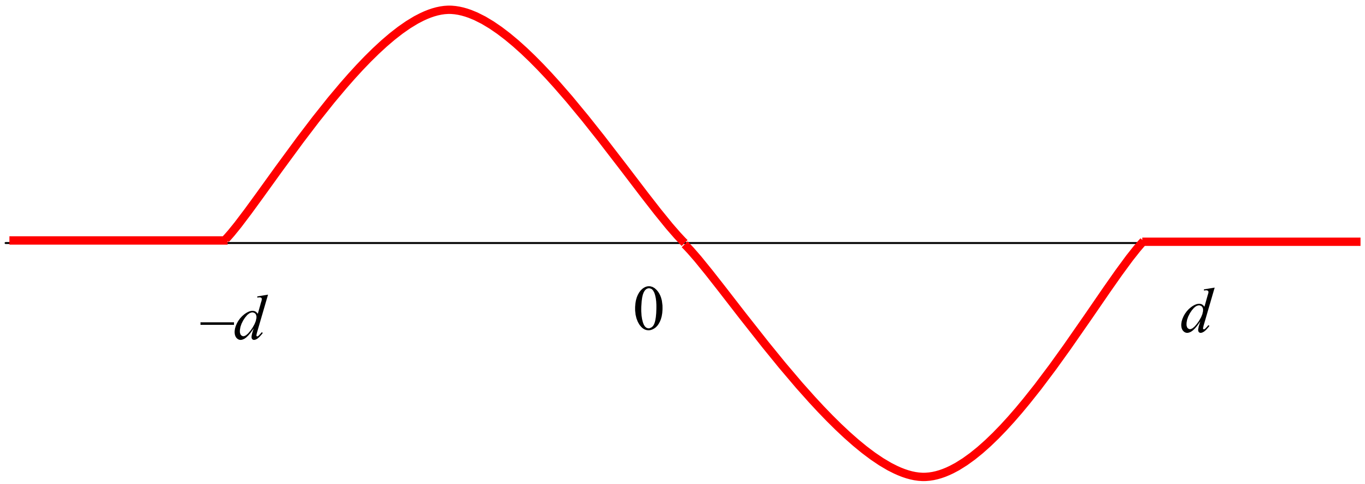
Computer simulation

Limitations of natural experiment

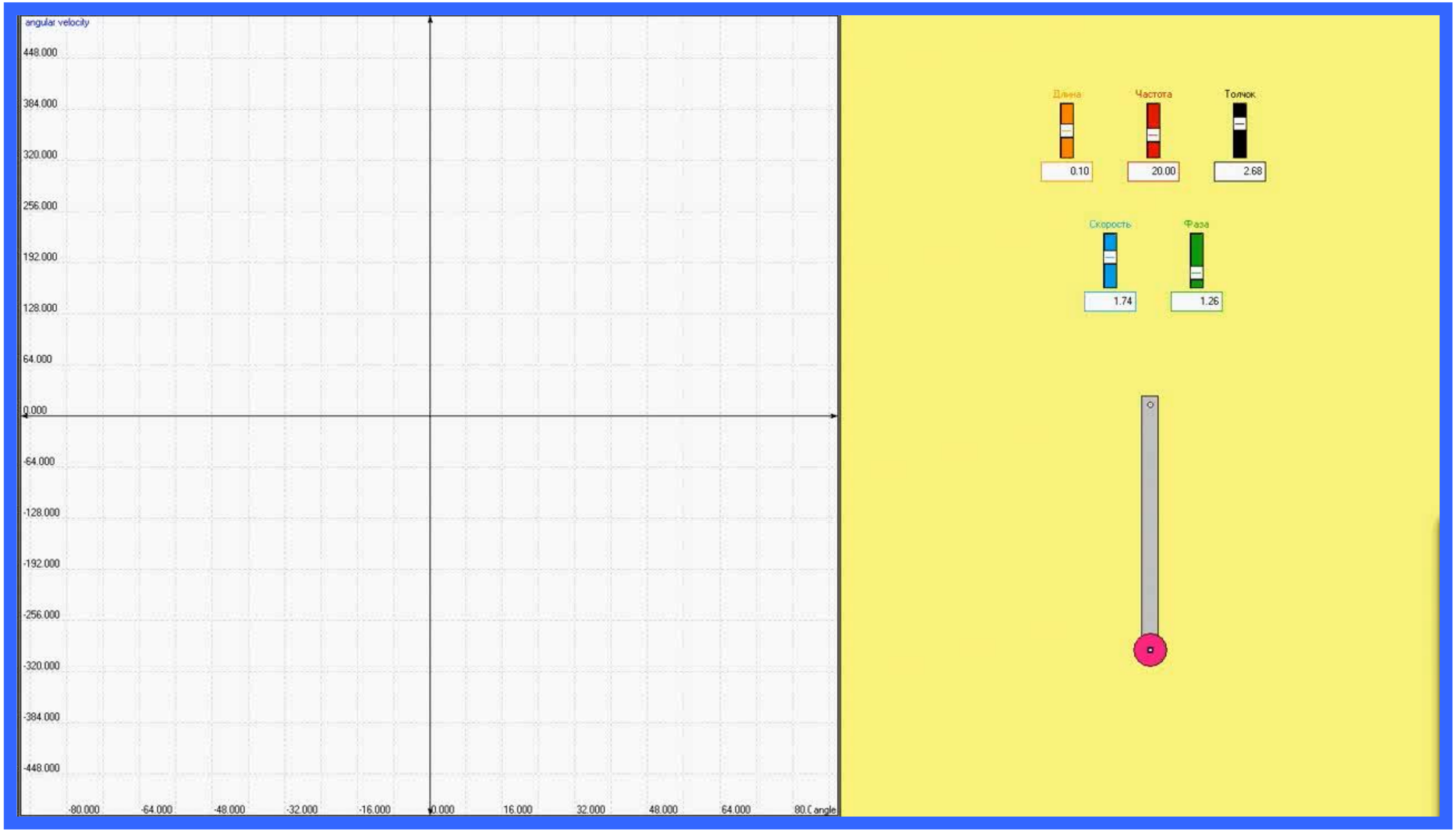
- There are limitations on the voltage and current in power supply.
- Fine effects are difficult to observe because of vibrations and external disturbances.

Model of magnets interaction

$$F(x) = F_0 \sin\left(\frac{\pi x}{d}\right)$$



Computer simulation



Limitations of the computer model

- We do not know which initial start conditions lead to the establishment of undamped oscillations.
- In each test, we need to wait 10 minutes to establish the stationary regime.
- To conduct all the necessary measurements it will take us more then decade of continuous work!

Poincare map

Poincare map

The pendulum interacts with an electromagnet in the narrow angle range. This allows to relate values (v, φ) on two consecutive spans.

$$\begin{pmatrix} v_n \\ \varphi_n \end{pmatrix} \qquad \begin{pmatrix} v_n \\ \varphi_n \end{pmatrix} \Rightarrow \begin{pmatrix} v_{n+1} \\ \varphi_{n+1} \end{pmatrix} \qquad \begin{pmatrix} v_{n+1} \\ \varphi_{n+1} \end{pmatrix}$$

Theoretical model

Change in velocity for half-period

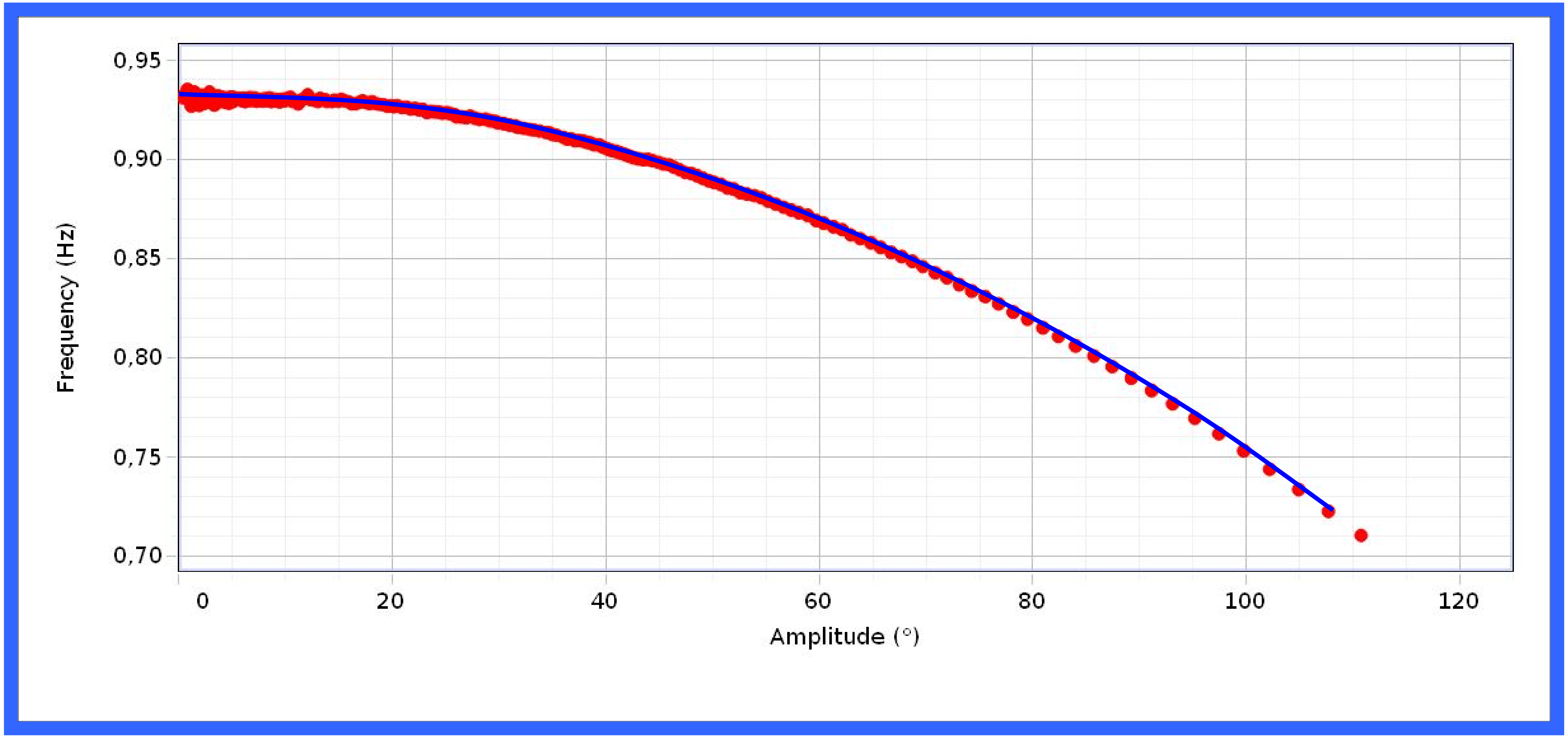
The electromagnet is switched off:

$$v_{n+1} = (1-\varepsilon)\cdot v_n$$

The electromagnet is switched on:

$$v_{n+1} = (1-\varepsilon)\cdot v_n + \frac{F_0}{m\omega}\cdot\frac{2\pi\left(\frac{\omega d}{v_n}\right)\cdot\sin\left(\frac{\omega d}{v_n}\right)}{\left(\pi^2-\left(\frac{\omega d}{v_n}\right)^2\right)}\cdot\cos\varphi_n$$

Oscillations with large amplitude



$$f(\alpha) = f_0 \cdot \frac{\pi / 2}{\int_0^{\alpha} \sqrt{2(\cos\theta - \cos\alpha)}\, d\theta}$$

Change in phase for half-period

Dependence of the oscillation period on the velocity.

$$T(v) = T_0 \cdot \left\{ 1 + \left(\frac{1}{2}\right)^2 \left(\frac{v^2}{2U^2}\right) + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 \left(\frac{v^2}{2U^2}\right)^2 + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 \left(\frac{v^2}{2U^2}\right)^3 + \dots \right\}$$

Our calculation takes into account first six expansion terms.

$$U^2 = 2gl$$

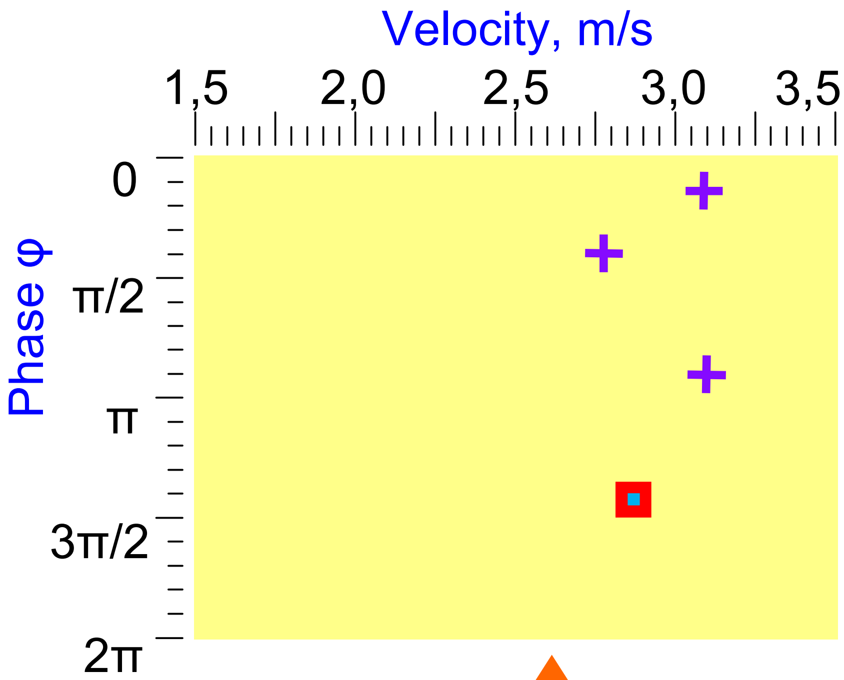
$$\varphi_{n+1} = \varphi_n + \omega \cdot \frac{T(v_{n+1})}{2}$$

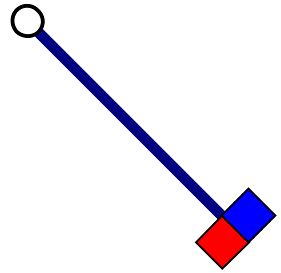
Final formulas

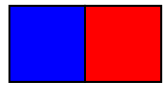
$$v_{n+1} = (1-\varepsilon)\cdot v_n + \frac{F_0}{m\omega}\cdot \frac{2\pi\left(\frac{\omega d}{v_n}\right)\cdot \sin\left(\frac{\omega d}{v_n}\right)}{\left(\pi^2 - \left(\frac{\omega d}{v_n}\right)^2\right)}\cdot \cos\varphi_n$$

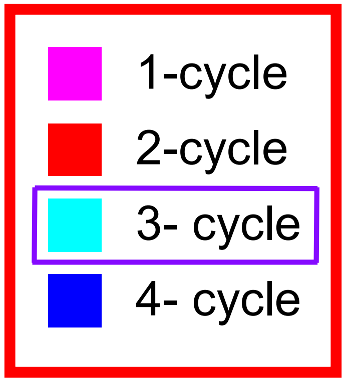
$$\varphi_{n+1} = \varphi_n + \omega\cdot\frac{T(v_{n+1})}{2}$$

Poincare mapping

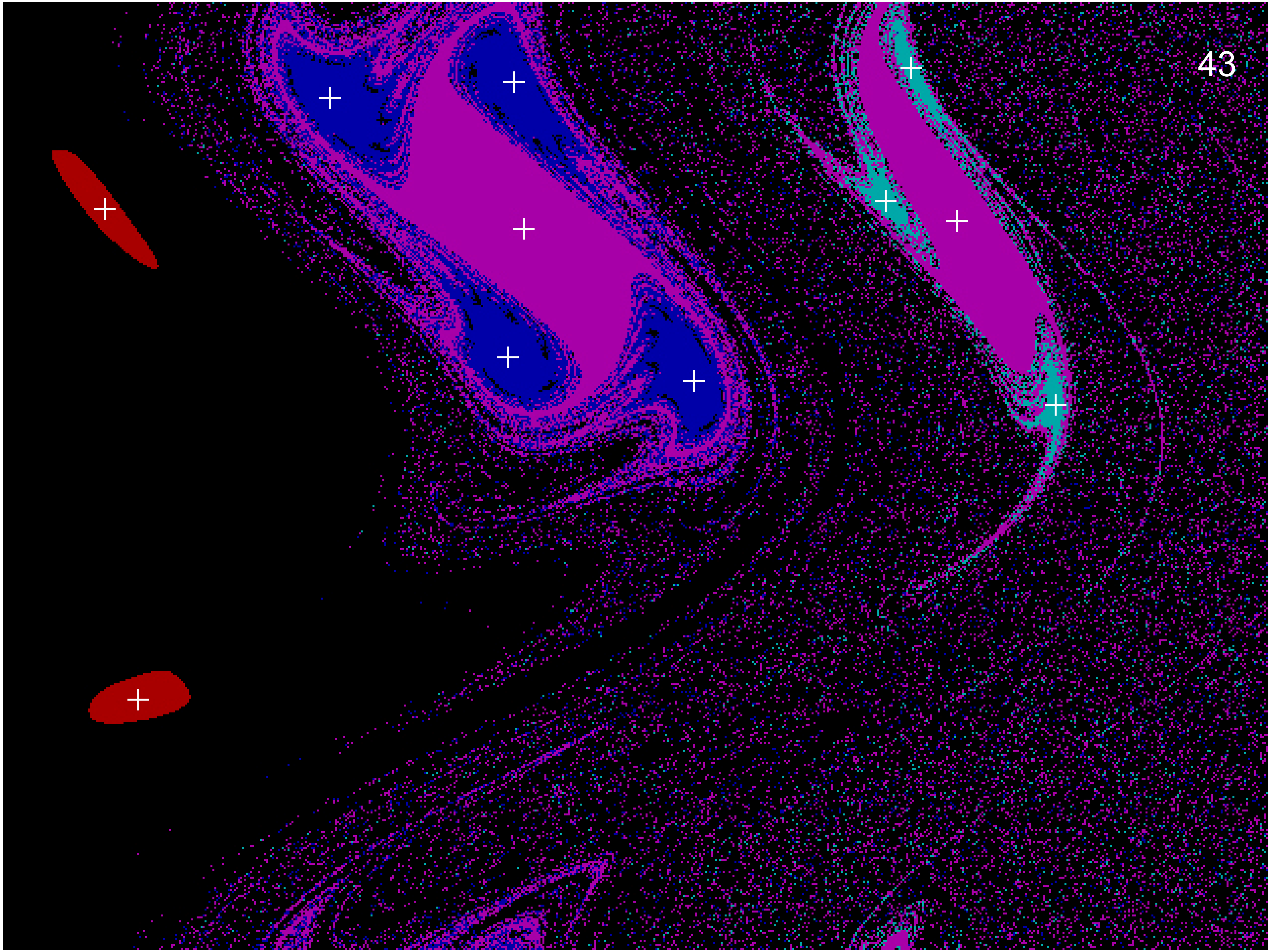




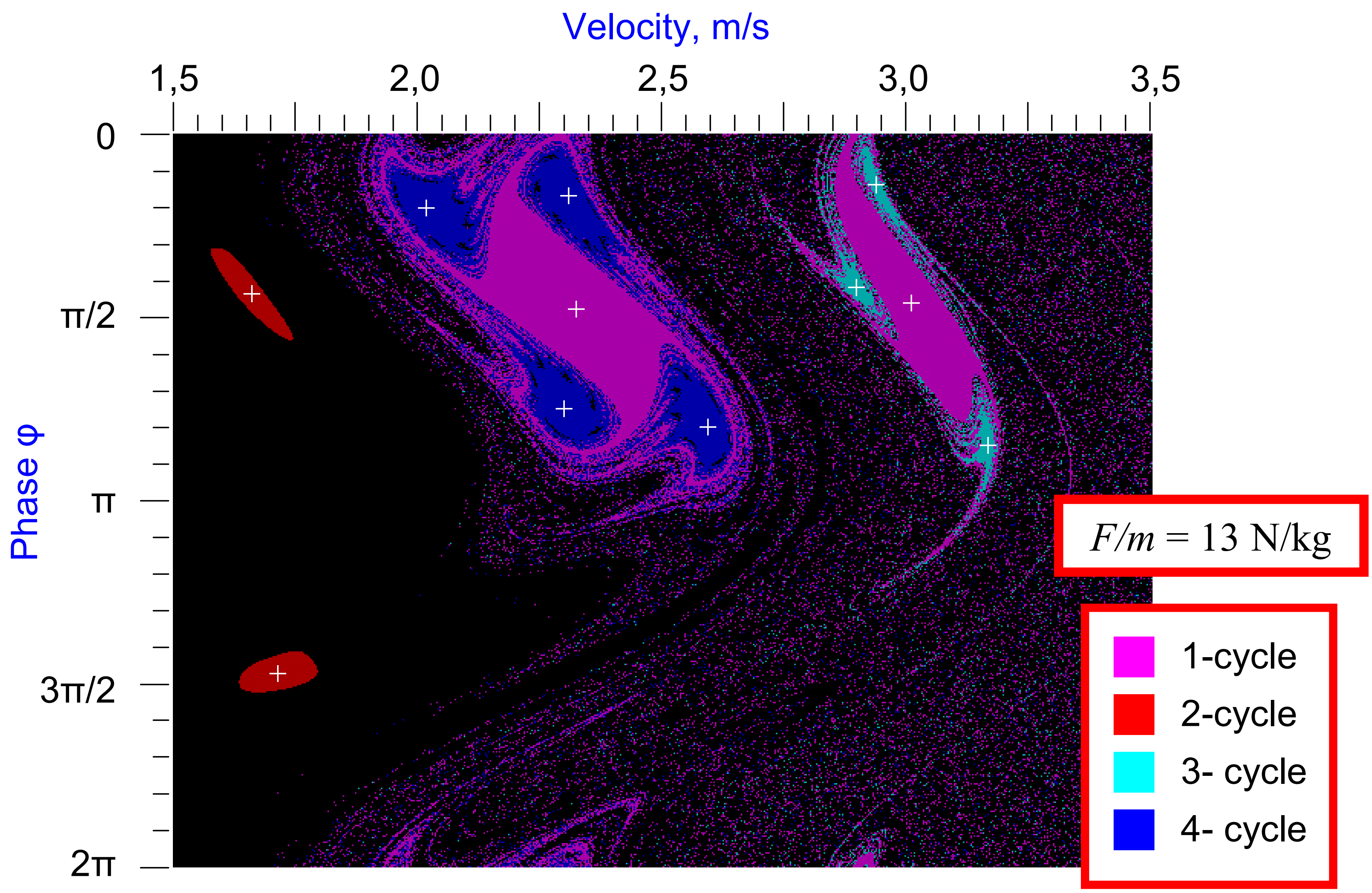




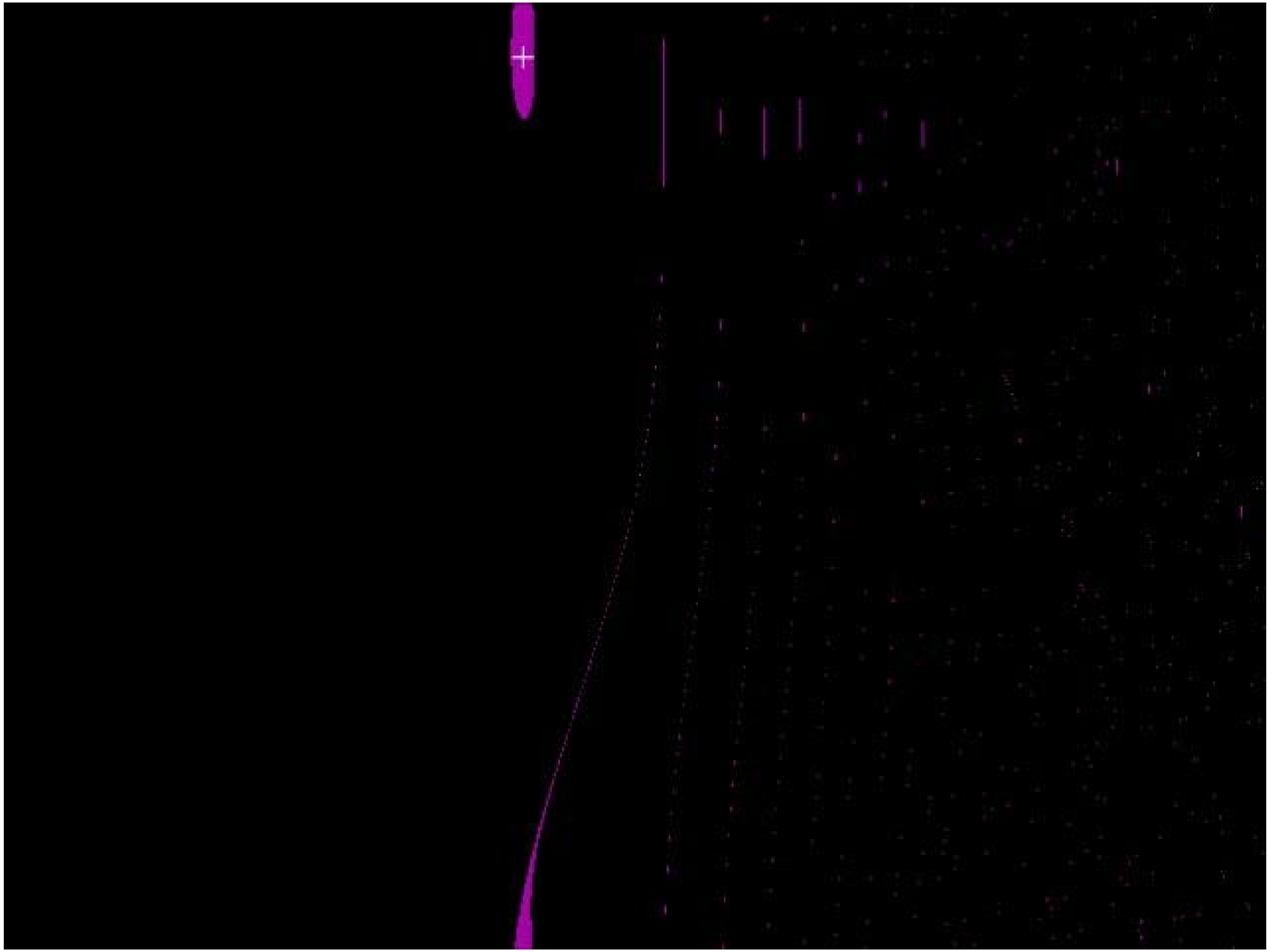
Velocity	Phase
2.92	0.31
2.73	1.57
3.22	3.06
2.92	0.31
2.73	1.57
3.22	3.06



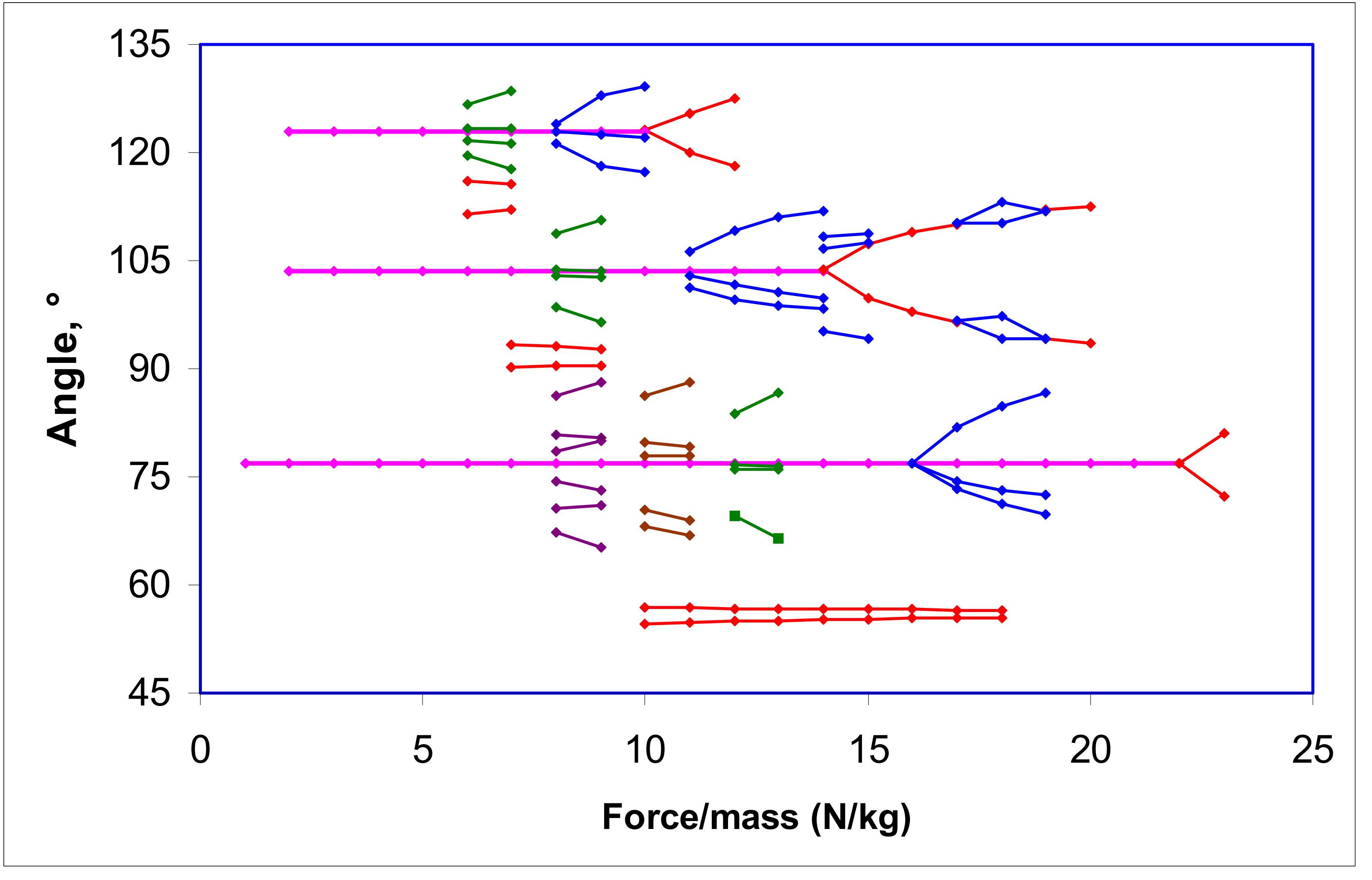
Poincare map



Increasing the strength (video)

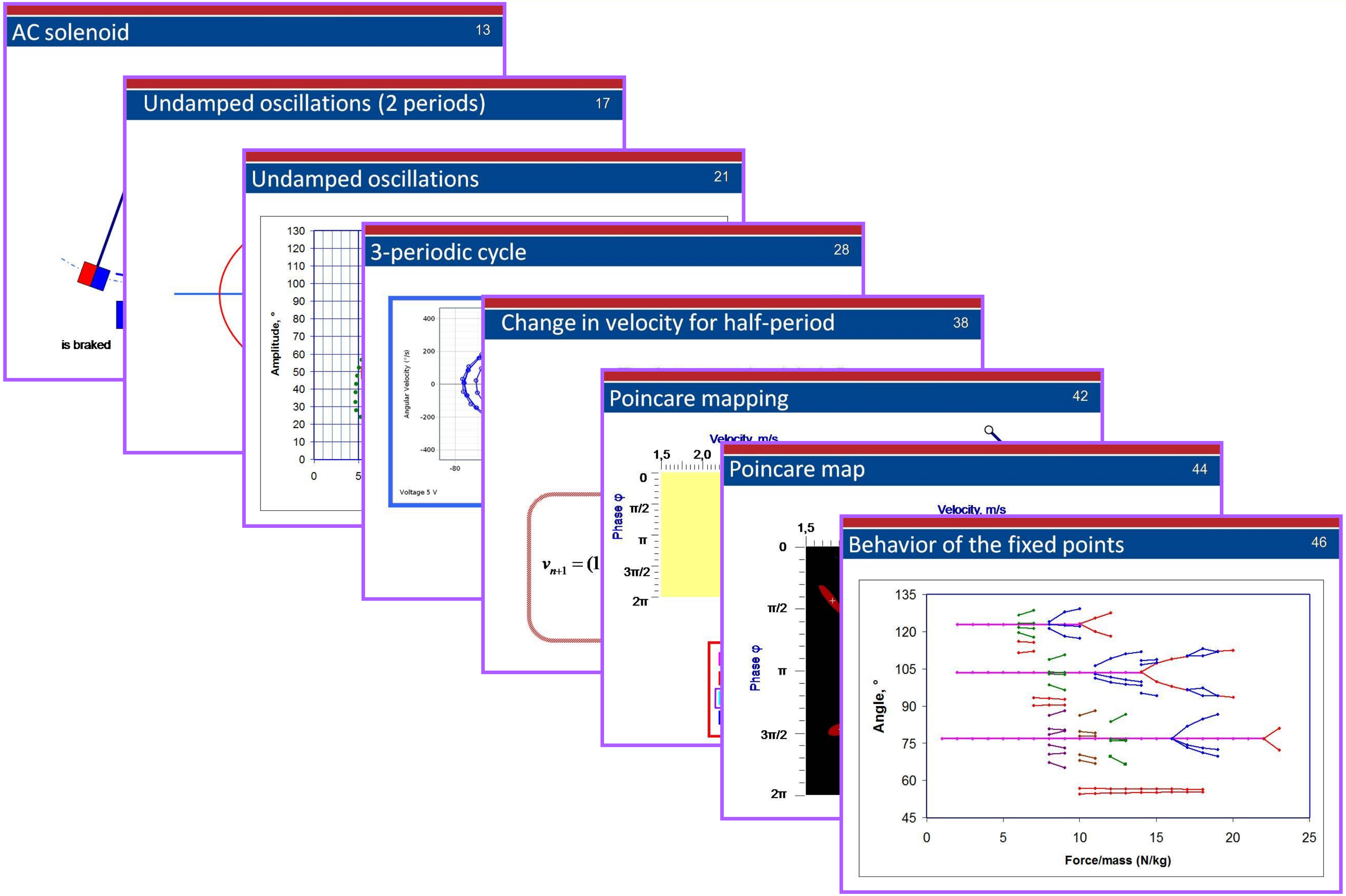


Behavior of the fixed points



Summary

Conclusions



References

- П.С.Ланда, Я.Б.Дубошинский (1989) “Автоколебательные системы с высокочастотными источниками энергии”. *УФН,* **158**, 729–742.
- V.Damgov, I.Popov (2000) “Discrete oscillations and multiple attractors in kick-excited systems”. *Discrete Dynamics in Nature and Society*, **4**, 99–124.

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Thank you for your attention!