Russia IYPT

Moving brush

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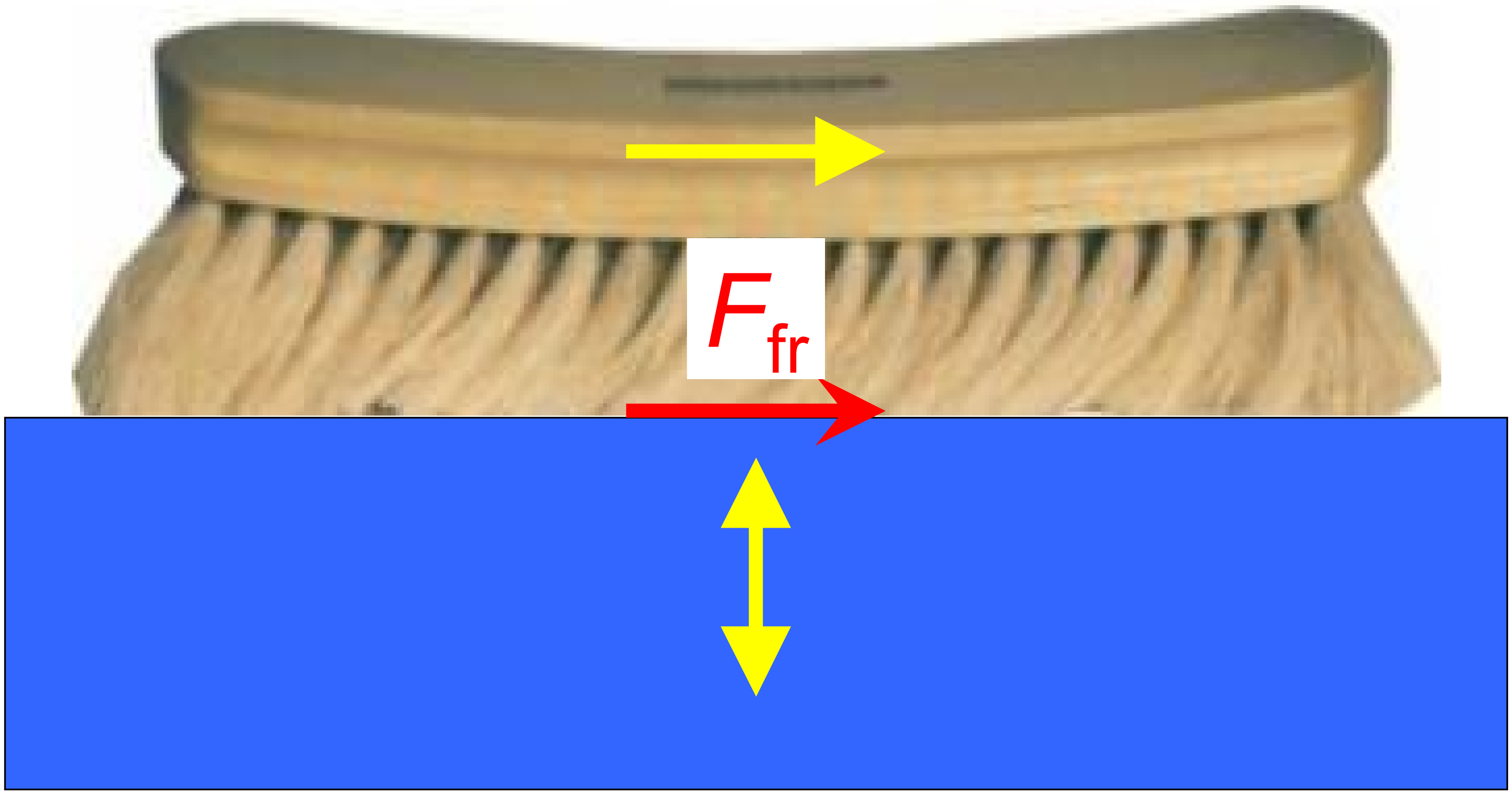
The problem

A brush may start moving when placed on a vibrating horizontal surface. Investigate the motion.

First observations

Simple brush (240 fps)

Conversion of vibration into motion



Outline of the report

Brush optimization

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Steady forward motion

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Computer & Theoretical modeling

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Forward motion experiments

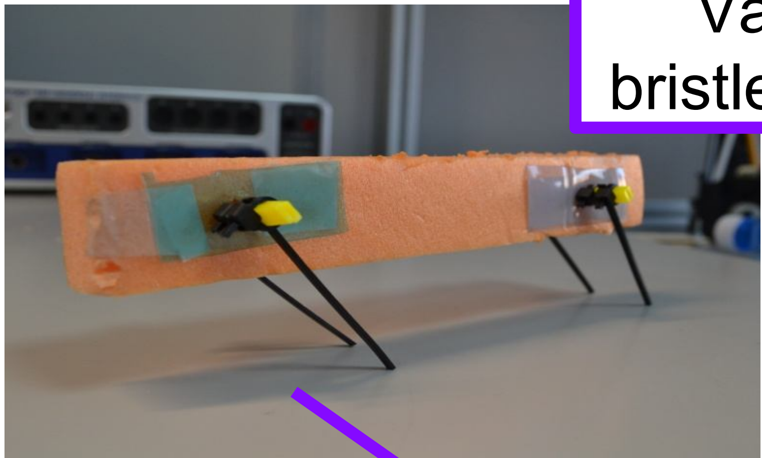
↓

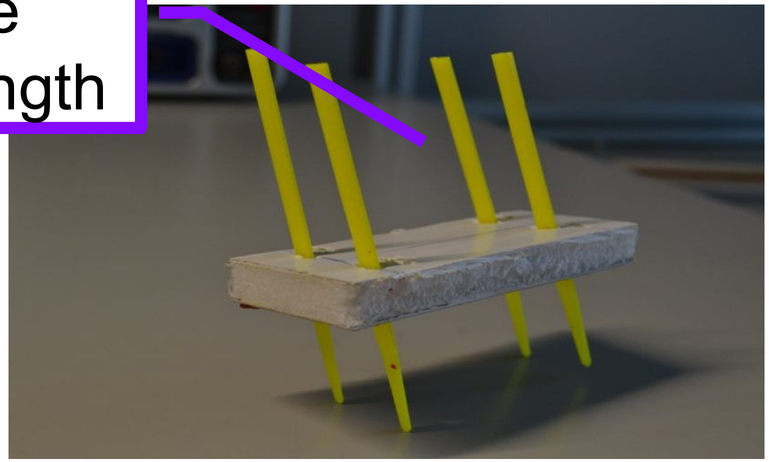
Backward motion

Optimized brush construction

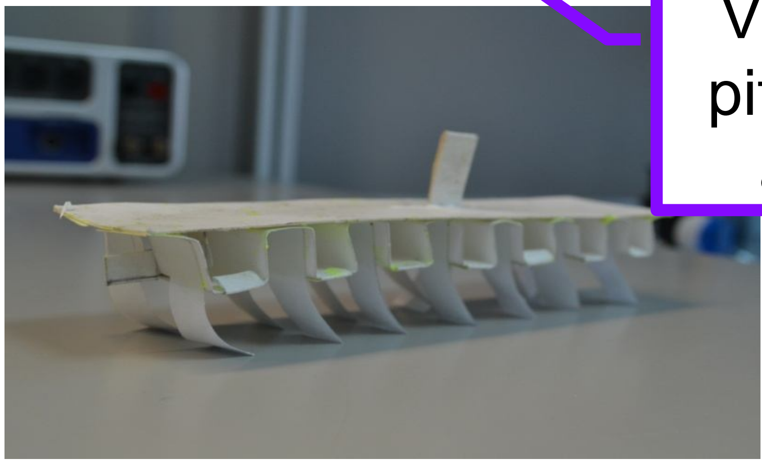
Different brushes

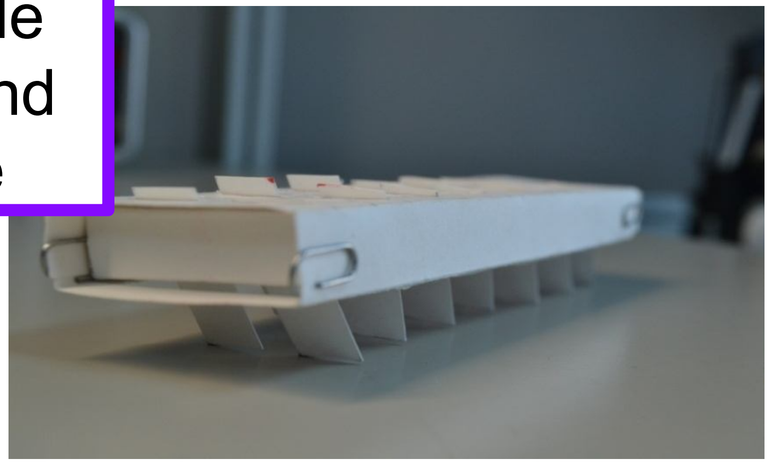
Variable bristles’ length



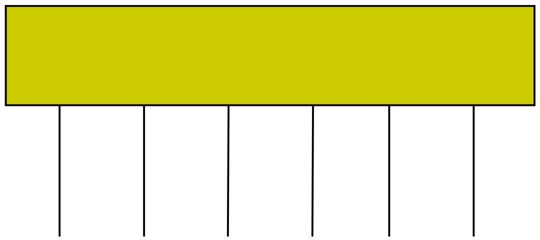


Variable pitch and angle

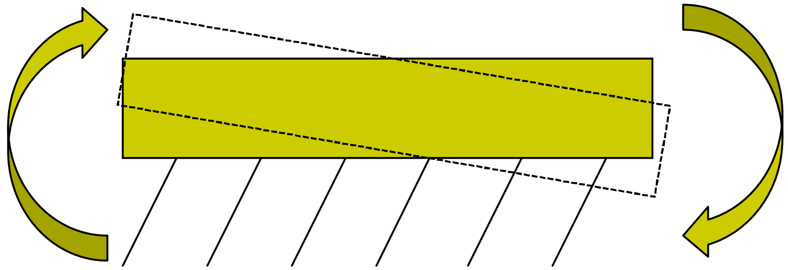




Search for the optimal design



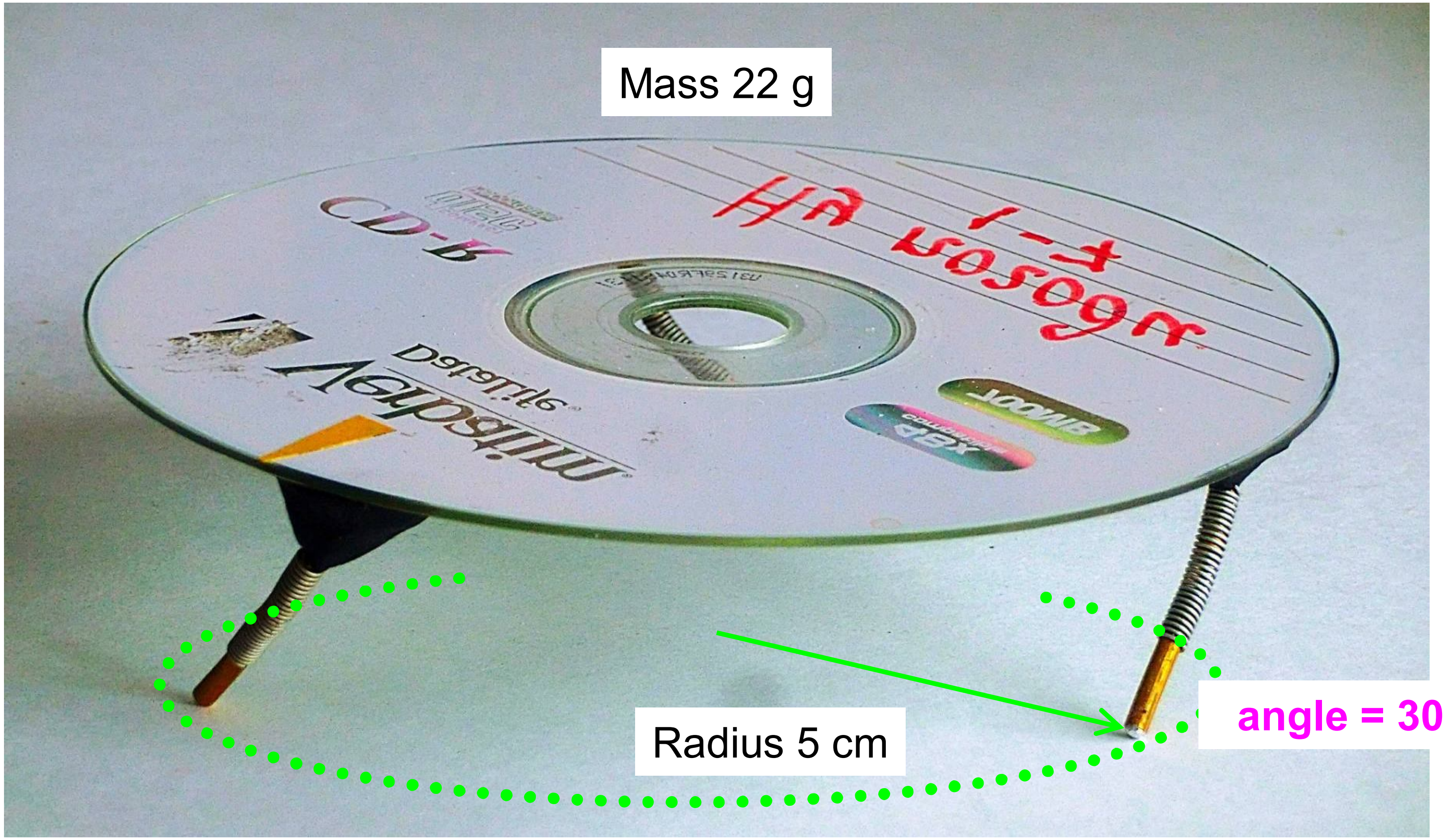
No preferred bristles' direction
Chaotic motion
Unstable construction



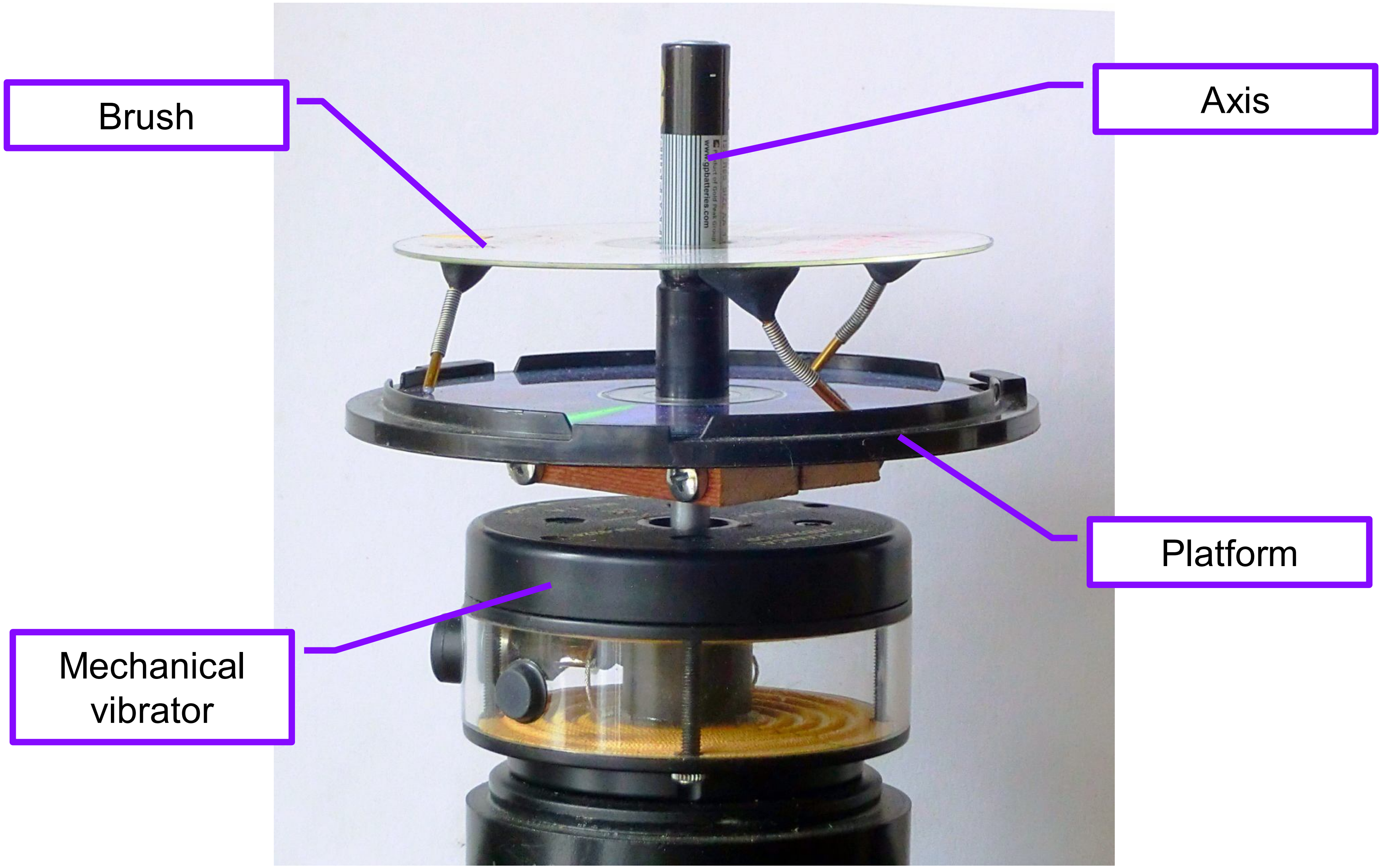
Non-uniform load distribution
Strong pitching
Not identical bristles

Experimental setup

Tripod rotating brush



Experimental setup



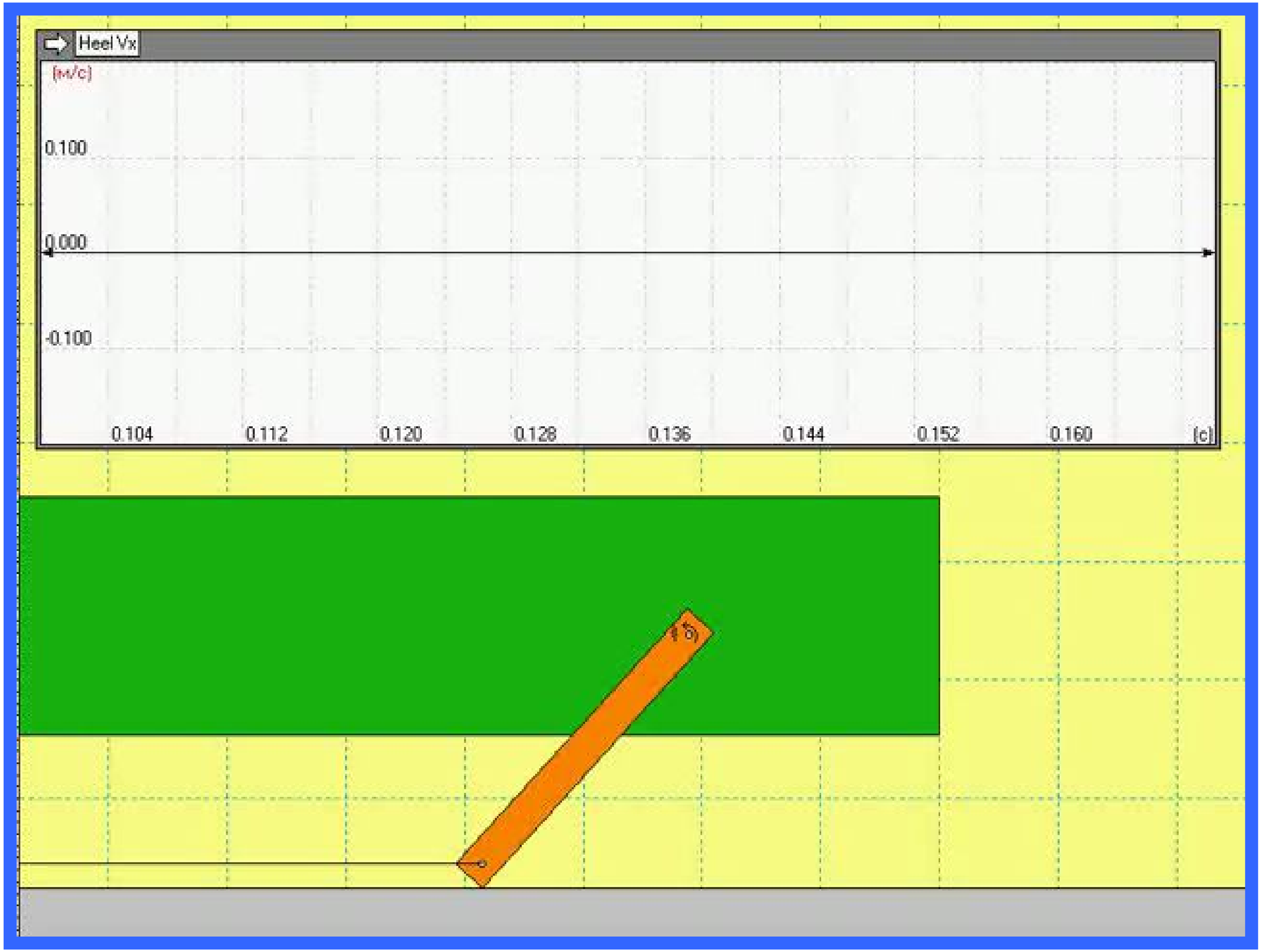
Starting of forward motion

Forward motion (240 fps)

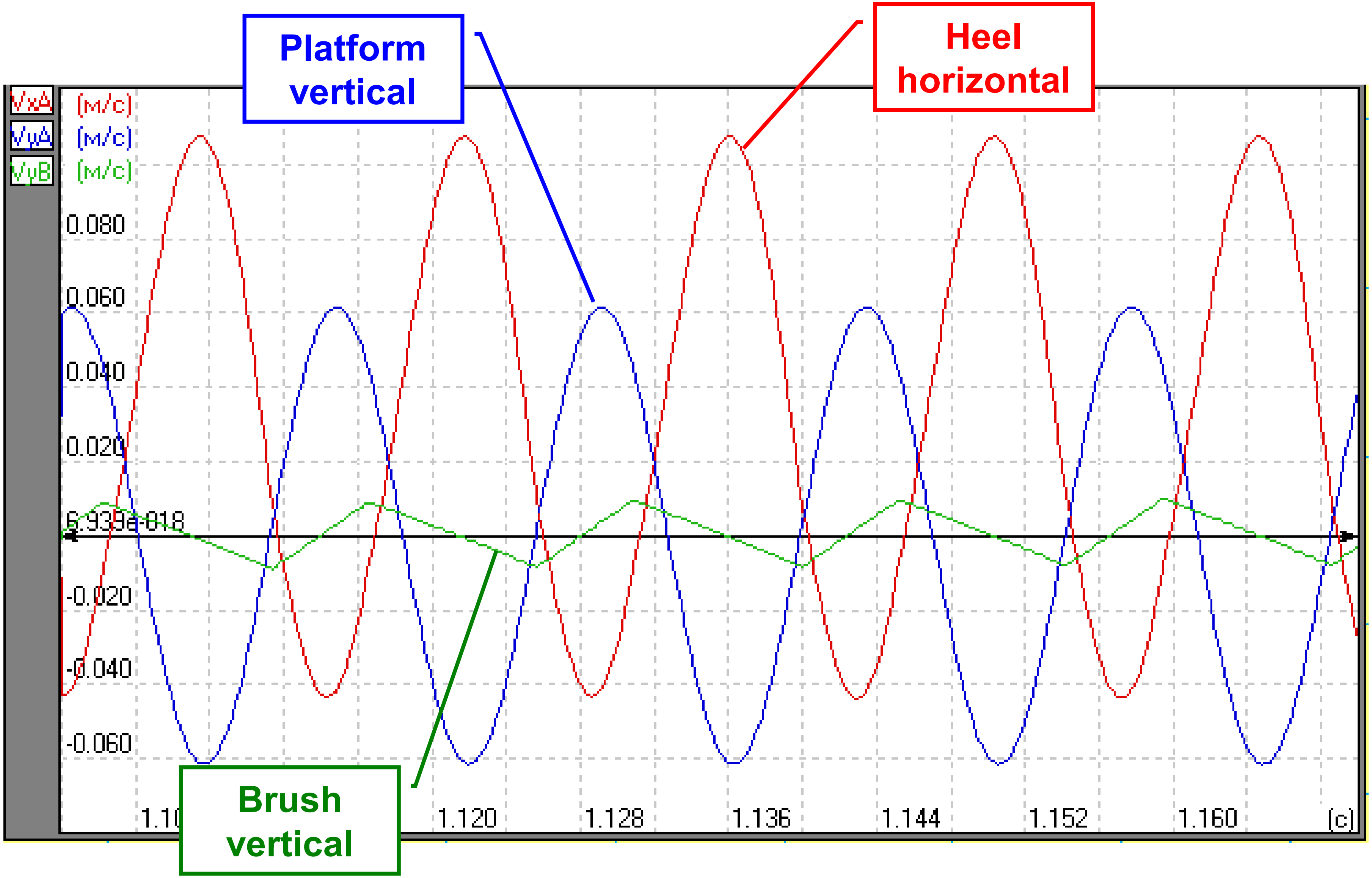
Frequency 40 Hz

Vertical motion of the brush is negligible

Permanent heel slippage

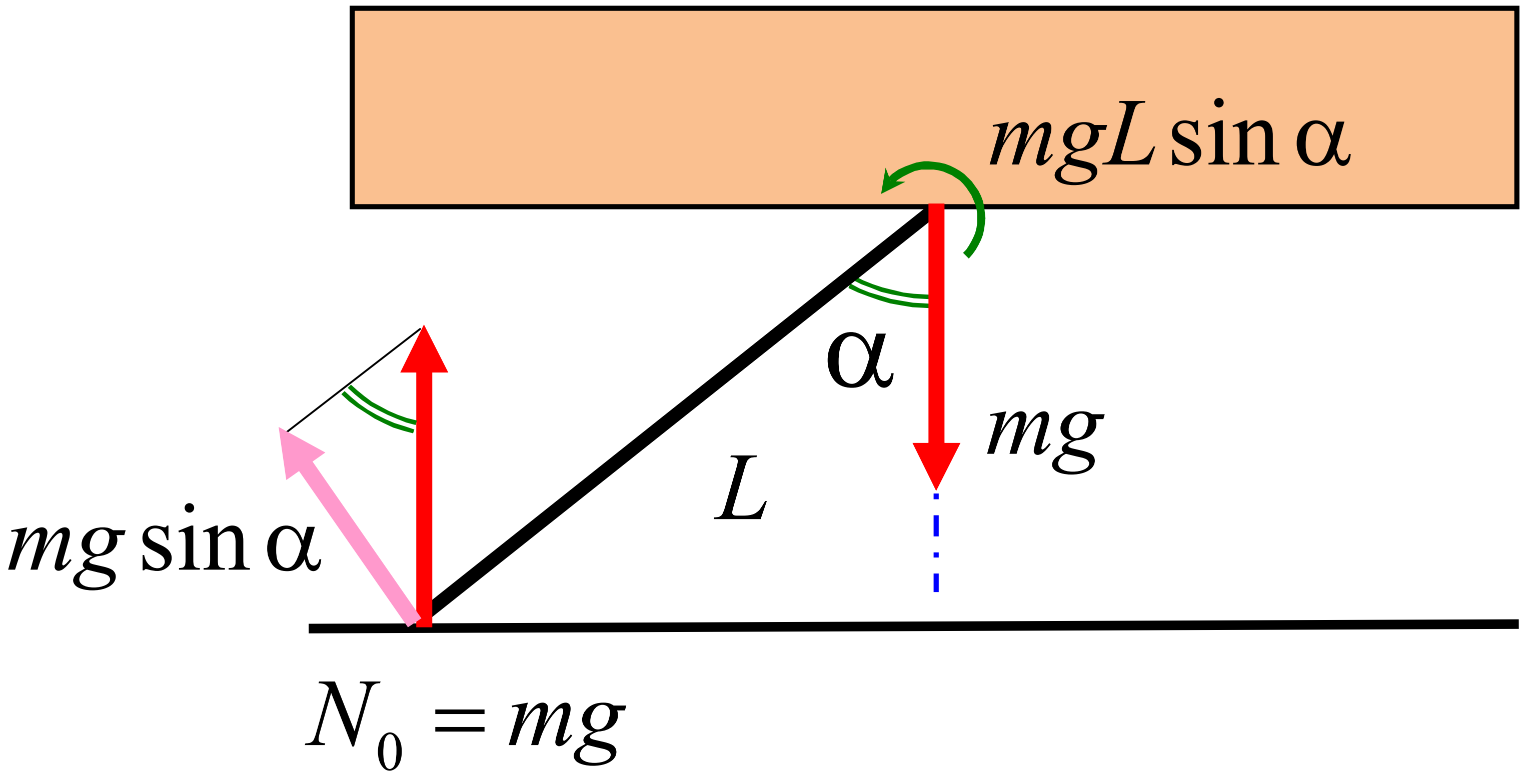


Brush and heel velocities

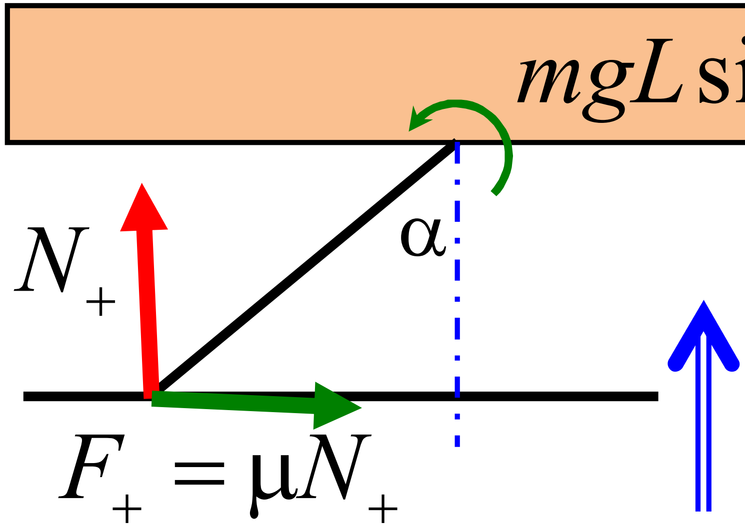


Theoretical model

Static equilibrium



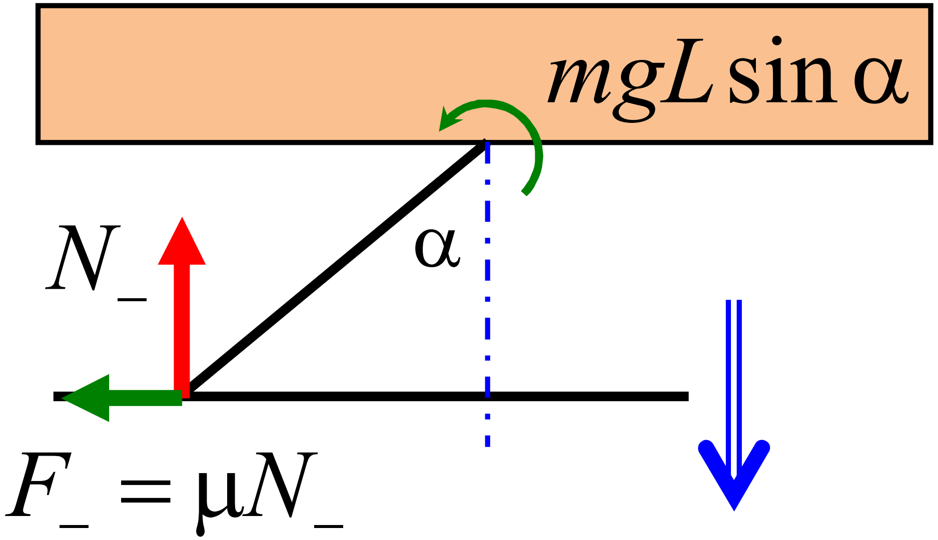
The platform goes up



$$(N_+ \sin\alpha - F_+ \cos\alpha)L = mgL\sin\alpha$$

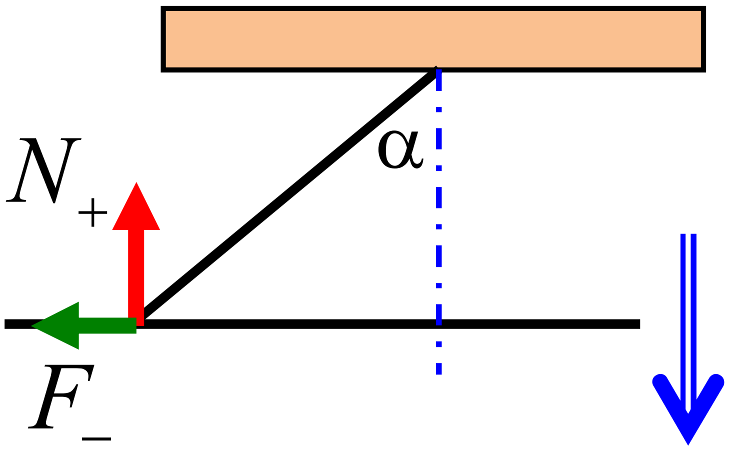
$$F_+ = \frac{\mu \cdot mg}{1 - \mu \operatorname{ctg}\alpha}$$

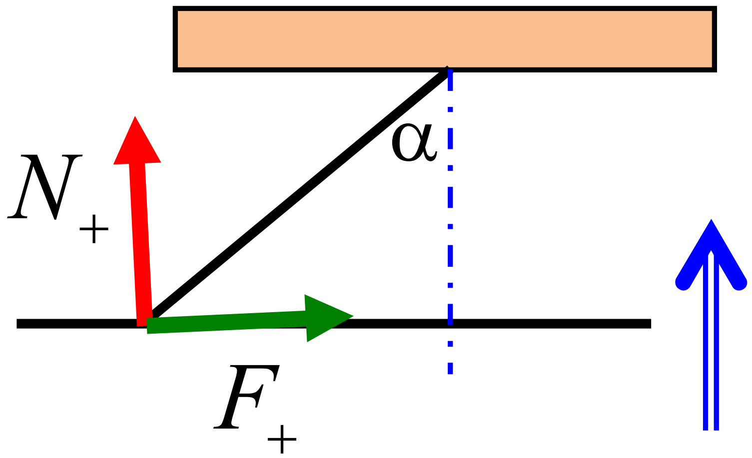
The platform goes down



$$(N_- \sin\alpha + F_- \cos\alpha)L = mgL\sin\alpha$$

$$F_- = \frac{\mu \cdot mg}{1 + \mu \operatorname{ctg}\alpha}$$





$$F_- = \frac{\mu \cdot mg}{1+\mu \operatorname{ctg}\alpha} < F_+ = \frac{\mu \cdot mg}{1-\mu \operatorname{ctg}\alpha}$$

F_+ defines the direction of motion

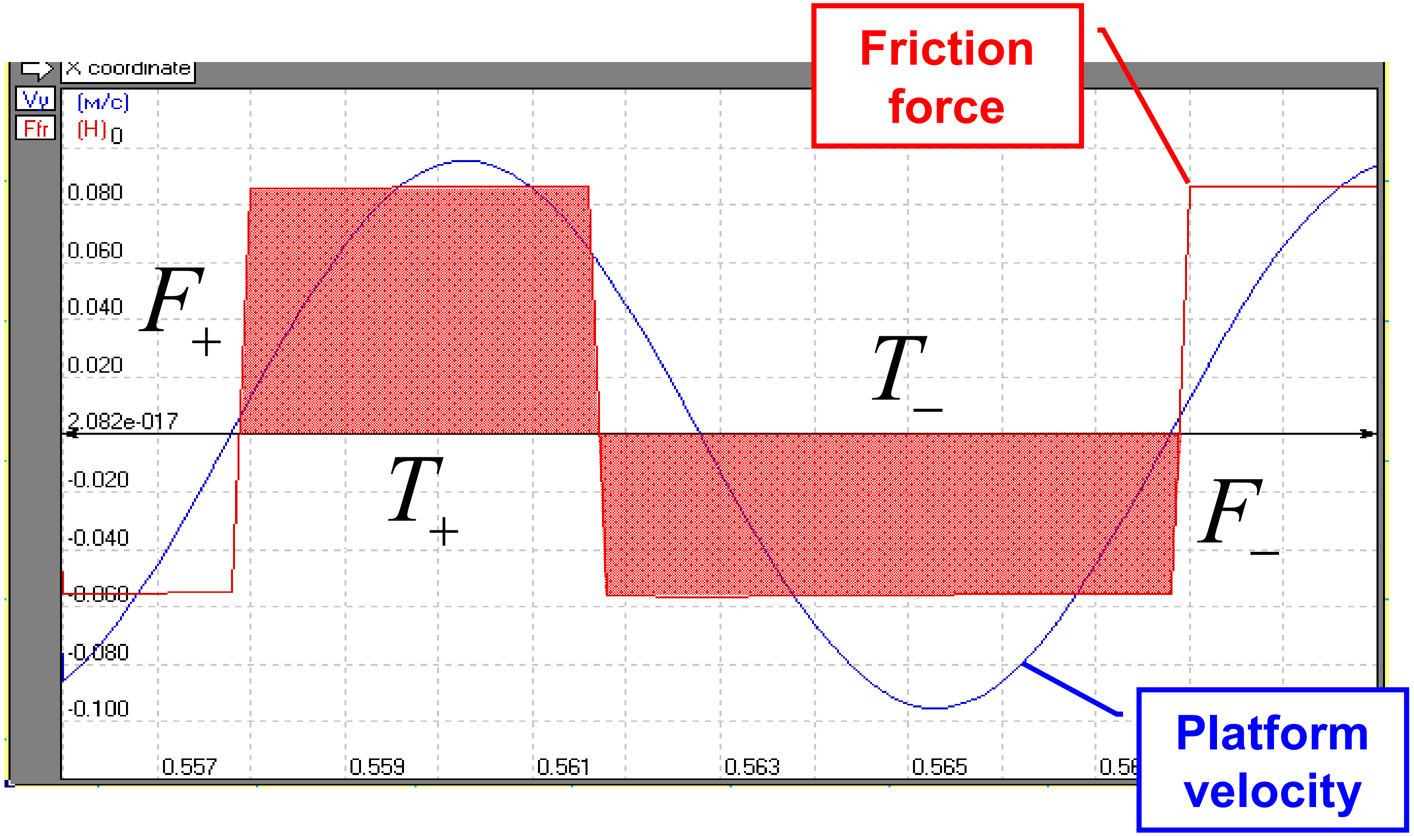
Paradox!

- Difference between $\boldsymbol{F}_-$ and $\boldsymbol{F}_+$ causes a brush's acceleration.

- Experimental results are in contradiction with theory – the brush moves with **constant speed!**

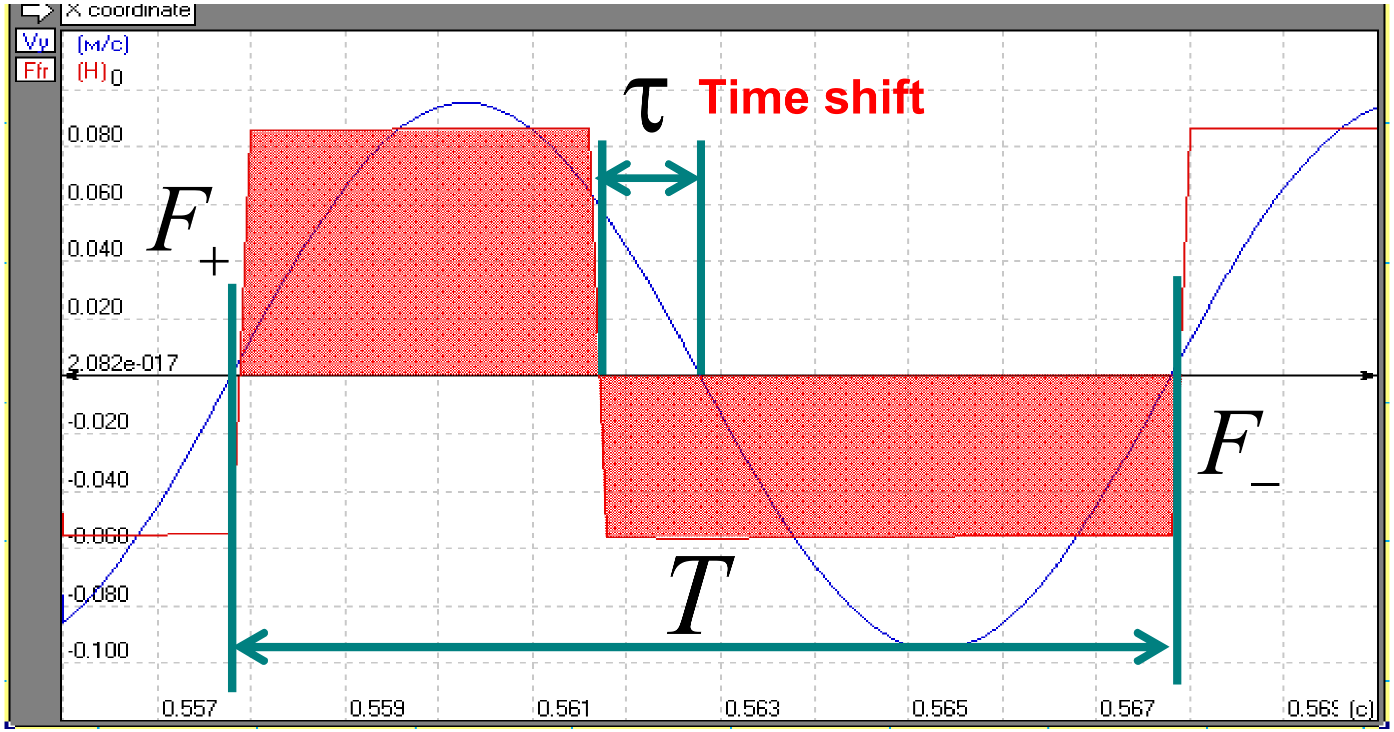
$$\vec{F} = \frac{d\vec{p}}{dt} = 0 \quad \Rightarrow \quad F_+ \cdot \Delta t_1 = F_- \cdot \Delta t_2$$

Equality of impulses



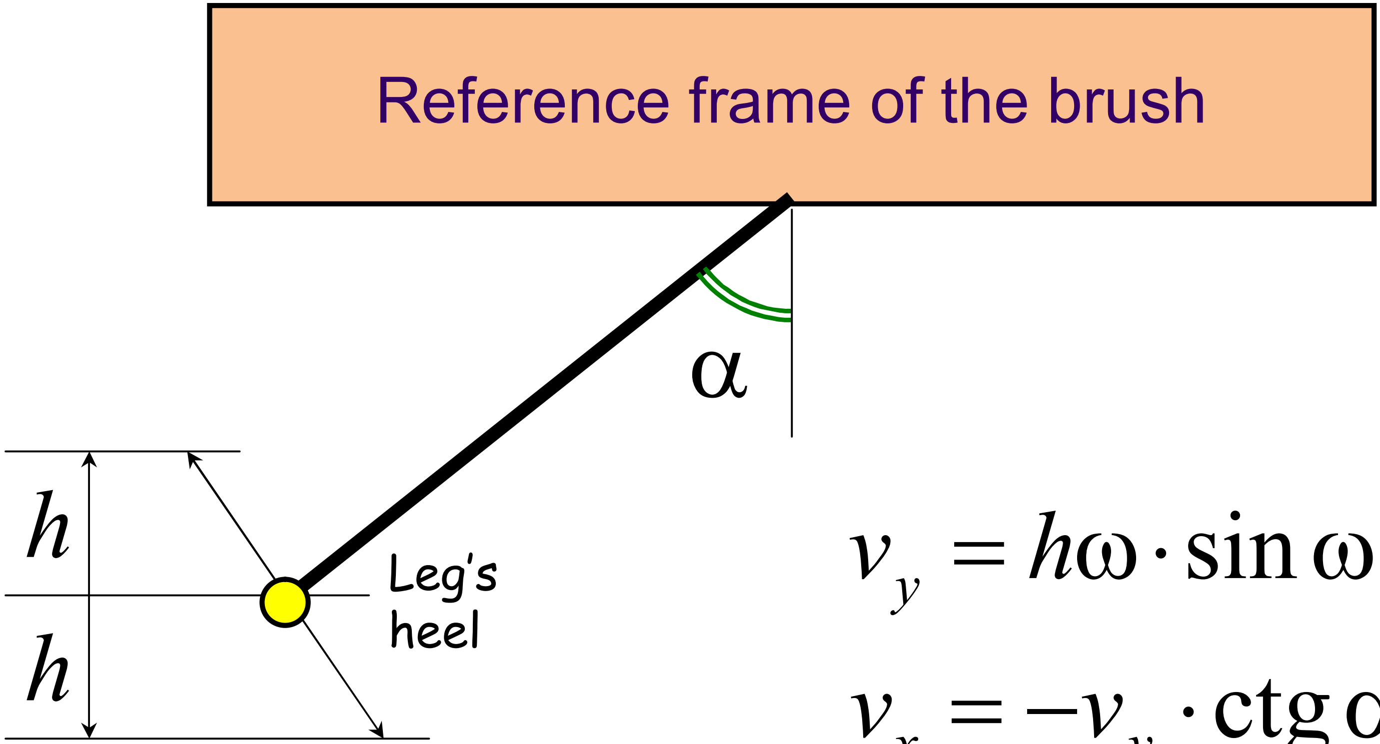
$$F_+T_+ = F_-T_-$$

Time shift



$$\tau = \frac{T}{2} \cdot \frac{F_+ - F_-}{F_+ + F_-} = \frac{T}{2} \cdot \mu \operatorname{ctg} \alpha$$

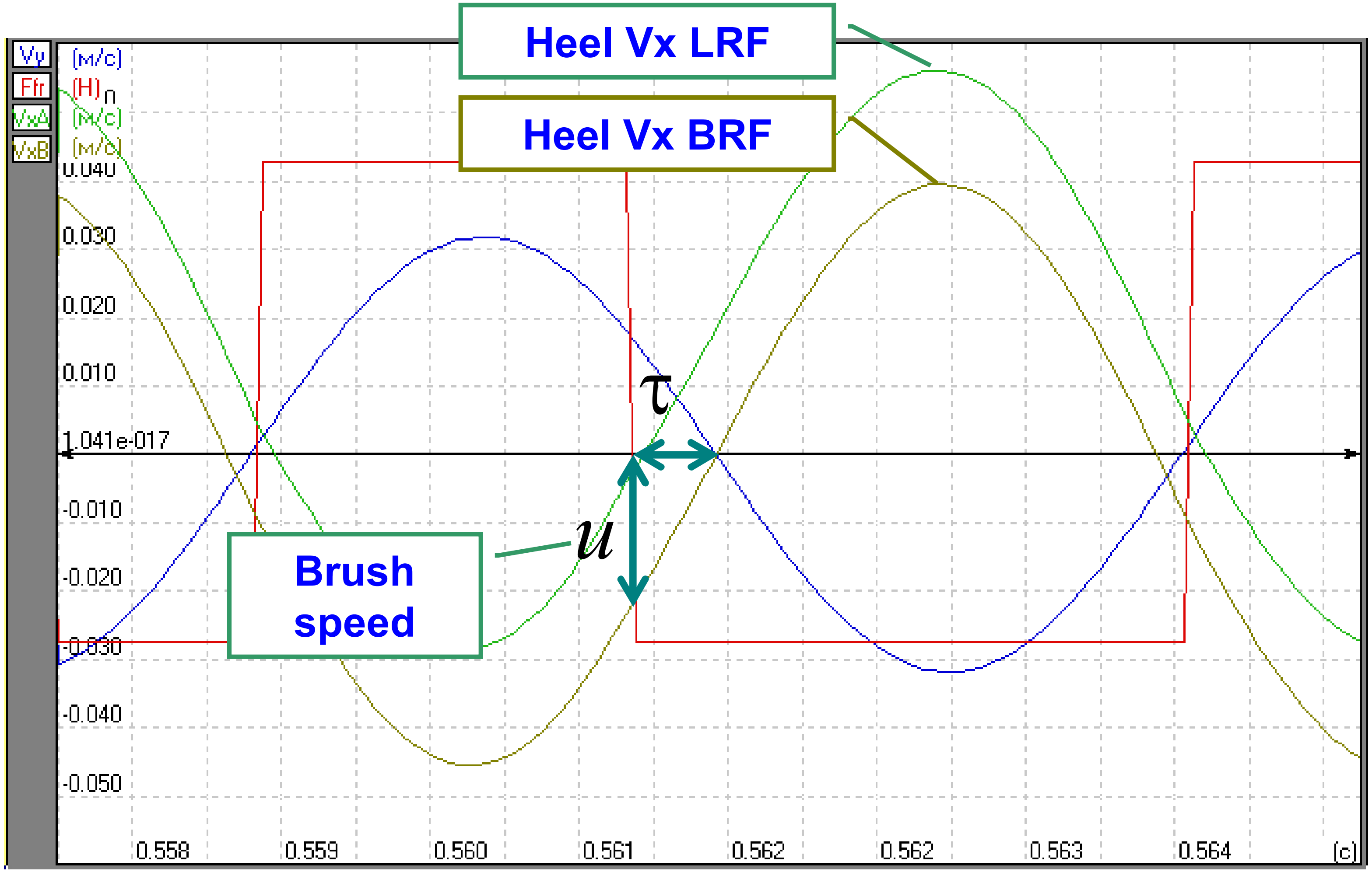
Heel velocity



$$v_y = h\omega \cdot \sin \omega t$$

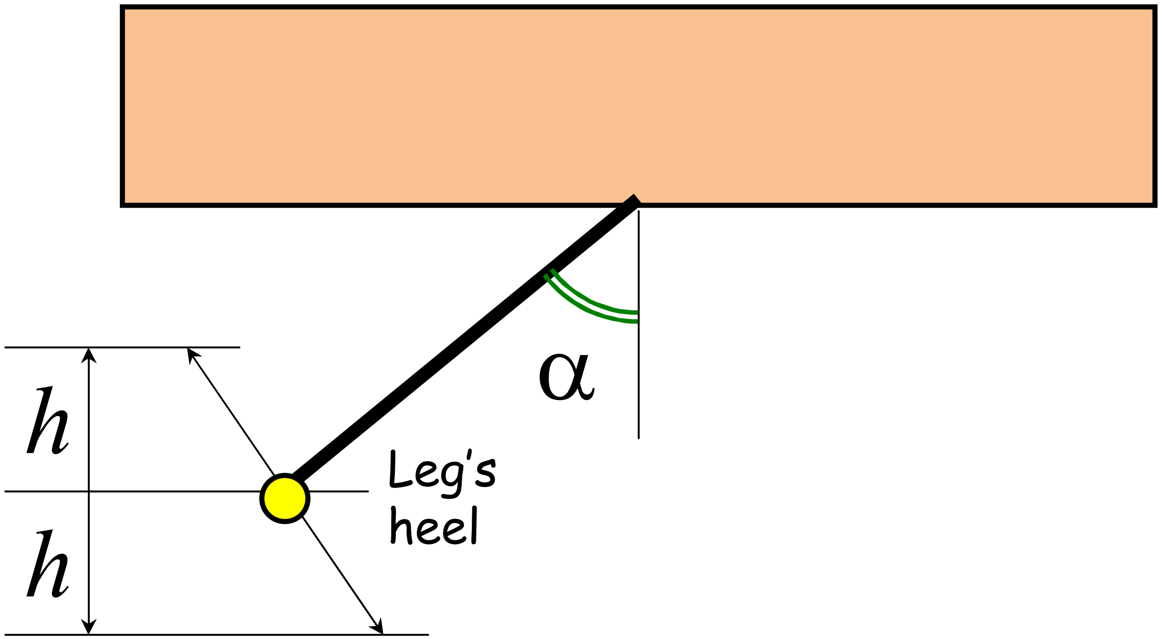
$$v_x = -v_y \cdot \operatorname{ctg} \alpha$$

Steady brush speed



$$u = h\omega \cdot \sin(\omega\tau) \cdot \mathrm{ctg}\,\alpha = h\omega \cdot \sin(\pi\mu\,\mathrm{ctg}\,\alpha) \cdot \mathrm{ctg}\,\alpha$$

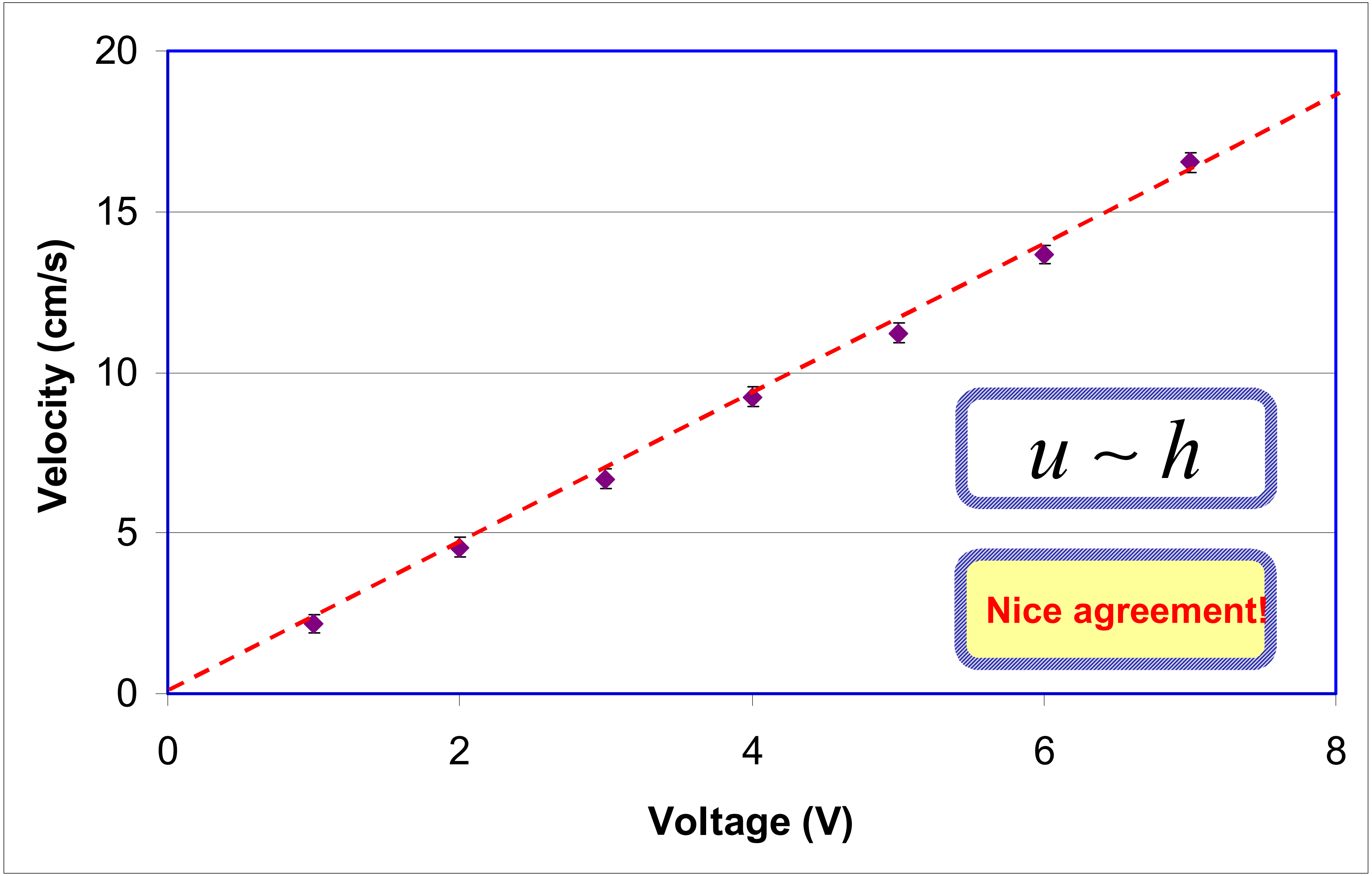
Theoretical prediction



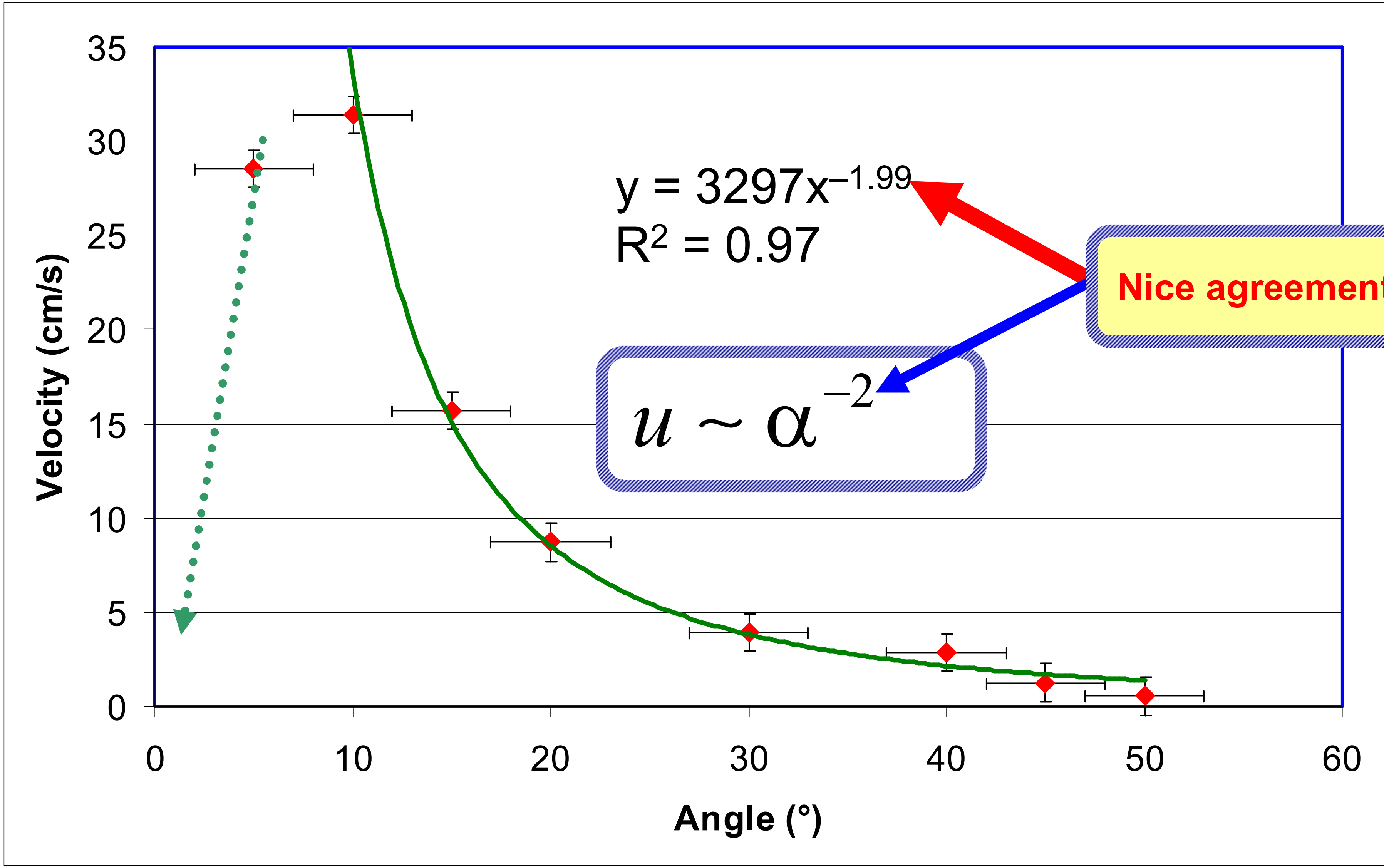
$$u \approx 2\pi^2 \mu \cdot \frac{hf}{\alpha^2}$$

Experiment

Velocity vs. amplitude (voltage)

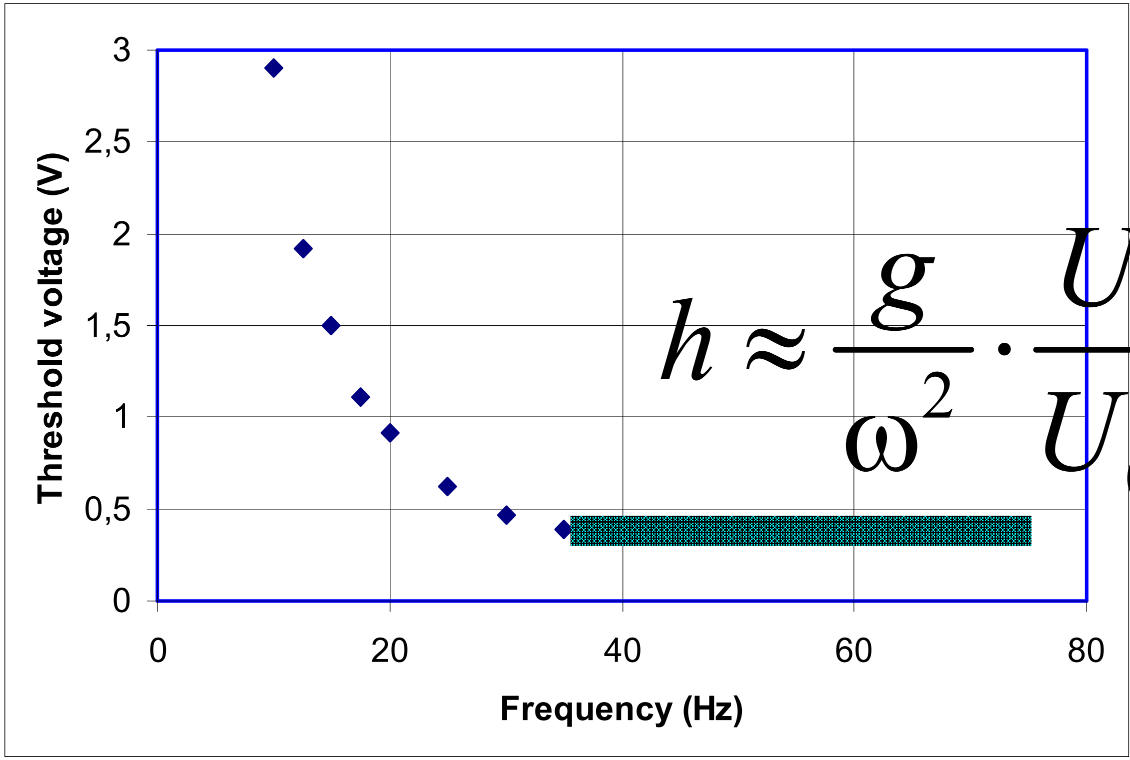


Velocity vs. angle of bristles



Vibrator frequency response

$$h\omega^2 = g$$



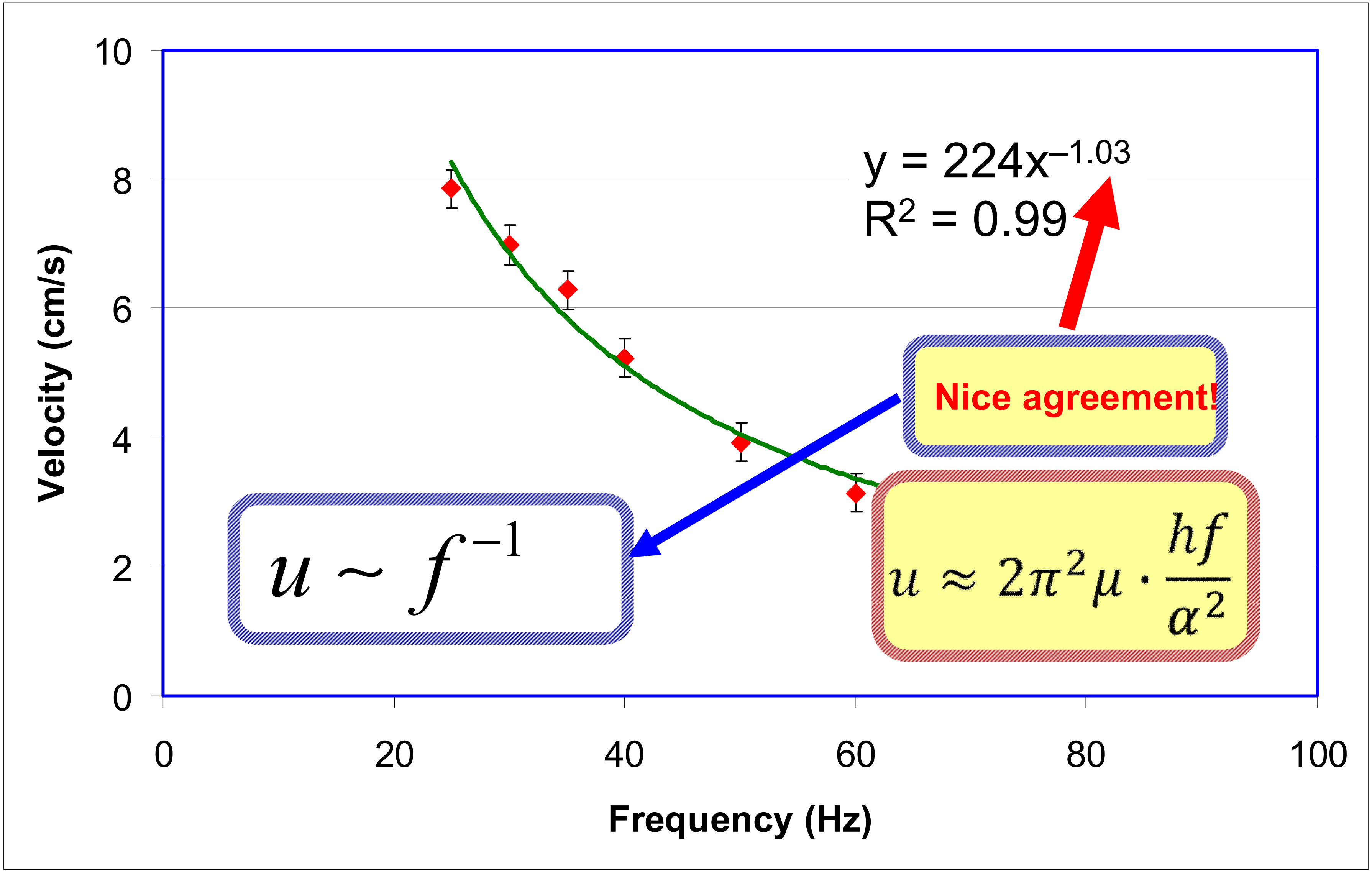
$$h \approx \frac{g}{\omega^2} \cdot \frac{U}{U_0}$$

Theoretical prediction

$$U = \text{const} \quad \Rightarrow \quad h \sim f^{-2}$$

$$u \sim hf \sim f^{-1}$$

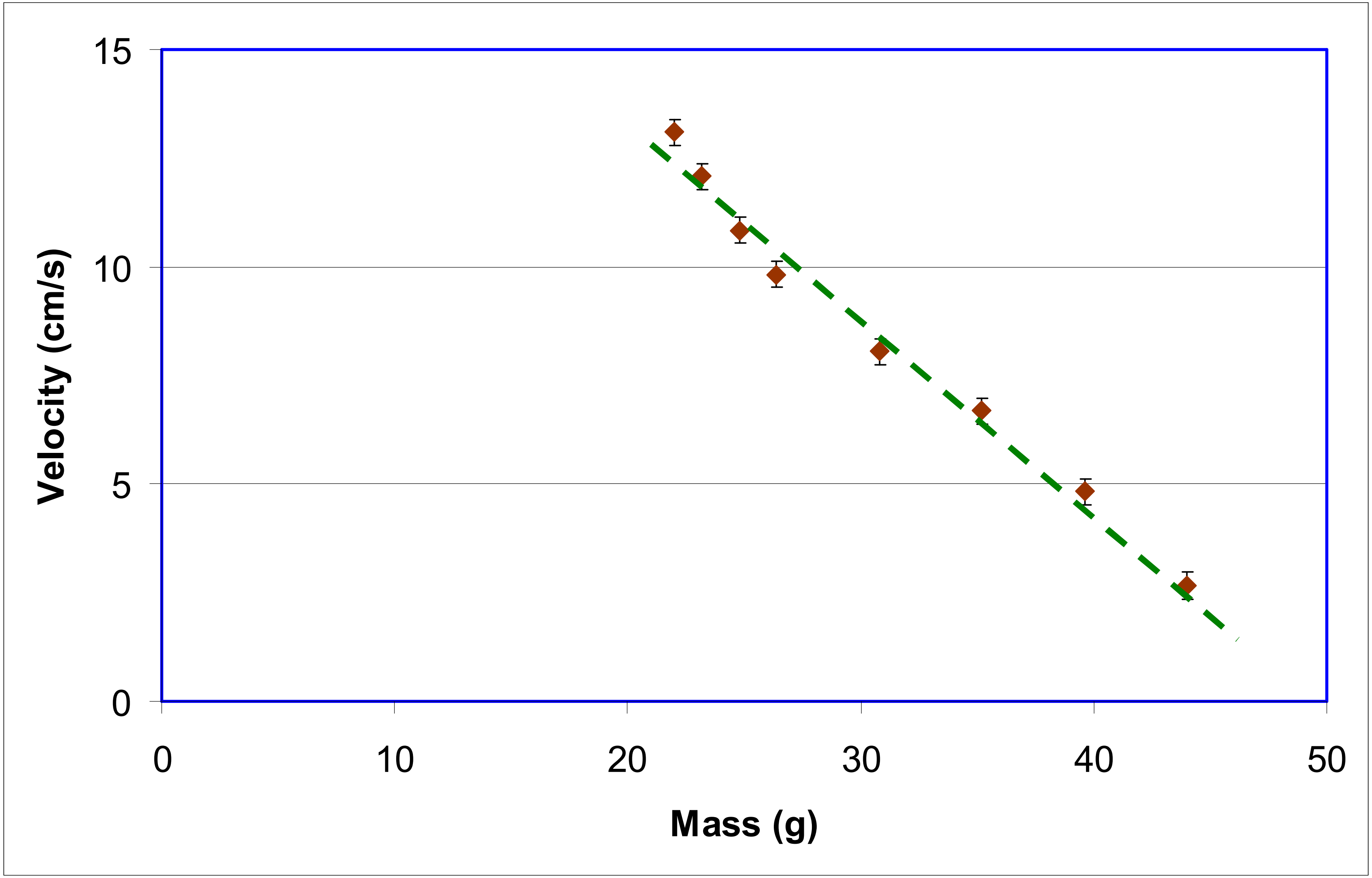
Velocity vs. frequency



$$u \sim f^{-1}$$

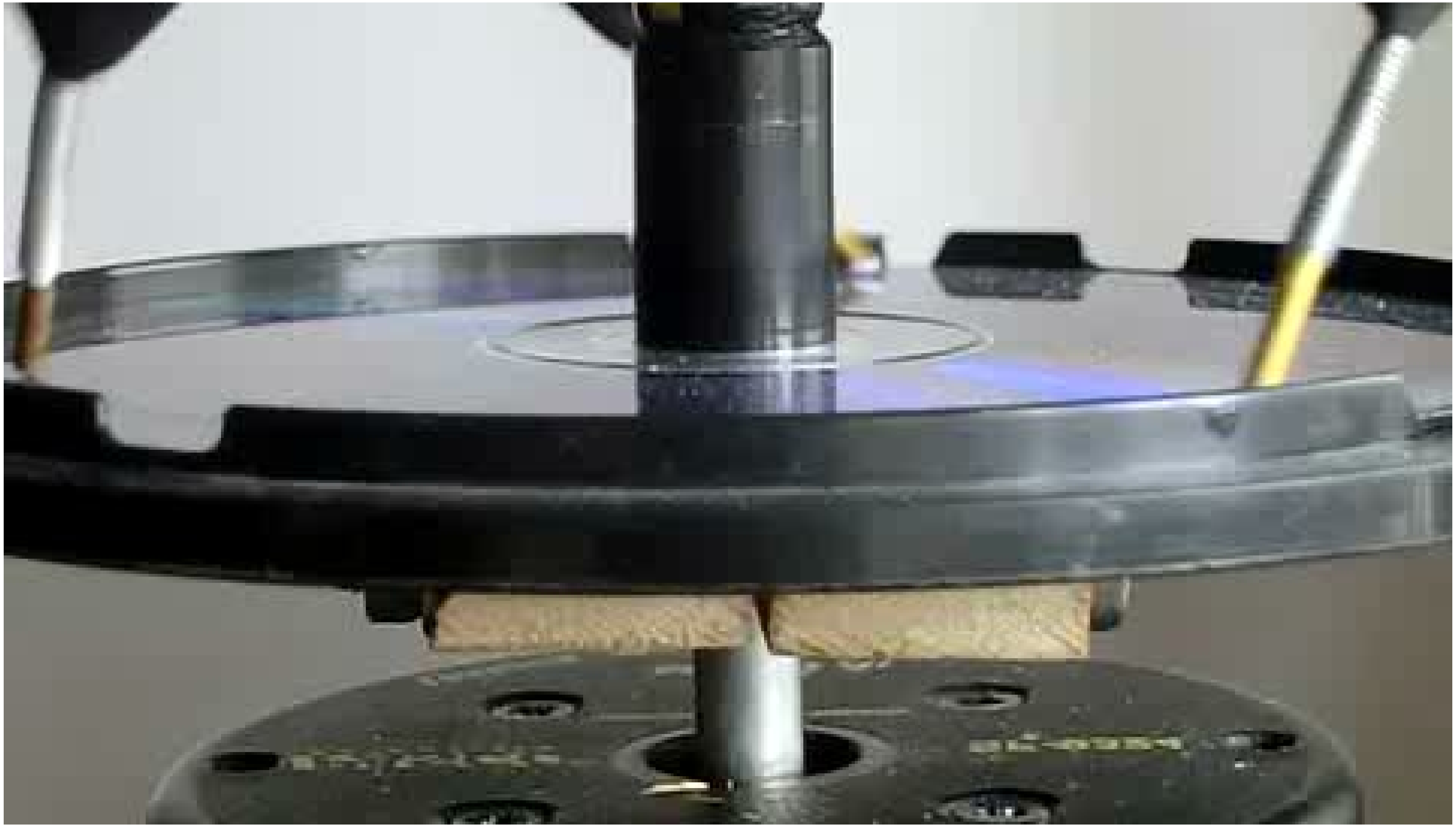
$$u \approx 2\pi^2 \mu \cdot \frac{hf}{\alpha^2}$$

Velocity vs. mass



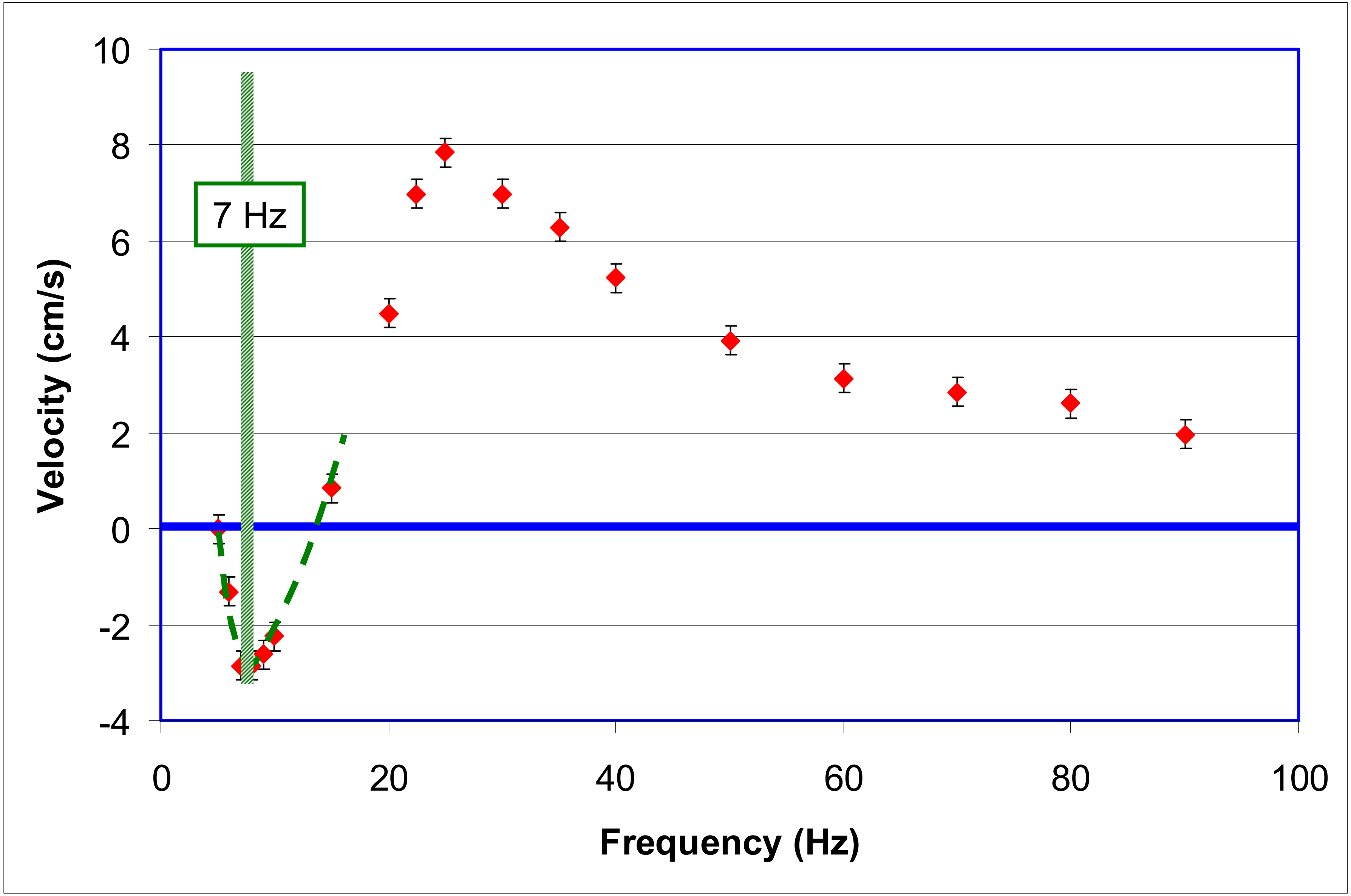
Backward motion

Backward motion (240 fps)

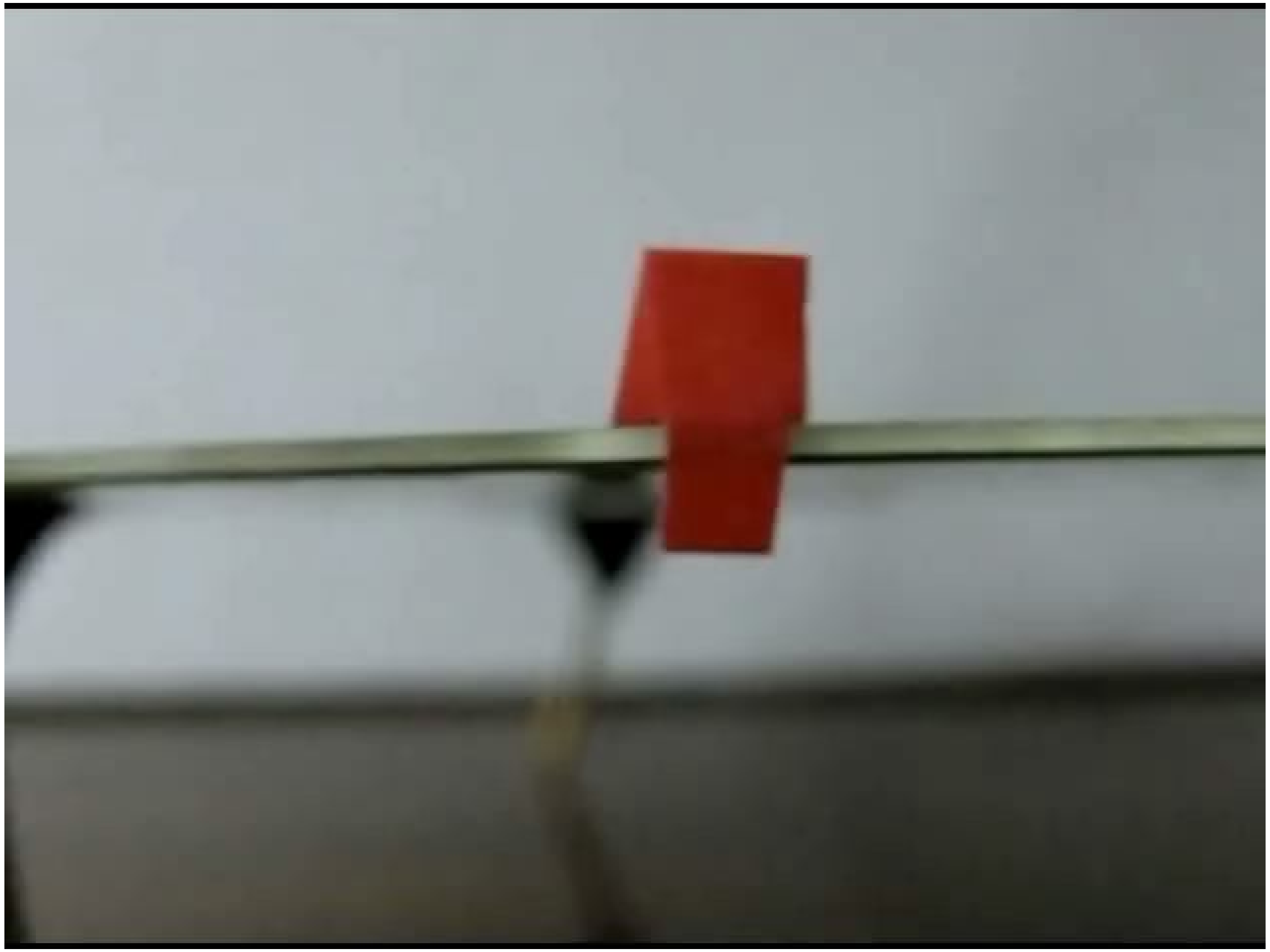


Frequency 7 Hz

Velocity vs. frequency



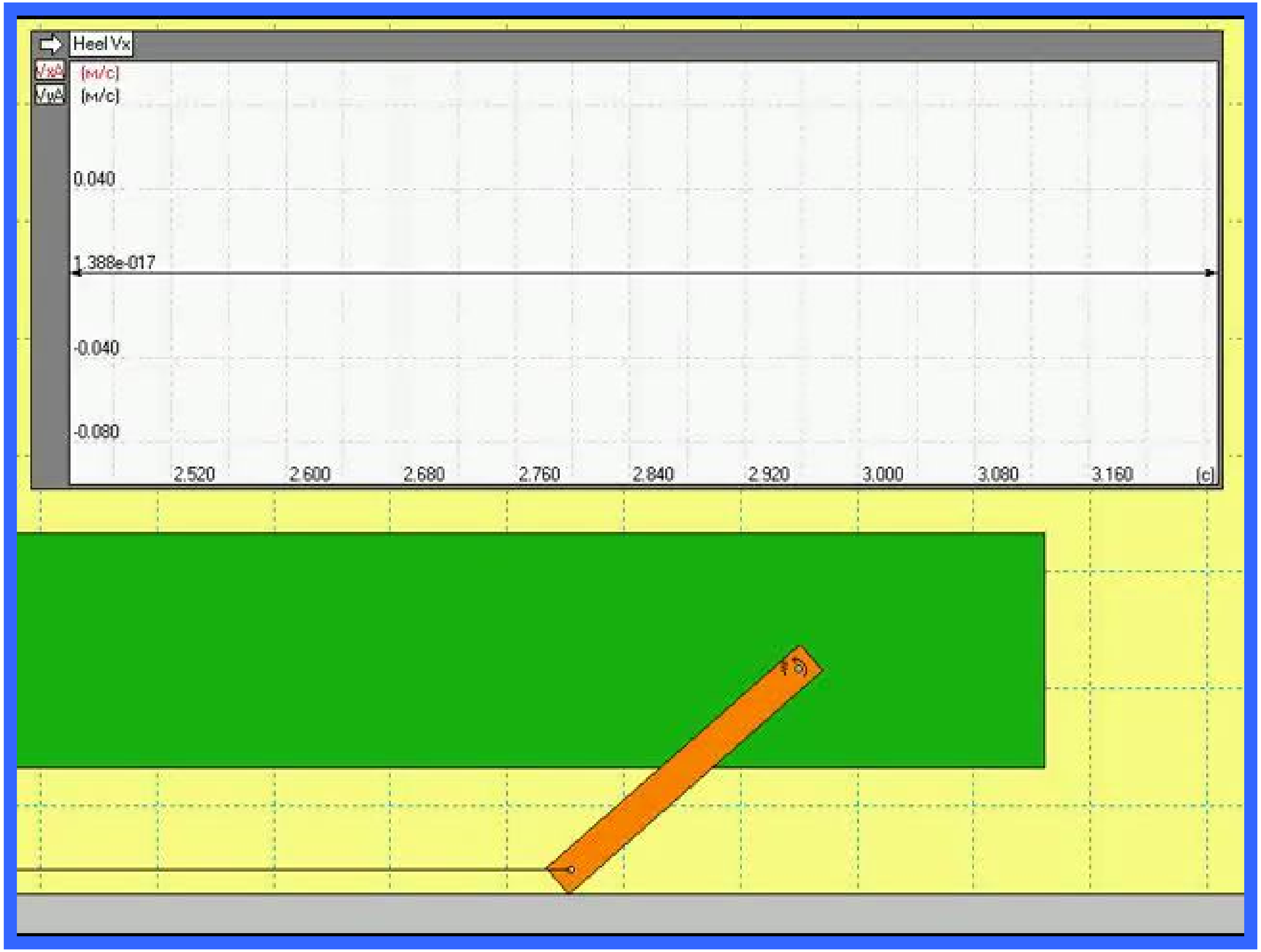
Natural oscillations (video 240 fps)



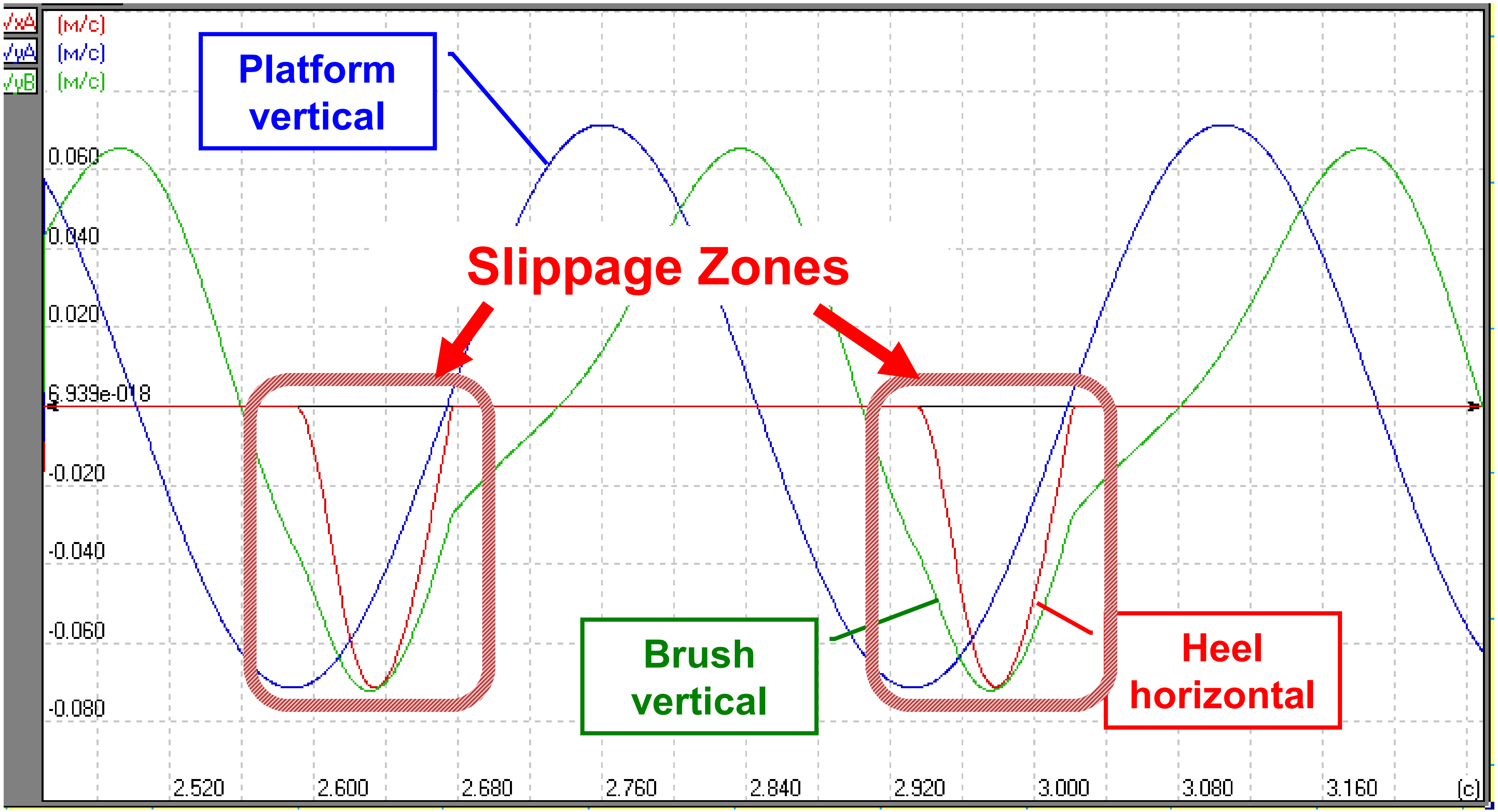
Frequency 7 Hz

Bristles' Behavior

Periodic heel slippage



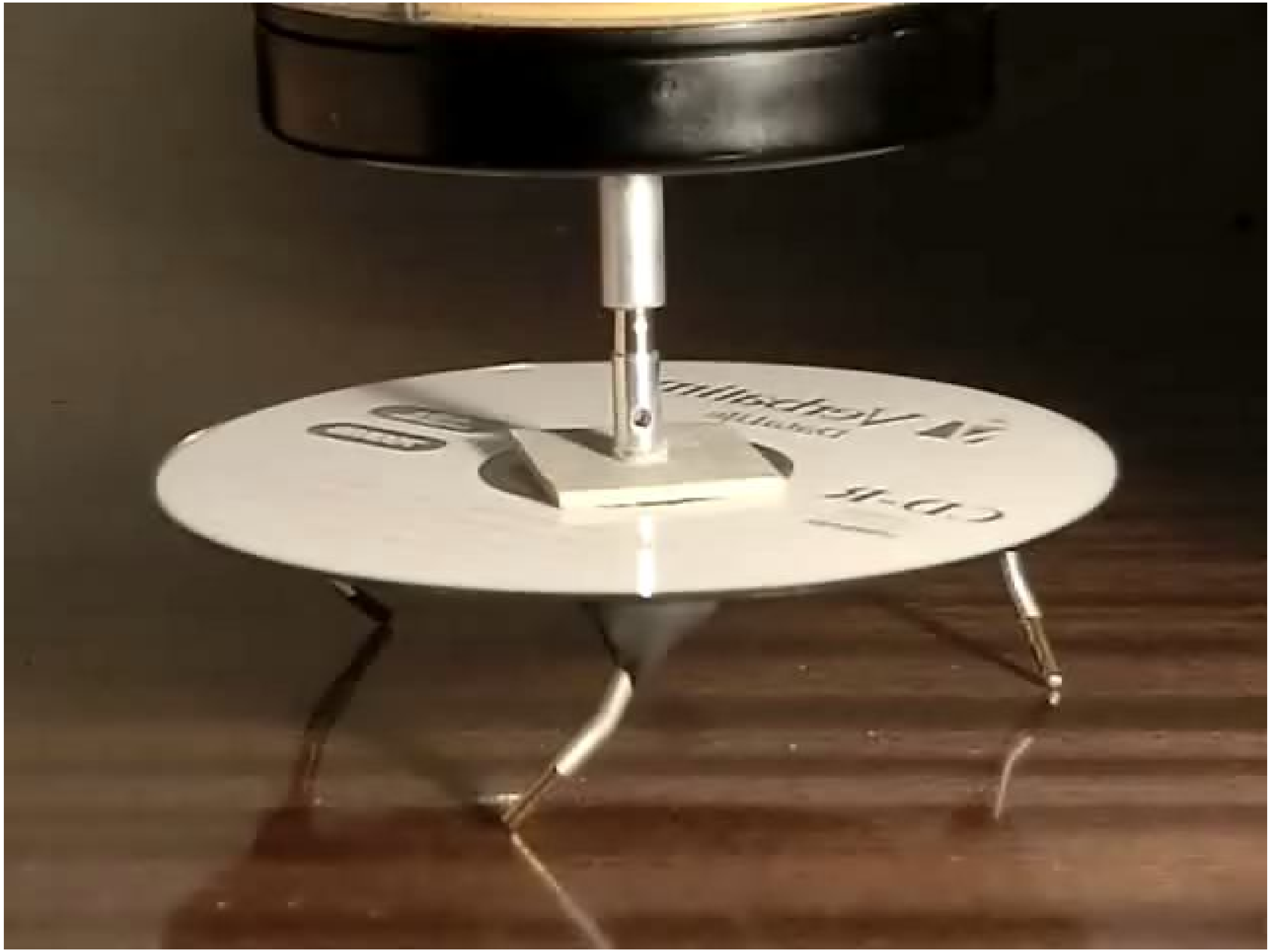
Brush and heel velocities



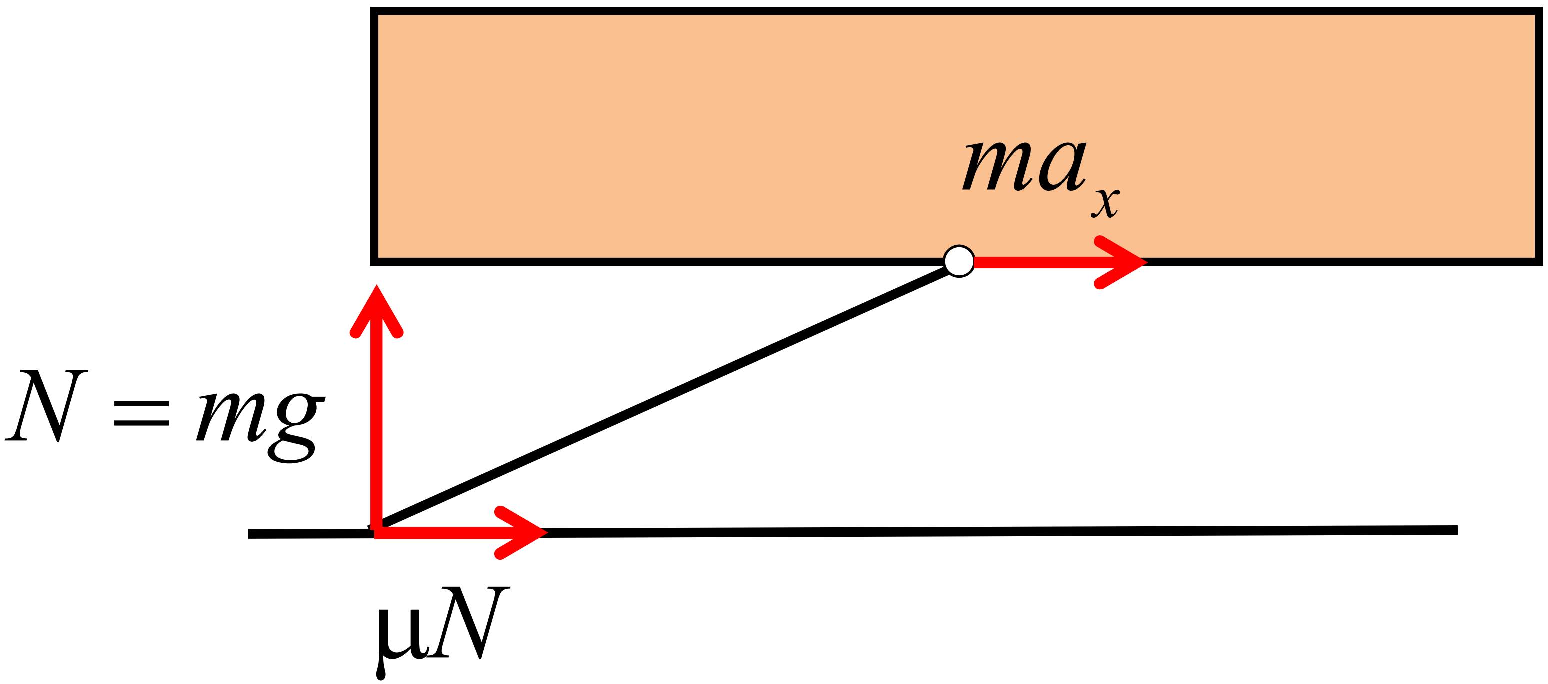
Surface's role

"…may start moving when placed on
a vibrating horizontal surface…"

Oscillating ceiling!

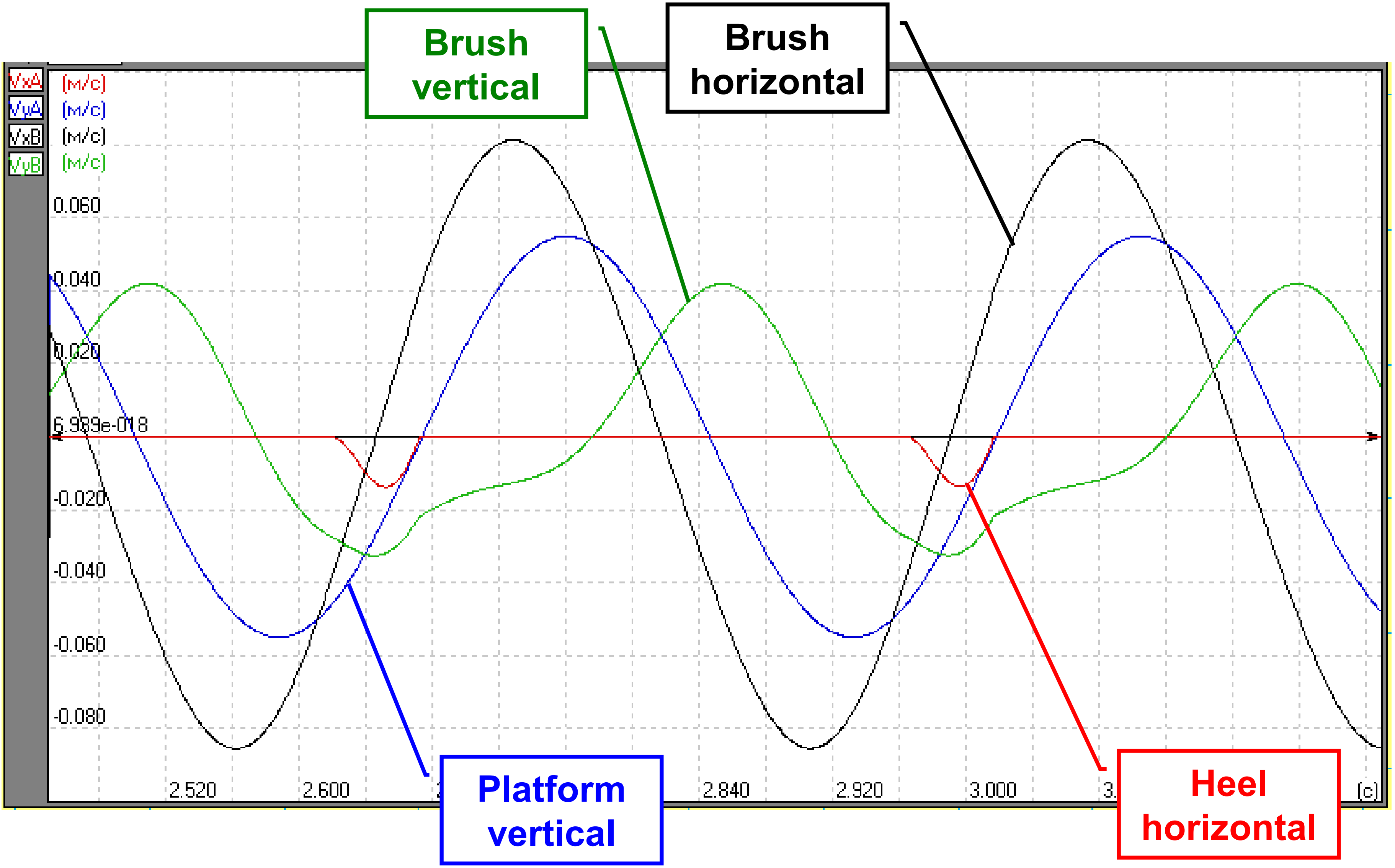


Brush goes down



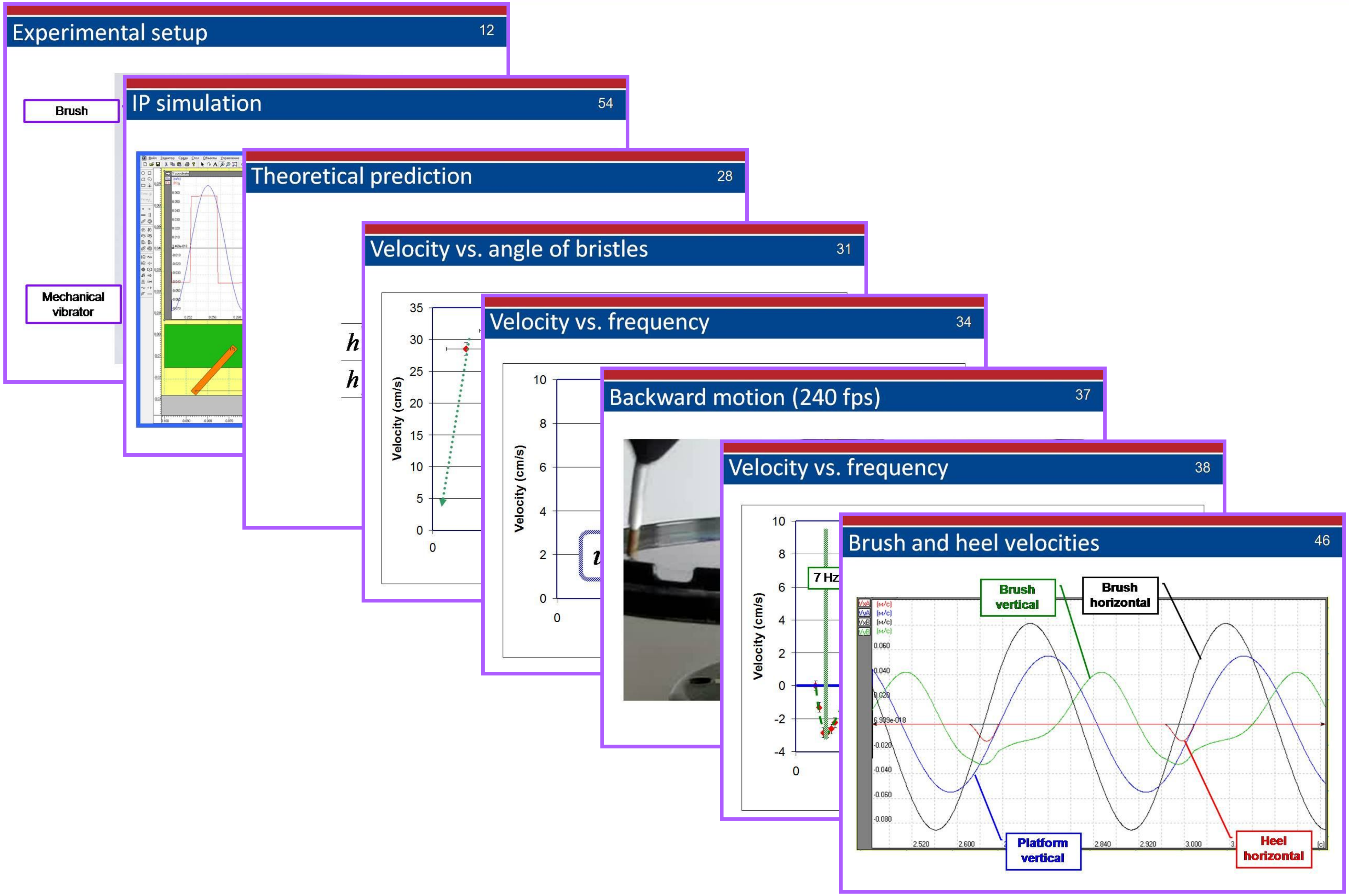
$$\mu N = ma_x$$

Brush and heel velocities



Summary

Main results



References

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Thank you for your attention!